Title: Geometry in Topological Quantum Matter and Beyond

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Abstract: <p style="margin:0cm;margin-bottom:.0001pt"><o:p></o:p>In the past few years we have witnessed a flurry of activity in the field of topological phases of matter. An outstanding theme in the field is the study of the interplay between geometry and topology of many-body wave functions, which has attracted the attention of condensed matter and high-energy physicists. In this talk, I will present the quantum field-theoretic descriptions of the fascinating novel phenomena emergent from intertwined geometry and topology, which are vividly exemplified by the geometric responses in fractional quantum Hall systems. For the strict topological limit, where only the global topology of space matters, the fractional quantum Hall systems are characterized by their universal properties such as fractional quantum Hall conductivity and a degeneracy on surfaces with the topology of a torus. Quite surprisingly, these topological fluids also couple to the geometry and have universal responses to the adiabatic deformations of the background geometry. These responses are given by a Wen-Zee term, Hall viscosity term, and gravitational Chern-Simons term. Through the field-theoretic approaches of the topological fluids, I will for the first time show how to derive all the universal geometric responses. To account for the coupling to the background geometry, I show that the concept of â€œflux attachmentâ€• must be modified in the curved space and use it to derive the responses from Chern-Simons theories for all the known fractional quantum Hall states. I also apply these results to the theory of the anisotropic quantum Hall systems, where the geometric responses play a central role in understanding their universal physics.</p>

Colloquium at Perimeter Institute (May 2nd, 2017)

Geometry in Topological Quantum Matter and Beyond

Gil Young Cho

Korea Institute for Advanced Study (KIAS)

Contents:

Modern Condensed Matter Physics

$\approx$ **Emergent phenomena enabled by quantum mechanics and/or many-body physics**

Quantum Mechanics   Many-body Physics

Modern Condensed Matter Physics

≈ **Emergent phenomena enabled by quantum mechanics and/or many-body physics**

Quantum Hall Effects

Topological Insulators

Spin Liquids
[R. Melko, G. Baskaran]

Superconductors

Quantum Criticality

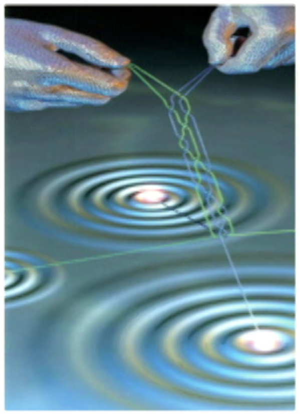
Non-Fermi Liquid
[S.-S. Lee]

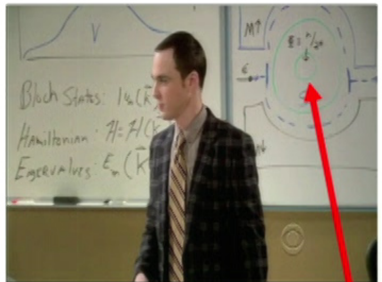
Entanglement
[G. Vidal, R. Melko, R. Myers]

Quantum Mechanics **Many-body Physics**

Why should I care about topological phases?

Topological Quantum Computation

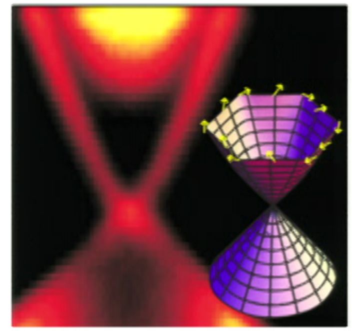




[Big bang theory]

Majorana fermion

Spin current in spin-based electronics



in topological insulator materials

Topological insulator

as good thermoelectric materials

Physically-interesting phases of matter!

Representative phases of matter of illustrating

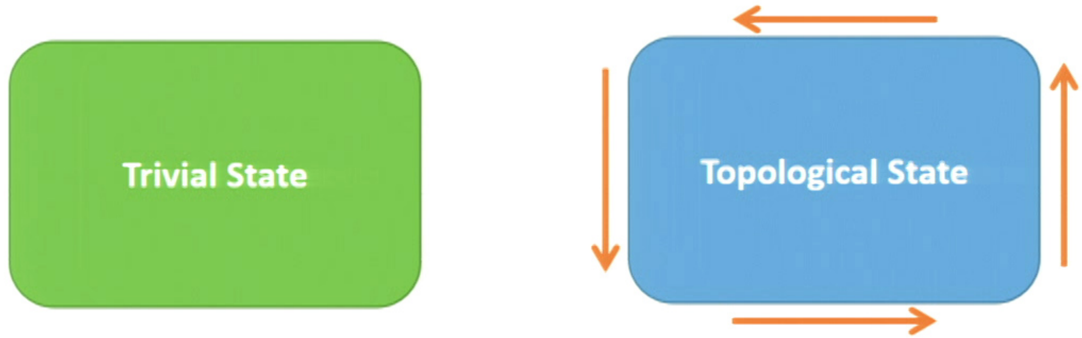
"how much quantum can be different from classical"

Nayak, Simon, Stern, Freedman, Das Sarma (2008), Ghaemi, Mong, Moore, PRL (2010), Hasan, Kane (2010)
Hasan, Moore, Ann. Rev. Cond. Matt. Phys. (2011), Moore, Nat. Phys. (2013), Pesin, McDonald, Nat. Mat. (2013)

Topological phase: a central theme in modern condensed matter physics

Topological phases: "intrinsically" quantum mechanical

EX:



Both states have the same symmetry ! (classically equivalent phases !)

Trivial : $\sigma_{xx} = \sigma_{xy} = 0$ **Topological:** $\sigma_{xx} = 0, \text{but } \sigma_{xy} = \frac{e^2}{h}$

> **Essential physics of the topological state comes from**
>
> **"topological structure" of the wave functions !**

Much effort has been devoted to understand the topological physics !

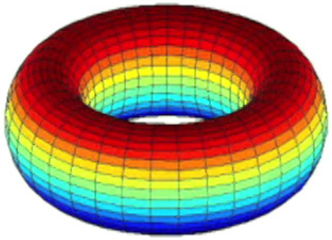
Reviews & Books

Wen (2004), Fradkin (2013), Qi, Zhang, RMP (2011), Hasan, Kane, RMP (2010), Hasan, Moore, Ann. Cond. Matt. Phys. (2013), Senthil, Ann. Cond. Matt. Phys. (2015), and many others

Some of the physics are well understood by now !

Geometry and Spatial symmetries on Topological Phases:

(Classical) topological phases: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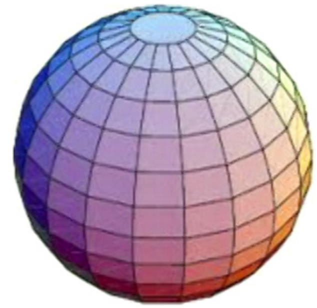
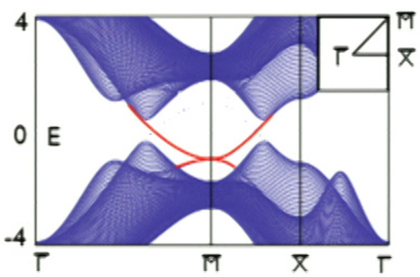


Only the global topology of space matters !

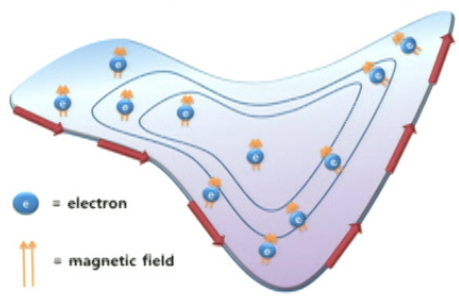
We are observing developments beyond this strict topological limit !

Topological crystalline insulator



Fu, PRL (2011)
GYC, Hsieh, Morimoto, and Ryu, PRB (2015)

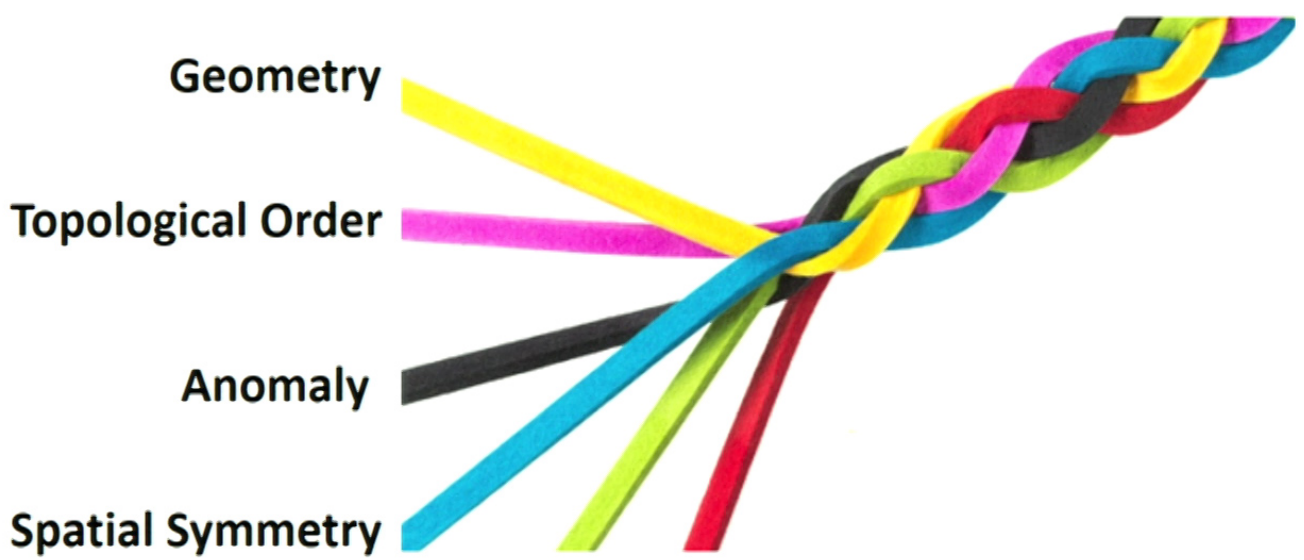
Geometry in Quantum Hall System



Entanglement Spectrum

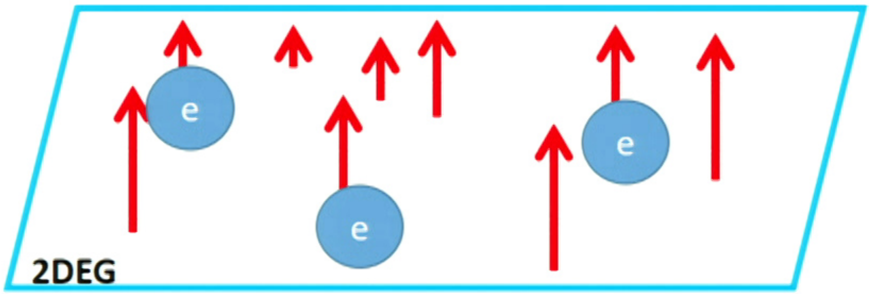
GYC, Ludwig, and Ryu (2016)
GYC, Shiozaki, Ryu, and Ludwig (2016)

