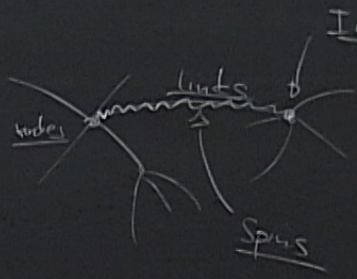
Title: PSI 2016/2017 Explorations in Quantum Gravity - Lecture 15

Date: Apr 07, 2017 10:15 AM

URL: http://pirsa.org/17040019

Abstract:

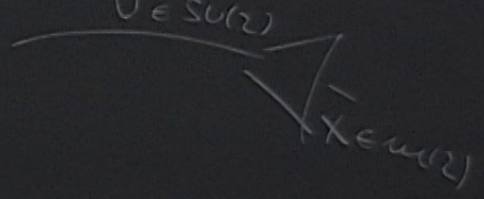
Spinors in LQG



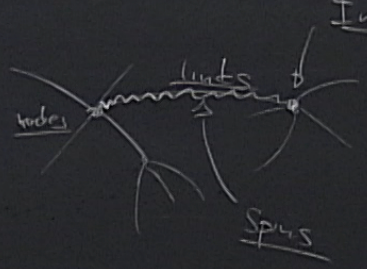
$\bar{\Psi}_{\underline{j},\underline{i},\Gamma}$

