Title: The chiral anomaly in Dirac-Weyl semimetals.*

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Abstract: <p>In 1983, Nielsen and Ninomiya predicted that the Adler-Bell-Jakiw (or chiral) anomaly should be observable in a crystal that has protected Dirac states in the bulk (3+1 D). Following recent progress in the field of Topological Quantum Matter, the anomaly has now been observed, most clearly in the two semimetals Na3Bi and GdPtBi. I will discuss the general problem of realizing Weyl Fermions in semimetals, and explain what the chiral anomaly is in condensed matter. I will remark on its historical context, starting with pion decay. Several tests that support our conclusion will be described.</p>

<p> </p>

<p>*Supported by the Moore Foundation, ARO and NSF.</p>

Dirac and Weyl Semimetals and the chiral anomaly

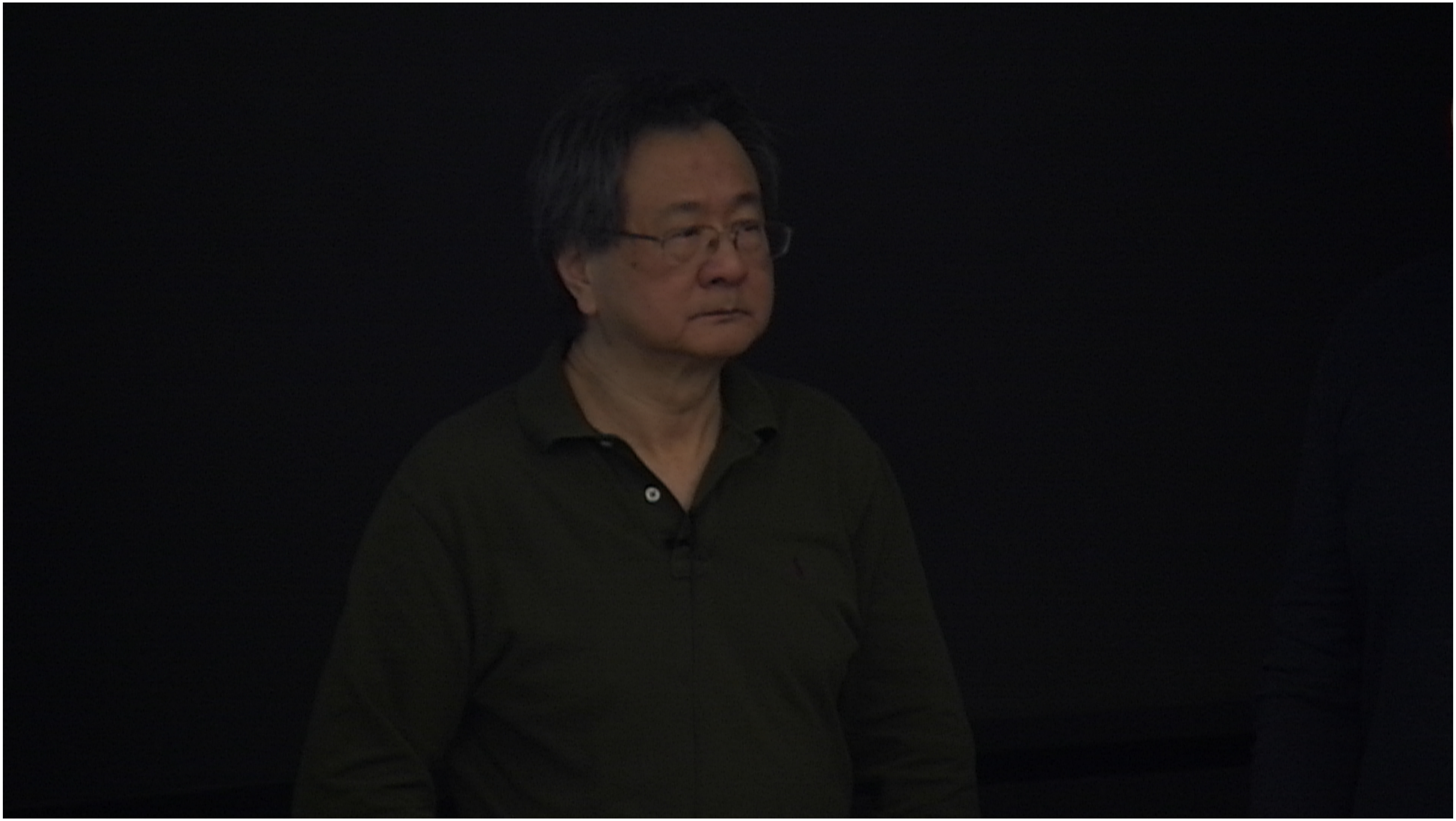
Jun Xiong | Kushwaha | Tian Liang | Jason Krizan | Hirschberger | S.H. Liang | Bob Cava | NP Ong

Jun Xiong, Tian Liang, Max Hirschberger, Sihang Liang, N. P. Ong
Department of Physics, Princeton Univ.
Satya Kushwaha, Jason Krizan, R. J. Cava
Department of Chemistry, Princeton Univ.

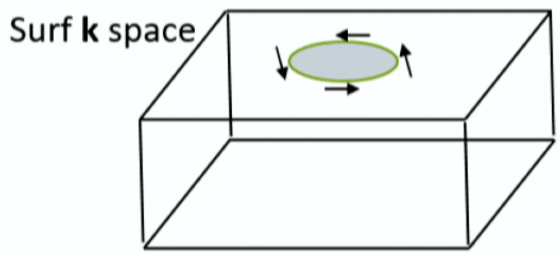
1. Symmetry protected states in topological matter
2. Dirac and Weyl states in semimetals
3. Introduction to chiral anomaly
4. The Chiral anomaly in Na_3Bi and in GdPtBi
5. Current jetting? The squeeze test

GORDON AND BETTY MOORE FOUNDATION

Support from
Moore Foundation, ARO, NSF



Survey of 2D and 3D Topological Matter

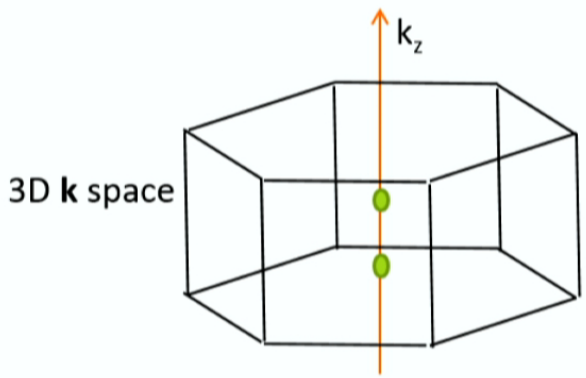


Topological Insulator

2D Spin-locked states on surface

Bulk insulating

Bi_2Se_3, Bi_2Te_3, Bi_2Te_2Se …



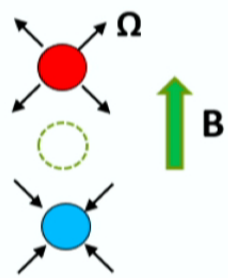
Topological Dirac semimetal

Bulk Dirac states are conducting

3D protected nodes on symmetry axis

Each Dirac node is comprised of 2 Weyls

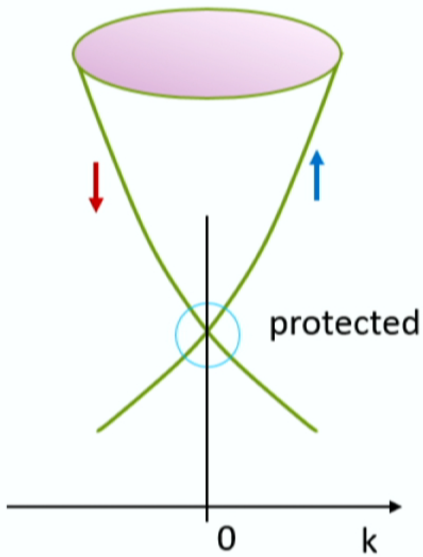
Na_3Bi, Cd_3As_2



Weyl nodes

In applied **B**, Weyl nodes move apart. Act as monopole source and sink of Berry curvature **Ω** (an eff. magnetic field in **k** space).

2D Dirac node at Γ protected by time-reversal symmetry (TRS)



Kramer's theorem

H is TRS

$$H\varphi = E\varphi, \qquad H\psi = E\psi$$

$$\Theta\varphi = \psi, \qquad \Theta\psi = -\varphi$$

$$(\Theta\varphi, \Theta\psi) = (\varphi, \psi)^* = (\psi, \varphi)$$ antiunitarity

$$-(\psi, \varphi) = (\psi, \varphi) = 0$$

φ, ψ must be orthogonal (hence 2-fold degenerate)

Slight generalization

$$M = (\varphi, V\psi) = 0$$

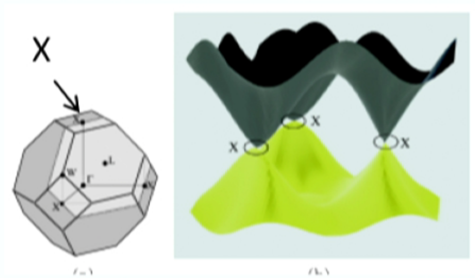
All matrix elements of TRS potentials *V* must vanish.
Therefore, node is protected against gap formation.

Search for (3+1)d Dirac cones with protected nodes

Kane Mele

Dirac Semimetal in Three Dimensions

Young, Zaheer, Teo, Kane, Mele and Rappe PRL 2012



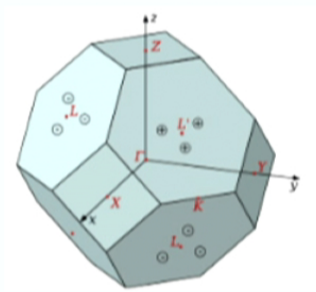
Time reversal symmetry (TRS) *and* Inversion symmetry (IS) protect a Dirac node if it is pinned at zone corner *X*

Candidate: β cristobalite BiO_2
(unfortunately, chem. unstable)

Vishwanath

Topological semimetal and Fermi-arc surface states in pyrochlore iridates

Wan, Turner, Vishwanath and Savrasov, PRB 2011

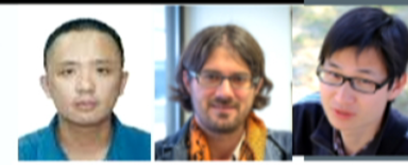


Predicted $Y_2Ir_2O_7$ should exhibit multiple **Weyl** nodes

Experimental progress has been slow

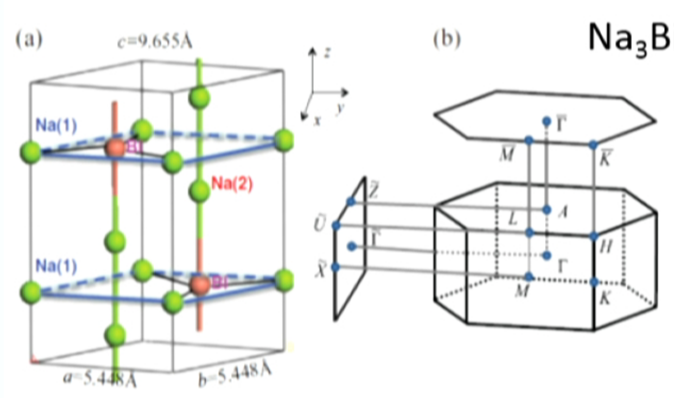
Prediction: Na3Bi and Cd3As2 are topological Dirac semimetals