"Geometry" in topological phases



This can be best illustrated in **quantum Hall systems**

Quantum Hall Systems:



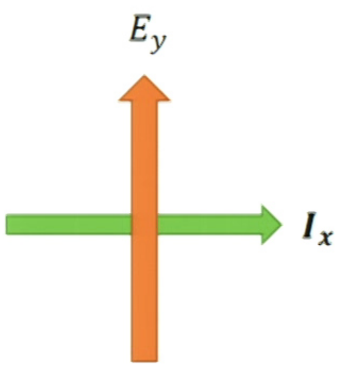
Electrons under uniform magnetic field. What should I do?

Classical Mechanics:

$$\rho_{xy} = \frac{E_y}{I_x} \sim \frac{B}{ne}$$

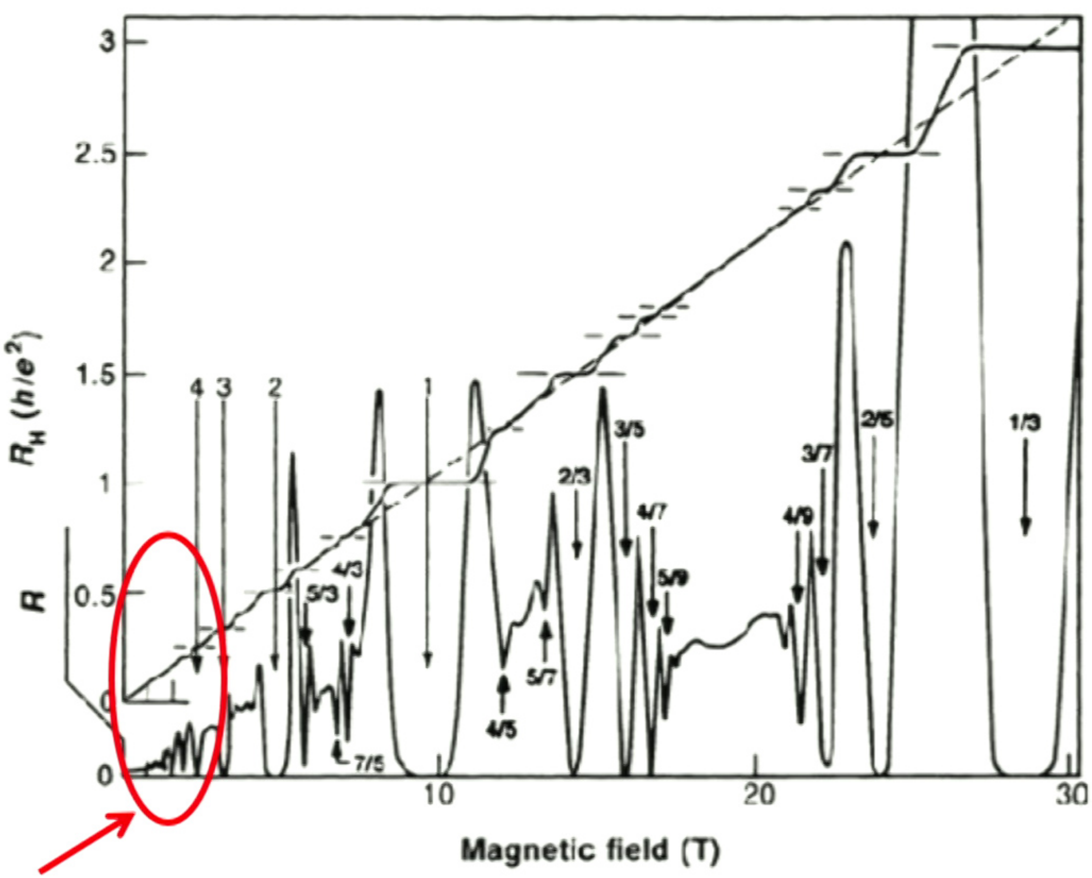
Newtonian Dynamics (Drude formula)

$$m\dot{\vec{v}} = q\vec{E} - \frac{m\vec{v}}{\tau} + q\vec{v} \times \vec{B}$$



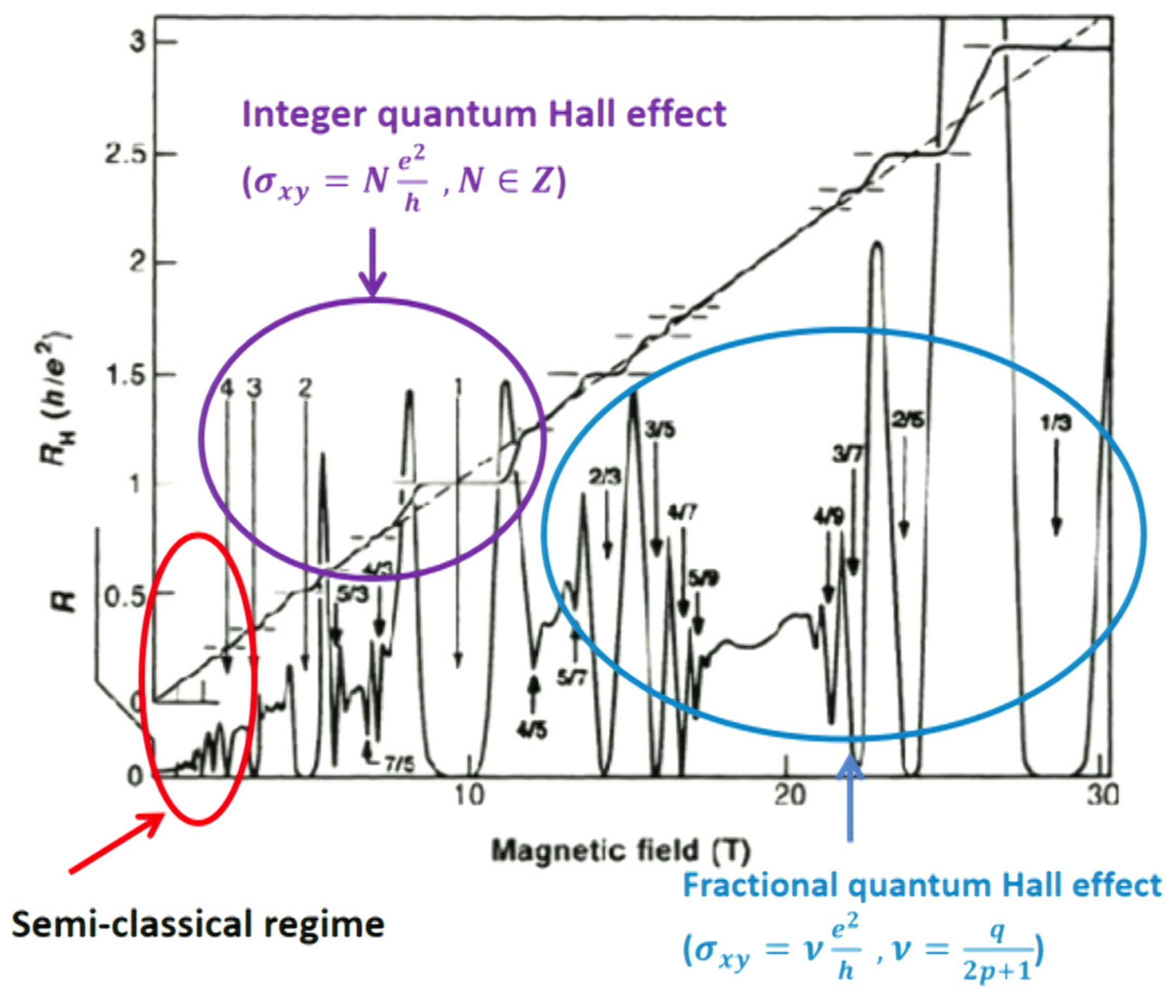
Cyclotron motion

Experiment? Striking deviations from classical mechanics !



Semi-classical regime

Experiment? Striking deviations from classical mechanics !



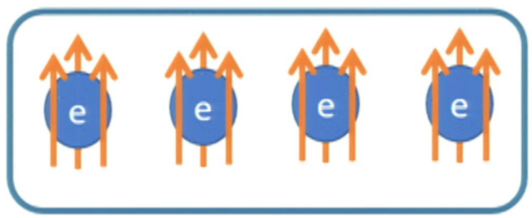
Some information from experiments:

I. Gapped insulating states appear at the filling, e.g., $\nu = \frac{1}{3}$

Filling ν? = unit flux quantum $\Phi_0 = 2\pi$ or $\Phi_0 = \frac{h}{e}$ e = electron

$$\nu = \frac{(\text{number of electron } e)}{(\text{number of arrow})}$$

Hence, $\nu = \frac{1}{3}$ state:



Some information from experiments:

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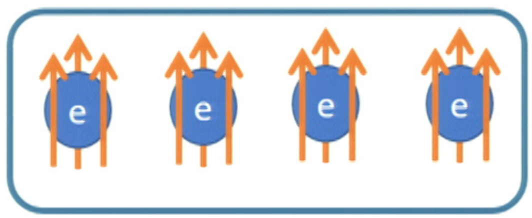


II. The quantum Hall fluids have $\sigma_{xy} = \nu \frac{e^2}{h} = \frac{e^2}{3h}$ and $\sigma_{xx} = 0$

How do we understand the quantum Hall states ?

