Title: Constructing Quantum Spacetime

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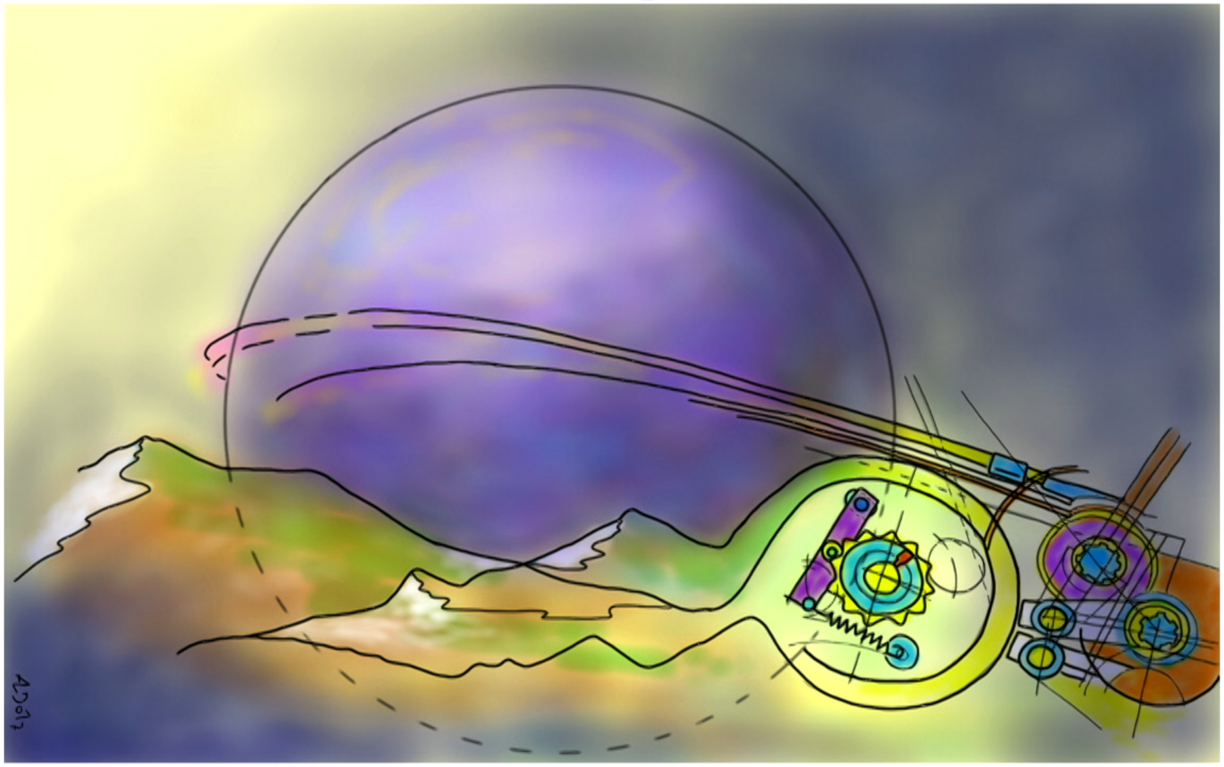
Abstract: <p>General relativity taught us that space time is dynamical and quantum theory posits that dynamical objects are quantum. Hence the Newtonian notion of space time as a passive stage where physics takes place needs to be replaced by a notion of quantum space time which is interacting with all other quantum matter fields.

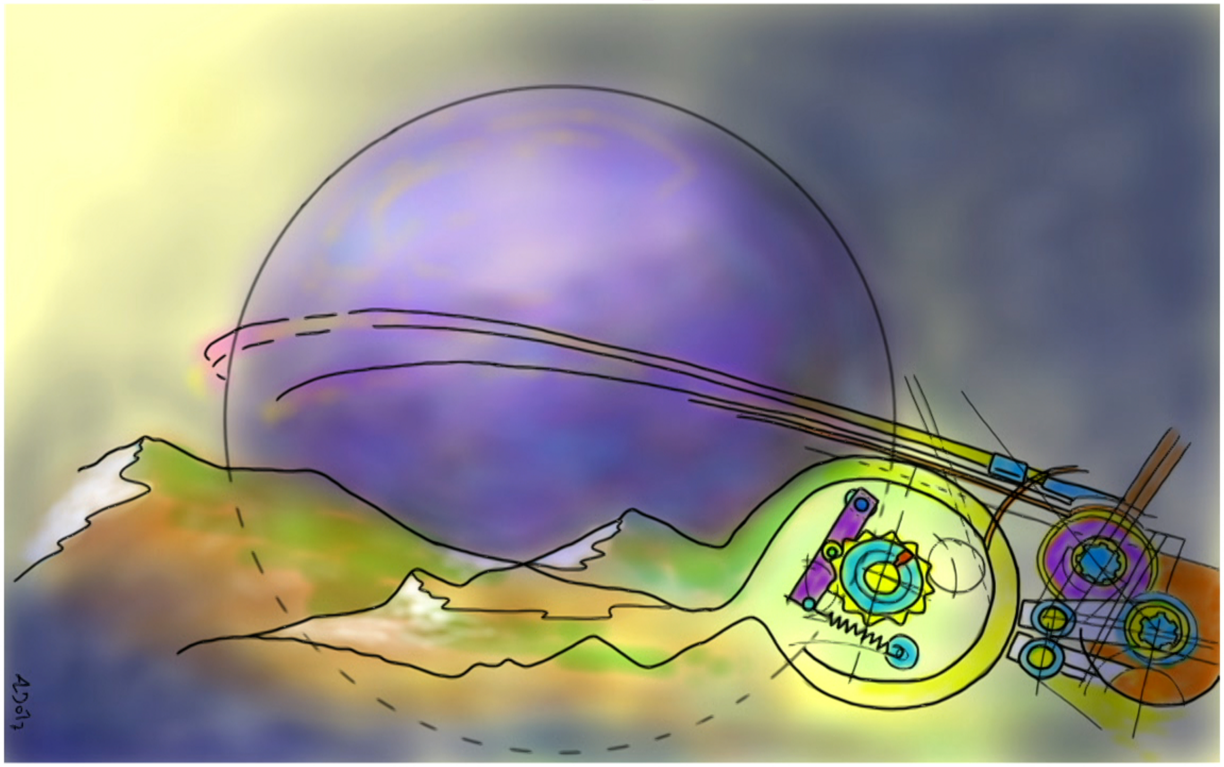
I will present recent developments that aim at constructing quantum space time. In particular I will explain how topological quantum field theories give rise to new quantum geometry realizations and how these serve as starting points for the construction of a dynamics of quantum gravity, which is consistent over all scales. Such a dynamics will then determine the properties of quantum space time. </p>

