Title: Analyticity in Spin in Conformal Theories

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Abstract: <p>The conformal bootstrap aims to calculate scaling dimensions and correlation functions in various theories, starting from general principles such as unitarity and crossing symmetry. I will explain that local operators are not independent of each other but organize into analytic functions of spin, and I will present a formula which quantifies the consequences of this fact. This will include a controlled approximation to the operator spectrum at large spin, as well as new bounds over the strength of bulk higher-derivative interactions in large-N theories with a sparse spectrum. These bounds, previously conjectured to exist using gauge-gravity duality, encode causality of the bulk theory. Based on 1703.00278.</p>

Analyticity in Spin: Causality and Bulk Locality in Conformal Theories

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PI, March 15 2017

A Froissart-Gribov Formula for CFT

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Context

Conjecture:

Any large-N CFT with a large gap of operator dimension has an AdS dual, down to lengths $\ell_{\mathrm{AdS}}/\Delta_{\mathrm{gap}}$

[Heemskerk,Penedones,Polchinski& Sully '09]

They proved: solutions to crossing in large-N CFTs w/gap $\longleftrightarrow$ local interactions in AdS

But why are higher-dim interactions suppressed by powers of Δ_{gap}?

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New CFT ingredient:

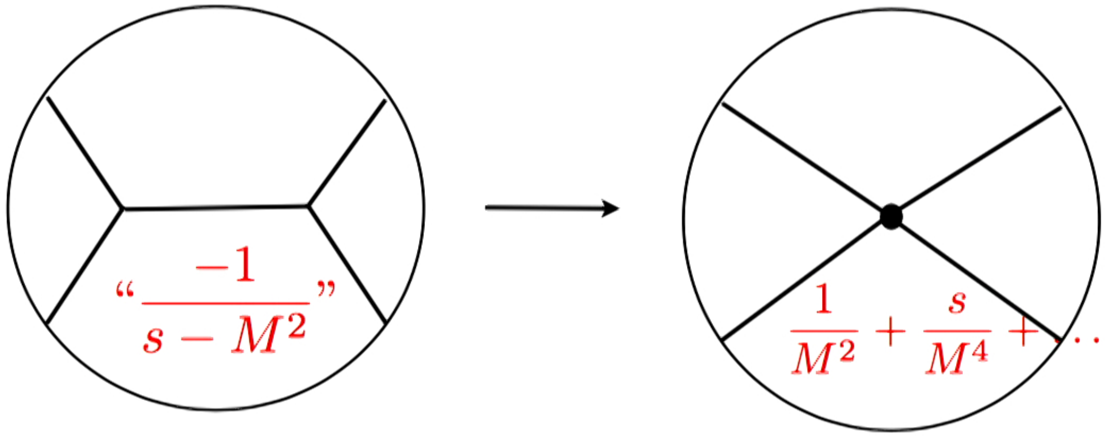
Control over high-energy (Regge) scattering in CFT

[Maldacena, Shenker& Stanford '15]

[Hartman,Kundu&Tajdini '16]

$\Longrightarrow$ Will ensure convergence of a 'dispersion relation'

- Effective field theory in AdS:

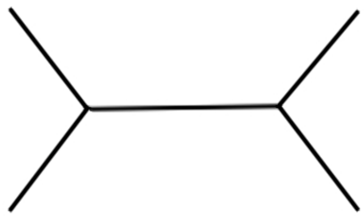


- In AdS5xS5, for example, $1/M^2 \sim \alpha' \ll L_{\mathrm{AdS}}^2$
- Suppression would be clear from a 'dispersion relation in the flat space limit of AdS':

$$\mathcal{M}(s) \sim \int_{M^2}^{\infty} \frac{ds'}{s' - s} \mathrm{Im}\mathcal{M}(s') \sim \frac{1}{M^2} + \frac{s}{M^4} + \ldots$$

Sigh, if only a CFT formula existed that read like this!

