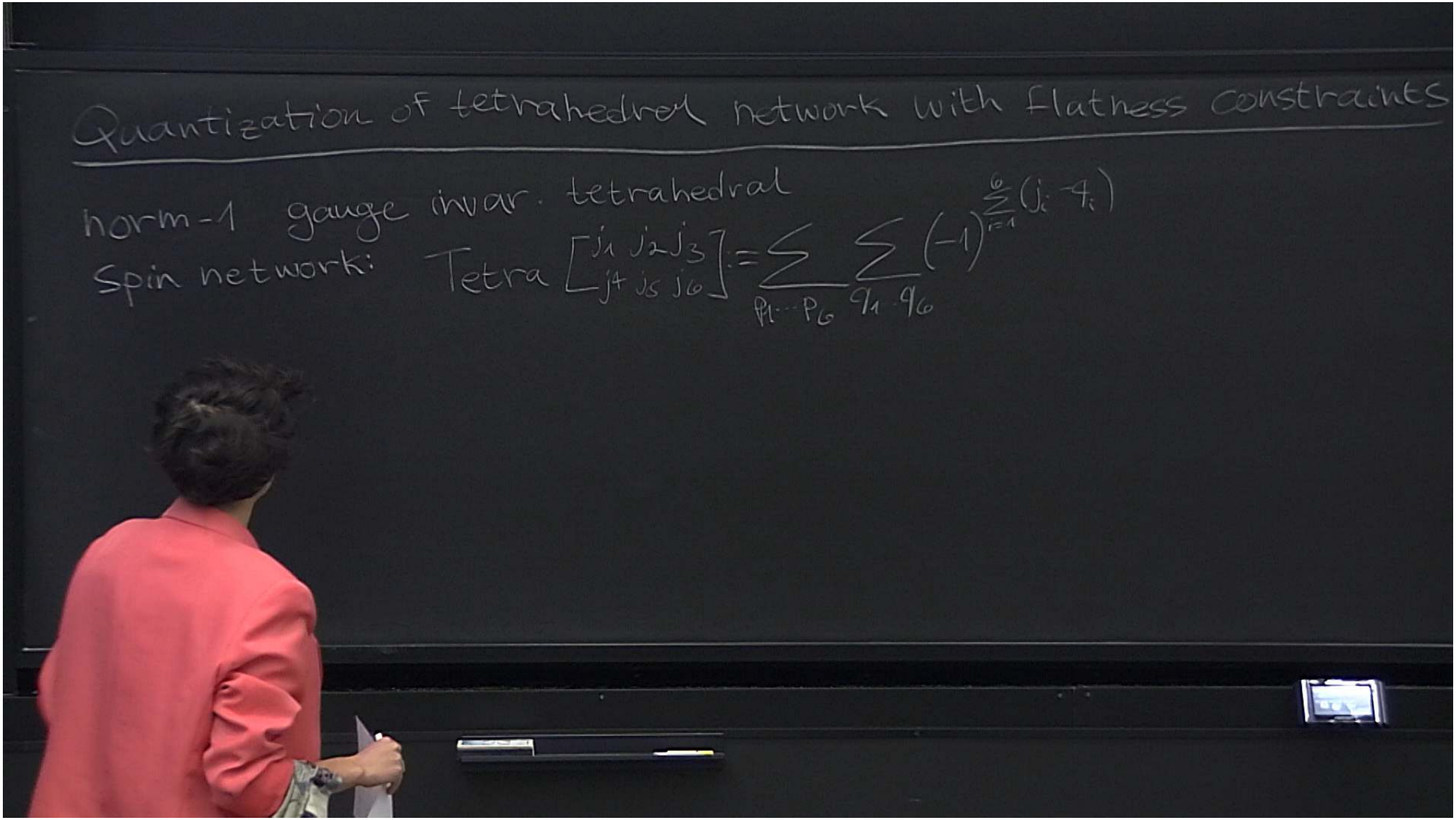
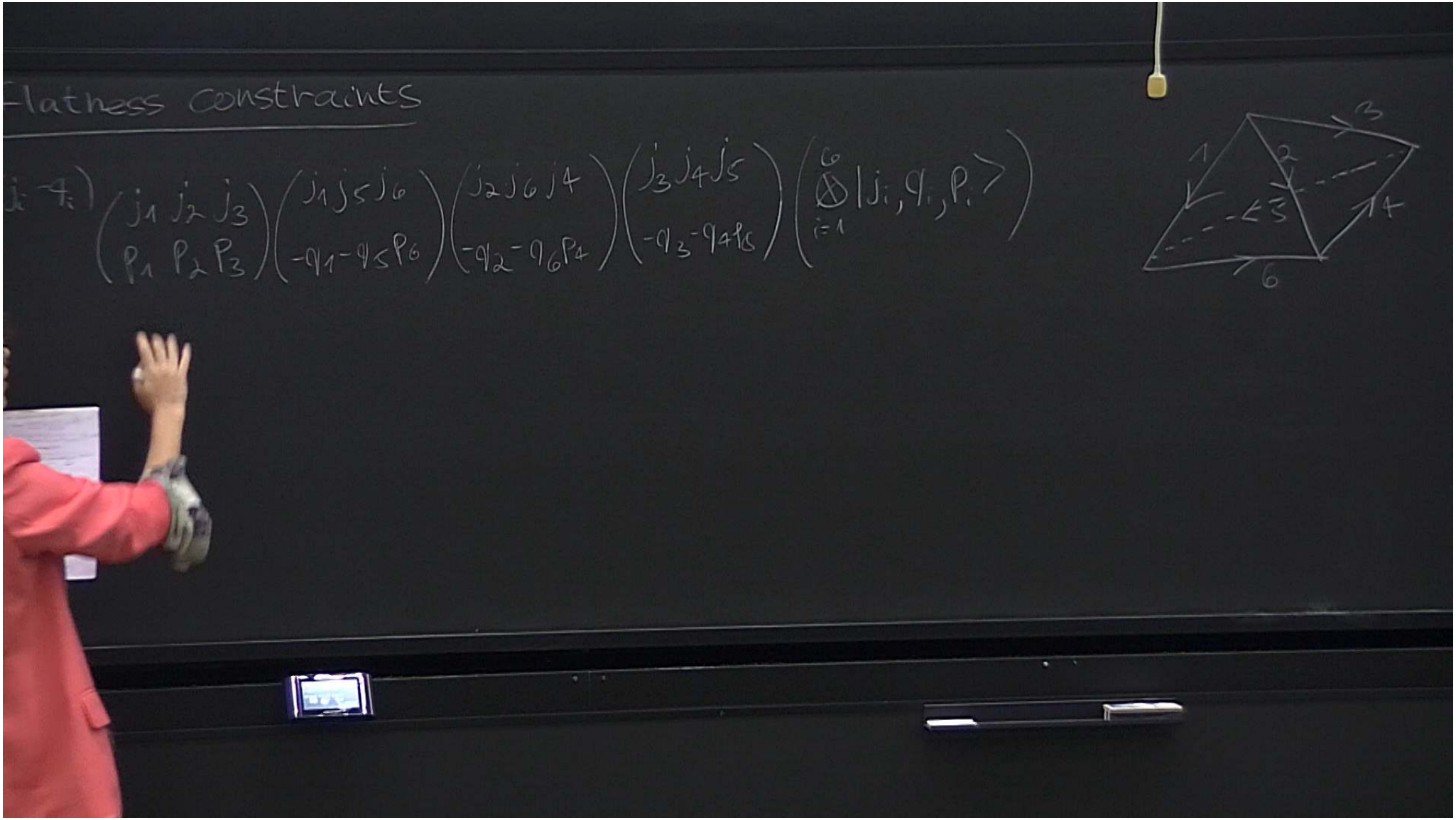
Title: PSI 2016/2017 Explorations in Quantum Gravity - Lecture 13