Constructing Quantum Space Time



Bianca Dittrich
Perimeter Institute

Constructing Quantum Space Time



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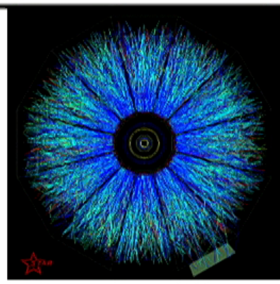
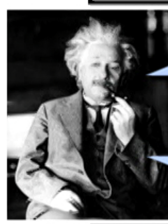
Quantum space time

Quantum (field) theory

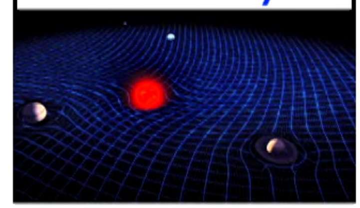


Quantum Gravity

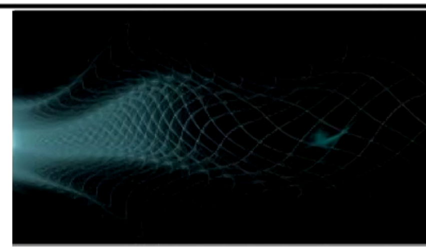


gravity = space time geometry

General Relativity



Quantum Space Time



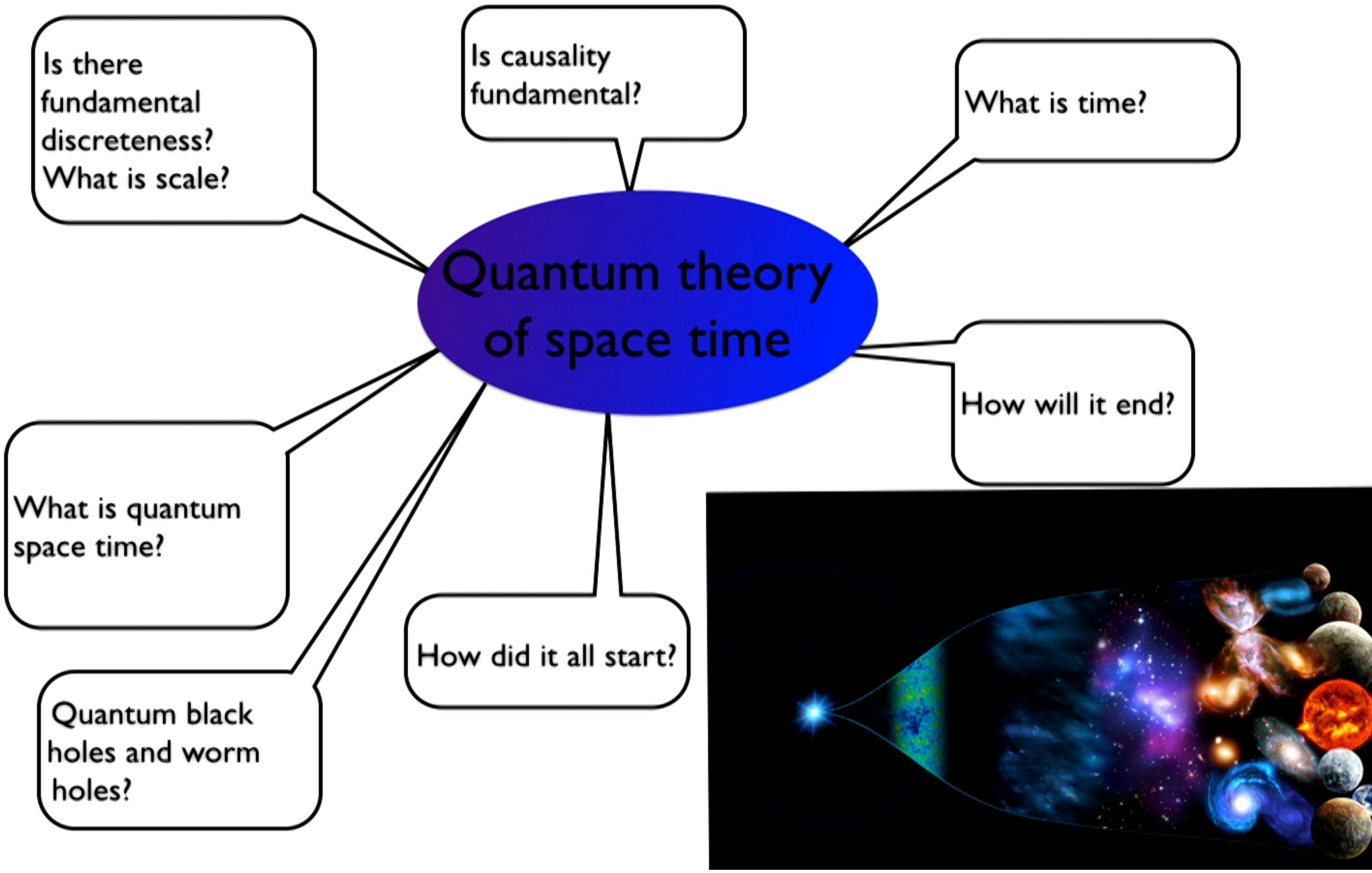
Do space time points exist?

