Title: SU(3) Landau-Zener-Stueckelberg-Majorana interferometry with quantum triangles

Date: Apr 04, 2017 03:30 PM

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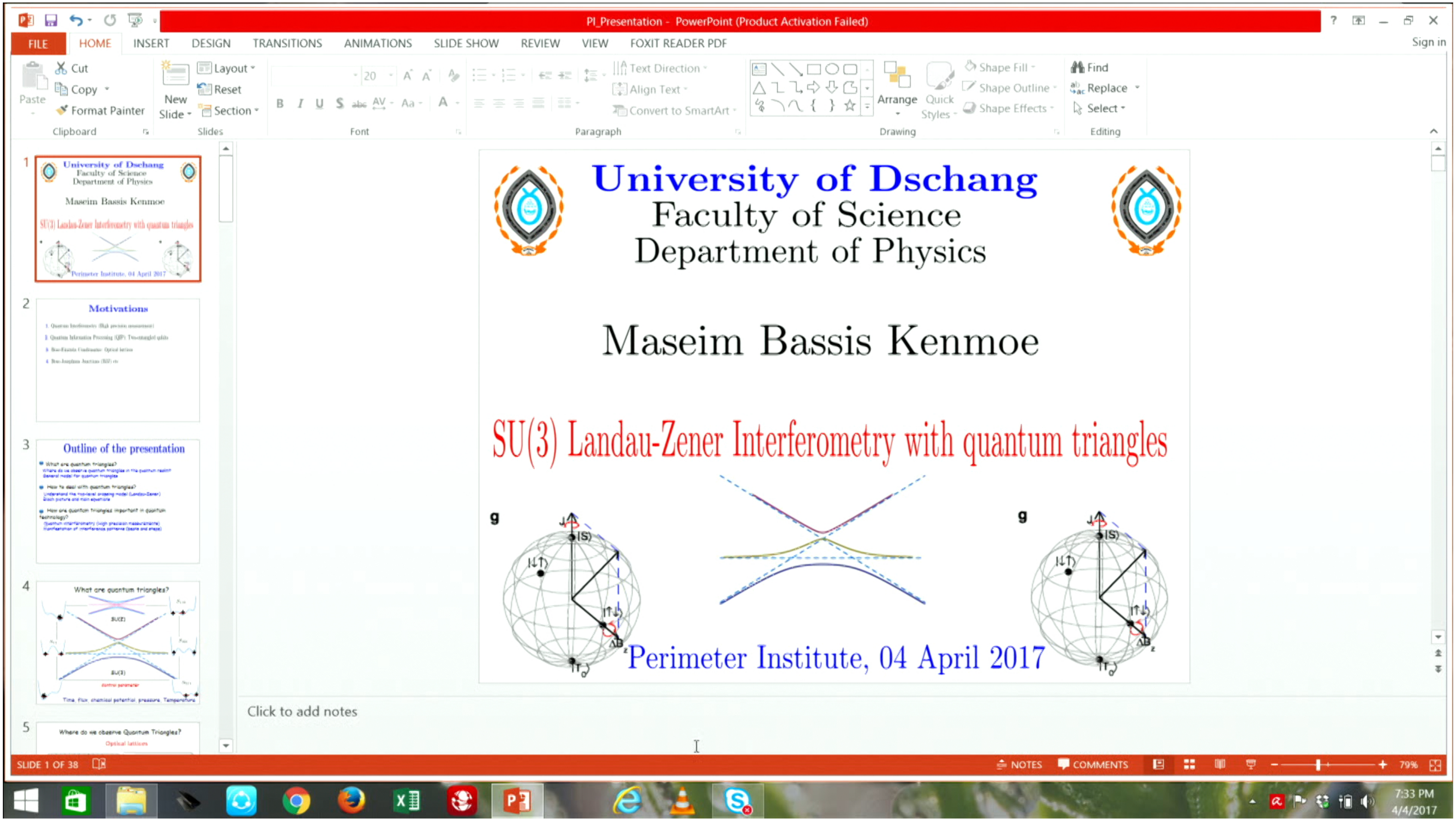
Abstract: <p>Quantum triangles can work as interferometers. Depending on their geometric size and interactions between paths, â€œbeatsâ€• and/or â€œstepsâ€•</p>

<p>patterns are observed. We show that when inter-level distances between level positions in quantum triangles periodically change with time, formation of beats and/or steps no longer depends only on the geometric size of the triangles but also on the characteristic frequency of the transverse signal. For large-size triangles, we observe the coexistence of beats and steps for moderated frequencies of the signal and for large frequencies a maximum of four steps instead of two as in the case with constant interactions are observed.</p>

<p>Small-size triangles also revealed counter-intuitive interesting dynamics for large frequencies of the field: unexpected two-step patterns are observed. When the frequency is large and tuned such that it matches the uniaxial anisotropy, three-step patterns are observed.</p>

<p>We have equally observed that when the transverse signal possesses a static part, steps maximize to six. These effects are semi-classically explained in terms of Fresnel integrals and quantum mechanically in terms of quantized fields with a photon-induced tunneling process. Our expressions for populations are in excellent agreement with the gross temporal profiles of exact numerical solutions. We compare the semi-classical and quantum dynamics in the triangle and establish the conditions for their equivalence.</p>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University of Dschang
Faculty of Science
Department of Physics
Maseim Bassis Kenmoe
SU(3) Landau-Zener Interferometry with quantum triangles
Perimeter Institute, 04 April 2017
Motivations
Outline of the presentation
What are quantum triangles?
Click to add notes
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University of Dschang

Faculty of Science

Department of Physics

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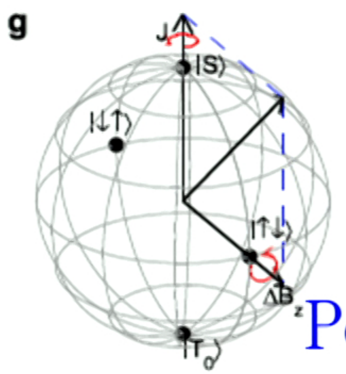
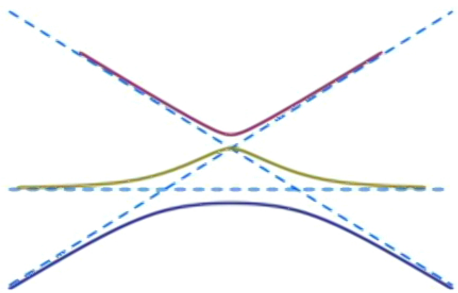
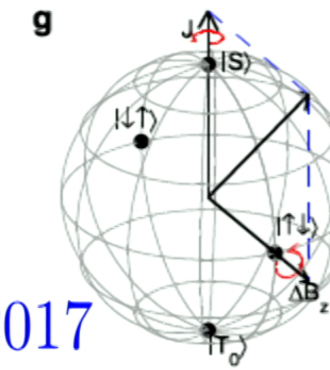
SU(3) Landau-Zener Interferometry with quantum triangles







Perimeter Institute, 04 April 2017

Motivations

1. Quantum Interferometry (High precision measurement)

2. Quantum Information Processing (QIP): Two-entangled qubits

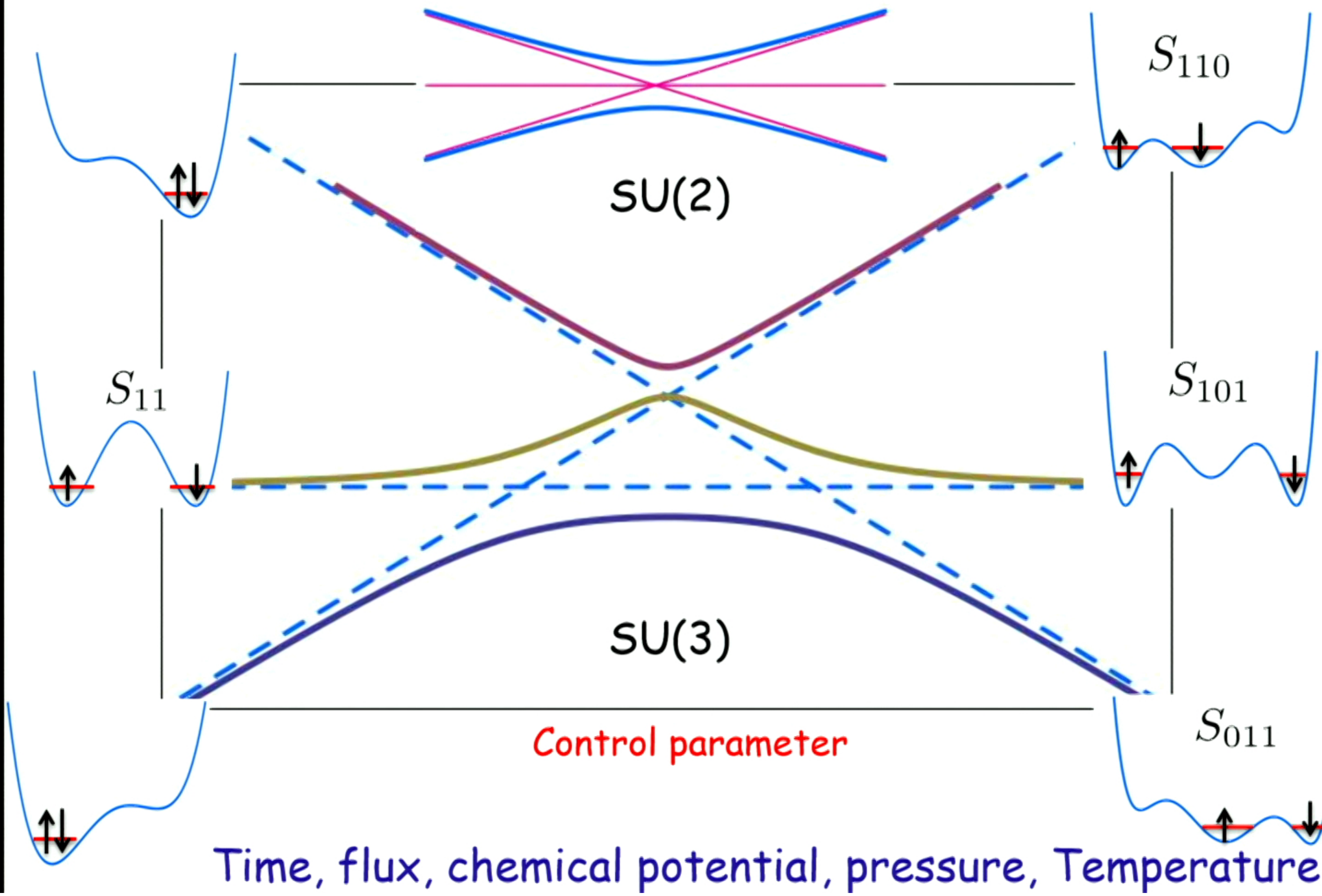
3. Bose-Einstein Condensates: Optical lattices

4. Bose-Josephson Junctions (BJJ) etc

Outline of the presentation

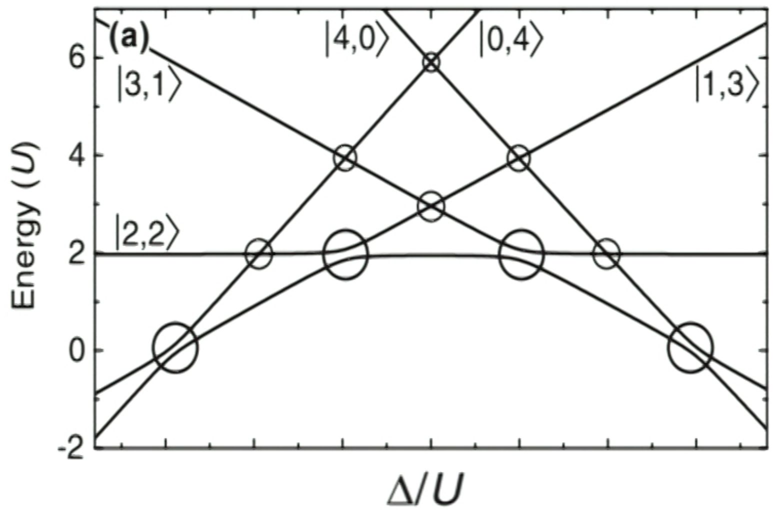
- What are quantum triangles?
  - Where do we observe quantum triangles in the quantum realm?
  - General model for quantum triangles
- How to deal with quantum triangles?
  - Understand the two-level crossing model (Landau-Zener)
  - Bloch picture and main equations
- How are quantum triangles important in quantum technology?
  - Quantum interferometry (High precision measurements)
  - Manifestation of interference patterns (beats and steps)

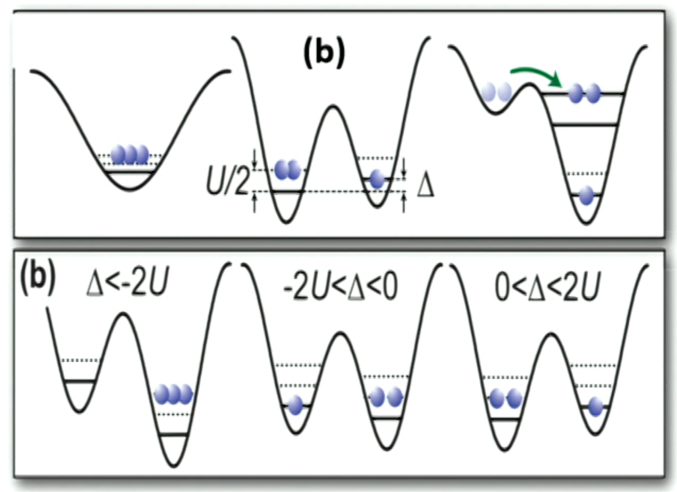
What are quantum triangles?



Where do we observe Quantum Triangles?

