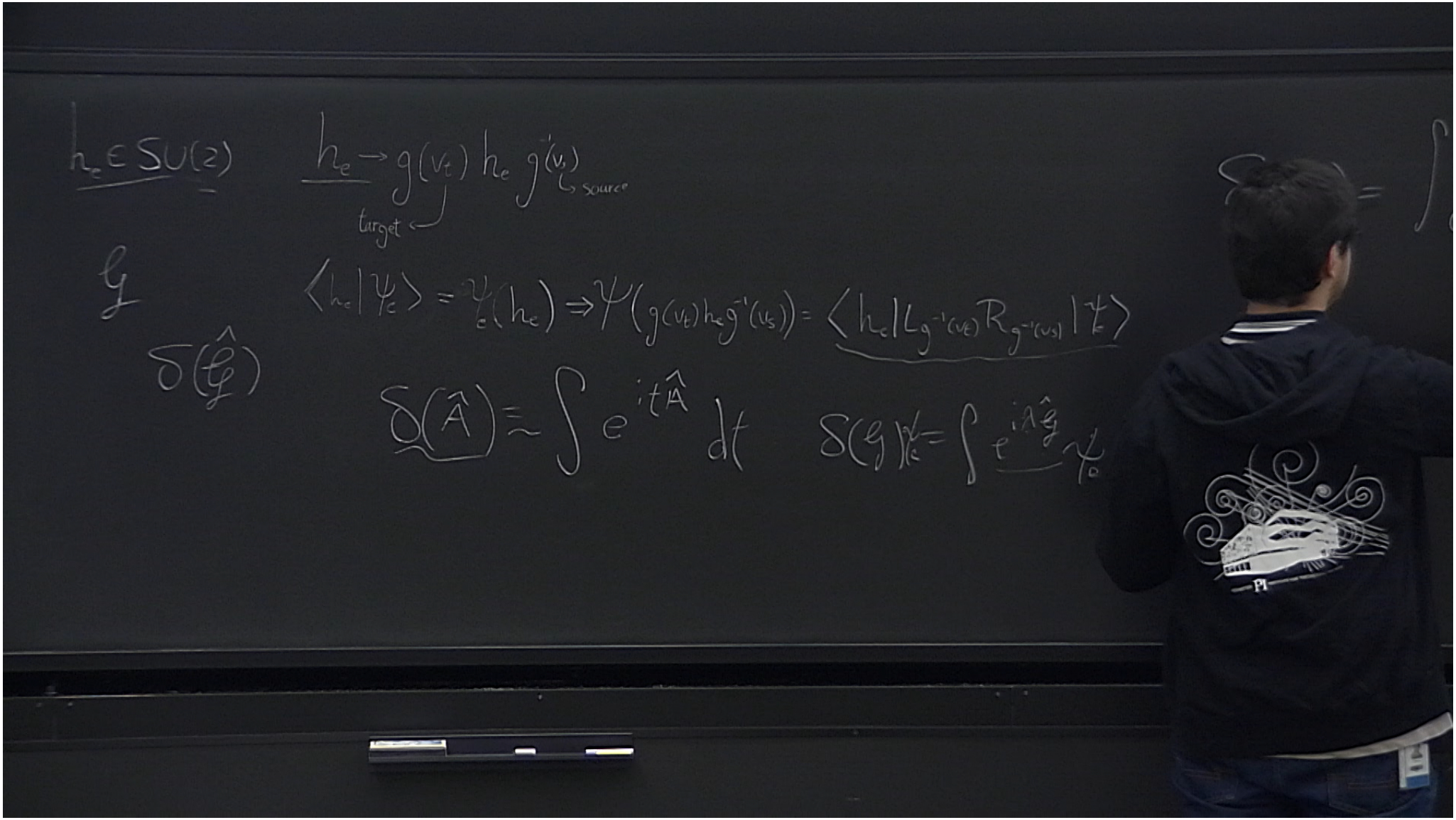
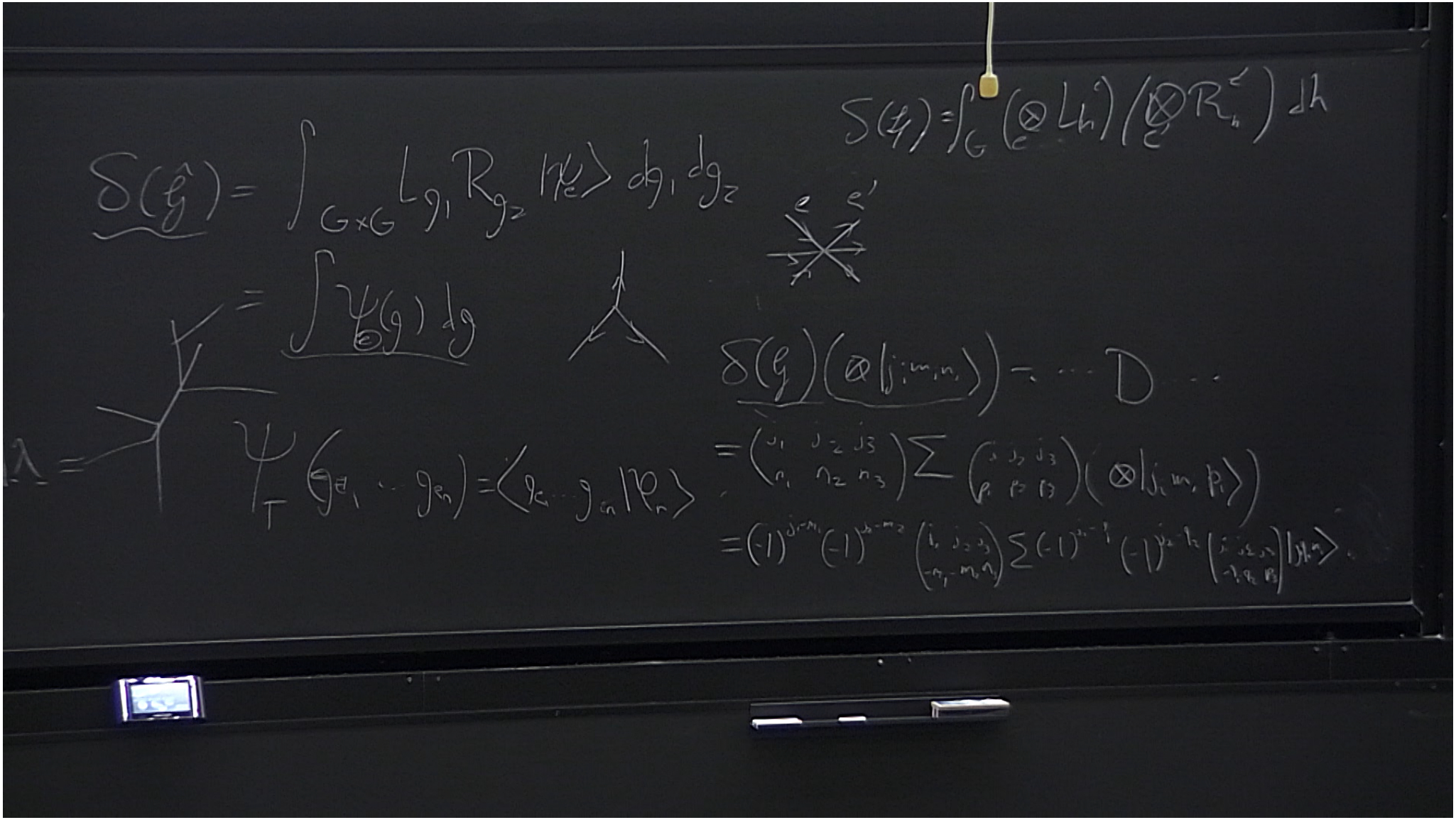
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