Title: Entropy measurement in quantum systems

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Abstract: <p>Entropy is an important information measure. A complete understanding of entropy flow will have applications in quantum thermodynamics and beyond; for example it may help to identify the sources of fidelity loss in quantum communications and methods to prevent or control them. Being nonlinear in density matrix, its evaluation for quantum systems requires simultaneous evolution of more-than-one density matrix. Recently in [1] a formalism for such an evolution has been proposed and [2] shows that the flow of entropy between two systems corresponds to the full counting statistics of physical quantities that are exchanged between them. Interestingly, in quantum systems with heat dissipations this will not be equivalent to the second law of thermodynamics. In this talk I will describe a consistent formalism to evaluate entropy and show how to measure it in some quantum systems; for example in quantum point contacts in nanoelectronics and in the quantum heat engines introduced to describe photosynthesis and photovoltaic cells. The entropy flow is made of two parts: 1) an incoherent part, which can be re-evaluated semiclassically from the second law, and 2) a coherent part, which has no semiclassical analogue and appear as a result of extending Kubo-Martin-Schwinger (KMS) correlations.</p>

<p> </p>

<p>[1] M.H.A. and Y. Nazarov, Phys. Rev. B 91, 104303 (2015)</p>

<p>[2] M.H.A. and Y. Nazarov, Phys. Rev. B 91, 174307 (2015)</p>

Entropy-noise correspondence:
The mathematics and physics of a new formalism

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Entanglement entropy is a fundamental characteristic describing quantum many-body correlations between 2 parts of the system

Entropy measurement proved **useful** in:

- Quantum critical phenomena
- Quantum quenches
- Topologically ordered states
- Strongly correlated systems
- Quantum heat engines, …
- Also a measure of available resources for quantum computation

Measuring entropy in systems made of a large number of particles requires full many-body density matrix, *difficult!*

*We made a general **connection** between **entropy** and **noise***

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Why bother?

Information content of these flows

- *Theoretical*: finally understand information in q.m.
- The scope of possible informational *applications* in quantum devices
- *Resources?*

Control information flows

Measure information flows using noise

- Entropy unphysical? Look at H_{AB} (or correlation)
- for charge transfers: Levitov & Klich idea
- for energy transfers: M.A. & Nazarov
- measuring entropy by noise is like measuring electric current with magnetic coil

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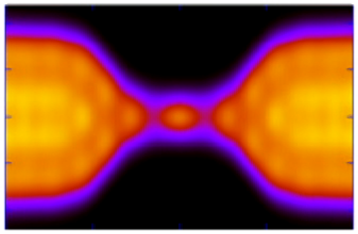
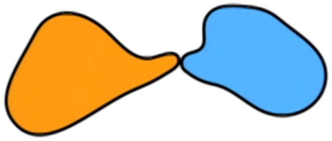
How to measure entropy?

- What is the **problem** with measuring entropy?
  - entropy is a nonlinear in density matrix therefore unphysical/non-observable.
  - measuring density matrix of probe environment is nontrivial procedure ➜ its characterization in an infinitely large system ➜ many repetition of measurements and precise re-initialization
- **An idea**: [Levitov, Klich] A surprising entropy-noise correspondence in charged systems at *low* temperature



- **Generalization**: [MA, Nazarov] Extending it to energy transfers at all temperature. ***(Early summary 1)***



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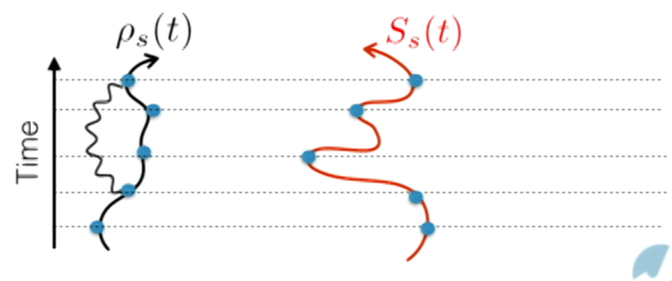
Entropy

- is a measure in quantum information theory,
- is nonlinear in density matrix, similar to other measures,
- its flow determines the information transfer from a system into another,
- **Early summary 2:** evaluating nonlinear measures (e.g. entropy) by taking the following steps gives rise to wrong results:
    - finding $\rho(t)$ from time-evolution equations,
    - tracing out all except the system of interest $\rho_s(t)$,
    - substituting $\rho_s(t)$ in entropy S=-$\rho_s(t)\ ln\rho_s(t)$ and its flow $dS(t)/dt$



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- Why does $\boxed{\text{nonlinearity}}$ lead to complications

  $\boxed{\text{allows}}$ for information exchange via **a new family of correlations** …

- How important are these correlators? … *as important as* standard ones

Questions:

1) *Given the nonlinearity in entropy*

How to consistently evaluate the flow of entropy?

2) *Knowing that*

…. physical quantities are linear in density matrix…

Is there any relation between the flow of information and physical quantities? (to make information measurable in the lab?)

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Question 1: **How to evaluate it?**

Question 2: **How to measure it?**

Question 1: **How to evaluate it?**

Needs a new formalism: e.g. ***multiple parallel worlds***

remains valid in all perturbation orders

Question 2: **How to measure it?**

Needs a new correspondence: ***entropy-noise correspondence***

Limited to the to the 2nd order

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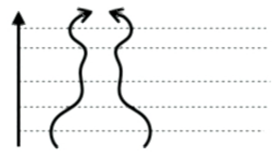
Question 1: **How to evaluate it?**

Simple example:

Consider a simple q.s. coupled to a thermal bath,
Interactions are so weak that density matrix is perturbed:
$\rho(t)=p_0+\rho^{(1)}(t)$ with $\rho^{(1)}/p_0 \ll 1$

$$dS/dt = -(1+\ln p_0)d\rho^{(1)}/dt - (1/2p_0)d(\rho^{(1)})^2/dt + \cdots$$

Simultaneous evolution of more-than-one world



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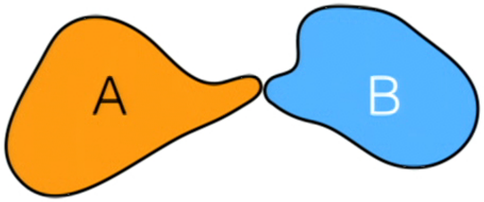
Question 1: **How to evaluate it?**

$$S_A = -\mathrm{Tr}_A\{\rho_A \ln \rho_A\} = -\lim_{M\to 1} \frac{d}{dM} \boxed{\underbrace{\mathrm{Tr}_A\{(\rho_A)^M\}}_{S_M}}$$