The key: Add *point-group symmetry* C_n to TRS and IS! (Bernevig, XiDai)


XiDai   Bernevig   Z. Wang

Called *Topological* Dirac Semimetal

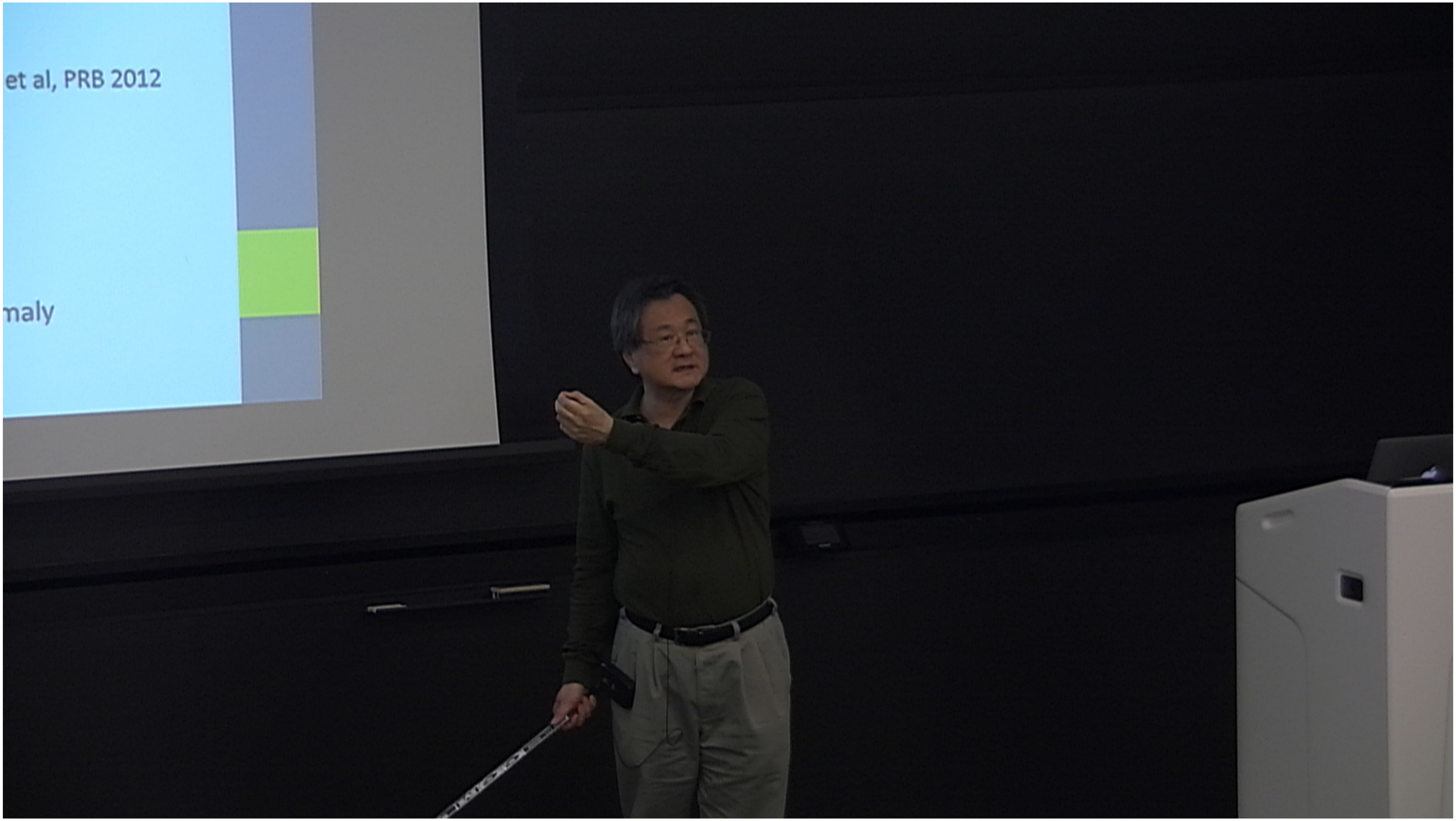
Na_3Bi and Cd_3As_2 (Wang *et al.*)



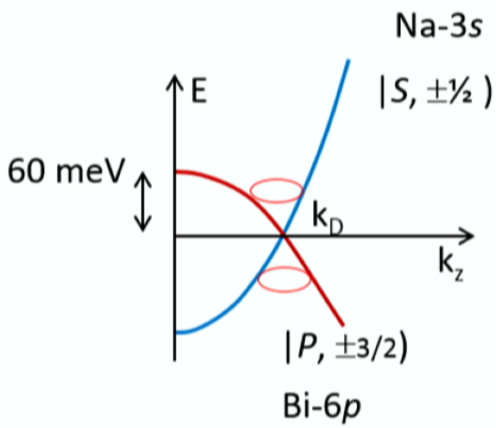
Zhijun Wang, Xi Dai et al, PRB 2012

Wan, Turner, Vishwanath, *PRB* 2011
Burkov, Hook, Balents, *PRB* 2011
Son, Spivak, *PRB* 2013

Sparked massive renewed interest in search for the chiral anomaly


et al, PRB 2012
maly

The band structure of Na_3Bi (Wang, Dai, Fang et al. *PRB* 2012)

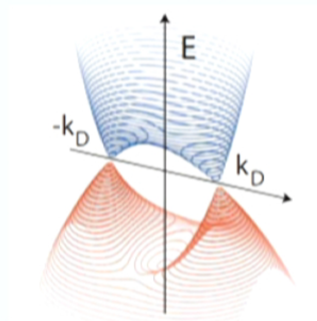


Only 2 bands, derived from Na-3*s* and Bi-6*p*, lie near Fermi energy.

Spin orbit interaction leads to crossings at $\mathbf{K}_{\pm} = (0, 0, \pm k_D)$

Crossings protected against gap formation --- |S) and |P) states belong to different irreducible representations of C_3.

We end up with 2 Dirac nodes centered at $\mathbf{K}_{\pm}$



Dirac Equation and Weyl states

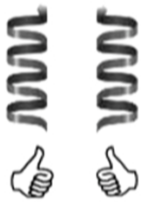

Herman Weyl

Dirac equation $(i\gamma^{\mu}\partial_{\mu} + m)\Psi = 0$

Weyl representn

$\Psi_R = \frac{1}{2}(1+\gamma^5)\Psi \qquad \Psi_L = \frac{1}{2}(1-\gamma^5)\Psi$

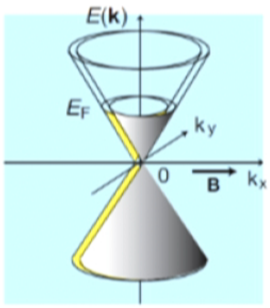
$L = \bar{\Psi}_L\, i\gamma^{\mu}\partial_{\mu}\Psi_L + \bar{\Psi}_R\, i\gamma^{\mu}\partial_{\mu}\Psi_R$

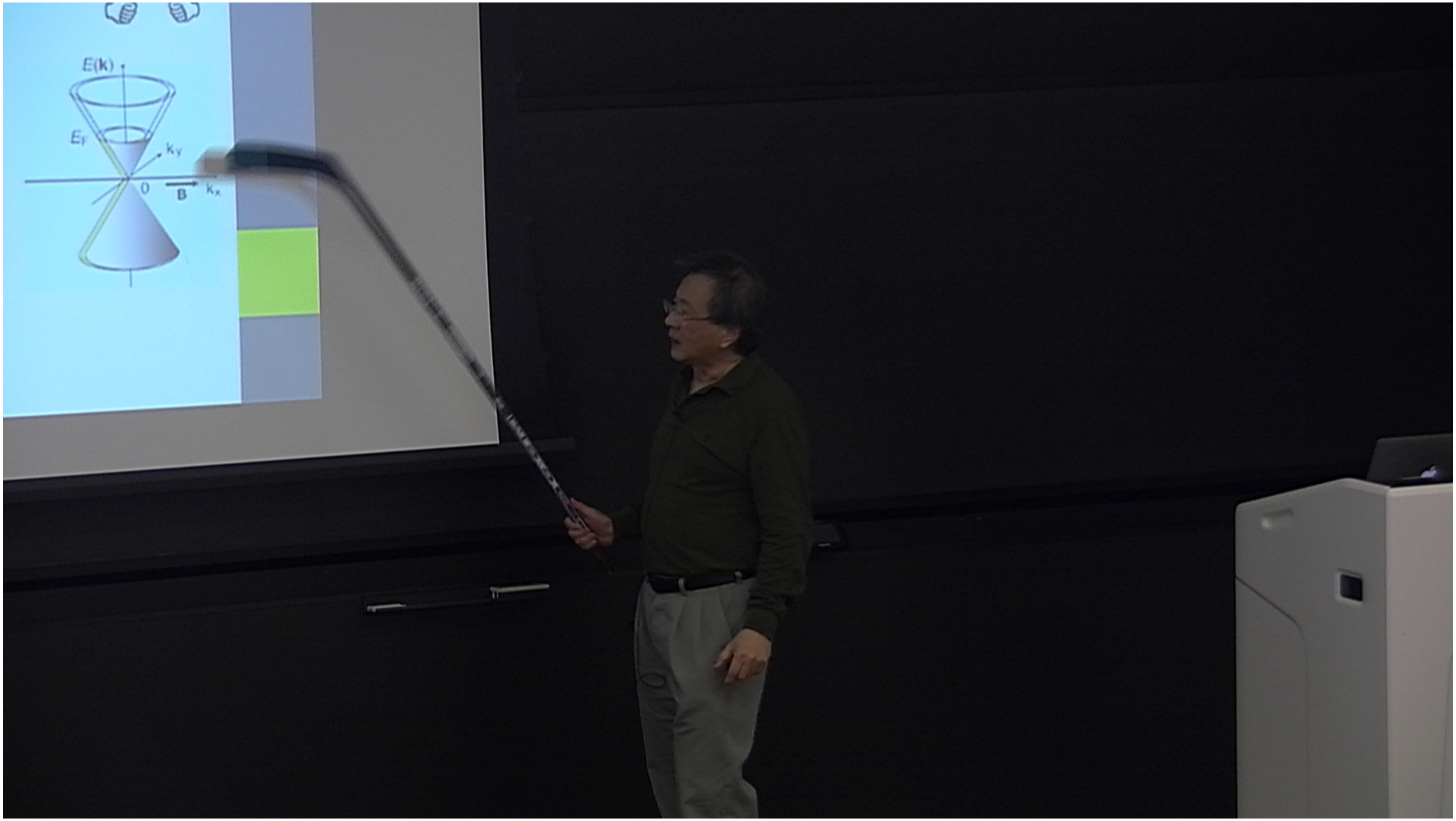


If m = 0, the Dirac Hamiltonian describes two massless populations.