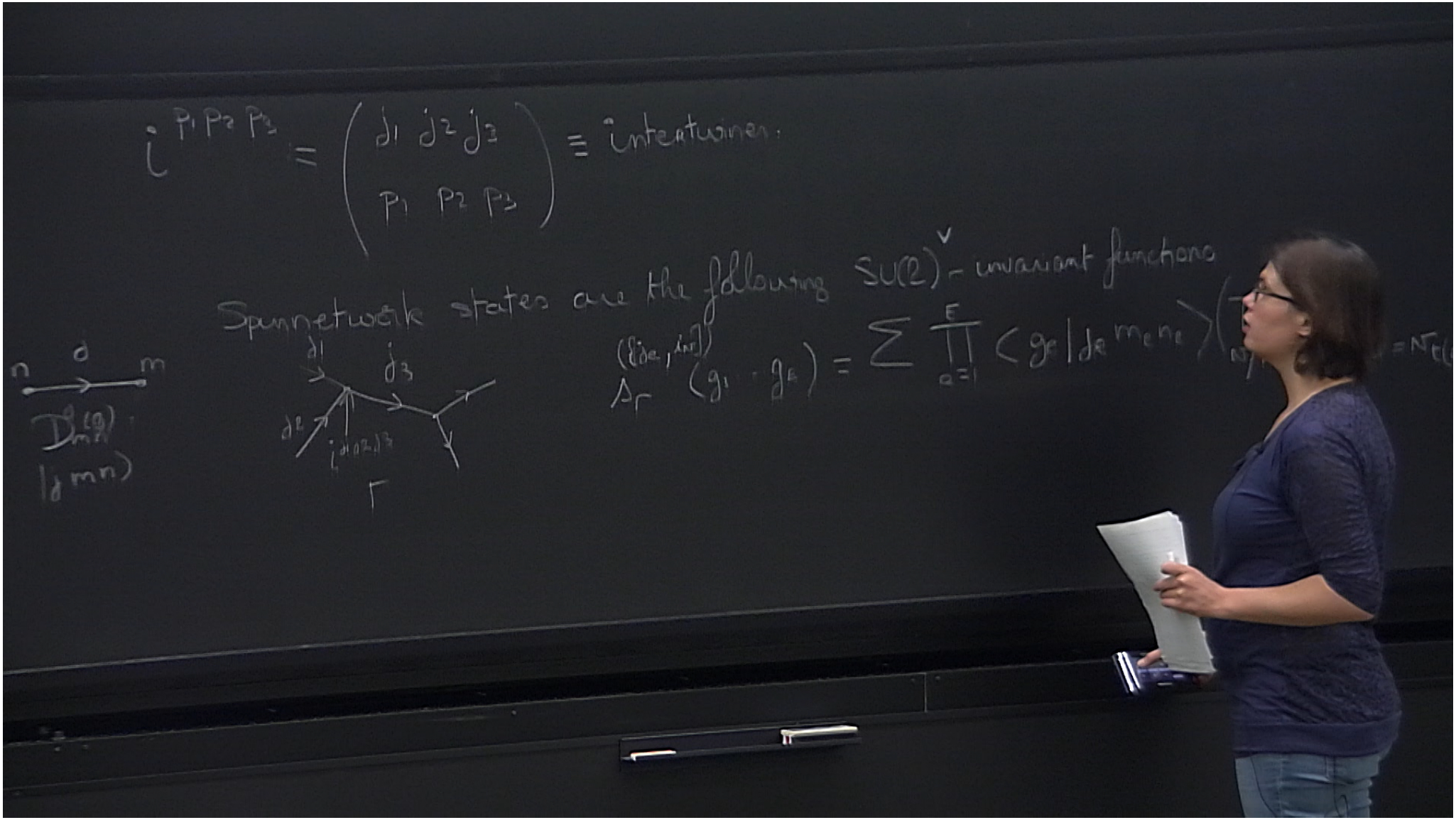
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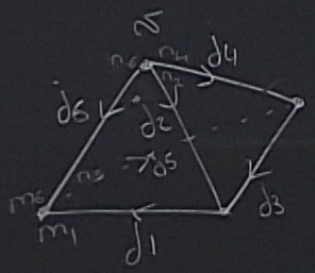
Abstract:


$h_e \in SU(2)$
$h_e \to g(v_t)\, h_e\, g^{-1}(v_s)$
target
source
$\mathcal{G}$
$\delta(\hat{\mathcal{G}})$
$\langle h_e | \psi_e \rangle = \psi_e(h_e) \Rightarrow \psi(g(v_t) h_e g^{-1}(v_s)) = \langle h_e | L_{g^{-1}(v_t)} R_{g^{-1}(v_s)} | \psi \rangle$
$\delta(\hat{A}) = \int e^{it\hat{A}}\, dt$
$\delta(\hat{\mathcal{G}})\psi_e = \int e^{i\Lambda\hat{\mathcal{G}}} \psi_e$


$\delta(\hat{g}) = \int_{G\times G} L_{g_1} R_{g_2} |e\rangle\, dg_1\, dg_2$
$= \int \psi_e(g)\, dg$
$\delta(g) = \int_G (\otimes L_h)(\otimes R_h)\, dh$
$\delta(g)(\otimes |j; m_i n_i\rangle) = \cdots D \cdots$
$= \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix} \Sigma \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} (\otimes |j_i, m_i, p_i\rangle)$
$\psi_T(g_1 \cdots g_n) = \langle e | g_1 \cdots g_n | e \rangle$

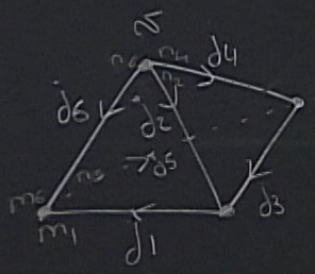

$i^{p_1 p_2 p_3} = \begin{pmatrix} j_1 & j_2 & j_3 \\ p_1 & p_2 & p_3 \end{pmatrix} \equiv$ intertwiner
Spinnetwork states are the following $SU(2)^V$ - invariant functions
$\Delta_\Gamma(g_1 \cdots g_E) = \sum_{\{j_e, i_n\}} \prod_{e=1}^{E}$
n e m
$D^{(j)}_{mn}(g)$
$|j\, m\, n\rangle$
$j_1$ $j_3$ $j_2$
$(j_1 j_2) j_3$
$\Gamma$

Concrete example 2-sphere as our canonical surface triangulated with 4 triangles which are the boundary of a tetrahedron




$$D^{\{de\}}_{tet}(g_1 \cdots g_6) = \sum ($$

Concrete example 2-sphere as our canonical surface triangulated with 4 triangles which are the boundary of a tetrahedron



