

Title: Schwinger-Keldysh effective field theory

Date: Mar 21, 2017 02:30 PM

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Abstract: <p>The subject of quantum field theory in mixed states of quantum matter is an old and rich one. The natural setting to discuss field theory in a mixed state is the Schwinger-Keldysh formalism. The subject of this talk is the set of peculiar symmetries that arise in Schwinger-Keldysh theories, and how they may be accounted for in effective field theory. In particular, when the mixed state is thermal, the effective description is constrained by two BRST-like supercharges which, at low energies, generate an algebra akin to minimal supersymmetric quantum mechanics. If time allows, I will also discuss a sort of Schwinger-Keldysh bootstrap for effective actions on more complicated closed-time-contours, which describe the out-of-time-ordered correlation functions that diagnose early-time chaotic growth in quantum systems.</p>

Schwinger-Keldysh EFT

based on 1701.07436

w/ N. Pinzani-Fokeeva & A. Yarom

(see also papers by P. Kovtun et al, H. Lee et al,
M. Rangamani et al)

Schwinger-Keldysh EFT

based on 1701.07436

w/ N. Iizumi-Fukuda & A. Yamamuro

(see also papers by P. Kovtun et al, H. Liu et al,
M. Rangamani et al)

hydrodynamics $\left\{ \begin{array}{l} \text{black branes in AdS} \\ \text{QFT @ T=0} \end{array} \right.$

1. "current algebra"
2. broad range of applicability

$$S[\varphi], \quad Z = \int [d\varphi] e^{iS[\varphi]}$$

$$\langle T^{\mu\nu} \rangle$$

$$\langle J^{\mu\nu} \rangle$$

$$S[\varphi], \quad Z = \int [d\varphi] e^{iS[\varphi]}$$

$$\langle T^{\mu\nu} \rangle$$

$$\langle J^{\nu} \rangle$$

$$\langle J^0 \rangle = \rho$$

$$\langle J^i \rangle = -\rho \nabla^i \rho + O(j^2)$$

$$S[\varphi], \quad Z = \int [d\varphi] e^{iS[\varphi]}$$

$$\langle T^{\mu\nu} \rangle$$

$$\langle J^{\mu\nu} \rangle$$

$$\langle J^0 \rangle = \rho$$

$$\langle J^i \rangle = -\rho \nabla^i \varphi + O(\partial^2)$$

$$\partial_\mu \langle J^{\mu\nu} \rangle = 0$$

$$S[\varphi], \quad Z = \int [d\varphi] e^{iS[\varphi]}$$

$$\langle T^{\mu\nu} \rangle$$

$$\langle J^{\mu} \rangle$$

$$\langle J^0 \rangle = \rho$$

$$\langle J^i \rangle = -\rho \nabla^i \varphi + O(j^2)$$

$$\partial_\rho \langle J^i \rangle = 0$$

slowly varying $(g_{\mu\nu} A_\mu)$

$$\langle J^i \rangle [A_\nu]$$

$$\frac{\delta \langle J^i(x) \rangle}{\delta A_\nu(y)} \Big|_{A=0} = \langle J^i(x) J^\nu(y) \rangle$$

- 1. action principle?
- 2. perturbation thy?
- 3. Second Law?

$$S^p = S u^p + o(\epsilon)$$
$$D_{\downarrow} S^p \geq 0$$

QFT
in mixed
states

1. action principle?
2. perturbation thy?
3. Second Law?

SK

$$S^{\mu\nu} = S_0^{\mu\nu} + O(\epsilon)$$
$$D_{\mu} S^{\mu\nu} \geq 0$$

\$ QFT mixed state $\rho_{-\infty}$

$$Z = \text{tr} \left(U[A_1]_{-\infty} U^\dagger[A_2] \right)$$

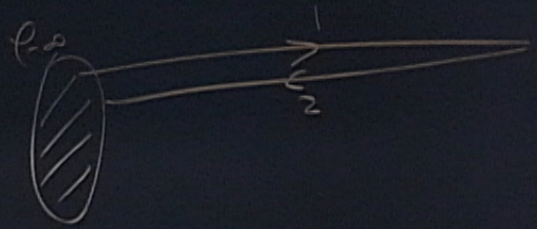
§ QFT mixed state $\rho_{-\infty}$

$$\mathcal{Z}[A_1, A_2] = \text{tr} \left(U_1 \rho_{-\infty} U_2^\dagger \right)$$

\$ QFT mixed state $\rho_{-\infty}$

$$Z[A_1, A_2] = \text{tr} \left(U_1 \rho_{-\infty} U_2^\dagger \right)$$

$$Z, S = \int [d\varphi_1] [d\varphi_2] e^{i(S[\varphi_1, A_1] - S[\varphi_2, A_2])}$$

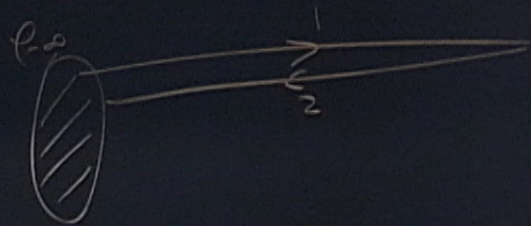


\$ QFT mixed state $\rho_{-\infty}$

$$Z[A_1, A_2] = \text{tr} \left(U_1 \rho_{-\infty} U_2^\dagger \right)$$

$$\text{L.S.} = \int [d\varphi_1] [d\varphi_2] e^{i(S[\varphi_1, A_1] - S[\varphi_2, A_2])}$$

$$Z^*[A_1, A_2] = \text{tr} \left(U_2 \rho_{-\infty} U_1^\dagger \right) = Z[A_2, A_1]$$

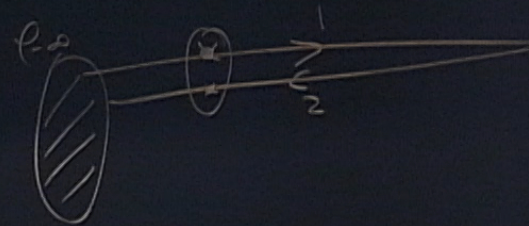


\$ QFT mixed state $\rho_{-\infty}$

$$Z[A_1, A_2] = \text{tr} \left(U_1 \rho_{-\infty} U_2^\dagger \right)$$

$$Z, S = \int [d\varphi_1] [d\varphi_2] e^{i(S[\varphi_1, A_1] - S[\varphi_2, A_2])}$$

$$Z^*[A_1, A_2] = \text{tr} \left(U_2 \rho_{-\infty} U_1^\dagger \right) = Z[A_2, A_1]$$



Self EC

QFT
in mixed
States

1. action principle?
2. perturbation thy?
3. Second Law?