CFT bootstrap



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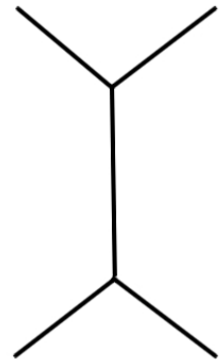

$$\sum_{\mathcal{O}} f_{\mathcal{O}}^2 G_{\mathcal{O}}(u,v) = \sum_{\mathcal{O}'} f_{\mathcal{O}'}^2 G_{\mathcal{O}'}(v,u)$$

[Rattazi, Rychkov, Toni & Vichi '08]

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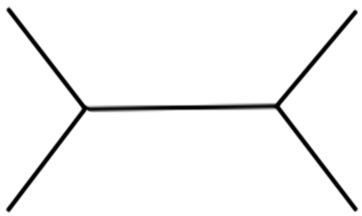
CFT bootstrap



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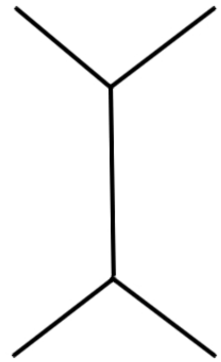


We'll invert one side:

$$f_{\mathcal{O}}^2 = \int K_{\mathcal{O}} \sum_{\mathcal{O}'} f_{\mathcal{O}'}^2 G_{\mathcal{O}'}(v,u)$$

Inverse is non unique. Goal: find one which:

- reads like a bulk dispersion relation
- makes certain limits clear

Outline

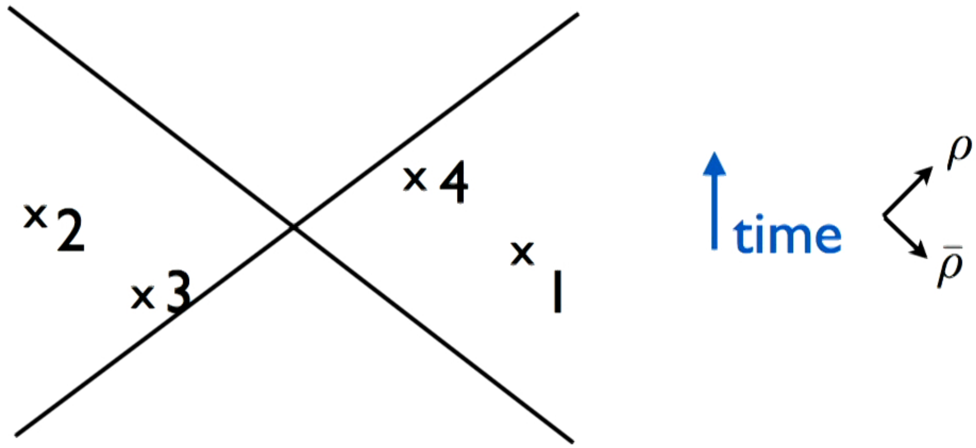
1. Baby dispersion relation, ingredients from:

   - 'A bound on chaos' [..., Maldacena,Shenker&Stanford '15]
   - 'ANEC from causality' [Hartman,Kundu&Tajdini '16]

2. Analytic inverse: Froissart-Gribov formula

3. Applications:
   -large spin expansions
   -large N, large gap → Witten diagrams

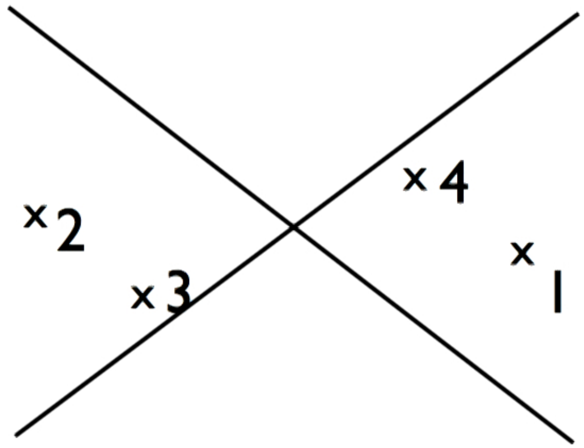
- We consider 4-point correlator in CFT$_d$



