

Title: Exposing the Global Landscape of Topological Quantum Matter

Date: Mar 28, 2017 09:00 AM

URL: <http://pirsa.org/17030090>

Abstract: **A central theme of modern condensed matter physics is the study of topological quantum matter enabled by quantum mechanics, which provides a further "topological" twist to the classical theory of ordered phases. These quantum-entangled phases of matter such as fractional quantum Hall phases, spin liquids, and some non-Fermi liquids, are typically strongly-correlated and thus cannot be studied within conventional perturbative approaches. Because of the spectacular emergent phenomena as well as their potential for realistic applications, there has been much recent interest in exploring the physics of these exotic phases. In this talk, I show that the powerful methods of quantum field theory, namely quantum anomaly and duality, can expose the global landscape in parameter space of these gapped and gapless topological quantum phases and lead to several novel insights on these phases. As a demonstration of this principle, we study clean fractional quantum Hall transitions, composite Fermi liquids, and the surface of fractional topological insulators. Despite long and storied histories, these three systems are at the frontier of our knowledge of two and three dimensional topological phases. I show that the non-perturbative approach for these systems, i.e., the duality, sheds some new light on these systems and allows us to resolve some longstanding puzzles, which have not been clear previously. Furthermore, it uncovers novel physics of these intrinsically strongly-correlated phases of matter.**



Gil Cho

Exploring the Landscape of Topological Quantum Matter

Gil Young Cho

Korea Advanced Institute of Science and Technology

(KAIST)



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오후 10:01
2017-03-28



my previous and current research:

1. Geometric Responses

- [1] You, and Fradkin, PRB (2014)
- [2] You, GYC, You, Abanov, and Fradkin, PRL (2014)
- [3] You, GYC, and Fradkin, PRX (2014)
- [4] GYC, Parrikar, You, Leigh, and Hughes, PRB (2014)
- [5] You, GYC, and Hughes, PRB (2016)
- [6] You, GYC, and Fradkin, PRB (2016)

2. Anomaly in Condensed Matter Systems

- [1] GYC, Teo, and Ryu, PRB (2014)
- [2] Hsieh, Sule, GYC, Ryu, and Leigh, PRB (2014)
- [3] GYC, Hsieh, Morimoto, and Ryu, PRB (2015)
- [4] Hsieh, GYC, and Ryu, PRB (2016)
- [5] GYC, Shiozaki, Ryu, and Ludwig, arxiv (2016)
- [6] GYC, and Ryu, in preparation

3. Novel Many-body Phenomena in Semimetals

- [1] GYC, and Moon, Sci. Rep. (2015)
- [2] Lapa, GYC, and Hughes, PRB (2016)
- [3] GYC, Han, and Moon, PRB (2017)
- [4] Han, GYC, and Moon, in preparation

4. Physical Realizations of Topological Phases

- [1] GYC, and Moore, PRB (2011)
- [2] GYC, arxiv (2011)
- [3] GYC, Bardarson, Lu, and Moore, PRB (2012)
- [4] Oon, GYC, and Xu, PRB (2013)
- [5] GYC, Soto-Garrido, and Fradkin, PRL (2014)
- [6] Wang, GYC, Hughes, and Fradkin, PRB (2016)

5. Entanglement

- [1] Chen, GYC, Faulkner, and Fradkin, JSTAT (2016)
- [2] Gu, Lee, Wen, GYC, Ryu, and Qi, PRB (2016)
- [3] Wen, GYC, Gu, Lopes, Qi, and Ryu, PRB (2016)
- [4] GYC, Ludwig, and Ryu, PRB (2017)

6. Duality & Topological Phase Transition

- [1] GYC, Teo, Fradkin, to appear
- [2] Sahoo, Sirota, GYC, and Teo, arxiv (2017)



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오후 10:02
2017-03-28



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- [3] Wen, GYC, Gu, Lopes, Qi, and Ryu, PRB (2016)
- [4] GYC, Ludwig, and Ryu, PRB (2017)

6. Duality & Topological Phase Transition

- [1] GYC, Teo, Fradkin, to appear
- [2] Sahoo, Sirota, GYC, and Teo, arxiv (2017)

My most recent interest !



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ents

1. Topological Phase Transition and Landscape of FQHE, CFL, and FTI

Ref. **GYC**-Teo-Fradkin, *to appear*

Sahoo-Sirota-**GYC**-Teo, arxiv:1701.08828 (2017), *submitted to PRL*

2. Future Research Directions



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Part 1. Topological Phase Transition and Landscape of FQHEs, CFLs, and FTIs

Ref. **GYC**-Teo-Fradkin, *to appear*



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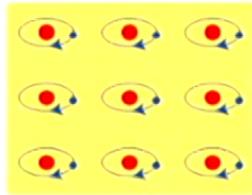
A focus of modern condensed matter physics: **Emergent Topological Phenomena**

≈ breaking down of classical intuitions by quantum mechanics & many-body physics

Before Topological Phases

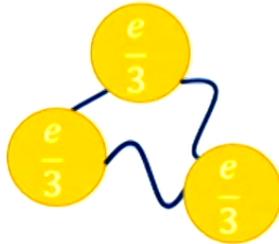


Trivial Insulators

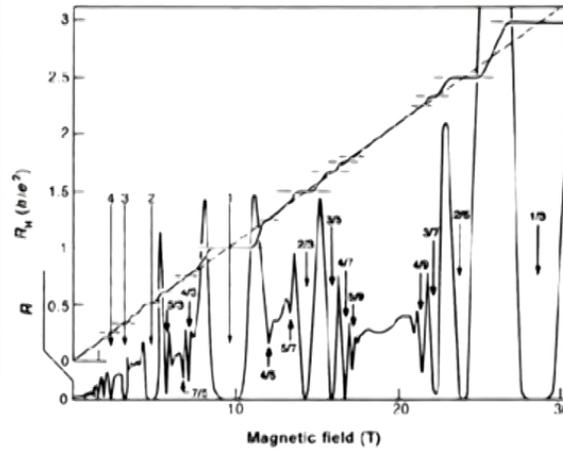


Nothing topological

Fractional Excitations:

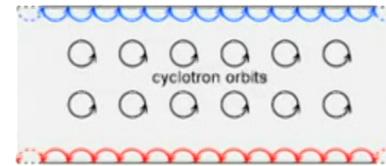


Universal & Quantized Response

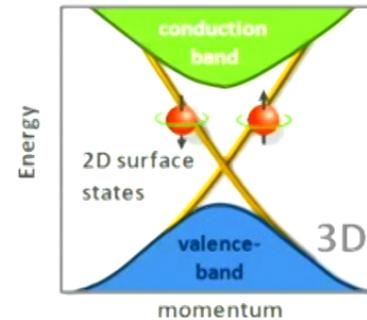


(e.g., σ_{xy} in QHEs)

Gapless Edge Modes



QHE



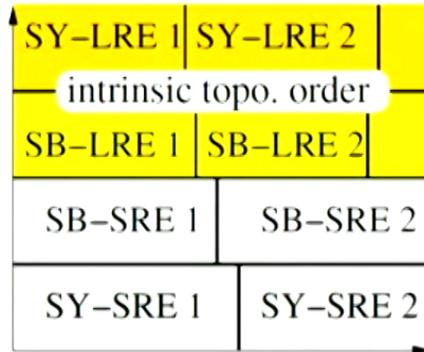
3D TI

Topological Phases



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Much effort has been devoted to classify...



Class	Symmetry			Spatial Dimension d								
	T	C	S	1	2	3	4	5	6	7	8	...
A	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	...
AIII	0	0	1	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	...
AI	1	0	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	...
BDI	1	1	1	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	...
D	0	1	0	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	...
DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	...
AII	-1	0	0	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	...
CII	-1	-1	1	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	...
C	0	-1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	...
CI	1	-1	1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	...

...and to understand their physics.

Reviews & Books: Wen (2004), Fradkin (2013), Qi, Zhang, RMP (2011), Hasan, Kane, RMP (2010),
 Hasan, Moore, Ann. Cond. Matt. Phys. (2013), Senthil, Ann. Cond. Matt. Phys. (2015), and many others



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Quantum Hall system as the representative member of topological phases

Motif for various topological phases, spin liquids, topological insulators etc. !

& one of **the oldest and best-understood topological phases** !



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오후 10:04
2017-03-28



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Quantum Hall system as the representative member of topological phases

Motif for various topological phases, spin liquids, topological insulators etc. !

& one of **the oldest and best-understood topological phases** !

FQHEs: **important open problems**



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Brief Review on FQHE

Quantum Hall System: 2DEG in magnetic field

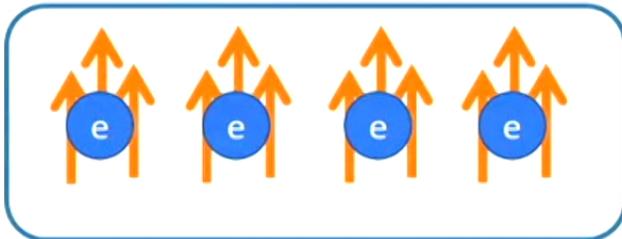
Defining the filling:

↑ = unit flux quantum Φ_0 ● e = electron

"Filling ν " = (# of electron) / (# of arrows)

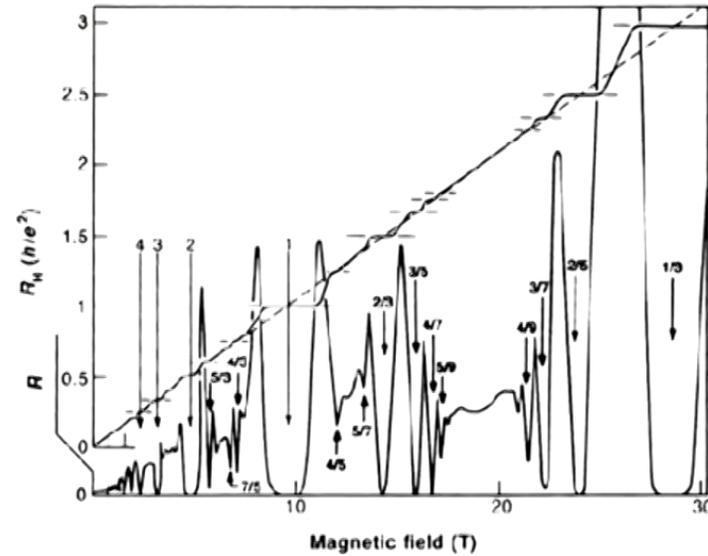
Stable gapped phases at $\nu = \frac{q}{2p+1}$

FQHE @ $\nu = \frac{1}{3}$:



(i) Universal Quantized Responses $\{\sigma_{xy}, \kappa_{xy}, \eta_H, S\}$

(ii) Fractional excitations and topological order



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FQHEs: **important open problems**



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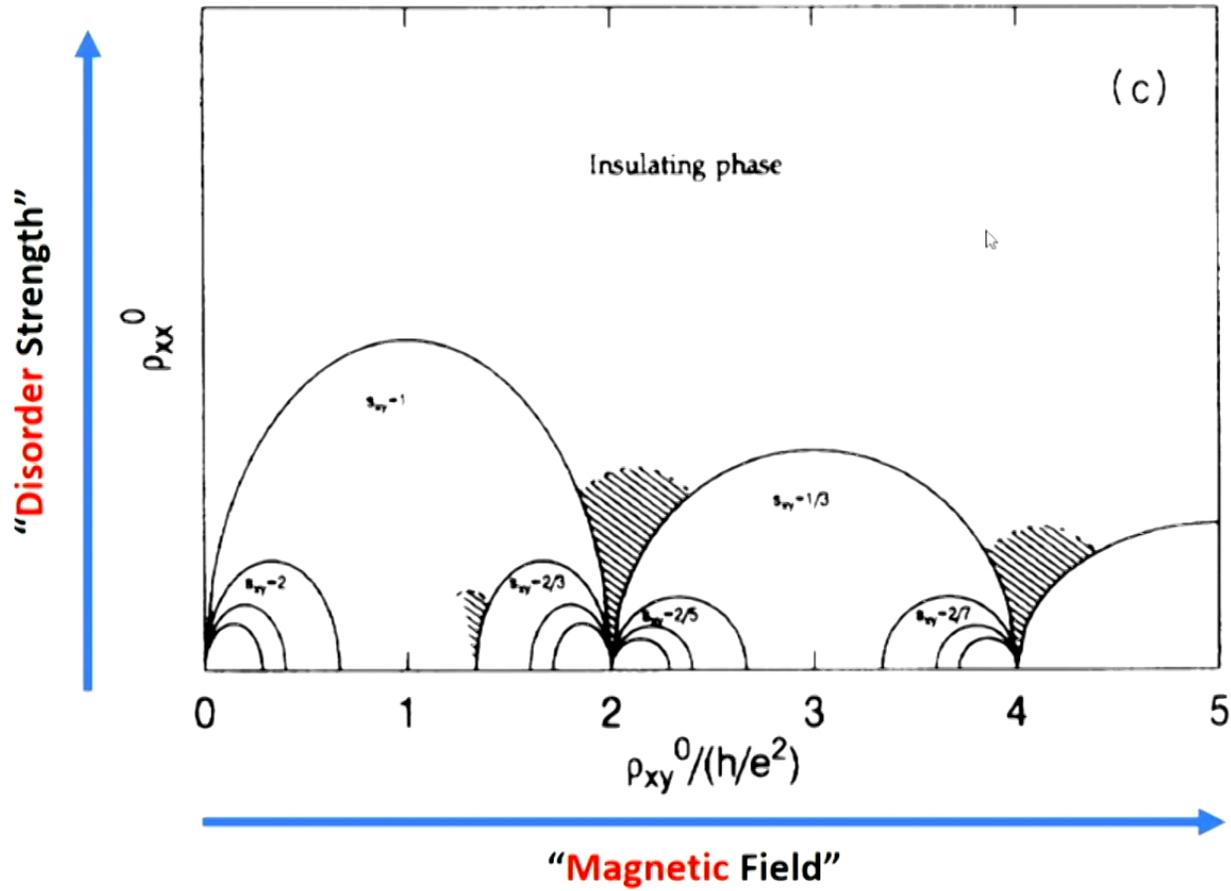