SK

$$\langle \tilde{T}(\theta \dots \theta) \tau(\theta \dots \theta) \rangle$$

ret
qdv
sym

- QFT in mixed states
1. action principle?
 2. perturbation thy?
 3. Second Law?
- SK

$$\langle \tilde{T}(\theta \dots \theta) \tau(\theta \dots \theta) \rangle$$

ret

qdv

sym

$$\phi_r = \frac{\phi_1 + \phi_2}{2}, \quad A_r = \frac{A_1 + A_2}{2}$$

$$\phi_a = \frac{\phi_1 - \phi_2}{2}, \quad A_a = \frac{A_1 - A_2}{2}$$

$$\int \delta A_1 \theta_1 - \delta A_2 \theta_2$$

QFT
in mixed
states

1
2
3

SK

action principle?

perturbation?

Second Lo

$$\langle \tilde{T}(\theta \dots \theta) \tau(\theta \dots \theta) \rangle$$

ret

q.d.v

sym

$$\phi_r = \frac{\phi_1 + \phi_2}{2}, \quad A_r = \frac{A_1 + A_2}{2}$$

$$\phi_a = \frac{\phi_1 - \phi_2}{2}, \quad A_a = \frac{A_1 - A_2}{2}$$

$$\delta A_1 \theta_1 - \delta A_2 \theta_2$$

$$\delta A_r \theta_a + \delta A_a \theta_r$$

- QFT in mixed states
1. action principle?
 2. perturbation thy?
 3. Second Law?
- SK

$$\langle \tilde{T}(\theta \dots \theta) T(\theta \dots \theta) \rangle$$

$$\text{ret} = \frac{\delta^2 W}{\delta A_q \delta A_r} \quad \varphi_r = \frac{\varphi_1 + \varphi_2}{2}, \quad A_r = \frac{A_1 + A_2}{2}$$

$$q.d.v. = \frac{\delta^2 W}{\delta A_r \delta A_q} \quad \varphi_q = \frac{\varphi_1 - \varphi_2}{2}, \quad A_q = \frac{A_1 - A_2}{2}$$

$$\text{SYM} = \frac{\delta^2 W}{\delta A_q \delta A_r}$$

$$\delta A_1 \Theta_1 - \delta A_2 \Theta_2$$

$$\delta A_r \Theta_q + \delta A_q \Theta_r$$

Schwinger-Keldysh EFT

based on 1701.07436
 w/ N. Pinzani-Fokeeva & A. Yarom
 (see also papers by P. Kovtun et al., H. Liu et al.)

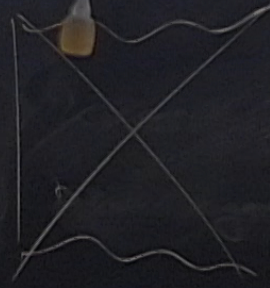
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Maldacena



"thermofield double's

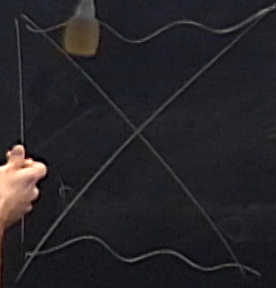
$$\rho_{-\infty} = e^{-\beta H}$$

Schwinger-Keldysh EFT

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Maldaena

"thermofield double"



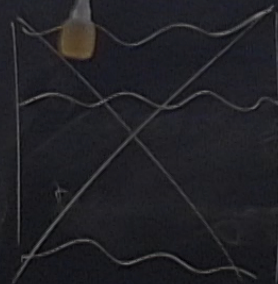
$$e^{-\infty} = e^{-BH}$$

$$\text{tr} \left(e^{-BH/2M} \left(\dots \right) e^{-BH/2M} \left(\dots \right) \right)$$

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"thermofield double's

$$e^{-\infty} = e^{-BH}$$

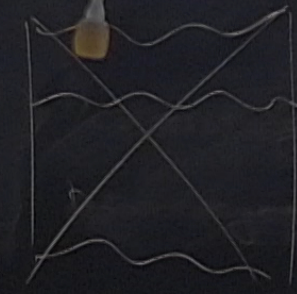
$$\text{tr} \left(e^{-BH/2M} \left(\dots \right) e^{-BH/2M} \left(\dots \right) \right)$$

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$\delta A_a \delta A_a$ $\delta H_a \delta H_a$

Maldacena



"thermofield double's

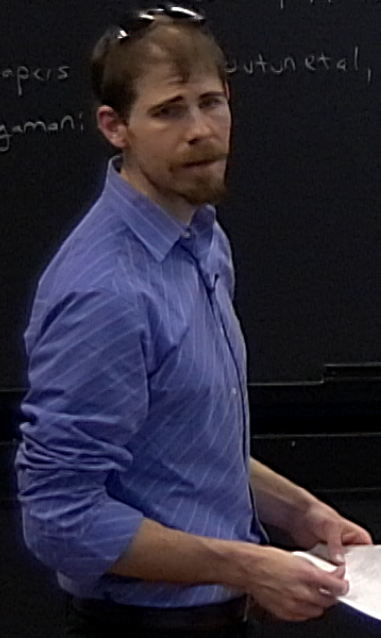
$$e_{-\infty} = e^{-\beta H}$$

$$\text{tr} \left(e^{-\beta H/2} \rho \left(\dots \right) e^{-\beta H/2} \rho \left(\dots \right) \right)$$