$[T^*SU(2)]^{\#\,\text{links}}$

$T^*SL(2,\mathbb{C})$



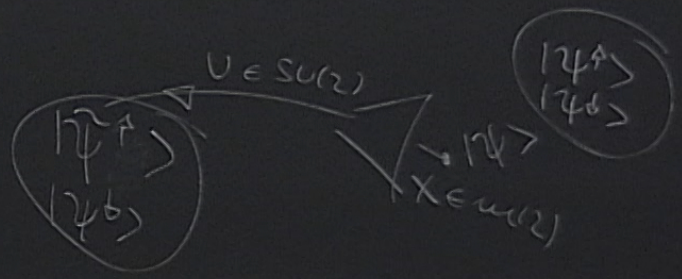
Spinors in LQG

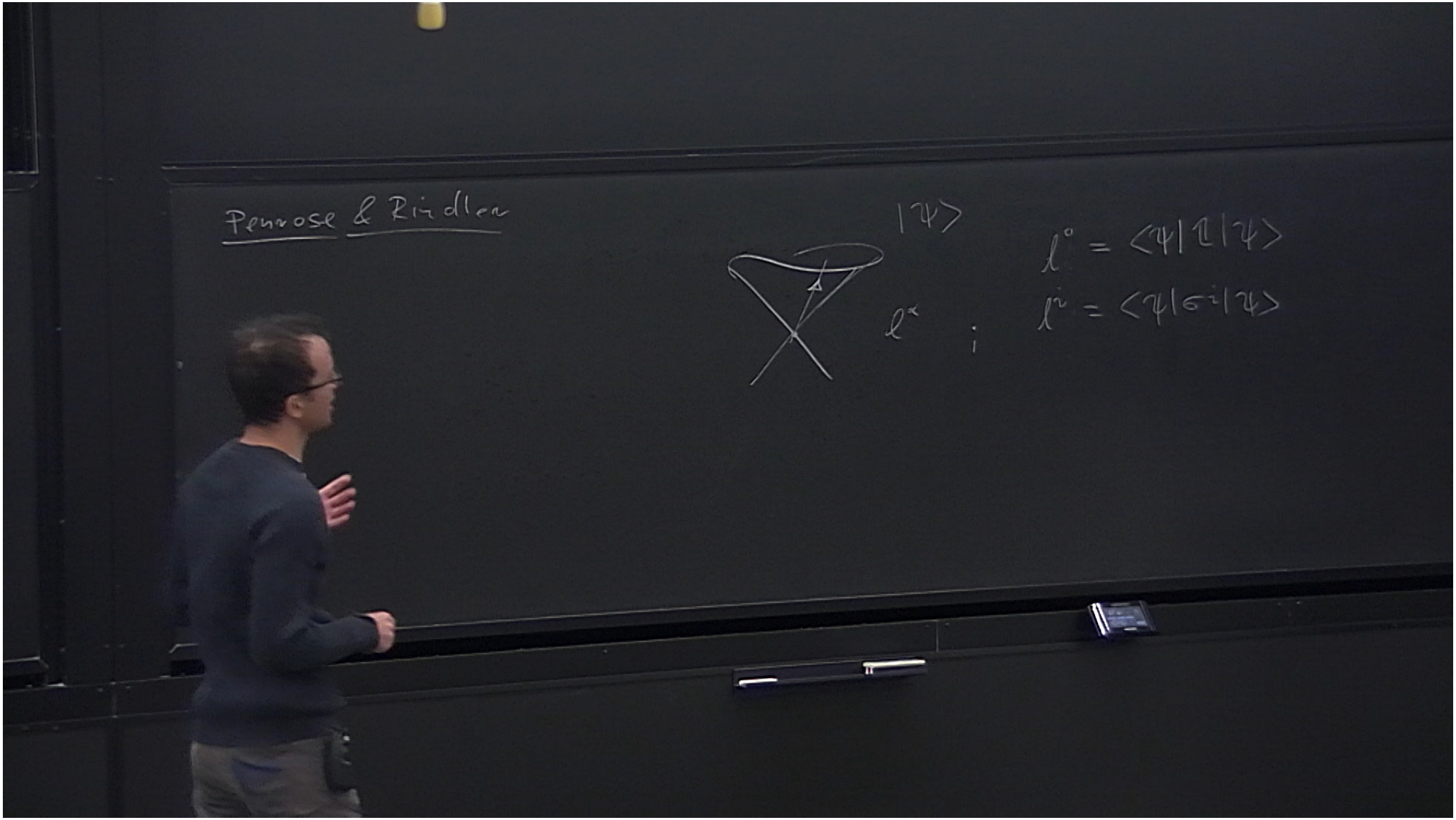


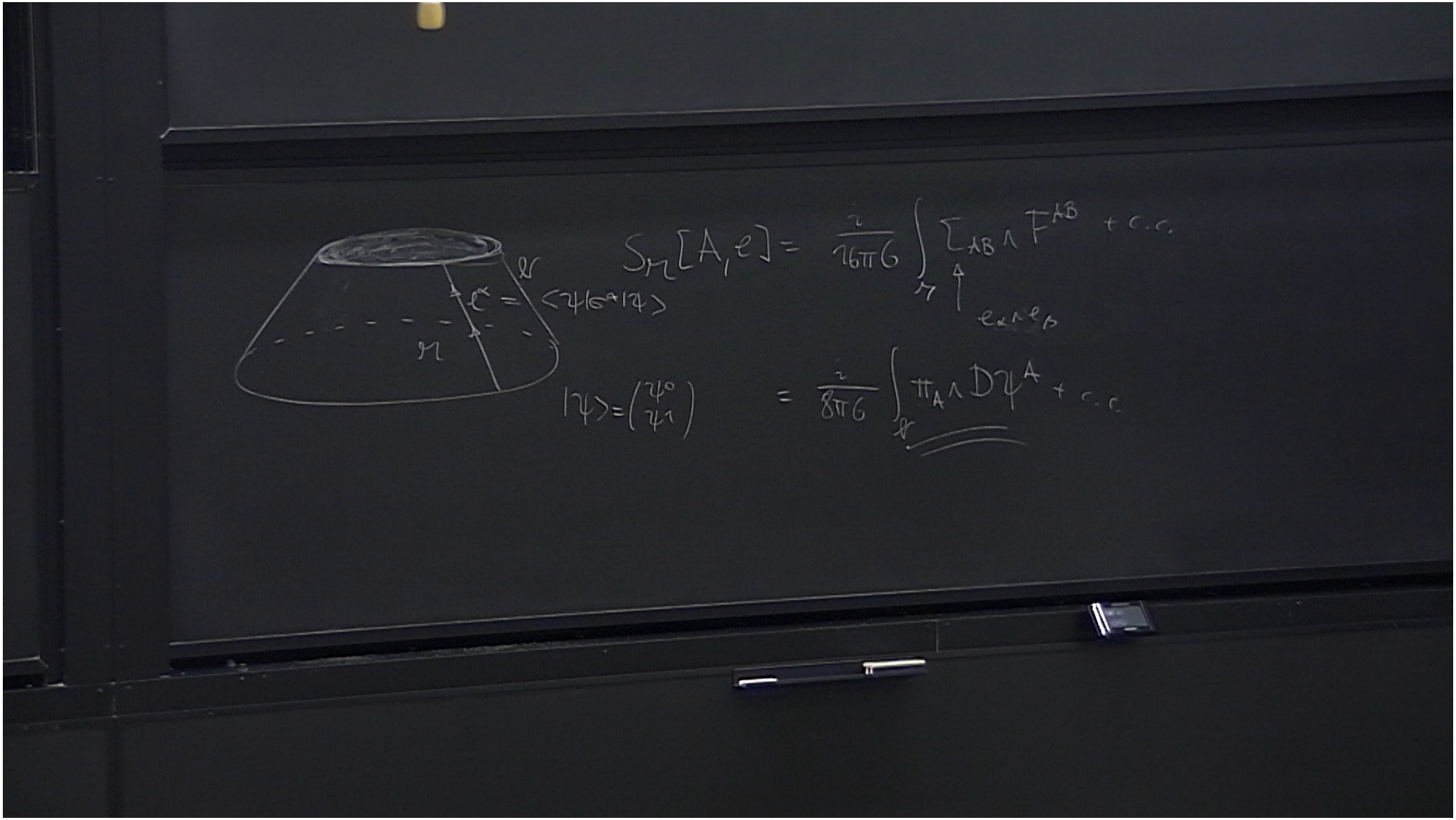
$\Psi_{j_1, j_2, \ldots}$

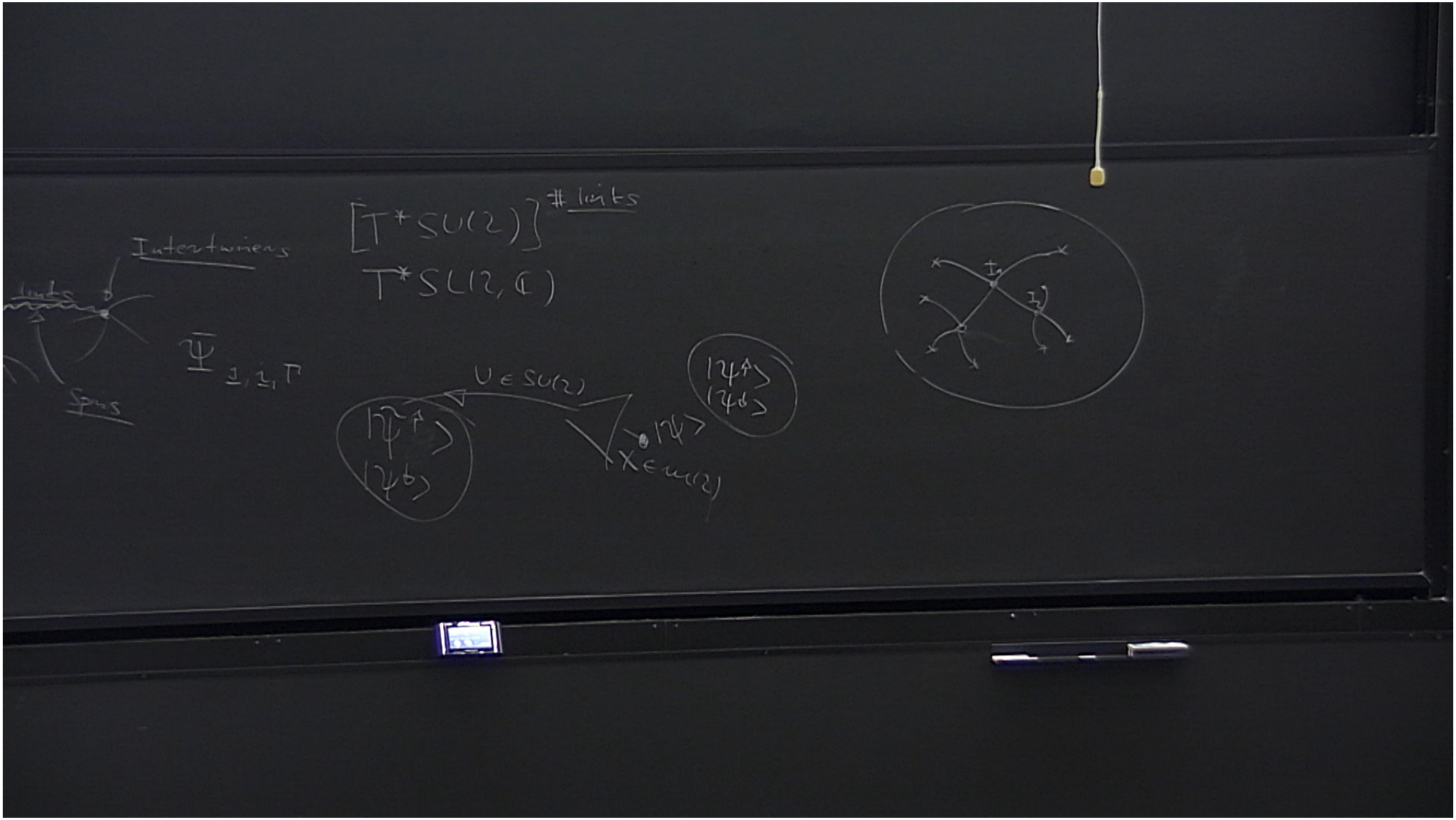
$[T^*SU(2)]^{\#\text{ links}}$

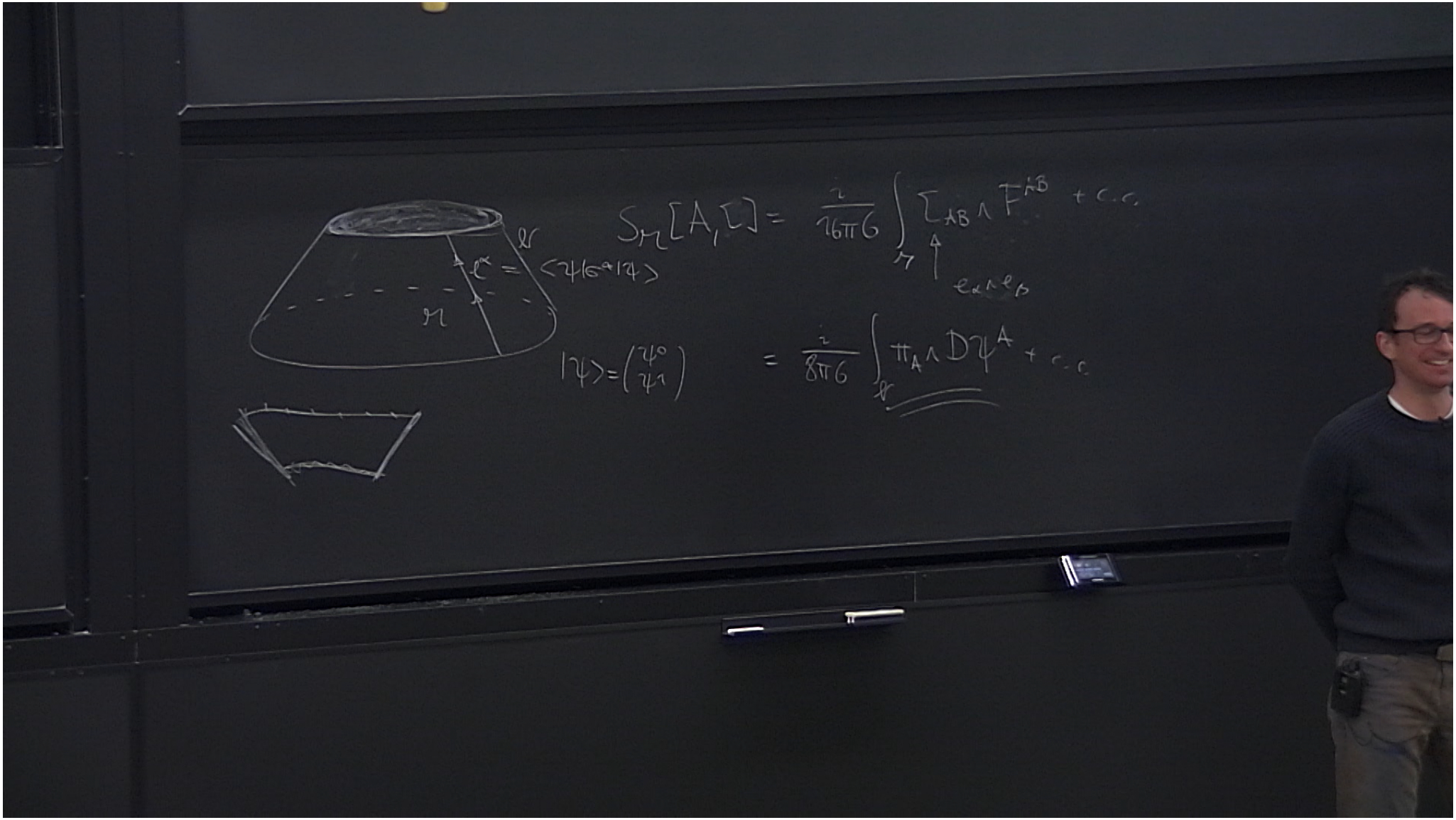
$T^*SU(2,\mathbb{C})$

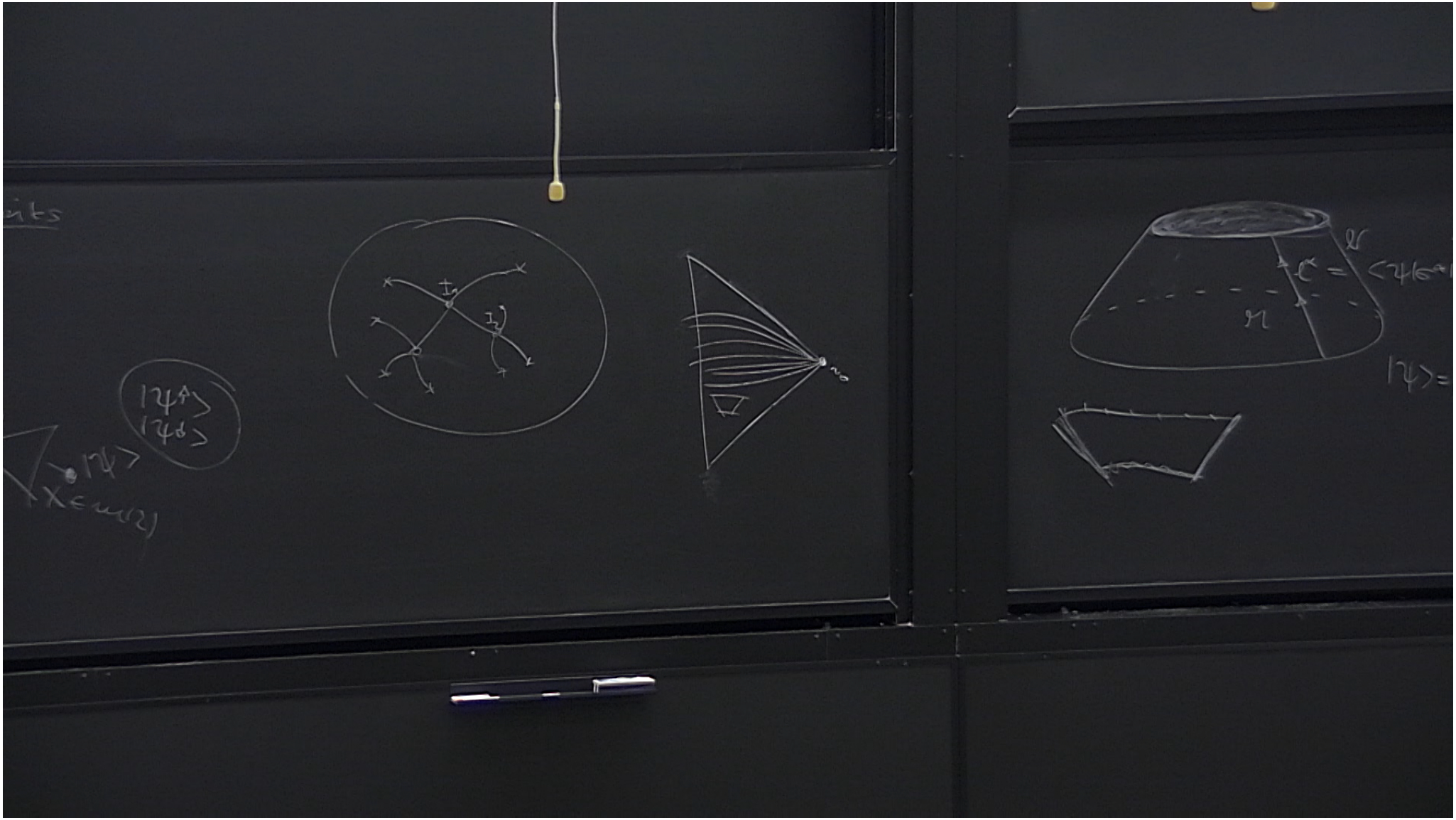



Penrose & Rindler
|ψ⟩
l^α ;
$l^0 = \langle\psi|\mathbb{1}|\psi\rangle$
$l^i = \langle\psi|\sigma^i|\psi\rangle$


$S_H[A,e] = \frac{2}{16\pi G}\int_M \Sigma_{AB}\wedge F^{AB} + c.c.$
$\Sigma_{AB} = e_A \wedge e_B$
$\ell^\alpha = \langle\psi|\sigma^\alpha|\psi\rangle$
M
$|\psi\rangle = \begin{pmatrix}\psi^0\\ \psi^1\end{pmatrix}$
$= \frac{2}{8\pi G}\int_M \pi_A \wedge D\psi^A + c.c.$


Intertwiners
links
spins
$\Psi_{\vec{\jmath},\vec{\iota},\Gamma}$
$[T^*SU(2)]^{\#\text{links}}$
$T^*SU(2,\mathbb{C})$
$U \in SU(2)$
$|\psi^A\rangle$
$|\psi^b\rangle$
$|\psi\rangle$
$X \in su(2)$