Composite Boson Theory

..which is an extremely successful theory !

[Zhang 1992, Wen 1992, 1995]

Fractional quantum Hall effects at $\nu = \frac{1}{3}$ state:

Flux attachment :

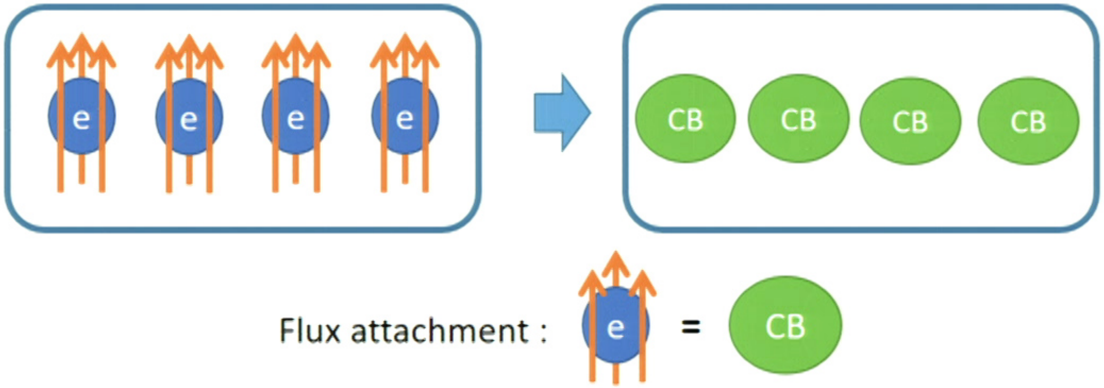
Bosons in the absence of background magnetic field

Naturally condense and become **a superconductor** !

Fractional quantum Hall effect of electrons	**=**	**Superconductor of composite bosons**

[Zhang 1992, Wen 1992, 1995]

From "superconductor" to Chern-Simons theory:



From this simple picture, we can derive a remarkably simple theory:

$$L = -\frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + A_\mu \frac{\epsilon^{\mu\nu\lambda}}{2\pi} \partial_\nu b_\lambda + \cdots$$

which is the effective Chern-Simons theory for the quantum Hall effects

[Zhang 1992, Wen 1992, 1995]

Fractional quantum Hall effects at $\nu = \frac{1}{3}$ state:

Composite boson theory is a simple and successful description !

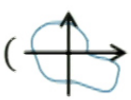
$$L = -\frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{2\pi} b_\mu \partial_\nu A_\lambda \, \epsilon^{\mu\nu\lambda} + \cdots$$

(i) Correct electromagnetic responses: $\sigma_{xy} = \frac{e^2}{3h}$ and $\sigma_{xx} = 0$

(ii) Correct fractional anyon excitations and topological order

(iii) Generating the correct ground state wave-function (from composite boson theory):

$$\Psi(z_1, z_2, z_3 \cdots) \propto \Pi_{i>j}(z_i - z_j)^3$$ [Laughlin (1983)]

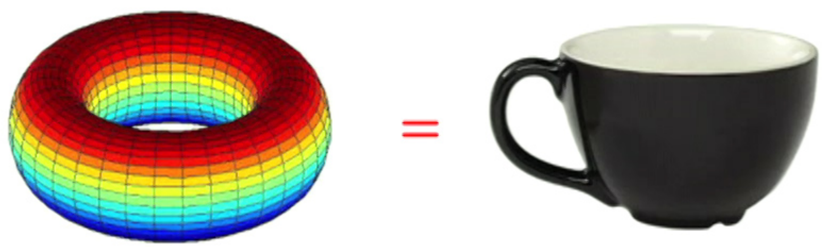
which is rotational invariant: Ψ () $\propto$ Ψ ()

[Wen 1992, 1995]

The Chern-Simons theory is topological !

$$L = -\frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{2\pi} b_\mu \partial_\nu A_\lambda \epsilon^{\mu\nu\lambda} + \cdots$$

I.e., the fractional quantum Hall state cannot distinguish



Its topological order is completely characterized by the two numbers

(i) Degeneracy on Torus = 3

(ii) Quantum Hall conductivity $\sigma_{xy} = \frac{1}{3}$

[Wen 1992, 1995]

Geometry in quantum Hall states

..which is the physics of the quantum Hall states

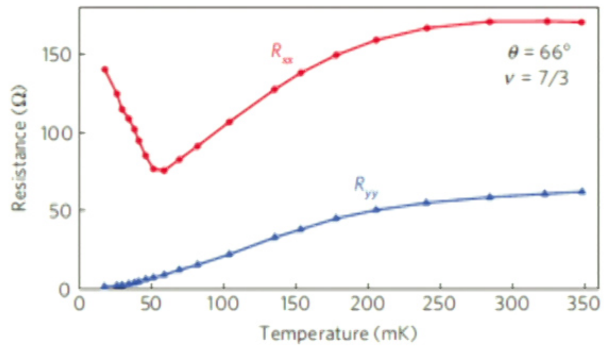
not captured within the topological Chern-Simons theory

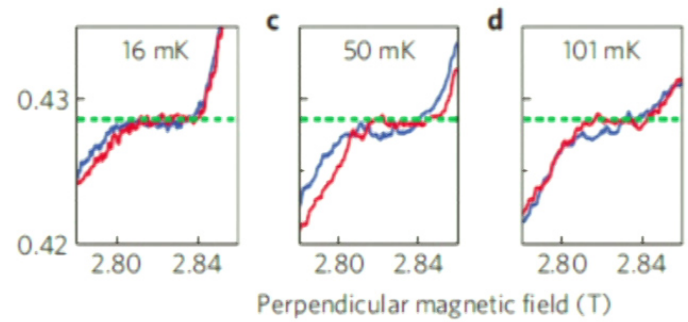
Anisotropic fractional quantum Hall effects in Nature:

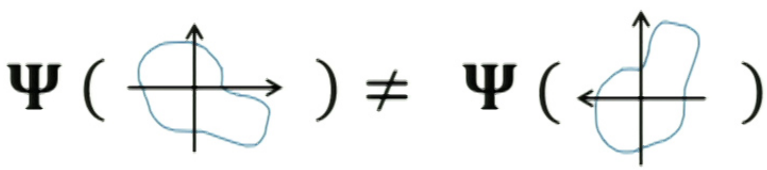
Evidence for a fractionally quantized Hall state with anisotropic longitudinal transport

Jing Xia[1*†], J. P. Eisenstein[1], L. N. Pfeiffer[2] and K. W. West[2] [Nature physics, 2011]

A quantum Hall state **breaking rotational symmetry** is found at $\nu = 2 + \frac{1}{3}$!







Ψ ( ) $\neq$ Ψ ( )

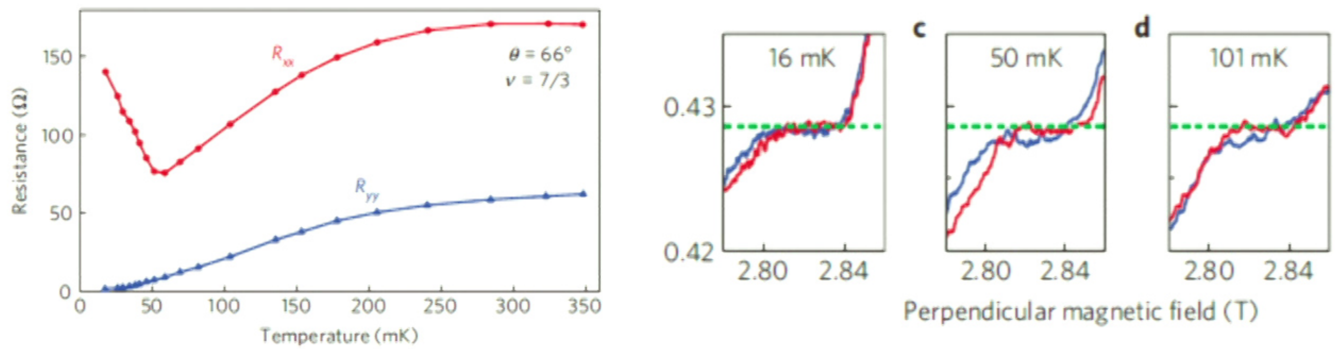
Anisotropy as Background Geometry:

I claim: $g^{ij} = \begin{bmatrix} 1+\delta g_{xx} & \delta g_{xy} \\ \delta g_{xy} & 1-\delta g_{xx} \end{bmatrix}$ with δg=order parameter for anisotropy

$$L_{\text{total}} = \left(\Psi^* i D_0 \Psi + \frac{1}{2m} (D_i \Psi)^* g^{ij} (D_j \Psi)\right) \text{ with } D_\mu = \partial_\mu + iA_\mu$$

Electromagnetic gauge field

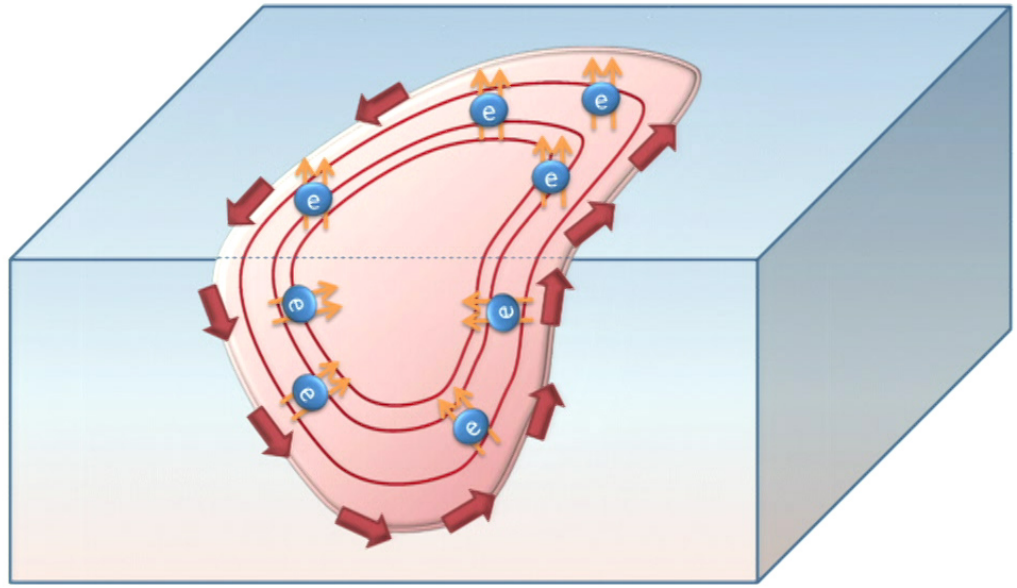
describes the anisotropic quantum Hall state:



Effectively, (isotropic) quantum Hall effect is on the curved space !