A: $e_{-\infty} = e^{-\beta H}$

$$\left. \begin{matrix} \text{---} Q_1 \text{---} \\ \text{---} \tilde{Q} \text{---} \end{matrix} \right\} = -\beta H$$

KMS

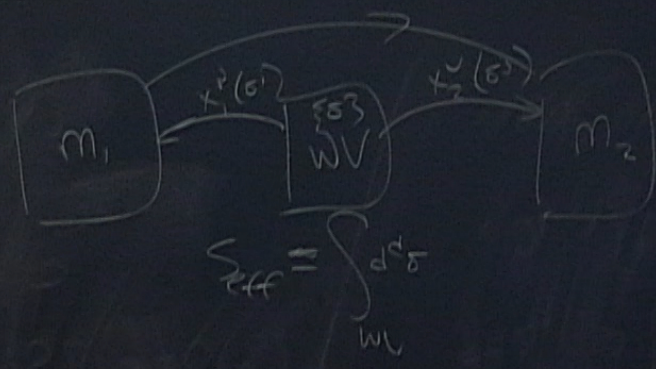
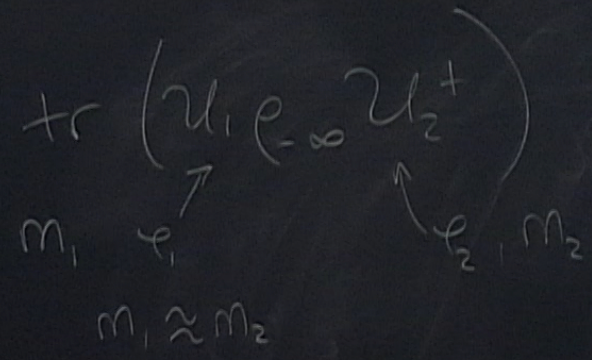


$$C(A_1, A_2) = \text{tr} \left(U_2 \begin{pmatrix} -\infty & 0 \\ 0 & a_1 \end{pmatrix} U_1 \right)$$

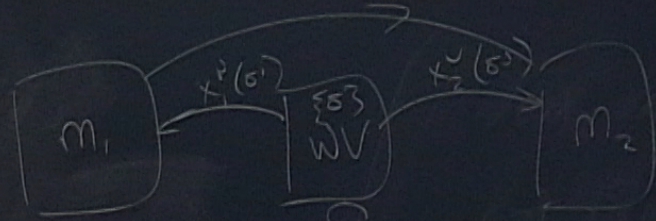
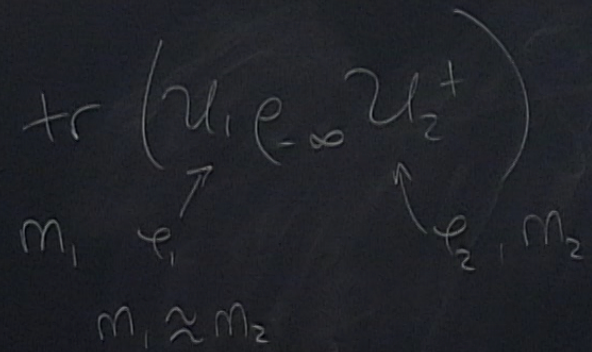
$$\text{tr} \left(U_1 \begin{pmatrix} -\infty & 0 \\ 0 & a_2 \end{pmatrix} U_2 \right)$$

$\nearrow \varphi_1$ $\nwarrow \varphi_2$

$$c(A_1, A_2) = T_1 (A_2 e^{-\infty} A_1)$$



$$c(A_1, A_2) = \frac{1}{2} (u_2(-\infty) u_1)$$



$$\mathcal{O}_r(\sigma) = \frac{\mathcal{O}_1(\sigma) + \mathcal{O}_2(\sigma)}{2} \approx \int_{WV} d^d \sigma$$

Schwinger-Keldysh symmetry

$$\mathcal{Z} \Rightarrow A_1 = A_2 = A$$

Schwinger-Keldysh symmetry

$$Z \xrightarrow{A_1=A_2=A} \text{tr} \left(e_{-\infty} \right) \longrightarrow \langle \mathcal{O}_a \mathcal{O}_a \rangle = 0$$

Schwinger-Keldysh symmetry

$$\mathcal{Z} \xrightarrow{A_1=A_2=A} \text{tr} \left(e_{-\infty} \right) \longrightarrow \langle \mathcal{O}_a \dots \mathcal{O}_a \rangle = 0$$

$(\psi_1, \psi_2, \psi_g, \psi_{\bar{g}})$

Schwinger-Keldysh symmetry

$$Z \xrightarrow{A_1=A_2=A} \text{tr}(\rho_{-\infty}) \longrightarrow \langle \mathcal{O}_a \dots \mathcal{O}_a \rangle = 0$$

$$(\psi_1, \psi_2, \psi_g, \psi_{\bar{g}}) \in \mathcal{Q} \quad \text{w/ } \mathcal{O}_a = \{ \mathcal{O}_1, \dots \}$$