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1. FQH Plateau Transition

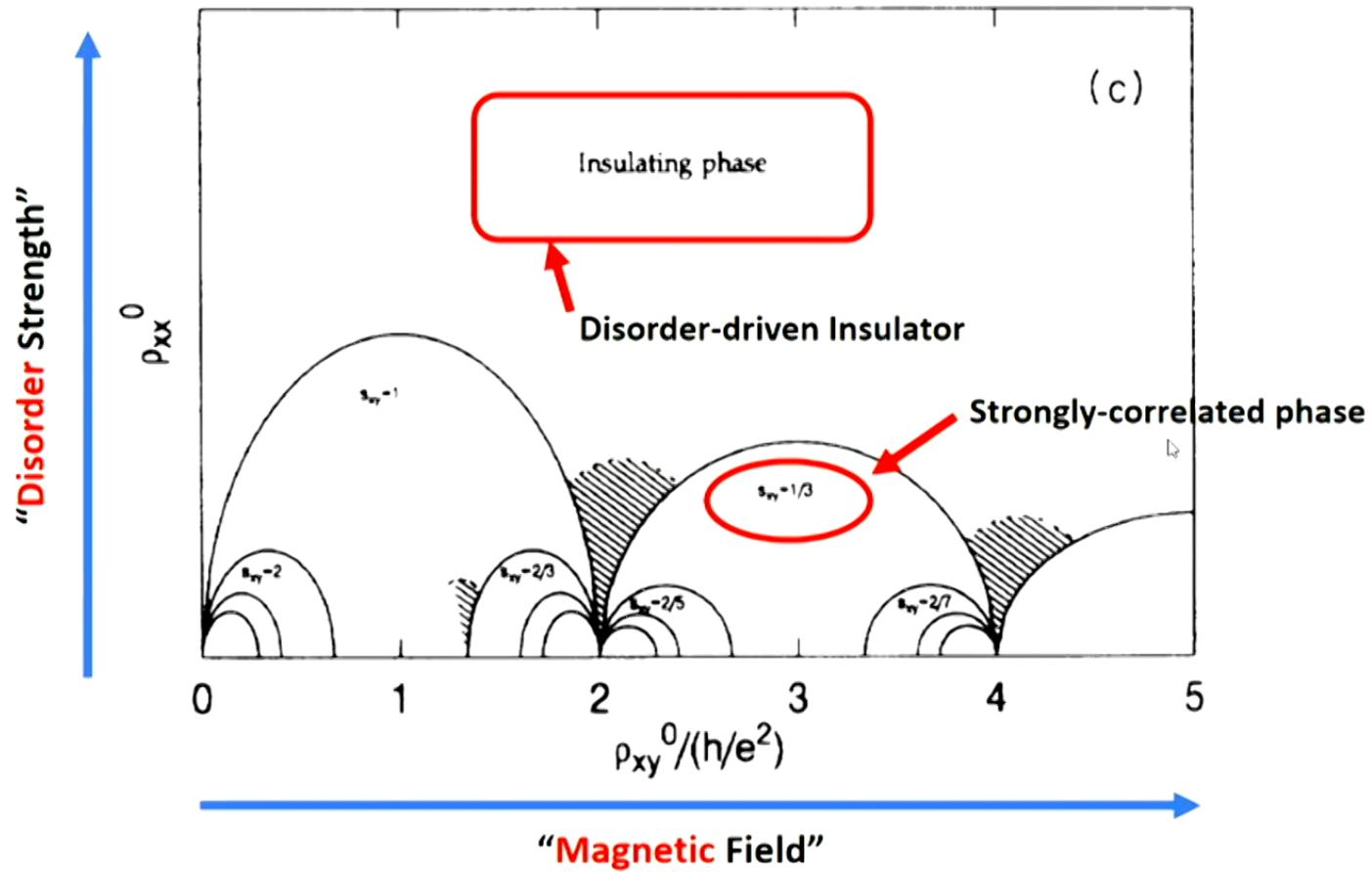


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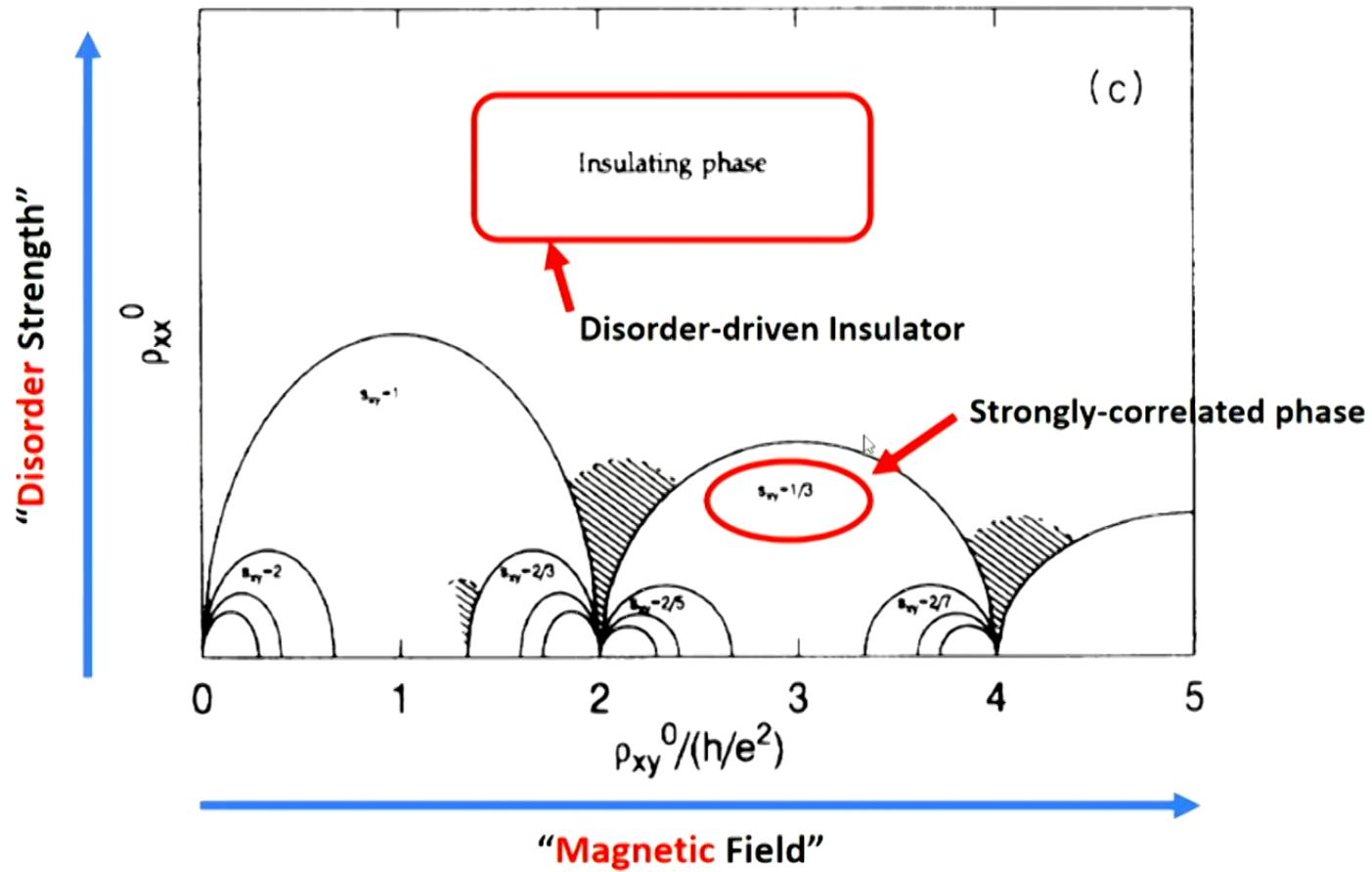
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1. FQH Plateau Transition



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1. FQH Plateau Transition



Problems of Strong disorders, Strong interactions, and Topology !



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1. FQH Plateau Transition

A. Super-universality:

Universal correlation exponent $\nu'_\zeta \approx 2.4$

Universal dynamical exponent $\nu' \approx 1$

} Independent of fillings !

All the plateau transitions belong to the same universality class !



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1. FQH Plateau Transition

A. Super-universality:

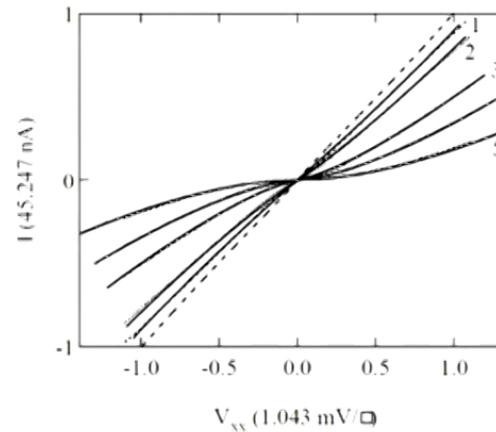
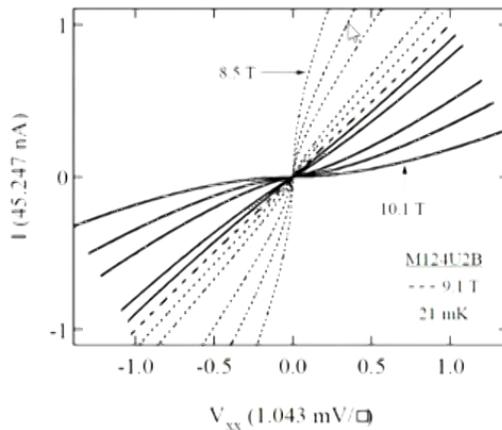
Universal correlation exponent $\nu'_z \approx 2.4$

Universal dynamical exponent $\nu' \approx 1$

Independent of fillings !

All the plateau transitions belong to the same universality class !

B. Hidden Particle-Vortex Duality near Transition:



Across the transition, 'I' (current) and 'V' (voltage) exchange roles !

(This is an incidence that duality plays an important role in topological phases!)



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1. FQH Plateau Transition

No successful microscopic theory & Largely unsolved problem!

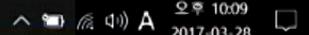
Strong Disorders

Change in Topology

Strong Interactions

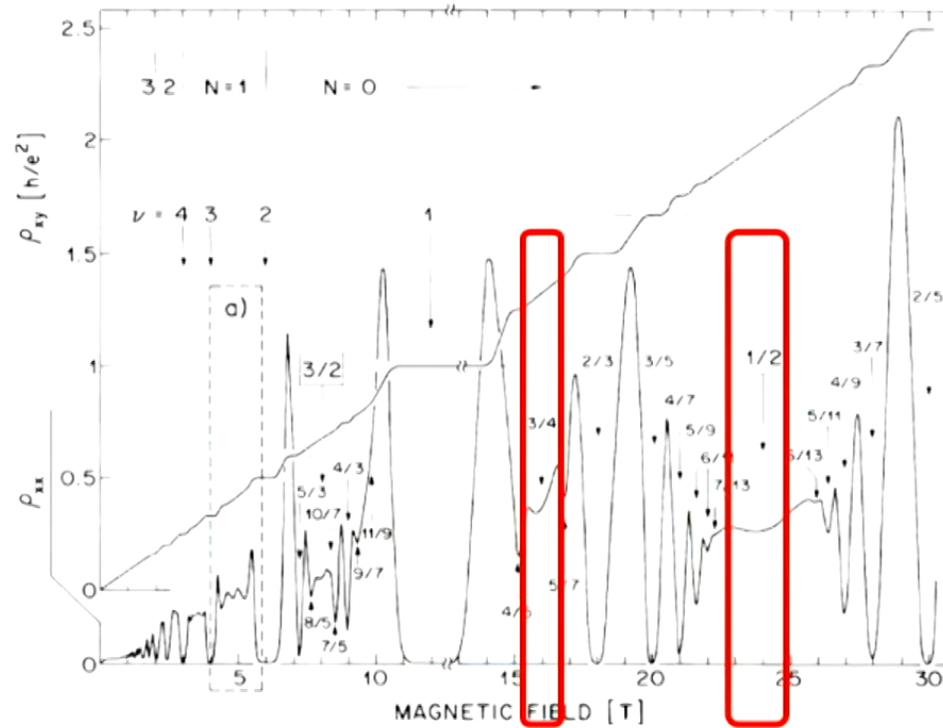


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2017-03-28

2. Gapless States: Composite Fermi Liquid



At the filling $\nu = \frac{1}{2n} = \frac{1}{2}, \frac{1}{4}, \frac{1}{6} \dots$, metallic states appear !

Nature of the states and their QFT descriptions?