...whose effective field theory is not known much.

You, GYC, and Fradkin, PRX (2014)



How do I describe the quantum Hall system on the curved space?

Does this topological fluid "see" the geometry ?

Anisotropy as Background Geometry:

I claim:

$$g^{ij} = \begin{bmatrix} 1+\delta g_{xx} & \delta g_{xy} \\ \delta g_{xy} & 1-\delta g_{xx} \end{bmatrix}$$ with δg=order parameter for anisotropy

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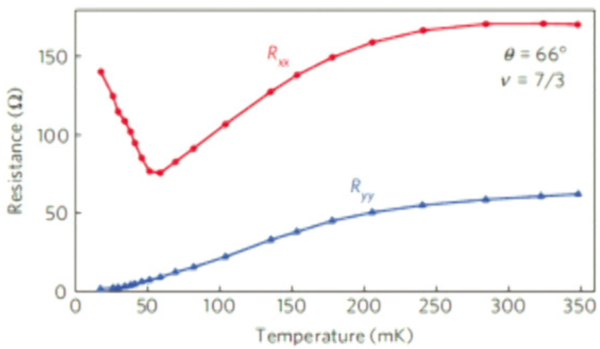
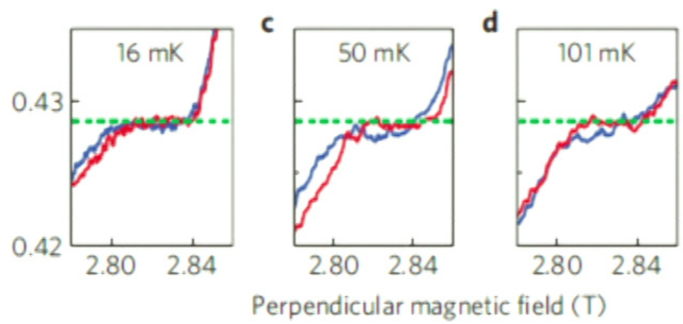




You, **GYC,** and Fradkin, PRX (2014)

Universal geometric responses of quantum Hall effect

Wen-Zee term

Hall Viscosity

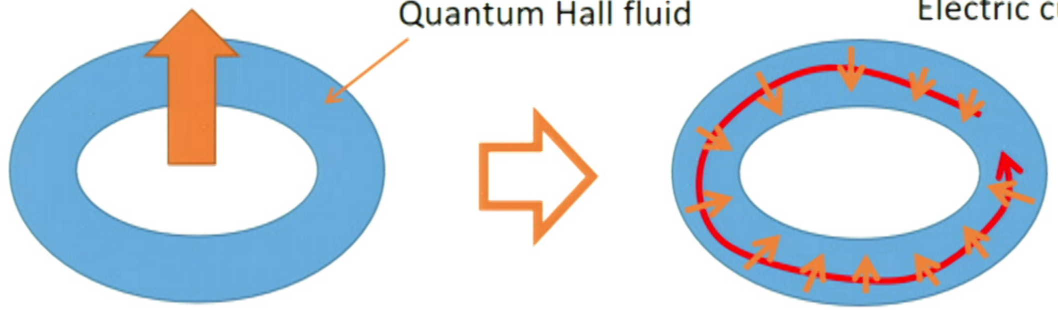
Partial incomplete list of reference :
Wen, Zee (1992), Avron, Seiler, Zograf (1995), Read (2009), Haldane (2009), Read, Rezayi (2011), Nicolis, Son (2011), Hoyos, Son (2012), Bradlyn, Goldstein, Read (2012), Son (2013), Can, Laskin, Wiegmann (2014), Wu, Wu (2014), many other references

Wen-Zee term : quantum Hall systems see curvature as ``magnetic flux''

[Wen, Zee (1992)]

Quantum Hall effect collects electric charge at the magnetic flux

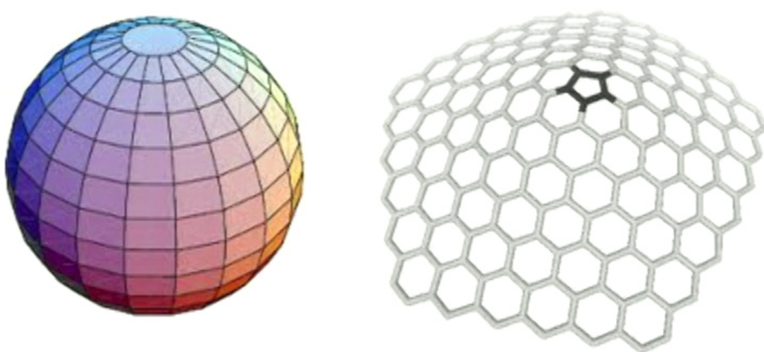
Imagine :



Induced $\vec{E}$ field! ($\partial_t B = \nabla \times \vec{E}$)

Thus quantum Hall fluids accumulate the charge at the magnetic flux !

On the **space with curvature**...



Quantum Hall wavefunctions

collect/deplete the charges

at the curvature !

Wen-Zee term: curvature as magnetic flux

Wen and Zee supplemented a **phenomenological term** into the effective theory:

$$L = -\frac{K}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + qA_\mu J^\mu + \boxed{s\omega_\mu J^\mu} \cdots$$

(J^μ: electron current)

Coupling constant: **spin S** (or shift = 2 x "spin")

[One of the most important characteristics of topological orders]

Here ω_λ = ``spin connection" describing the (scalar) curvature of background geometry

i.e., Local curvature $R = \nabla \times \vec{\omega}$ (similar to local magnetic field $\mathrm{B} = \nabla \times \vec{A}$)

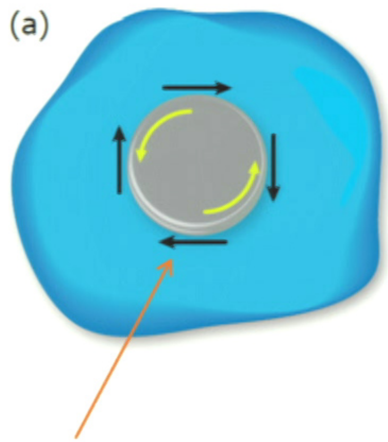
From the coupling (i.e., A_μ and ω_μ couple to J_μ in the identical way),

$R = \nabla \times \vec{\omega}$ will collect charge as like $\mathrm{B} = \nabla \times \vec{A}$ collect charge !

No derivation so far !

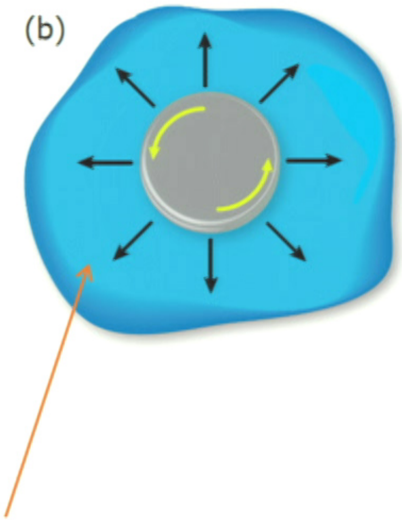
Wen, Zee (1992)

Hall viscosity : force perpendicular to fluid motion



Conventional fluid :

frictional force ∥ the motion



Quantum Hall fluid :

force ⊥ the motion

(force · fluid motion = 0 i.e. dissipationless)

Hall viscosity : force perpendicular to fluid motion

The Hall viscosity can be **phenomenologically** captured by the term

$$L_{eff} = 2\eta_H \omega_t = \bar{\rho}\, S\, \omega_t$$ [Hoyos, Son (2012)]

In which ω_t is the time component of **spin connection** ω_μ

S is the spin entering into **Wen-Zee term**

Hall viscosity : force perpendicular to fluid motion

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$$L_{eff} = 2\eta_H \omega_t = \bar{\rho}\, S\, \omega_t$$ [Hoyos, Son (2012)]

In which ω_t is the time component of **spin connection** ω_μ

S is the spin entering into **Wen-Zee term**

No field-theoretic derivation !

Geometric responses of quantum Hall effects:

1. Wen-Zee term: $L = \frac{S}{2\pi} \epsilon^{\mu\nu\lambda} b_\mu \partial_\nu \omega_\lambda$

2. Hall Viscosity term: $L = \bar{\rho} \, S \, \omega_t$

A. Both terms contain spin connection ω_μ and spin S (or "shift")

B. Both terms (or spin S) have been known mainly from ideal wavefunctions studies and claimed to be protected (though it is not a priori clear !)

C. These responses are known piece-by-piece, but not in a unified fashion !

Partial incomplete list of reference :
Wen, Zee (1992), Avron, Seiler, Zograf (1995), Read (2009), Haldane (2009), Read, Rezayi (2011), Nicolis, Son (2011), Hoyos, Son (2012), Bradlyn, Goldstein, Read (2012), Son (2013), Can, Laskin, Wiegmann (2014), Wu, Wu (2014), many other references

Physically-clearer and unified description of geometric responses is desired !

Very successful approaches for quantum Hall states !

Can we use the composite particle theories to derive the geometric responses in a unified fashion?

GYC, You, and Fradkin (2014)

Fractional quantum Hall effects at $\nu = \frac{1}{3}$ state:

Flux attachment : 

Bosons in the absence of background magnetic field

Naturally condense and become **a superconductor** !