Schwinger-Keldysh symmetry

$$Z \xrightarrow{A_1=A_2=A} \text{tr}(\rho_{-\infty}) \longrightarrow \langle \mathcal{O}_a \mathcal{O}_a \rangle = 0$$

$$(\psi_1, \psi_2, \psi_g, \psi_{\bar{g}}) \xrightarrow{\uparrow} \exists Q \quad \text{w/ } \mathcal{O}_a = \{Q, \dots\}$$

$$= (\psi_r, \psi_{\bar{g}}), (\psi_g, \psi_a)$$

Schwinger-Keldysh symmetry

$$Z \xrightarrow{A_1=A_2=A} \text{tr}(\rho_{-\infty}) \longleftrightarrow \langle \mathcal{O}_a \mathcal{O}_a \rangle = 0$$

$$(\psi_1, \psi_2, \psi_g, \psi_{\bar{g}}) \xrightarrow{\uparrow} \exists Q \quad \text{w/ } \mathcal{O}_a = \{Q, \dots\}$$

$$(\psi_r, \psi_{\bar{g}}), (\psi_g, \psi_a)$$

Schwinger-Keldysh EFT

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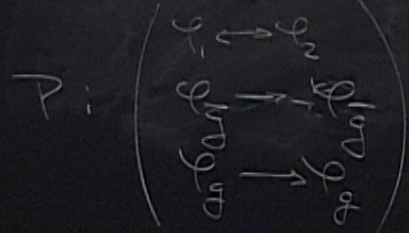
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(see also papers by P. Kovtun & H. Liu et al,
M. Rangamani et al)

$\delta A_1 \delta A_2$ $\delta H_1 \delta H_2$

$$Z^{\leftarrow}[A_1, A_2] = Z[A_2, A_1]$$

$$S_{\text{eff}}\{\{\varphi\}; A_1, A_2\}^{\leftarrow} = -S_{\text{eff}}\{\{\varphi\}; A_2, A_1\}$$



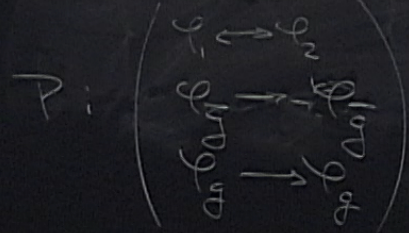
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 (see also papers by ... et al, H. Ge et al,
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reality condition:

$$Z^{\leftarrow}[A_1, A_2] = Z[A_2, A_1]$$

$$S_{\text{eff}}\left[\left\{\varphi\right\}; A_1, A_2\right]^{\leftarrow} = -S_{\text{eff}}\left[\left\{\varphi\right\}; A_2, A_1\right]$$



$$\begin{aligned}
 & \tau \rightarrow +r(\rho_{-\infty}) \quad \leftarrow \langle \partial_a, \partial_a \rangle = 0 \\
 & A_1 = A_2 = A \\
 & (\varphi_1, \varphi_2, \varphi_g, \varphi_a) \quad \exists Q \quad w/ \partial_a \varphi_i \\
 & (\varphi_r, \varphi_g), (\varphi_g, \varphi_a) \quad \theta^2 = 0 \quad \left. \begin{array}{l} \varphi_r = \varphi_r + \theta \varphi_g \\ \varphi_a = \varphi_g + \theta \varphi_a \end{array} \right\} Q \rightarrow \frac{\partial}{\partial \theta}
 \end{aligned}$$

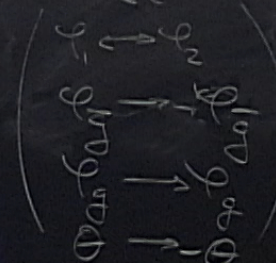
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reality condition:

$$\mathcal{Z}^{\leftarrow}[A_1, A_2] = \mathcal{Z}[A_2, A_1]$$

$$S_{\text{eff}}\left[\left\{\varphi\right\}; A_1, A_2\right]^{\leftarrow} = -S_{\text{eff}}\left[\left\{\varphi\right\}; A_2, A_1\right]$$



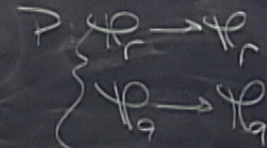
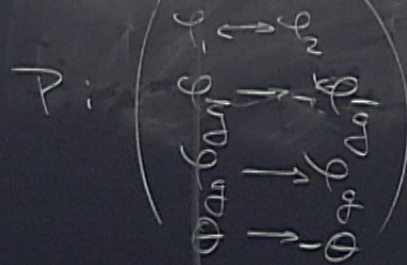
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reality condition:

$$Z^* [A_1, A_2] = Z [A_2, A_1]$$

$$S_{\text{eff}} \left[\{ \varphi \}, A_1, A_2 \right]^* = - S_{\text{eff}} \left[\{ \varphi \}, A_2, A_1 \right]$$



- QFT in mixed states
1. action principle?
 2. perturbation theory?
 3. Second Law?

$$A_0 = A_1 - A_2$$

nciple?
n thy?
?

$$\int d^d \sigma d\theta L(\varphi_r, \varphi_{qj}, \partial_{ij} \frac{\varphi}{\partial \theta})$$

Schwinger-Keldysh symmetry

$$Z \xrightarrow{A_1=A_2=A} \text{tr}(\rho_{-\infty})$$

$$\begin{aligned} & \uparrow \\ & (\varphi_1, \varphi_2, \varphi_3) \\ & \uparrow \\ & (\varphi_r, \varphi_g, \varphi_b, \varphi_a) \end{aligned}$$

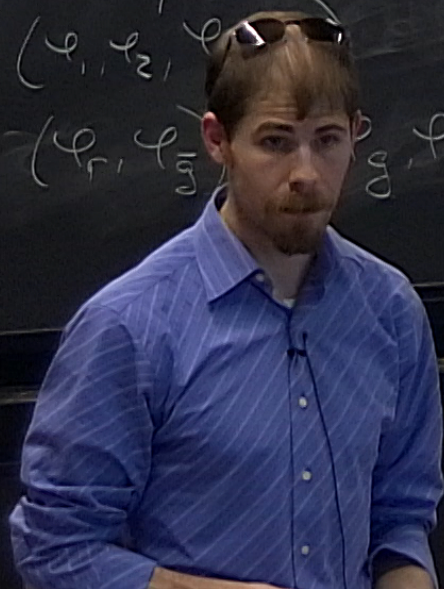
$$\langle \partial_a \dots \partial_a \rangle = 0$$