Optical lattices





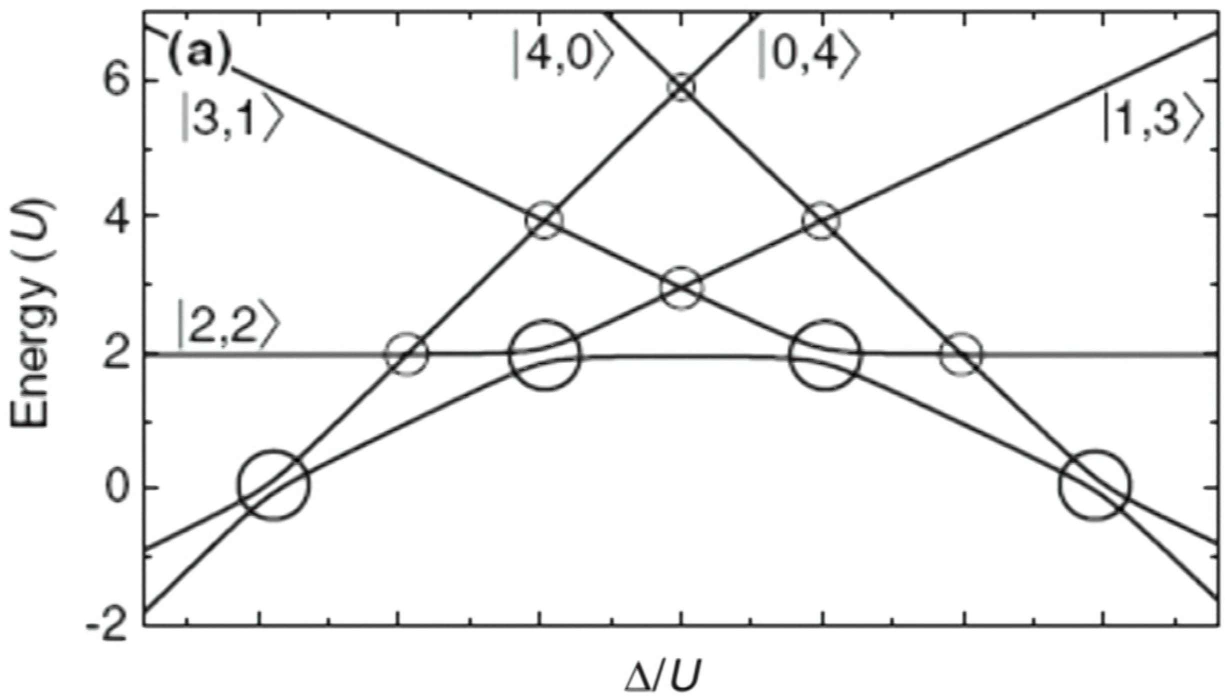
Lattice sites converted into double-well potentials

Two-mode Hubbard model

$$\mathcal{H}(t) = -\sum_{\nu=x,z} \mathbf{B}_\nu(t) S^\nu + D(S^z)^2 + D\mathbf{n}(\mathbf{n}-1)$$

I. Bloch et al 2008

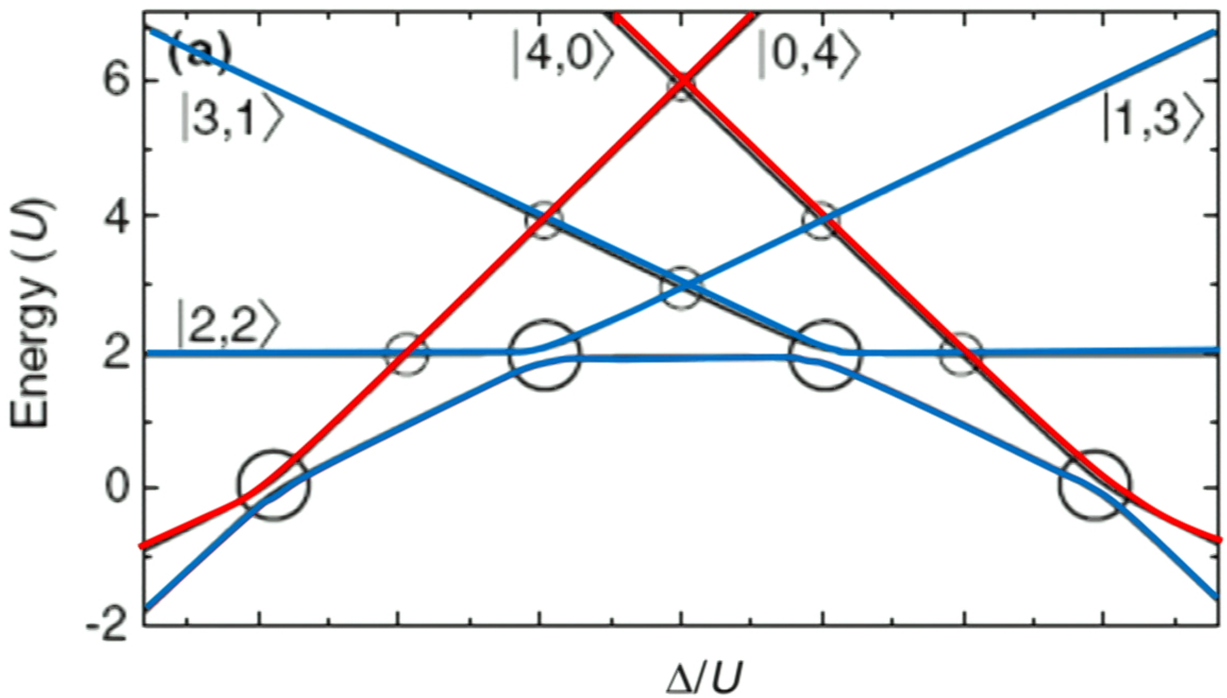
Effectiveness of triangles in experiments (Non-Adiabatic evolution)



level crossings

I. Bloch et at, 2008

Effectiveness of triangles in experiments (Adiabatic evolution)

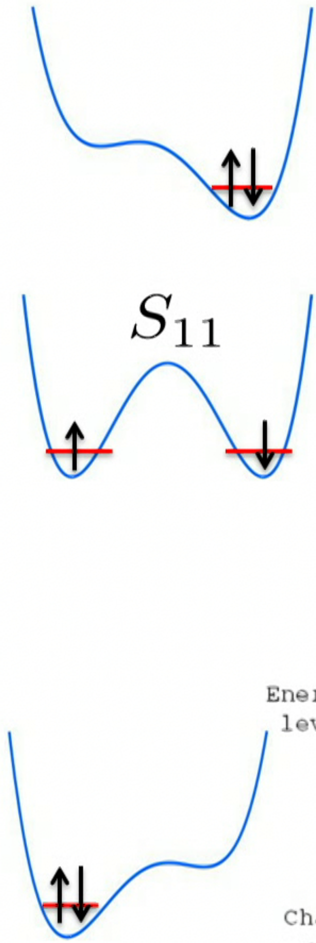


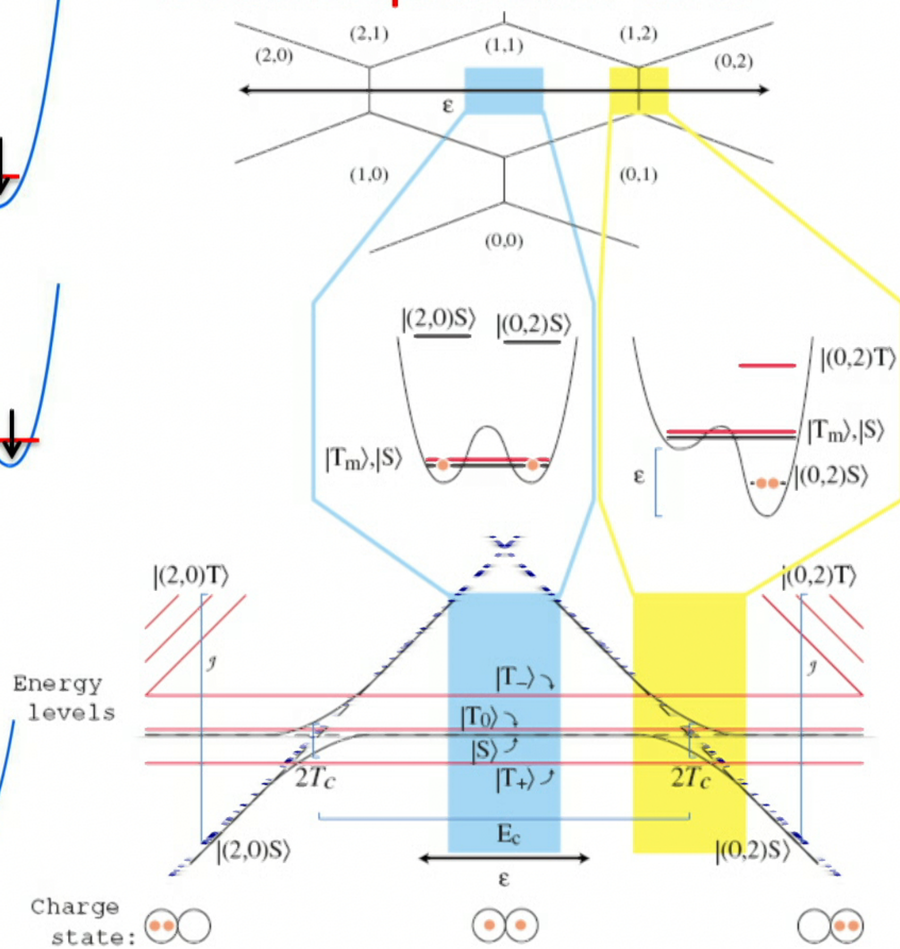
Avoided level crossings

I. Bloch et at, 2008

Where do we observe Quantum Triangles? Cont'd

Double quantum dots

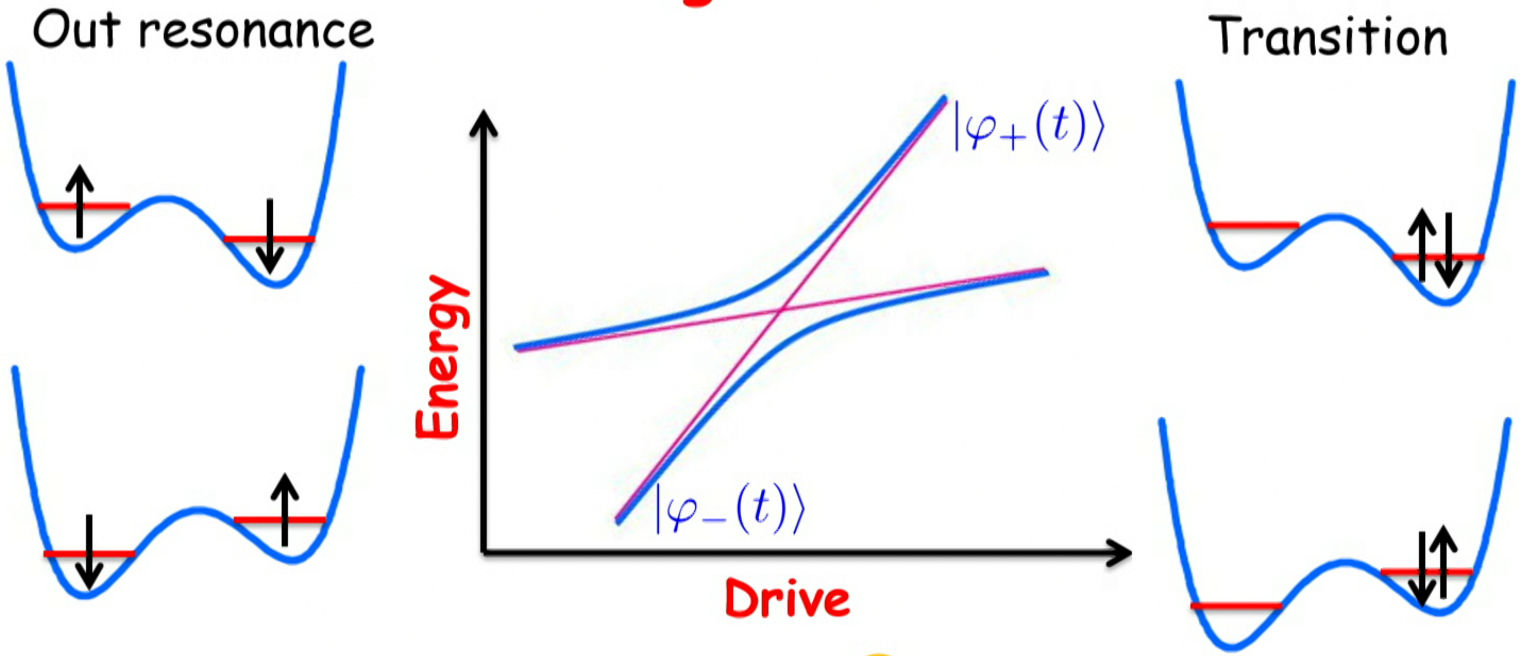




J.Petta et al 2007

How to deal with Quantum Triangles?

Two level crossing: Landau-Zener model



Hamiltonian: $\mathbf{H}(t) = \alpha t \boldsymbol{\sigma}_z + \Delta \boldsymbol{\sigma}_x,$

Eigen-states(adiabatic states): $|\varphi_+(t)\rangle$ and $|\varphi_-(t)\rangle$

Eigen-energies: $\lambda_\pm(t) = \pm\sqrt{\alpha^2 t^2 + \Delta^2}$

What is the Landau-Zener effect?

(Landau, Zener, Stuckelberg, Majorana 1932)

$$\mathbf{H}(t) = \vec{b}(t) \cdot \vec{\boldsymbol{\sigma}},$$

$$\vec{b}(t) = \Big[b_x(t), 0, b_z(t)\Big], \qquad \vec{\boldsymbol{\sigma}} = \Big[\boldsymbol{\sigma}_x, 0, \boldsymbol{\sigma}_z\Big]^T,$$

Zeeman field

Pauli matrices

$$\mathbf{H}(t) = b_z(t)\boldsymbol{\sigma}_z + b_x(t)\boldsymbol{\sigma}_x,$$

Transition time: $\tau_{zee} = \left|\frac{b_z}{\dot{b}_z}\right|$ Field variation time: $\tau_{FV} = \left|\frac{\dot{b}_z}{\ddot{b}_z}\right|$

Condition of short transition time: $\tau_{zee} \ll \tau_{FV}$ ➡ $b_z \ll \frac{(\dot{b}_z)^2}{|\ddot{b}_z|}$

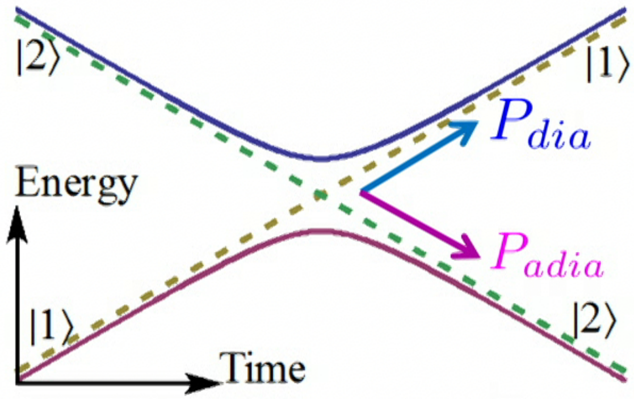
$b_z(t) = \dot{b}_z(t)t,$ ➡ $\mathbf{H}(t) = \dot{b}_z(t)t\boldsymbol{\sigma}_z + b_x(t)\boldsymbol{\sigma}_x,$

Landau-Zener model

Populations

Diabatic basis
(unperturbed basis)

(Fast drive)



$$i\frac{d}{dt}c(t) = \mathbf{H}(t)c(t),$$

$$\mathbf{H}(t) = \begin{bmatrix} \alpha t & \Delta \\ \Delta & -\alpha t \end{bmatrix}, \qquad c(t) = [c_1(t), c_2(t)]^T$$

$$P_{dia}(t) = |c_1(t)|^2,$$ Survival probability

$$P_{adia}(t) = |c_2(t)|^2,$$ Transition probability

$$P_{dia}(t) + P_{adia}(t) = 1$$

From Diabatic to Adiabatic Basis

Diabatic basis $c(t) = [c_1(t), c_2(t)]^T$ (Fast drive)
(unperturbed basis)

$$i\frac{d}{dt}c(t) = \mathbf{H}(t)c(t), \qquad \mathbf{H}(t) = \begin{bmatrix} \alpha t & \Delta \\ \Delta & -\alpha t \end{bmatrix},$$

Passage

⬇

$$c(t) = \mathbf{W}(t)a(t), \qquad \mathbf{W} = \begin{bmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{bmatrix},$$