$S_M[A,\Sigma] = \frac{i}{16\pi G}\int_M \Sigma_{AB}\wedge F^{AB} + c.c.$
$\Sigma^{\alpha\beta} = \langle\chi|\sigma^{\alpha\beta}|\chi\rangle$
$e_\alpha \wedge e_\beta$
M
$|\chi\rangle = \begin{pmatrix}\chi^0\\ \chi^1\end{pmatrix}$
$= \frac{i}{8\pi G}\int_M \pi_A \wedge D\chi^A + c.c.$


$|\psi_A\rangle$
$|\psi_B\rangle$
$|\psi\rangle$
$\chi \in \omega(\mathcal{U})$
I_1
I_2
$|\psi\rangle =$

LQG

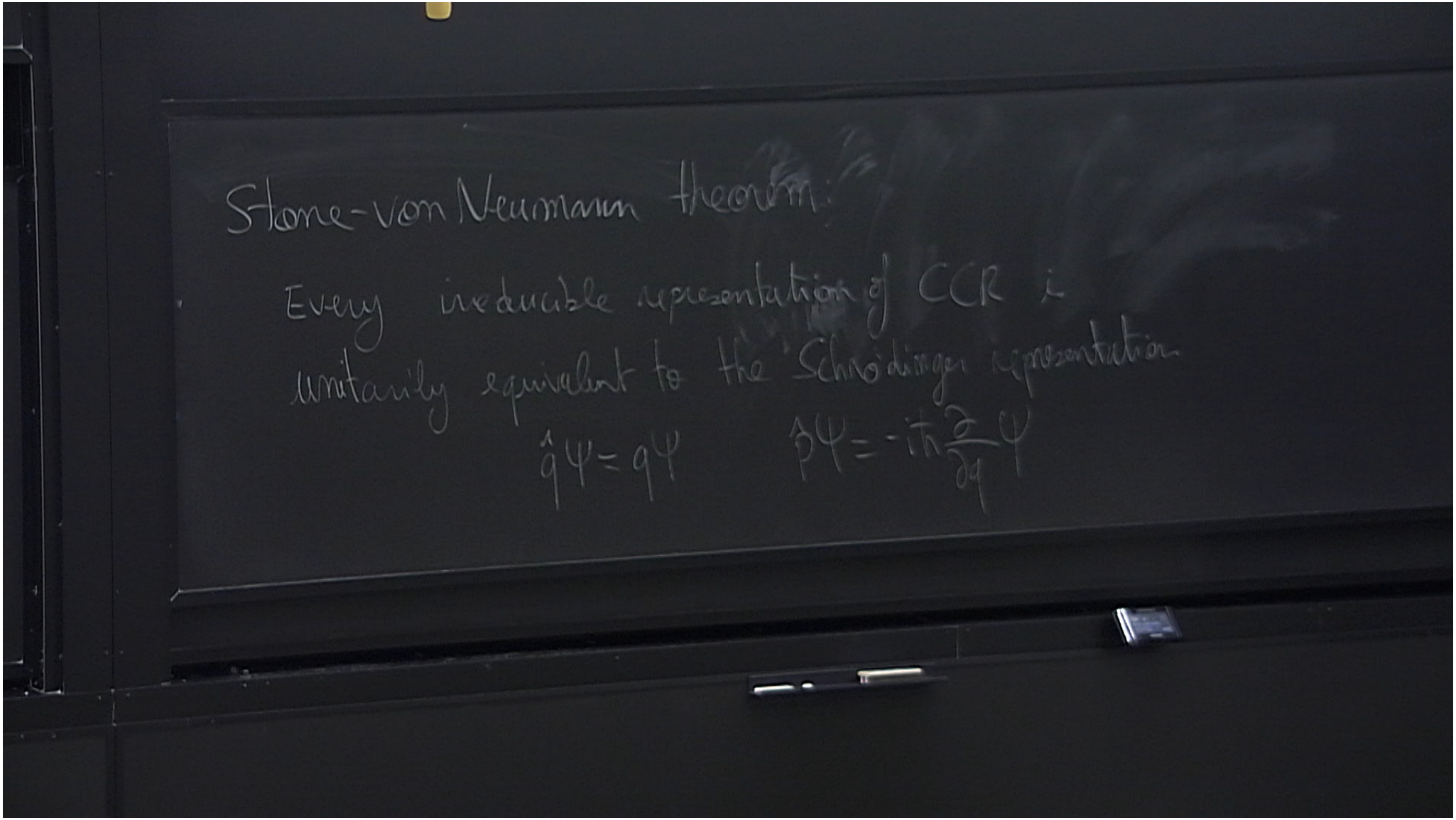
1 d of QM. Heisenberg CCR $[q,p] = i\mathbb{1}$