Date: Apr 05, 2017 10:15 AM

URL: http://pirsa.org/17040017

Abstract:

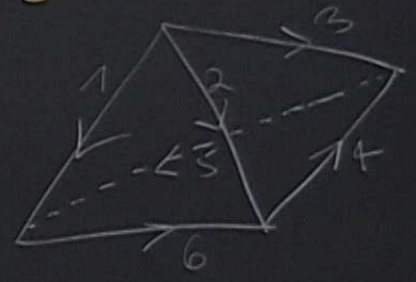

Quantization of tetrahedral network with flatness constraints
norm-1 gauge invar. tetrahedral
spin network:
Tetra $\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} := \sum_{p_1 \cdots p_6} \sum_{q_1 \cdots q_6} (-1)^{\sum_{i=1}^{6} (j_i - q_i)}$


Flatness constraints
$\begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -q_1 & -q_5 & p_6 \end{pmatrix} \begin{pmatrix} j_2 & j_6 & j_4 \\ -q_2 & -q_6 & p_4 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & j_5 \\ -q_3 & -q_4 & p_5 \end{pmatrix} \left(\bigotimes_{i=1}^{6} |j_i, q_i, p_i\rangle \right)$
1
2
3
4
5
6

Flatness constraints

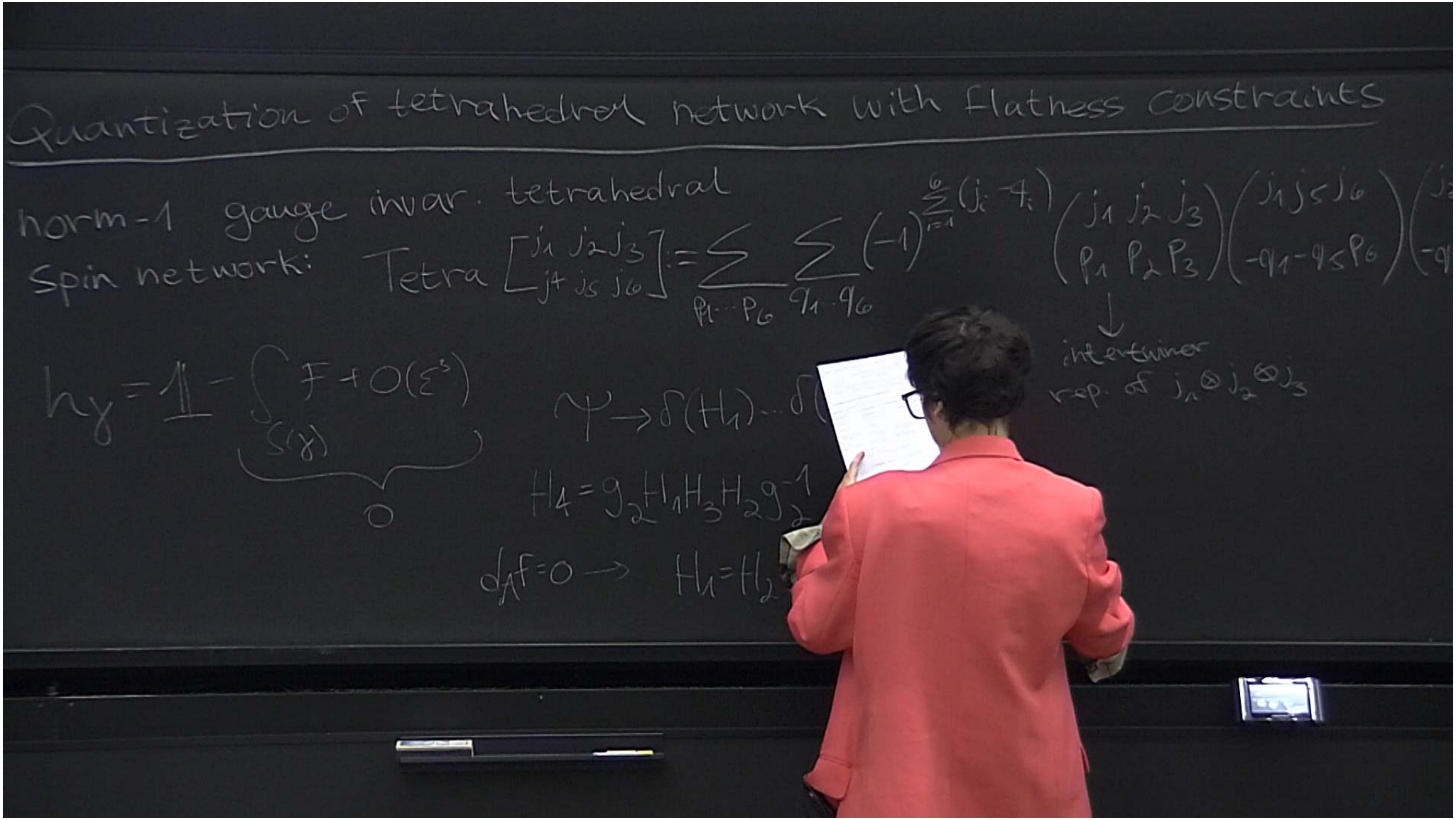
$$\left(j_i \to 4_i\right)\begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix}\begin{pmatrix} j_1 & j_5 & j_6 \\ -g_1 & -g_5 p_6 & \end{pmatrix}\begin{pmatrix} j_2 & j_6 & j_4 \\ -g_2 & -g_6 p_4 & \end{pmatrix}\begin{pmatrix} j_3 & j_4 & j_5 \\ -g_3 & -g_4 p_5 & \end{pmatrix}\left(\bigotimes_{i=1}^{6} |j_i, g_i, p_i\rangle\right)$$