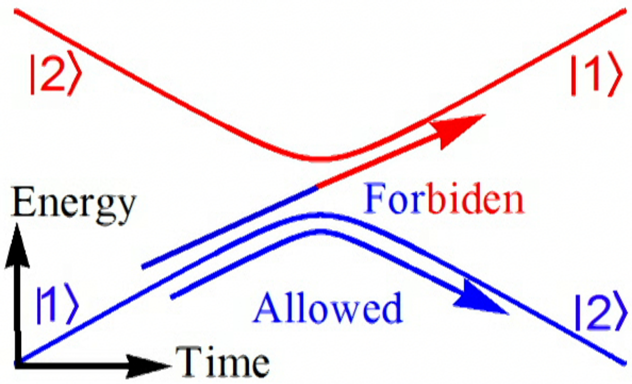
Adiabatic basis $a(t) = [a_1(t), a_2(t)]^T$ (Slow drive)
(dressed states)

$$i\frac{d}{dt}a(t) = \mathbf{H}_a(t)a(t), \qquad \mathbf{H}_a(t) = \begin{bmatrix} \lambda_- & -i\dot{\vartheta} \\ i\dot{\vartheta} & \lambda_+ \end{bmatrix},$$

⬇

$$\mathbf{H}_a(t) = \mathbf{W}^T\mathbf{H}(t)\mathbf{W} - i\mathbf{W}^T\frac{d}{dt}\mathbf{W}$$

Condition for Adiabatic Evolution



Adiabatic theorem

A slowly driven system remains in the same adiabatic state

Coupling less than splitting

$$-i\mathbf{W}^T \frac{d}{dt}\mathbf{W} \ll |\lambda_- - \lambda_+|$$

Superadiabatic evolution

$$\dot{\vartheta}(t) \to 0$$

Results for two-state systems

$$P_{adia}(\tau,\tau_0) = \frac{1}{2} - \frac{\tau\tau_0}{\omega(\tau)\omega(\tau_0)} - \frac{2\lambda}{\omega(\tau)\omega(\tau_0)}\cos\Big[\Lambda_{12}(\tau,\tau_0)\Big]$$

$$Q_{adia}(\tau,\tau_0) = \frac{1}{2} + \frac{\tau\tau_0}{\omega(\tau)\omega(\tau_0)} + \frac{2\lambda}{\omega(\tau)\omega(\tau_0)}\cos\Big[\Lambda_{12}(\tau,\tau_0)\Big]$$

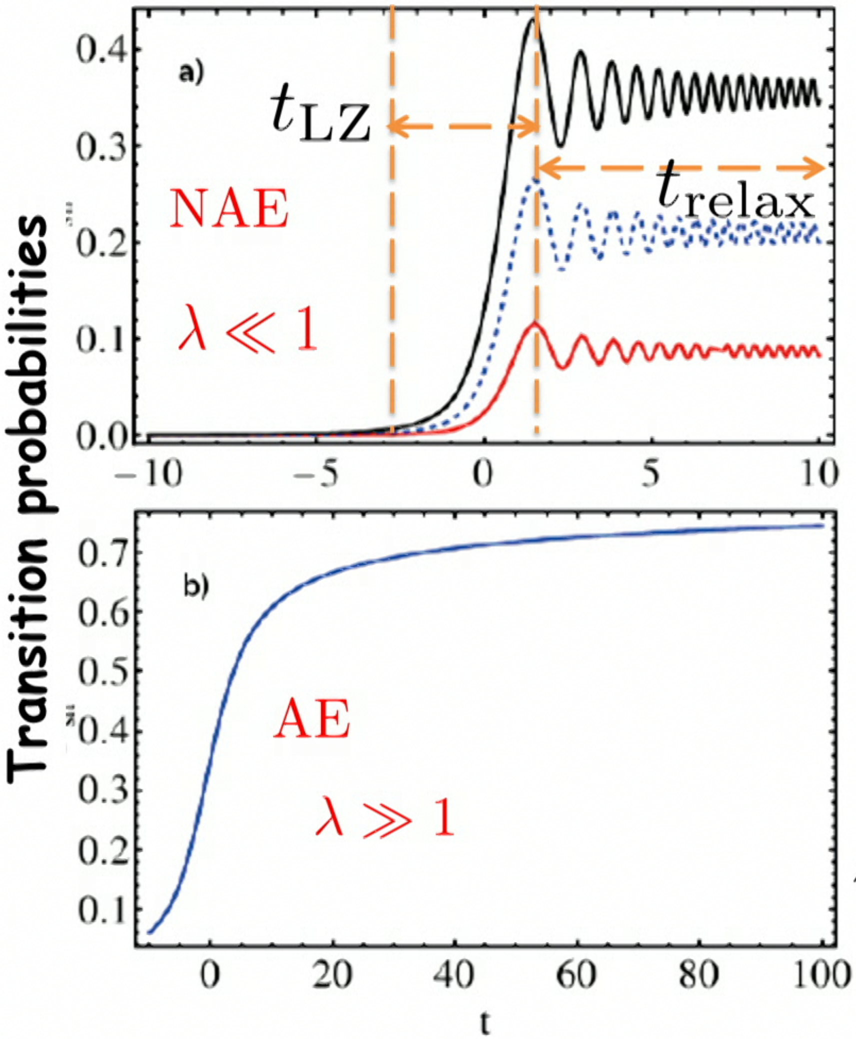
$$\phi(\tau) = \frac{1}{2}\Big(\tau\sqrt{\tau^2+4\lambda} + 4\ln(\tau+\sqrt{\tau^2+4\lambda})\Big).$$

$$\Lambda_{12}(\tau,\tau_0) = \phi(\tau) - \phi(\tau_0),$$

$$\omega(\tau) = \sqrt{\tau^2+4\lambda},$$

$$\tau = t\sqrt{\alpha}, \qquad \lambda = \Delta^2/\alpha.$$

Landau–Zener times



$$t_c = 1/\Delta_{max}$$

$$t_{\mathrm{LZ}} = 1/\sqrt{\alpha}$$

Non-adiabatic

$$P_{\uparrow\rightarrow\uparrow} = \exp(-2\pi\lambda)$$

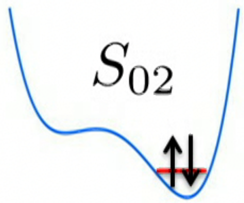
$$\lambda = \frac{\Delta^2}{\alpha}$$

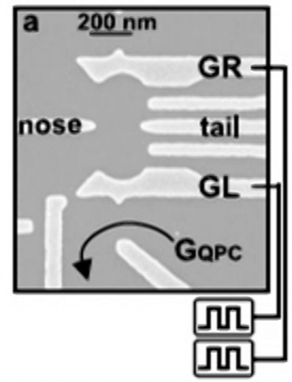
$$t_{adia} = \Delta/\alpha$$ **Adiabatic**

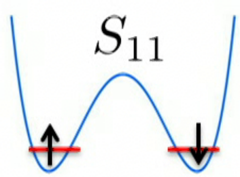
$$\tau_{\mathrm{LZ}} = \max\left(\frac{1}{\sqrt{\alpha}}, \frac{\Delta}{\alpha}\right)$$

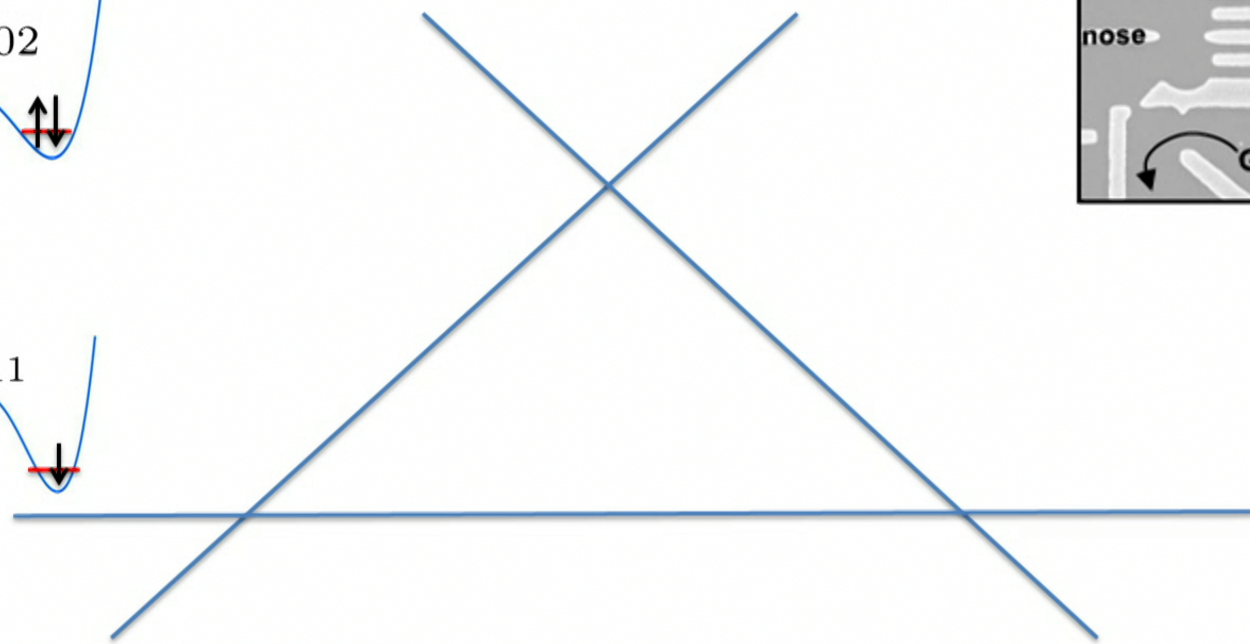
Mullen, Ben-Jacob, Gefen, Schuss, 1989

What is so exciting in QTs?





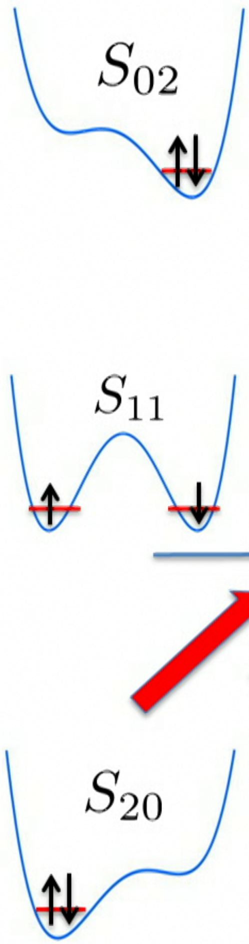


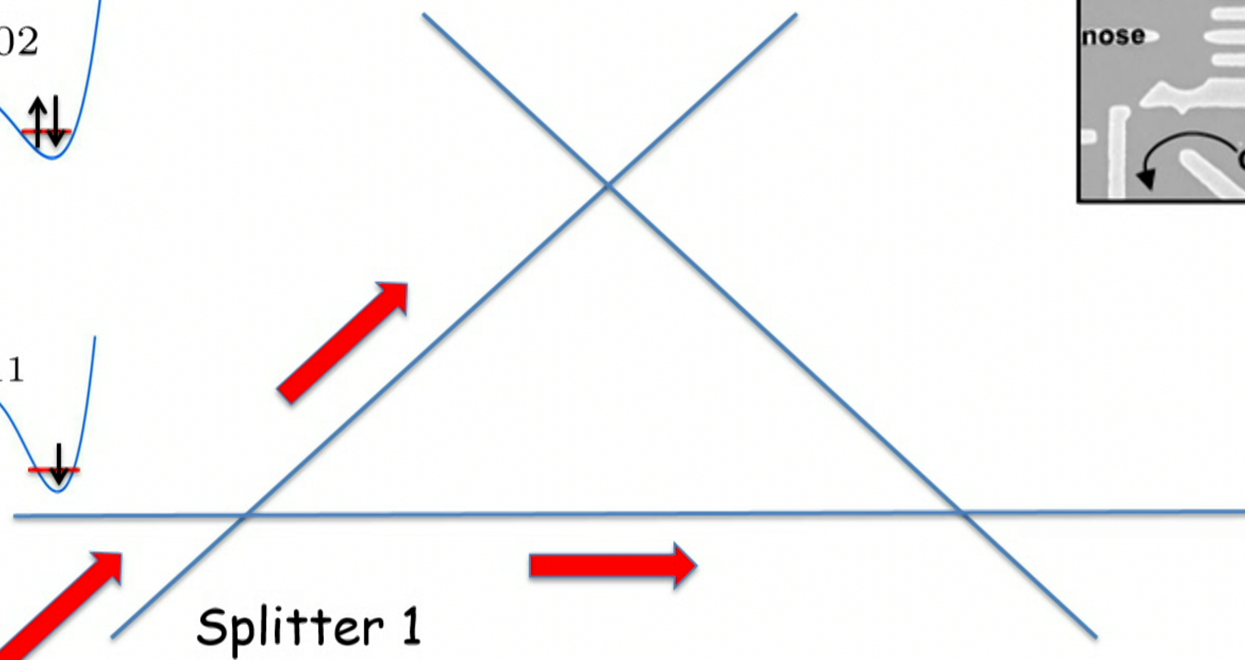


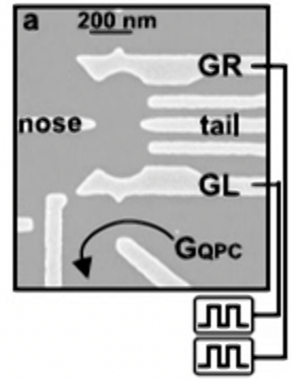
$$\mathcal{H}(t) = \alpha t S^z + f(t) S^x + D(S^z)^2$$

$$f(t) = \Delta$$

What is so exciting in QTs?