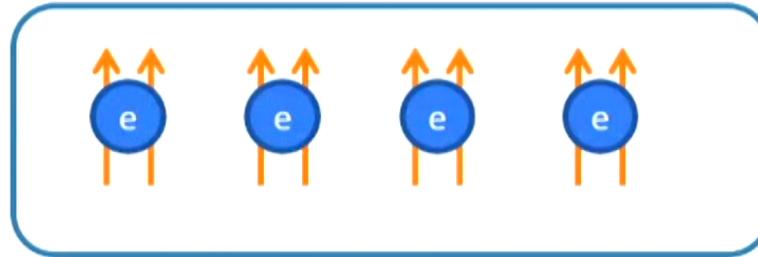
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2. Gapless States: Composite Fermi Liquid

QH State @ $\nu = \frac{1}{2}$:

 = unit flux quantum Φ_0

 = electron, charge-1



Natural to work with composite fermion theory :

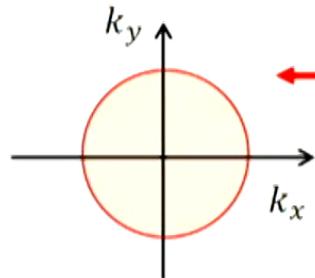
 = 



Finite density of composite fermions
with no background magnetic field !

= **Fermi liquid** (composite Fermi liquid)

Halperin-Lee-Read wrote down the QFT for the state:



$$\mathcal{L} = \mathcal{L}(\Psi_{e, \uparrow + \downarrow}) + \frac{1}{8\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$

Problem: Lack of particle-hole symmetry, etc.



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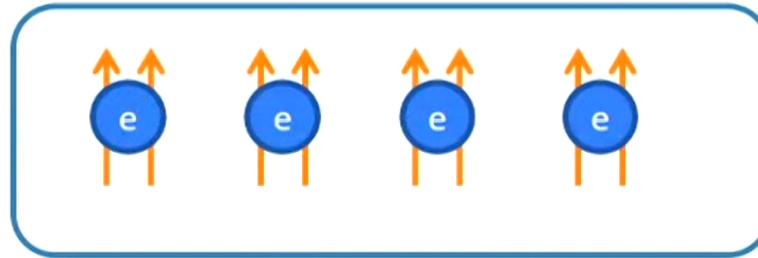
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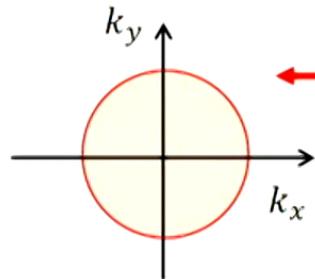
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Halperin-Lee-Read wrote down the QFT for the state:



$$\mathcal{L} = \mathcal{L}(\Psi_{e, \uparrow} + A) + \frac{1}{8\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$

Problem: **Lack of particle-hole symmetry**, etc.



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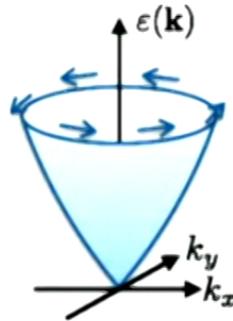
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2. Gapless States: Composite Fermi Liquid

Son's theory for Composite Fermi Liquid @ $\nu = \frac{1}{2}$:

To solve the problem of particle-hole symmetry,



$$\mathcal{L} = \bar{\chi} \left(i\mathcal{D}_a + \mu\gamma_0 \right) \chi - \frac{1}{4\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{8\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

Neutral Dirac fermion at the finite chemical potential

Support from numerics !

And argument based on **fermion-fermion duality**

(which is another incidence of duality playing important role!)

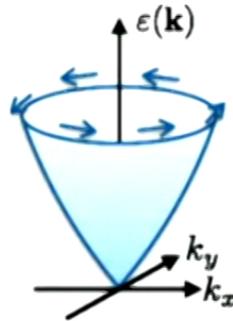


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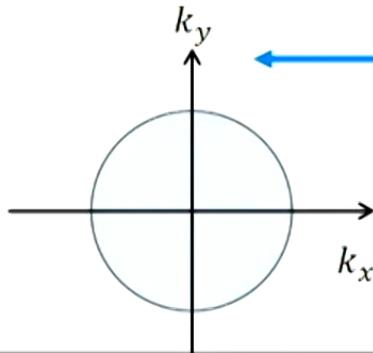
Neutral Dirac fermion at the finite chemical potential

Support from numerics !

And argument based on **fermion-fermion duality**

(which is another incidence of duality playing important role!)

Generalized for $\nu = \frac{1}{2n}, n > 1$ (conjecture based on some phenomenology of CFLs):



$$\mathcal{L} = \mathcal{L}_{\text{eff}}(\chi, a) - \frac{1}{4n\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{8n\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

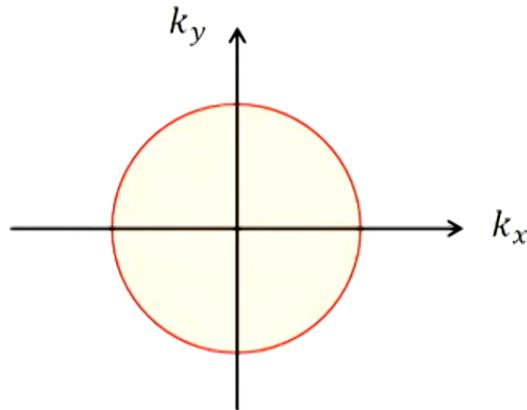
Neutral fermion at the finite chemical potential



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2. Gapless States: Composite Fermi Liquid

1. Halperin-Lee-Read Descriptions

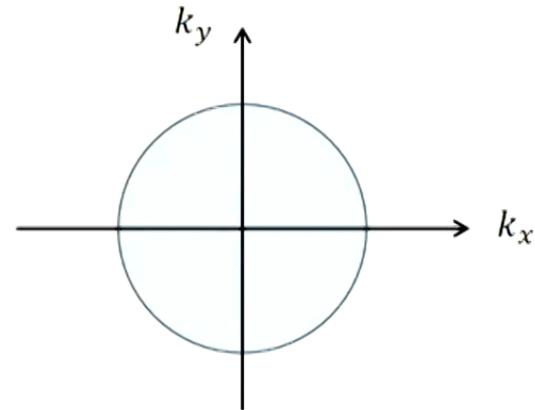


$$\mathcal{L} = \mathcal{L}(\Psi_c, a + A) + \frac{1}{8n\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$

- (i) Fermion carries EM charge 1
- (ii) Fermion is coupled to Chern-Simons term

Ref. Halperin, Lee, Read, (1993), DH Lee, (1998)
 (See, however, Wang, Cooper, Halperin, Stern, 2017)

2. Son/Wang-Senthil Descriptions



$$\mathcal{L} = \mathcal{L}_{\text{low-E}}(F, a) - \frac{\varepsilon^{\mu\nu\lambda}}{4\pi n} a_\mu \partial_\nu A_\lambda + \frac{\varepsilon^{\mu\nu\lambda}}{8\pi n} A_\mu \partial_\nu A_\lambda$$

- (i) Fermion is neutral under EM gauge A
- (ii) No self Chern-Simons term for a

Ref. Rezayi-Read, (1994), Read, (1994), Son, (2015)
 Wang-Senthil (2015, 2016)

Not clear which one is correct, nor if they are different or same



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We will shed some new light on these problems !



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오후 10:13
2017-03-28

We will consider :



Trivial Insulator

$$\sigma_{xy} = 0$$



QHEs

Strong Disorders

Change in Topology

Strong Interactions

Ref. GYC-Teo-Fradkin, *to appear*



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2017-03-28

We will consider :



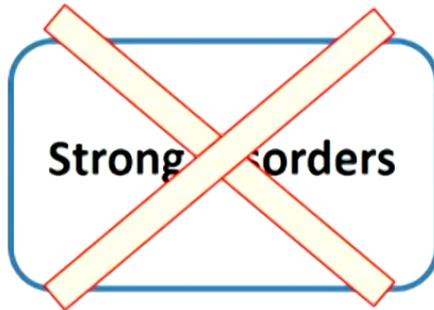
Trivial Insulator

$$\sigma_{xy} = 0$$



QHEs

By focusing only on...



Clean quantum Hall transition !

Ref. GYC-Teo-Fradkin, *to appear*



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We consider the clean topological phase transition of



Trivial Insulator

$$\sigma_{xy} = 0$$



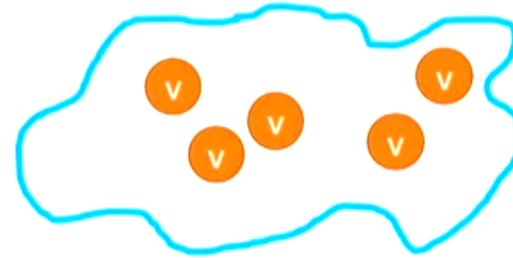
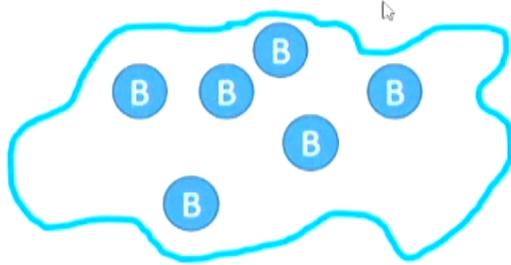
QHEs

with the help of the **duality**

which have played important roles in
physics of plateau transitions & CFLs

Duality?

e.g. Particle-vortex duality: theory of bosons \approx theory of interacting **vortices**



$$L = \frac{1}{2} |\partial_\mu \Psi|^2 - \frac{r}{2} |\Psi|^2 - \frac{u}{4} |\Psi|^4 + \dots$$

$$L = \frac{1}{2} |(\partial_\mu - ia_\mu) \tilde{\Psi}|^2 - \frac{\tilde{r}}{2} |\tilde{\Psi}|^2 - \frac{\tilde{u}}{4} |\tilde{\Psi}|^4 + \dots$$

$\Psi \sim "2\pi\text{-flux}"$ of a_μ

ref. Peskin 1978, Dasgupta, Halperin 1981



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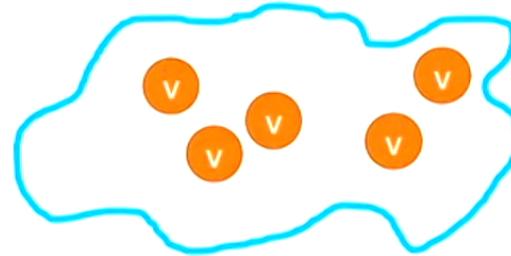
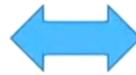
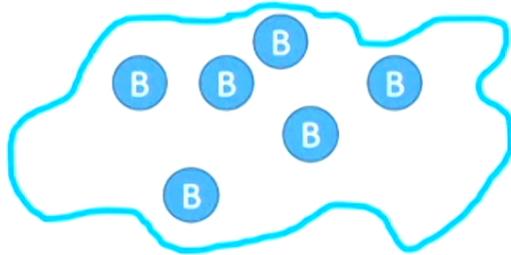
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오후 10:14
2017-03-20

Duality?

e.g. Particle-vortex duality: theory of bosons \approx theory of interacting **vortices**



$$L = \frac{1}{2} |\partial_\mu \Psi|^2 - \frac{r}{2} |\Psi|^2 - \frac{u}{4} |\Psi|^4 + \dots$$

$$L = \frac{1}{2} |(\partial_\mu - ia_\mu) \tilde{\Psi}|^2 - \frac{\tilde{r}}{2} |\tilde{\Psi}|^2 - \frac{\tilde{u}}{4} |\tilde{\Psi}|^4 + \dots$$

$\Psi \sim$ "2 π -flux" of a_μ

ref. Peskin 1978, Dasgupta, Halperin 1981

Recent progress in more broader classes of theories:

Seiberg, Senthil, Wang, Witten, Ann. Phys. (2016),
 Tong, Karch, PRX (2016)
 Mross, Aicea, Motrunich, PRL (2016)
 Hsin, Seiberg, arxiv (2016)
 Aharony, Benini, Hsin, Seiberg, arxiv (2016)
 Wang, Nahum, Metlitski, Vishwanath, arxiv (2017)

with many ancestors of these dualities

Gapless matter+ internal gauge fields

+ **Chern-Simons terms**

i.e., **Ready-made theories**

for topological phase transitions !

As a result of the cross-fertilization between condensed matter & high-energy physics !



PDDZC

Duality?

$$\mathcal{L} = \bar{\Psi} i \not{D}_A \Psi + \frac{1}{4\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \longleftrightarrow \mathcal{L} = \bar{\chi} i \not{D}_a \chi - \frac{2}{4\pi} \varepsilon^{\mu\nu\lambda} b_\mu \partial_\nu b_\lambda + \frac{1}{2\pi} \varepsilon^{\mu\nu\lambda} (A_\mu - a_\mu) \partial_\nu b_\lambda$$

Dirac fermion

Another Dirac fermion

with gauge fields {a,b}

$\Psi \sim$ "4 π -flux of gauge field a"

Ref. Seiberg, Senthil, Wang, Witten (2016); Tong, Karch (2016); Mross, Aicea, Motrunich (2016)



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Duality?