↓ intertwiner rep. of $j_1 \otimes j_2 \otimes j_3$



$H_1 = g_2^{-1} g_6 g_1$

$H_2 =$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Spin network:
Tetra
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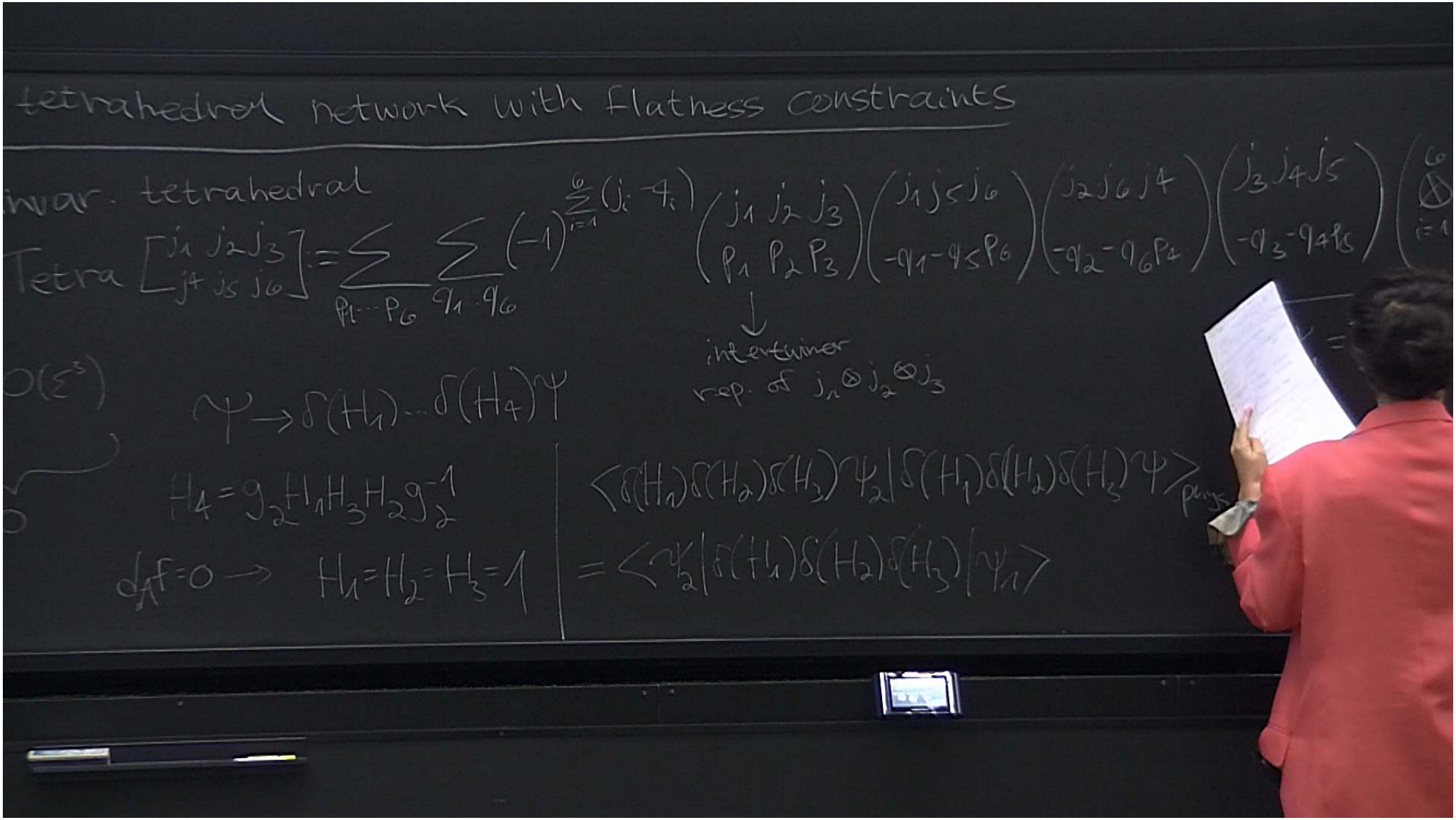
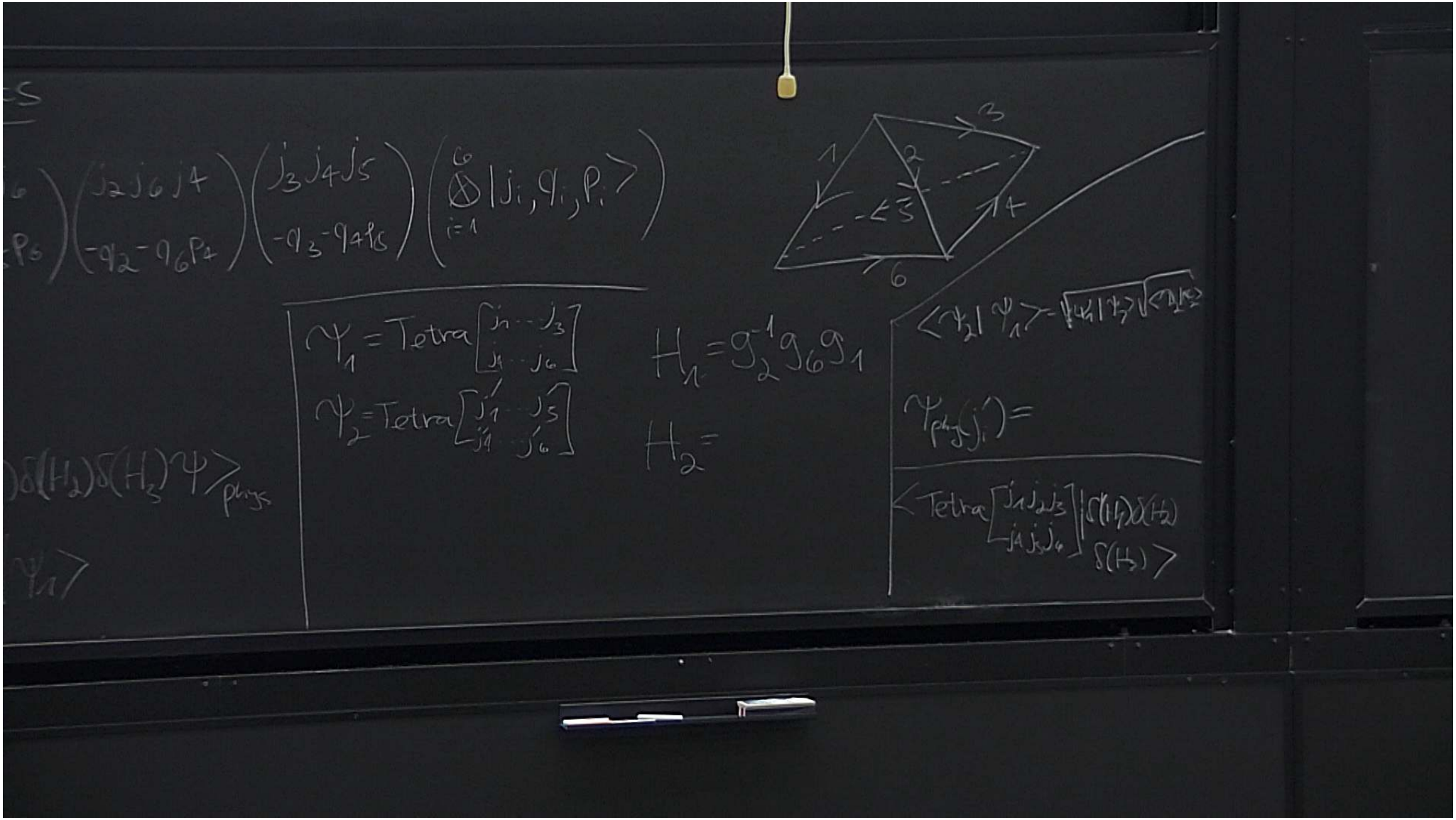
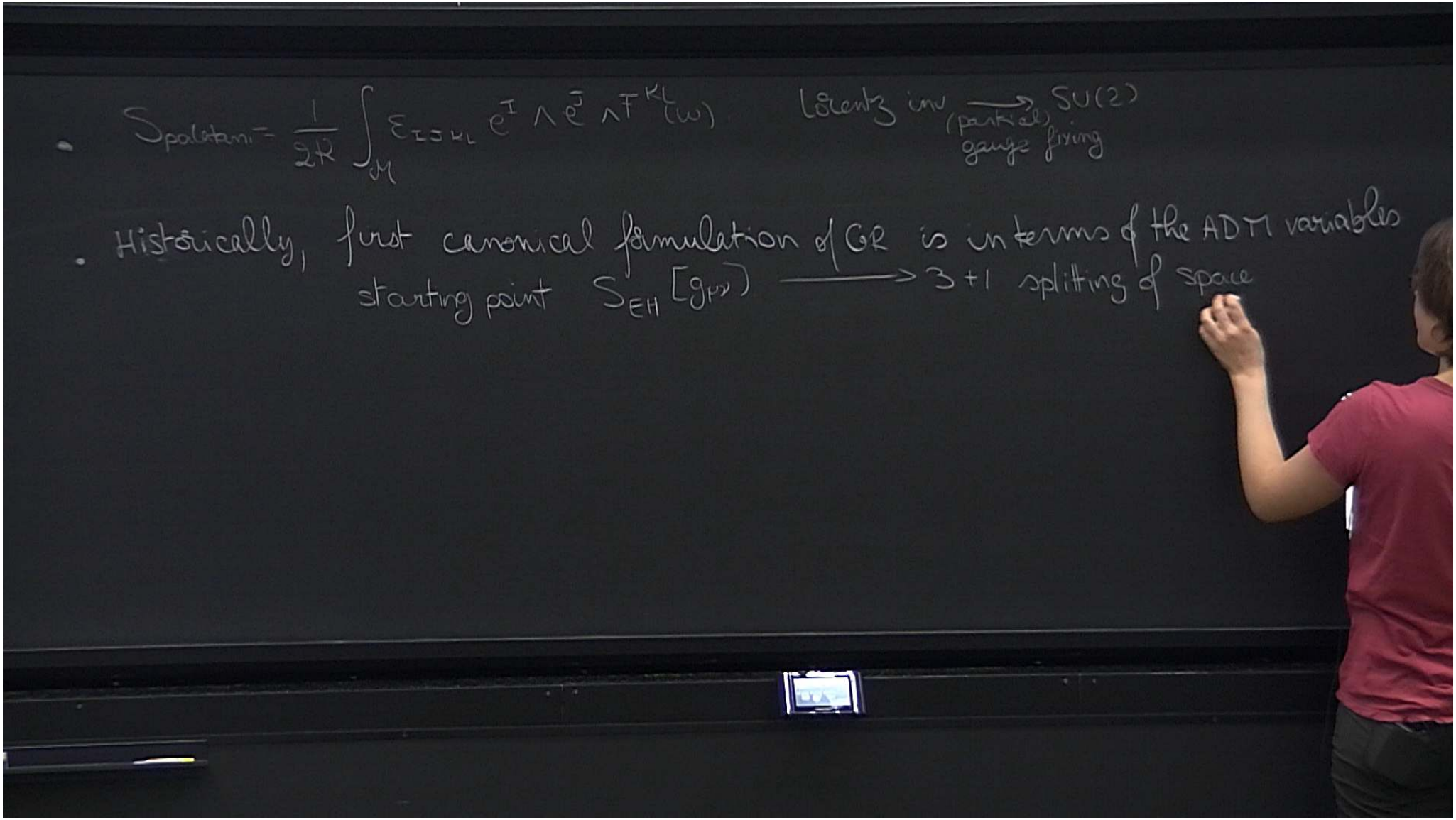