To resolve two nearby space time points: need fast clocks, that is high energy.

A high energy density will lead to formation of black holes.

Need to change classical notion of space time.

Many exciting questions



How to construct a theory of quantum space time?

A quantum field theory needs a regulator.
But usually regulators need a space time background:

- energy: needs notion of time, breaks diffeomorphism symmetry
- lattice: assumes usually a background, breaks diffeomorphism symmetry

How to define a theory of quantum space time?

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But usually regulators need a space time background:

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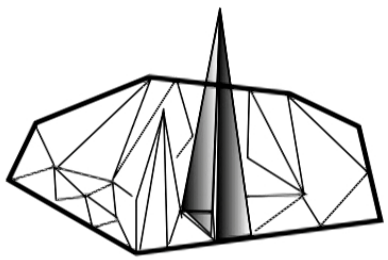
- lattice: assumes usually a background, breaks diffeomorphism symmetry

A diffeomorphism invariant regulator?
⇨Sum over geometries assigned to a (random) lattice (Quantum Regge calculus)

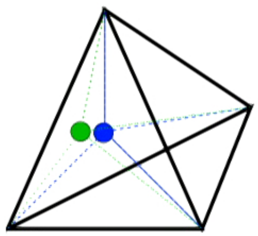
$$Z(\mathrm{bdry}) = \sum_{\mathrm{length}} \exp(iS(\mathrm{length}))$$



- Where is the cut-off? Here we seem to integrate over arbitrarily small and large length.

We will see how we can obtain a quantum induced cut-off.
In the main part of the talk.

Diffeomorphism symmetry?



- Diffeomorphism symmetry is typically broken! [Bahr, BD CQG 09]
  This cannot define a fundamental theory of quantum general relativity.

- Reason:
  Diffeomorphism action can change locally scales of discretization: probes all scales.
  Discrete amplitudes usually constructed as approximations (reliable on scales larger than the discretization scale).

Diffeomorphism symmetry requires
to construct the dynamics on all scales.

Consistent boundary formalism

[BD NJP 12, BD 14]

A formalism to construct better and better approximations to the exact dynamics of the theory.
Emphasis on boundary amplitudes allows to deal with non-local features, which are unavoidable for obtaining diffeomorphism invariance.

[BD, Kaminski, Steinhaus CQG 2014]

Important point:
Getting reliable approximations for "low excited states" (~large scales) first.
Contrary to the usual viewpoint, which rather defines theory at UV.