- Symmetrical, within Rindler wedges:

$$-\rho_2 = \rho_1 = 1$$
$$-\rho_3 = \rho_4 \equiv \rho$$

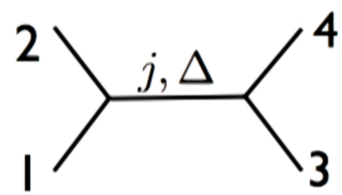
- We consider 4-point correlator in CFT$_d$



$$-\rho_2 = \rho_1 = 1$$
$$-\rho_3 = \rho_4 \equiv \rho$$

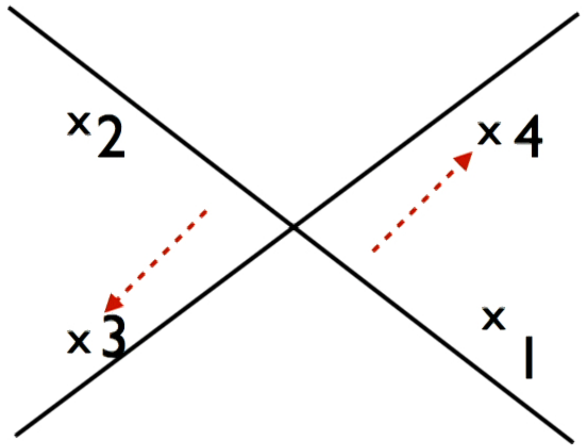
- at small ρ, s-channel OPE:

$$G(\rho, \bar{\rho}) = \sum_{j,\Delta} c_{j,\Delta} \rho^{\frac{\Delta - j}{2}} \bar{\rho}^{\frac{\Delta + j}{2}} =$$



- Suppose we had a dispersion relation in CFT.
- What would be ‘$\mathrm{Im}\,\mathcal{M}$’ ?

- Take x_{41} and x_{23} time-like:



- Certainly looks like a 'scattering amplitude'
- Claim:

$$S \equiv \frac{G}{G_{\text{Eucl}}} \quad \text{satisfies} \quad |S| \le 1$$

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proof

- s-channel OPE diverges upon entering light-cone [Hogervorst&Rychkov '13]

- Use OPE around t-channel (timelike one)

$$G(\rho,\bar\rho) = \sum_{j,\Delta} c_{j,\Delta} \left(\frac{1-\sqrt{\rho}}{1+\sqrt{\rho}}\right)^{\Delta-j} \left(\frac{1-\sqrt{\bar\rho}}{1+\sqrt{\bar\rho}}\right)^{\Delta+j}$$

- All that happens timelike, $\rho > 1$, is extra phases:

$$\begin{aligned} \left|G(\rho,\bar\rho)\right| &= \left|\sum (\text{positive}) e^{i\pi(\Delta-j)}\right| \\ &\leq \sum (\text{positive}) = G(1/\rho,\bar\rho) \equiv G_{\text{Eucl}} \end{aligned}$$

- This means that an 'imaginary part' is positive:

$$S = 1 + i\mathcal{M} \qquad |S| \leq 1 \qquad \Rightarrow \mathrm{Im}\,\mathcal{M} > 0$$

- Since S contains the '1', this is *double discontinuity*:

$$G_{\mathrm{Eucl}} \propto 1$$
$$G_{\mathrm{below}} \propto 1 + i\mathcal{M}$$
$$G_{\mathrm{above}} \propto 1 - i\mathcal{M}^*$$

$$\Rightarrow 2\mathrm{Im}\,\mathcal{M} \propto 2G_{\mathrm{Eucl}} - G_{\mathrm{above}} - G_{\mathrm{below}}$$
$$\equiv \mathrm{dDisc}\,G$$
$$> 0$$

- This double-discontinuity has played an important role in recent literature
- Recent 'onset of chaos' studies use comm. squared:

$$F(t) = \mathrm{tr}\left[e^{-\beta H/2}[A(t), B(0)]e^{-\beta H/2}[B(0), A(t)]\right]$$