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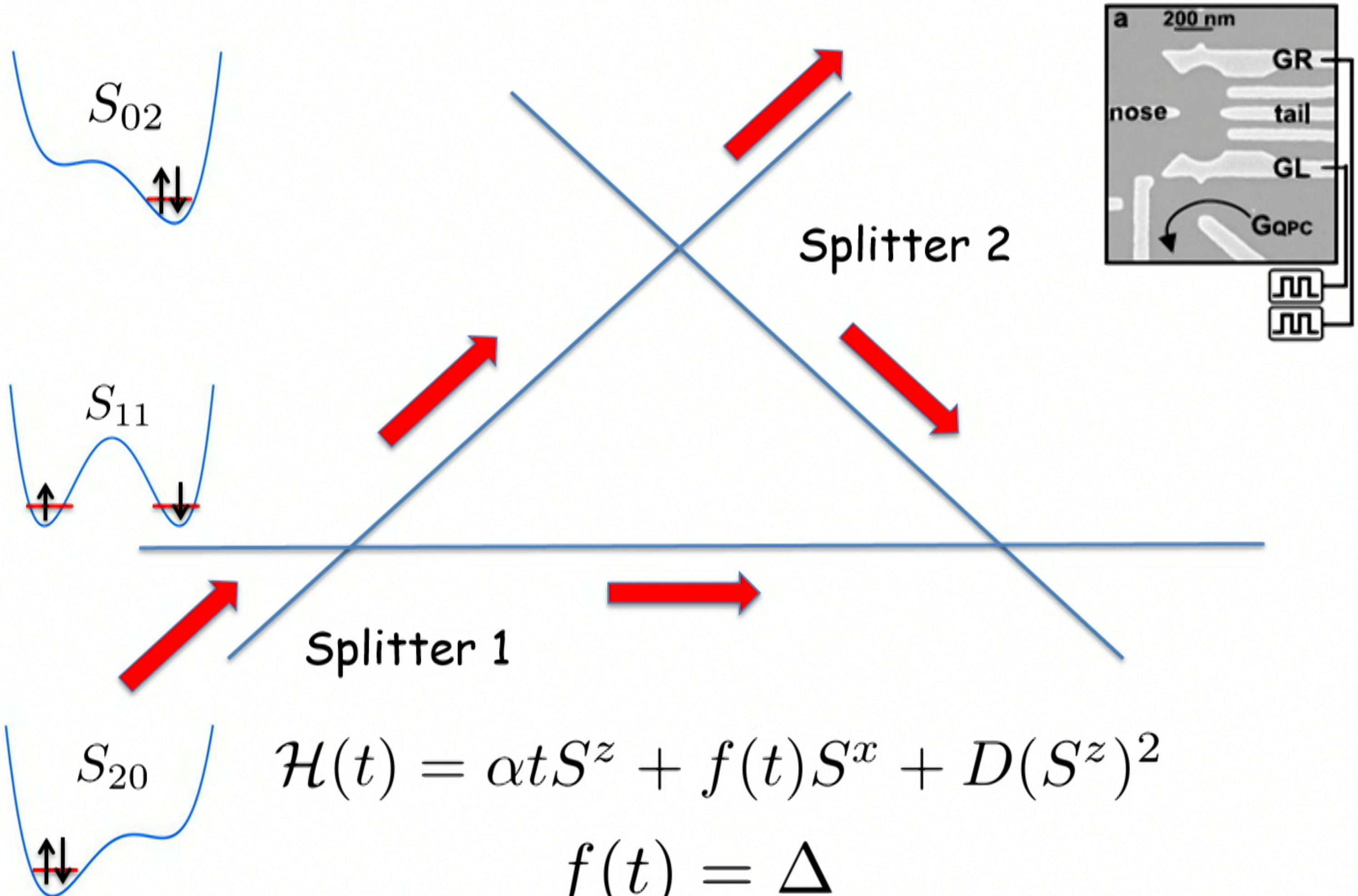

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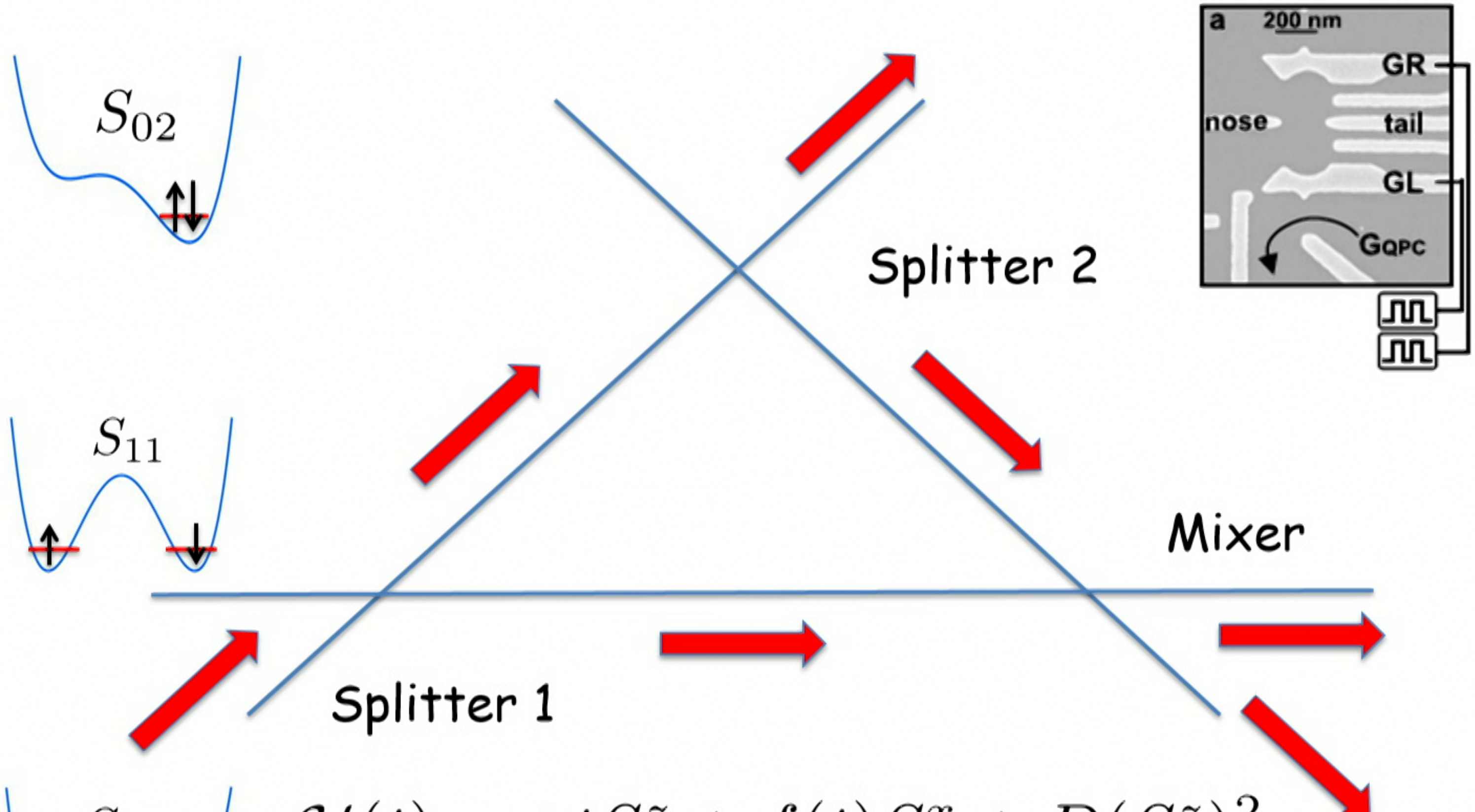


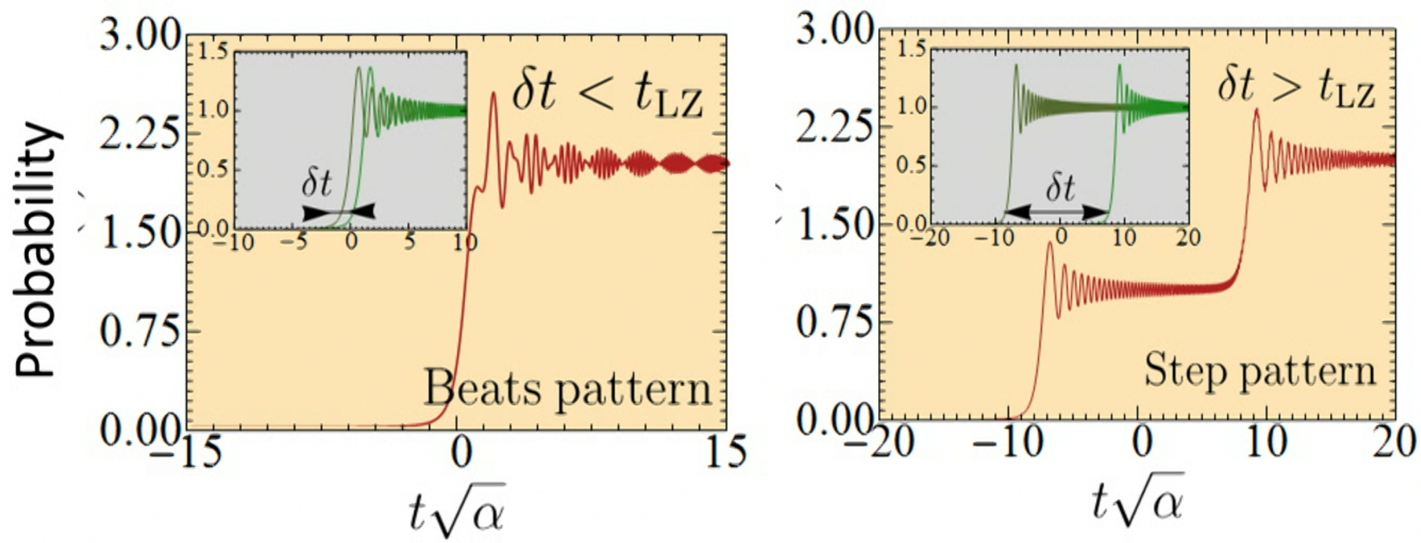
$$\mathcal{H}(t) = \alpha t S^z + f(t) S^x + D(S^z)^2$$

$$f(t) = \Delta$$

"Beats" and "Steps" pattern

Time difference between two crossings

$\delta t < t_{\rm LZ}$ $\delta t > t_{\rm LZ}$



M. N. Kiselev et al 2013

General Model for Quantum Triangles

$$\mathcal{H}(t) = -\sum_{\nu=x,z} \mathbf{B}_\nu(t) S^\nu + D(S^z)^2$$

- Optical lattices (Two-mode Hubbard model)

$$\mathcal{H}(t) = -B_x(t)(a_\sigma^\dagger b_{\sigma'} + b_{\sigma'}^\dagger a_\sigma) - B_z(t)(n_\sigma - n_{\sigma'}) + \frac{D}{2}[n_\sigma(n_\sigma - 1) + n_{\sigma'}(n_{\sigma'} - 1)]$$

$$S^x = \frac{1}{2}\left(a_\sigma^\dagger b_{\sigma'} + b_{\sigma'}^\dagger a_\sigma\right), \quad \mathcal{K} = (S^x)^2 + (S^y)^2 + (S^z)^2 = \frac{n}{2}(\frac{n}{2} + 1)$$

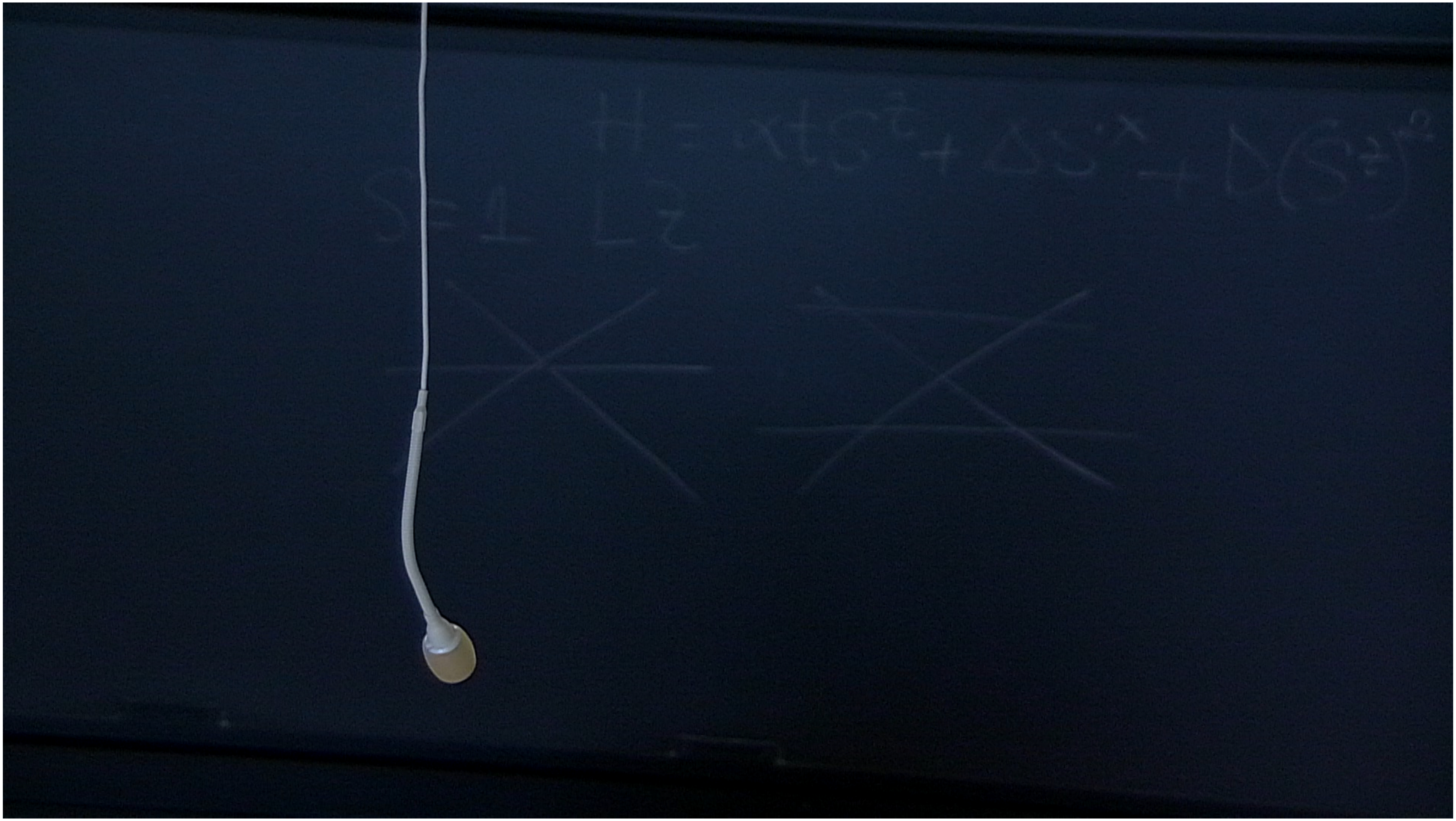
$$S^y = \frac{1}{2i}\left(a_\sigma^\dagger b_{\sigma'} - b_{\sigma'}^\dagger a_\sigma\right), \quad n_\sigma = a_\sigma^\dagger a_\sigma, \quad n = n_\sigma + n_{\sigma'},$$

$$S^z = \frac{1}{2}\left(a_\sigma^\dagger a_\sigma - b_{\sigma'}^\dagger b_{\sigma'}\right), \quad n_{\sigma'} = b_{\sigma'}^\dagger b_{\sigma'}, \quad [n, S^\nu] = 0$$

$$\mathcal{H}(t) = -\sum_{\nu=x,z} \mathbf{B}_\nu(t) S^\nu + D(S^z)^2 + \underbrace{D\frac{n}{2}(\frac{n}{2} - 1)}_{\text{Abelian term}}$$

- Two entangled qubits

$$\mathcal{H}(t) = \sum_{\nu=x,z} \mathbf{B}_\nu(t)(\boldsymbol{\sigma}_\nu^{(1)} + \boldsymbol{\sigma}_\nu^{(2)}) + J\boldsymbol{\sigma}_z^{(1)}\boldsymbol{\sigma}_z^{(2)}$$ Kibble-Zurek model



General Model for Quantum Triangles

$$\mathcal{H}(t) = -\sum_{\nu=x,z} \mathbf{B}_\nu(t) S^\nu + D(S^z)^2$$

- **Optical lattices** (Two-mode Hubbard model)

$$\mathcal{H}(t) = -B_x(t)(a^\dagger_\sigma b_{\sigma'} + b^\dagger_{\sigma'} a_\sigma) - B_z(t)(n_\sigma - n_{\sigma'}) + \frac{D}{2}[n_\sigma(n_\sigma - 1) + n_{\sigma'}(n_{\sigma'} - 1)]$$

$$S^x = \frac{1}{2}\left(a^\dagger_\sigma b_{\sigma'} + b^\dagger_{\sigma'} a_\sigma\right), \quad \mathcal{K} = (S^x)^2 + (S^y)^2 + (S^z)^2 = \frac{n}{2}(\frac{n}{2} + 1)$$

$$S^y = \frac{1}{2i}\left(a^\dagger_\sigma b_{\sigma'} - b^\dagger_{\sigma'} a_\sigma\right), \quad n_\sigma = a^\dagger_\sigma a_\sigma, \quad n = n_\sigma + n_{\sigma'},$$

$$S^z = \frac{1}{2}\left(a^\dagger_\sigma a_\sigma - b^\dagger_{\sigma'} b_{\sigma'}\right), \quad n_{\sigma'} = b^\dagger_{\sigma'} b_{\sigma'}, \quad [n, S^\nu] = 0$$

$$\mathcal{H}(t) = -\sum_{\nu=x,z} \mathbf{B}_\nu(t) S^\nu + D(S^z)^2 + \underbrace{D\frac{n}{2}(\frac{n}{2} - 1)}_{\text{Abelian term}}$$