Renyi entropy of degree M

$$\rho_A = \mathrm{Tr}_B(\rho)$$



- Generalization of information entropy (Alfred Renyi 1964)

$$S_M = \sum_n p_n^M; \quad S_1 = 1; \quad S = \lim_{M\to 1} \frac{\partial S_M}{\partial M}$$

- Renyi entropies has applications on
  - multi-fractality: spectrum of Renyi dimensions
  - Tsallis: q-statistics, alternatives to Gibbs distributions
  - generalisation for reduced density matrix
  - determines Shannon entropy

In quantum: $S_M = \mathrm{Tr}\hat{\rho}^{M}$ **M : "parallel worlds"**

- **Renyi** entropy flow

$$F_M = \frac{d \ln S_M}{dt} = \frac{1}{S_M} \frac{d\mathrm{Tr}_A\{(\rho_A)^M\}}{dt}$$

$$\frac{d(\rho_A)^M}{dt} = ?$$

Evaluation needs a new formalism!

- Why Renyi and not something else

We can define quantities such as $\sum_{m,n,l} \rho_{mn}\rho_{nl}\rho_{lm}$

  - Renyi entropy in the limit of M→1 seems to be working well to obtain tunneling entropy.

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- Let us see how $d(\rho_s)^M/dt$ can be evaluated (let's drop $-_s$ for now)

$$\rho(t) = \rho_0 + \rho^{(1)}(t) + O(2), \qquad \rho^{(1)} = -i \int_0^t dt' [H(t'), \rho(t)]$$

$$d\rho(t)/dt = \delta^{(1)}(t) + \delta^{(2)}(t) + O(3) \qquad \delta^{(1)}(t) = -i[H(t), \rho(t)]$$

$$d(\rho)^M/dt = (d\rho/dt)(\rho)^{M-1} + \rho(d\rho/dt)(\rho)^{M-2} + \cdots + (\rho)^{M-1}(d\rho/dt).$$

$$\frac{d\rho^M}{dt} = \left\{\delta^{(2)}\rho_0^{M-1} + \rho_0\delta^{(2)}\rho_0^{M-2} + \cdots + \rho_0^{M-1}\delta^{(2)}\right\} +$$

$$\Big\{\delta^{(1)}\left[\rho^{(1)}\rho_0^{M-2} + \rho_0\rho^{(1)}\rho_0^{M-3} + \rho_0^2\rho^{(1)}\rho_0^{M-4} + \cdots\right]$$

$$+\rho_0\delta^{(1)}\left[\rho^{(1)}\rho_0^{M-3}\rho_0\rho^{(1)}\rho_0^{M-4} + \cdots\right]$$

$$+\rho_0^2\delta^{(1)}\left[\rho^{(1)}\rho_0^{M-4}\rho_0\rho^{(1)}\rho_0^{M-5} + \cdots\right]$$

$$+ \cdots + \rho_0^{M-2}\delta^{(1)}\rho^{(1)}\Big\}$$

⬅ **new correlators**

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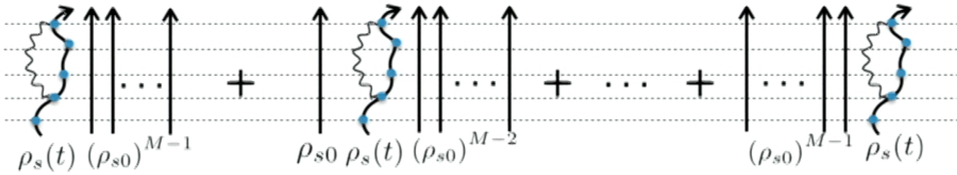
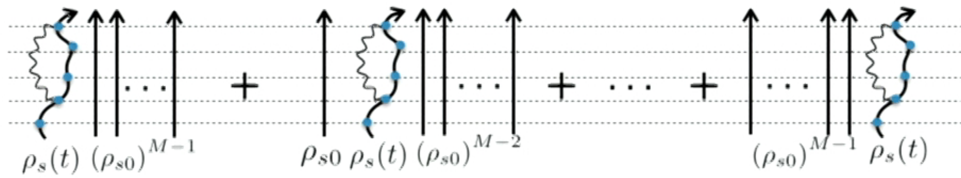
$$\frac{d\rho_s}{dt}(\rho_{s0})^{M-1} + \rho_{s0}\frac{d\rho_s}{dt}(\rho_{s0})^{M-2} + \cdots + (\rho_{s0})^{M-1}\frac{d\rho_s}{dt}$$

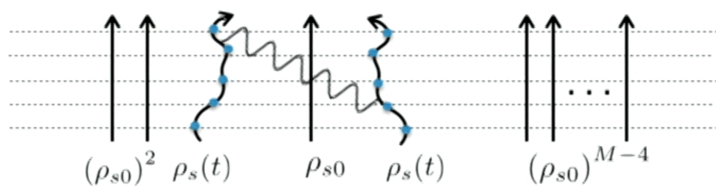


$$\frac{d\rho_s}{dt}(\rho_{s0})^{M-1} + \rho_{s0}\frac{d\rho_s}{dt}(\rho_{s0})^{M-2} + \cdots + (\rho_{s0})^{M-1}\frac{d\rho_s}{dt}$$



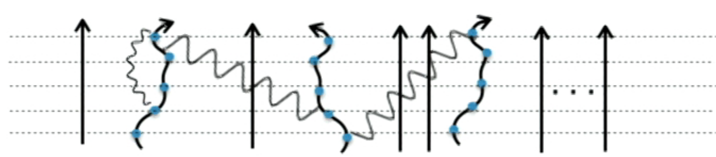
A typical term from the yellow box:

$$(\rho_{s0})^2 \frac{d\rho_s^{(1)}(t)}{dt} \rho_{s0}\rho_s^{(1)}(t)\,(\rho_{s0})^{M-4}$$