tetrahedral network with flatness constraints
invar. tetrahedral
Tetra
intertwiner
rep. of
O(ε³)
plus


$\Psi_1 = \mathrm{Tetra}\begin{bmatrix} j_1 & \cdots & j_3 \\ j_4 & \cdots & j_6 \end{bmatrix}$
$\Psi_2 = \mathrm{Tetra}\begin{bmatrix} j_1' & \cdots & j_5 \\ j_4 & \cdots & j_6' \end{bmatrix}$
$H_1 = g_2^{-1} g_6 g_1$
$H_2 =$
$\langle \Psi_2 | \Psi_1 \rangle = \sqrt{\{4\}\{2\}} \sqrt{\langle \Psi_1 | \Psi_2 \rangle}$
$\Psi_{\mathrm{phys}}(j_i) =$
$\langle \mathrm{Tetra}\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} | \delta(H_1)\delta(H_2)\delta(H_3) \rangle$

- $S_{\text{palatini}} = \frac{1}{2k} \int_M \varepsilon_{IJKL}\, e^I \wedge e^J \wedge F^{KL}(\omega)$ Lorentz inv $\xrightarrow[\text{gauge fixing}]{\text{(partial)}}$ SU(2)

- Historically, first canonical formulation of GR is in terms of the ADM variables
  starting point $S_{EH}[g_{\mu\nu}]$ ⟶ 3+1 splitting of space



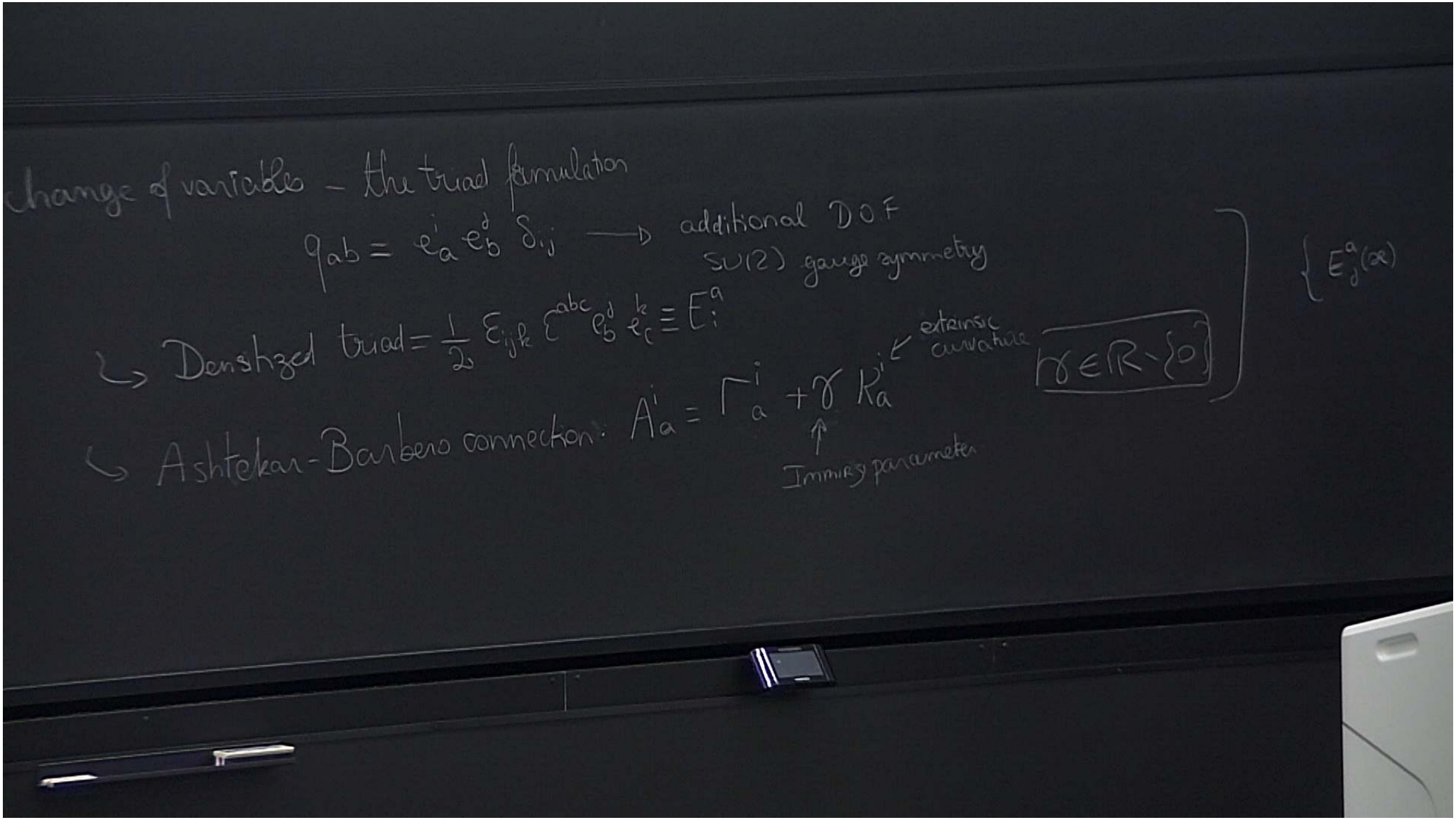
$$\text{Ham} = -\frac{1}{2k}\int_M \varepsilon_{IJKL}\, e^I \wedge e^J \wedge F^{KL}(\omega)$$ Lorentz inv $\xrightarrow[\text{gauge fixing}]{\text{(partial)}}$ SU(2)