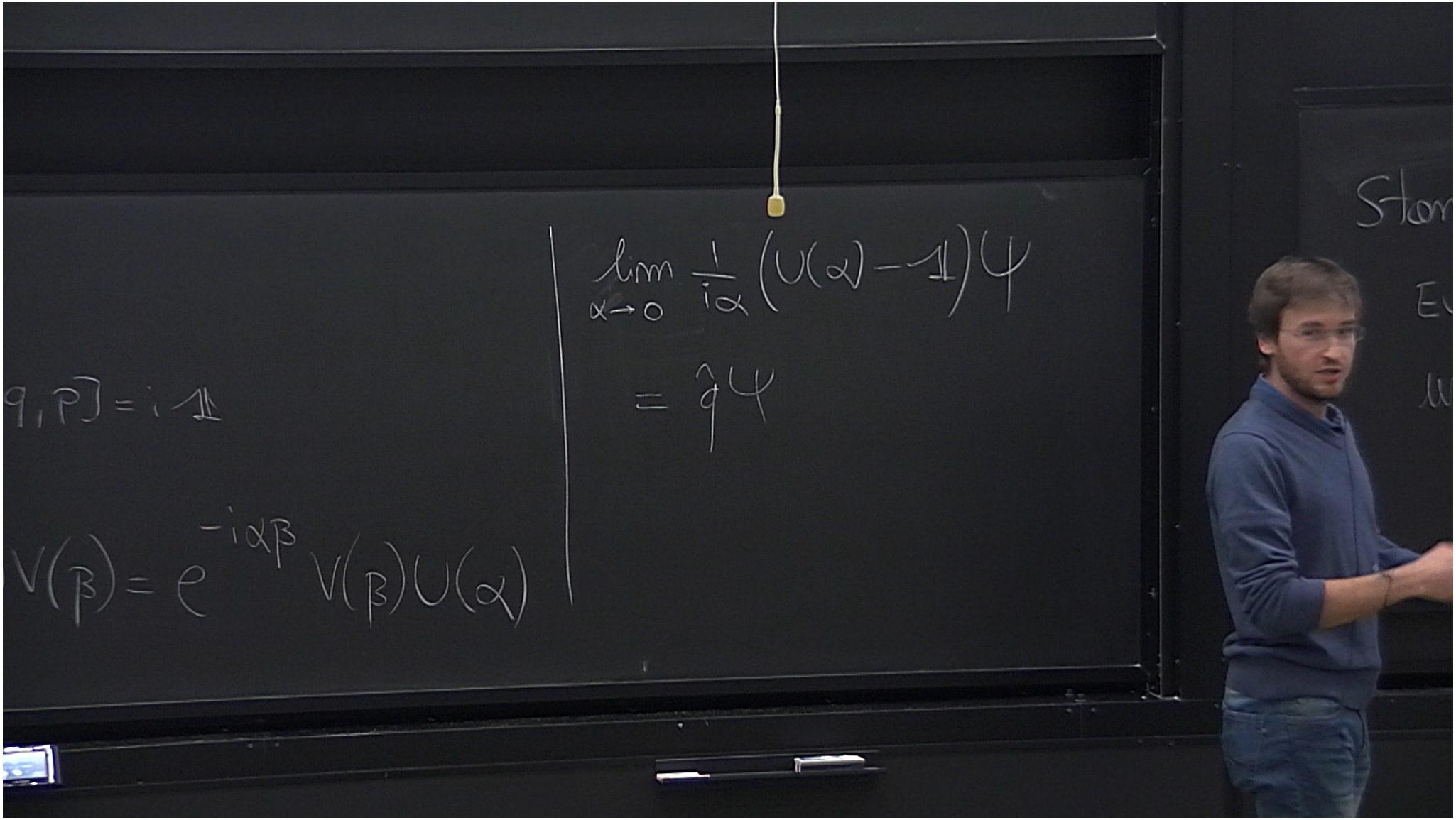
$\hbar = 1$

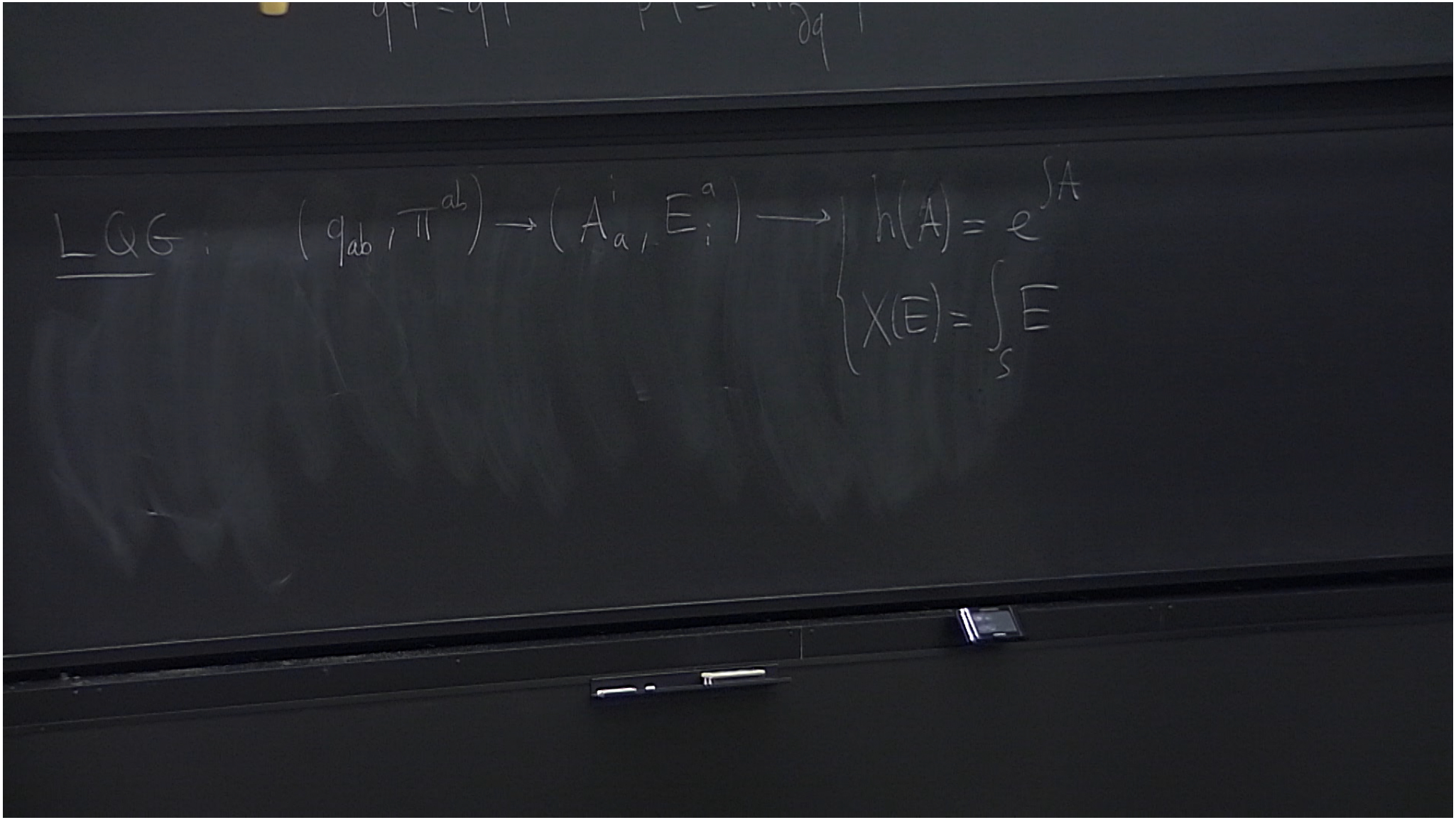
$U(\alpha) = e^{i\alpha q}$

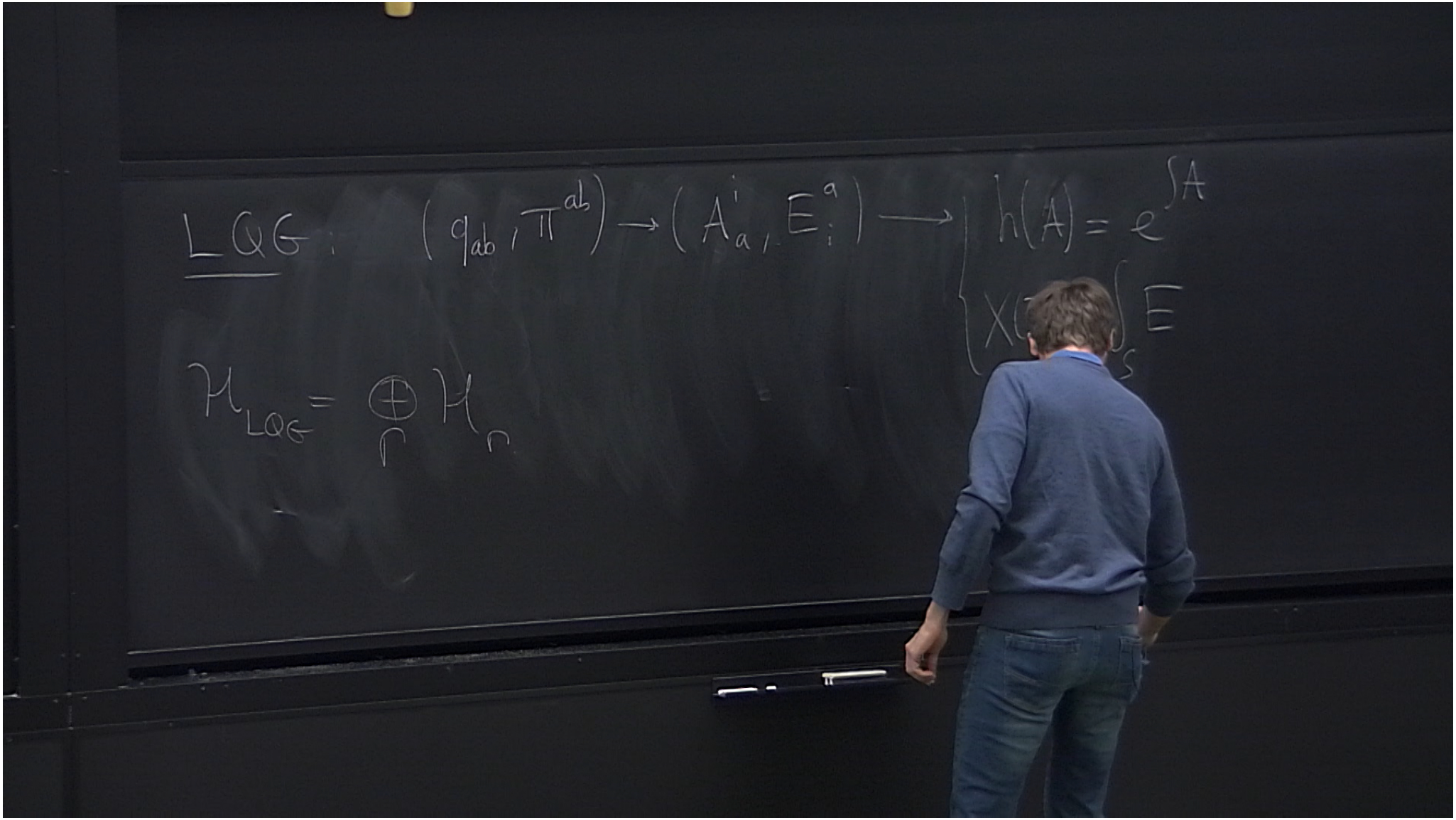
$V(\beta) = e^{i\beta p}$

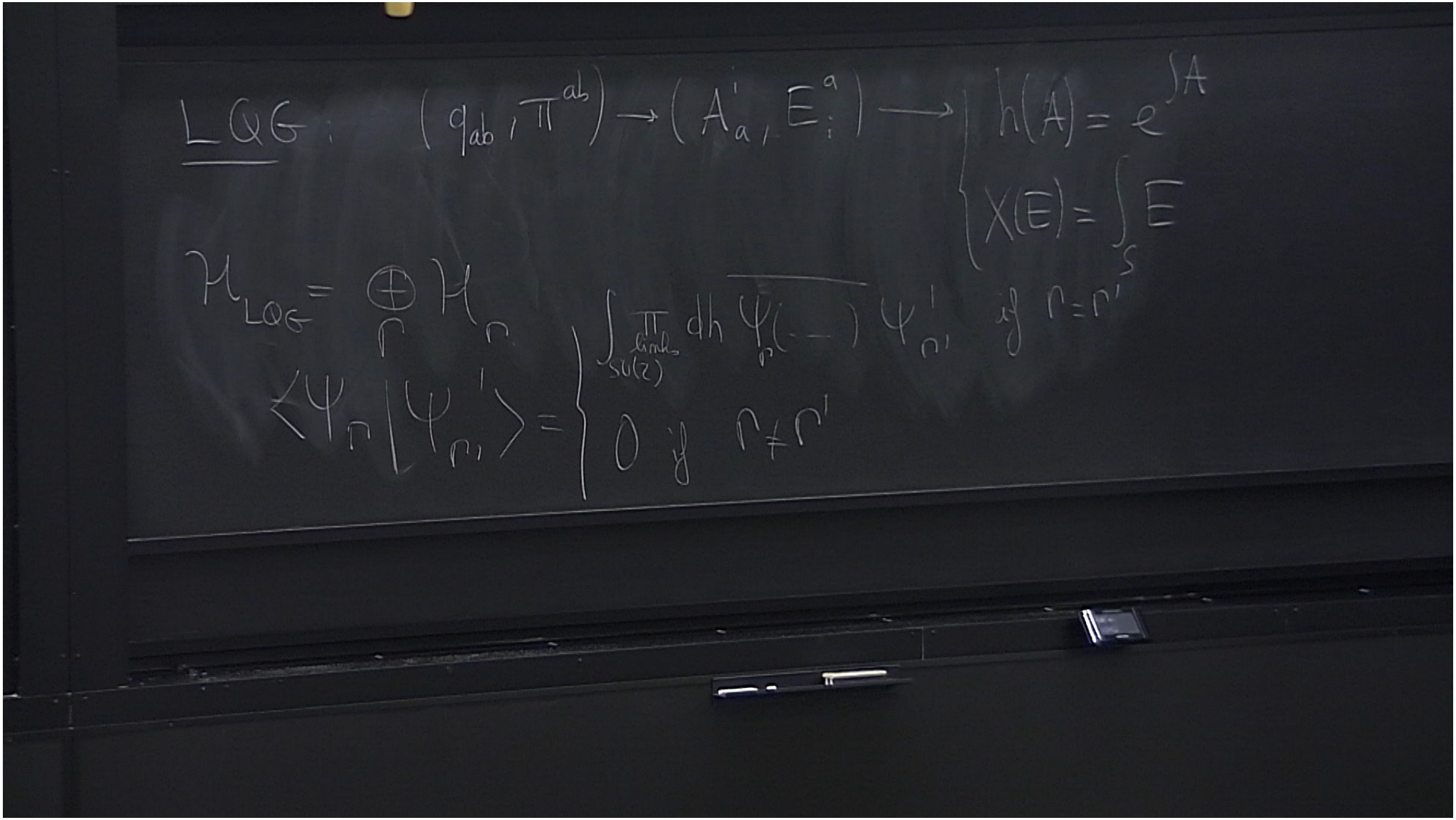
Weyl CCR: $U(\alpha)V(\beta) = e^{-i\alpha\beta}V(\beta)U(\alpha)$