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Dirac fermion

Another Dirac fermion

with gauge fields {a,b}

$\Psi \sim$ "4 π -flux of gauge field a"

Evidences: The same set of symmetries

The same set of nearby massive phases

The same gauge invariant operator $\sim \Psi$

Explicit derivation from quasi-1d limit

Ref. Seiberg, Senthil, Wang, Witten (2016); Tong, Karch (2016); Mross, Aicea, Motrunich (2016)

Arora, Bernevig, Vishwanath (2017); Ahum, Metlitski, Vishwanath (2017)



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PDDZC

With the help of the duality, we consider:

0. IQH transition and CFL @ $\nu = \frac{1}{2}$

1. FQH transition and CFLs

2. Relations between different descriptions of FQH transitions

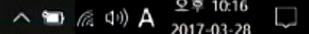
3. Surface of Fractional Topological Insulators



PDDZC



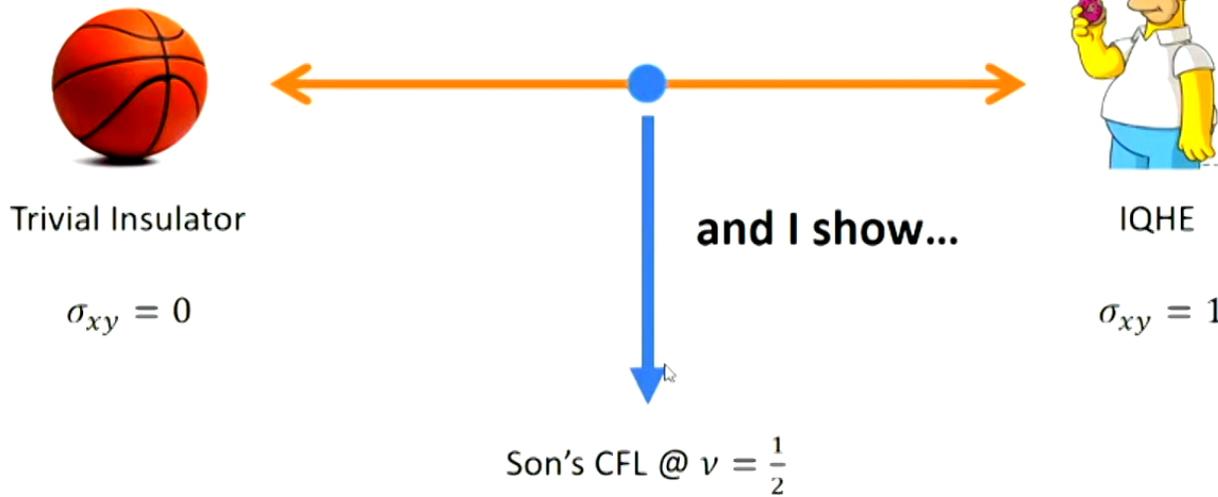
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2017-03-28

Result 0.

To see if the duality is useful, I consider....



Ref. GYC-Teo-Fradkin, *to appear*



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A Lattice Model for Topological Phase Transition:

$\sigma_{xy} = C_1$ with band structure

$C_1 = \text{Chern number} \approx \text{filling } \nu \in \mathbb{Z}$



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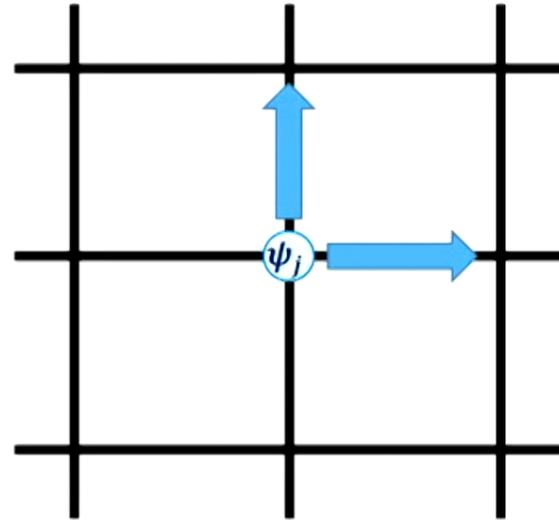
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2017-03-28

A Lattice Model for Topological Phase Transition:

$\sigma_{xy} = C_1$ with band structure

$C_1 = \text{Chern number} \approx \text{filling } \nu \in \mathbb{Z}$

Model with $C_1 = 1 \rightarrow C_1 = 0$?



$$H_{\text{MFT}} = \sum_{\vec{k}} \psi_{\vec{k}}^* \left[\lambda (\sigma_x \sin k_x + \sigma_y \sin k_y) + t \sigma_z (2 - \cos k_x - \cos k_y) + M \sigma_z \right] \psi_{\vec{k}}$$

1. $M > 0$: $C_1 = 1$ and IQHE
2. $M < 0$: $C_1 = 0$ and trivial insulator

We have a desired transition at $M = 0$!



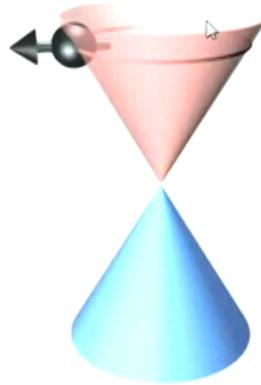
PDDZC

At $M = 0$, the band gap closes at $\vec{k} = 0$

$$H_{\text{MFT}} = \sum_{\vec{k}} \psi_{\vec{k}}^* [\lambda(\sigma_x \sin k_x + \sigma_y \sin k_y) + t\sigma_z(2 - \cos k_x - \cos k_y)] \psi_{\vec{k}},$$

We expand near $\vec{k} = 0$ and find:

$$H_{\text{MFT}} \approx \psi_{\vec{k}}^* \lambda(\sigma_x k_x + \sigma_y k_y) \psi_{\vec{k}}$$



A single Dirac fermion !



PDDZC



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Trivial Insulator

$$\sigma_{xy} = 0$$



A single Dirac fermion !

$$\mathcal{L} = \bar{\Psi} i \not{D}_A \Psi + \frac{1}{4\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$



IQHE

$$\sigma_{xy} = 1$$



What do we do? Let's perform the duality !

$$\mathcal{L} = \bar{\Psi} i \not{D}_A \Psi + \frac{1}{4\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \longleftrightarrow \mathcal{L} = \bar{\chi} i \not{D}_a \chi - \frac{2}{4\pi} \epsilon^{\mu\nu\lambda} b_\mu \partial_\nu b_\lambda + \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} (A_\mu - a_\mu) \partial_\nu b_\lambda$$

Dirac fermion

Another Dirac fermion



PDDZC

Result 0.



Trivial Insulator

$$\sigma_{xy} = 0$$



A single Dirac fermion !

$$\mathcal{L} = \bar{\Psi} i \not{D}_A \Psi + \frac{1}{4\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$



$$\mathcal{L} = \bar{\chi} i \not{D}_a \chi - \frac{1}{4\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{8\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

Applying magnetic field \bar{B}_z :

$$\mathcal{L} = \bar{\chi} \left(i \not{D}_a + \mu \gamma_0 \right) \chi - \frac{1}{4\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{8\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

with the finite chemical potential $\mu \propto \bar{B}_z$

This is Son's theory of CFL @ $\nu = \frac{1}{2}$!



IQHE

$$\sigma_{xy} = 1$$



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Ref. GYC-Teo-Fradkin, to appear

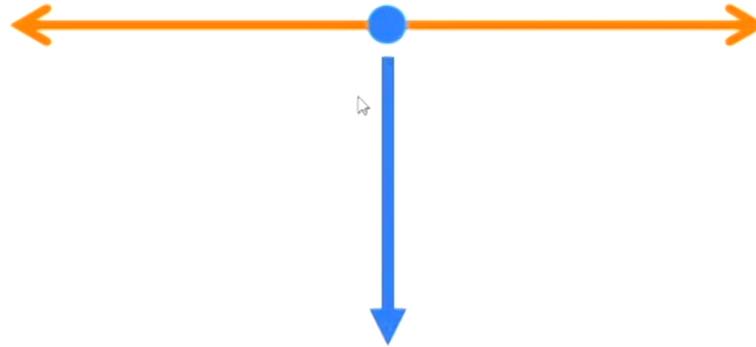
Result 0.

I showed that...



Trivial Insulator

$$\sigma_{xy} = 0$$



IQHE

$$\sigma_{xy} = 1$$

Son's CFL @ $\nu = \frac{1}{2}$

$$\mathcal{L} = \bar{\chi} \left(i \not{D}_a + \mu \gamma_0 \right) \chi - \frac{1}{4\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{8\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

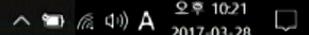
Ref. GYC-Teo-Fradkin, *to appear*



PDDZC



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오후 10:21
2017-03-28

With the help of the duality, we consider:

0. IQH transition and CFL @ $\nu = \frac{1}{2}$

1. FQH transition and CFLs

2. Relations between different descriptions of FQH transitions

3. Surface of Fractional Topological Insulators



PDDZC



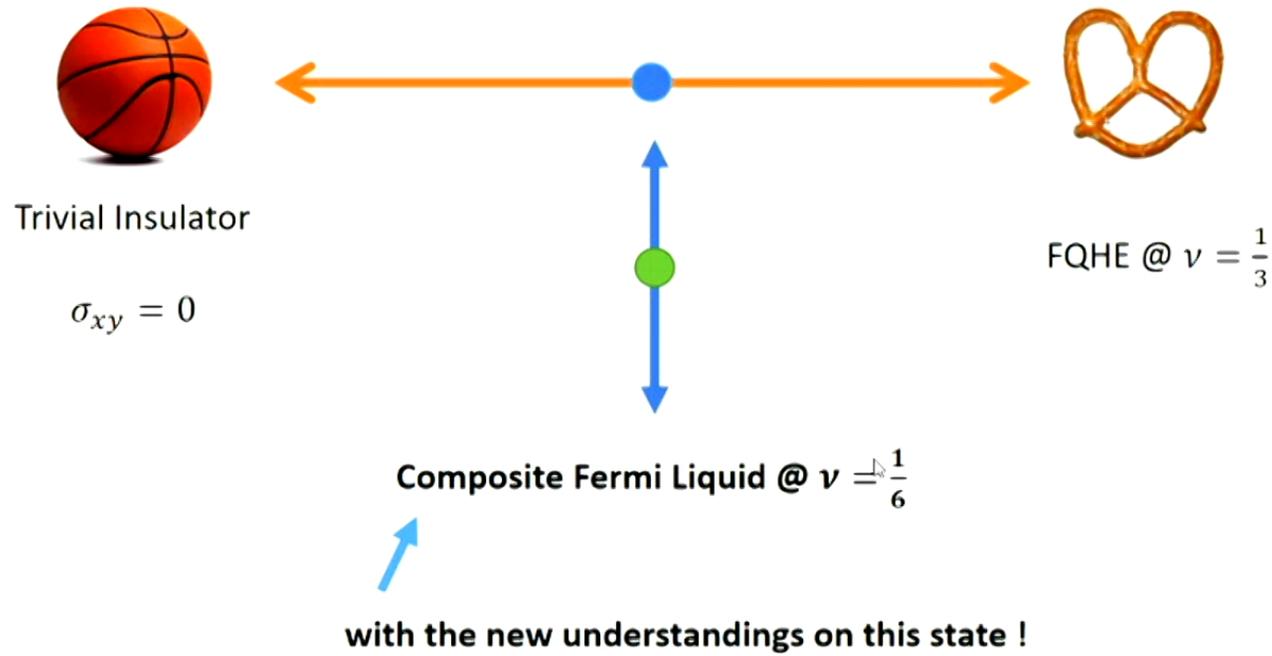
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Result 1.

For the FQHE, I show...



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Trivial Insulator

$$\sigma_{xy} = 0$$



FQHE @ $\nu = \frac{1}{3}$

To see the physics clearly,

we use the parton picture of FQHE and Transition.

Ref. GYC-Teo-Fradkin, *to appear*
Wen (1999)



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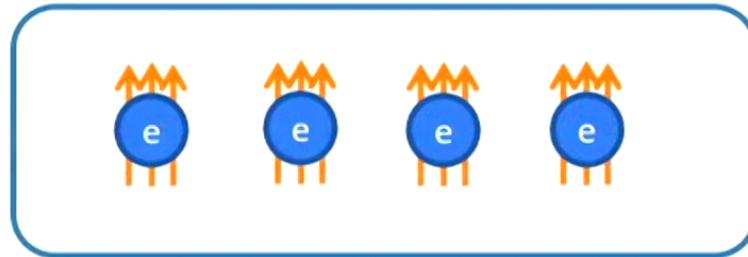


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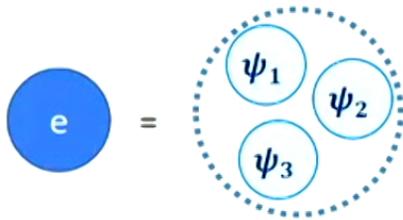
Parton Picture of FQHE at $\nu = \frac{1}{3}$: Ref. Wen (1999)

 = unit flux quantum Φ_0

 = electron, charge -1

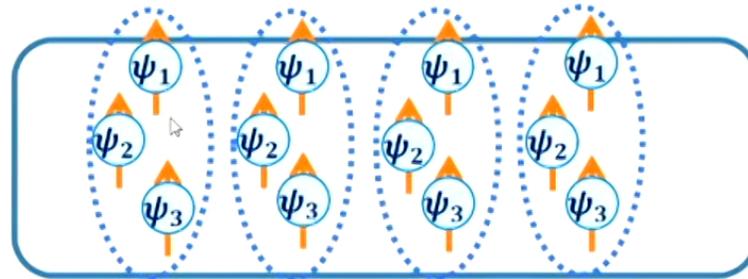


This state can be represented by...



$$\Psi_e = \psi_1 \psi_2 \psi_3$$

 = fermion, charge $-\frac{1}{3}$



Each ψ_j in the integer quantum Hall effect at $\nu = 1$!