⬅ **new correlators**

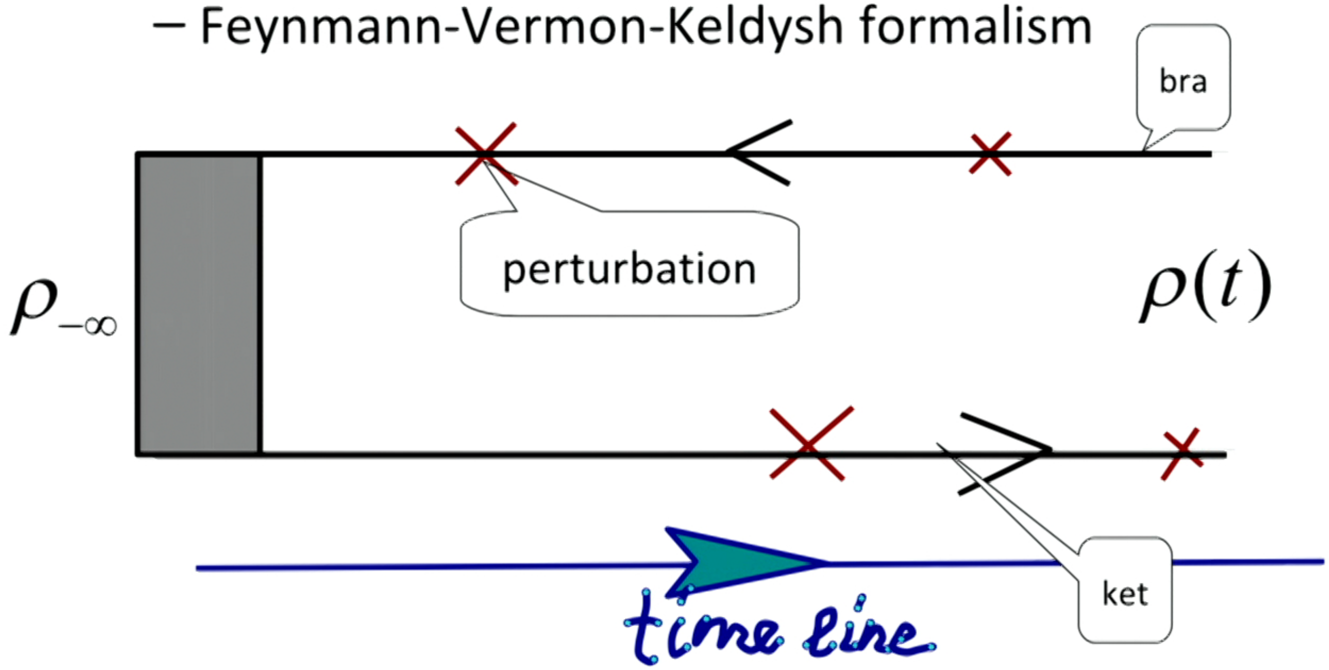
in higher than second order:



⬅ **new correlators**

Keldysh Formalism

- Evolution of density matrix in physical world:
  - Feynmann-Vermon-Keldysh formalism



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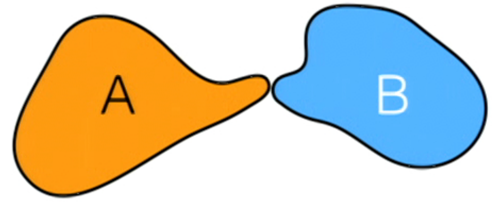
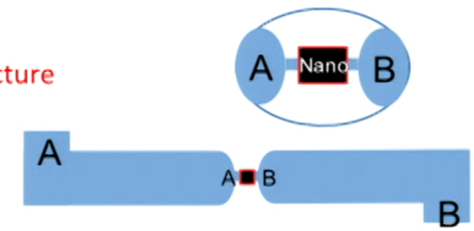
Interacting systems: double contours



- Once all physical quantities are calculated…
- (Quantum) information quantities
- Natural bipartition: AxB (leads)

nanostructure



$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{H}_{AB}$$

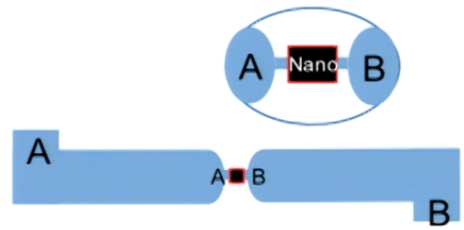
- Invariants U_A x U_B are <u>conserved quantities</u>

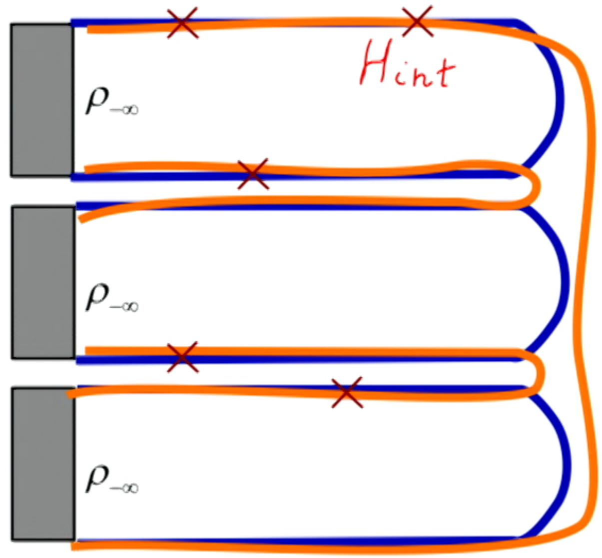
For instance, $\mathrm{Tr}\{\hat{\rho}_A^2\}, \mathrm{Tr}\{\hat{\rho}_B^2\}$

- Quantities infinite: the flows are finite
  - Originate from H_{AB}

AxB

Extended Keldysh formalism (parallel worlds)





AxB

$$Tr_A\left(Tr_B\rho(t)\right)^3$$
$$= S_A^{(3)}$$

Connecting differently:

More (interesting) invariants

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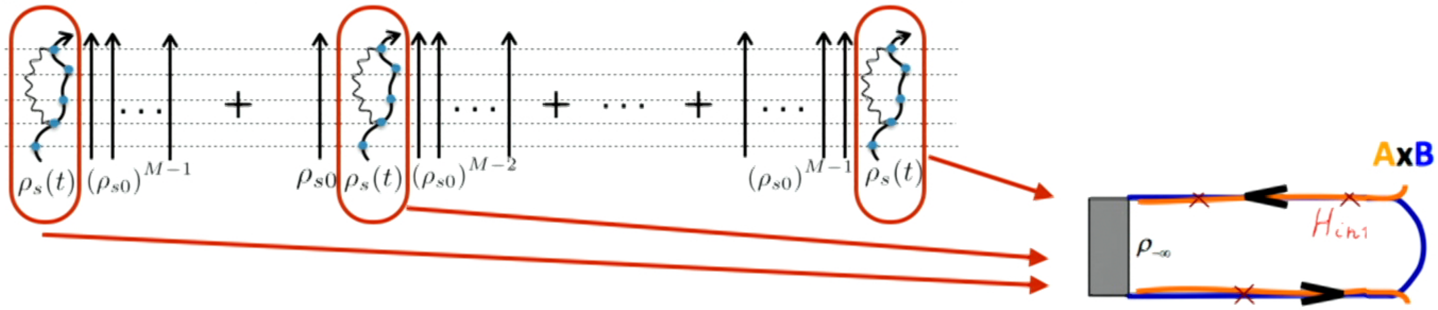
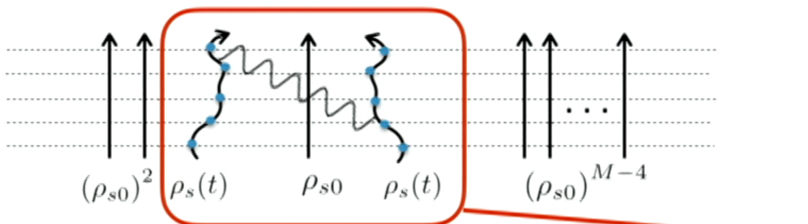
$$\frac{d\rho_s}{dt}(\rho_{s0})^{M-1} + \rho_{s0}\frac{d\rho_s}{dt}(\rho_{s0})^{M-2} + \cdots + (\rho_{s0})^{M-1}\frac{d\rho_s}{dt}$$

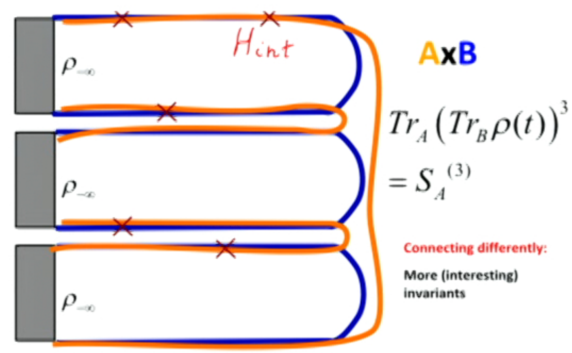


$$(\rho_{s0})^2 \frac{d\rho_s^{(1)}(t)}{dt} \rho_{s0}\rho_s^{(1)}(t)\,(\rho_{s0})^{M-4}$$

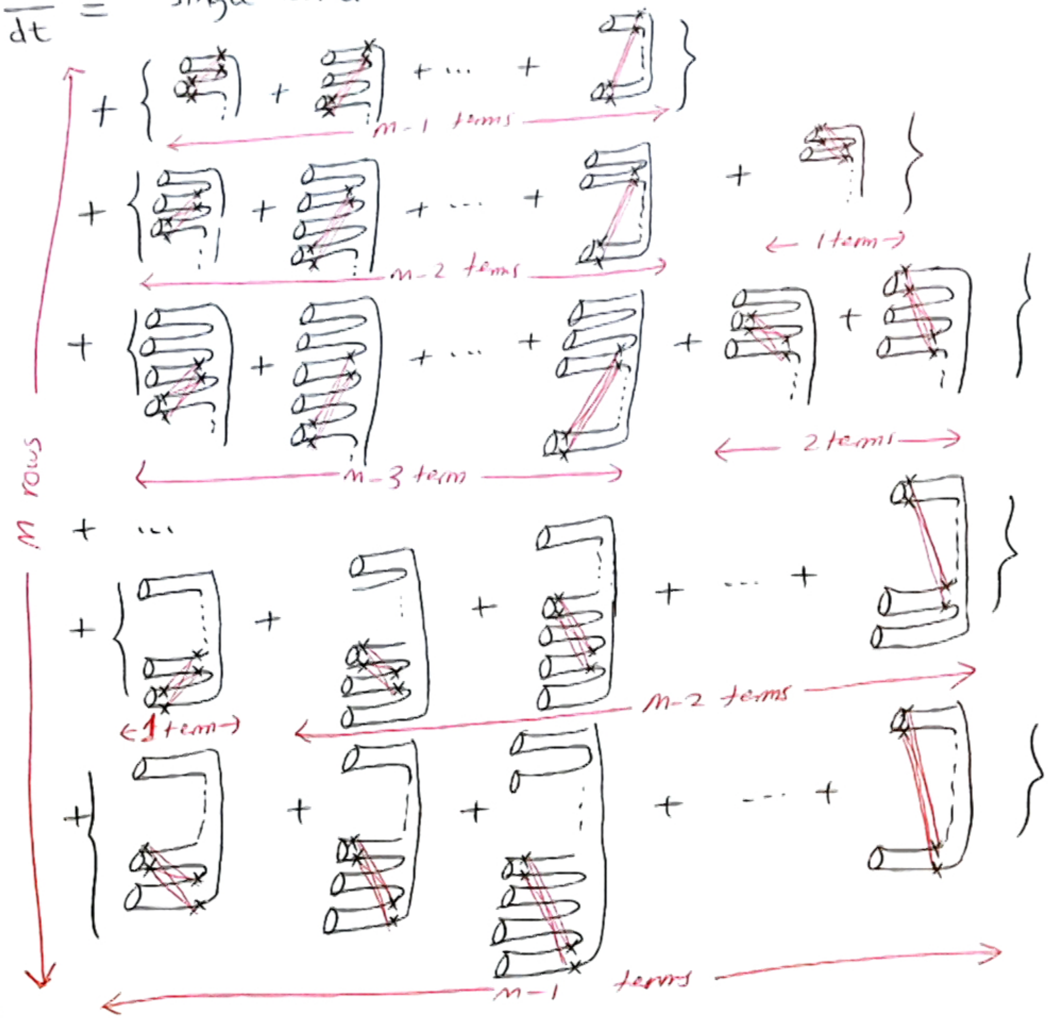


new correlators

Extended Keldysh technique on multiple parallel worlds



$$\frac{d\rho^M}{dt} = \text{single world terms}$$



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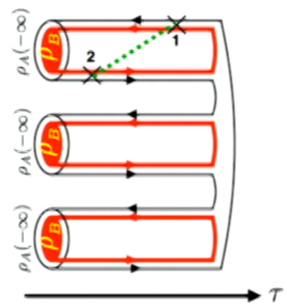
interaction Hamiltonian $\hat{H}_{int} = \hat{A}\hat{B}$