- **Two entangled qubits**

$$\mathcal{H}(t) = \sum_{\nu=x,z} \mathbf{B}_\nu(t)(\boldsymbol{\sigma}^{(1)}_\nu + \boldsymbol{\sigma}^{(2)}_\nu) + J\boldsymbol{\sigma}^{(1)}_z \boldsymbol{\sigma}^{(2)}_z \quad \text{Kibble-Zurek model}$$

Correspondence between SU(2) and SU(3)

SU(2)		SU(3)
Pauli Matrices $\boldsymbol{\sigma}^\alpha, \quad \alpha = 1 + 3$		Gell-Mann Matrices $\boldsymbol{\lambda}^\alpha, \quad \alpha = 1 + 8$
$b^\alpha(t) = \mathrm{Tr}(\rho(t) \cdot \boldsymbol{\sigma}^\alpha)$	Bloch vector	$b^\alpha(t) = \mathrm{Tr}(\rho(t) \cdot \boldsymbol{\lambda}^\alpha)$
$\vec{b}^2(t) = 1$	Surface	$\vec{b}^2(t) = 1$

Equation of Motion for the Density Matrix = Bloch equation

$$i\frac{d}{dt}b^\alpha = \mathrm{Tr}([H,\rho] \cdot \boldsymbol{\sigma}^\alpha) \qquad i\frac{d}{dt}b^\alpha = \mathrm{Tr}([H,\rho] \cdot \boldsymbol{\lambda}^\alpha)$$

$$i\frac{d}{dt}\vec{b} = -\vec{\Theta} \times \vec{b}$$

$$(\vec{\Theta} \times \vec{b})^\alpha = \epsilon^{\alpha\beta\gamma}\Theta^\beta b^\gamma \qquad (\vec{\Theta} \times \vec{b})^\alpha = f^{\alpha\beta\gamma}\Theta^\beta b^\gamma$$

$$\epsilon^{\alpha\beta\gamma} = \frac{1}{4i}\mathrm{Tr}([\boldsymbol{\sigma}^\alpha, \boldsymbol{\sigma}^\beta]) \cdot \boldsymbol{\sigma}^\gamma), \;\; f^{\alpha\beta\gamma} = \frac{1}{4i}\mathrm{Tr}([\boldsymbol{\lambda}^\alpha, \boldsymbol{\lambda}^\beta]) \cdot \boldsymbol{\lambda}^\gamma)$$

Correspondence between SU(2) and SU(3)

SU(2)		SU(3)
Pauli Matrices $\boldsymbol{\sigma}^\alpha, \quad \alpha = 1+3$		Gell-Mann Matrices $\boldsymbol{\lambda}^\alpha, \quad \alpha = 1+8$
$b^\alpha(t) = \mathrm{Tr}(\rho(t)\cdot\boldsymbol{\sigma}^\alpha)$	Bloch vector	$b^\alpha(t) = \mathrm{Tr}(\rho(t)\cdot\boldsymbol{\lambda}^\alpha)$
$\vec{b}^2(t) = 1$	Surface	$\vec{b}^2(t) = 1$

Equation of Motion for the Density Matrix = Bloch equation

$$i\frac{d}{dt}b^\alpha = \mathrm{Tr}([H,\rho]\cdot\boldsymbol{\sigma}^\alpha) \qquad\qquad i\frac{d}{dt}b^\alpha = \mathrm{Tr}([H,\rho]\cdot\boldsymbol{\lambda}^\alpha)$$

$$i\frac{d}{dt}\vec{b} = -\vec{\Theta}\times\vec{b}$$

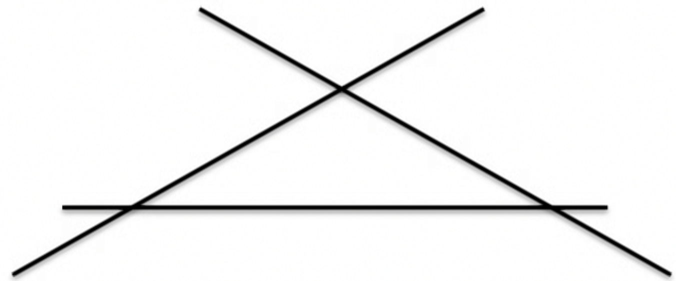
$$(\vec{\Theta}\times\vec{b})^\alpha = \epsilon^{\alpha\beta\gamma}\Theta^\beta b^\gamma \qquad\qquad (\vec{\Theta}\times\vec{b})^\alpha = f^{\alpha\beta\gamma}\Theta^\beta b^\gamma$$

$$\epsilon^{\alpha\beta\gamma} = \frac{1}{4i}\mathrm{Tr}([\boldsymbol{\sigma}^\alpha,\boldsymbol{\sigma}^\beta])\cdot\boldsymbol{\sigma}^\gamma), \;\; f^{\alpha\beta\gamma} = \frac{1}{4i}\mathrm{Tr}([\boldsymbol{\lambda}^\alpha,\boldsymbol{\lambda}^\beta])\cdot\boldsymbol{\lambda}^\gamma)$$

Welcome to the 8-dimensional world !

Crash course of SU(3)

$$3 \times SU(2) \left\{ \begin{array}{l} \vec{s}_1 = \frac{1}{2}(\lambda_1 \lambda_2 \lambda_3) \\ \vec{s}_2 = \frac{1}{2}(\lambda_4 \lambda_5 \lambda_+) \\ \vec{s}_3 = \frac{1}{2}(\lambda_6 \lambda_7 \lambda_-) \end{array} \right.$$



$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_+ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda_\pm = (\sqrt{3}\lambda_8 \pm \lambda_3)/2$$

Reduction of the 8-dimensional world

$$\rho_{11}(t) = \frac{1}{3}(1 + \frac{R(t)}{2} + \frac{3Q(t)}{2}) \qquad \rho_{33}(t) = \frac{1}{3}(1 + \frac{R(t)}{2} - \frac{3Q(t)}{2})$$

$$\rho_{22}(t) = \frac{1}{3}(1 - R(t))$$

$$\frac{dQ}{dt} = -\int_{-\infty}^{t} f(t)f(t_1)\Big[Kr^-(t,t_1)R(t_1) + Kr^+(t,t_1)Q(t_1)\Big]dt_1 + \Phi_-(t),$$

$$\frac{dR}{dt} = -3\int_{-\infty}^{t} f(t)f(t_1)\Big[Kr^+(t,t_1)R(t_1) + Kr^-(t,t_1)Q(t_1)\Big]dt_1 + 3\Phi_+(t),$$

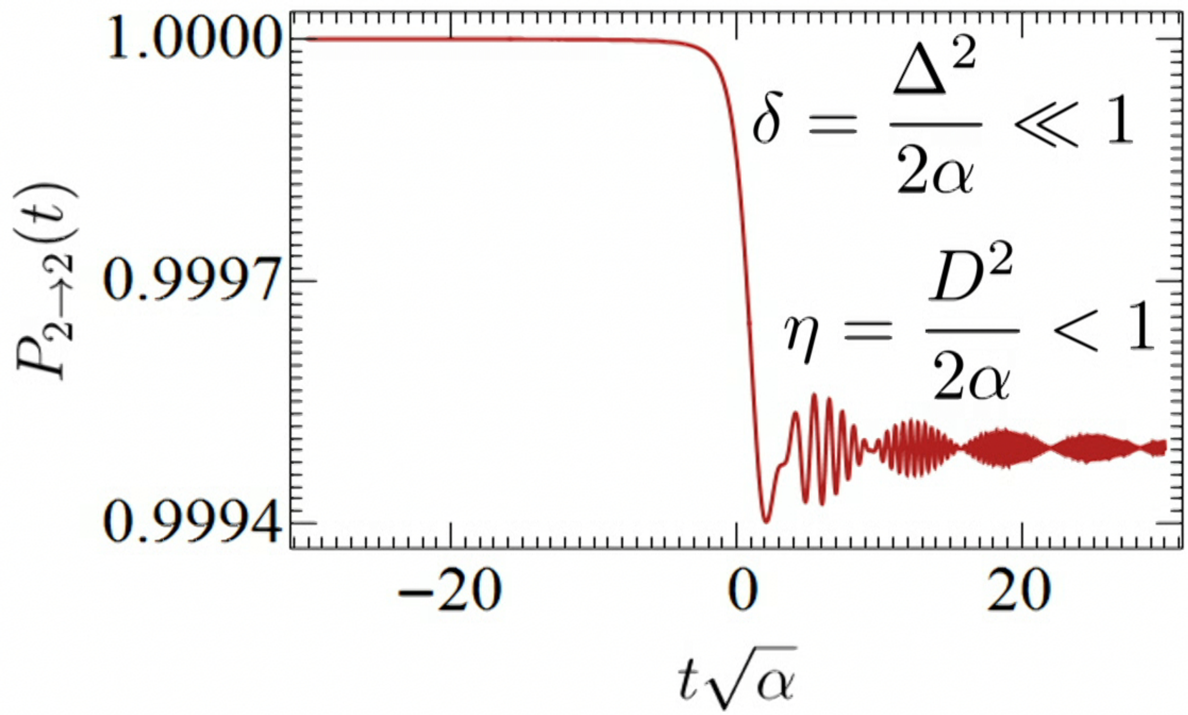
$$\frac{dW}{dt} = \int_{-\infty}^{t} f(t_1)\Big[Ki^+(t,t_1)R(t_1) + Ki^-(t,t_1)Q(t_1)\Big]dt_1 + \Phi_0(t).$$

$$K\mu^{\pm}(t,t_1) = K\mu^{\Omega^+}(t,t_1) \pm K\mu^{\Omega^-}(t,t_1) \qquad K\mu^{\xi}(t,t_1) = \mathrm{L}\mu[\exp[i(\xi(t) - \xi(t_1))]]$$

$$\xi(t) = (\Omega^+(t), \Omega^-(t)) \qquad \Omega^{\pm}(t) = \pm\frac{\alpha}{2}(t \pm \frac{D}{\alpha})^2 \mp \frac{D^2}{2\alpha}$$

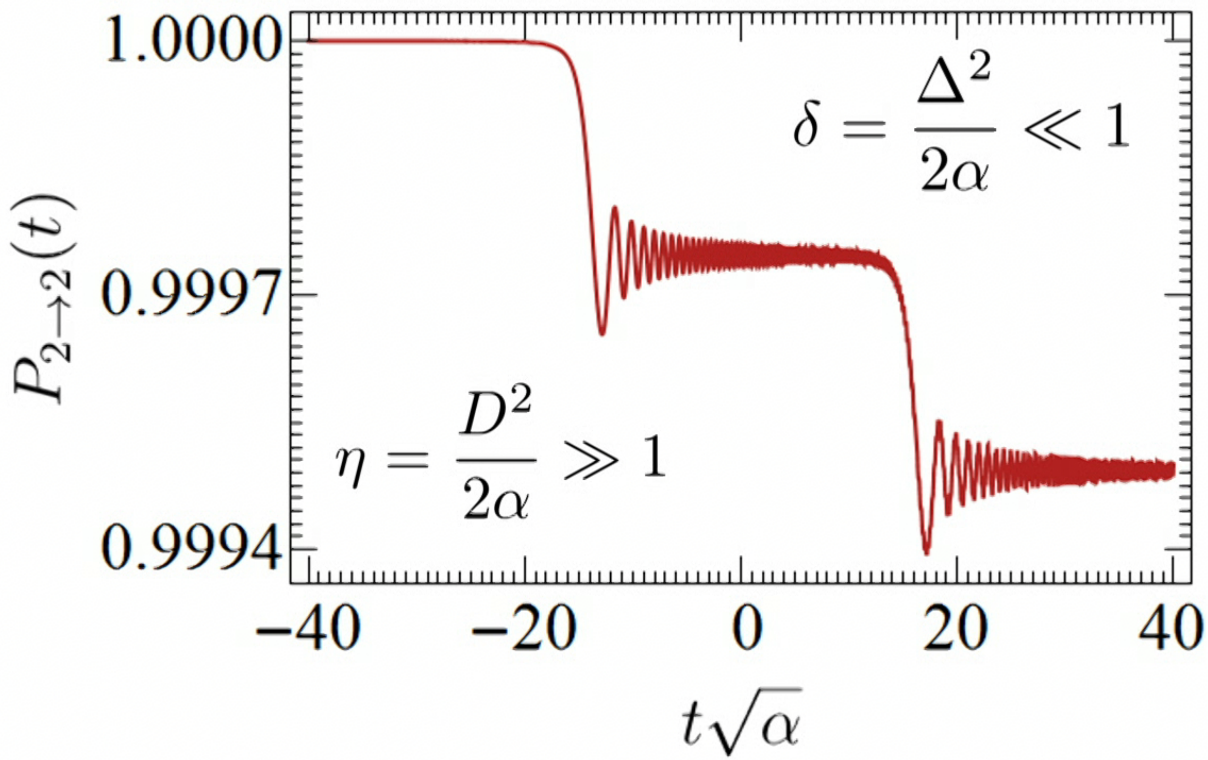
$$\mu = r, i \text{ and } \mathrm{L}r = \mathrm{Re},\ \mathrm{L}i = \mathrm{Im}$$

SU(3) LZ interferometer : the beats



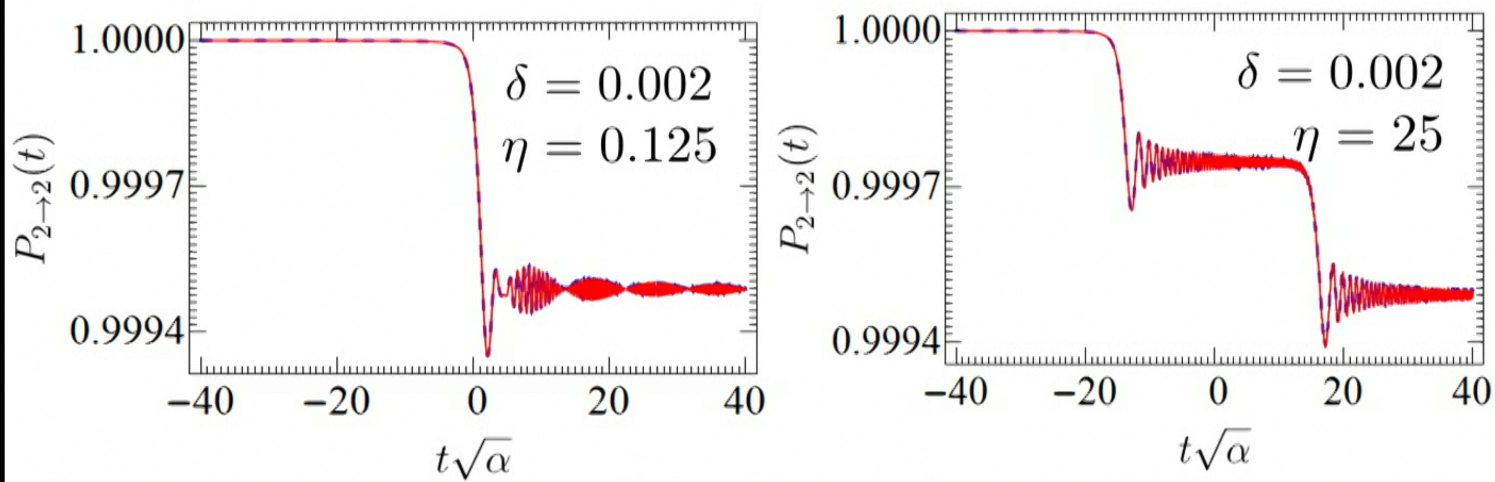
What is the period of the beats ?