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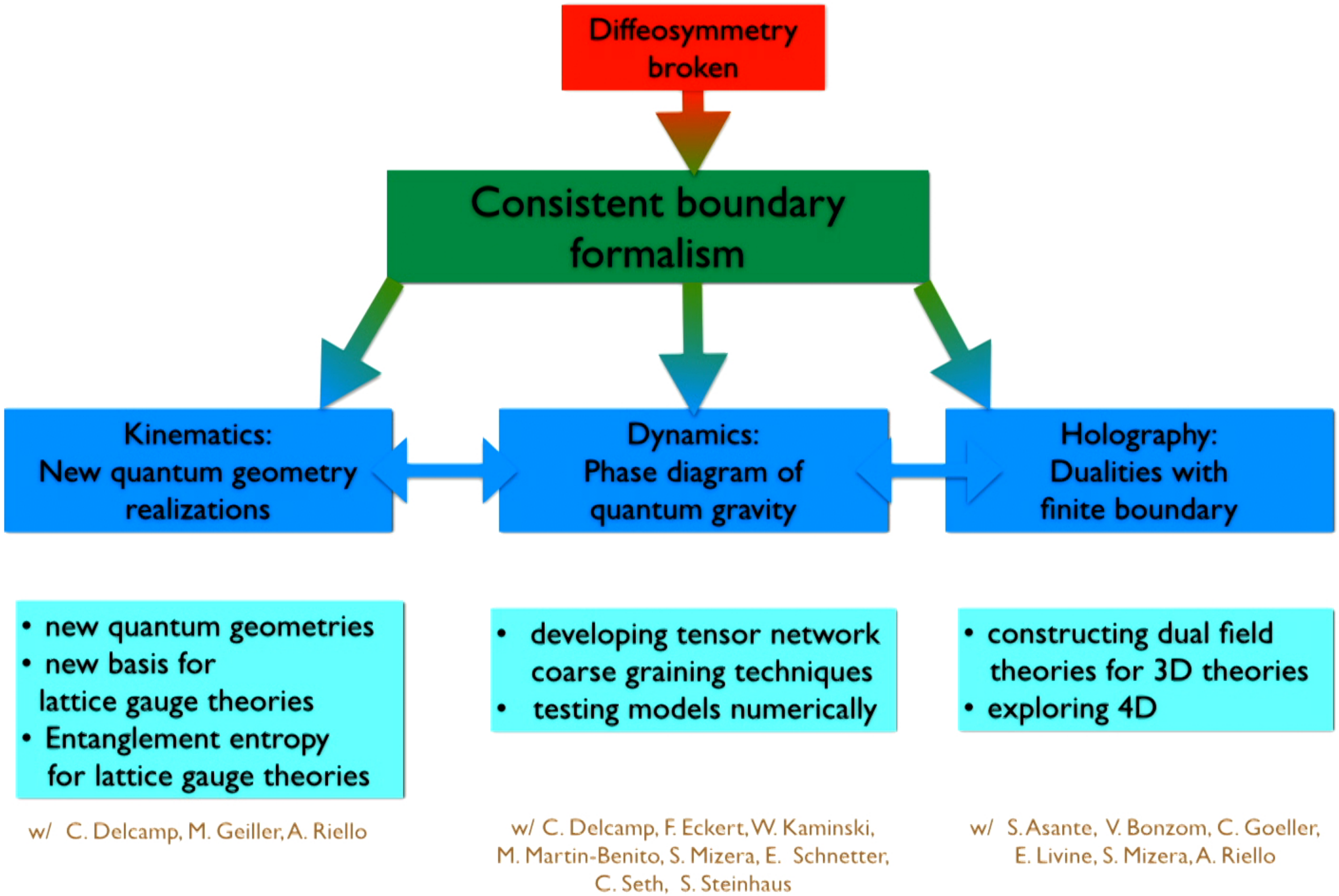
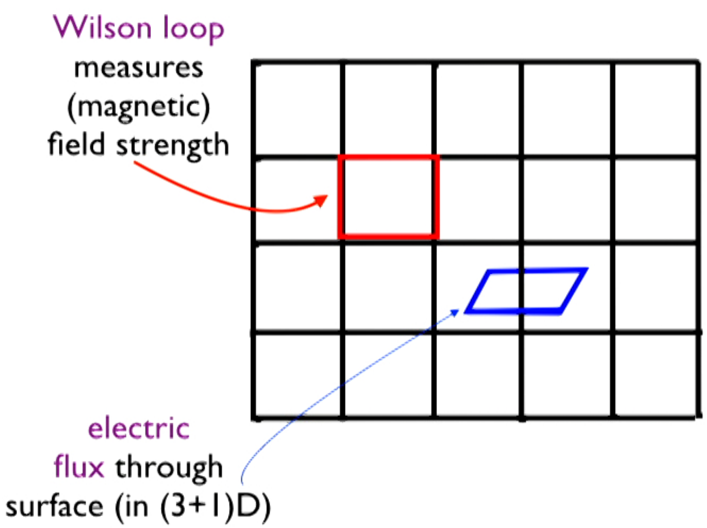
What do vacuum and low excitations mean?
In the main part of the talk.

Mind map


Diffeosymmetry broken
Consistent boundary formalism
Kinematics: New quantum geometry realizations
Dynamics: Phase diagram of quantum gravity
Holography: Dualities with finite boundary
• new quantum geometries • new basis for lattice gauge theories • Entanglement entropy for lattice gauge theories
• developing tensor network coarse graining techniques • testing models numerically
• constructing dual field theories for 3D theories • exploring 4D
w/ C. Delcamp, M. Geiller, A. Riello
w/ C. Delcamp, F. Eckert, W. Kaminski, M. Martin-Benito, S. Mizera, E. Schnetter, C. Seth, S. Steinhaus
w/ S. Asante, V. Bonzom, C. Goeller, E. Livine, S. Mizera, A. Riello

Discrete observables in a continuum theory: Hilbert space formalism

Using a lattice to pick a set of observables.

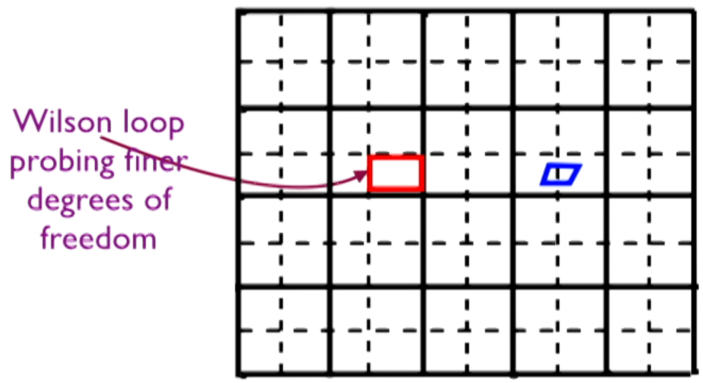


Such (generalized to SU(2)) electromagnetic variables can also describe phase space of gravity .