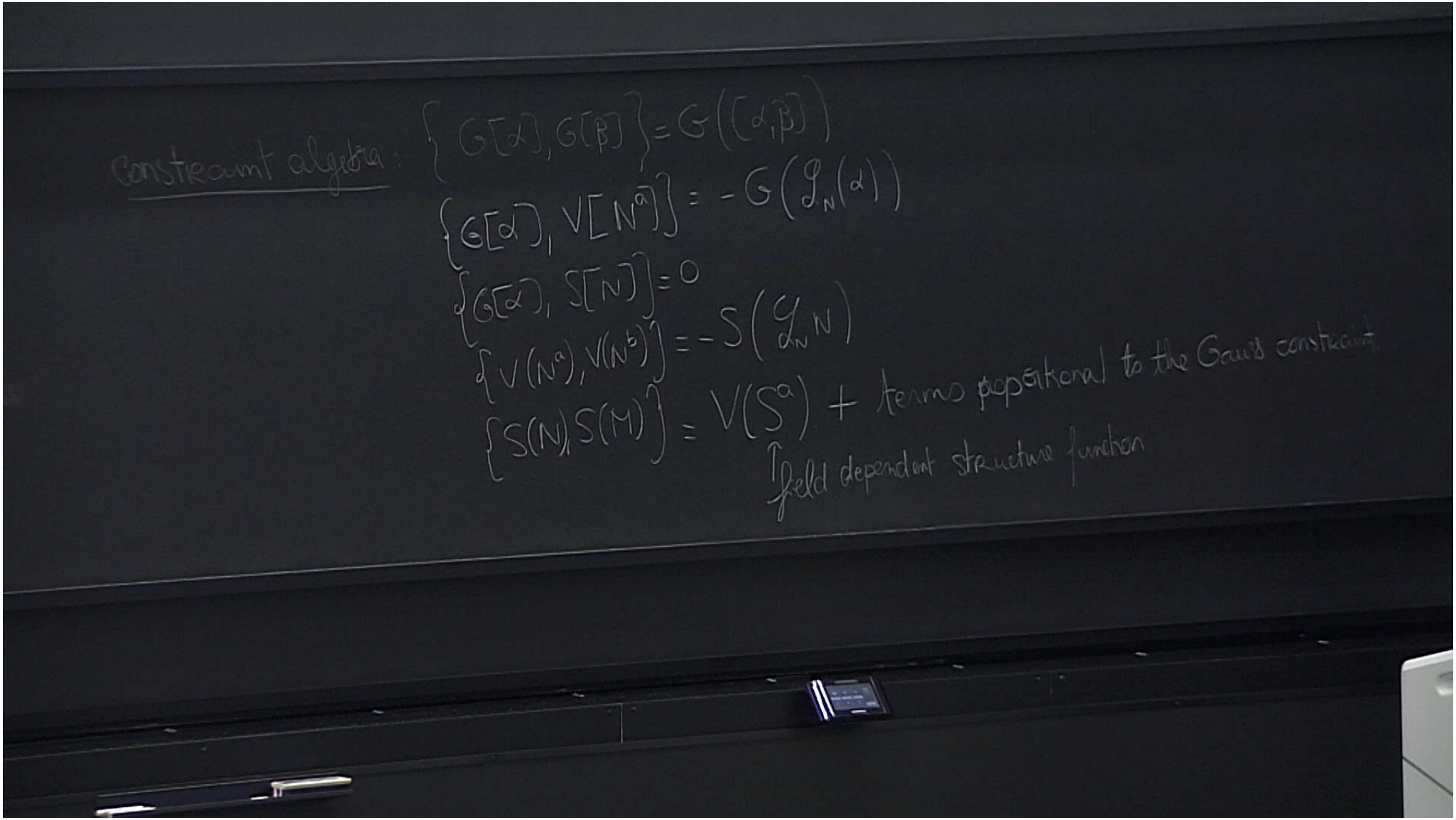
ically, first canonical formulation of GR is in terms of the ADM variables

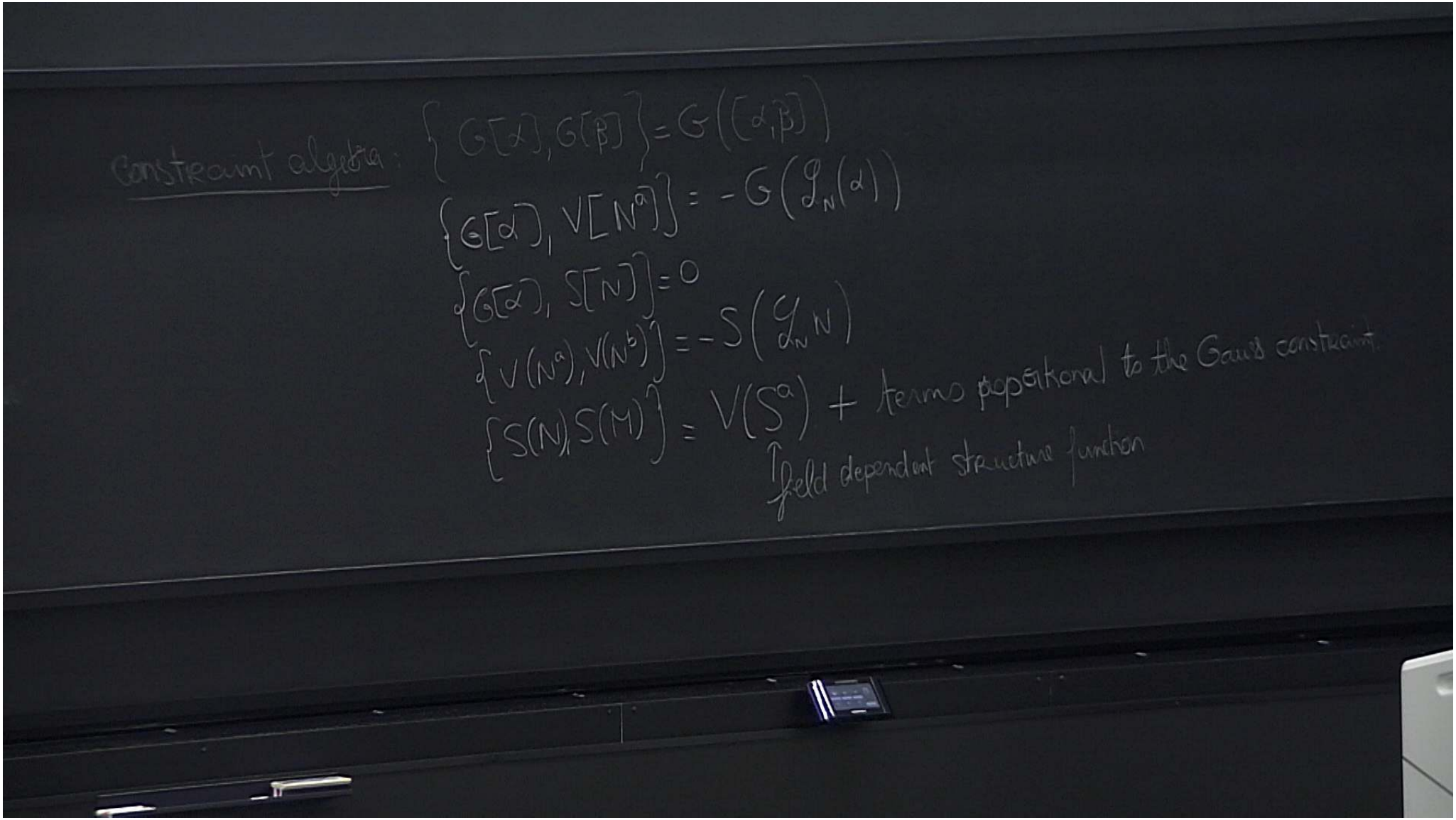
starting point $S_{EH}[g_{\mu\nu}]$ $\longrightarrow$ 3+1 splitting of spacetime. $\Sigma \times \mathbb{R}$