SU(3) LZ interferometer : steps



What is the time scale for the steps ?

SU(3) beats and steps: non-adiabatic passage



Blue - numerical solution of SE. Red - perturbative analytic solution of BE.

$$P_{2\to 2}(t) \approx 1 - p_+(t) - p_-(t) + \mathcal{O}(\delta^2)$$

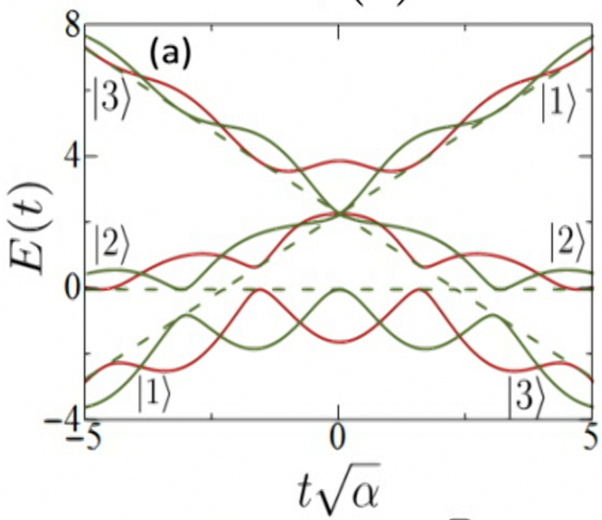
$$p_+(t) = \pi\delta F\Big(t + \frac{D}{\alpha}, t + \frac{D}{\alpha}\Big) \qquad p_-(t) = \pi\delta F\Big(t - \frac{D}{\alpha}, t - \frac{D}{\alpha}\Big)$$

$$F(x,y) = \frac{1}{2}\Big[\Big(\frac{1}{2} + C\Big(\sqrt{\frac{\alpha}{\pi}}x\Big)\Big)\Big(\frac{1}{2} + C\Big(\sqrt{\frac{\alpha}{\pi}}y\Big)\Big) + \Big(\frac{1}{2} + S\Big(\sqrt{\frac{\alpha}{\pi}}x\Big)\Big)\Big(\frac{1}{2} + S\Big(\sqrt{\frac{\alpha}{\pi}}y\Big)\Big)\Big]$$

$$G(x,y) = \frac{1}{2}\Big[\Big(\frac{1}{2} + C\Big(\sqrt{\frac{\alpha}{\pi}}x\Big)\Big)\Big(\frac{1}{2} + S\Big(\sqrt{\frac{\alpha}{\pi}}y\Big)\Big) - \Big(\frac{1}{2} + S\Big(\sqrt{\frac{\alpha}{\pi}}x\Big)\Big)\Big(\frac{1}{2} + C\Big(\sqrt{\frac{\alpha}{\pi}}y\Big)\Big)\Big]$$

SU(3) LZ interferometry with transverse drive

$$\mathcal{H}(t) = \alpha t S^z + f(t) S^x + D(S^z)^2,$$



Monochromatic signal

$$f(t) = A\cos(\omega t + \phi)$$

$$P_{2\to 2}(t) \approx 1 - p_+(t) - p_-(t) + \mathcal{O}(\delta^2)$$

$$p_\pm(t) = \pi\delta\Big[F\Big(t \pm \frac{D\mp\omega}{\alpha}, t \pm \frac{D\mp\omega}{\alpha}\Big) + F\Big(t \pm \frac{D\pm\omega}{\alpha}, t \pm \frac{D\pm\omega}{\alpha}\Big)$$

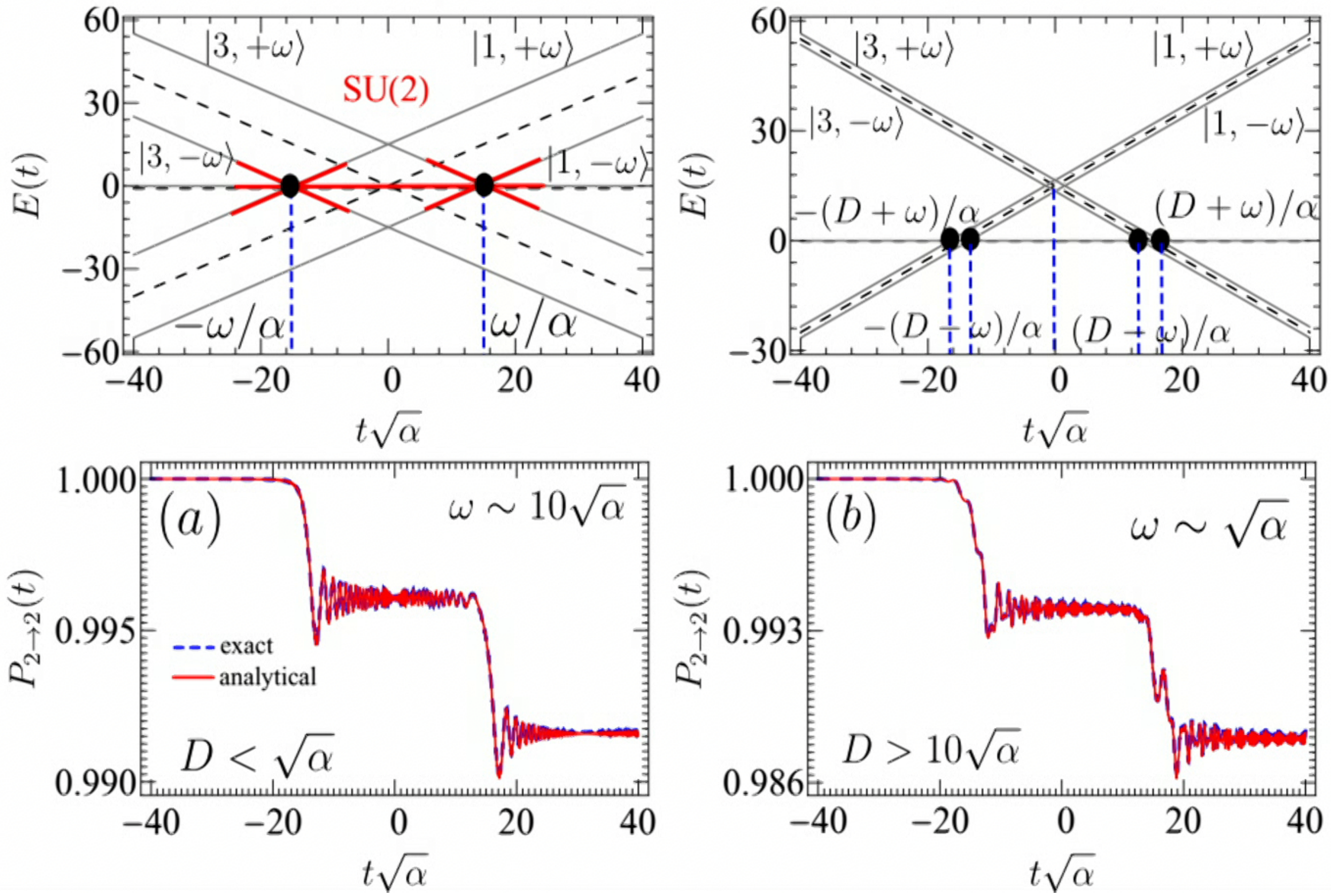
$$+2F\Big(t \pm \frac{D\pm\omega}{\alpha}, t \pm \frac{D\mp\omega}{\alpha}\Big)\cos 2\vartheta^\mp + 2G\Big(t \pm \frac{D\pm\omega}{\alpha}, t \pm \frac{D\mp\omega}{\alpha}\Big)\sin 2\vartheta^\mp\Big]$$

$$\delta = \frac{A^2}{4\alpha}, \qquad \vartheta^\mp = \phi \mp D\omega/\alpha$$

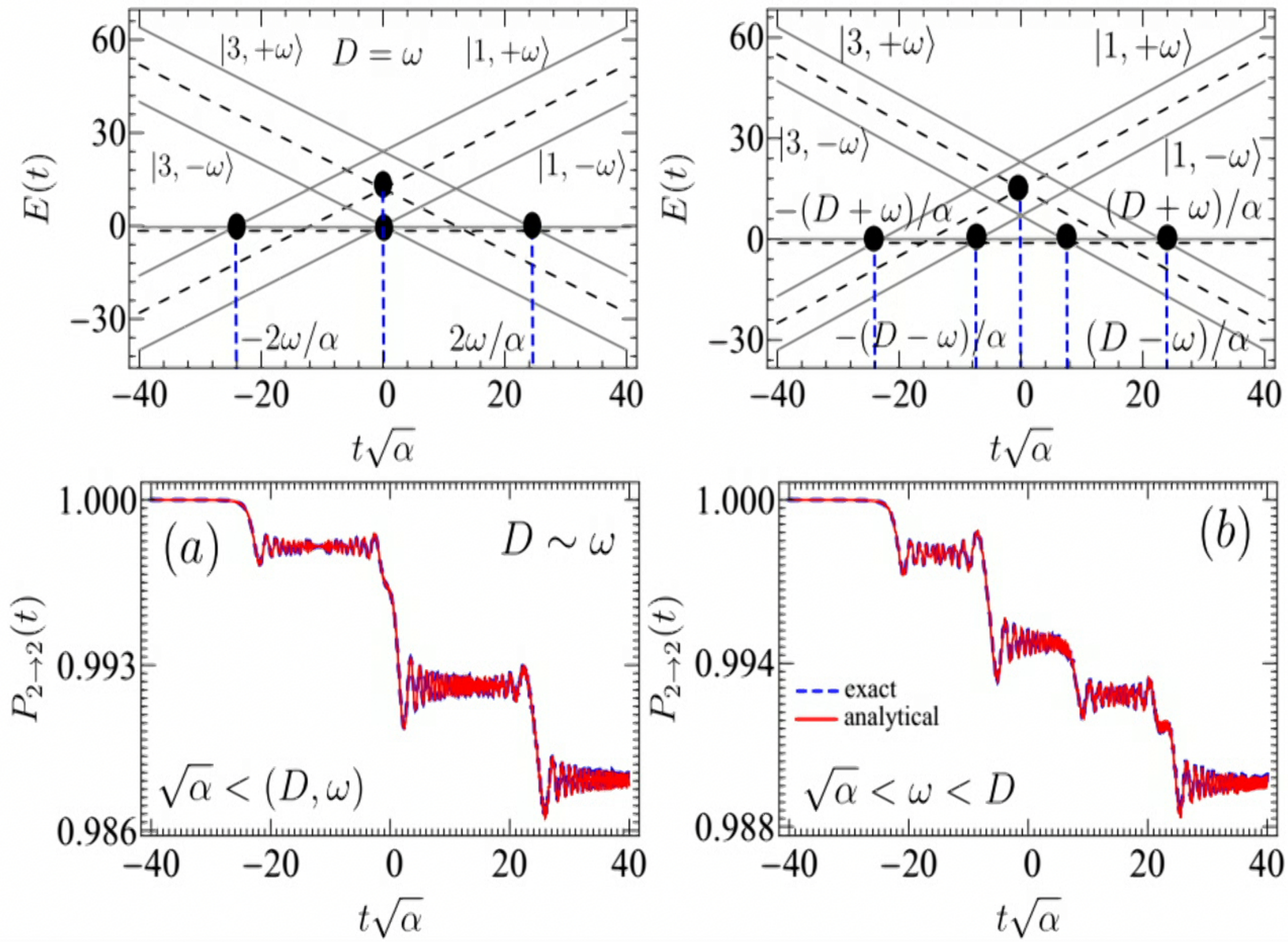
Phase accumulated during a linear sweep

Numerical versus analytical results

Two-step, coexistence of beat and steps



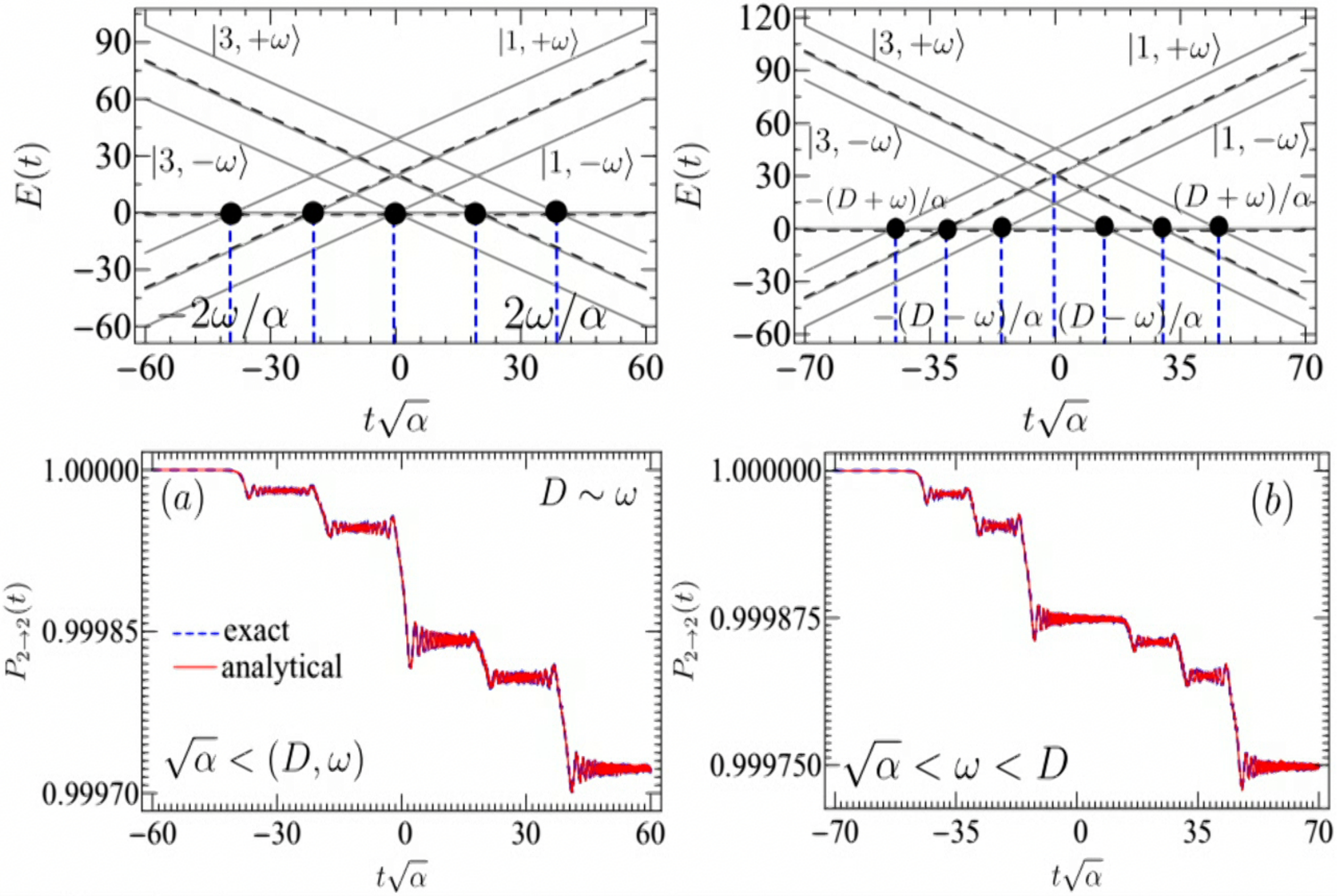
Numerical versus analytical results



Numerical versus analytical results

Five- and Six- Steps

$$f(t) \rightarrow f(t) = \Delta + A\cos(\omega t + \phi)$$



How do we understand these behaviors?