[Ashtekar 86]

Discrete observables in a continuum theory

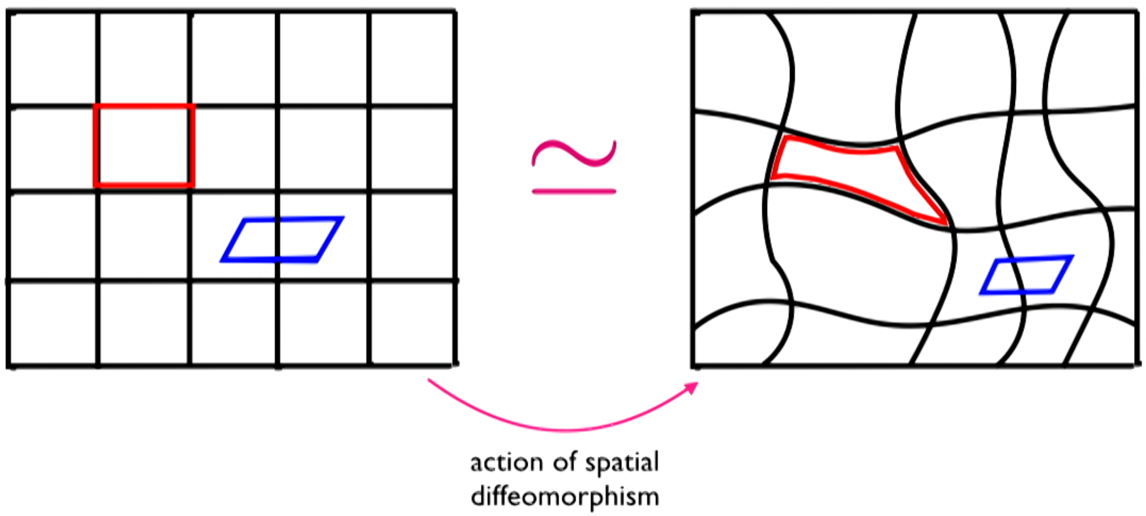
To specify a continuum state need to specify behaviour of observables on all possible lattices.



Need to specify the state for the finer degrees of freedom.
Ideally: find vacuum state.

Impose symmetries

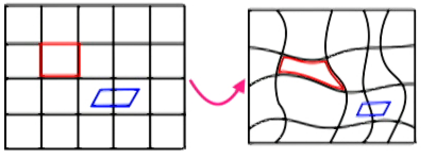
For a background independent theory we want to impose that:



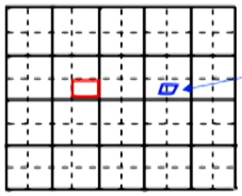
Only one choice (?)

F-LOST [Fleischhack; Lewandowski-Okolow-Sahlmann-Thiemann 06] uniqueness theorem:

- If you want a (irreducible) representation of
  - Wilson loops and electric flux operators
    (kinematical observable algebra of general relativity)
  - with an action of spatial diffeomorphism



- there is only one choice:
  - the Ashtekar-Lewandowski representation [Ashtekar-Lewandowski-Isham 92-95]
  - which prescribes that all finer electric flux operators vanish sharply

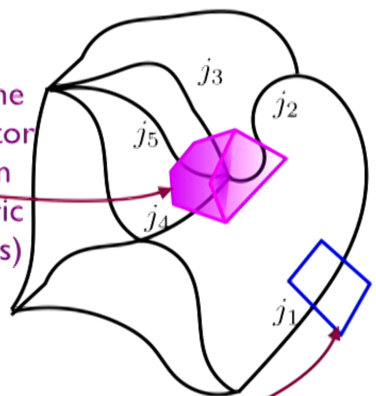


$$\hat{E}_S \, \psi = 0$$

First spatially diffeomorphism invariant realization of quantum geometry.
Since the 90's basis of loop quantum gravity.
And it seemed to be the only one.

Key accomplishment of loop quantum gravity

A quantum geometry



Determined 'typical LQG states'.

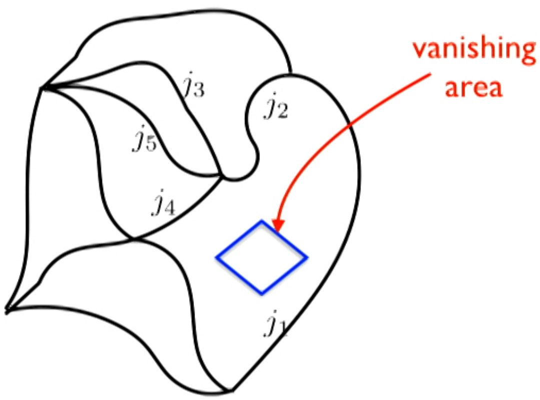
Key property and structure:

- spatial geometry operators (length, area, volume) had discrete spectra: [Rovelli,Smolin 94]

⇨Quantization-induced UV cut-off.

- spin network basis: [Rovelli,Smolin 95]
  also gauge invariant basis for lattice gauge theory, diagonalizing electric flux operators

Typical LQG states: degenerate geometries



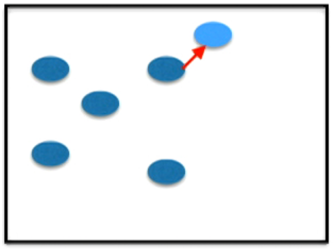
States are generated (by Wilson loop operators) from a vacuum state, describing totally degenerate spatial geometry E=0.

States describing 'smooth geometry' need to be very highly excited states.

Makes investigations of large scale physics extremely difficult.

Is there another possibility?

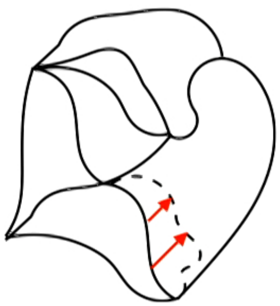
Topological QFTs with defects



Topological QFT: do not need background metric.