- **Standard Kubo-Martin-Schwinger (KMS) relation:**

for reservoir at equilibrium

Defining $S(\tau) = Tr_B \left\{ \hat{B}(t-\tau)\hat{B}(t)\rho_B \right\}$

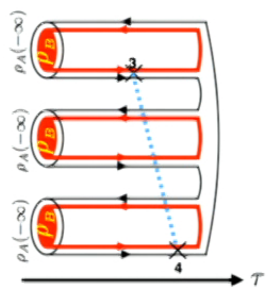
$S(\tau) = \bar{n}(\omega/T)\tilde{\chi}(\omega)$



- **Generalized KMS relation:**

Defining $S^{N,M}(\tau) = \dfrac{Tr_B \left\{ \hat{B}(t-\tau)\rho_B^N \hat{B}(t)(\rho_B)^{M-N} \right\}}{Tr_B \left\{ (\rho_B)^M \right\}}$

$S^{N,M}(\tau) = \bar{n}(M\omega/T)e^{n\omega/k_B T}\tilde{\chi}(\omega)$



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If you like to rename this **parallel world** a **quantum replica trick,** feel free to do so, however notice

the differences:

1. In replica trick the "replicas" do not define new measure, while parallel worlds define a standard information measure called "Renyi entropies"
2. In most applications, replicas bear no dynamics
3. **"**Replicas" do not give a sense of simultaneous evolution
4. The usual use of replicas involve an averaging over a static random potential. Parallel world formalism do not have it.
5. Other detailed reasons…

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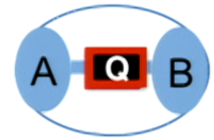
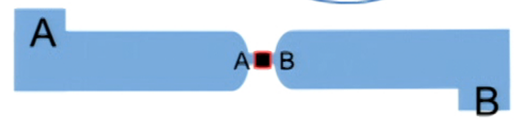
Example: Interacting reservoirs via a quit





$$H = H_0 + H_{int} + H_{dr}$$

$$H_0 = H_A + H_B + H_Q$$

$$H_A = \sum_q \hbar\omega_q \hat{b}_q^{(A)\dagger} \hat{b}_q^{(A)}$$

$$H_Q = \sum_n E_n |n\rangle\langle n|$$

$$H_{dr} = \textstyle\sum_{m,n} \Omega_{mn} |m\rangle\langle n| e^{-i\omega t} + \text{H.c.},$$

$$\omega \approx E_1 - E_0$$

$$H_{int} = \sum_{\alpha = A,B} \sum_{m,n} |m\rangle\langle n| \hat{X}_{mn}^{(\alpha)}$$

We assume linear response of each environment on the state of quantum system. ➜ environments characterised by a set of frq-dep. susceptibilities $\chi_{mn,pq}^{(a)}(\nu)$ ➜ related to standard KMS correlators

$$S_{mn,pq}^{(a)}(t) \equiv \mathrm{Tr}_a\{\hat{X}_{mn}^a(0)\hat{X}_{pq}^a(t)\rho_a\}$$

in fluctuation-dissipation theorem:

$$S_{mn,pq}^{(a)}(\nu) = n_B(\nu/T)\tilde{\chi}_{mn,pq}(\nu)$$

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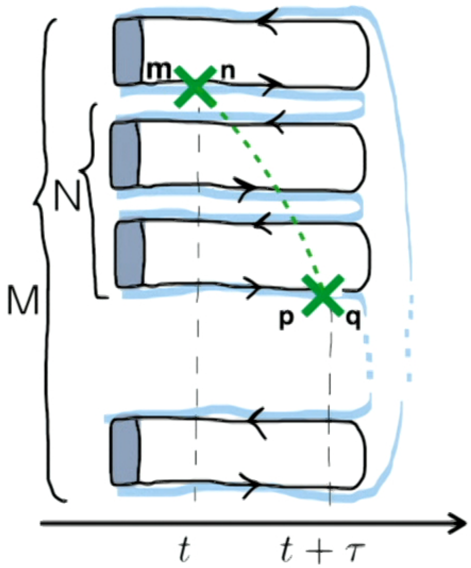
An example

$$S^{N,M}_{mn,pq}(\tau) \equiv Tr_b\{\hat{X}_{mn}(t)\rho_b^N \hat{X}_{pq}(t+\tau)\rho_b^{M-N}\}/Tr_b\{\rho_b^M\}$$



Multiple parallel world diagrams (MA & Nazarov)

$$\int_0^\infty d\tau\, e^{i\omega\tau}\, Tr_B\left(\hat{X}_{mn}(0)\,\rho_B^N\, \hat{X}_{pq}(\pm\tau)\,\rho_B^{M-N}\right)/Tr_B\,\rho_B^N$$

$$= \frac{1}{2}\mathbf{S}^{N,M}_{mn,pq}(\pm\omega) \pm i\,\boldsymbol{\Pi}^{N,M}_{mn,pq}(\pm\omega)$$

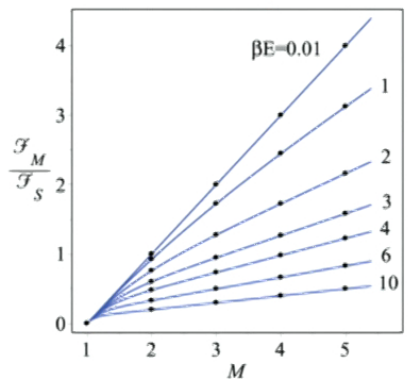
$$\mathbf{S}^{N,M}_{mn,pq}(\omega) = \int d\tau\, e^{i\omega\tau}\, Tr_B\Big(\hat{X}_{mn}(0)\,\underbrace{\rho_B^N}_{\frac{e^{-\beta E_n N}}{Z^N}}\, \hat{X}_{pq}(+\tau)\,\underbrace{\rho_B^{M-N}}_{\frac{e^{-\beta E_m(M-N)}}{Z^{M-N}}}\Big)/Tr_B\,\rho_B^M$$

$$\boldsymbol{\Pi}^{N,M}(\omega) = -\frac{1}{2\pi}\frac{\int dz\,\mathbf{S}^{N,M}(z)}{z-\omega}.$$