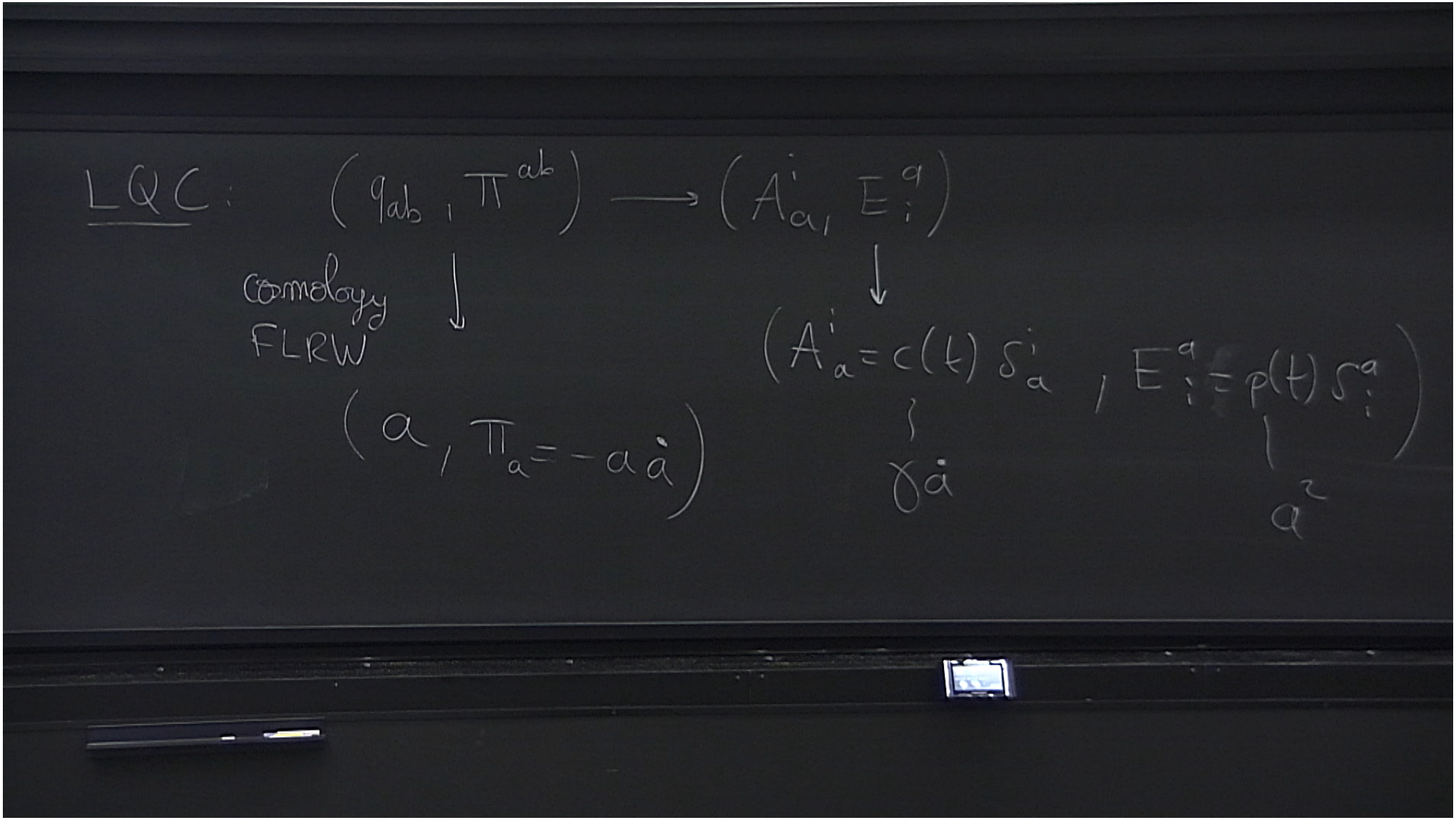

Stone-von Neumann theorem:
Every irreducible representation of CCR is
unitarily equivalent to the 'Schrödinger representation
$\hat{q}\psi = q\psi$
$\hat{p}\psi = -i\hbar\frac{\partial}{\partial q}\psi$

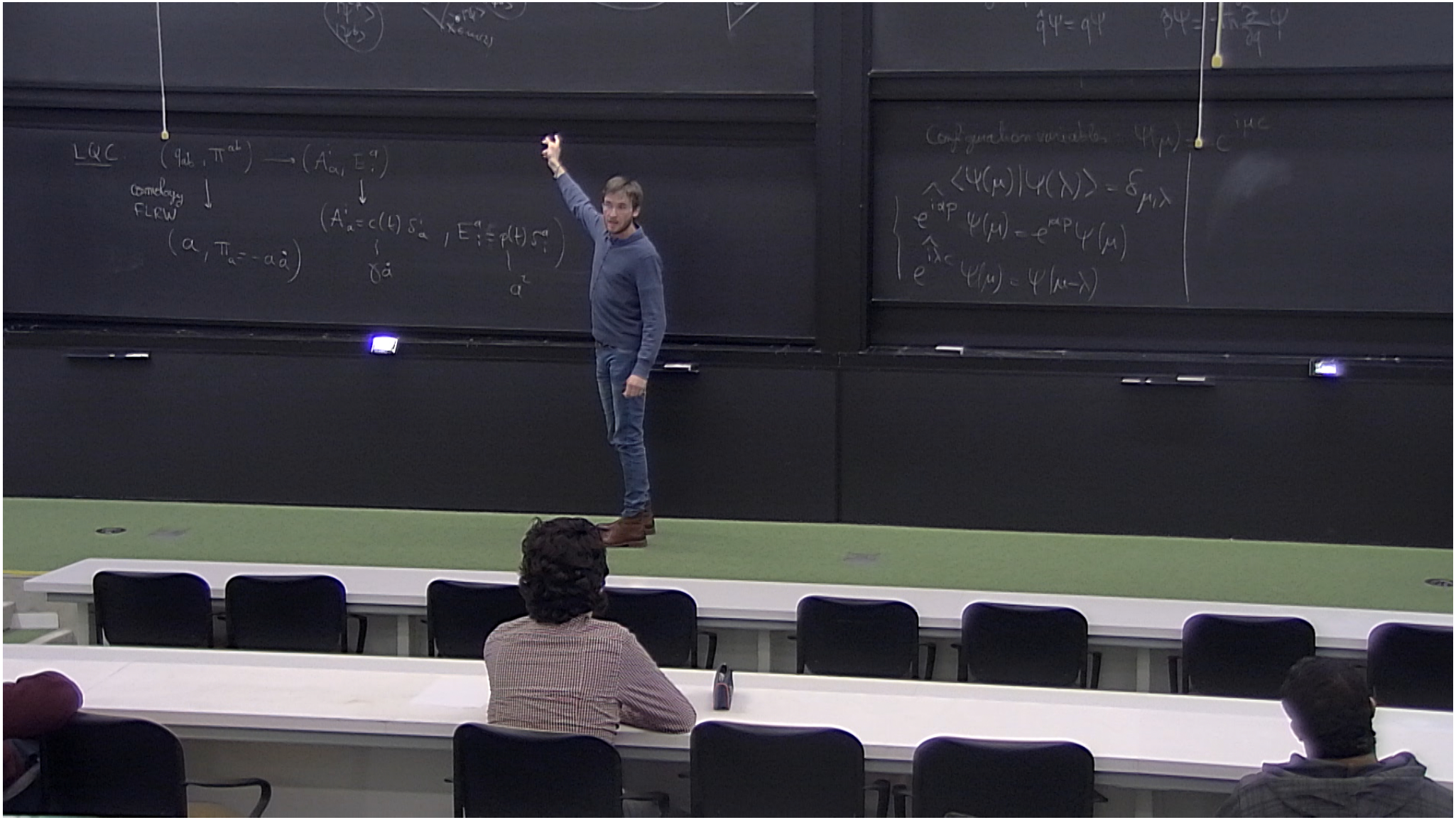

$\lim_{\alpha \to 0} \frac{1}{i\alpha} (U(\alpha) - \mathbb{1})\psi$
$= \hat{q}\psi$
$q, p] = i\mathbb{1}$
$V(\beta) = e^{-i\alpha\beta} V(\beta) U(\alpha)$

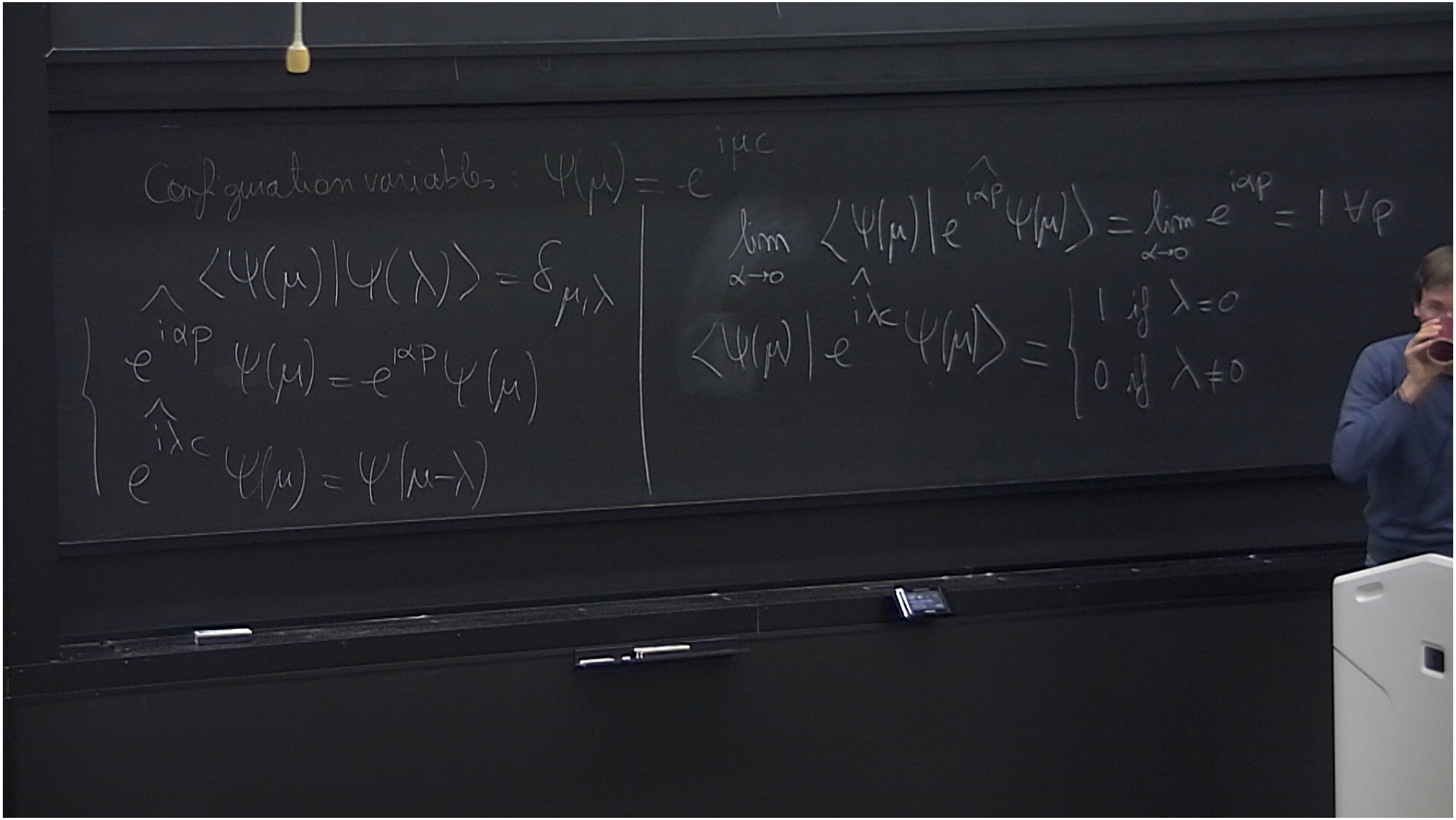

LQG: $(q_{ab}, \pi^{ab}) \to (A_a^i, E_i^a) \longrightarrow \begin{cases} h(A) = e^{\int A} \\ X(E) = \int_S E \end{cases}$