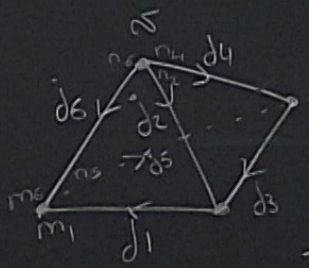

$$\partial^{\{de\}}_{tet}(g_1 \cdots g_6) = \sum \begin{pmatrix} d_6 & d_2 & d_4 \\ n_6 & n_2 & n_4 \end{pmatrix} \begin{pmatrix} d_1 & d_5 & d_6 \\ -m_1 & n_5 & -m_6 \end{pmatrix} \begin{pmatrix} d_1 & d_2 & d_3 \\ n_1 & -m_2 & -m_3 \end{pmatrix} \begin{pmatrix} d_3 \\ n_3 \end{pmatrix}$$

Concrete example 2-sphere as our canonical surface triangulated with 4 t
boundary of a tetrahedron

$\Psi(g)$

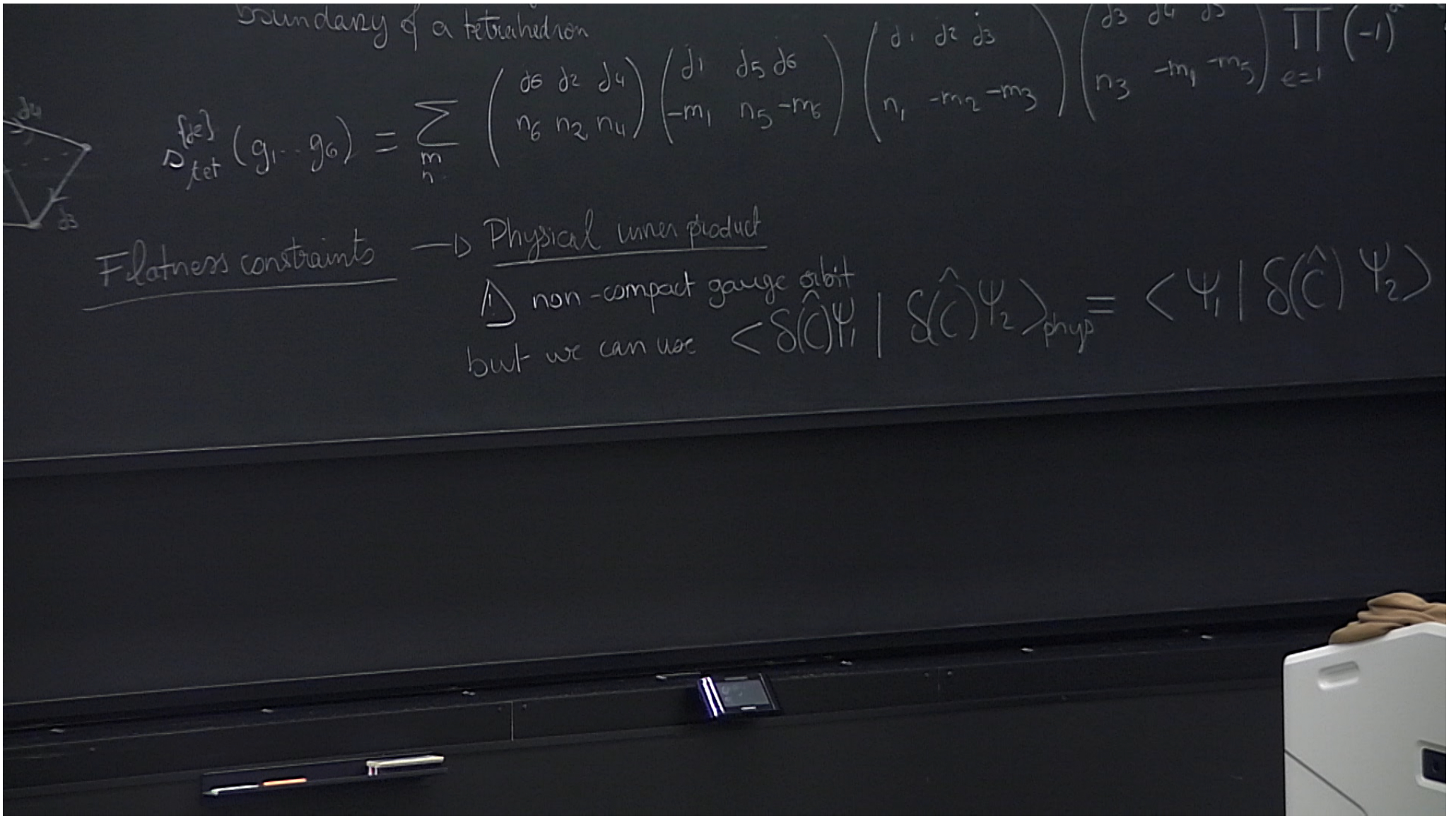
$|j\ m\ n\rangle$

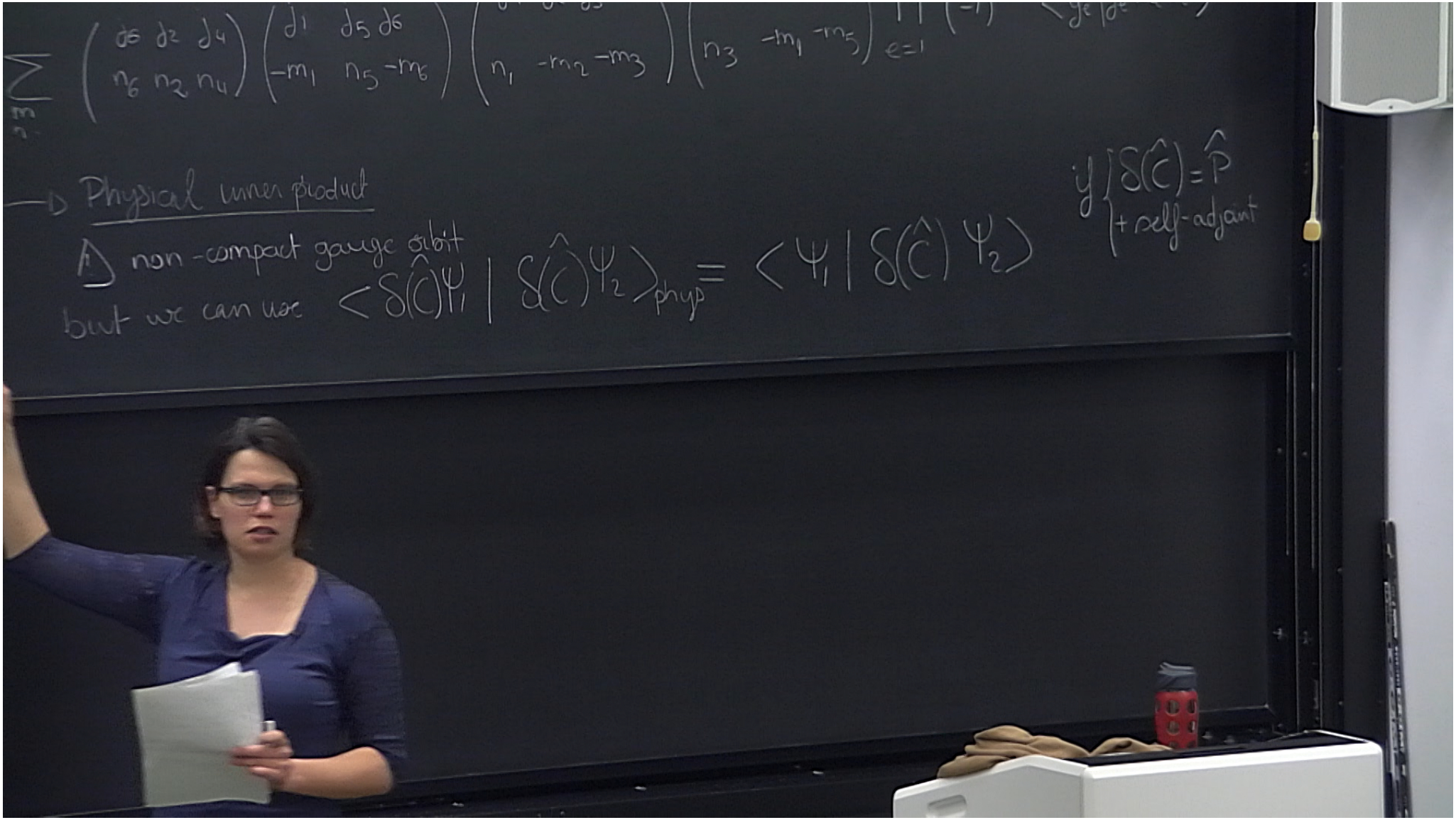
$\langle g|jmn\rangle = D^j_{mn}(g)$



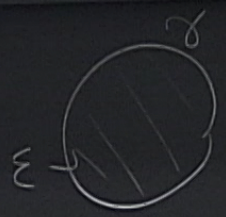

$$D^{\{j_e\}}_{\{m_e\}}(g_1 \dots g_6) = \sum_{\{n_e\}} \begin{pmatrix} j_6 & j_2 & j_4 \\ n_6 & n_2 & n_4 \end{pmatrix} \begin{pmatrix} j_1 & j_5 & j_6 \\ -m_1 & n_5 & -m_6 \end{pmatrix}$$

Flatness constraints


boundary of a tetrahedron
Flatness constraints → Physical inner product
non-compact gauge orbit
but we can use


Physical inner product
non-compact gauge orbit
but we can use $\langle \hat{\delta}(C)\Psi_1 \,|\, \hat{\delta}(C)\Psi_2 \rangle_{phys} = \langle \Psi_1 \,|\, \hat{\delta}(C)\Psi_2 \rangle$
if $\hat{\delta}(C) = \hat{P}$ + self-adjoint