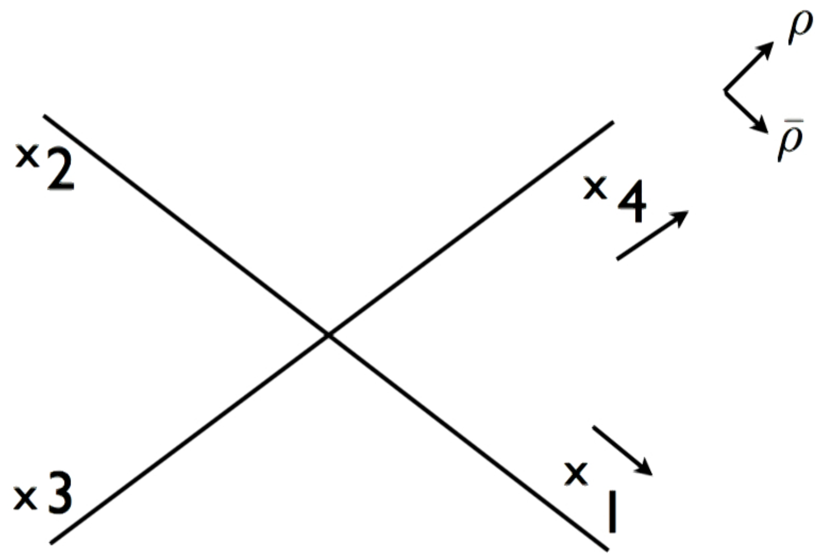
[..., Maldacena,Shenker&Stanford '15]

- Positive by Cauchy-Schwartz

- Take H=Rindler boost& $\beta = 2\pi$, the four terms give:

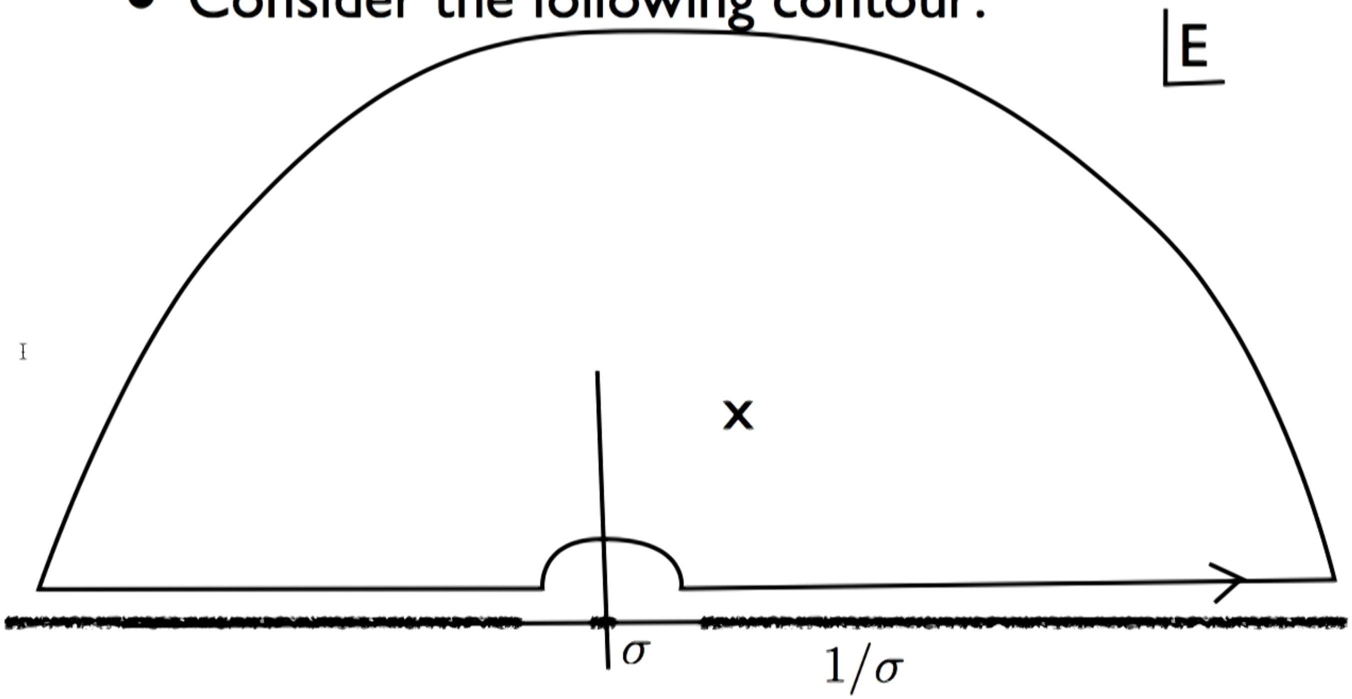
$$F(t) = \mathrm{dDisc}\, G(\rho, \bar{\rho})$$