Fractional quantum Hall effect of electron	**=**	**Superconductor of composite bosons**

[Zhang 1992, Wen 1992, 1995]

Composite Boson Theory for the fractional quantum Hall state:

Problem: Electrons under the uniform magnetic field

Electromagnetic gauge field

$$L = \sqrt{g}\left(\Psi^* iD_0\, \Psi + \frac{1}{2m}\,(D_i\,\Psi)^*\; g^{ij}\,(D_j\Psi)\right) \quad \text{with } D_\mu = \partial_\mu + iA_\mu$$

$(A_\mu = \bar{A}_\mu + \delta A_\mu)$

Uniform magnetic field

e

CB

$$L = \sqrt{g}\left(\Psi^* iD_0\, \Psi + \frac{1}{2m}\,(D_i\,\Psi)^*\; g^{ij}\,(D_j\Psi)\right) + \frac{1}{4\pi\times 3}\, a_\mu \partial_\nu a_\lambda\, \epsilon^{\mu\nu\lambda} \quad \text{with } D_\mu = \partial_\mu + iA_\mu + ia_\mu$$

Attaching three flux quantum to Ψ

Now: $\bar{A}_i + \bar{a}_i = 0$ in which $\bar{A}_i$ generates the constant uniform magnetic field

$$L = \sqrt{g}\left(\Psi^* iD_0\, \Psi + \frac{1}{2m}\,(D_i\,\Psi)^*\; g^{ij}\,(D_j\Psi)\right) + \frac{1}{4\pi\times 3}\, \delta a_\mu \partial_\nu \delta a_\lambda\, \epsilon^{\mu\nu\lambda}$$

fluctuation

with $D_\mu = \partial_\mu + i\delta A_\mu + i\delta a_\mu$

Probe field

Bosons coupled to fluctuating gauge fields !

Composite Boson Theory for the fractional quantum Hall state:

$$L = \sqrt{g}\left(\Psi^* i D_0 \Psi + \frac{1}{2m}(D_i \Psi)^* \, g^{ij} \, (D_j \Psi)\right) + \frac{\epsilon^{\mu\nu\lambda}}{4\pi \times 3} \delta a_\mu \partial_\nu \delta a_\lambda \text{ with } D_\mu = \partial_\mu + i\delta A_\mu + i\delta a_\mu$$

Superconductor: $\Psi = \sqrt{\bar{\rho} + \delta\rho}\, e^{i\theta} \neq 0$

Performing the standard boson-vortex duality, we find:

$$L = \sqrt{g}\left(\bar{\rho}\,\delta A_0 + \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}(\delta a_\mu + \delta A_\mu)\partial_\nu b_\lambda + \frac{1}{12\pi}\epsilon^{\mu\nu\lambda}\delta a_\mu \partial_\nu \delta a_\lambda + \cdots\right)$$

Charging energy

with $J^\mu = \frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\nu b_\lambda$ as the electron current

Coupling of electron current to gauge field $= \boldsymbol{J^\mu(\delta A_\mu + \delta a_\mu)}$

Integrating out δa_μ and expanding for small $\delta g^{ij} = g^{ij} - \delta^{ij}$, we find:

$$L = \bar{\rho}\delta A_0 - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{2\pi} b_\mu \partial_\nu \delta A_\lambda \, \epsilon^{\mu\nu\lambda} + \cdots$$

The topological Chern-Simons theory !

Failure of the composite particle theories?

which made extreme success, e.g., ground state, excitations, etc !

Only fail to capture the very particular two geometric responses ?

I fix this by revisiting the flux attachment !

I ask if the flux attachment changes only the statistics or spin, too !

GYC, You, and Fradkin (2014)

Revisiting the flux attachment 2.

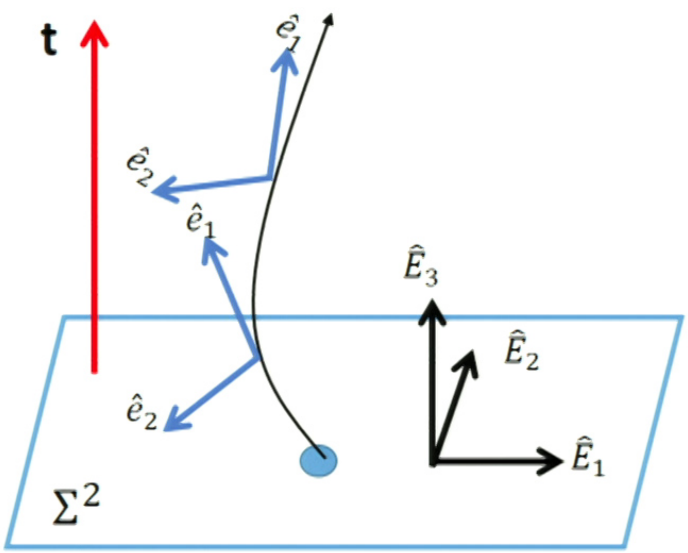
$$\Phi[C] = \langle e^{i\int_C A_\mu} \rangle = e^{i\theta_{stat} W[C]} = e^{i\theta_{\text{stat}} L} e^{-i\theta_{\text{stat}} T[C]}$$

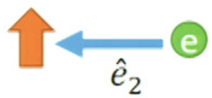
$$T[C] = \frac{1}{2\pi} \int d\boldsymbol{r} \cdot \left[\boldsymbol{e}_2 \times \frac{\partial \boldsymbol{e}_2}{\partial s} \right]$$

= "how much $\hat{e}_2$ rotates"

= "internal motion of particle and flux"

$$C : s \to \boldsymbol{r}$$





GYC, You, and Fradkin (2014)

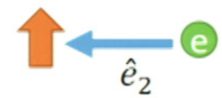
Revisiting the flux attachment 2.

$$\Phi[C] = \langle e^{i\int_C A_\mu} \rangle = e^{i\theta_{stat} W[C]} = e^{i\theta_{\rm stat} L} e^{-i\theta_{\rm stat} T[C]}$$

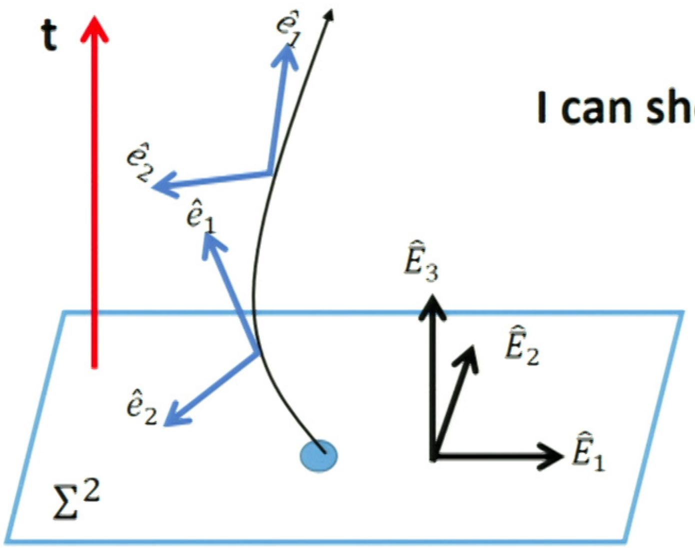
$$T[C] = \frac{1}{2\pi} \int d\boldsymbol{r} \cdot \left[\boldsymbol{e}_2 \times \frac{\partial \boldsymbol{e}_2}{\partial s} \right]$$

= “how much $\hat{e}_2$ rotates”

= “internal motion of particle and flux”



$$C\ :\ s \to \boldsymbol{r}$$



I can show that….

$$= \frac{1}{2\pi} \int d\boldsymbol{r} \cdot \boldsymbol{\omega}$$

Spin connection

The torsion knows the background geometry

through the spin connection !

GYC, You, and Fradkin (2014)

Revisiting the flux attachment 3.

By plugging this back to the expression :

$$\Phi[C] = \langle e^{i\int_C A_\mu} \rangle = e^{i\theta_{stat} W[C]} = e^{i\theta_{\text{stat}} L} e^{-i\theta_{\text{stat}} T[C]}$$

$$= \exp(iL\theta_{\text{stat}}) \exp\left(-iS_z \int d\boldsymbol{r} \cdot \boldsymbol{\omega}\right)$$ with $S_z = \frac{\theta_{stat}}{2\pi}$ (induced spin)

Thus the composite particles' covariant derivative : $D_\mu = \partial_\mu + ia_\mu + iS_z\omega_\mu$

GYC, You, and Fradkin (2014)

Revisiting the flux attachment 3.

By plugging this back to the expression :

$$\Phi[C] = \langle e^{i\int_C A_\mu} \rangle = e^{i\theta_{stat} W[C]} = e^{i\theta_{\text{stat}} L} e^{-i\theta_{\text{stat}} T[C]}$$

$$= \exp(iL\theta_{\text{stat}}) \exp\left(\boxed{-iS_z \int d\boldsymbol{r} \cdot \boldsymbol{\omega}} \right)$$ with $S_z = \frac{\theta_{stat}}{2\pi}$ (induced spin)

Thus the composite particles' covariant derivative : $D_\mu = \partial_\mu + ia_\mu + \boxed{iS_z\omega_\mu}$

I find a new piece which has been missing since 1992 !

This will give the correct geometric responses.

GYC, You, and Fradkin (2014)

Revisiting the flux attachment 4.