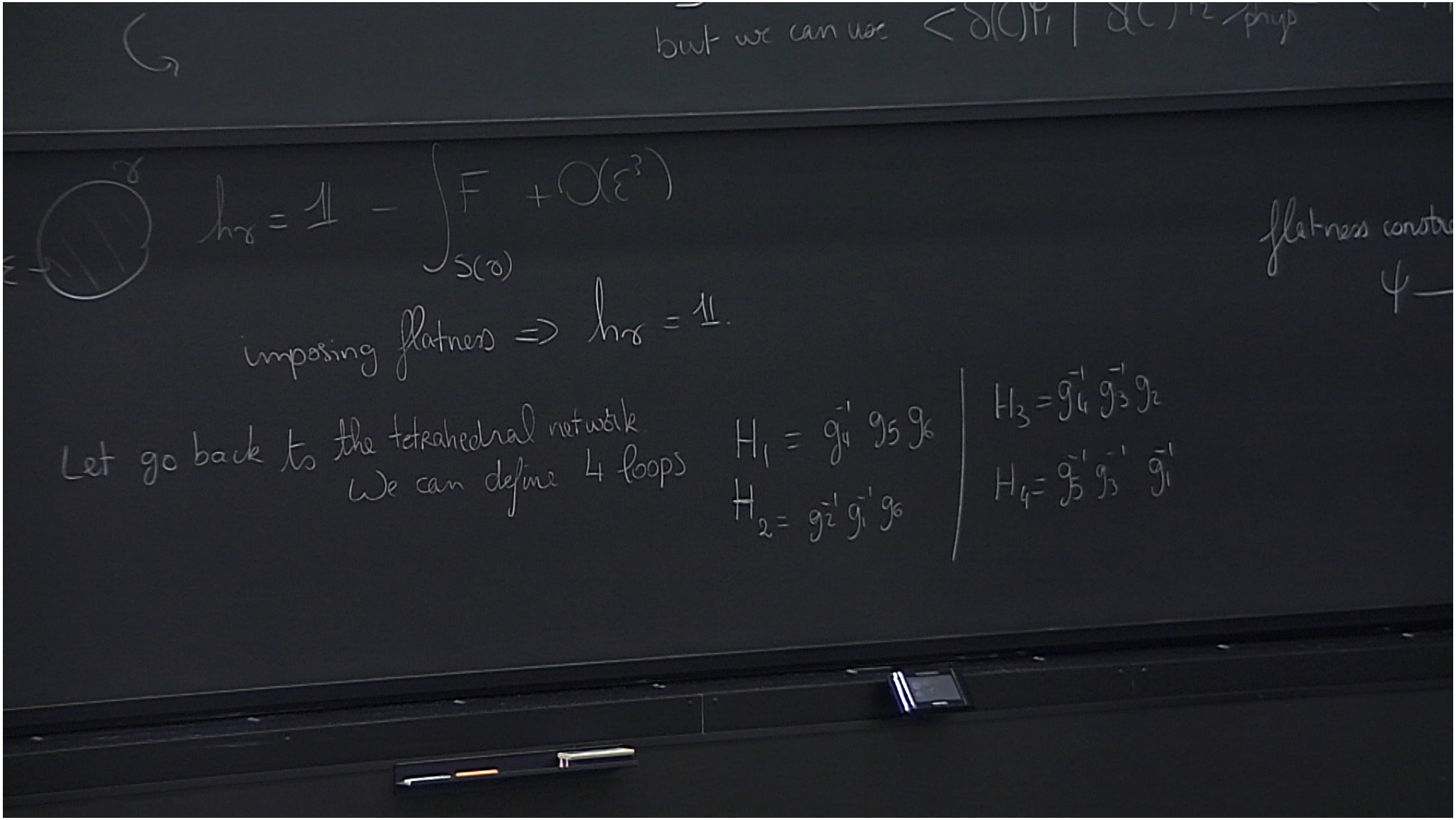
but we can use $< \delta(\Phi_i$

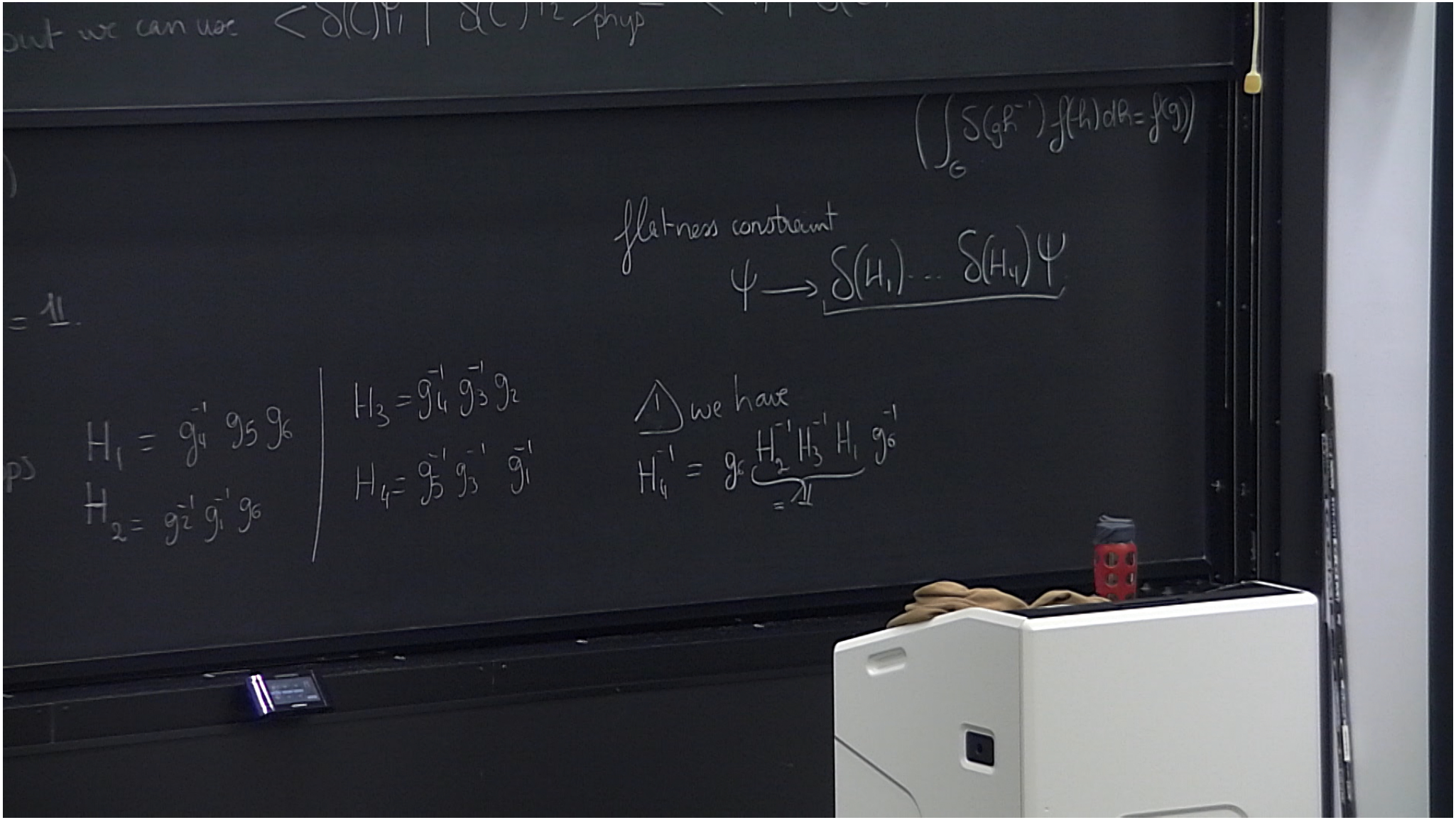


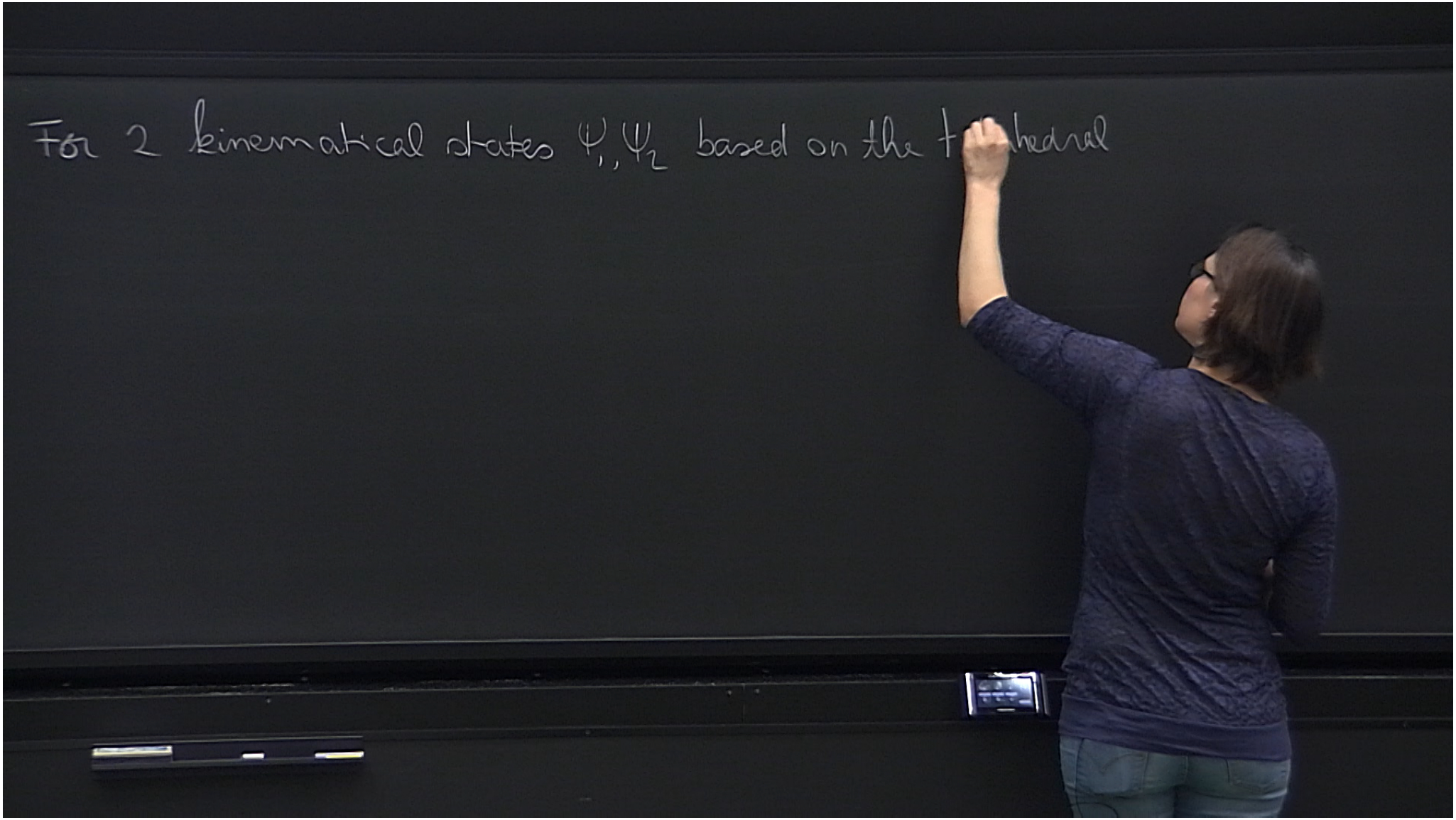

$$h_\gamma = \mathbb{1} - \int_{S(\gamma)} F + \mathcal{O}(\varepsilon^3)$$

imposing flatness $\Rightarrow h_\gamma = \mathbb{1}$.