Defects: by definition do not interact and thus do not see the distance between each other.

⇨Ideal setting for the kinematics of a background independent theory.
Defects encode degrees of freedom of a gravitational theory.



Indeed one can interpret the Ashtekar-Lewandowski representation as a

- (trivial) TQFT which imposes vanishing electric fluxes
- where arbitrarily many line defects (non-vanishing fluxes) are allowed

[BD NJP 2012; BD, Steinhaus NJP 2014]

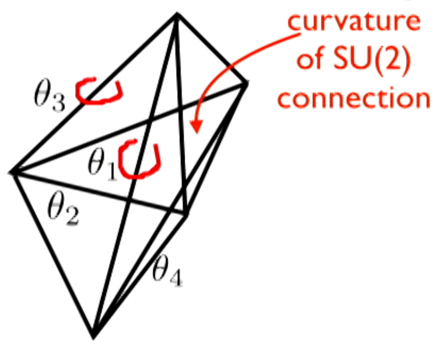
We can use TQFTs with suitable defect structure for the construction of quantum geometry realizations by:

- using TQFT amplitudes to define vacuum state and associated refinement maps

A new quantum geometry realization

[BD, Geiller CQG 2014; BD, Geiller CQG 2015; Bahr, BD, Geiller 2015]

A very different quantum geometry



curvature basis: eigenstates of Wilson loop operators

Based on BF theory:
vacuum state peaked on vanishing curvature F=0.

States with curvature defects are generated by exponentiate electric flux operators (t'Hooft operators) from the vacuum state.

For (2+1)D: solution to dynamics (with point particles).

Can now describe large scale geometries as low excited states.

Much stronger connection to spin foams, which provides a covariant definition of dynamics for loop quantum gravity.

Spectra of geometric observables

Lattice gauge theory and Ashtekar-Lewandowski representation

Phase space

Configuration space:
curvature via holonomies -
exponentiated connection

Momentum space:
electrical fluxes

$$\exp \int_{\text{path}} A \quad \in \mathrm{SU}(2)$$

$$\int_S E \ \in \ (\mathrm{su}(2))^*$$

compact and curved

non-compact and flat

Holonomy operators:
continuous spectra and commutative

electric flux operators:
discrete spectra and non-commutative

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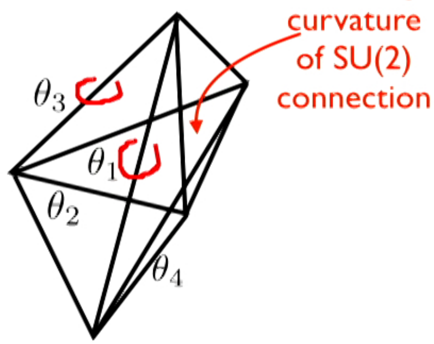

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Spectra of geometric observables

The BF representation

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exponentiated connection

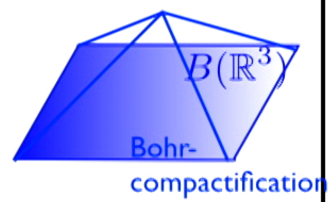
$$\exp \int_{\text{path}} A \quad \in \mathrm{SU}(2)$$

compact and curved

Momentum space:
exponentiated electrical fluxes

circumvents
uniqueness
theorem

$$\exp \alpha \int_S E$$

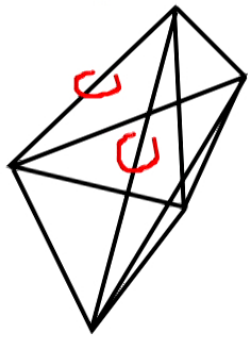


compact

exp. electric flux operators:
spectra looking discrete but
mathematically continuous (mostly)

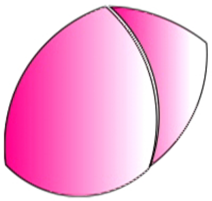
Is there a more natural way of compactifying?

BF representation:



- piecewise flat geometry
- pieces can be arbitrary large
- appropriate for approximating general relativity without a cosmological constant (Regge calculus)

Wish list for the new representation:



- piecewise homogeneously curved geometry
- with positive curvature: pieces cannot be arbitrarily large: Induces a compactification!
- appropriate for approximating general relativity with a cosmological constant (Regge calculus with curved building blocks)

[Bahr, BD NJP 09]

Can we implement this in the quantum theory?

Yes, we can!

[BD, Geiller NJP 2017] **A (2+1)D quantum geometry based on Turaev-Viro TQFT**

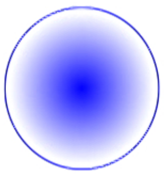
Configuration space:
curvature via holonomies -
exponentiated connection

SU(2)

compact and curved

Momentum space:
exponentiated electrical fluxes

SU(2)



compact and curved

Holonomy operators:
spectra discrete and bounded,
operators non-commutative

exp. electric flux operators:
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Yes, we can!