$\exists Q$

$$\omega / \partial_a \left\{ \begin{aligned} & \varphi_1 \\ & \varphi_2 \\ & \varphi_3 \end{aligned} \right\}$$

$$Q \rightarrow \frac{\partial}{\partial \theta}$$

$$\theta^2 = 0 \quad / \quad \varphi_r = \varphi_r + \theta \varphi_g, \quad \varphi_a = \varphi_a + \theta \varphi_b$$



$$e^{-\infty} = e^{-\beta H}$$

$$e^{-\beta H} = e^{-\beta H}$$

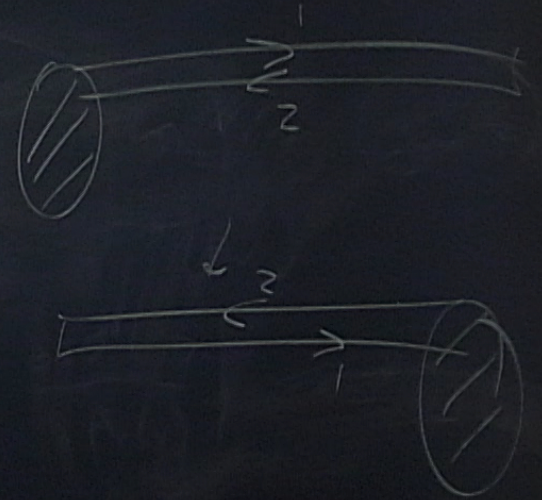
$$e^{-\beta H} \mathcal{U}_2^\dagger [A_2(t)] = \mathcal{U}_2^\dagger [A_2(t-i\beta)] e^{-\beta H}$$

$$+ (\mathcal{U}_1 e^{-\beta H} \mathcal{U}_2^\dagger) = + (\mathcal{U}_2^\dagger [A_2(t-i\beta)] e^{-\beta H} \mathcal{U}_1)$$

$$e^{-\infty} = e^{-\beta t}$$

$$e^{-\beta t} \mathcal{U}_2^+ [A_2(t)] = \mathcal{U}_2^+ [A_2(t - i\beta)] e^{-\beta t}$$

$$+ \mathcal{U}_1 (e^{-\beta t} \mathcal{U}_2^+) = + \mathcal{U}_2^+ [A_2(t - i\beta)] e^{-\beta t} \mathcal{U}_1$$

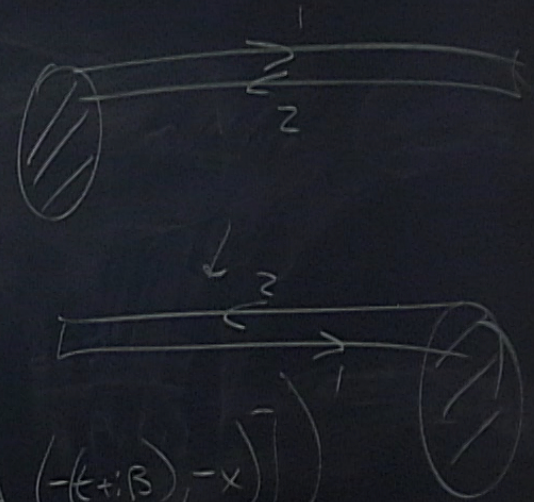


$$e^{-\infty} = e^{-\beta H}$$

$$e^{-\beta H} \mathcal{Z}_2^+ [A_2(t)] = \mathcal{Z}_2^+ [A_2(t-i\beta)] e^{-\beta H}$$

$$+ r (\mathcal{Z}_1 e^{-\beta H} \mathcal{Z}_2^+) = + r (\mathcal{Z}_2^+ [A_2(t-i\beta)] e^{-\beta H} \mathcal{Z}_1)$$

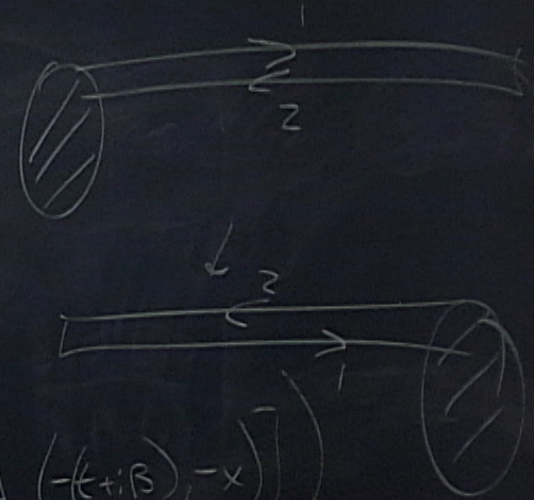
$$= + r (\mathcal{Z}_1 [\eta_A A_1(-t, -x)] e^{-\beta H} \mathcal{Z}_2^+ [A_2(-(t+i\beta), -x)])$$



$$e^{-\infty} = e^{-\beta H}$$

$$e^{-\beta H} \mathcal{U}_2^+ [A_2(t)] = \mathcal{U}_2^+ [A_2(t-i\beta)] e^{-\beta H}$$

$$\begin{aligned} r(\mathcal{U}_1 e^{-\beta H} \mathcal{U}_2^+) &= +r(\mathcal{U}_2^+ [A_2(t-i\beta)] e^{-\beta H} \mathcal{U}_1) \\ &= +r(\mathcal{U}_1 [\eta_A A_1(-t, -x)] e^{-\beta H} \mathcal{U}_2^+ [\eta_A A_2(-(t+i\beta), -x)]) \end{aligned}$$