In summary, I have shown you

1. **Flux attachment** accompanies ``**induced spin**'' to the composite particle

2. ``**induced spin**'' is determined by number N_{ϕ_0} of flux, i.e., $S_z = \frac{N_{\phi_0}}{2}$

   (statistical angle $\theta_{stat} = \pi N_{\phi_0} \rightarrow \; S_z = \frac{\theta_{stat}}{2\pi} = \frac{N_{\phi_0}}{2}$)

3. ``**induced spin**'' of the composite particle couples to the **spin connection**

   of the background geometry! (Though the particle is seemingly scalar!)

$$D_\mu = \partial_\mu + i a_\mu + i S_z \omega_\mu$$

GYC, You, and Fradkin (2014)

Again, composite Boson theory:

Previously:

$$L = \sqrt{g}\left(\Psi^* i D_0 \Psi + \frac{1}{2m}(D_i \Psi)^* g^{ij}(D_j \Psi)\right) + \frac{1}{4\pi\times 3} a_\mu \partial_\nu a_\lambda \epsilon^{\mu\nu\lambda} \text{ with } D_\mu = \partial_\mu + iA_\mu + ia_\mu$$

which has generated:

$$L = \bar{\rho}\delta A_0 - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{2\pi} b_\mu \partial_\nu \delta A_\lambda \epsilon^{\mu\nu\lambda} + \cdots$$

With the correct covariant derivative: $D_\mu = \partial_\mu + iA_\mu + ia_\mu + i\frac{3}{2}\omega_\mu$ ← Induced spin by the flux attachment

GYC, You, and Fradkin (2014)

Again, composite Boson theory:

Previously:

$$L = \sqrt{g}\left(\Psi^* iD_0 \Psi + \frac{1}{2m}(D_i \Psi)^* g^{ij}(D_j\Psi)\right) + \frac{1}{4\pi\times 3} a_\mu \partial_\nu a_\lambda \epsilon^{\mu\nu\lambda} \text{ with } D_\mu = \partial_\mu + iA_\mu + ia_\mu$$

which has generated:

$$L = \bar{\rho}\delta A_0 - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{2\pi} b_\mu \partial_\nu \delta A_\lambda \epsilon^{\mu\nu\lambda} + \cdots$$

With the correct covariant derivative: $D_\mu = \partial_\mu + iA_\mu + ia_\mu + i\frac{3}{2}\omega_\mu$

Induced spin by the flux attachment

We find the following (by shifting $\delta A_\mu \to \delta A_\mu + \frac{3}{2}\omega_\mu$):

$$L = \bar{\rho}\delta A_0 + \frac{3\bar{\rho}}{2}\omega_0 - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{2\pi} b_\mu \partial_\nu \delta A_\lambda \epsilon^{\mu\nu\lambda} + \frac{3}{4\pi} b_\mu \partial_\nu \omega_\lambda \epsilon^{\mu\nu\lambda} \cdots$$

Charging energy

Correct Hall viscosity

Topological Chern-Simons terms for σ_{xy} and deg. on torus

Correct Wen-Zee term

GYC, You, and Fradkin (2014)

Again, composite Boson theory 2.

$$L = \bar{\rho}\delta A_0 + \frac{3}{2}\,\bar{\rho}\omega_0 - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{2\pi}\, b_\mu \partial_\nu \delta A_\lambda\, \epsilon^{\mu\nu\lambda} + \frac{3/2}{2\pi}\, b_\mu \partial_\nu \omega_\lambda\, \epsilon^{\mu\nu\lambda} \ldots$$

I successfully derived Hall viscosity and Wen-Zee terms in a unified fashion !

(1) It can be generalized to any abelian fractional quantum Hall state (not only $\nu = \frac{1}{3}$ state)

(2) I can investigate the effect of perturbations on the Wen-Zee term and Hall viscosity !

(3) Am I missing **the gravitational Chern-Simons term** [another geometric response]?!

$$L = -c\,\frac{\epsilon^{\mu\nu\lambda}}{48\,\pi}\,\omega_\mu \partial_\nu \omega_\lambda$$

whose coefficient `**c**' is the central charge, e.g., c= 1 for $\nu = \frac{1}{3}$ state

Thermal Hall conductivity: $\kappa_{xy} = c\left(\frac{\pi^2 k_B^2}{3h}\right)T$ along the edge !

GYC, You, and Fradkin (2014)

Again, composite Boson theory 2.

$$L = \bar{\rho}\delta A_0 + \frac{3}{2}\,\bar{\rho}\omega_0 - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{2\pi}\, b_\mu \partial_\nu \delta A_\lambda\, \epsilon^{\mu\nu\lambda} + \frac{3/2}{2\pi}\, b_\mu \partial_\nu \omega_\lambda\, \epsilon^{\mu\nu\lambda} \ldots$$

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GYC, You, and Fradkin (2014)

Gravitational Chern-Simons term 1:

The gravitational Chern-Simons term appears in the effective response theory:

$$L = \bar{\rho}\delta A_0 + s\bar{\rho}\omega_0 + \frac{\nu}{4\pi}\epsilon^{\mu\nu\lambda}(\delta A_\mu + s\omega_\mu)\partial_\nu(\delta A_\lambda + s\omega_\lambda) - \frac{\boldsymbol{c}}{\boldsymbol{48\pi}}\boldsymbol{\epsilon^{\mu\nu\lambda}\omega_\mu\partial_\nu\omega_\lambda}$$

..where $\boldsymbol{s}$=spin, $\boldsymbol{\nu}$ = filling, and $\boldsymbol{c}$= (chiral) central charge

Now, from the effective theory,

$$L = \bar{\rho}\delta A_0 + \frac{3\bar{\rho}}{2}\omega_0 - \frac{\mathbf{3}}{4\pi}b_\mu\partial_\nu b_\lambda\epsilon^{\mu\nu\lambda} + \frac{1}{2\pi}b_\mu\partial_\nu\delta A_\lambda\,\epsilon^{\mu\nu\lambda} + \frac{3}{4\pi}b_\mu\partial_\nu\omega_\lambda\,\epsilon^{\mu\nu\lambda}\dots$$

Naively, if I integrate out b_μ to find the effective response:

$$\Rightarrow\quad L = \bar{\rho}\delta A_0 + \frac{3}{2}\bar{\rho}\omega_0 + \frac{1/3}{4\pi}\epsilon^{\mu\nu\lambda}\left(\delta A_\mu + \frac{3}{2}\omega_\mu\right)\partial_\nu\left(\delta A_\lambda + \frac{3}{2}\omega_\lambda\right)$$

$c = 0$ for the Laughlin state?

Gromov, **GYC,** You, Abanov, and Fradkin, PRL (2015)

Gravitational Chern-Simons term 2:

In fact, I need to be careful when I integrate out b_μ in curved spacetime !

[Witten (1989), Bar-Natan and Witten (1991)]

$$Z[\omega_\mu] = \int Db_\mu \exp\left[-i\frac{k}{4\pi}\int b_\mu \partial_\nu b_\lambda\ \epsilon^{\mu\nu\lambda}\right] \propto \exp\left[-i\frac{\mathrm{sgn}(k)}{48\pi}\int \omega_\mu \partial_\nu \omega_\lambda\right]$$

So-called framing anomaly !

With this into account,

$$L = \bar{\rho}\delta A_0 + \frac{3\bar{\rho}}{2}\omega_0 - \frac{3}{4\pi}b_\mu\partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \frac{1}{2\pi}\ b_\mu \partial_\nu \delta A_\lambda\ \epsilon^{\mu\nu\lambda} + \frac{3}{4\pi}\ b_\mu \partial_\nu \omega_\lambda\ \epsilon^{\mu\nu\lambda} \cdots$$

I can integrate out b_μ to find the effective response theory:

$$L = \bar{\rho}\delta A_0 + \frac{3}{2}\bar{\rho}\omega_0 + \frac{1/3}{4\pi}\epsilon^{\mu\nu\lambda}\left(\delta A_\mu + \frac{3}{2}\omega_\mu\right)\partial_\nu\left(\delta A_\lambda + \frac{3}{2}\omega_\lambda\right) - \frac{1}{48\pi}\epsilon^{\mu\nu\lambda}\omega_\mu\partial_\nu\omega_\lambda$$

...which contains **the correct spin $s = \frac{3}{2}$ and correct central charge $c = 1$** !

Gromov, **GYC,** You, Abanov, and Fradkin, PRL (2015)

I can also show that:

Composite fermion theory

Projective Parton theory

..surprisingly generate the same correct answers !

Only when the correct covariant derivatives and framing anomalies are included !

I calculated the geometric responses for all the known quantum Hall states

[e.g., para-fermion and pfaffian states]

GYC, You, and Fradkin, PRB (2014)
Gromov, **GYC,** You, Abanov, and Fradkin, PRL (2015)

Comments:

1. The results can be generalized to **many other topological phases in 2d** (e.g., spin liquid), which can be constructed by composite particle theories and parton method.

2. Equivalence between composite fermion/boson theories beyond topological description

For example, near the topological transition..

> expansions.[12] As we show below, in the fermion limit the model is soluble and exhibits a "gap-closing" transition between a band insulator and an integer quantum Hall state. We analyze the critical properties in this case, which are described by a massless 2+1 Dirac equation, and find that they are most certainly different from the 3D *XY* model,

Composite fermion theory

Composite boson theory

[From Chen, Fisher, Wu (1993)]

"Composite Fermion Theory = Composite Boson Theory" is questioned

Further support comes from (2+1)d fermion-boson duality, **GYC,** Teo, and Fradkin, in preparation!

GYC, You, and Fradkin (2014), Gromov, **GYC,** You, Abanov, and Fradkin (2015), **GYC,** Teo, and Fradkin, in preparation

In summary,

I have shown that:

1. **Naïve approach** of composite particle theories **fails** to reproduce

   the Hall viscosity, Wen-Zee term, and Gravitational Chern-Simons term.

2. **I have modified the flux attachment and Chern-Simons theory in curved space**

   [I have corrected the covariant derivative !]

3. With this, I can derive Wen-Zee term and Hall viscosity for any quantum Hall states

4. I have used **framing anomaly** to find the correct central charge !

The results have been shown to be consistent with other approaches !