[BD, Geiller NJP 2017]

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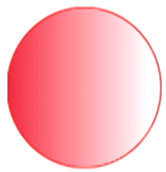

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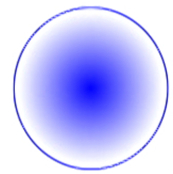

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Feeding two birds with one scone:

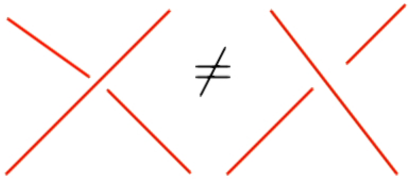
- **UV and IR finiteness: discrete and bounded spectra for all geometric operators**
- **implementation of a (positive) cosmological constant**

Two main achievements:

- UV and IR finiteness:
  discrete and bounded spectra for all geometric operators
- implementation of a (positive) cosmological constant

But what about (3+1)D?

Towards (3+1)D:

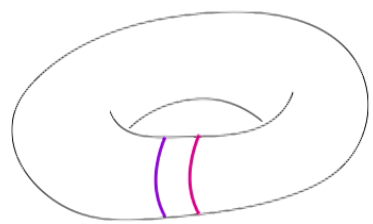


Two problems for (3+1)D:

- the non-commutativity of Wilson loops needs 2D space:
  How to generalize to three dimensions?

- need a polarization: set of commuting operators

  →connection and flux combined to a pair of new (still non-commuting) connections
  →parallel Wilson loops in these two connections define a polarization (in (2+1)D)

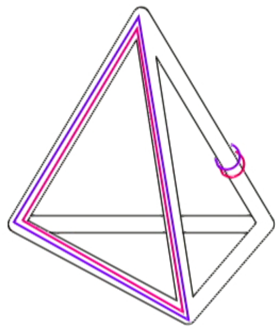


Non-contractible curves encode degrees of freedom.

How to apply this in (3+1) dimensions?

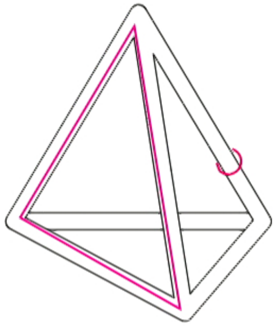
Lifting (2+1)D TQFTs to (3+1)D theories

[Delcamp, BD JMP 2017, BD JHEP 2017]



- Consider surface surrounding a defect graph in 3D: here tetrahedron.
- Use state space of the (2+1)D Turaev-Viro theory for this surface.

- Impose constraints that ensure:
  No degrees of freedom are associated to curves contractible in surrounding 3D space.

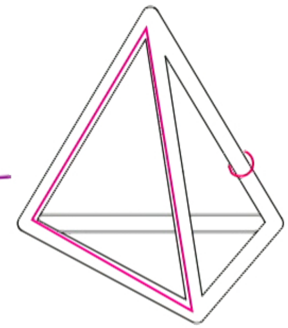


- These constraints trivialize one copy in the pair of Wilson loops.

  (We are left with the state space of the Witten-Reshetikhin-Turaev TQFT.)

A finite and self-dual quantum geometry

[BD JHEP 2017]



Wilson loops around a face measure area of this face.

Wilson loops around an edge measures (excess) curvature on this edge.

Diagonalized by spin network basis.

generalized S-trafo ⟷

Diagonalized by curvature basis.

Spectrum of (exponentiated) area and curvature operator

$$\mathrm{k} = \frac{6\pi}{\ell_p^2 \Lambda}$$

$$\frac{\sin\left(\frac{\pi}{\mathrm{k}+2}(2j+1)(2l+1)\right)\sin\left(\frac{\pi}{\mathrm{k}+2}\right)}{\sin\left(\frac{\pi}{\mathrm{k}+2}(2j+1)\right)\sin\left(\frac{\pi}{\mathrm{k}+2}(2l+1)\right)} \xrightarrow{\mathrm{k}\to\infty} 1 - \frac{8}{3}j(j+1)\,l(l+1)\left(\frac{\pi}{\mathrm{k}+2}\right)$$

What does this achieve?

- A new family of (3+1)D quantum geometry realizations based on vacuum peaked on homogeneously curved geometry: Crane-Yetter TQFT.

- Rigorous implementation of quantum group structure into (3+1)D LQG.

  [Smolin, Major, Noui, Perez, Pranzetti, Dupuis, Girelli, Bonzom, Livine, Haggard, Han, Kaminski, Riello, Rovelli, Vidotto, ...]

  Quantum group: $\mathrm{SU}(2)_\mathrm{k}$

  where $$\mathrm{k} = \frac{6\pi}{\ell_p^2 \, \Lambda}$$

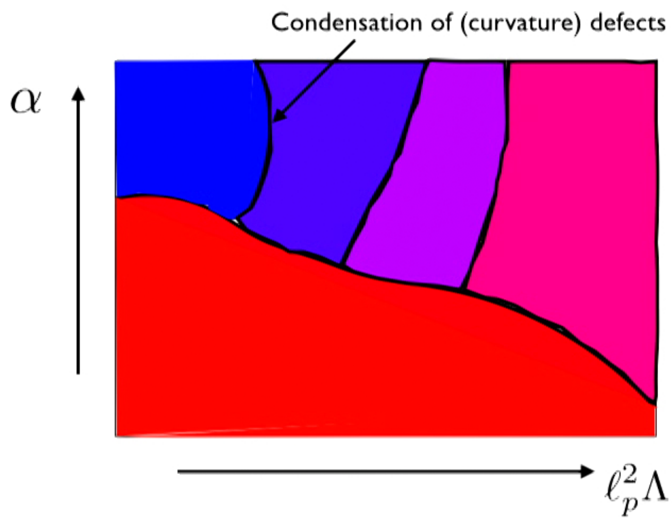
  [Smolin, Major 95]

- Hilbert spaces (associated to fixed triangulations/ graphs) are finite dimensional:
  - which is important for (numerical) coarse graining efforts.
  - and leads to geometric operators having discrete and bounded spectra: UV and IR cut-off

- New bases adapted to coarse graining.