Quantized fields: Three-level system in a QED cavity

$$\mathcal{H}(t) = \alpha t S^z + \mathcal{H}_{\rm cav} + \mathcal{H}_{\rm ThLS-cav} + D(S^z)^2,$$

$$\mathcal{H}_{\rm cav} = \omega(\hat{b}_1^\dagger \hat{b}_1 - \hat{b}_2^\dagger \hat{b}_2),$$

$$\mathcal{H}_{\rm ThLS-cav} = \sum\nolimits_{j=1,2} g_j (\hat{b}_j^\dagger + \hat{b}_j) S^x,$$

$$\hat{b}_{1,2} = \sqrt{n_{1,2}} e^{i(\omega t + \phi_q)},$$

Mean field approximation

$$\lambda_{\kappa,\kappa'}^{\mathbf{p_a}/\mathbf{p_b}} = \frac{A}{\sqrt{4 n_{1,2}}}$$

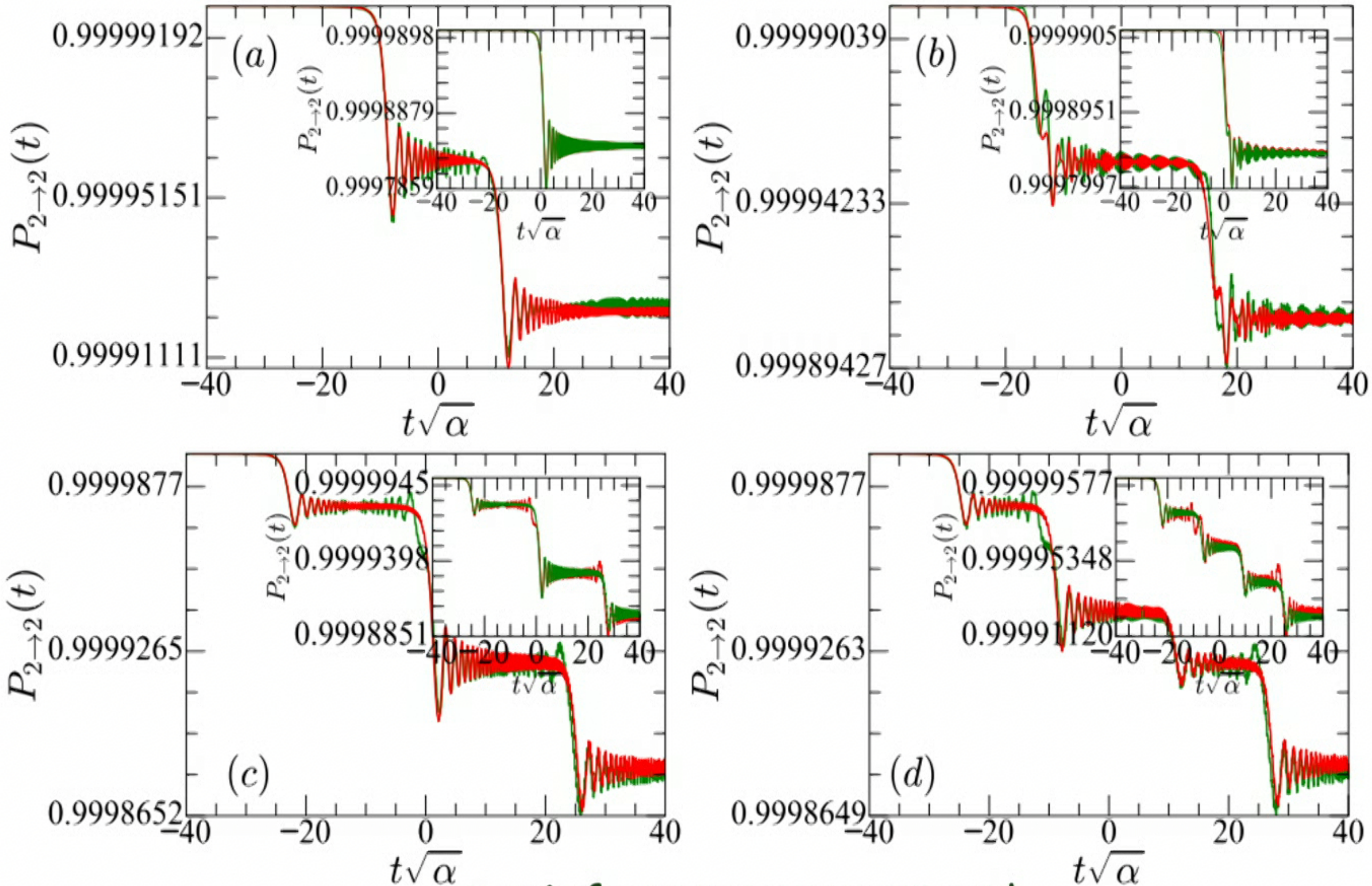
$$SU(3) \rightarrow SU(5)$$

$$\{|1,\omega\rangle, |1,-\omega\rangle, |2\rangle, |3,-\omega\rangle, |3,\omega\rangle\}$$

$$\mathcal{H}(t) = \begin{bmatrix} \alpha t + (D+\omega) & 0 & \lambda_{1,2}^{\mathbf{p_a}}/\sqrt{2} & 0 & 0 \\ 0 & \alpha t + (D-\omega) & \lambda_{\bar{1},2}^{\mathbf{p_a}}/\sqrt{2} & 0 & 0 \\ \lambda_{2,1}^{\mathbf{p_a}}/\sqrt{2} & \lambda_{2,\bar{1}}^{\mathbf{p_a}}/\sqrt{2} & 0 & \lambda_{2,\bar{3}}^{\mathbf{p_b}}/\sqrt{2} & \lambda_{2,3}^{\mathbf{p_b}}/\sqrt{2} \\ 0 & 0 & \lambda_{\bar{3},2}^{\mathbf{p_b}}/\sqrt{2} & -\alpha t + (D-\omega) & 0 \\ 0 & 0 & \lambda_{3,2}^{\mathbf{p_b}}/\sqrt{2} & 0 & -\alpha t + (D+\omega) \end{bmatrix}$$

Quantum versus Semi-classical treatment

SU(3) versus SU(5)



Satisfactory agreement!

SU(3) LZ interferometry with transverse drive

Polychromatic signal

$$f(t) = \sum_{n=0}^{N} A_n \cos(\omega_n t + \phi_n),$$

$$p_\pm(t) = \sum_{n=0}^{N}\sum_{m=0}^{N} \pi\delta_{mn}\Big(\cos\left[\Psi_n^\mp - \Psi_m^\mp\right] F\Big(t \pm \frac{D\mp\omega_n}{\alpha}, t \pm \frac{D\mp\omega_m}{\alpha}\Big) + \cos\left[\Psi_n^\mp + \varphi_m^\pm\right] F\Big(t \pm \frac{D\mp\omega_n}{\alpha}, t \pm \frac{D\pm\omega_m}{\alpha}\Big)$$

$$+\cos[\varphi_n^\pm + \Psi_m^\mp] F\Big(t \pm \frac{D\pm\omega_n}{\alpha}, t \pm \frac{D\mp\omega_m}{\alpha}\Big) + \cos[\varphi_n^\pm - \varphi_m^\pm] F\Big(t \pm \frac{D\pm\omega_n}{\alpha}, t \pm \frac{D\pm\omega_m}{\alpha}\Big)$$

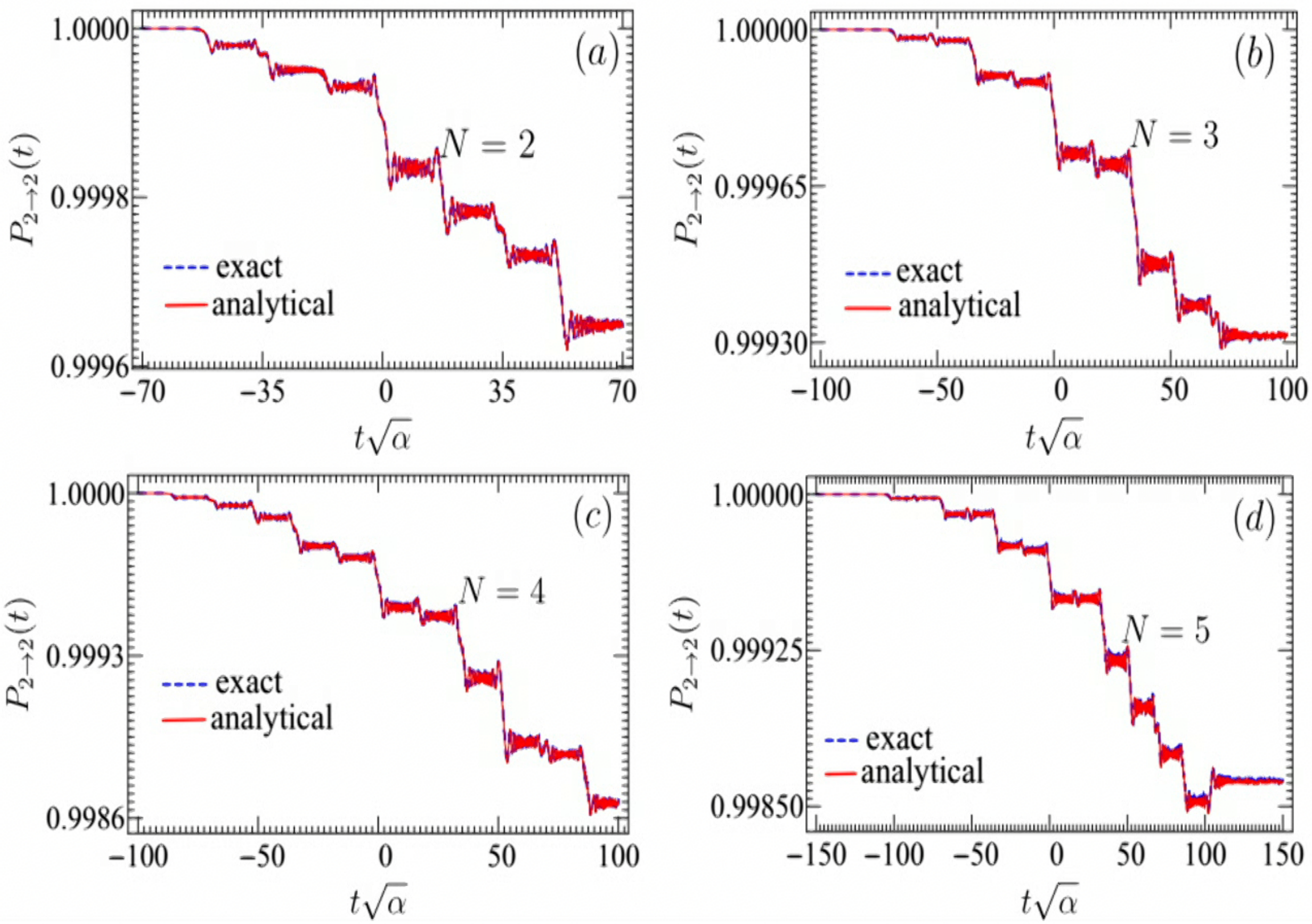
$$-\sin[\Psi_n^\mp - \Psi_m^\mp] G\Big(t \pm \frac{D\mp\omega_n}{\alpha}, t \pm \frac{D\mp\omega_m}{\alpha}\Big) + \sin[\varphi_n^\pm + \Psi_m^\mp] G\Big(t \pm \frac{D\pm\omega_n}{\alpha}, t \pm \frac{D\mp\omega_m}{\alpha}\Big)$$

$$-\sin[\Psi_n^\mp + \varphi_m^\pm] G\Big(t \pm \frac{D\mp\omega_n}{\alpha}, t \pm \frac{D\pm\omega_m}{\alpha}\Big) + \sin[\varphi_n^\pm - \varphi_m^\pm] G\Big(t \pm \frac{D\pm\omega_n}{\alpha}, t \pm \frac{D\pm\omega_m}{\alpha}\Big)\Big)$$

$$\Psi_n^{(i)} = \phi_n + \int_0^{t_{\Psi,n}^{(i)}} \alpha t' dt' \qquad \varphi_n^{(i)} = \phi_n - \int_0^{t_{\varphi,n}^{(i)}} \alpha t' dt'$$

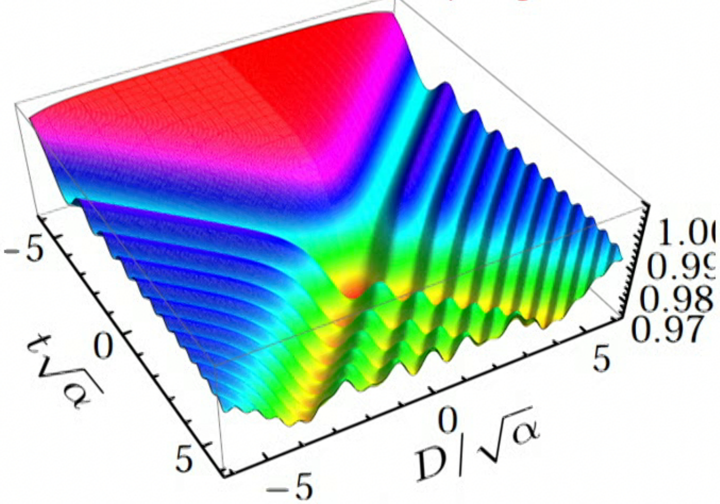
Phases picked up by the ThLS during a linear sweep