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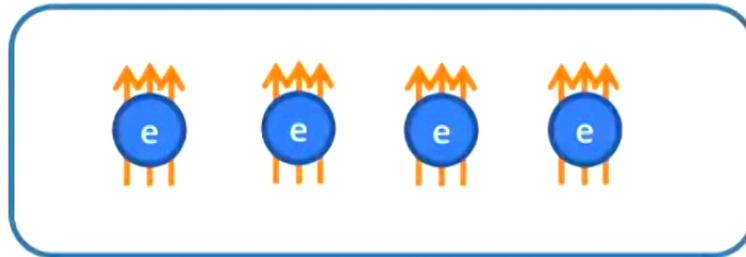
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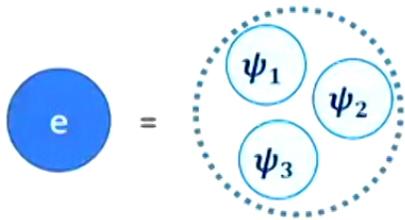
Parton Picture of FQHE at $\nu = \frac{1}{3}$: Ref. Wen (1999)

 = unit flux quantum Φ_0

 = electron, charge-1

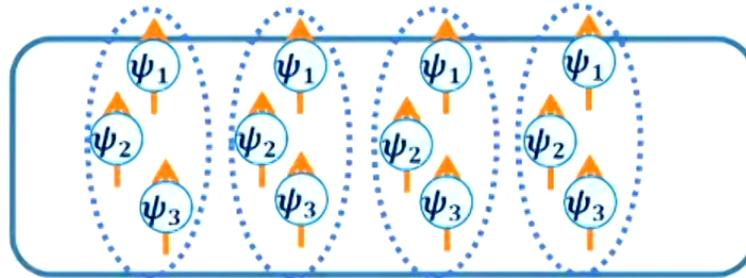


This state can be represented by...



$$\Psi_e = \psi_1 \psi_2 \psi_3$$

 = fermion, charge $-\frac{1}{3}$



Each ψ_j in the integer quantum Hall effect at $\nu = 1$!

Using $\sigma_{xy}^Q = \nu \times Q^2$ for charge-Q particle at filling $\nu \in \mathbb{Z}$

$$\sigma_{xy}^{\text{total}} = \sum_{j=1,2,3} \sigma_{xy}^j = \sum_{j=1,2,3} \left(1 \times \left(\frac{1}{3} \right)^2 \right) = \frac{1}{3} !$$



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Parton Picture of FQH Transition

From : $e = \begin{matrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{matrix}$ $\Psi_e = \psi_1\psi_2\psi_3$

$\psi_j = \text{fermion, charge } \frac{1}{3}$

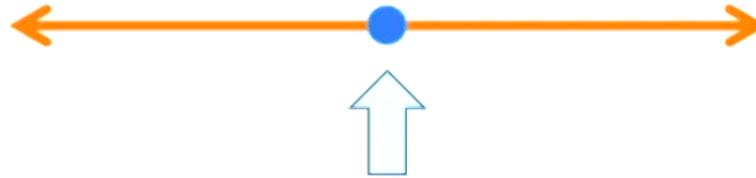


Trivial Insulator

$\sigma_{xy} = 0$: How?

By putting all ψ_j into $\nu = 0$:

$$\sigma_{xy}^{\text{total}} = \sum_{j=1,2,3} \left(0 \times \left(\frac{1}{3} \right)^2 \right) = 0!$$



ψ_j undergoes IQH transition!

Low-E Theory?



FQHE @ $\nu = \frac{1}{3}$

$$\sigma_{xy}^{\text{total}} = \sum_{j=1,2,3} \left(1 \times \left(\frac{1}{3} \right)^2 \right) = \frac{1}{3}$$

where all ψ_j in $\nu = 1$



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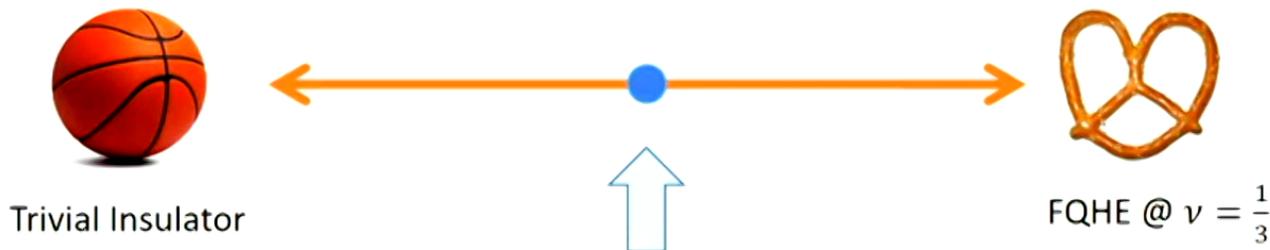
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Each ψ_j forms a Dirac fermion !

$$\mathcal{L} = \sum_{j=1, \dots, 3} \left[\bar{\psi}_j i \not{D}_{\frac{1}{3}A + q_j^I \alpha_I} \psi_j + \frac{1}{4\pi} \left(\frac{1}{3}A + q_j^I \alpha_I \right) d \left(\frac{1}{3}A + q_j^I \alpha_I \right) \right].$$

Which is now ready for application of the duality !

$$\mathcal{L} = \bar{\Psi} i \not{D}_A \Psi + \frac{1}{4\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \quad \longleftrightarrow \quad \mathcal{L} = \bar{\chi} i \not{D}_{a\lambda} \chi - \frac{2}{4\pi} \varepsilon^{\mu\nu\lambda} b_\mu \partial_\nu b_\lambda + \frac{1}{2\pi} \varepsilon^{\mu\nu\lambda} (A_\mu - a_\mu) \partial_\nu b_\lambda$$

Dirac fermion

Another Dirac fermion

where we perform the fermion-fermion duality for each ψ_j !



Trivial Insulator



FQHE @ $\nu = \frac{1}{3}$

$\Psi_e = \psi_1 \psi_2 \psi_3$ with ψ_j in a Dirac fermion !

$$\mathcal{L} = \sum_{j=1, \dots, 3} \left[\bar{\psi}_j i \not{D}_{\frac{1}{3}A + q_j^I \alpha_I} \psi_j + \frac{1}{4\pi} \left(\frac{1}{3}A + q_j^I \alpha_I \right) d \left(\frac{1}{3}A + q_j^I \alpha_I \right) \right].$$



Duality

$$\mathcal{L} = \sum_{j=1}^3 \left(\bar{\chi}_j i \not{D}_{a_j} \chi_j + \frac{1}{2\pi} \varepsilon^{\mu\nu\lambda} a_{j,\mu} \partial_\nu b_\lambda \right) - \frac{6}{4\pi} \varepsilon^{\mu\nu\lambda} b_\mu \partial_\nu b_\lambda + \frac{1}{2\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu b_\lambda$$

$\Psi_e = B \cdot \chi_1 \chi_2 \chi_3$ with χ_j in a Dirac fermion !

B = boson with charge-e @ the filling $\nu = \frac{1}{6}$ (here $J_B = \frac{1}{2\pi} db$)

χ_j = neutral Dirac fermions (dual to ψ_j)

with three dynamical U(1) gauge fields a_j ("glues")



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Duality?

$$\mathcal{L} = \bar{\Psi} i \not{D}_A \Psi + \frac{1}{4\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \longleftrightarrow \mathcal{L} = \bar{\chi} i \not{D}_a \chi - \frac{2}{4\pi} \varepsilon^{\mu\nu\lambda} b_\mu \partial_\nu b_\lambda + \frac{1}{2\pi} \varepsilon^{\mu\nu\lambda} (A_\mu - a_\mu) \partial_\nu b_\lambda$$

Dirac fermion

Another Dirac fermion

with gauge fields {a,b}

$\Psi \sim$ "4 π -flux of gauge field a"

Evidences: The same set of symmetries

The same set of nearby massive phases

The same gauge invariant operator $\sim \Psi$

Explicit derivation from quasi-1d limit

Useful ?

Ref. Seiberg, Senthil, Wang, Witten (2016); Tong, Karch (2016); Mross, Aicea, Motrunich (2016)

Microsoft Edge, Bing, Google, YouTube, GitHub, Metlitski, Vishwanath (2017)

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2017-03-28



Trivial Insulator



$$\Psi_e = \psi_1 \psi_2 \psi_3!$$



FQHE @ $\nu = \frac{1}{3}$



$$\Psi_e = B \cdot \chi_1 \chi_2 \chi_3!$$

B = boson with charge-e @ the filling $\nu = \frac{1}{6}$

χ_j = neutral Dirac fermions (dual to ψ_j)

For this metallic state, $\sigma_{xy}^{\text{MFT}} = \frac{1}{6} \times 1^2 = \frac{1}{6}$ (only B contributes!)

Any relation with Composite Fermi Liquid at $\nu = \frac{1}{6}$?



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Trivial Insulator



$$\Psi_e = B \cdot \chi_1 \chi_2 \chi_3!$$

B = boson with charge-e @ $\nu = \frac{1}{6}$

χ_j = neutral fermions



FQHE @ $\nu = \frac{1}{3}$



Confinement

$$\chi_1 \chi_2 \chi_3 \rightarrow F$$

Composite Fermi Liquid @ $\nu = \frac{1}{6}$

$$\Psi_e \sim B \cdot F$$

B: boson with charge-e @ $\nu = \frac{1}{6}$

F: neutral fermion

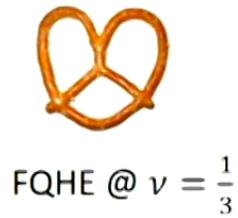
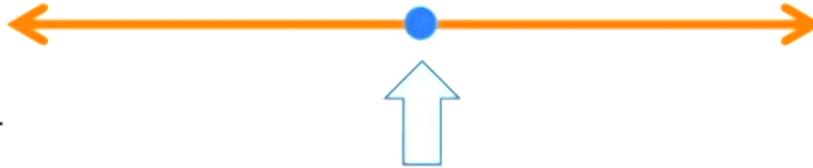


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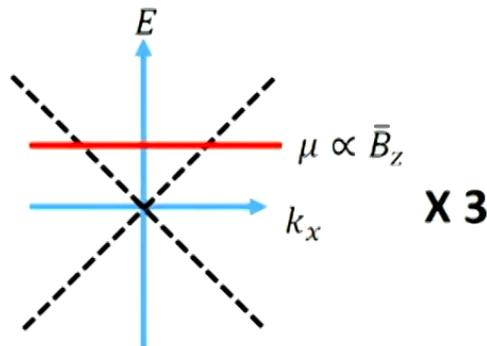
Looking closely into:



$$\mathcal{L} = \sum_{j=1}^3 \bar{\chi}_j (i\mathcal{D}_{a_j} + \mu\gamma_0)\chi_j - \frac{1}{12\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{24\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

with the finite chemical potential $\mu \propto B_z$

(here, each χ_j carries charge- $\frac{1}{3}$ of the gauge field 'a')

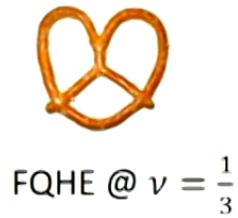
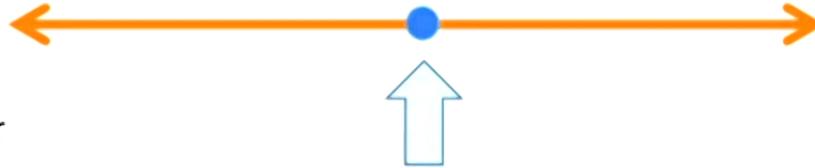


Charge	a	α	β
χ_1	1/3	1	
χ_2	1/3	-1	1
χ_3	1/3		-1



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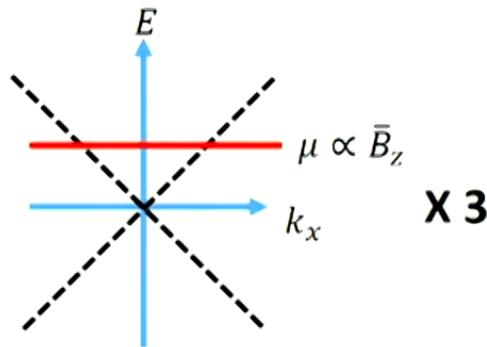
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with the finite chemical potential $\mu \propto B_z$

(here, each χ_j carries charge- $\frac{1}{3}$ of the gauge field 'a')



Charge	a	α	β
χ_1	1/3	1	
χ_2	1/3	-1	1
χ_3	1/3		-1

How do we confine? Condensation of monopoles in gauge fields $\{\alpha, \beta\}$



PDDZC

What can we learn from this?



Trivial Insulator

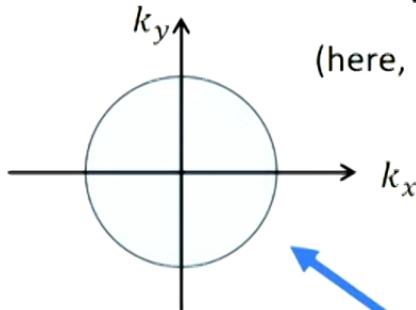


FQHE @ $\nu = \frac{1}{3}$

$$\mathcal{L} = \sum_{j=1}^3 \bar{\chi}_j (i\mathcal{D}_{a_j} + \mu\gamma_0)\chi_j - \frac{1}{12\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda + \frac{1}{24\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

with the finite chemical potential $\mu \propto \bar{B}_z$

(here, each χ_j carries charge $-\frac{1}{3}$ of the gauge field 'a')



Inheriting finite density of χ_j 's

by monopole condensation

$$\mathcal{L} = \mathcal{L}_{eff}(F, a) - \frac{1}{12\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda + \frac{1}{24\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

Looks different from standard Halperin-Lee-Read theory !?