Let go back to the tetrahedral network
We can define 4 loops $H_i =$


but we can use
$h_\gamma = \mathbb{1} - \int_{S(\gamma)} F + \mathcal{O}(\varepsilon^3)$
imposing flatness $\Rightarrow h_\gamma = \mathbb{1}$.
Let go back to the tetrahedral network
We can define 4 loops
$H_1 = g_4^{-1} g_5 g_6$
$H_2 = g_2^{-1} g_1^{-1} g_6$
$H_3 = g_4^{-1} g_3^{-1} g_2$
$H_4 = g_5^{-1} g_3^{-1} g_1^{-1}$


but we can use
flatness constraint
$\psi \longrightarrow \delta(H_1) \dots \delta(H_4)\psi$
$H_1 = g_4^{-1} g_5 g_6$
$H_2 = g_2^{-1} g_1^{-1} g_6$
$H_3 = g_4^{-1} g_3^{-1} g_2$
$H_4 = g_5^{-1} g_3^{-1} g_1^{-1}$
we have
$H_4^{-1} = g_6 H_2^{-1} H_3^{-1} H_1 g_6^{-1}$


For 2 kinematical states $\Psi_1, \Psi_2$ based on the t

For 2 kinematical states $\Psi_1, \Psi_2$ based on the tetrahedral graph satisfying t
but not the flatness constraints
physical inner pdt given us

$$< \delta(H_1)\,\delta(H_2)\,\delta(H_3)\,\Psi_1 \,|\, \delta(H_1)\,\delta(H_2)\,\delta(H_3)\,\Psi_2 >_{phy}$$

tes $\Psi_1, \Psi_2$ based on the tetrahedral graph satisfying the Gauss constraints

flatness constraints

pdt given us

$$< \delta(H_1)\,\delta(H_2)\,\delta(H_3)\,\Psi_1 \,|\, \delta(H_1)\,\delta(H_2)\,\delta(H_3)\,\Psi_2 >_{phys} = < \Psi_1 \,|\, \delta(H_1)\delta(H_2)\,\delta($$

$$= \text{Tetra}\begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} =$$

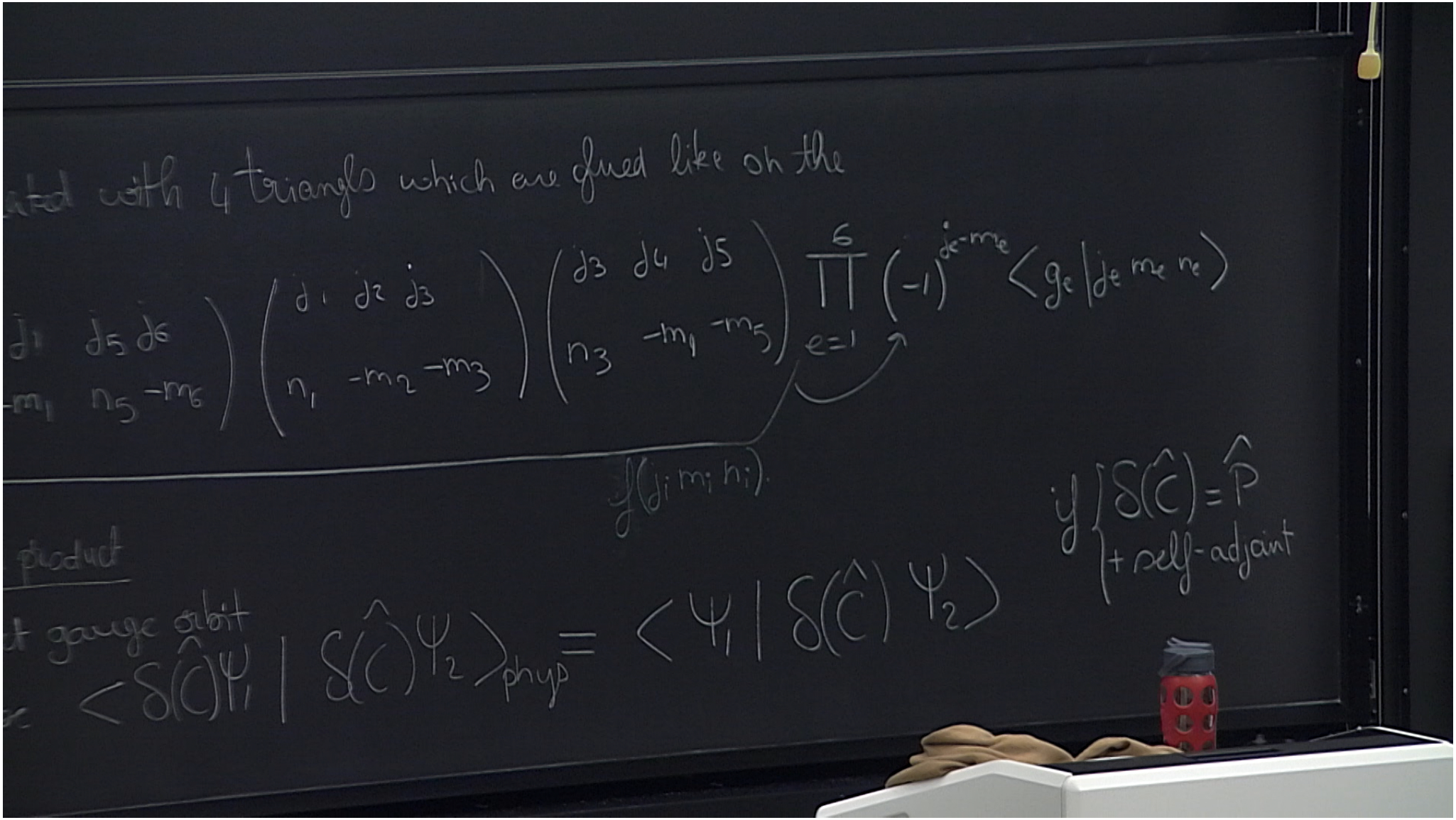
$$= \text{Tetra}\begin{bmatrix} j_1' & j_2' & j_3' \\ j_4 & j_5' & j_5' \end{bmatrix}$$

ed on the tetrahedral graph satisfying the Gauss constraints

to

$$\hat{\delta}(H_3)\Psi_1 \,|\, \hat{\delta}(H_1)\hat{\delta}(H_2)\hat{\delta}(H_3)\Psi_2\rangle_{phys} = \langle \Psi_1 \,|\, \hat{\delta}(H_1)\hat{\delta}(H_2)\hat{\delta}(H_3)\Psi_2\rangle$$