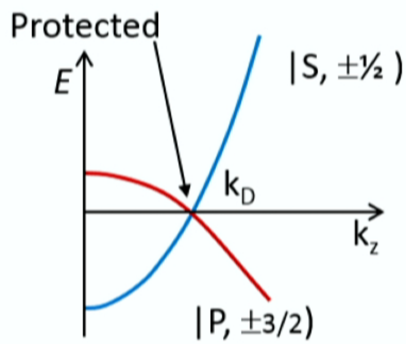
$$H = \begin{bmatrix} H_+ & 0 \\ 0 & H_- \end{bmatrix}$$

i) Each has permanent handedness (chirality = ± 1)
ii) Opposites do not mix (chiral symmetry holds)




E(k)
E_F
k_y
0
B
k_x

Dirac cone resolves into two Weyl nodes with opposite chiralities $\chi = \pm 1$



The low-E Hamiltonian, close to node $\mathbf{K}_+$, reduces to

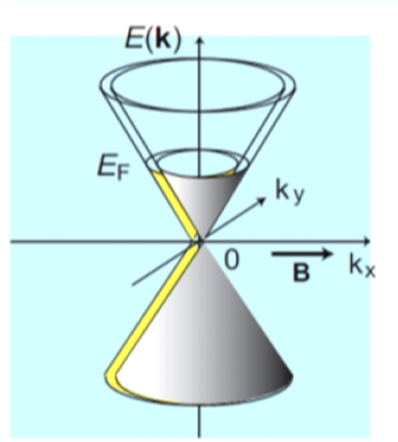
$$H = v \begin{bmatrix} k_z & k_+ & 0 & 0 \\ k_- & -k_z & 0 & 0 \\ 0 & 0 & k_z & -k_- \\ 0 & 0 & -k_+ & -k_z \end{bmatrix} \begin{pmatrix} S, ½ \\ P, 3/2 \\ S, -½ \\ P, -3/2 \end{pmatrix}$$

H resolves into two 2x2 Weyl Hamiltonians H_1, H_2

Calculate chirality from velocity matrix $\tilde{v}$

$$H_1 = \mathbf{k} \cdot \tilde{\mathbf{v}}_1 \cdot \boldsymbol{\tau} = \mathrm{v}(k_x \tau_1 - k_y \tau_2 + k_z \tau_3)$$

$$\tilde{\mathbf{v}}_1 = v \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



The chirality is $\chi = \dfrac{\det[\tilde{\mathbf{v}}]}{\mathrm{v}}$

$\chi_1 = -1, \qquad \chi_2 = +1$

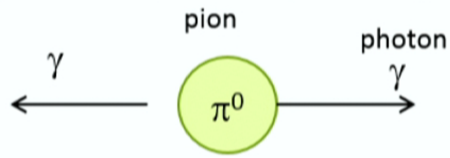
We have a superposition of two Weyl nodes at $B = 0$

Chiral anomaly in pion decay and beyond

The Adler Bell Jackiw (or chiral, axial) anomaly – 300 million factor

An anomaly in QFT is the breaking of a classically allowed symmetry by quantum effects.

First appeared in pion decay -- discrepancy of 300 million between neutral and charged pions

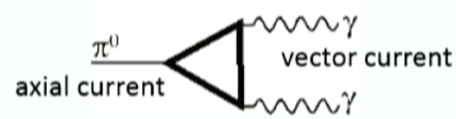


Pions, the lightest hadrons, are long-lived.
Charged pions can decay only into leptons $\pi^+ \rightarrow \mu^+ + \nu$

However, *neutral* pions can decay into 2 photons (3x10^8 faster!)

$$\pi^0 \rightarrow \gamma + \gamma$$

(Adler, Bell, Jackiw, 1969)[1]

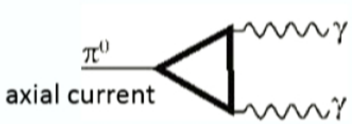


Adler Bell Jackiw anomaly

Coupling to EM field **globally** breaks chiral symmetry of pions
Leads to decay of axial current into photons

1. Axial Anomaly, R. Jackiw, *Scholarpedia*

Chiral anomaly as spoiler (*local* chiral symmetry breaking)



$$A = \frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}\, F_{\mu\nu}F_{\alpha\beta}$$

Spoiler role and Anomaly-free condition (local chiral symmetry breaking)

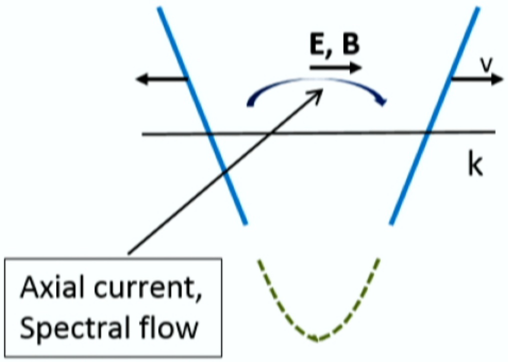
All the anomalies must cancel for a theory to be renormalizable.

In Glashow-Weinberg-Salam theory, exact cancellation of all anomalies in each generation of quarks and leptons has been called "magical" (Peskin[1]).

Triangle diagram has no further corrections to infinite order in perturb theory (Adler Bardeen)

1. *Intro to QFT*, Peskin Schroeder
2. *Anomalies in QFT,* Bertlmann
3. *Geometry* ... , Nakahara

Landau quantized



Adler Bell Jackiw anomaly

$$A = \frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu}F_{\alpha\beta}$$

B quantizes Dirac states into Landau levels

Rate at which charge is pumped in **E** field

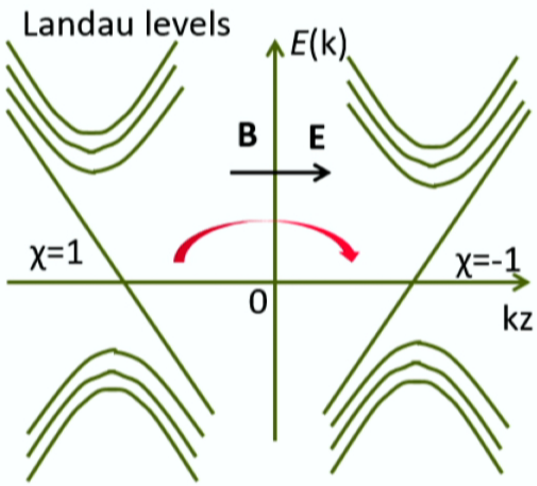
$$A = -\left(\frac{L^2}{2\pi\ell_B^2}\right)\left(\frac{Le\dot{k}_z}{2\pi}\right) = -V\frac{e^3}{4\pi^2\hbar^2}\mathbf{E}.\mathbf{B}$$

DOS of one Landau level

Rate of increase of states along k_z in **E** field

Chiral anomaly is observable as a large, negative longitudinal magnetoresistance (Nielsen and Ninomiya, *Phys. Lett.* 1983)

Charge pumping and the chiral anomaly



Nielsen, Ninomiya, *Phys. Lett.* 1983
Wan, Turner, Vishwanath, *PRB* 2011
Burkov, Hook Balents, *PRB* 2011
Son, Spivak, *PRB* 2013
Parameswaran et al. *PRX* 2014

Chiral anomaly engenders large, negative longitudinal MR
Locked to B field

In large-*B* regime, with **E**||**B,** charge is pumped between Weyl nodes at the rate

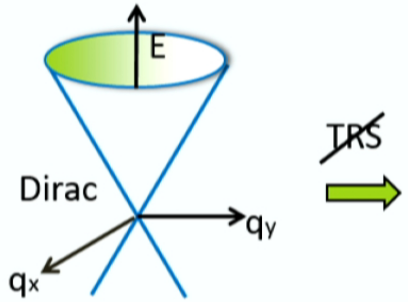
$$A = -\frac{L^2}{2\pi\ell_B^2}\frac{Le\dot{k}}{2\pi} = -V\frac{e^3}{4\pi^2\hbar^2}\mathbf{E}.\mathbf{B}$$

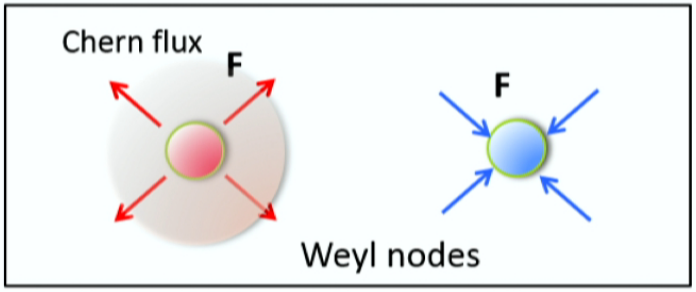
In weak *B*, charge pumping gives (Son and Spivak, *PRB* 2013)

$$\sigma_\chi = \frac{e^2}{4\pi^2\hbar c}\frac{v}{c}\frac{(eBv)^2}{\epsilon_F^2}\tau_a$$

τ_a is relaxation time for pumped current

Berry curvature of Weyl nodes





$$\mathbf{A} = -i\langle u_{\mathbf{k}}|\nabla_{\mathbf{k}}|u_{\mathbf{k}}\rangle$$

Berry curvature $\mathbf{\Omega(k)} = \nabla \times \mathbf{A(k)}$

Chirality $\chi = \frac{1}{2\pi}\oint \mathbf{\Omega} \cdot \mathbf{dS(k)}$

Proposed (2011) existence of Weyl nodes in iridates (Vishwanath et al.) reawakened strong interest in the Nielsen Ninomiya prediction.