- Let's try to write a 'dispersion relation' using $\mathrm{Im}\,\mathcal{M}$
- In the Regge limit (large boost) this is easy



$$\rho_4/\rho_1 = \sigma E \to \infty$$
$$\bar{\rho}_4/\bar{\rho}_1 = \sigma/E \to 0$$

- Consider the following contour:



$$\mathcal{M}(E) = \frac{1}{2\pi i}\oint_C \frac{dE'}{E'-E}\mathcal{M}(E')$$

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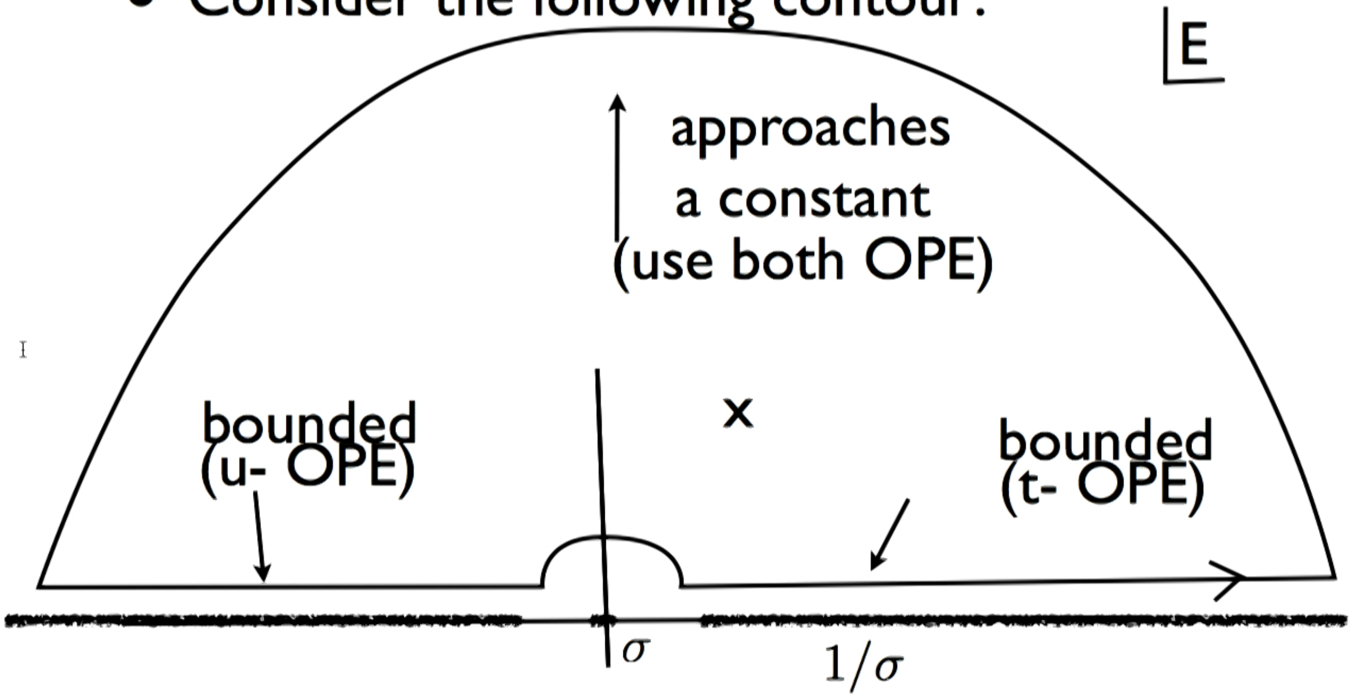