[ref. Geracie, Son, Wu, Wu (2015), Wu, Wu (2015), Bradlyn, Read (2015), Can, Laskin, Wiegmann (2015) and others]

GYC, You, and Fradkin, PRB (2014)
Gromov, **GYC,** You, Abanov, and Fradkin, PRL (2015)

Quantum Criticality :

$$L = \Psi^* i D_0 \Psi + \frac{1}{2m} (D_i \Psi)^* \delta^{ij} (D_j \Psi)$$

$$+\delta g_{xx}\left[\frac{1}{2m} (D_x \Psi)^* (D_x \Psi) - \frac{1}{2m} (D_y \Psi)^* (D_y \Psi)\right] + \delta g_{xy}\left[\frac{1}{2m} (D_x \Psi)^* (D_y \Psi) + \frac{1}{2m} (D_y \Psi)^* (D_x \Psi)\right]$$

$$+\frac{\delta g_{xx}^2 + \delta g_{xy}^2}{2V}$$

[Result of Hubbard-Stratonovich Transformation on :

$$L = -V\left[\frac{1}{2m} (D_x \Psi)^* (D_x \Psi) - \frac{1}{2m} (D_y \Psi)^* (D_y \Psi)\right]^2 - V\left[\frac{1}{2m} (D_x \Psi)^* (D_y \Psi) + \frac{1}{2m} (D_y \Psi)^* (D_x \Psi)\right]^2]$$

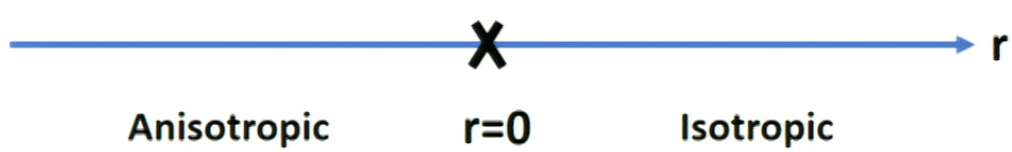
[Oganesyan, Kivelson, and Fradkin, PRB (2001)]

Integrating out fermions to find effective action for the order parameter:

$$L = \eta_H \delta g_{xy}\, \partial_t\, \delta g_{xx} - \frac{\kappa}{2} [|\nabla\, \delta g_{xx}|^2 + |\nabla\, \delta g_{xy}|^2] - r\, (\delta g_{xx}^2 + \delta g_{xy}^2) + \cdots$$

Hall viscosity

z=2 quantum criticality



You, GYC, and Fradkin, PRX (2014)

On the topological order of the anisotropic state:

Given the transition between the two phases,

it is not a priori clear what topological order the anisotropic phase will have.

What topological order will the anisotropic state have ?

It must have the same topological order as the isotropic quantum Hall state !

Hence, the anisotropic quantum Hall state must have

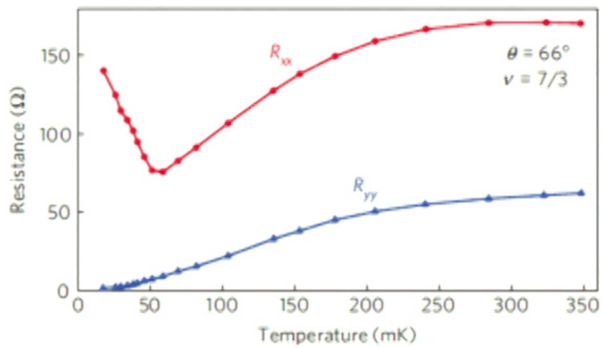
1. The same fractional excitation as in isotropic fractional quantum Hall state

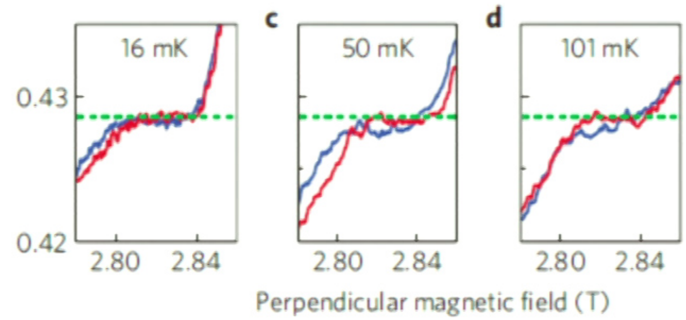
2. $\sigma_{xy} = \frac{1}{3}$ and $R_{xx} \rightarrow 0$ and $R_{yy} \rightarrow 0$ at the lowest temperature

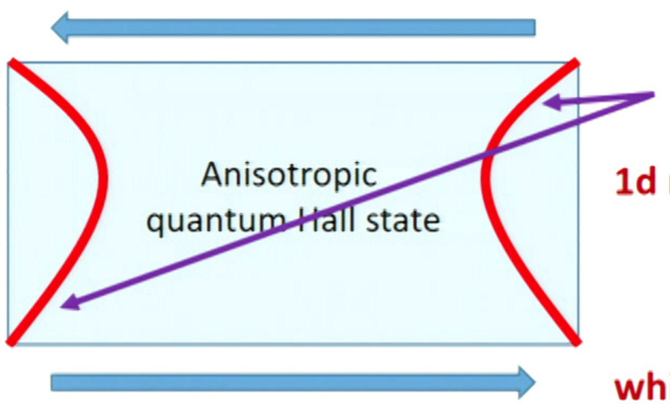
You, GYC, and Fradkin, PRX (2014)

DC Resistivity inside the anisotropic quantum Hall state:

It should have : $R_{xx} \to 0$ and $R_{yy} \to 0$ but $R_{xy} = \frac{h}{\nu e^2}$







Domain walls in anisotropic order parameter

1d modes located inside the domain wall !

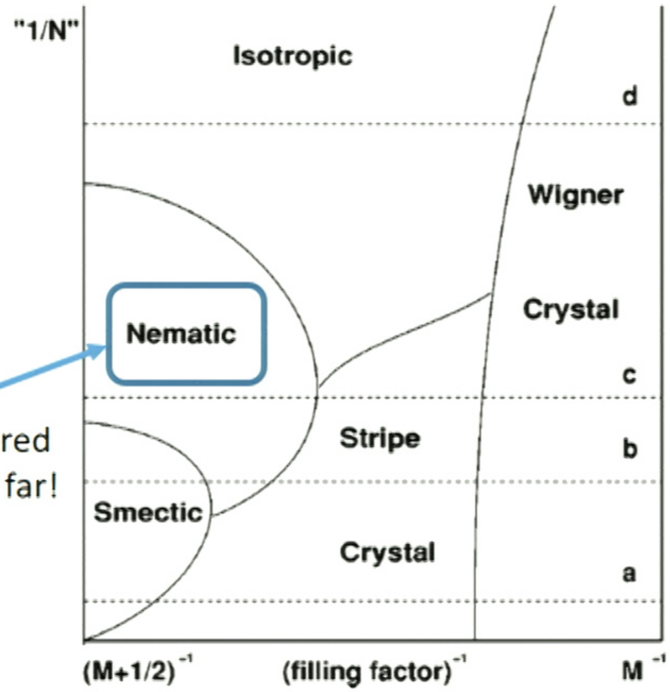
$$H = K(\partial_x \theta)^2 + \frac{1}{K}(\partial_x \phi)^2 + \text{“disorders”}$$

which will localize due to disorders as $T \to 0$,

GYC, and Fradkin, in preparation

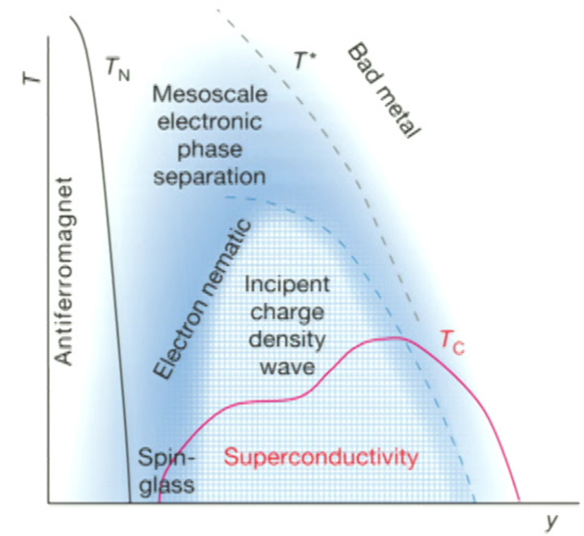
Extension to other strongly correlated systems:

Strong interactions often lead to (i) spatial symmetry breaking and (ii) topological orders



We considered only this so far!

[Fradkin, Kivelson PRB (1999)]



[Fradkin, Kivelson, Nat. Phys. (2012)]

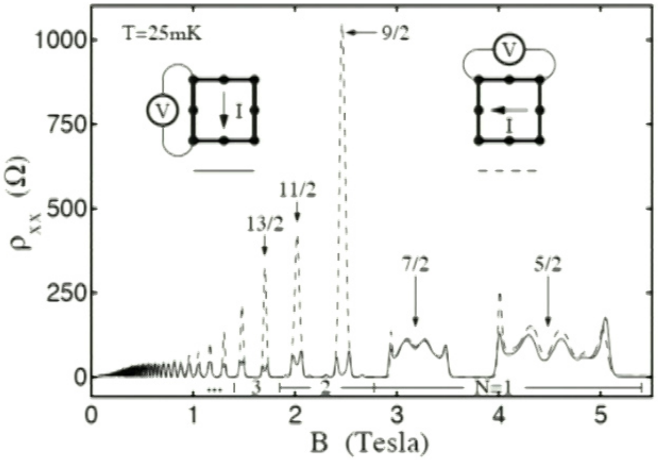
Spatial symmetry (i.e., geometry) and topology are naturally intertwined !

[**GYC,** Soto-Garrido, and Fradkin, PRL (2015)]

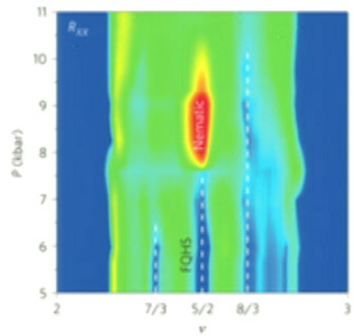
The physics out of the interplay in the topological phases is largely unexplored !

Extension to gapless phases:

Anisotropic metallic state, composite Fermi liquid, appear at $\nu = \frac{1}{2}$!