$$\left(\begin{matrix} d_3 \\ d_6 \end{matrix}\right) =$$

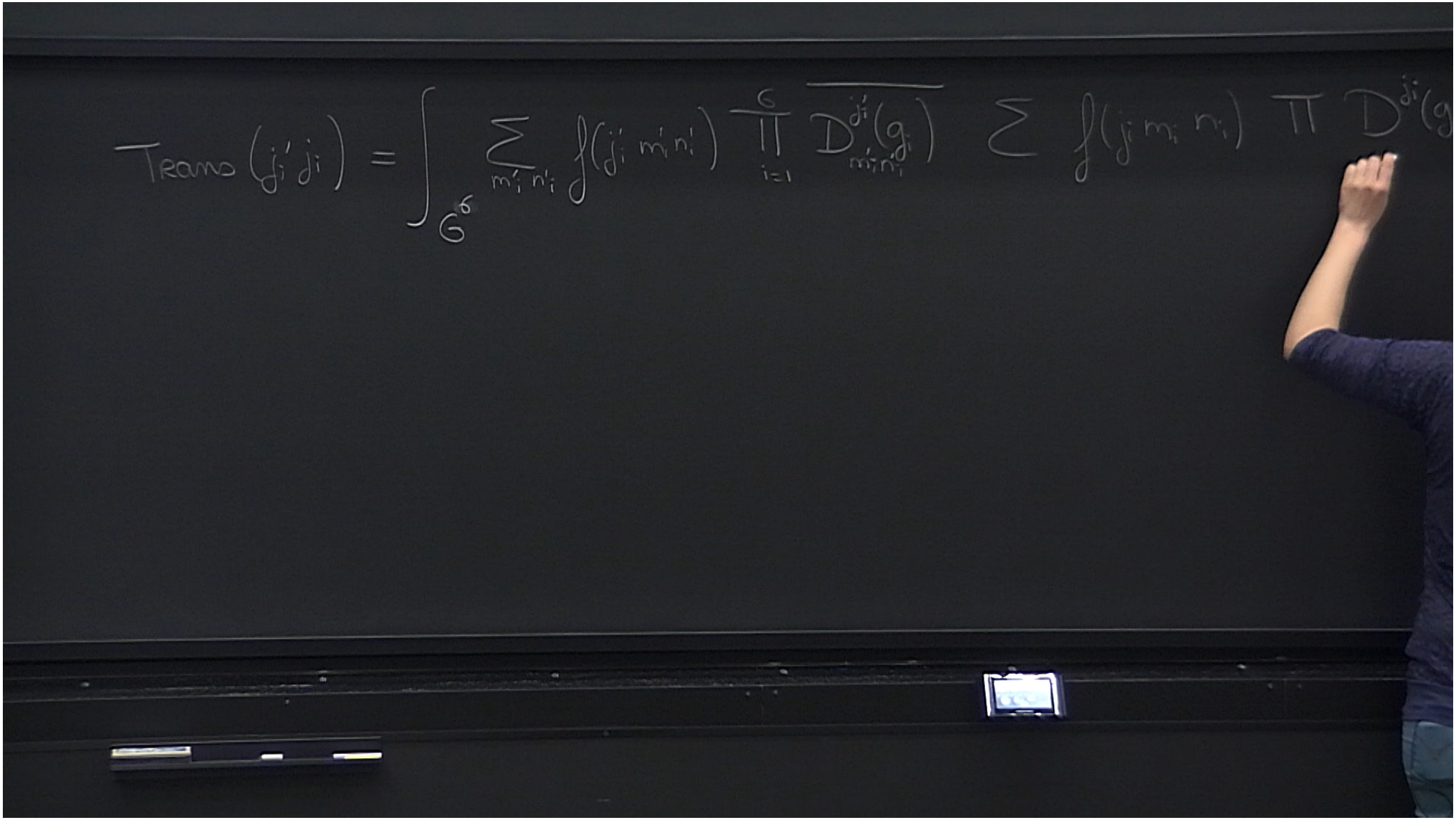

ated with 4 triangles which are glued like on the
$\begin{pmatrix} j_1 & j_5 & j_6 \\ -m_1 & n_5 & -m_6 \end{pmatrix} \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & -m_2 & -m_3 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & j_5 \\ n_3 & -m_4 & -m_5 \end{pmatrix} \prod_{e=1}^{6} (-1)^{j_e - m_e} \langle g_e | j_e m_e n_e \rangle$
$f(j_i m_i n_i)$.
product
t gauge orbit
$\langle \hat{\delta}(\hat{C}) \Psi_1 | \hat{\delta}(\hat{C}) \Psi_2 \rangle_{phys} = \langle \Psi_1 | \hat{\delta}(\hat{C}) \Psi_2 \rangle$
$\hat{y}$ { $\hat{\delta}(\hat{C}) = \hat{P}$ + self-adjoint

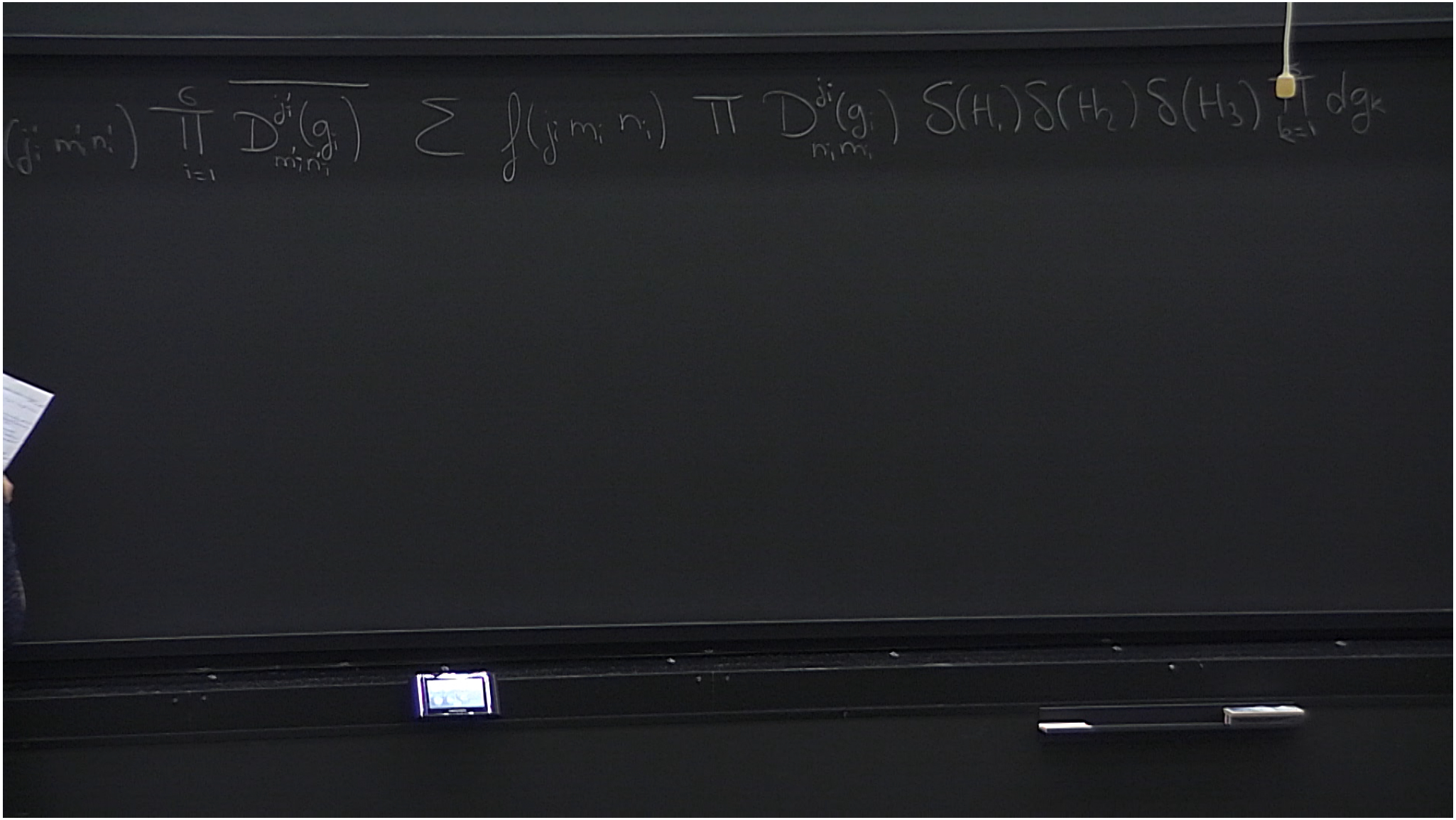
based on the tetrahedral graph satisfying the Gauss constraints

$$\delta(H_2)\,\delta(H_3)\,\Psi_1 \,|\, \delta(H_1)\,\delta(H_2)\,\delta(H_3)\,\Psi_2\rangle_{phys} = \langle \Psi_1 \,|\, \delta(H_1)\,\delta(H_2)\,\delta(H_3)\,\Psi_2$$