Ref. GYC-Teo-Fradkin, to appear



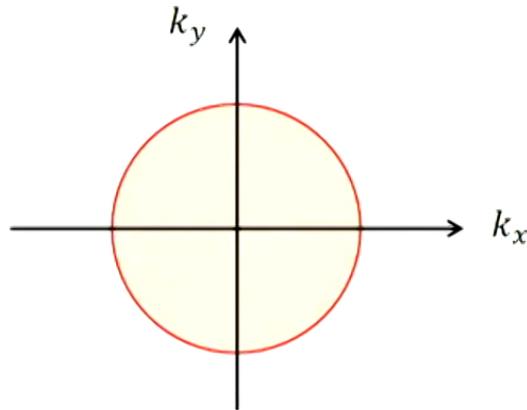
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Two (proposed) QFT descriptions of CFL at $\nu = \frac{1}{2n}$

1. Halperin-Lee-Read Descriptions

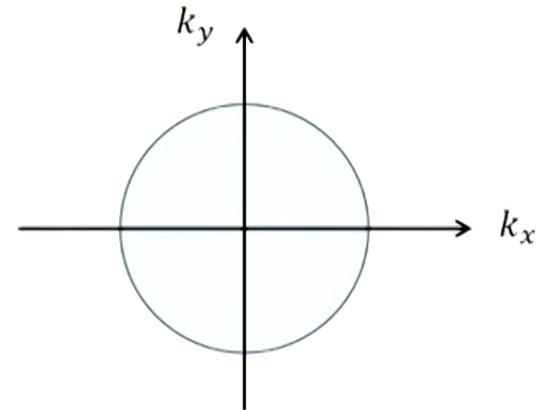


$$\mathcal{L} = \mathcal{L}(\Psi_c, a + A) + \frac{1}{8n\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$

- (i) Fermion carries EM charge 1
- (ii) Fermion is coupled to Chern-Simons term

Ref. Halperin, Lee, Read, (1993), DH Lee, (1998)
(See, however, Wang, Cooper, Halperin, Stern, 2017)

2. Son/Wang-Senthil Descriptions



$$\mathcal{L} = \mathcal{L}_{\text{low-E}}(F, a) - \frac{\varepsilon^{\mu\nu\lambda}}{4\pi n} a_\mu \partial_\nu A_\lambda + \frac{\varepsilon^{\mu\nu\lambda}}{8\pi n} A_\mu \partial_\nu A_\lambda$$

- (i) Fermion is neutral under EM gauge A
- (ii) No self Chern-Simons term for a

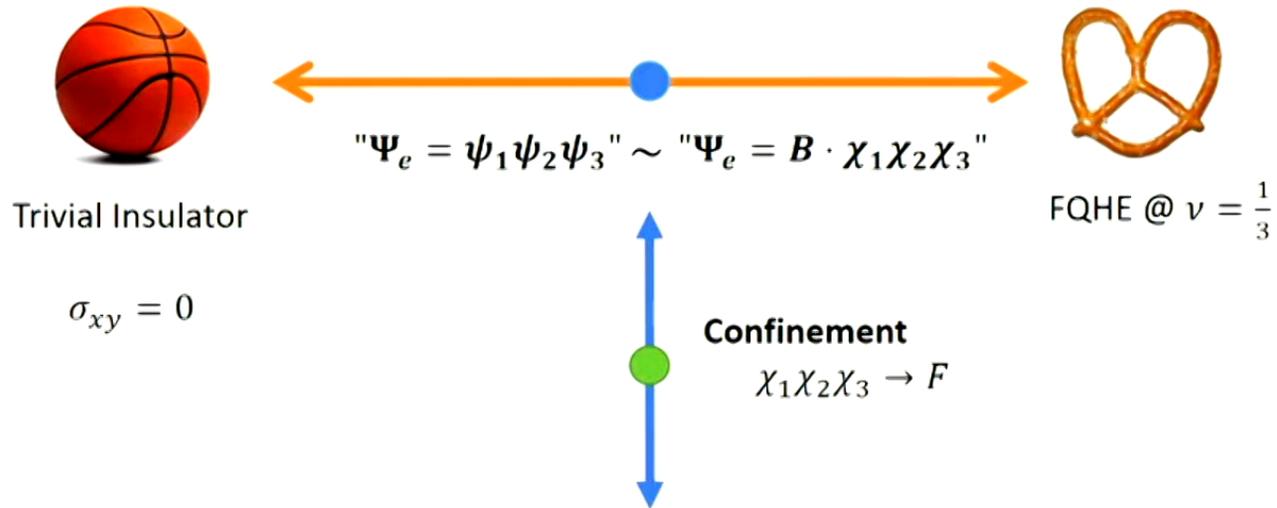
Ref. Rezayi-Read, (1994), Read, (1994), Son, (2015)
Wang-Senthil (2015, 2016)

Consistent with our QFT result !



Result 1.

Using the duality:



Composite Fermi Liquid @ $\nu = \frac{1}{6}$

$\Psi_e \sim B \cdot F$

With the derivation of conjectured QFT description

$$\mathcal{L} = \mathcal{L}_{eff}(F, a) - \frac{1}{12\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{24\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

(we also have coupled wire model for this)



PDDZC

Result 1. Generalizations



Trivial Insulator

$$\sigma_{xy} = 0$$

Using the duality:



$$|\Psi_e\rangle = \prod_{j=1 \dots 2p+1} \psi_j \sim |\Psi_e\rangle = B \cdot \prod_{j=1 \dots 2p+1} \chi_j$$



FQHE @ $\nu = \frac{1}{2p+1}$

Confinement

$$\prod_{j=1 \dots 2p+1} \chi_j \rightarrow F$$

Composite Fermi Liquid @ $\nu = \frac{1}{2(2p+1)}$

$$|\Psi_e\rangle \sim B \cdot F$$

With the derivation of conjectured QFT description

$$\mathcal{L} = \mathcal{L}_{\text{low-E}}(F, a) - \frac{\epsilon^{\mu\nu\lambda}}{4\pi n} a_\mu \partial_\nu A_\lambda + \frac{\epsilon^{\mu\nu\lambda}}{8\pi n} A_\mu \partial_\nu A_\lambda$$

(with $n = 2p+1$)



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With the help of the duality, we consider:

0. IQH transition and CFL @ $\nu = \frac{1}{2}$

1. FQH transition and CFLs

2. Relations between different descriptions of FQH transitions

3. Surface of Fractional Topological Insulators



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오후 10:38
2017-03-28

Result 1.

Using the duality:



Trivial Insulator

$$\sigma_{xy} = 0$$



$$"\Psi_e = \psi_1\psi_2\psi_3" \sim "\Psi_e = B \cdot \chi_1\chi_2\chi_3"$$



FQHE @ $\nu = \frac{1}{3}$



Confinement

$$\chi_1\chi_2\chi_3 \rightarrow F$$

Composite Fermi Liquid @ $\nu = \frac{1}{6}$

$$\Psi_e \sim B \cdot F$$

With the derivation of conjectured QFT description

$$\mathcal{L} = \mathcal{L}_{eff}(F, a) - \frac{1}{12\pi} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda + \frac{1}{24\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

(we also have coupled wire model for this)



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What can we learn from this?



Trivial Insulator

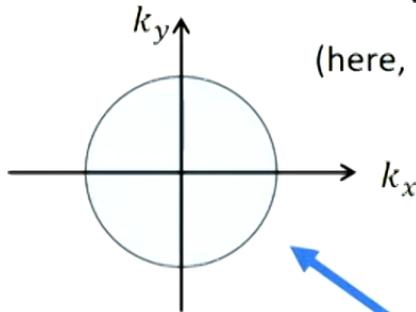


FQHE @ $\nu = \frac{1}{3}$

$$\mathcal{L} = \sum_{j=1}^3 \bar{\chi}_j (i\mathcal{D}_{a_j} + \mu\gamma_0)\chi_j - \frac{1}{12\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda + \frac{1}{24\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

with the finite chemical potential $\mu \propto \bar{B}_z$

(here, each χ_j carries charge $-\frac{1}{3}$ of the gauge field 'a')



Inheriting finite density of χ_j 's



by monopole condensation

$$\mathcal{L} = \mathcal{L}_{eff}(F, a) - \frac{1}{12\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda + \frac{1}{24\pi}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

Looks different from standard Halperin-Lee-Read theory !?

Ref. GYC-Teo-Fradkin, to appear



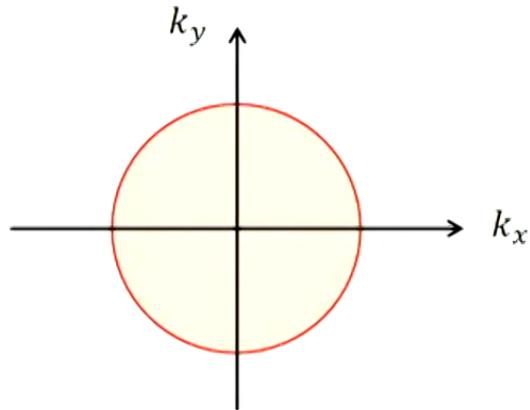
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Two (proposed) QFT descriptions of CFL at $\nu = \frac{1}{2n}$

1. Halperin-Lee-Read Descriptions

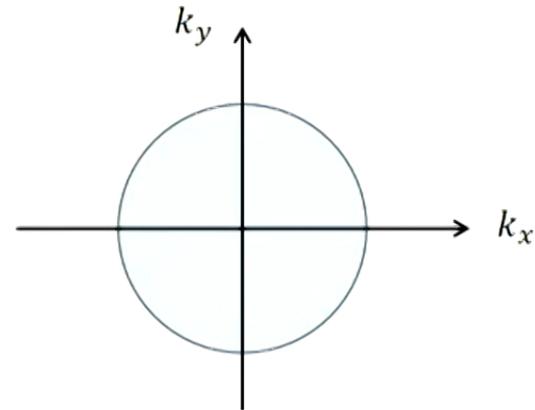


$$\mathcal{L} = \mathcal{L}(\Psi_c, a + A) + \frac{1}{8n\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$

- (i) Fermion carries EM charge 1
- (ii) Fermion is coupled to Chern-Simons term

Ref. Halperin, Lee, Read, (1993), DH Lee, (1998)
 (See, however, Wang, Cooper, Halperin, Stern, 2017)

2. Son/Wang-Senthil Descriptions



$$\mathcal{L} = \mathcal{L}_{\text{low-E}}(F, a) - \frac{\varepsilon^{\mu\nu\lambda}}{4\pi n} a_\mu \partial_\nu A_\lambda + \frac{\varepsilon^{\mu\nu\lambda}}{8\pi n} A_\mu \partial_\nu A_\lambda$$

- (i) Fermion is neutral under EM gauge A
- (ii) No self Chern-Simons term for a

Ref. Rezayi-Read, (1994), Read, (1994), Son, (2015)
 Wang-Senthil (2015, 2016)

Consistent with our QFT result !



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Using the duality:



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(we also have coupled wire model for this)



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Result 2.

Relations between Different Descriptions of FQH Transition:

Parton Descriptions &

Composite Particle Theories

1. Composite Boson Theory
2. Composite Fermion Theory



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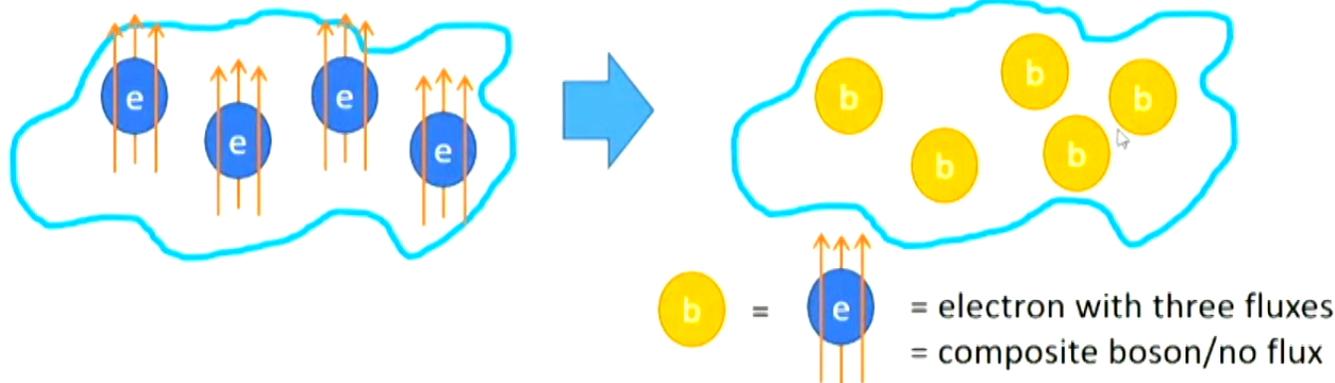


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오후 10:40
2017-03-28

Composite Boson Theory at $\nu = \frac{1}{3}$



- ➔ Boson in the absence of magnetic field will condense (BEC)!
- ➔ FQHE is the **superfluid** of the composite boson!