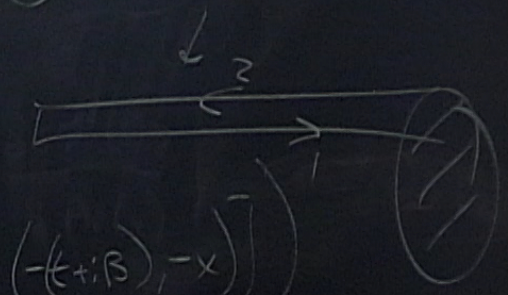
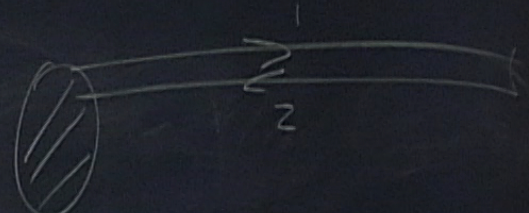


$$e^{-\infty} = e^{-\beta H}$$

$$e^{-\beta H} \mathcal{U}_2^+ [A_2(t)] = \mathcal{U}_2^+ [A_2(t-i\beta)] e^{-\beta H}$$

$$+ r (\mathcal{U}_1 e^{-\beta H} \mathcal{U}_2^+) = + r (\mathcal{U}_2^+ [A_2(t-i\beta)] e^{-\beta H} \mathcal{U}_1)$$

$$A_2(t) = A_1(t+i\beta) = + r (\mathcal{U}_1 [\eta_A A_1(-t, -x)] e^{-\beta H} \mathcal{U}_2^+ [A_2(-t+i\beta, -x)])$$



- QFT in mixed States
1. action principle?
 2. perturbation thy?
 3. Second Law?
- SK

$$\int d^d \sigma d\theta L(\Psi_r, \Psi_q, \dots; i \frac{\partial}{\partial \theta})$$

$$\tilde{A}_r = \frac{A_1(t) + A_2(t - i\beta)}{2} \rightarrow \tilde{\Theta}_q = \frac{\Theta_1(t) - \Theta_2(t - i\beta)}{2}$$

$$\tilde{A}_q = \frac{A_1(t) - A_2(t - i\beta)}{2} \rightarrow \tilde{\Theta}_r = \frac{\Theta_1(t) + \Theta_2(t - i\beta)}{2}$$

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SK

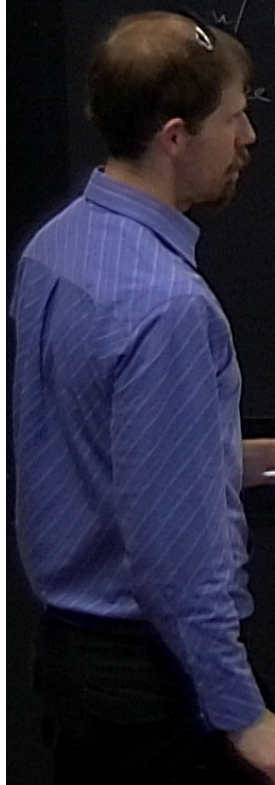
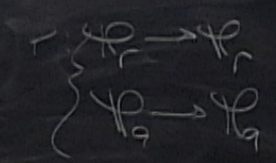
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(see also papers by P. Koutouetal, H. Li et al, Mirzamani et al)

$$\langle \hat{\mathcal{O}}_a \dots \hat{\mathcal{O}}_a \rangle = 0$$



→ SK

2

Schwinger-Keldysh EFT

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(see also papers by P. Kouturek et al, H. Li et al,
M. Rangamani et al)

$$\langle \tilde{\mathcal{O}}_a \dots \mathcal{O}_a \rangle = 0$$

$\tilde{\mathcal{O}}$

Sk

Schwinger-Keldysh EFT

based on 1701.07436
w/ M. Mari-Fokeeva & A. Yarom
(see also papers by P. Kovtun et al, H. Liu et al, M. Mari et al)

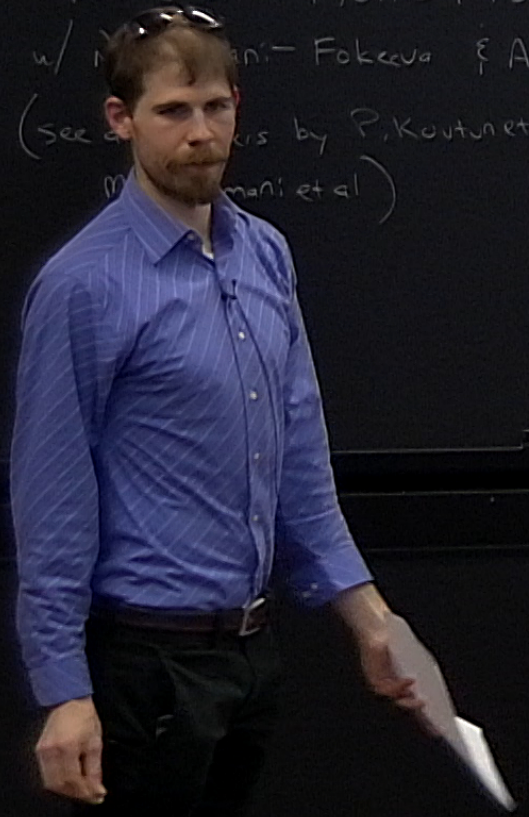
$$\langle \tilde{\Theta}_a \Theta_a \rangle = 0$$

$$\tilde{Q} \rightarrow \tilde{\Theta}$$

$$\Psi = \tilde{H} \rho_r + \tilde{\Theta} \rho_g + \Theta \rho_{\tilde{g}} + \tilde{\Theta} \Theta A \rho_g$$

$$\tilde{Q} \rightarrow \frac{\partial}{\partial \Theta}$$

$$Q \rightarrow \frac{\partial}{\partial \tilde{\Theta}} + i \Theta \beta \frac{\partial}{\partial t}$$



$$\frac{A}{R} = \coth\left(\frac{i\beta\omega}{2}\right) \frac{i\beta\omega}{2}$$