Result:

The flow of Renyi entropy

$$F_M = \boxed{\frac{M n_B(M\omega/T)}{n_B((M-1)\omega/T) n_B(\omega/T)\omega}}(Q_i - Q_c)$$



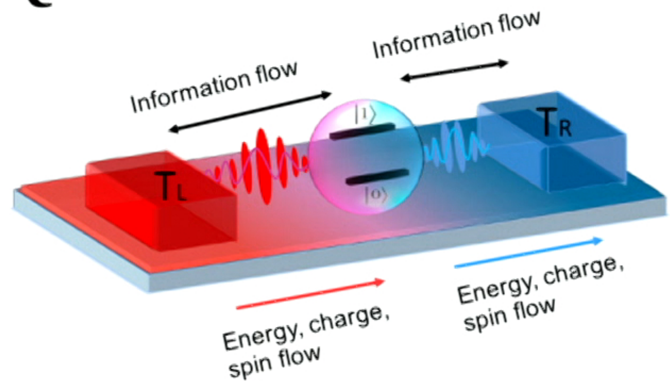
$$Q_i = \omega \left\{ \sum_{mnp;\eta_{np}=1} \rho_{mn} \tilde{\chi}_{pm,np}(\omega)(1 + n_B(\omega/T)) - \sum_{mnp;\eta_{pm}=1} \rho_{mn} \tilde{\chi}_{np,pm}(\omega) n_B(\omega/T) \right\},$$

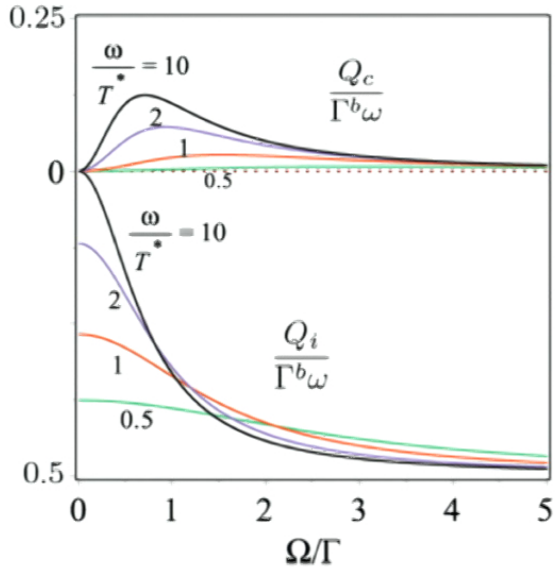
$$Q_c = \omega \sum_{mnpq;\eta_{pq}=1} \rho_{nm}\rho_{qp} \tilde{\chi}_{mn,pq}(\omega).$$

Flow of entropy: $F_S = (Q_i - Q_c)/T$

example: two level QHE

$$F_S = (Q_i - Q_c)/T$$





coherent flow

$$Q_c/\omega = \Gamma^b |\rho_{01}|^2$$

incoherent flow

$$Q_i/\omega = \Gamma^b_{\downarrow} p_1 - \Gamma^b_{\uparrow} p_0$$

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Question 2) How to measure entropy?

Is there any relation between information and physical quantities?

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- Exact correspondences between seemingly different concepts play an important role in all fields of physics, AdS/CFT, etc.
- *Example*: fluctuation-dissipation theorem

linear response of a system to externally applied forces ⟷ **correspondences** ⟷ system fluctuations

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- **Entropy-noise correspondence in charged systems**: Levitov & Klich

Entanglement entropy in charged systems (QPC) ⟷ **correspondences** ⟷ Full Counting statistics of charge transfer

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- **Entropy-noise correspondence in *all systems***: MA & Nazarov

Entanglement entropy ⟷ **correspondences** ⟷ Full Counting Statistics of energy transfers

More precisely

Renyi entropies ⟷ **correspondences** ⟷ FCS of energy transfers (rescaled temperature)

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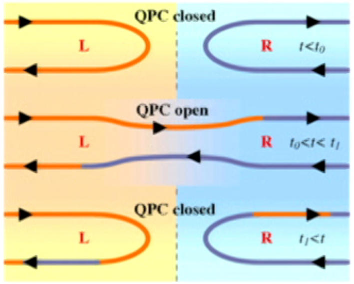
Entropy in Quantum Point Contact (QPC) - Levitov & Klich

- QPC is the simplest nanoelectronics device: a constriction connecting large electron reservoirs.
- A door between two reservoirs
- Surprisingly! they found that measurement of electric current fluctuations flowing through the QPC is sufficient for determining entropy.
- Consider QPC is brought to steady state
  ➜ the probability to transmit n charges is P_n



$$\chi(\lambda) = \sum_{n=-\infty}^{\infty} P_n e^{i\lambda n}, \qquad \log\chi(\lambda) = \sum_{m=1}^{\infty} \frac{(i\lambda)^m C_m}{m!},$$

The surprising relation between information and physics in charged systems:

$$\boxed{\mathcal{S} = \sum_{m>0} \frac{\alpha_m}{m!} C_m,} \qquad \alpha_m = \begin{cases} (2\pi)^m |B_m|, & m \text{ even} \\ 0, & m \text{ odd} \end{cases},$$

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- The validity of Levitov-Klich relation is restricted to
  - zero temperature
  - Interactions occurring by means of charge transfer
- A generalization requires to understand the possibility of noise-entropy relation
  - at all temperatures
  - in system where interactions takes places by means of energy transfer