Wan, Turner, Vishwanath, *PRB* 2011
Burkov, Hook, Balents, *PRB* 2011
Son, Spivak, *PRB* 2013

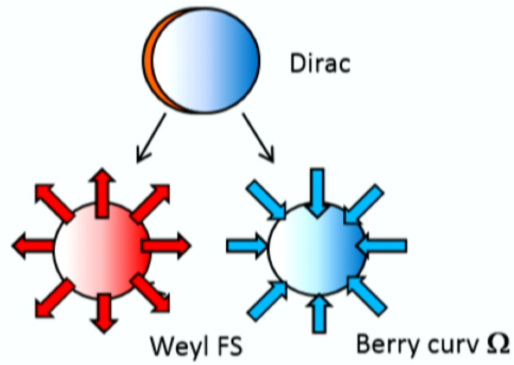
... and 80+ theory uploads on arXiv

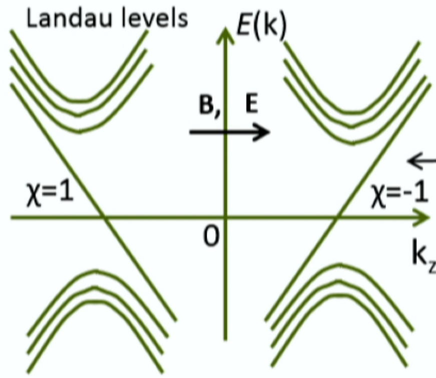
Experiments on Dirac-Weyl semimetals

Chiral anomaly in Na_3Bi

Creation of Weyl states in applied magnetic field

B field





Lowest Landau level
Schwinger model

Detect axial current

$$A = -\left(\frac{L^2}{2\pi\ell_B^2}\right)\left(\frac{Le\dot{k}_z}{2\pi}\right) = -V\frac{e^3}{4\pi^2\hbar^2}\mathbf{E}.\mathbf{B}$$

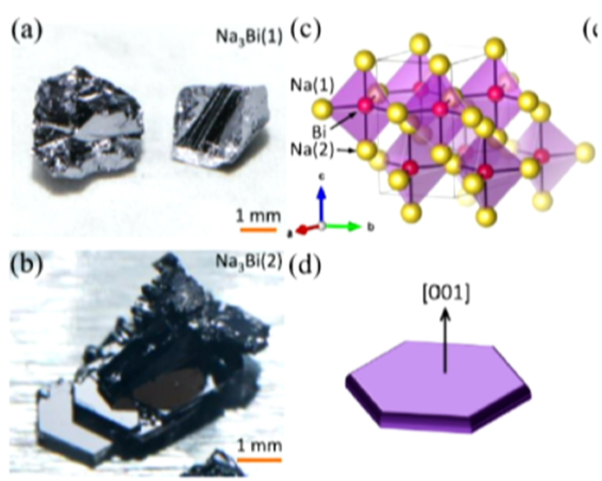
Bulk crystal growth and electronic characterization of the 3D Dirac semimetal Na_3Bi

S. Kushwaha, J. Krizan … **Yazdani**, **Ong** and **Cava**, *APL Materials* 2015


S. Kushwaha


J. Krizan


Cava



Deep purple crystals, that rapidly oxidize in ambient air (30 s)

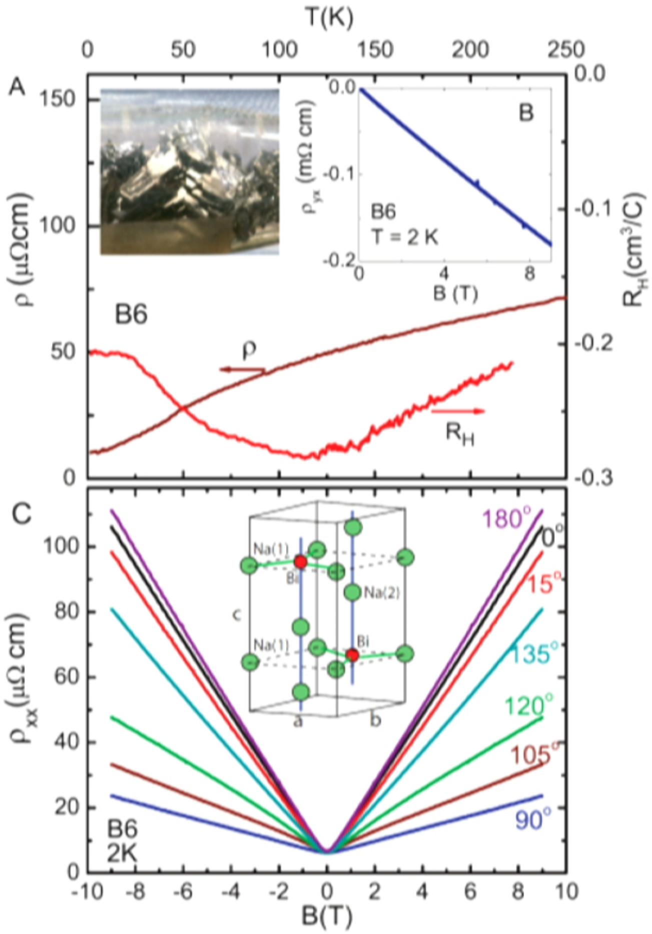
Initial growth produced highly metallic crystals, but no anomaly

Initial results on Na_3Bi

S. Kushwaha *et al.*, *APL* 2015

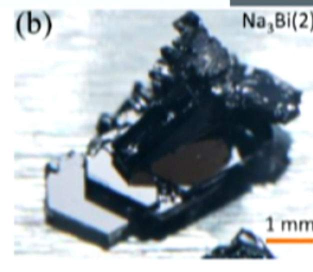
Jun Xiong *et al.*, submitd

Jun Xiong Kushwaha Krizan



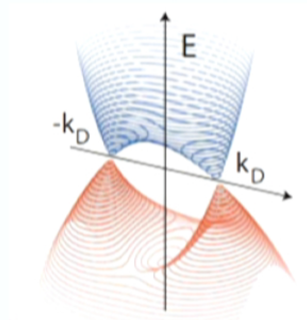
Deep purple crystals

Rapidly oxidizes in ambient air (30 s)

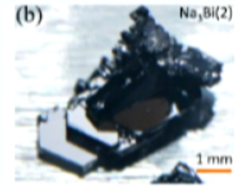


Large linear MR similar to Set B Cd_3As_2 samples

E_F 400 mV above node



Non-metallic Crystals of Na_3Bi with lower carrier density

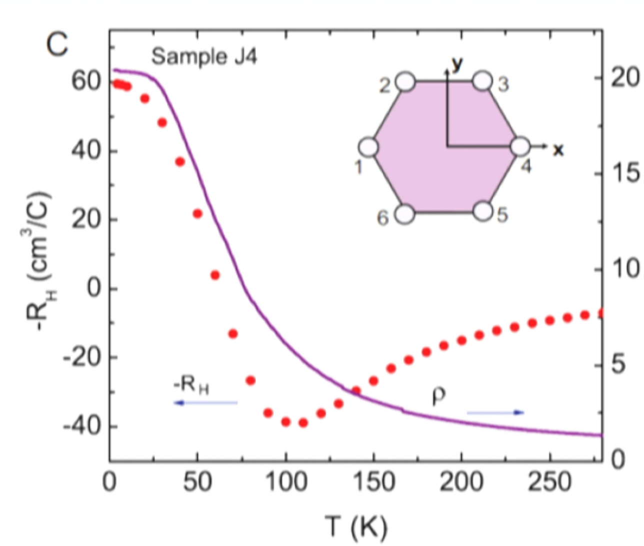


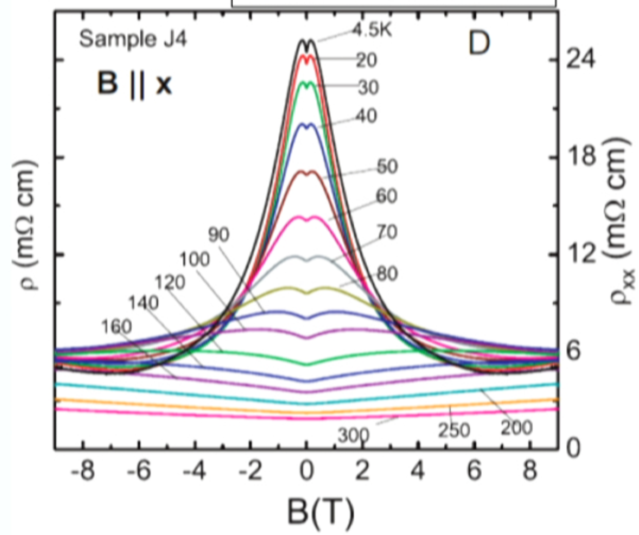
Jun Xiong, S. Kushwaha et al., Science 2015

Jun Xiong, S. Kushwaha, Krizan,

Long-term annealed crystals with E_F much closer to node

$$A = -V \frac{e^3}{4\pi^2\hbar^2} \mathbf{E}.\mathbf{B}$$





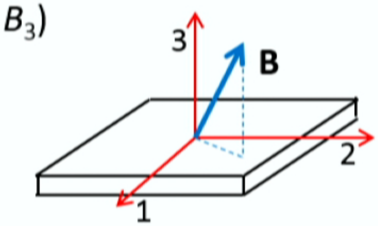
Fermi energy lies 30 meV above node

Striking negative longitud. MR (LMR)

Longitudinal MR is rare in conventional metals

A conventional 1-band model does not lead to MR, regardless of band anisotropy.

Standard Boltzmann equation with anisotropic mass tensor (elliptical FS) in *arbitrarily large*, tilted, field **B** = (B_1, B_2, B_3)



$$e\mathbf{E}\cdot\mathbf{v}\frac{\partial f^0_{\mathbf{k}}}{\partial\epsilon_{\mathbf{k}}} + e\mathbf{v}\times\mathbf{B}\cdot\frac{\partial g_{\mathbf{k}}}{\partial\mathbf{k}} = -\frac{g_{\mathbf{k}}}{\tau},$$

$$\hat{m} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

Solution for resistivity tensor

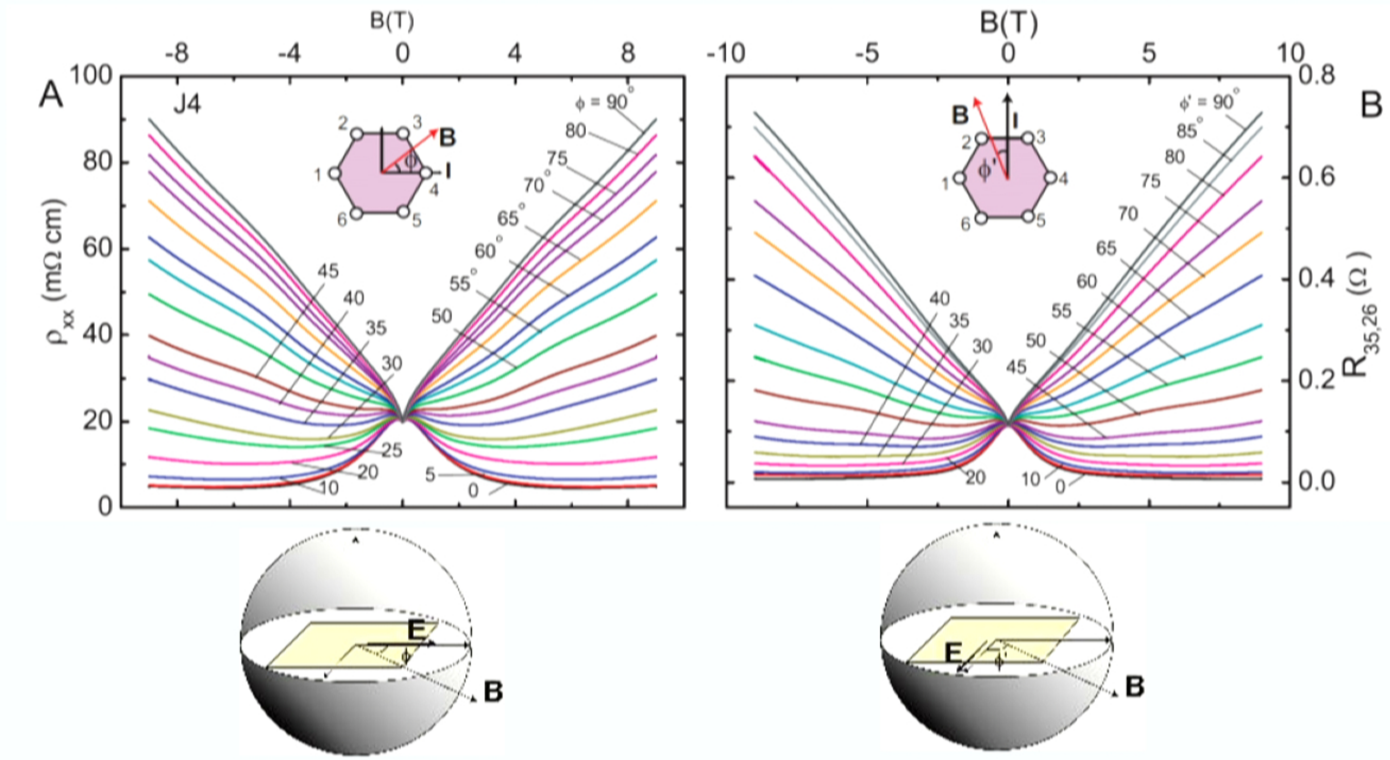
$$\hat{\rho} = \begin{bmatrix} \rho_1 & -B_3/ne & 0 \\ B_3/ne & \rho_2 & -B_1/ne \\ 0 & B_1/ne & \rho_3 \end{bmatrix}$$