Phase space variables: . spacial metric q_{ab} on the surface Σ

. $\pi^{ab} = \frac{\partial L_{EH}}{\partial \dot{q}_{ab}}$ $\Big)$ +


change of variables – the triad formulation
$q_{ab} = e^i_a e^j_b \delta_{ij}$ → additional DOF
SU(2) gauge symmetry
↳ Densitized triad $= \frac{1}{2} \varepsilon_{ijk} \varepsilon^{abc} e^j_b e^k_c \equiv E^a_i$
↳ Ashtekar-Barbero connection: $A^i_a = \Gamma^i_a + \gamma K^i_a$
extrinsic curvature
Immirzi parameter
$\gamma \in \mathbb{R} - \{0\}$
$\{E^a_j(x)$


Constraint algebra:
$\{G[\alpha], G[\beta]\} = G([\alpha,\beta])$
$\{G[\alpha], V[N^a]\} = -G(\mathcal{L}_N(\alpha))$
$\{G[\alpha], S[N]\} = 0$
$\{V(N^a), V(N^b)\} = -S(\mathcal{L}_{\alpha_N} N)$
$\{S(N), S(M)\} = V(S^a)$ + terms proportional to the Gauss constraint
field dependent structure function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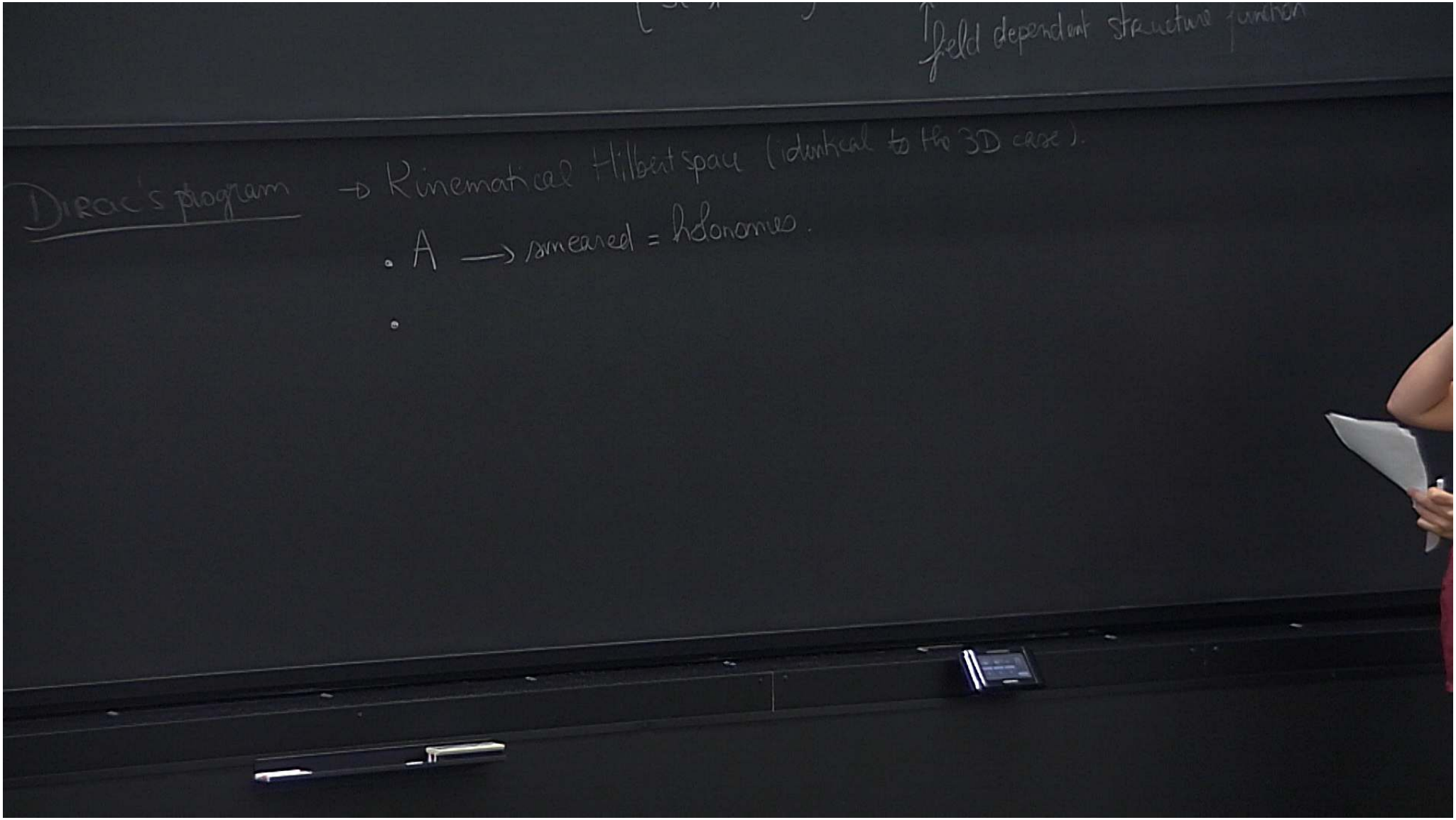
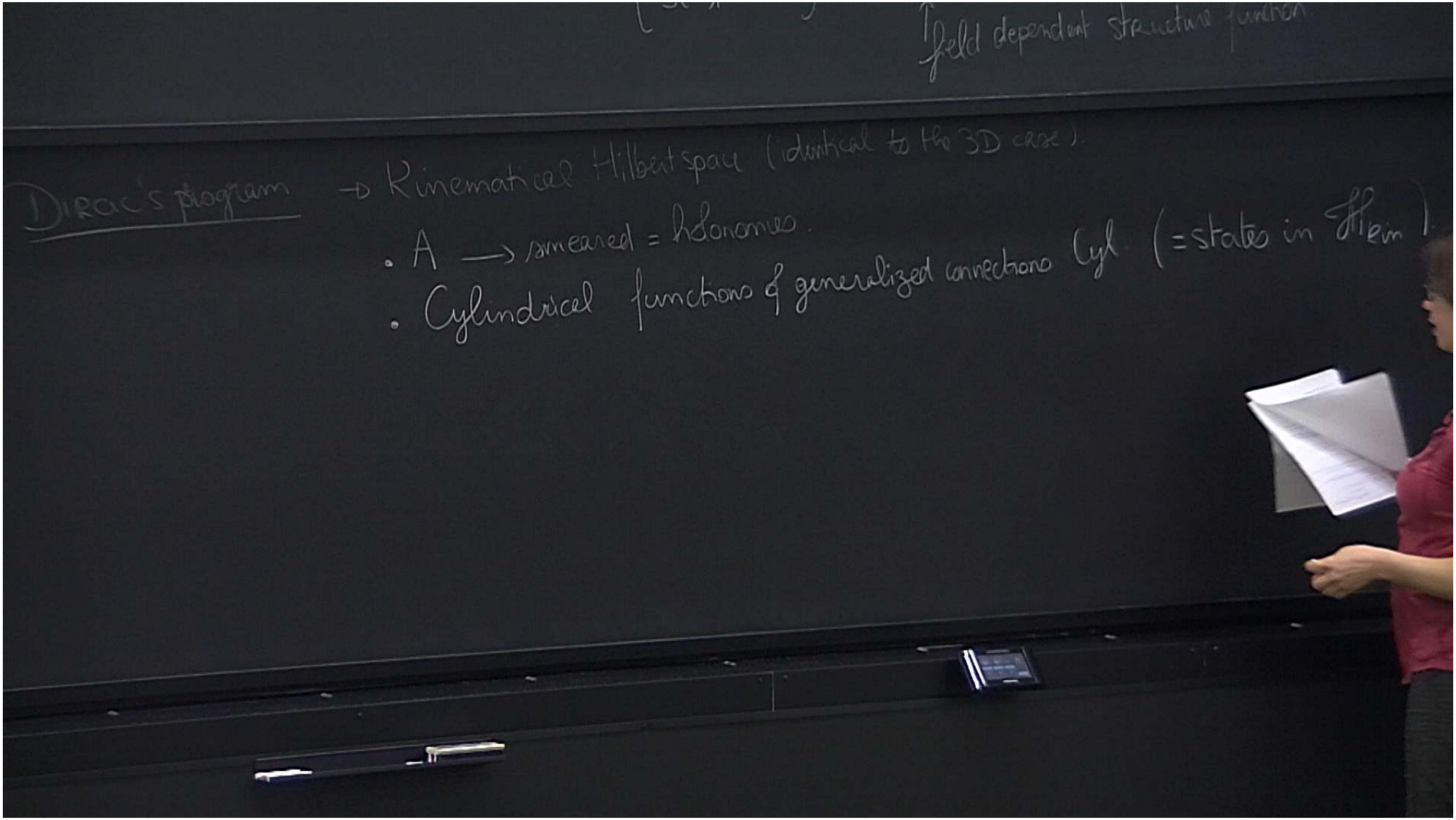
↑ field dependent structure function


field dependent structure function
Dirac's program → Kinematical Hilbert space (identical to the 3D case).
. A → smeared = holonomies.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. Cylindrical functions of generalized connections Cyl (= states in $\mathcal{H}_{kin}$).