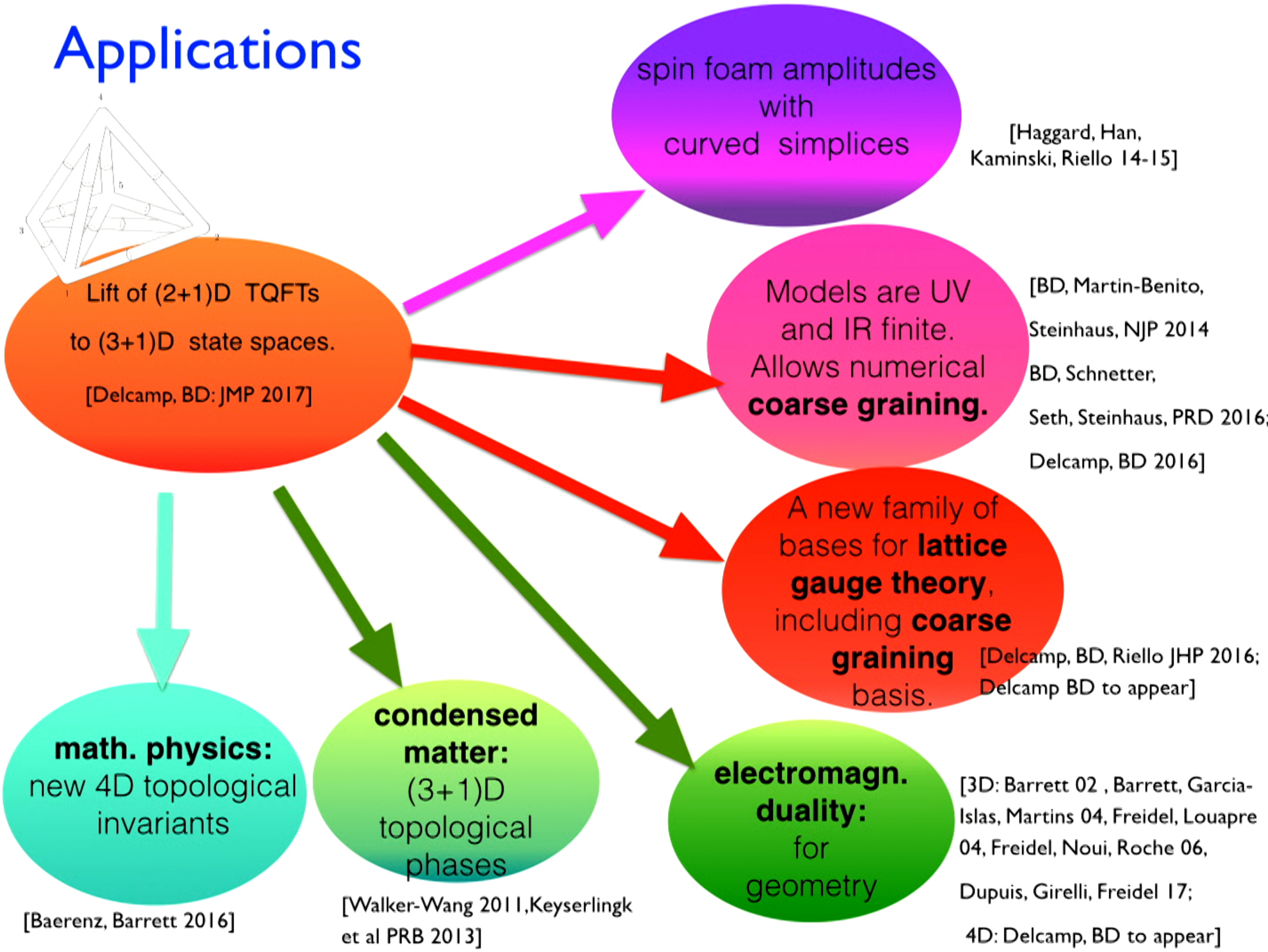
How does it help for the dynamics of quantum gravity ?

- spin foam models: relatives of TQFTs that encode quantum gravity amplitudes
- the different quantum geometries will show up in a phase diagram for spin foams; encodes their behaviour in the refinement limit



- Are there even more geometrically interesting phases?
  - explicit numerical investigation
  - construction via lifting technique
- Phase transition:
  - condensation of defects
  - propagating degrees of freedom
  - diffeomorphism symmetry

Applications



Conclusions

- restore close relation of LQG to TQFT [Barrett, Crane, Smolin]
  - could be crucial for continuum limit (do we already have a geometric phase?)
  - exchange of elegant techniques between (now also canonical) quantum gravity and TQFT

- new vacua can serve as starting point of approximation scheme for dynamics [BD 2012-14] (Consistent Boundary Framework)

- the new family of quantum geometry realization offers many advantages
  - spectra of intrinsic and extrinsic geometric operators are discrete and bounded
  - self-duality
  - finiteness properties important for (numerical) coarse graining schemes
  - new bases important for coarse graining

- new view on quantum geometries [BD, Steinhaus 2013: From TQFT to quantum geometry]
  - many new directions
  - are there other quantum geometries (4D TQFTs) out there?
  - how do predictions depend on choice of representation?

Thank you!