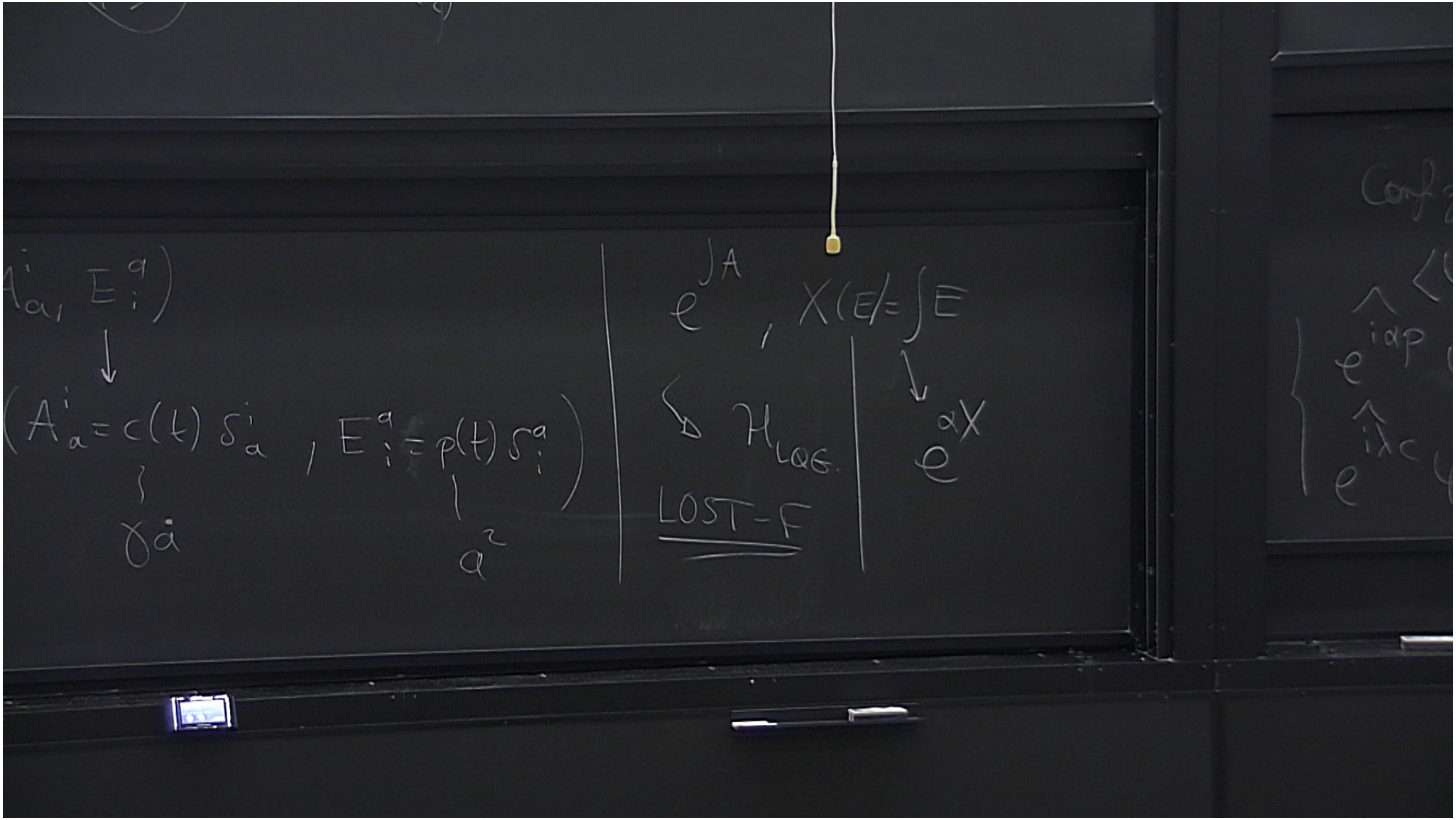

LQG: $(q_{ab}, \pi^{ab}) \to (A_a^i, E_i^a) \longrightarrow$
$h(A) = e^{\int A}$
$\mathcal{H}_{LQG} = \bigoplus_\Gamma \mathcal{H}_\Gamma$

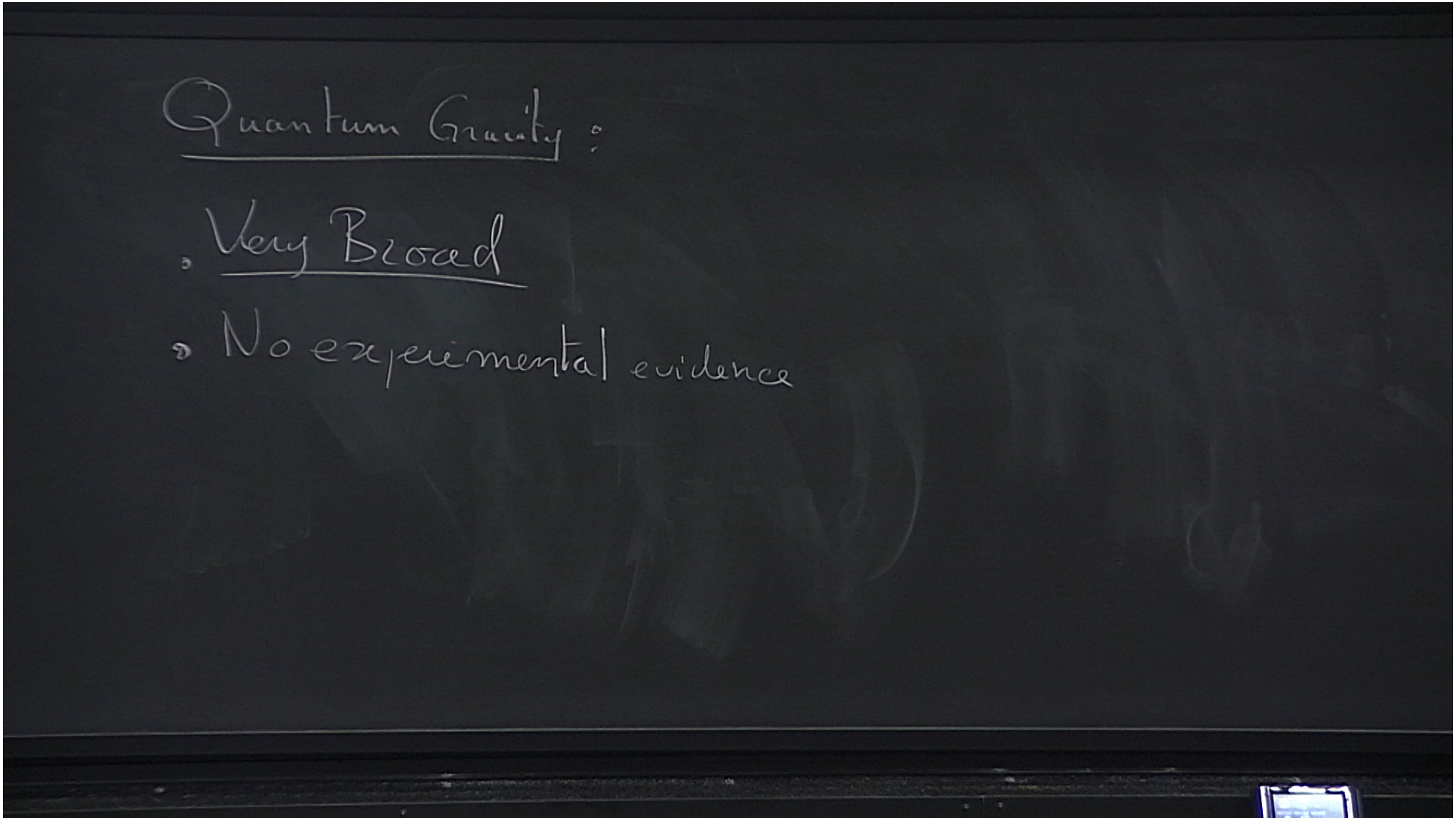

LQG: $(q_{ab}, \pi^{ab}) \to (A_a^i, E_i^a) \longrightarrow \begin{cases} h(A) = e^{\int A} \\ X(E) = \int_S E \end{cases}$
$\mathcal{H}_{LQG} = \bigoplus_\Gamma \mathcal{H}_\Gamma$
$\langle \psi_\Gamma | \psi_{\Gamma'} \rangle = \begin{cases} \int_{SU(2)} \prod_{\text{Graph}} dh \, \overline{\psi_\Gamma(\dots)} \, \psi_{\Gamma'} & \text{if } \Gamma = \Gamma' \\ 0 \text{ if } \Gamma \neq \Gamma' \end{cases}$

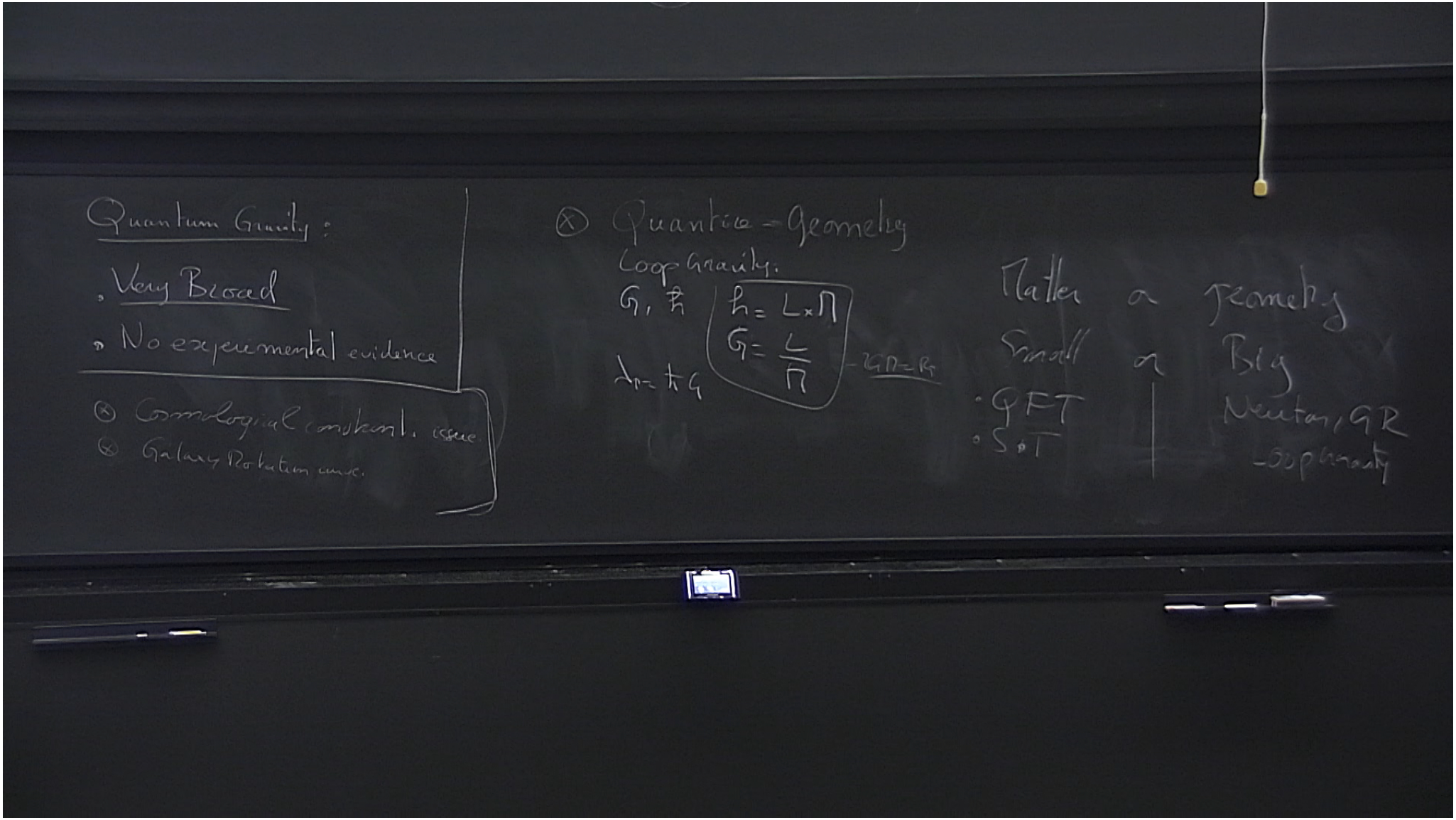

LQC: $(q_{ab}, \pi^{ab}) \longrightarrow (A_a^i, E_i^a)$
cosmology FLRW
$(a, \pi_a = -a\dot{a})$
$(A_a^i = c(t)\, s_a^i \;,\; E_i^a = p(t)\, s_i^a)$
$c = \gamma\dot{a}$
$p = a^2$