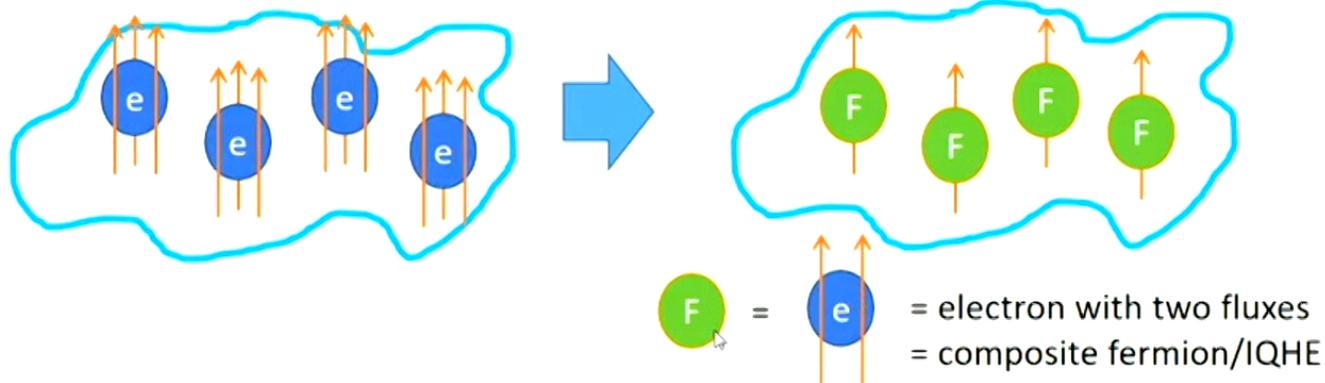


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Composite Fermion Theory at $\nu = \frac{1}{3}$



- ➔ Composite Fermion : Flux = 1:1, i.e., it is at the filling 1
- ➔ FQHE is the **IQHE** of composite fermion !

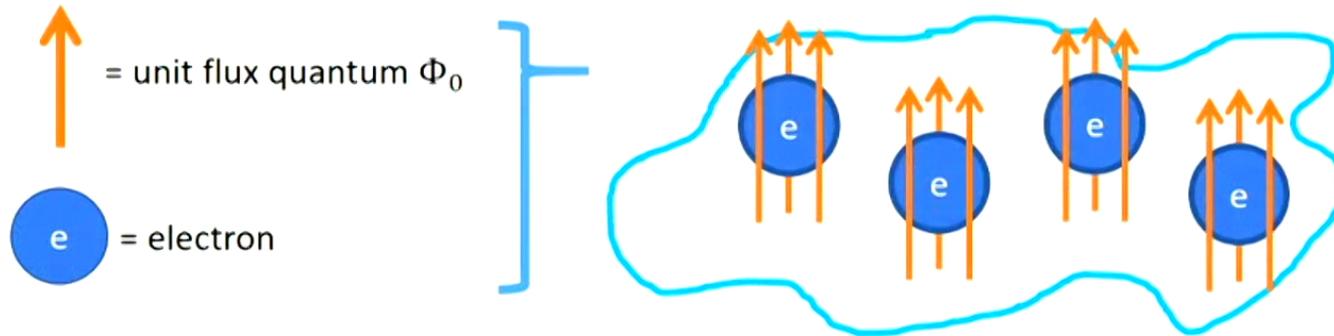


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➔ A careful look at FQHE @ $\nu = \frac{1}{3}$:



A remarkable equivalence between three descriptions of FQHEs !!:

- (i) **Composite Boson Approach:** Boson " b " = e " condenses
- (ii) **Composite Fermion Approach:** Fermion " cf " = e " forms IQHE
- (iii) **Projective Parton Approach:** Fermions " p " = $\frac{1}{3} \times e$ " forms 3-copies of IQHE



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A Key Ingredient in the **success & equivalence** of three different Mean Field Descriptions:

Gap to the excitations in FQH state

What about the transition out of the FQH state?

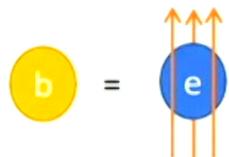


Trivial Insulator



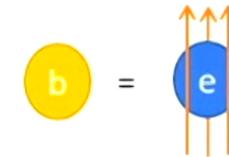
FQHE @ $\nu = \frac{1}{3}$

(i) Composite Boson Approach:



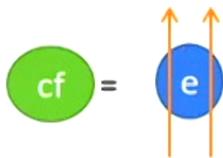
Mott Insulator

SC-MI transition



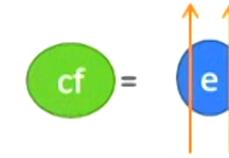
Superconductor

(ii) Composite Fermion Approach:



Trivial Band Insulator

Topological (Band) transition



IQHE



PDDZC

A Key Ingredient in the **success & equivalence** of three different Mean Field Descriptions:

Gap to the excitations in FQH state

What about the transition out of the FQH state?

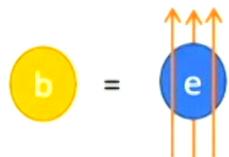


Trivial Insulator



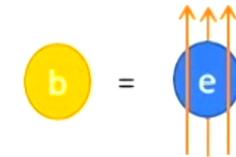
FQHE @ $\nu = \frac{1}{3}$

(i) Composite Boson Approach:



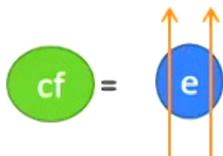
Mott Insulator

SC-MI transition



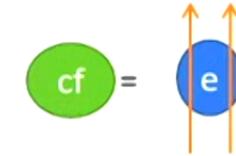
Superconductor

(ii) Composite Fermion Approach:



Trivial Band Insulator

Topological (Band) transition



IQHE



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Previously, it has been expected that :

expansions.¹² As we show below, in the fermion limit the model is soluble and exhibits a “gap-closing” transition between a band insulator and an integer quantum Hall state. We analyze the critical properties in this case, which are described by a massless 2+1 Dirac equation, and find that they are most certainly different from the 3D XY model,

Composite fermion theory

Composite boson theory

[From Chen, Fisher, Wu (1993)]

Composite Fermion Theory \neq Composite Boson Theory



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오후 10:44
2017-03-28



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With the help of the duality,
I revisit the problem again !

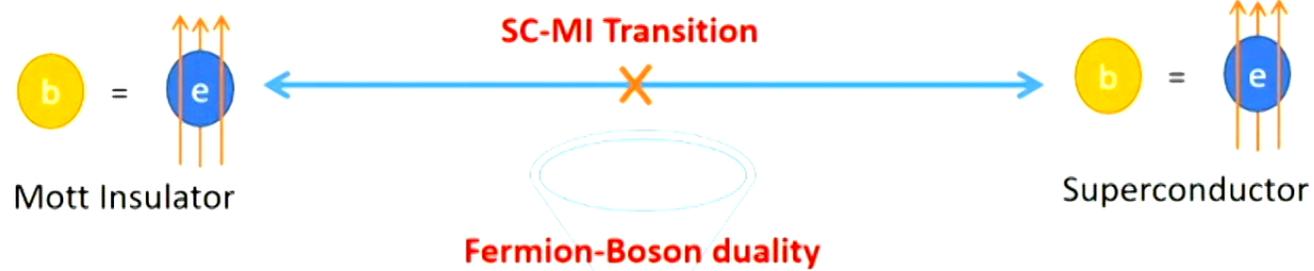
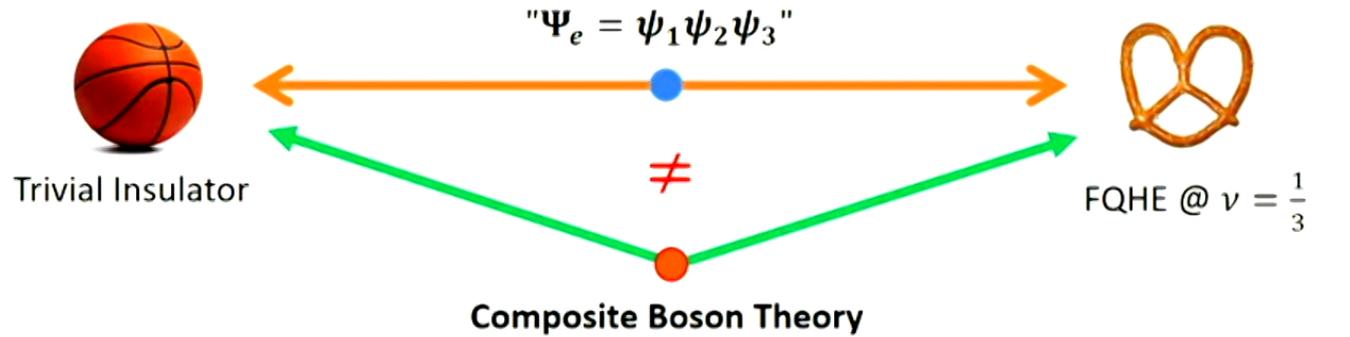


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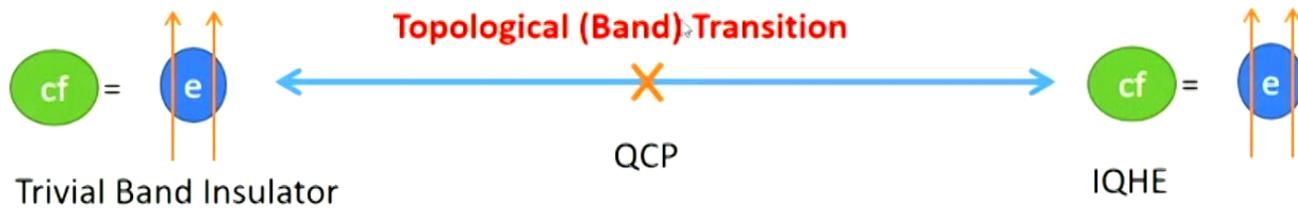


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2017-03-28

Composite Particle Theories for the FQH transition:



Composite Fermion Theory:



Ref. GYC-Teo-Fradkin, to appear

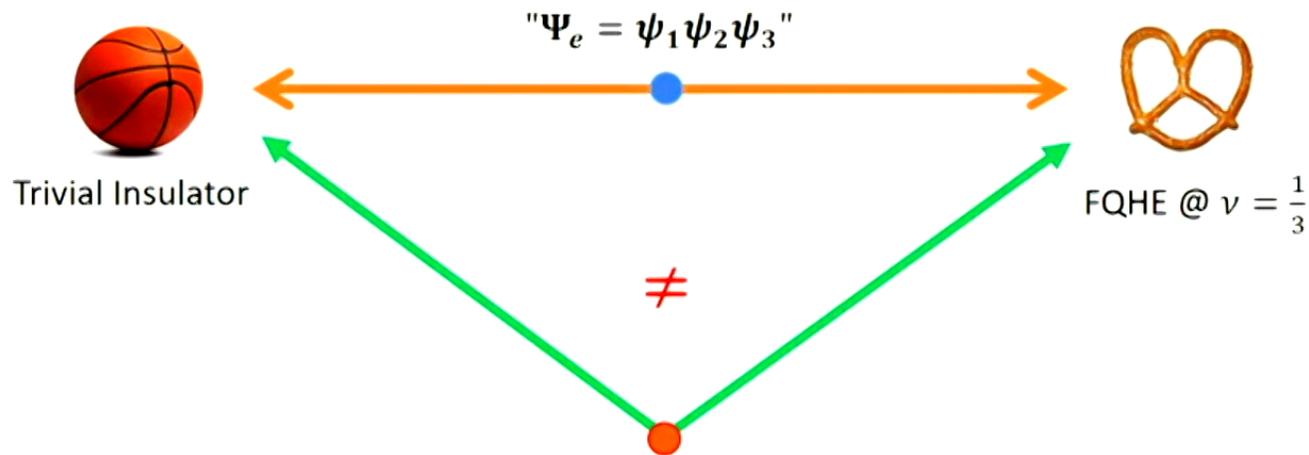


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Result 2. Composite Particle Theories for the FQH transition:



Composite Boson Theory = Composite Fermion Theory

Contrary to the previous expectations !