$$\rho_i = \frac{m_i}{ne^2\tau_0}$$

Diagonal elements are independent of **B** → absence of MR

A test for the chiral anomaly -- **B** is locked to **E**

Jun Xiong, S. Kushwaha et al., submitted



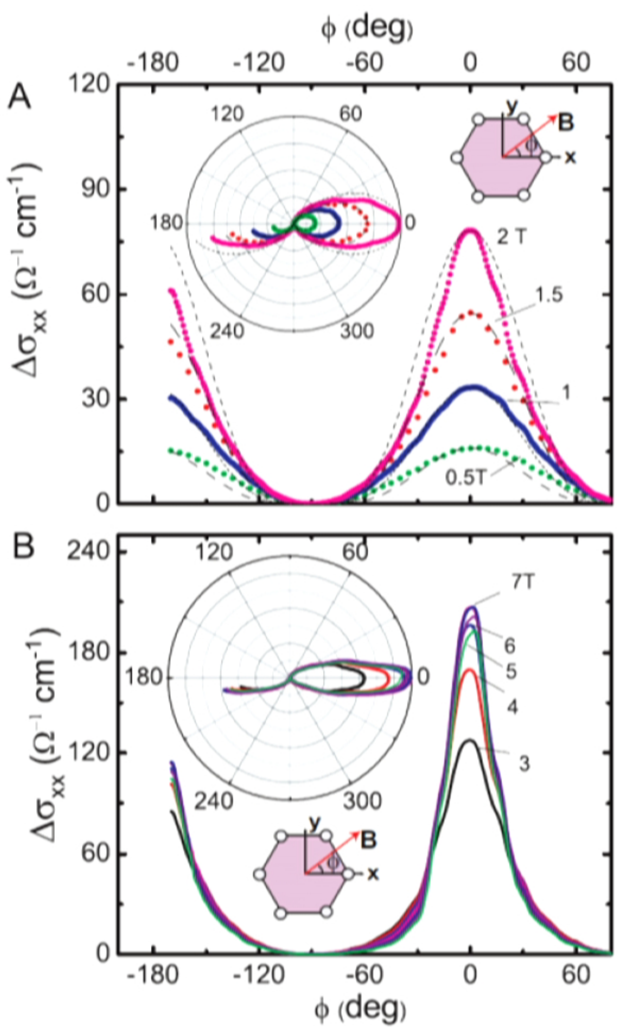
Negative MR appears only when **B** is locked to **E.**

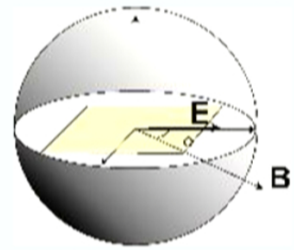
Test: if **E** is rotated by 90^{o} (right panel), neg. MR shifts to new direction of **E.**

For weak B, this locking is novel and unexpected in semiclasscl transport

A narrow plume of chiral current, B in-plane

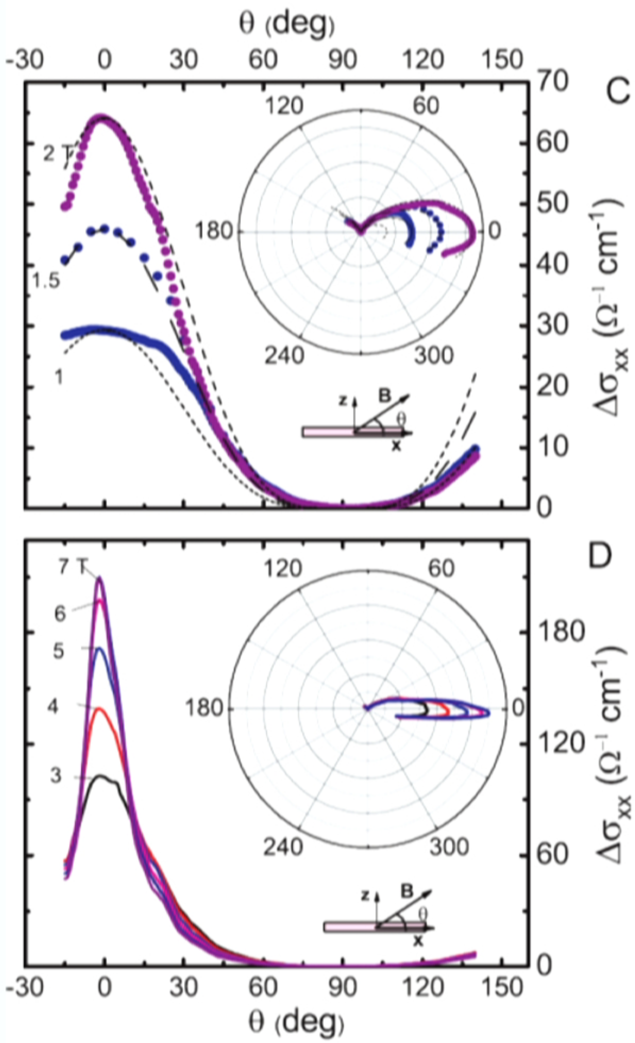
Jun Xiong, S. Kushwaha et al., submitted

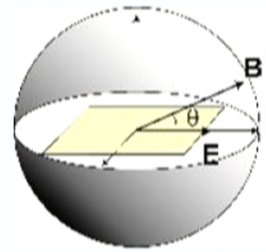




Enhanced cond. in a narrowly collimated beam for **B** in the *x*-*y* (horizontal) plane

Width of chiral conductivity "plume", B normal to plane





Enhanced cond. in a narrowly collimated beam for **B** rotated in the *x-z* (vertical) plane

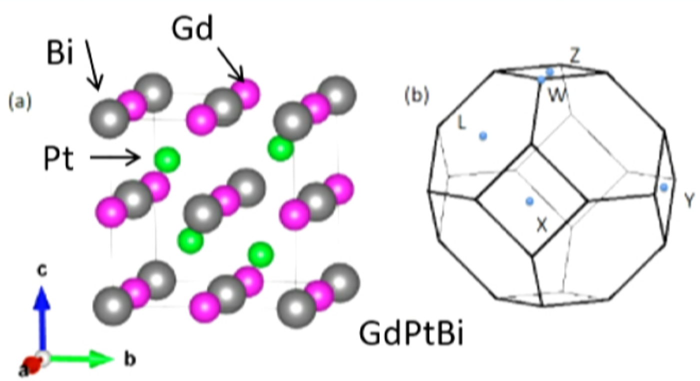
Chiral anomaly in half Heusler GdPtBi

The chiral anomaly and thermopower of Weyl fermions in the half-Heusler GdPtBi

Max Hirschberger, S. Kushwaha, ZJ Wang, Quinn Gibson, C. Belvin, B. A. Bernevig, R. J. Cava and N. P. O

Nature Materials, online Jun 2016

M. Hirschberger S. Kushwaha Zhijun Wang



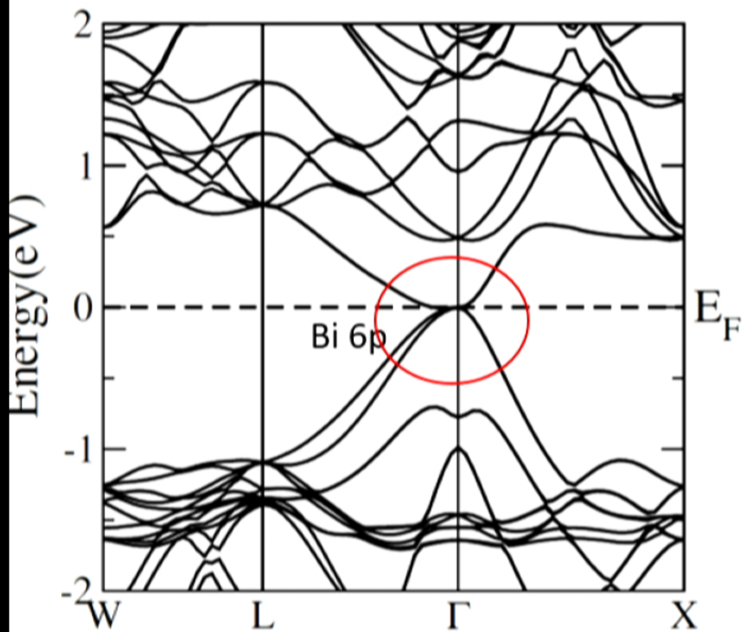
Zinc blende structure (like GaAs)

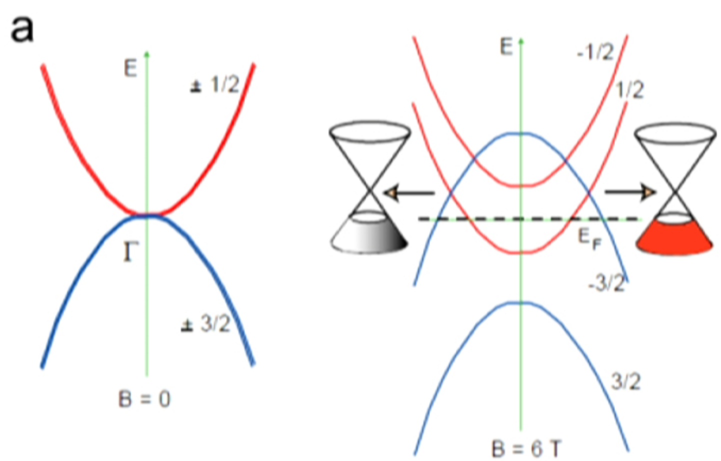
Except Bi is "stuffed" in fcc sublattice