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Dirac's program → Kinematical Hilbert space (identical to the 3D case).

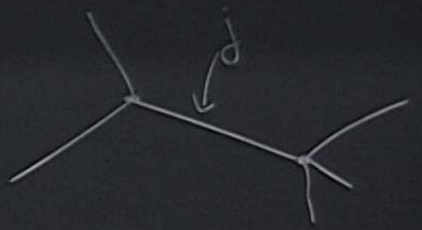
- $\bar{A}$ → smeared = holonomies.
- Cylindrical functions of generalized connections Cyl. (= states in $\mathcal{H}_{kin}$).
- Measure = Ashtekar-Lewandowski measure

$$\mu_{AL}(\psi_{\gamma,f}) = \int_{SU(2)^E} \prod_{e\in\gamma} dh_e \, f(h_{e_1}, h_{e_2} \dots h_{e_E})$$

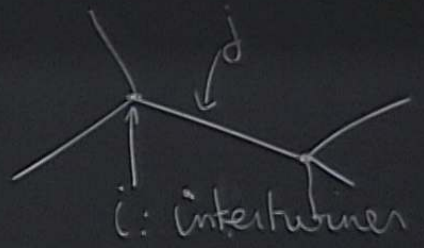
$\mathcal{H}_{kin}$

- $\langle \psi^{(1)}_{(\Gamma_1,f_1)} \,|\, \psi^{(2)}_{(\Gamma_2,f_2)} \rangle = \int d\mu_{AL}[\bar{A}]\; \overline{\psi^1_{\Gamma_1,f_1}[A]}\; \psi^2_{\Gamma_2,f_2}[A]$

• Gauss constraints ⇒ fundamental existations of spae
= spinnetwoiks

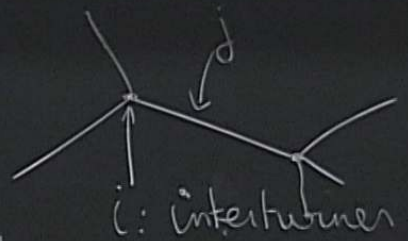


• Gauss constraints ⇒ fundamental excitations of space
= spin networks



s ⇒ fundamental existations of space

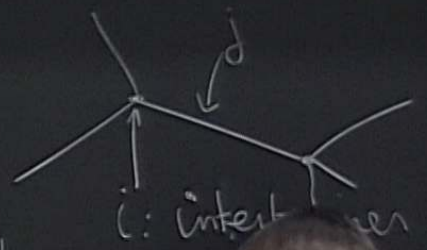
= spinnetworks



↳ form of complete basis

of the Hilbert space of solutions of the quantum Gauss law $\mathcal{H}^G_{kin}$

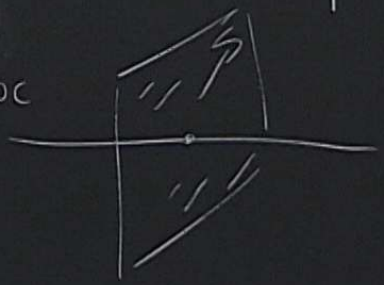
• Gauss constraints ⇒ fundamental existations of space
= spinnetworks



↳ form of complete basis
of the Hilbert space of solutions

AREA op

• function of the triad: $E^a_i \, \varepsilon_{abc}$

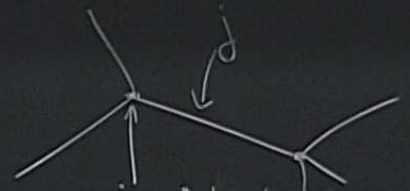


→ D area op

$\hat{A}_S$

⇒ fundamental existations of space

= spinnetwoiks



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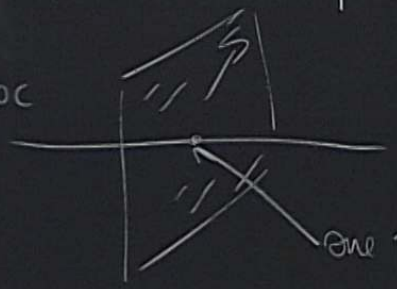
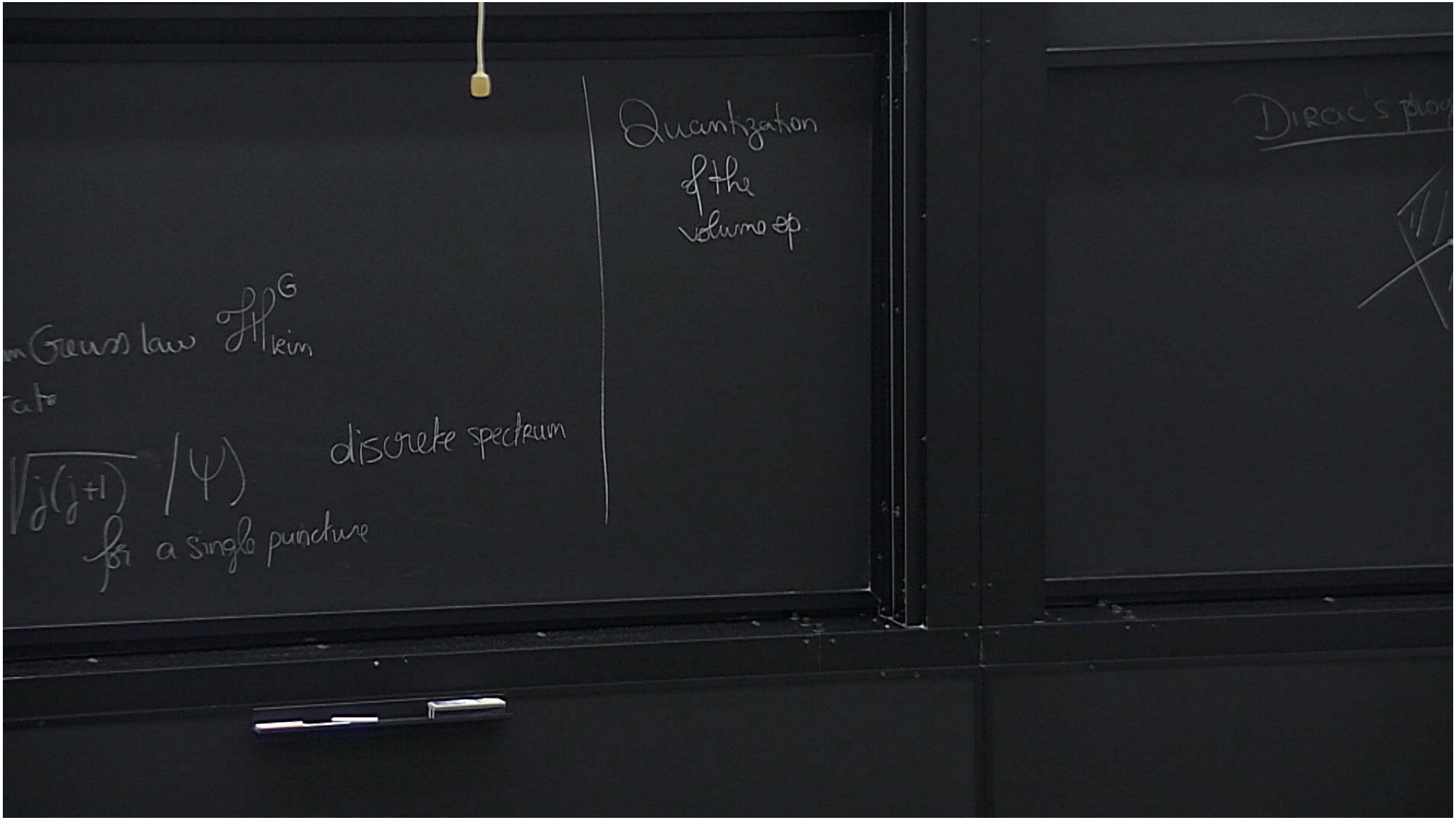