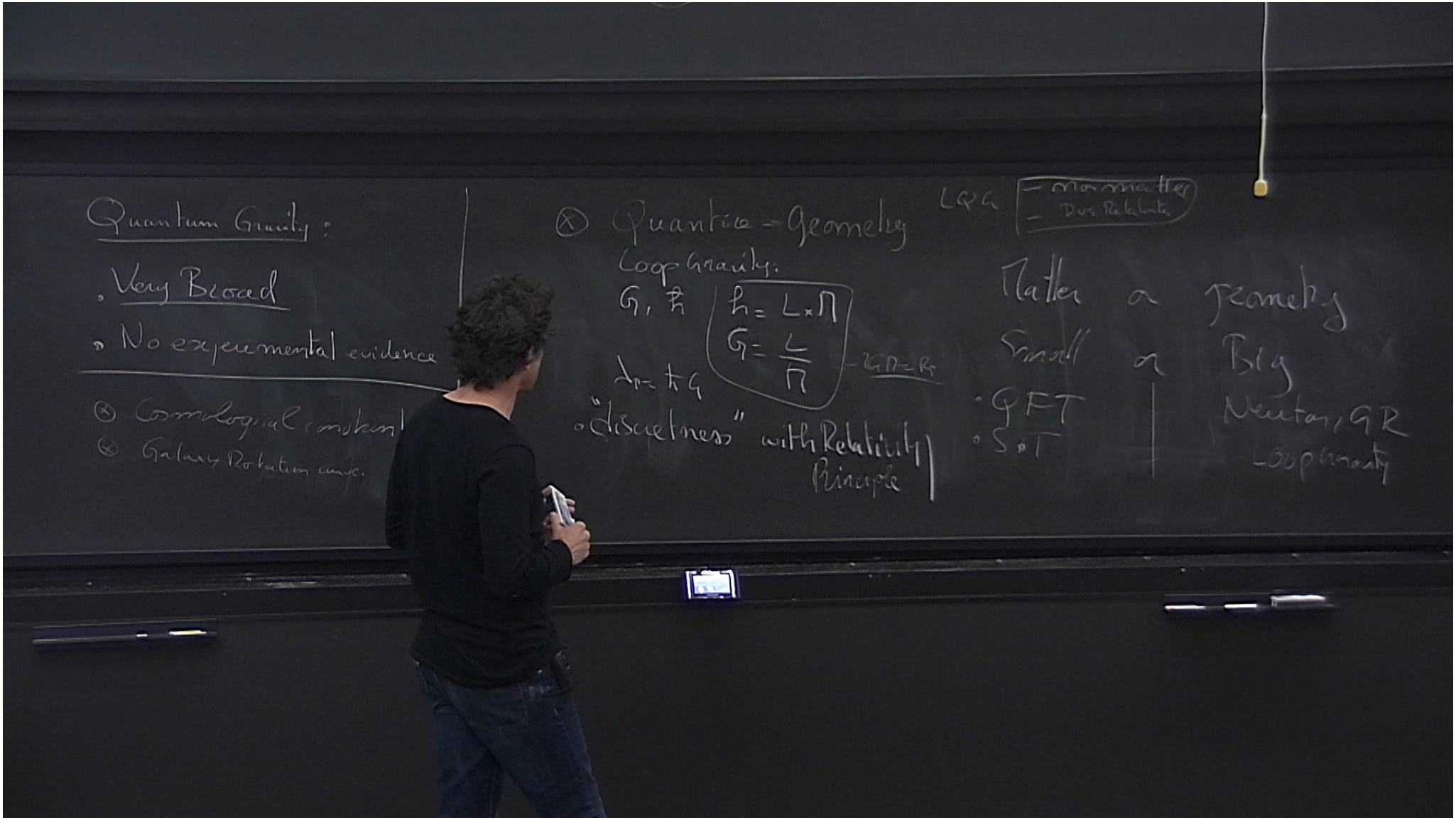

LQC
Cosmology FLRW
Configuration variables

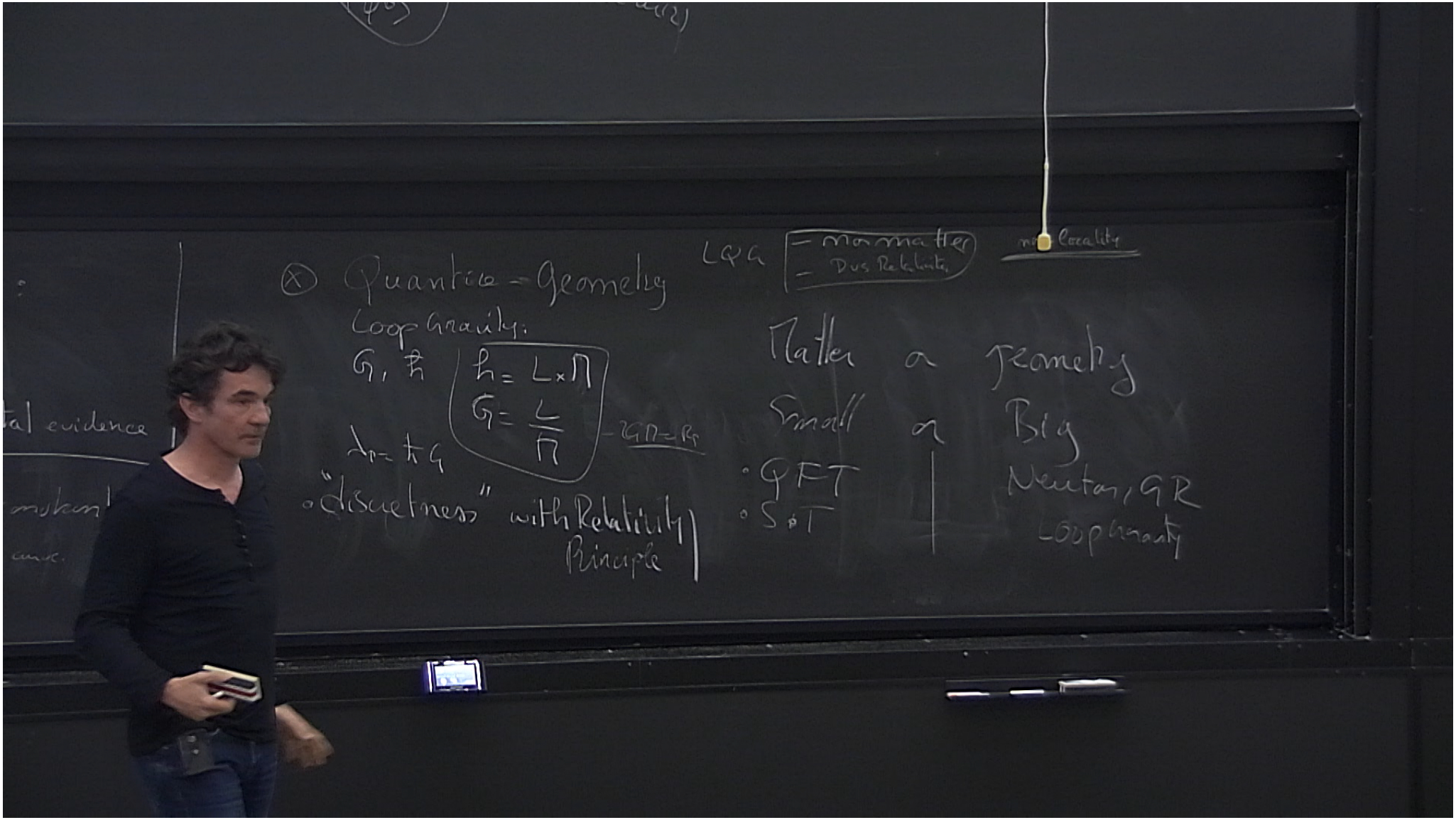

Configuration variables: $\psi(\mu) = e^{i\mu c}$
$\langle \psi(\mu) | \psi(\lambda) \rangle = \delta_{\mu,\lambda}$
$e^{i\alpha \hat{p}} \psi(\mu) = e^{i\alpha p} \psi(\mu)$
$e^{i\lambda \hat{c}} \psi(\mu) = \psi(\mu - \lambda)$
$\lim_{\alpha \to 0} \langle \psi(\mu) | e^{i\alpha \hat{p}} \psi(\mu) \rangle = \lim_{\alpha \to 0} e^{i\alpha p} = 1 \; \forall p$
$\langle \psi(\mu) | e^{i\lambda \hat{c}} \psi(\mu) \rangle = \begin{cases} 1 \text{ if } \lambda = 0 \\ 0 \text{ if } \lambda \neq 0 \end{cases}$