N-dependence of the number of steps
Analytical versus Numerics

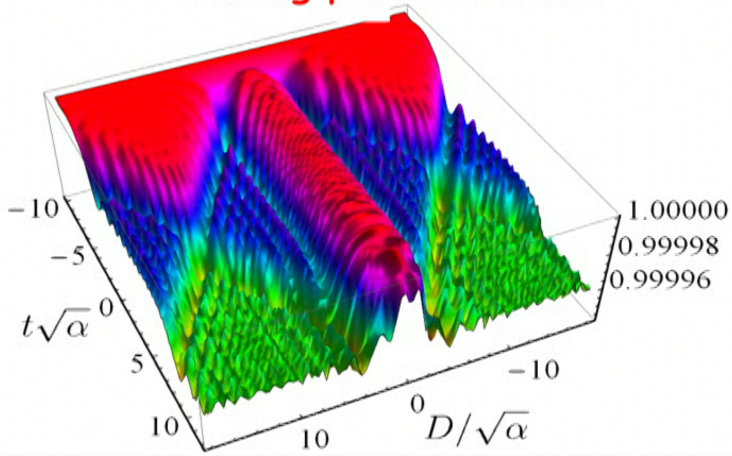


SU(3) interference patterns

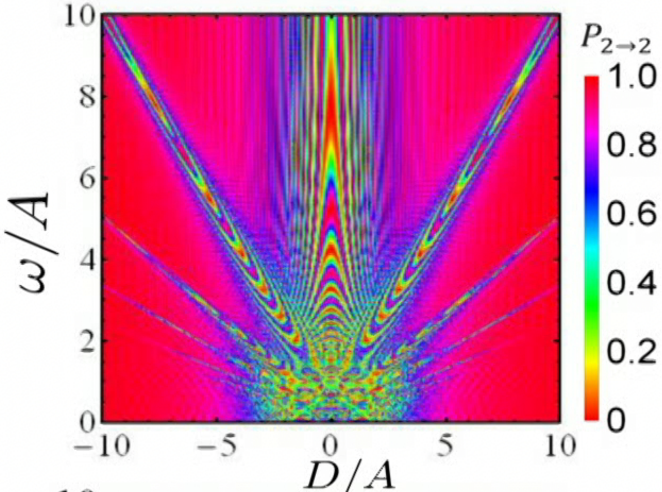
Constant couplings

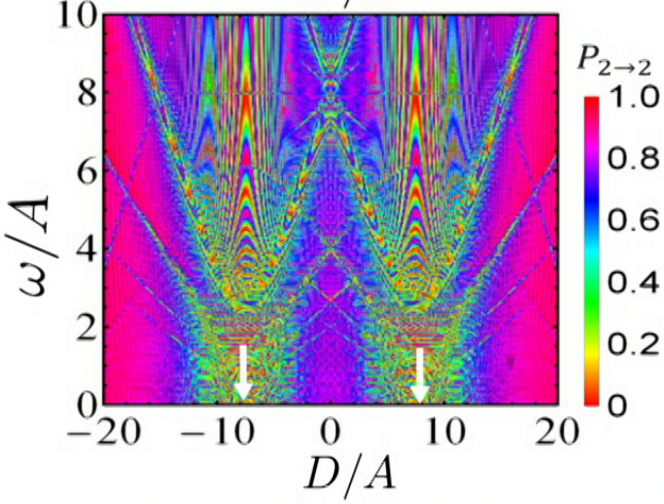


Strong periodic drive



Double Periodic Drive





Concluding Remarks

- When in a QT the couplings are constants:
the number of steps maximizes to 2

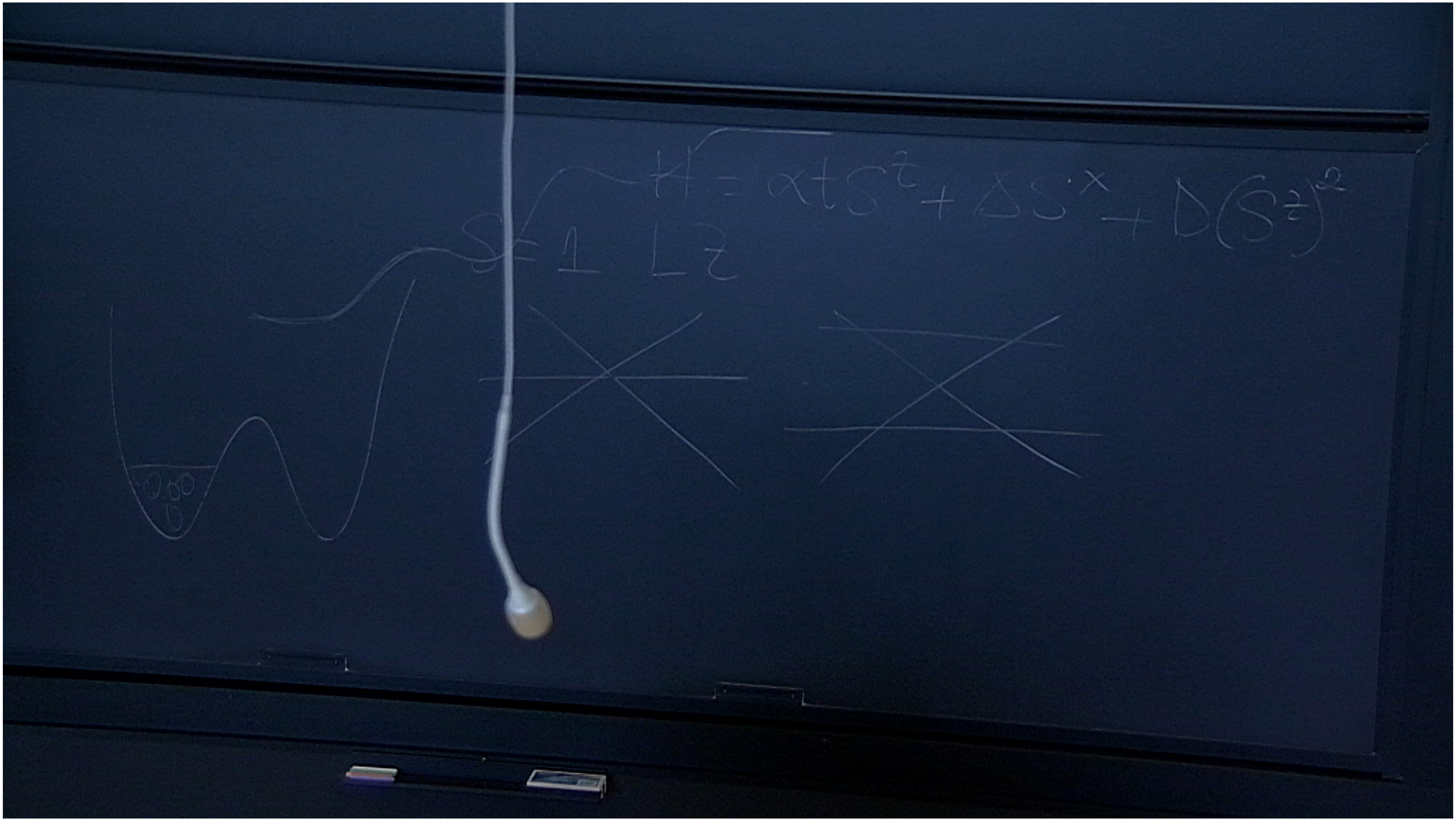
- When the couplings periodically change as a monochromatic signal: the number of steps maximize two 4

- When the couplings periodically change as a shifted monochromatic signal: the number of steps maximize two 6

- When the couplings periodically change as a polychromatic signal : the number of steps increases with the number N of monochromatic signals composing the main signal

- Steps are useful for the statistics of atoms in a Bose-Einstein condensate

- Beats are useful markers for manipulating spins for Quantum Information Processing


H = αt S^z + Δ S^x + D(S^z)^2
S = 1 L Z

Outlook (to do list)

- SU(3) Landau-Zener Interferometry with dissipation
- SU(3) Landau-Zener Interferometry with "Longitudinal" and "transverse" drives
- Statistics of atoms in Bose-Einstein Condensate
- Dynamics of two entangled qubits
- Dynamics of two entangled qutrits
- etc

Thanks

To pioneers: M. N. Kiselev (ICTP) K. K. Kikoin (University of Telaviv)

To collaborators: A. B. Tchapda and L. C. Fai (UDs, Cameroon)

To institutions: Perimeter Institute AIMS-Ghana

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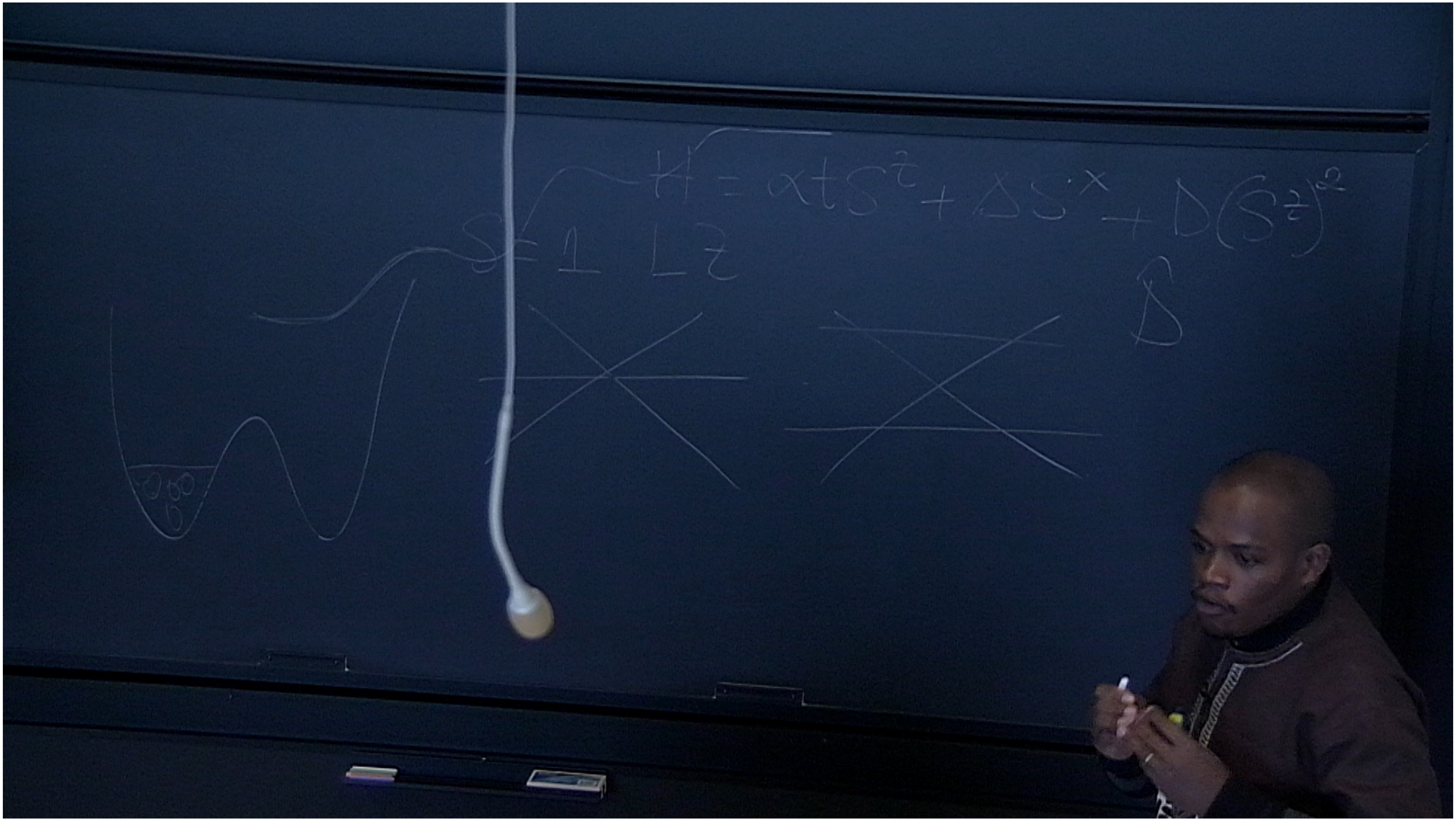
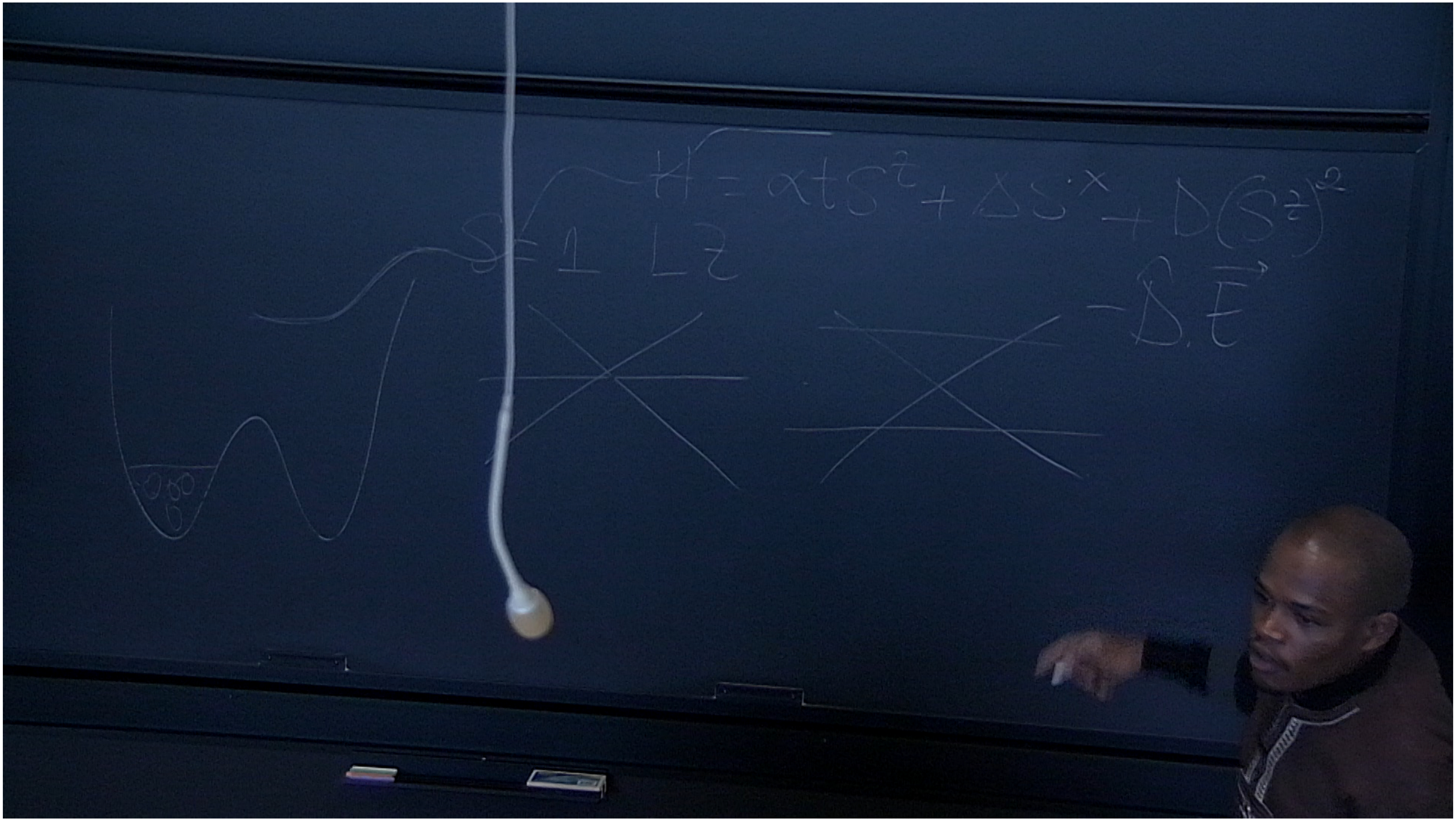

H = αtS^z + ΔS^x + D(S^z)^2
S = 1 LZ


H = αtS^z + ΔS^x + D(S^z)^2
S = 1 LZ
−D.E