See also, Checkelsky *Nat. Mat.* 2016

LDA calculations – effect of **B** on bands in GdPtBi

Zhijun Wang

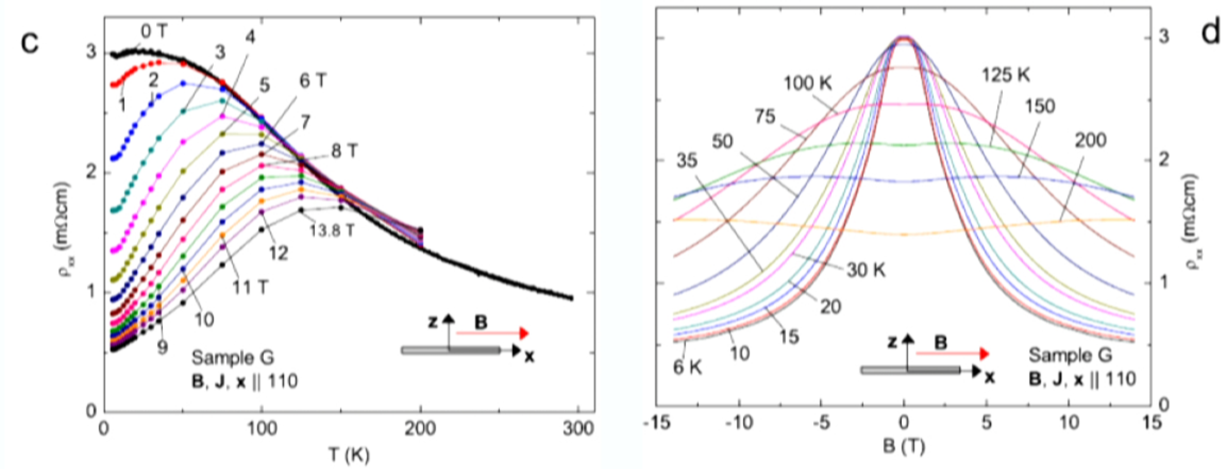




Low-lying states (4) are derived from Bi 6p
Quadratic bands touch at Γ to form zero gap
Large spin orbit coupling

In finite B, large Zeeman field lifts degeneracies
Leads to creation of two Weyl nodes

Evidence for chiral anomaly in GdPtBi



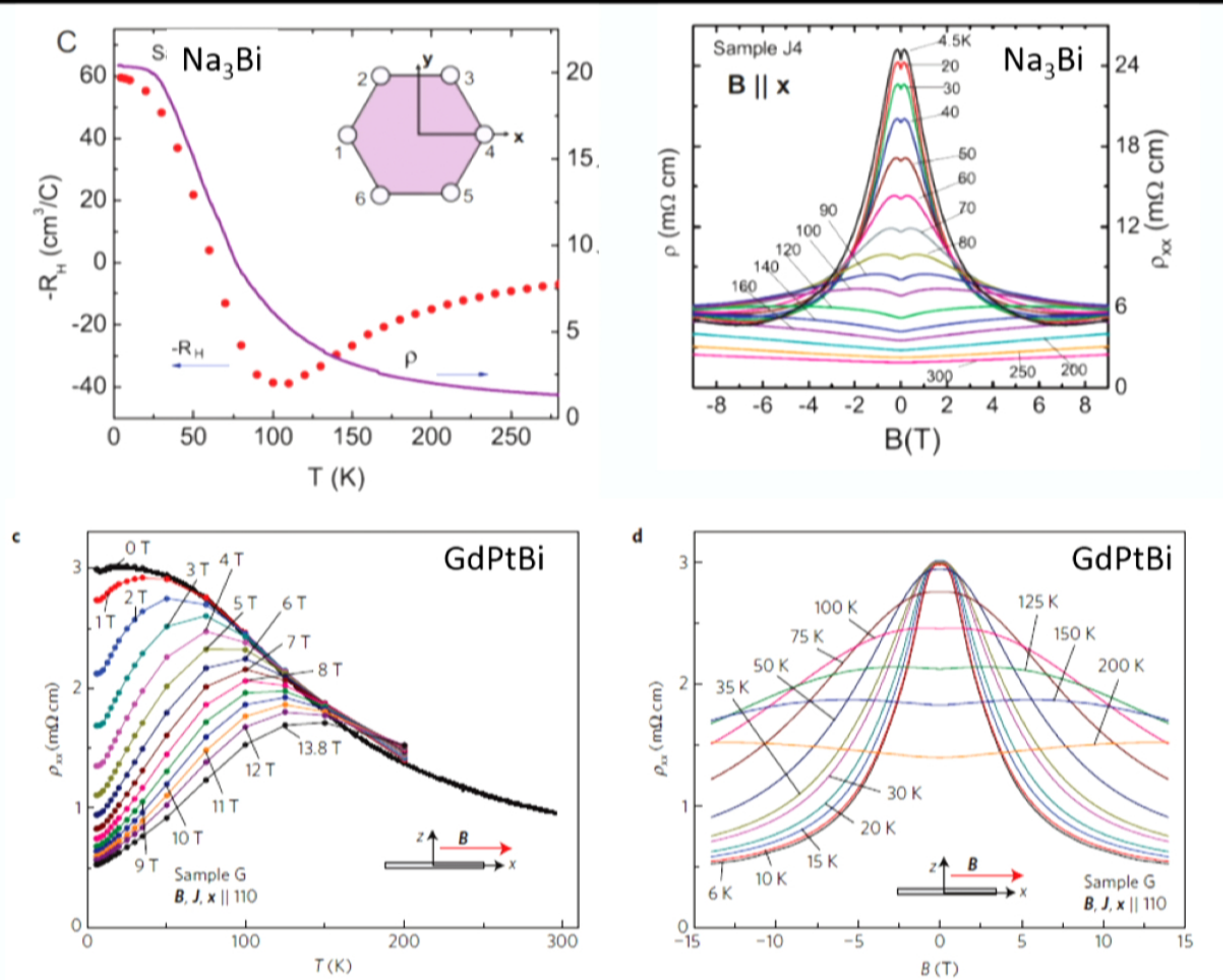
Longitudinal resistivity ρ_{xx} vs. *T* at selected *B*

Large suppression of ρ_{xx} at low *T* and large *B*

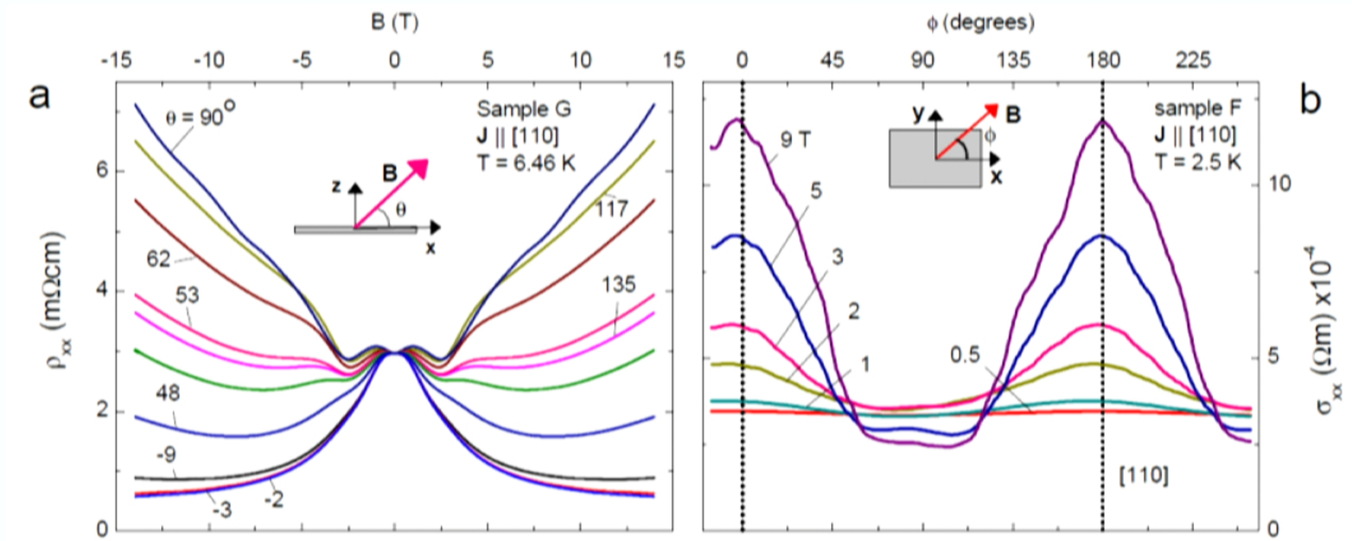
MR profile shows large suppression of ρ_{xx} when *B* exceeds ~3 T