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{pmatrix} = \sum_{m_i} \sum_{n_i} f(j_i\, m_i\, n_i) \left( \bigotimes_{i=1}^{6} |j_i\, m_i\, n_i\rangle \right)$$

$$\begin{pmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{pmatrix} = \sum_{m_i'} \sum_{n_i'} f(j_i'\, m_i'\, n_i') \left( \bigotimes_{i=1}^{6} |j_i'\, m_i'\, n_i'\rangle \right)$$

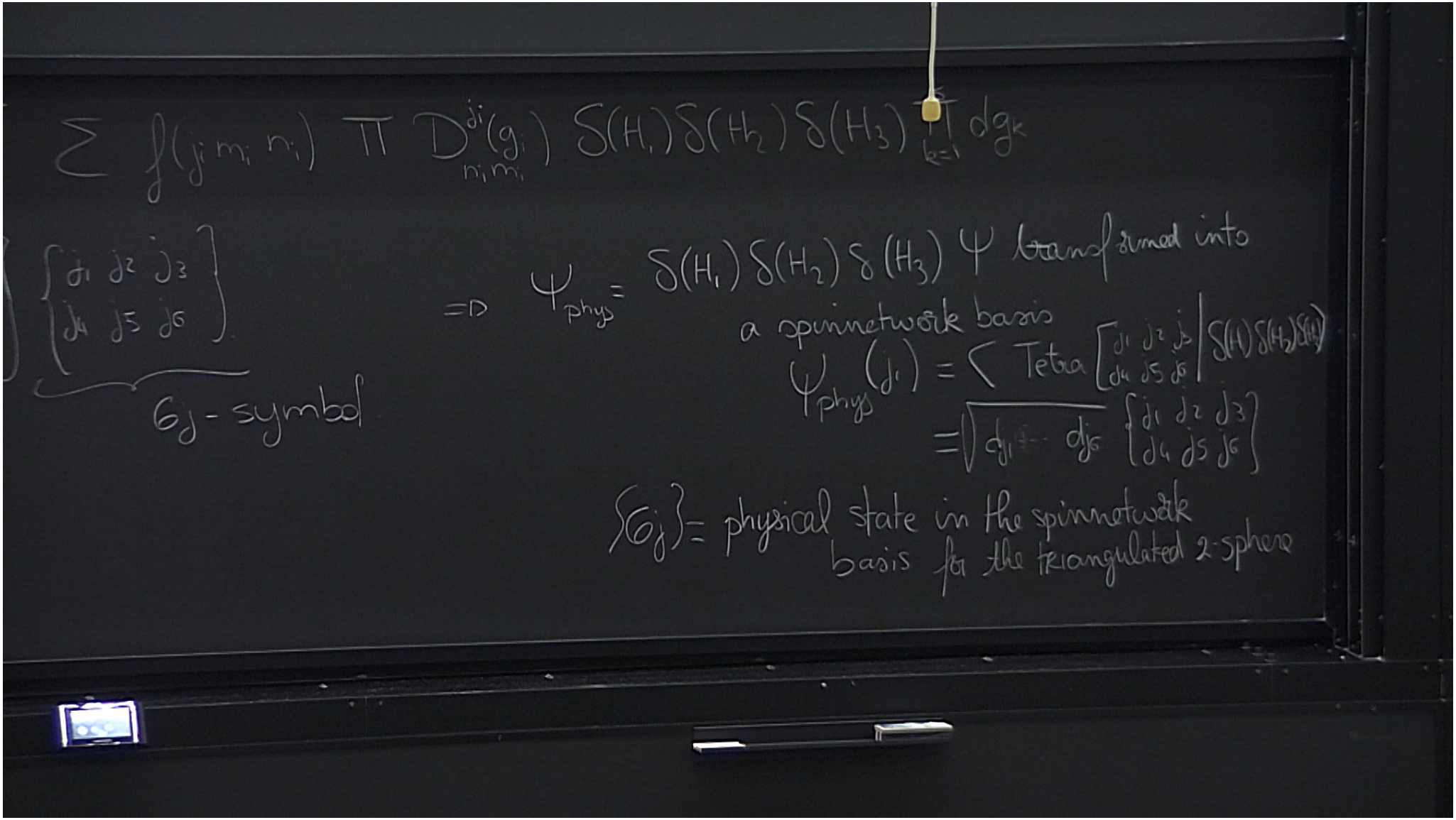

$T_{\text{trans}}(j_i' j_i) = \int_{G^6} \sum_{m_i' n_i'} f(j_i' m_i' n_i') \prod_{i=1}^{6} \overline{D^{j_i'}_{m_i' n_i'}(g_i)} \sum f(j_i m_i n_i) \prod D^{j_i}(g$


$(j_i m_i n_i) \prod_{i=1}^{6} \overline{D^{j_i}_{m_i n_i}(g_i)} \sum f(j_i m_i n_i) \prod D^{j_i}_{n_i m_i}(g_i) \delta(H_1)\delta(H_2)\delta(H_3) \prod_{k=1}^{6} dg_k$

$$\text{Trans}(j_i', j_i) = \int_{G^6} \sum_{m_i' n_i'} f(j_i' m_i' n_i') \prod_{i=1}^{6} \overline{D^{j_i'}_{m_i' n_i'}(g_i)} \sum f(j_i m_i n_i) \prod D$$

$$= \prod_{i=1}^{6} \sqrt{d_{j_i} d_{j_i'}} \begin{Bmatrix} j_1' & j_2' & j_3' \\ j_4' & j_5' & j_6' \end{Bmatrix} \underbrace{\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}}_{6j\text{-symbol}}$$

$$\langle \psi_2 | \psi_1 \rangle = \sqrt{\langle \psi_1 | \psi_1 \rangle} \sqrt{\langle \psi_2 | \psi_2}$$


$\sum f(j_i m_i n_i) \prod D^{j_i}_{n_i m_i}(g_i) \delta(H_1)\delta(H_2)\delta(H_3) \prod_{k=1} dg_k$
$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$
6j - symbol
$\Rightarrow \Psi_{phys} = \delta(H_1)\delta(H_2)\delta(H_3)\Psi$ transformed into a spinnetwork basis
$\Psi_{phys}(j_i) = \langle \text{Tetra} \begin{bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{bmatrix} | \delta(H_1)\delta(H_2)\delta(H_3) \rangle$
$= \sqrt{d_{j_1} \cdots d_{j_6}} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$
$\{6_j\}$ = physical state in the spinnetwork basis for the triangulated 2-sphere