- Consider quantum system is brought to steady state ➜ exchange of physical quantities (energy, charge, spin etc) is measured over time T ➜ the probability for energy transfer E over the time is $P(E,T)$
- FCS generating function is

$$\chi(\lambda) = \int dE\ P(E,T)\ e^{i\lambda E} \approx e^{-Tf(\lambda)}$$

- Cumulants

$$C_k = \lim_{\lambda \to 0} \frac{1}{i^k} \frac{\partial^k f}{\partial \lambda^k} \qquad \langle E \rangle = \lim_{\lambda \to 0} \frac{1}{i} \frac{\partial f}{\partial \lambda}$$

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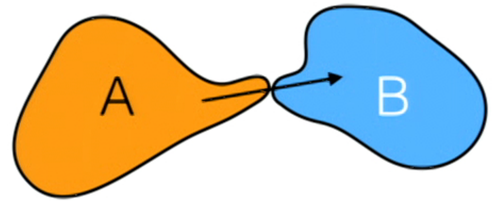
Full Counting Statistics of Energy Transfer

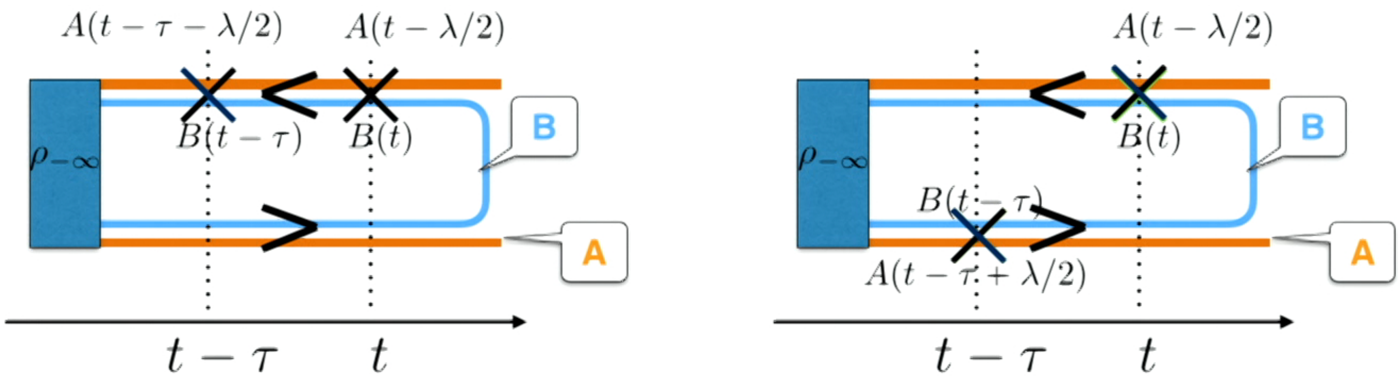


- There is a Keldysh technique to calculate $f(\lambda)$ by considering the following psudo-density matrix R' via the Hamiltonian $H^{+,-}$:

$$\hat{\boldsymbol{R}}'(t) = \mathrm{T}e^{i\int_{-\infty}^{t} d\tau \hat{H}^{+}(\tau)} \hat{\boldsymbol{R}}'(-\lambda) \tilde{\mathrm{T}}e^{-i\int_{-\infty}^{t} d\tau \hat{H}^{-}(\tau)}$$

$$H_{int} = \hat{A}\hat{B} \qquad H^{\pm}(t) = A(t \hat{\pm} \lambda/2)\hat{B}(t)$$

- Statistics of energy transfer in system A is determined from $Tr(\mathrm{R}')$



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$Tr[R'(t)] =$ 

$$f(\lambda) = -Tr[dR'(t)/dt]/Tr[R'(t)]$$

- FCS of energy transfer by means of interaction hamiltonian between A and B

$$\bar{f}^{(T)}(\lambda) = -\sum \int \frac{d\omega}{2\pi}\left(e^{-i\omega\lambda} - 1\right) S^{(\beta)}_{xy,zt}(\omega)\, \mathcal{B}_{xy,zt}(\omega)$$

$$\mathcal{B}_{xy,zt}(\omega) \equiv \frac{1}{\mathcal{T}}\int_0^{\mathcal{T}} dt \int_{-\infty}^{t} dt' \Big\{ \left\langle \hat{B}_{xy}(t')\, \hat{B}_{zt}(t) \right\rangle e^{-i\omega(t-t')} + \left\langle \hat{B}_{xy}(t)\, \hat{B}_{zt}(t') \right\rangle e^{i\omega(t-t')} \Big\},$$

we call this part incoherent energy transfer

$$f^{(T)}_{incoh}(\lambda)$$

- Comparing this result with Renyi entropy flow of degree M shows similarities between the two specially if we take $\lambda = i\beta(M-1)$

- However something is missing in the equality

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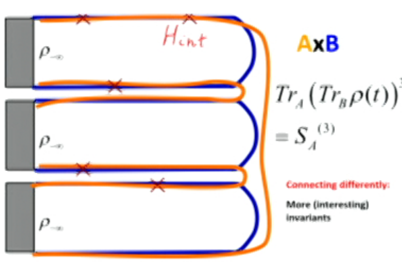
- What is missing is another FCS of energy transfer via the interaction Hamiltonian in which we replace $\hat{B} \rightarrow \langle \hat{B} \rangle$

we call this part coherent energy transfer $f_{coh}^{(T)}(\lambda)$

- **exact** correspondence between FCS of energy transfer and Renyi entropy

$$\boxed{F_M^{(T)}/M = f_{incoh}^{(T/M)}(\lambda) - f_{coh}^{(T/M)}(\lambda)} \qquad \lambda = i\beta(M-1)$$



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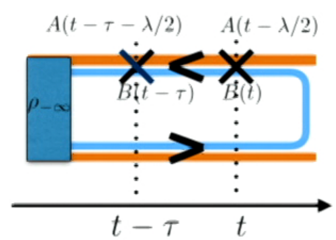

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Entropy-Noise correspondence (R/FCS corr.)