Comparison with Na_3Bi suggests existence of chiral anomaly

Resemblance between long. MR in Na_3Bi and GdPtBi

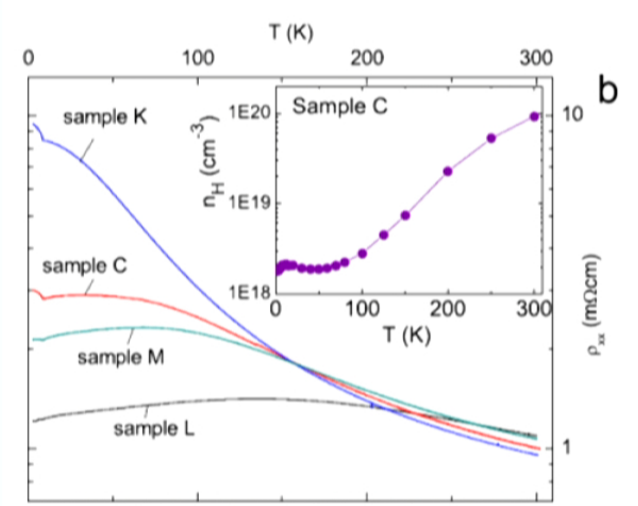


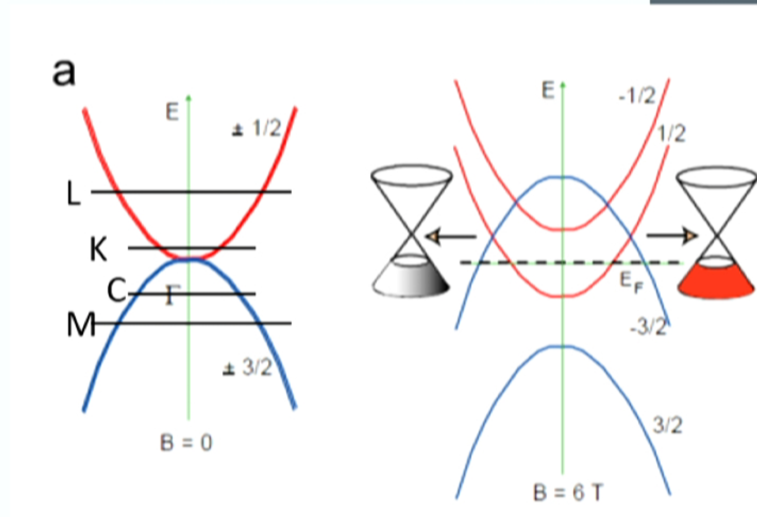
Angular dependence of current plume



Axial current plume is largest when **B** approaches alignment with **J**

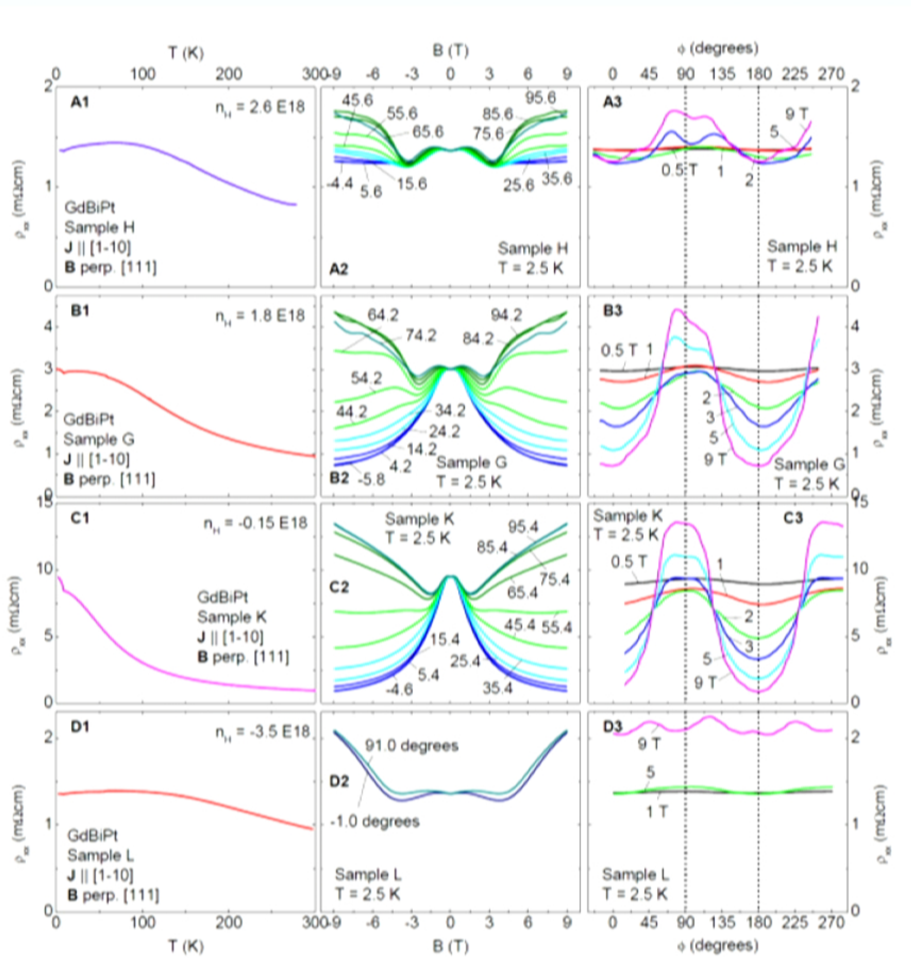
Dependence of resistivity profile on distance of E_F from node





Resistivity profiles are most non-metallic close to node

Dependence of chiral anomaly on distance of E_F from node



Maximum anomaly ampl. when closest to node

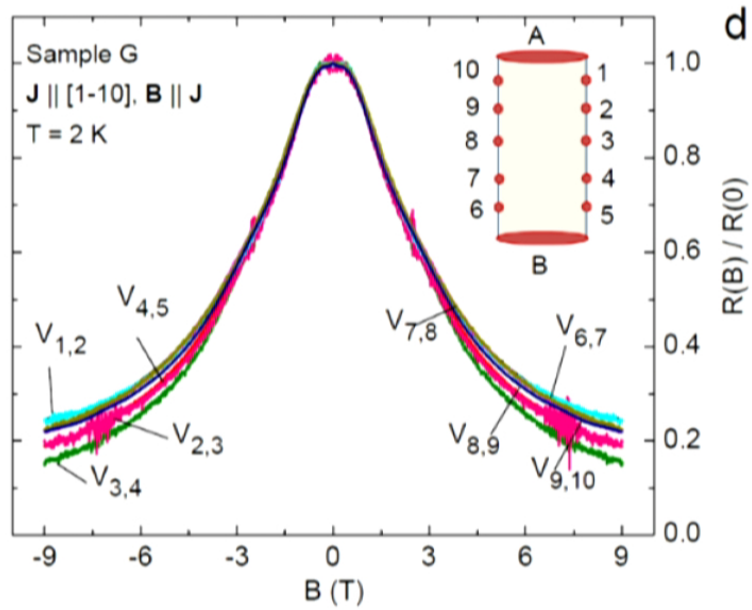
Hole density $+2.6 \times 10^{18}$

Hole density $+1.8 \times 10^{18}$

Electron density -0.15×10^{18}

Electron density -3.5×10^{18}

Check for uniformity of current density



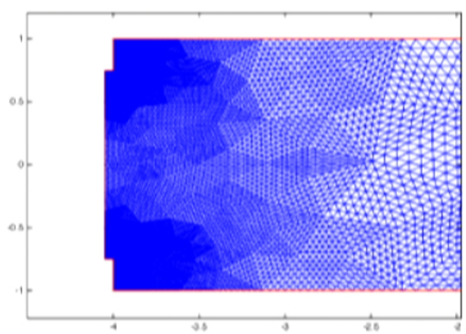
Repeat measmt. on Sample G with 10 voltage contacts
Longitud. MR profiles plotted as relative change are closely similar across all 8 nearest neighbor pairs of contacts

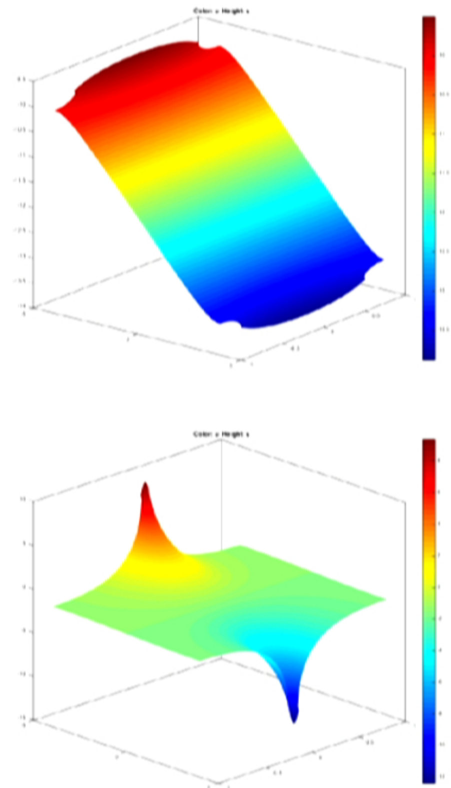
Conclusion: Negative longitude. MR is an intrinsic electronic effect, not a spurious result of inhomogeneity.

"Current jetting"

Current jetting can produce spurious, negative longitud. MR but only in high mobility samples in intense B (μB>>1)

$$[\partial_x \sigma_{xx} \partial_x + \partial_y \sigma_{yy} \partial_y] \psi(x, y) = 0.$$

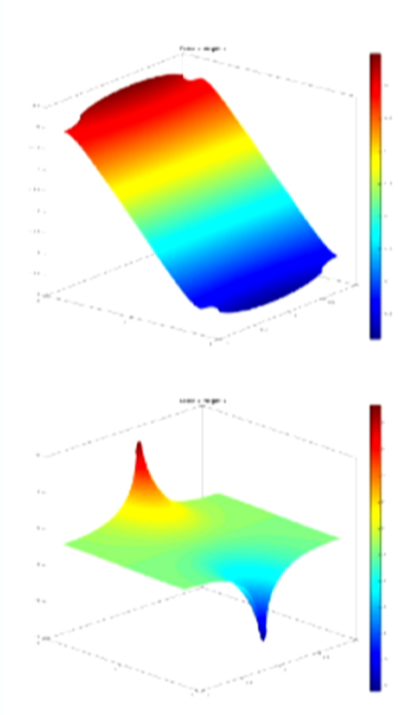


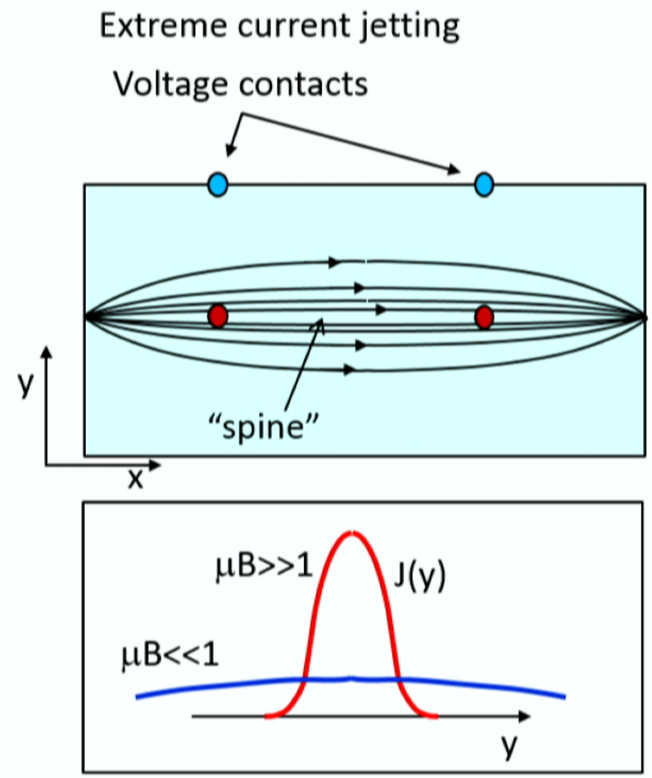


Numerically calculate potential function ψ(x,y) with Drude conductivity tensor in **B** ‖ **E**.
Current jetting is unimportant for small μB and broad current contacts (upper panel). However, for point contacts and large μB (lower panel), get current focusing and jetting (imitates very large contact resistance).