$\left\langle \partial_a \dots \partial_a \right\rangle = 0$

$Q \rightarrow \frac{\omega}{2}$

$\theta^2 = 0$ / $\phi_r = \phi_r + \theta \phi_g, \phi_a = \phi_g + \theta \phi_a$

$$\frac{A}{R} = \coth\left(\frac{i\beta\omega}{2}\right) \frac{i\beta\omega}{2}$$

$$A = 1 + O(\omega^2)$$

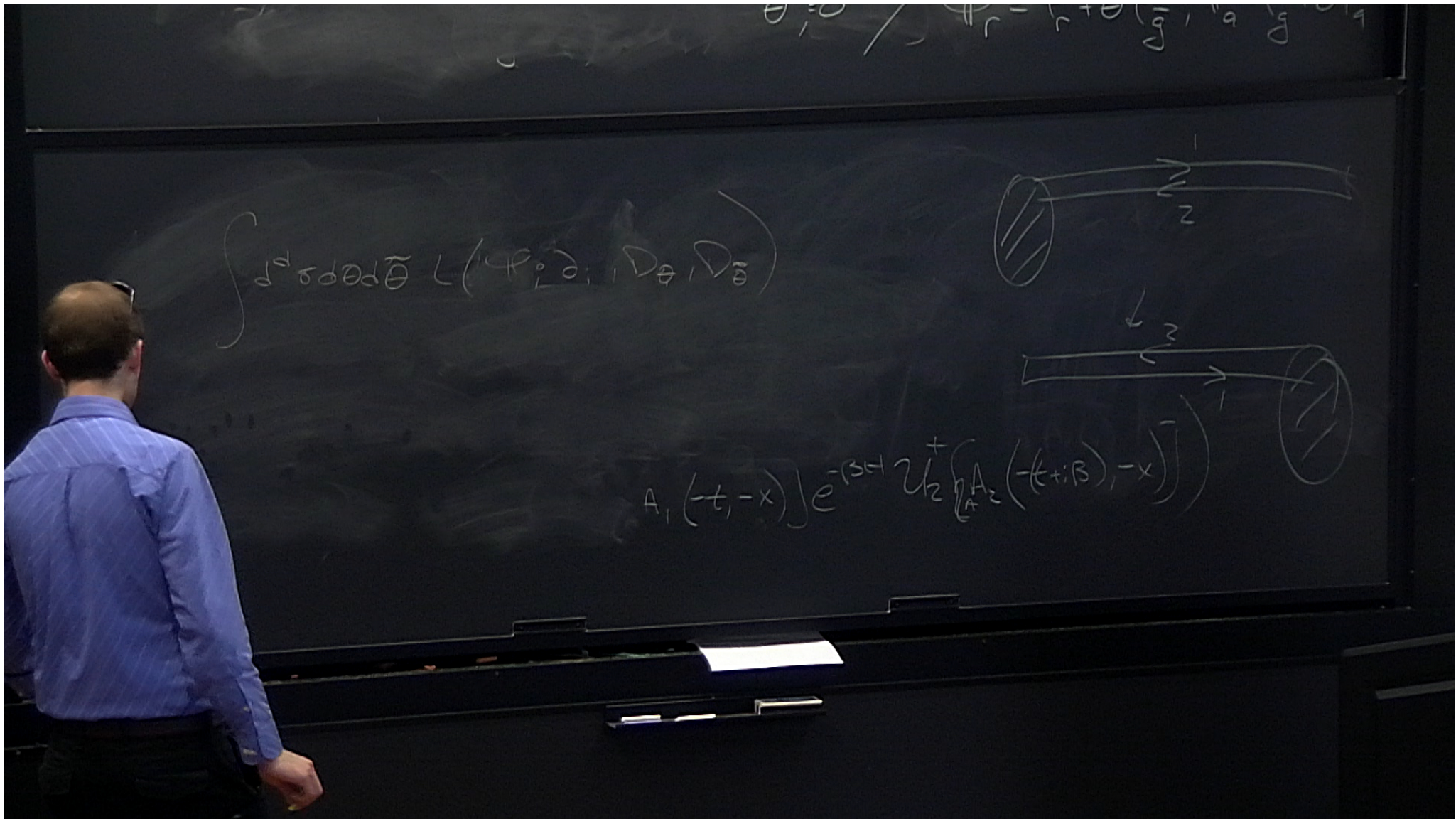
$$R = 1 + O(\omega^2)$$

$$\langle \partial_a \dots \partial_a \rangle = 0$$

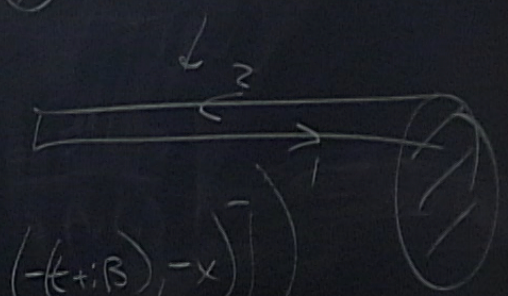
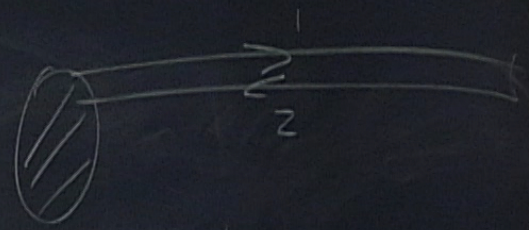
$$w/ \partial_a \{Q_i\}$$

$$\theta^2 = 0 \quad / \quad \varphi_r = \varphi_r + \theta \varphi_g, \quad \varphi_a = \varphi_g + \theta \varphi_f$$

$$Q \rightarrow \frac{\omega}{2}$$



$$\int d^d \sigma d\theta d\tilde{\theta} L(\varphi, \partial, D\varphi, D\tilde{\theta})$$