Eisenstein et.al., (1998)



Nematic state at $\nu = 2 + \frac{1}{2}$

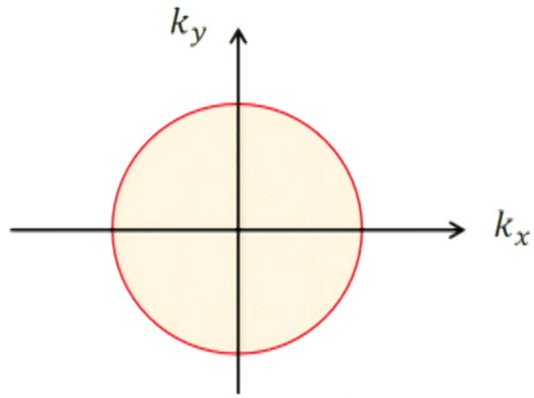
Ref. Samkharadze et.al., Nat. Phys. (2016)

Are they going to have the (quantized) Hall viscosity, Wen-Zee term?

GYC, You, and Fradkin (2014), You, GYC, and Fradkin (2016)

Geometric Responses of Composite Fermi liquid at $\nu = \frac{1}{2}$

1. Halperin-Lee-Read Description



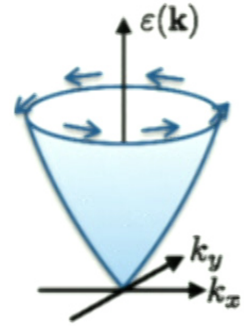
$$\mathcal{L} = \mathcal{L}(\Psi_e, a + A) + \frac{1}{8n\pi}\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$

(i) Fermion carries electromagnetic charge-1

(ii) Fermion is coupled to Chern-Simons term

[Halperin, Lee, Read, (1993), DH Lee (1998)]

2. Son's Description



$$\mathcal{L} = \bar{\chi}\left(i\not{D}_a + \mu\gamma_0\right)\chi - \frac{1}{4\pi}\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda + \frac{1}{8\pi}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

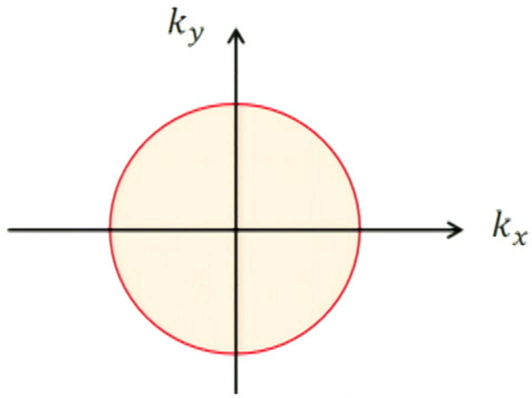
(i) Fermion is neutral under A_μ

(ii) No self Chern-Simons term for a_μ

[Son (2015)]

Geometric Responses of Composite Fermi liquid at $\nu = \frac{1}{2}$

1. Halperin-Lee-Read Description



$$\mathcal{L} = \mathcal{L}(\Psi_e, a + A) + \frac{1}{8n\pi}\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$$

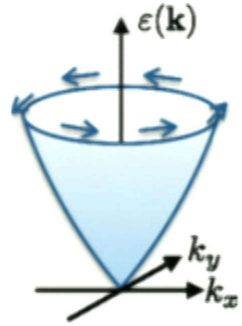
(i) Fermion carries electromagnetic charge-1
(ii) Fermion is coupled to Chern-Simons term

[Halperin, Lee, Read, (1993), DH Lee (1998)]

$$S = 1 + O\left(\frac{\Lambda}{E_F}\right)$$

[**GYC,** You, and Fradkin (2014), You, **GYC,** and Fradkin (2016)]

2. Son's Description



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(i) Fermion is neutral under A_μ
(ii) No self Chern-Simons term for a_μ

[Son (2015)]

$$S = \frac{1}{2}\ (!?)$$

[Levin and Son (2017)]

It is not clear which one is "more" correct, or even not sure if they are different !

Two descriptions *seem to* agree in many observables except the spin S (or Hall viscosity) !

[Wang, Cooper, Halperin, Stern, 2017]

Quantum Phases of Matter

Gapped Phases

Topological Phases

- Anomaly
- Symmetries
- Entanglement
- Geometry
- …

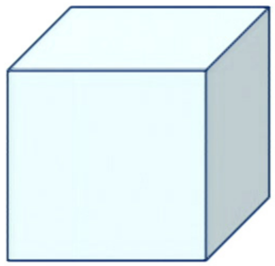
Gapless States

GYC, Teo, and Fradkin, in preparation; GYC and Ryu, in preparation

How? Natural gapless states "near" topological phases !

Two obvious metallic states:

(1) Surface states:

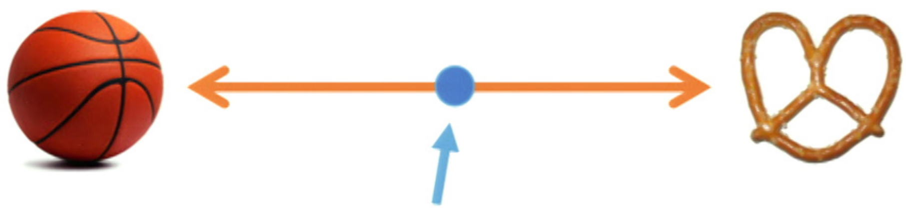


Gapless, symmetry-protected, anomalous

[GYC, Teo, and Ryu (2014), Hsieh, GYC, and Ryu (2015), Witten (2016)]

(e.g., parity anomaly $\sigma_{xy} = \frac{1}{2}$ on topological insulators)

(2) Topological Phase Transition out of topological phase:



We expect that this knows about the "anomalous" nature of topological phase!

(e.g., integer quantum Hall transition has $\sigma_{xy} = \frac{1}{2}$)

GYC, Teo, and Fradkin, in preparation; GYC and Ryu, in preparation

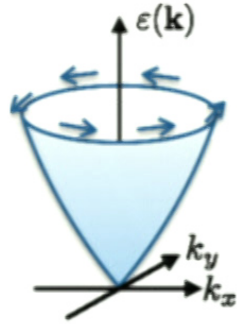
Why do I care about gapless states "near" topological phases?

In some cases,

the gapless states **have the same "anomaly"** and **symmetry** as conventional metallic states

This could mean that **they may be "dual" or the same theory**!

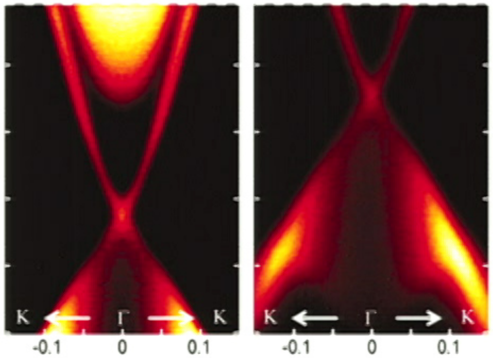
Son's theory for $\sigma_{xy} = \frac{1}{2}$ composite Fermi liquid



$$\mathcal{L} = \bar{\chi}\left(i\not{D}_a + \mu\gamma_0\right)\chi - \frac{1}{4\pi}\varepsilon^{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda + \frac{1}{8\pi}\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

$\approx$

Surface of topological insulators



Bi_2Se_3

Parity anomaly: $\sigma_{xy} = \frac{1}{2}$

Son's composite Fermi liquid is born out of topological insulators!

It can emerge from integer quantum Hall transition, too !

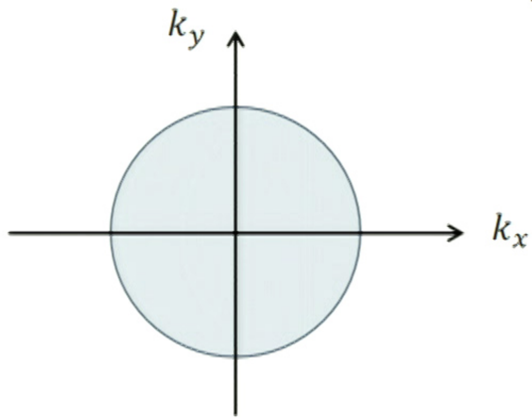
[**GYC,** Teo, and Fradkin, in preparation]

Guided by **anomaly, symmetry** and **duality,**

I have derived the effective theory of composite Fermi liquid at "$\sigma_{xy} = \frac{1}{2n}$"

[**GYC,** Teo, and Fradkin, in preparation]

Phenomenological theory for "$\sigma_{xy} = \frac{1}{2n}$"

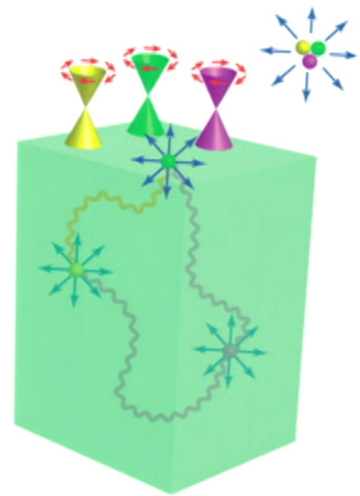


$$\mathcal{L} = \mathcal{L}_{\text{low-E}}(F, a) - \frac{\varepsilon^{\mu\nu\lambda}}{4\pi n} a_\mu \partial_\nu A_\lambda + \frac{\varepsilon^{\mu\nu\lambda}}{8\pi n} A_\mu \partial_\nu A_\lambda$$

(It is not clear how to derive this !)

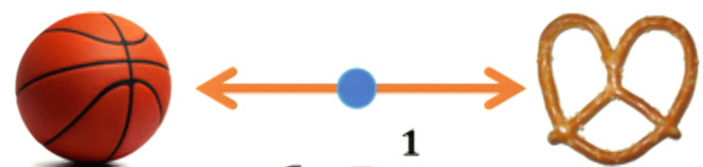
[Son (2015), Wang, Senthil (2015, 2016)]

Surface of fractional topological insulators



"$\sigma_{xy} = \frac{1}{2n}$"

Fractional quantum Hall transitions



$\sigma_{xy} = 0$ $\quad$ $\sigma_{xy} = \frac{1}{2n}$ $\quad$ $\sigma_{xy} = \frac{1}{n}$

So, many important questions are left to be discussed:

1. What kind of the gapless states can I study from this approach?

2. How far can I learn about the gapless states from this approach?

But at least, this signals that:

Knowledge from topological phases

may have bigger impacts on general quantum matters !

GYC, Teo, and Fradkin, in preparation; **GYC** and Ryu, in preparation