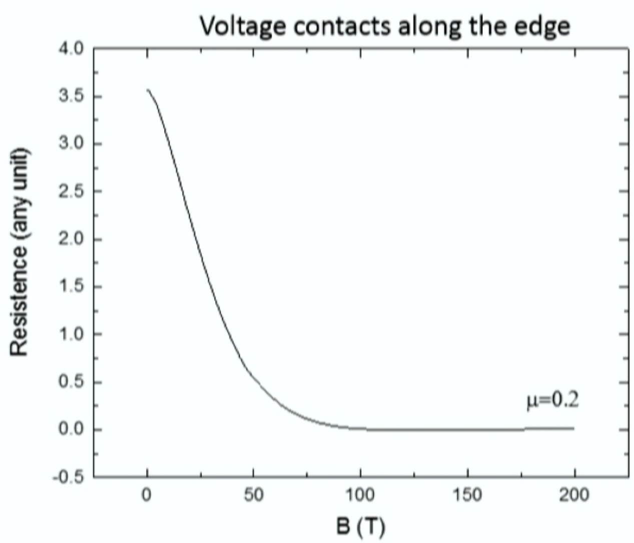
The “squeeze” test

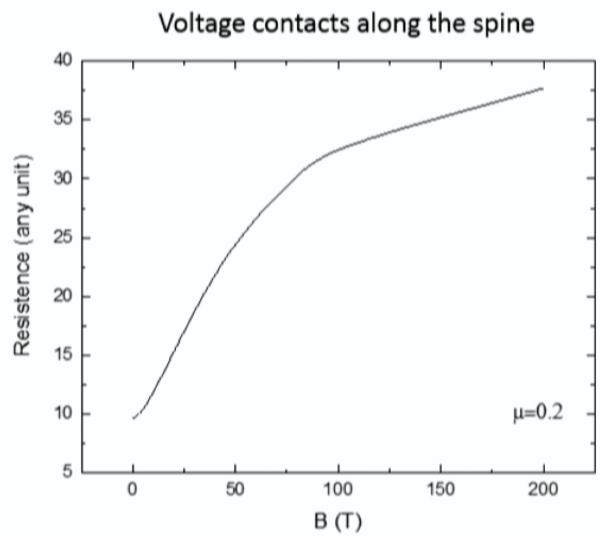
Chiral anomaly effect: Voltage decreases with B along spine and side
Pure current jetting: Voltage decr along side, **increases** along spine



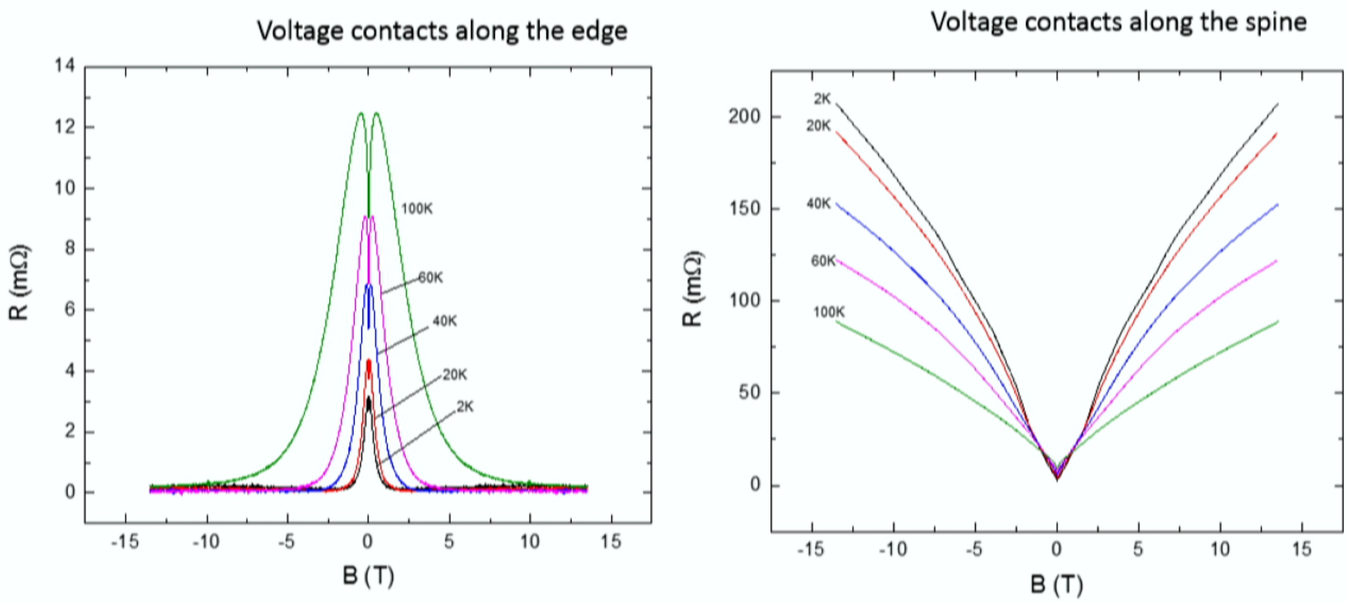


Numerical 2D simulation of squeeze test for pure current jetting





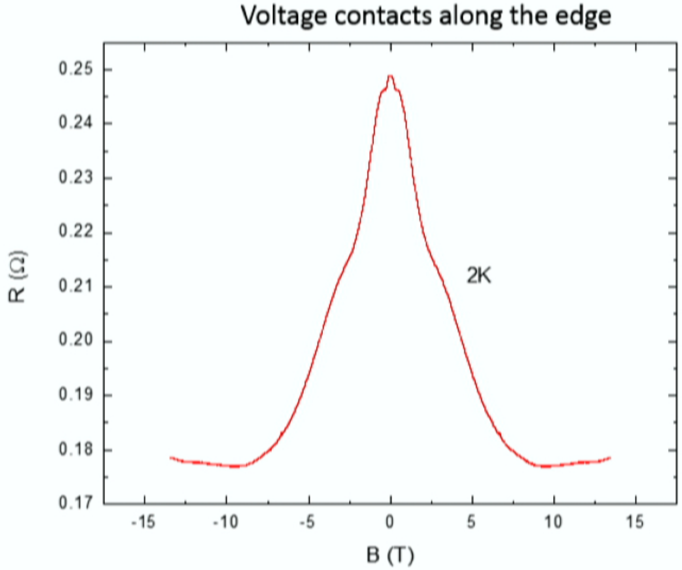
Squeeze test on high purity elemental bismuth

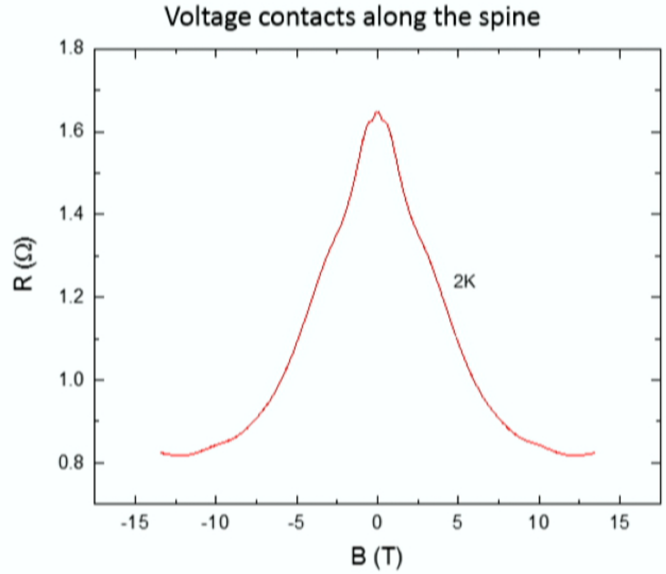


Negative LMR caused by extreme current jetting

The "squeeze" test applied to GdPtBi

Compare MR measured with spine and edge contacts in GdPtBi

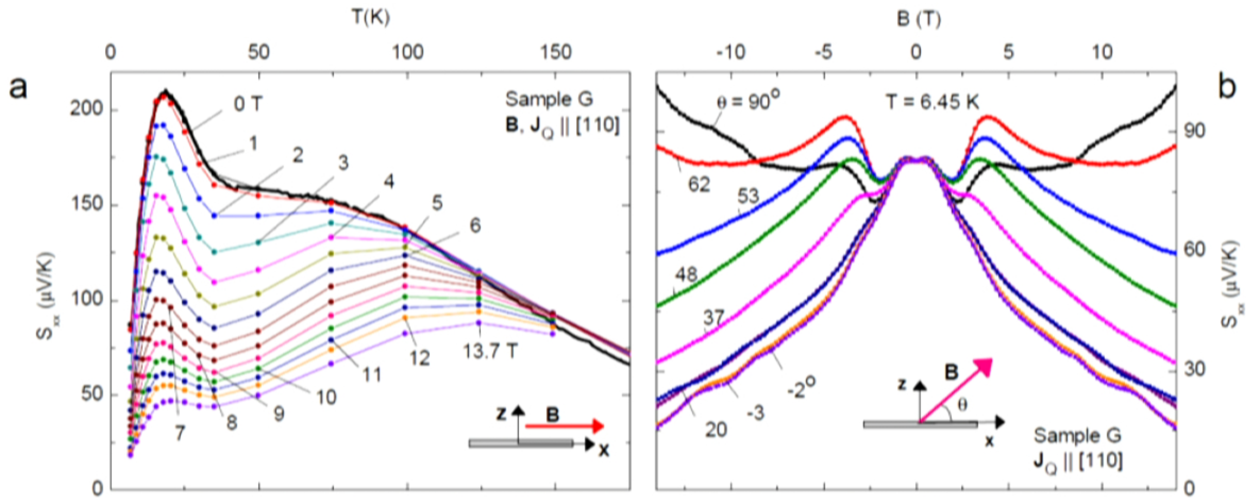




Negative LMR is intrinsic and uniform → GdPtBi

However, tests on TaAs and NbP fail (to date)

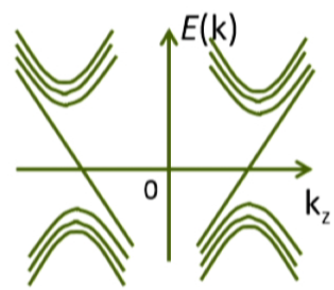
Thermopower of Weyl fermions in GdPtBi



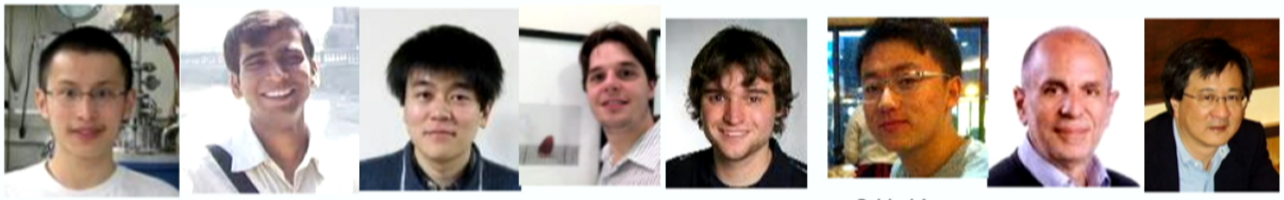
Thermoelectric response is strongly suppressed when axial current appears.

A consequence of chiral n=0 Landau level?

Reflects the “flat” DOS vs. E in the lowest Landau level



End



Jun Xiong Kushwaha Tian Liang Jason Krizan Hirschberger S.H. Liang Bob Cava NP Ong

Thank you