Flow of Renyi entropies $\longleftrightarrow$ (correspondences) Difference of two FCS of energy transfers (rescaled temperature)

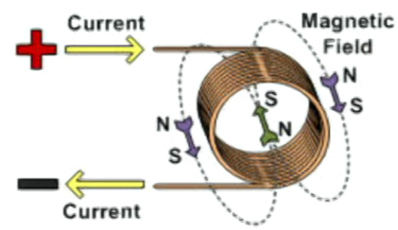
$$\boxed{F_M^{(T)}/M = f_{incoh}^{(T/M)}(\lambda) - f_{coh}^{(T/M)}(\lambda)} \qquad \lambda = i\beta(M-1)$$

Special case:

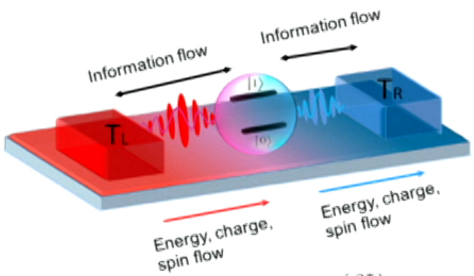
Flow of Shannon entropy $\longleftrightarrow$ (correspondences) Difference of two FCS of energy transfers

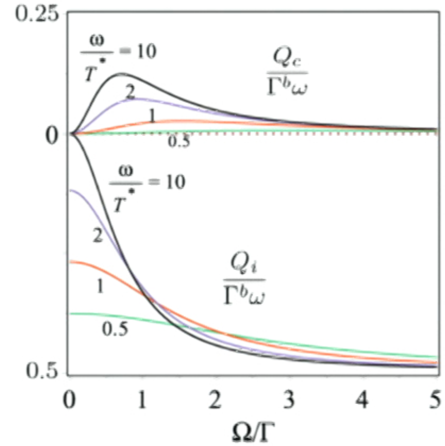
$$\boxed{F_S = f_{incoh}^{(T)}(\lambda) - f_{coh}^{(T)}(\lambda)}$$

like measuring electric current with magnetic coil



Ex 1: 2-level Quantum Heat Engine





$$\bar{f}_i^{(\beta^*)}(\xi^*) = \left(e^{-i\xi^*\Omega} - 1\right) \frac{\bar{n}(M\Omega/T)}{\bar{n}(\Omega/T)} \left[\Gamma_\downarrow p_1 - \Gamma_\uparrow p_0\right]$$

$$\bar{f}_c^{(\beta^*)}(\xi^*) = \left(e^{-i\xi^*\Omega} - 1\right) \frac{\bar{n}(M\Omega/T)}{\bar{n}(\Omega/T)} (\Gamma_\downarrow - \Gamma_\uparrow)\rho_{01}\rho_{10}$$

Using R/FCS correspondence ➜

$$\frac{dS}{dt} = \frac{Q_i - Q_c}{T}$$

incoherent flow $Q_i/\omega = \Gamma^b_\downarrow p_1 - \Gamma^b_\uparrow p_0;\quad Q_c/\omega = \Gamma^b |\rho_{01}|^2$ **coherent flow**

Ex 2: Harmonic oscillator Quantum Heat Engine

$$\bar{\mathcal{F}}_M^{(\beta)} = M(e^{\beta(M-1)\omega_0} - 1)\, S^{(M\beta)}(\omega_0)$$
$$\times \{\langle\langle a^\dagger a\rangle\rangle e^{\beta\omega_0} - \langle\langle a a^\dagger\rangle\rangle\}.$$

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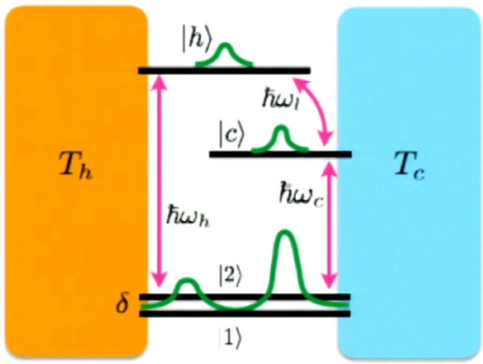
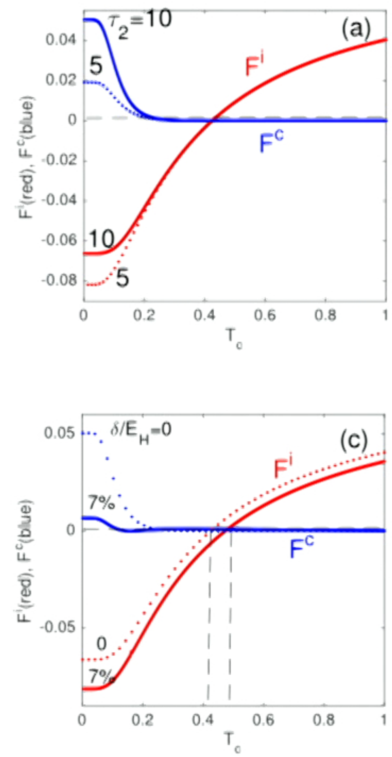
Ex 3: 4-level Quantum Heat Engine





This model and its modified versions are used to describe light-harvesting biocells, photovoltaic cells, etc. as well as several phenomena: Lasing without inversion, elevated output power via quantum coherence, and work extraction from single heat bath, etc..

$$
\begin{aligned}
\frac{dS}{dt} = & \left\{\gamma p_h - E_{h2}\tilde{\chi}_{h2}\bar{n}\left(\frac{E_{h2}}{T_h}\right)p_2 - E_{h1}\tilde{\chi}_{h1}\bar{n}\left(\frac{E_{h1}}{T_h}\right)p_1\right. \\
& -\tilde{\chi}_{1h,h2}\left[E_{h1}\bar{n}\left(\frac{E_{h1}}{T_h}\right) + E_{h2}\bar{n}\left(\frac{E_{h2}}{T_h}\right)\right]\mathrm{Re}\rho_{12} \\
& \left. -\frac{1}{2}\sum_{i=1,2} E_{hi}\tilde{\chi}_{1h,h2}|\rho_{12}|^2\right\}/T_h
\end{aligned}
$$

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Summary

- A large body of information flow is **missing** in the literature.
- We introduced **a formalism** to consistently evaluate them
  - valid in weak coupling regime
  - valid for any time-depending (stationary) external drive (not necessarily periodic)
- We introduced **a correspondence** that makes entropy flow physically accessible.

Open problems

- Extending the formalism to devices working with **strong** coupling
- Developing it on **curved spacetime**, e.g. studying 2nd law on black holes
- Studying **other** information measure, such as fidelity loss
- **Applications**: controlling information flow may help the efficiency of quantum heat engines and multi-qubit decoherence, etc.

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2 positions (1 PhD + 1postdoc) available at my group in Germany

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