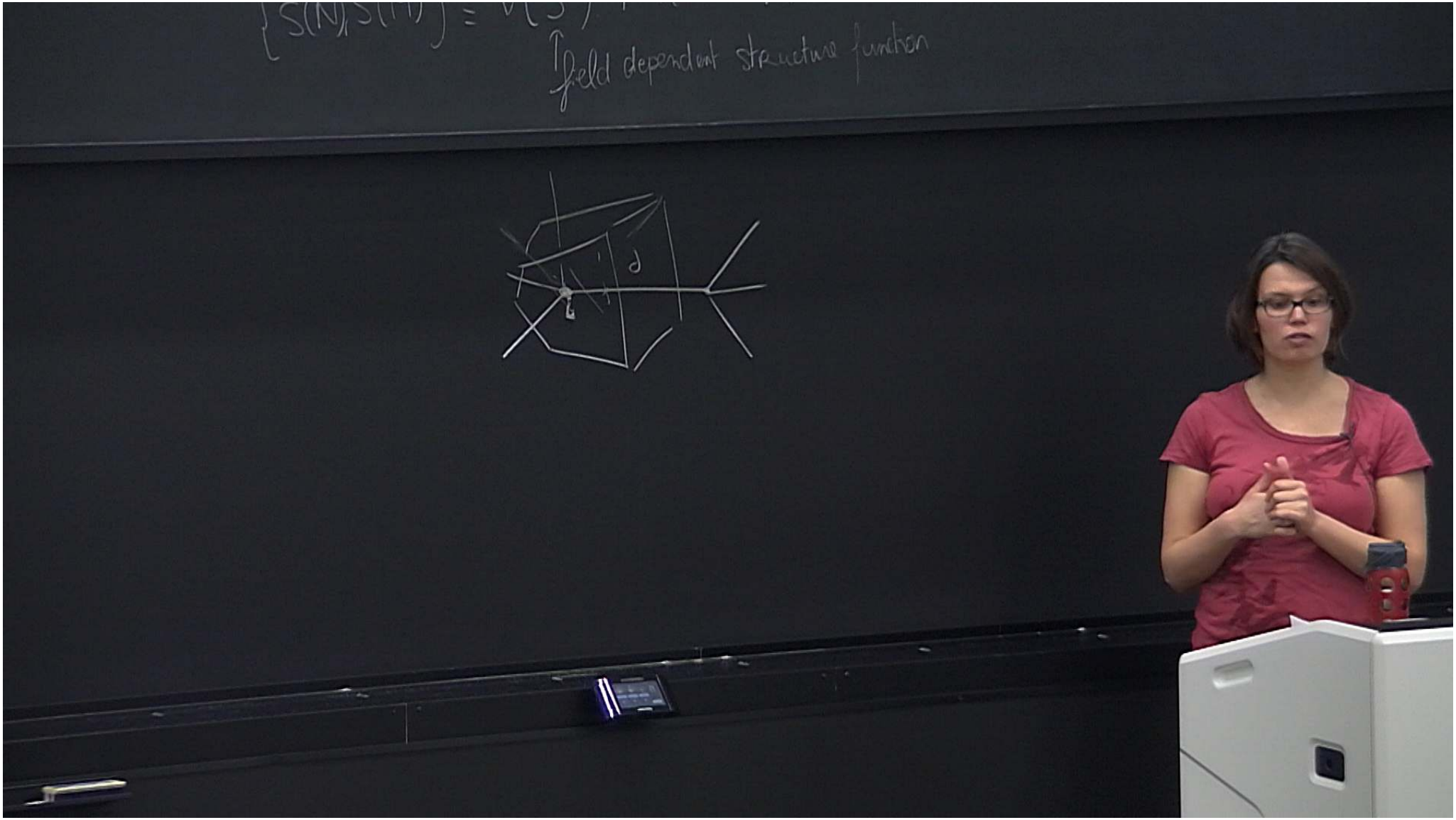
→ D area op

spinnetwork state

$$\hat{A}_S |\psi\rangle = 8\pi \ell_p^2 \gamma \sqrt{j(j+1)}\, |\psi\rangle$$

disc


Quantization
of the
volume op
Dirac's
Gauss law
discrete spectrum
for a single puncture

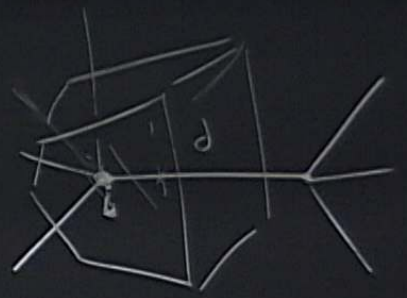

field dependent structure function

$\{S(N), S(M)\} = V(S)$

field dependent structure funct

- Spatial diffeo

$$\hat{V}^a \Psi_S[A] = 0$$



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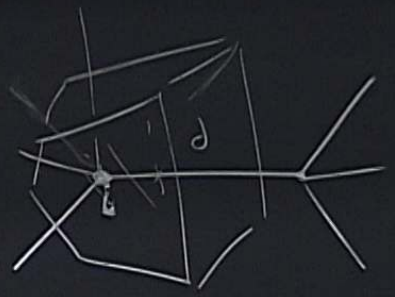


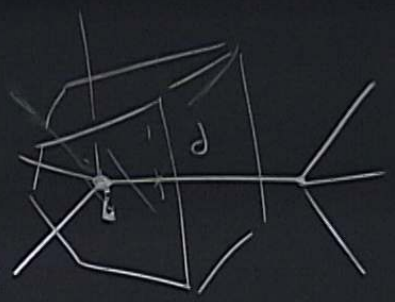
s-knots or abstract spinnetwork states

ot is an equivalence class of embedded spinnetworks S under the action of diffeo. Diff(Σ).

i.e $S, S' \in$ o if $\exists \phi \in Diff(\Sigma) / S' = \phi S$

$\{S(N), S(M)\} = V(S)$

↑ field dependent structure function



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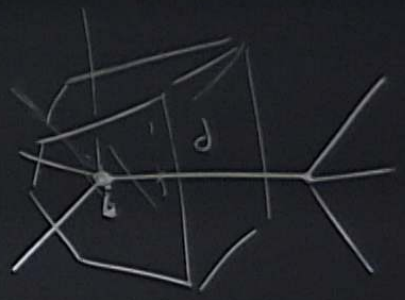
↑ field dependent structure

- Spatial diffeo

$$\hat{V}^a \Psi_S[A] = 0$$



natural basis = s-knots or abstract spinnetwork states

A s-knot is an equivalence class of embedded spinnetworks S

i.e $S, S' \in s$ if

- S → Master constraint
  → S.F.