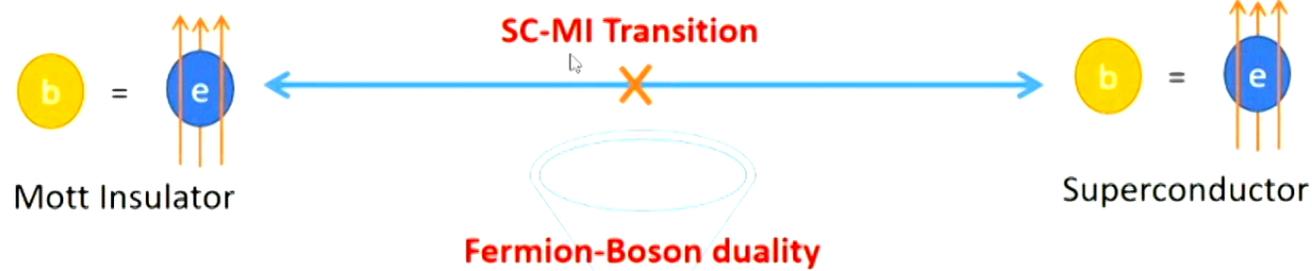
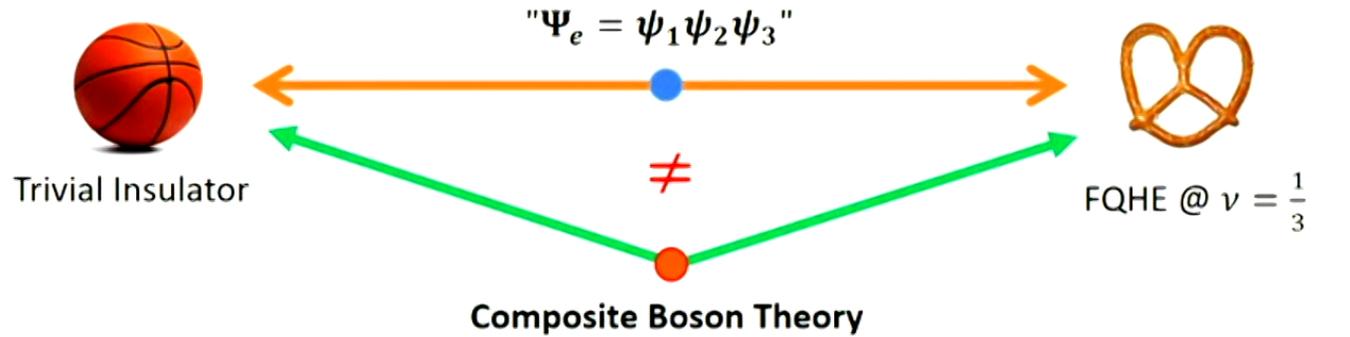


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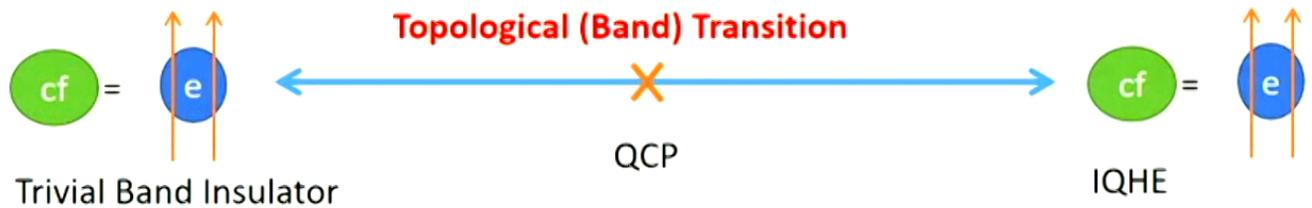
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Composite Particle Theories for the FQH transition:



Composite Fermion Theory:



Ref. GYC-Teo-Fradkin, to appear



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With the help of the duality, we consider:

0. IQH transition and CFL @ $\nu = \frac{1}{2}$

1. FQH transition and CFLs

2. Relations between different descriptions of FQH transitions

3. Surface of Fractional Topological Insulators



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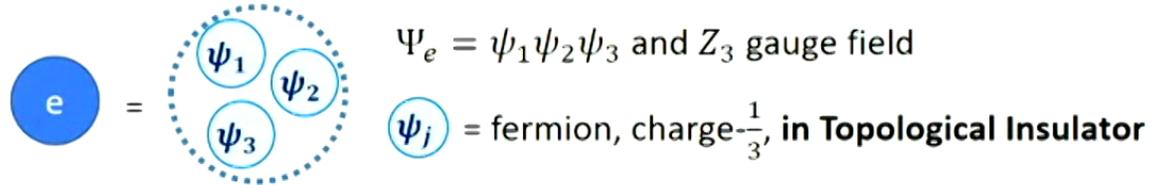


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2017-03-28

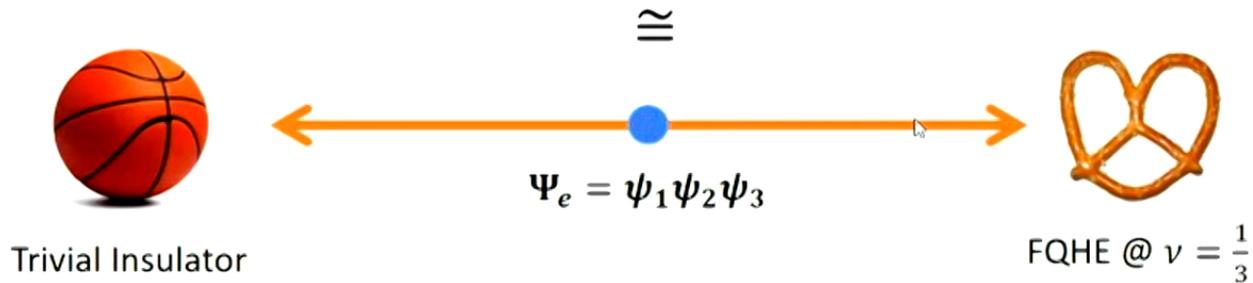
Result 3.

Can we find **a new topological state** with the help of duality?

The surface of Fractional Topological Insulators



On the surface, there are three Dirac fermions, each from ψ_j



With the help of the duality,

We construct **the symmetric gapped surface state** of this fractional phase !

Ref. GYC-Teo-Fradkin, *to appear*



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Conclusion

With the help of the duality, I have:

1. Derived the conjectured QFT of CFL @ $\nu = \frac{1}{6}$
2. Shown that CFL @ $\nu = \frac{1}{6}$ is proximate to the parton FQH transition
(and CFL @ $\nu = \frac{1}{2}$ emerges from IQH transition)
3. Shown the equivalence between composite boson and fermion theories
4. Constructed the novel symmetric gapped surface state of FTIs

Ref. **GYC-Teo-Fradkin**, *to appear*

Sobor Sirete, **GYC-Teo**, arXiv:1701.08828 (2017), *submitted to PRL*



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오후 10:50
2017-03-20

Conclusion

With the help of the duality, I have:

1. Derived the conjectured QFT of CFL @ $\nu = \frac{1}{6}$
2. Shown that CFL @ $\nu = \frac{1}{6}$ is proximate to the parton FQH transition
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Ref. **GYC-Teo-Fradkin**, *to appear*

Sehee Seiro, **GYC-Teo**, arXiv:1701.08828 (2017), *submitted to PRL*



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오후 10:51
2017-03-28

Part 2. Future Research Directions

I am interested in condensed matter systems with topology and strong-correlations

1. Exploring Consequences of Duality in Condensed Matter Systems
2. Anomaly in Condensed Matter Systems
3. Other Directions: Physics of Topological Materials & Entanglement



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오후 10:56
2017-03-28

Future Research Directions

1. Exploring Consequences of Duality in Condensed Matter Systems

: The duality as a new way to study strongly-correlated states of matter !



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2017-03-28

Future Research Directions

1. Exploring Consequences of Duality in Condensed Matter Systems

: The duality as a new way to study strongly-correlated states of matter !

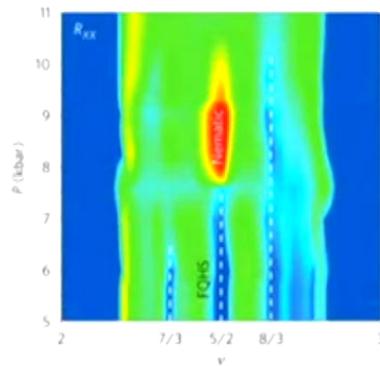
A. Topological Phase Transitions

➡ Dualities are ready-made theories for topological phase transitions !

e.g., versions with non-abelian gauge field $SU(N)$, $SO(N)$, and $SP(N)$

(Ref. Metlitski, Vishwanath & Xu, arxiv 2016; Aharony, Benini, Hsin, Seiberg, arxiv 2016)

May be relevant for non-abelian FQH transitions



Topological phase transition @ $\nu = 2 + \frac{1}{2}$

between nematic CFL & Isotropic paired state

Ref. Samkharadze et.al., Nat. Phys. (2016)



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Future Research Directions

1. Exploring Consequences of Duality in Condensed Matter Systems

B. Composite Fermi Liquids

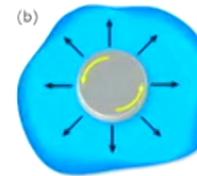
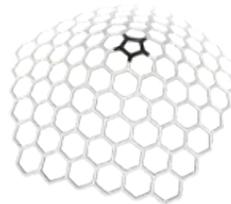
We have new theories for CFL states !

➡ Need to revisit the problems of pairing, nematic, and disorders

➡ **Geometric responses of half-filled states are hotly debated !**

Ref. You, GYC, and Fradkin (2016), Levin and Son (2016)

Geometric Response to background curvature and shear deformations



They are represented by Hall viscosity and Wen-Zee term in QFT !

- [1] GYC, You, and Fradkin, *Phys. Rev. B*, **90**, 115139 (2014)
- [2] Gromov, GYC, You, Abanov, and Fradkin, *Phys. Rev. Lett.*, **114**, 016805 (2014)
- [3] You, GYC, and Fradkin, *Phys. Rev. X*, **4**, 041050 (2014)
- [4] GYC, Parrikar, You, Leigh, and Hughes, *Phys. Rev. B*, **91**, 035122 (2014)
- [5] You, GYC, and Fradkin, *Phys. Rev. B*, **93**, 205401, (2016)

Extensions of my works in FQHEs to the composite Fermi liquids !



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Future Research Directions

2. Anomaly in Condensed Matter Systems



“break down of classical expectation (symmetry) at quantum level”

Anomaly has been playing a crucial role in understanding physics of topological phases !

From previous studies,

Symmetry-enforced gapless boundary of topological phases is anomalous !

Ref.

- [1] GYC, Teo, and Ryu, PRB (2014) [2] Hsieh, Sule, GYC, Ryu, and Leigh, PRB (2014) [3] GYC, Hsieh, Morimoto, and Ryu, PRB (2015)
- [4] Hsieh, GYC, and Ryu, PRB (2016) [5] GYC, Shiozaki, Ludwig, and Ryu, arxiv (2016) [6] GYC, Ryu, and Ludwig, PRB (2016)
- [7] Witten, RMP (2016)

Extend to other systems with symmetry-enforced gapless spectrum?

E.g., Many systems with symmetry-enforced gapless-ness **beyond topological insulators !**

Examples: 1d chains with fractional filling due to Lieb-Schultz-Mattis theorem
3d Dirac & Weyl semimetals (and many other topological semimetals)

 **Are they anomalous?** relevant for stability of the semimetals with correlations



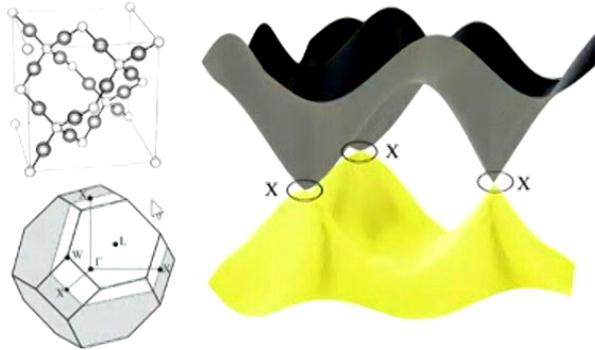
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Future Research Directions

2. Anomaly in Condensed Matter Systems

EX: Dirac semimetal in Diamond lattice:

(ref. Young et.al., PRL 2012)



$$H = \sum_{a=1,2,3} (\psi_{a,R}^* \vec{\sigma} \cdot \vec{k} \psi_{a,R} - \psi_{a,L}^* \vec{\sigma} \cdot \vec{k} \psi_{a,L})$$

Emergent SU(3) symmetries at low-energy

➔ Anomalous & must be gapless
(Ref. t'Hooft, PRL 1976)



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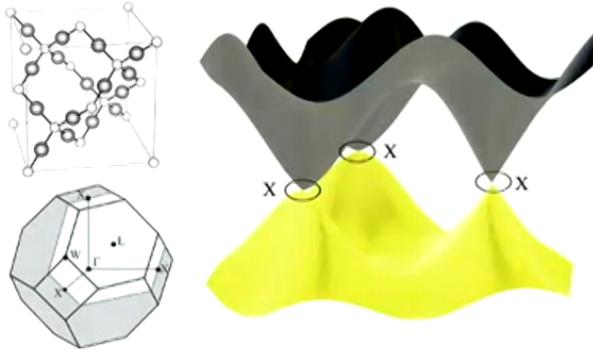
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Future Research Directions

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Emergent SU(3) symmetries at low-energy

➔ Anomalous & must be gapless
(Ref. t'Hooft, PRL 1976)

➔ In reality, only the discrete subgroup of SU(3) symmetry survives: is it still gapless?
(see Csaki and Murayama 1998)

Or, can it be trivially gapped? Does it allow gapped state with degeneracy?

Relation of anomaly and Hastings-Oshikawa-Lieb-Schultz-Mattis Theorems?
[Theorems concern about existence of "trivially gapped" state.]



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Future Research Directions

3. Other Directions: Physics of Topological Materials & Entanglement

A. Emergent Many-body Phenomena in Topological Systems

We search for the effect of correlations in various novel topological insulators & semimetals including symmetry-breaking (superconductivity, density waves) & quantum criticalities, etc. e.g., topology may lead to the novel quantum critical behaviors.

(Ref. GYC & Moon, Sci. Rep. 2016; GYC, Han & Moon, PRB 2017; Han, GYC & Moon, in preparation)

B. Realization of Topological Phases in Strongly-correlated Systems

We search for topological superconductivities in strongly-correlated systems !

E.g. strongly-correlated 1d wires, systems with multipolar fluctuations

(ref. GYC, Soto-Garrido, Fradkin, PRL 2014, Wang, GYC, Hughes, Fradkin, PRB 2016, GYC, Bardarson, Lu, Moore, PRB 2012)

C. Entanglement

Entanglement in topological phases & conformal field theories !

E.g. entanglement spectrum of gapped phase, emergent geometry, subleading terms

(Ref. GYC, Ludwig, Ryu, PRB 2017, GYC, Shiozaki, Ryu, Ludwig, arxiv 2016, Chen, GYC, Faulkner, Fradkin, JSTAT 2015 Wen, GYC, Lopes, Gu, Qi, Ryu, PRB 2016; Gu, Lee, Wen, GYC, Ryu, Qi, RPB 2016; Hu & Vidal, arxiv 2017)



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Thanks!



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오후 11:02
2017-03-28