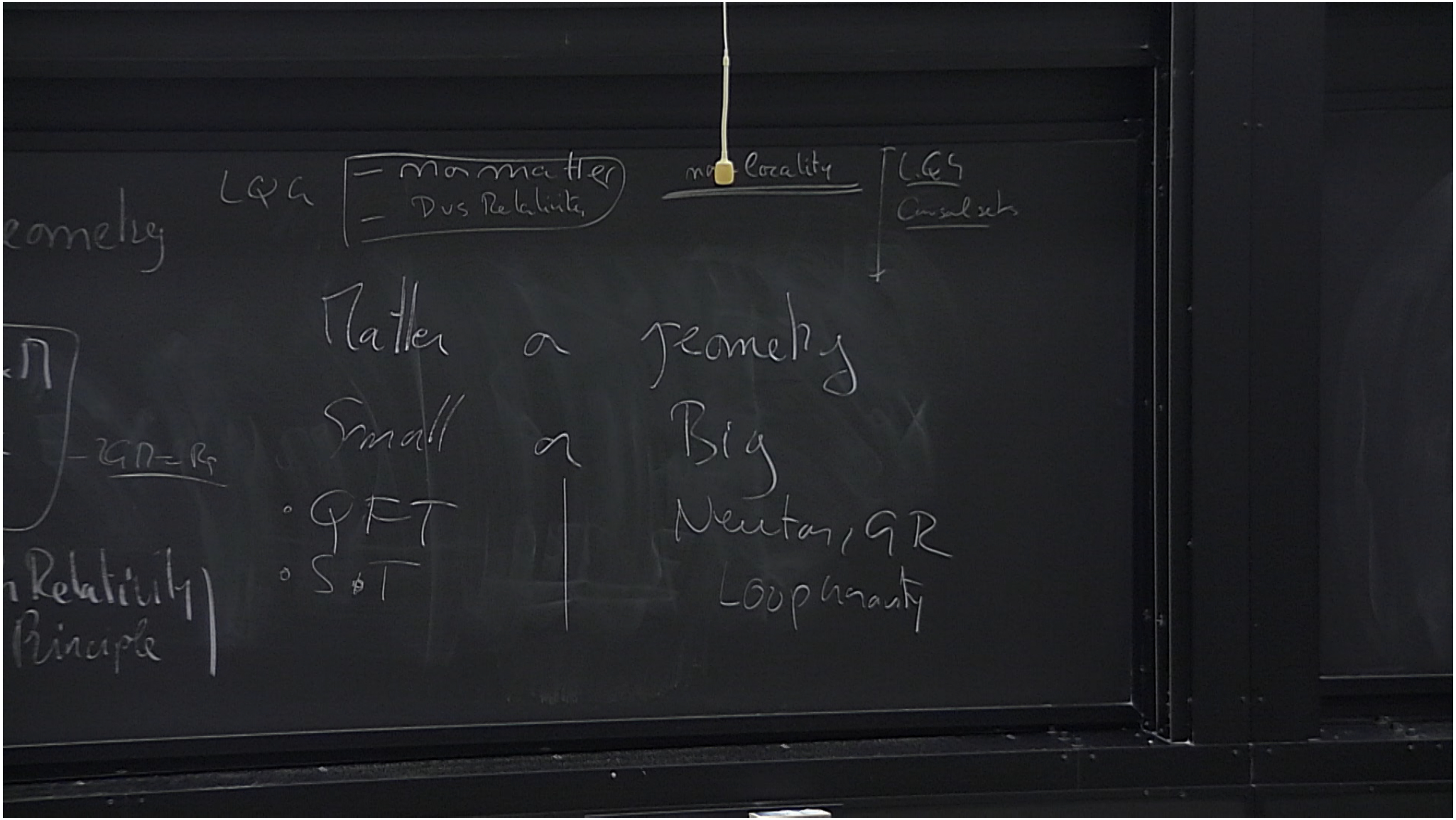

$A_a^i, E_i^a)$
$(A_a^i = c(t)\,\delta_a^i \;,\; E_i^a = p(t)\,\delta_i^a)$
γa
a^2
e^{JA}, $X(E) = \int E$
$\mathcal{H}_{LQG}$
LOST-F
$e^{\alpha X}$
e^{ip}
e^{ix}

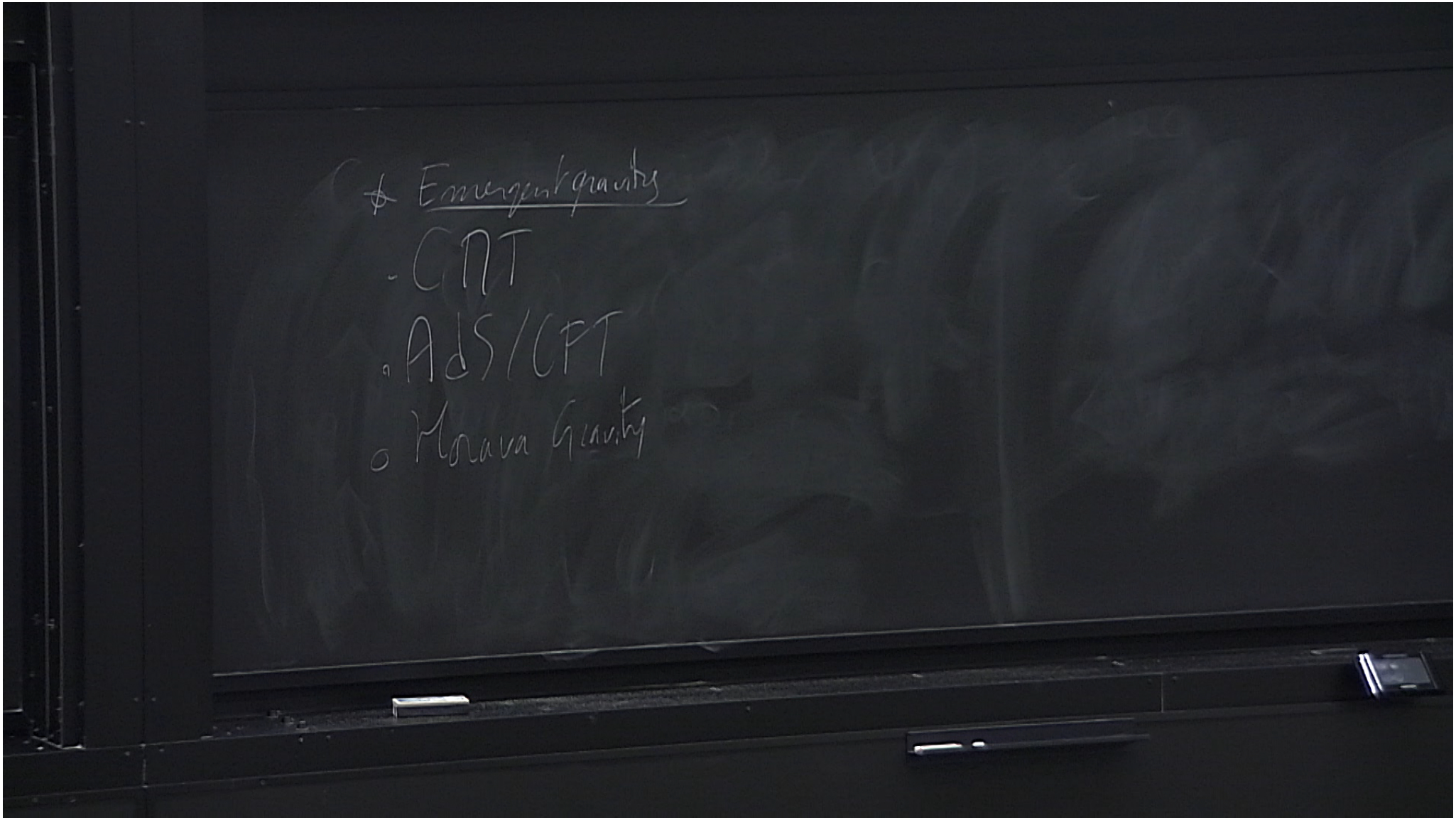

Quantum Gravity:
Very Broad
No experimental evidence

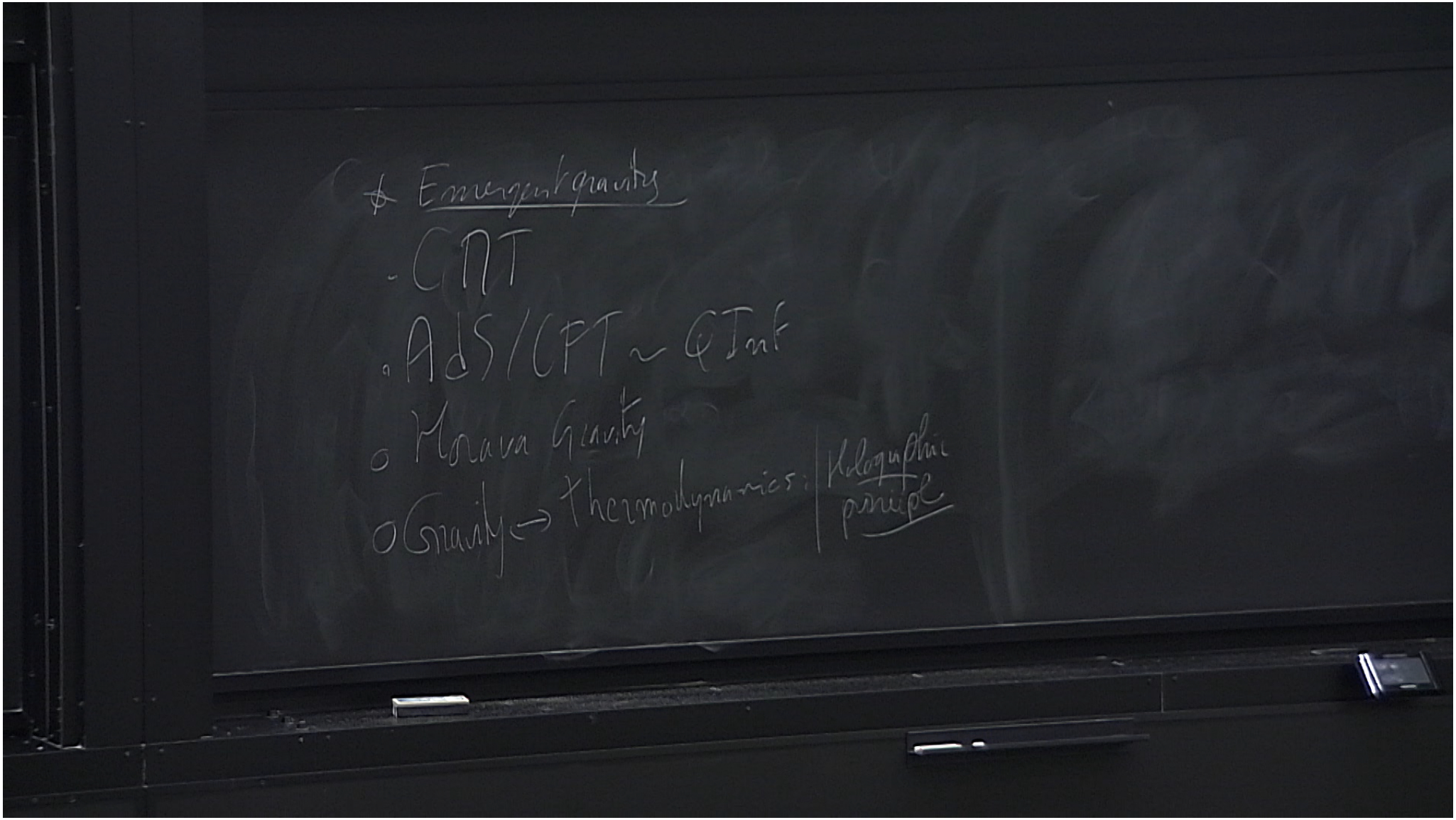

Quantum Gravity :
. Very Broad
» No experimental evidence
⊗ Cosmological constant issue
⊗ Galaxy Rotation curve.
⊗ Quantum - Geometry
Loop Gravity.
G, ħ
$\lambda_P = \sqrt{\hbar G}$
$\hbar = L \times \Pi$
$G = \frac{L}{\Pi}$
$\frac{1}{G\Pi} = R$
Matter a geometry
Small a Big
· QFT Newton, GR
· S.M -Singularity


Quantum Gravity:
Very Broad
No experimental evidence
Cosmological constant
Galaxy Rotation curve
Quantice - Geometry
Loop Gravity.
"discreteness" with Relativity
Principle
LQG
Rather a geometry
Small
Big
QFT
S*T
Neutan, GR


⊗ Quantum = Geometry
Loop Gravity
LQG
= matter
= Dus Relativity
no locality
$\hbar = L \times \Pi$
$G = \frac{L}{\Pi}$
$\Delta r = \pm G$
"discreetness" with Relativity Principle
Matter a geometry
Small a
Big
Newton, GR
Loop Gravity
• QFT
• S.T


LQG
= no matter
= Dus Relativity
no locality
LQG
Causal sets
Matter a geometry
Small a Big
• QFT
• S.T
Newton, GR
Loop Gravity


Emergent gravity
CFT
AdS/CFT
Horava Gravity


* Emergent gravity
- CNT
- AdS/CFT ~ QInf
- Horava Gravity
- Gravity → thermodynamics: Holographic principle