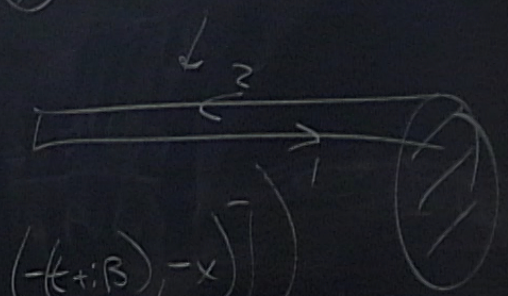
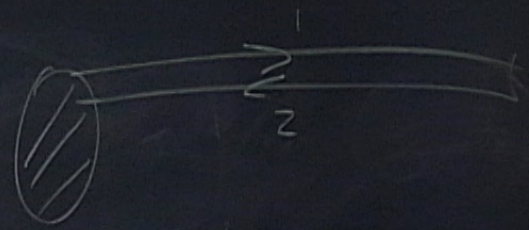


$$A_1(-t, -x) e^{-iSt} \mathcal{U}_2^+ \left(A_2 \left(-(t+\beta), -x \right) \right)$$

$$\int d^d \sigma d\theta d\tilde{\theta} L(\varphi, \partial, \mathcal{D}\varphi, \mathcal{D}\tilde{\theta})$$

$$\tilde{\varphi} = R\tilde{\varphi}_r + \tilde{\theta}\tilde{\varphi}_g + \theta\tilde{\varphi}_{\tilde{g}} + \tilde{\theta}\theta A\tilde{\varphi}_a$$

$$A_1(-t, -x) e^{-i\beta t} \mathcal{U}_2^+ \left(A_2 \left(\frac{-t+i\beta}{A}, -x \right) \right)$$



$$\int d^d \sigma d\theta d\tilde{\theta} \left(L(\varphi; \partial_t, \nabla_{\theta}, \nabla_{\tilde{\theta}}) + L(\tilde{\varphi}; \partial_t, \nabla_{\tilde{\theta}}) \right)$$

$$\tilde{\varphi} = R\tilde{\varphi}_r + \tilde{\theta}\tilde{\varphi}_g + \theta\tilde{\varphi}_{\tilde{g}} + \tilde{\theta}\theta A\tilde{\varphi}_a$$

$$K: \varphi \leftrightarrow \tilde{\varphi}$$

$$A_1(-t, -x) e^{-iS\varphi} \mathcal{U}_2^+ \left(A_2 \left(-(t+i\beta), -x \right) \right)$$

$$\int d^d x d\theta d\tilde{\theta} \left(L(\psi, \partial, D_\theta, D_{\tilde{\theta}}) + L(\tilde{\psi}; \partial, \dots) \right)$$

$$\tilde{\psi} = R\tilde{\psi}_r + \tilde{\theta}\tilde{\psi}_g + \theta\tilde{\psi}_{\bar{g}} + \tilde{\theta}\theta A\tilde{\psi}_a$$

$$\{Q, \tilde{Q}\} = -\beta H$$

↻
K

$$K: \psi \leftrightarrow \tilde{\psi}$$

$$K: \begin{matrix} Q & \rightarrow & \tilde{Q} \\ \tilde{Q} & \rightarrow & Q \end{matrix}$$

$$\pi: \begin{array}{ccc} \mathcal{O} & \rightarrow & \mathcal{O} \\ \mathcal{O} & \rightarrow & \mathcal{O} \end{array}$$

$+_1 (u_2^+ u_1, \rho_{-\infty} u_4^+ u_3)$

$\langle \partial_a \dots \partial_a \rangle = 0$

w/ $\partial_a \{ Q_i \}$

$Q \rightarrow \frac{\partial}{\partial \theta}$

$\theta^2 = 0$ / $\varphi_r = \varphi_r + \theta \varphi_g, \varphi_a = \varphi_g + \theta \varphi_g$

$$K: \begin{array}{ccc} \mathcal{Q} & \rightarrow & \mathcal{Q} \\ \mathcal{Q} & \rightarrow & \mathcal{Q} \end{array}$$

$+_1 (u_2^+ u_1, p_{-\infty} u_4^+ u_3)$
 $\langle \theta_1(t) \theta_2(0) \theta_1(t) \theta_2(0) \rangle$
 $\mathcal{Q} \rightarrow \frac{\partial}{\partial t}$
 $\langle \partial_a \dots \partial_a \rangle = 0$
 $\mathcal{Q} \rightarrow \frac{\partial}{\partial t}$
 $\theta^2 = 0$
 $\mathcal{P}_r = \mathcal{P}_r + \theta \mathcal{P}_g, \mathcal{P}_g = \mathcal{P}_g + \theta \mathcal{P}_g$

$$\pi: \begin{matrix} \mathcal{O} & \rightarrow & \mathcal{O} \\ \mathcal{O} & \rightarrow & \mathcal{O} \end{matrix}$$

$+_1 (u_2^+ u_1, p_{-\infty} u_4^+ u_3)$
 $p_{-\infty} \langle \theta_1(t) \theta_2(0) \theta_1(t) \theta_2(0) \rangle$

$\langle \theta_a \dots \theta_a \rangle = 0$
 $Q \rightarrow \frac{2}{30}$

$\theta^2 = 0$ / $\left. \begin{matrix} \text{w/ } \theta_a \neq 0 \\ \theta_1 \end{matrix} \right\}$
 $\varphi_r = \varphi_r + \theta \varphi_g, \varphi_a = \varphi_g + \theta \varphi_1$

$$\pi: \begin{matrix} \mathcal{O} & \rightarrow & \mathcal{O} \\ \mathcal{O} & \rightarrow & \mathcal{O} \end{matrix}$$

$+_1 (u_2^+ u_1, p_{-\infty} u_4^+ u_3)$
 $\langle \theta_1(t) \theta_2(0) \theta_1(t) \theta_2(0) \rangle$
 $\theta_1^2 = 0$

$\langle \partial_a \dots \partial_a \rangle = 0$
 $Q \rightarrow \frac{2}{30}$
 $\Phi_r = \phi_r + \theta \phi_g, \Phi_a = \phi_g + \theta \phi_r$