$$\mathcal{M}(E) = \frac{1}{2\pi i}\oint_C \frac{dE'}{E'-E}\mathcal{M}(E')$$

[Hartman,Kundu&Tajdini '16]

- Add '0' =similar contour below

$$\mathcal{M}(E) = C + \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{dE' \operatorname{Im} \mathcal{M}(E')}{E - E'}$$

- Add '0' =similar contour below

$$\mathcal{M}(E) = C + \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dE'\,\mathrm{Im}\,\mathcal{M}(E')}{E-E'}$$

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- This dispersion encodes the MSS chaos bound
- Look at local growth rate away from the real axis:

$$\mathrm{Im}\,\mathcal{M}(x+iy) = \int \frac{dx'\,\mathrm{Im}\mathcal{M}(x')}{(x'-x)^2+y^2} > 0$$

$$(y\partial_y - 1)\left[\log \mathrm{Im}\mathcal{M}(x+iy)\right] = -2\,\frac{\int \frac{dx'\,y^2\,\mathrm{Im}\mathcal{M}(x')}{((x'-x)^2+y^2)^2}}{\int \frac{dx'\,\mathrm{Im}\mathcal{M}(x')}{(x'-x)^2+y^2}} \leq 0$$

$\Rightarrow$ can't grow faster than linear in energy!

- (In Rindler time $E=e^{t/(2\pi)}$ → Lyapunov $\lambda < 2\pi T$)
- Bound can only be saturated if all spectral density is in the UV (→need large gap, if so in forward region)

Why dDisc is awesome

- t-channel expansion at large N:

$$G \sim \sum_{\text{double trace}} c_i |1-z|^{\overset{\sim 1/N}{\gamma_i}} + \sum_{\text{single trace}} c_j |1-z|^{\Delta_j}$$

G_{conn} contains at most single logs, killed by dDisc!

- dDisc kills all double-trace in the cross-channel!

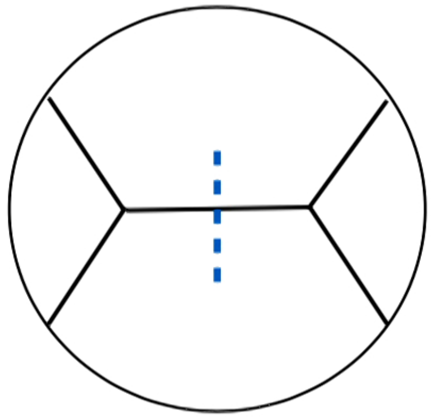
$$G \sim \int \mathrm{dDisc}\, G$$

double trace

single trace

Simple interpretation in theories with AdS dual:

$$\mathrm{Im}\,\mathcal{M} \quad =$$



dDisc saturated by propagating modes in AdS
(=single traces)

- If there's a gap in the spectrum:

$$((1-z)(1-\bar{z}))^{\Delta_{\rm gap}/2} \approx e^{-(z+\bar{z})\Delta_{\rm gap}/2}$$
$$\sim e^{-\Delta_{\rm gap}/E}$$

- Heavy ops only visible deep into Regge limit!
- 'low-energy' expansion will decaying coefficients:

$$\mathcal{M} = \int_{\Delta_{\rm gap}}^{\infty} \frac{\mathrm{Im}\,\mathcal{M}}{\pi(E-E')} \sim E + \frac{E^2}{\Delta_{\rm gap}} + \dots$$

Conclusion so far:

- Look at $\mathcal{M}$ = nontrivial part of four-point correlator in Lorentzian kinematics
- 'Dispersion relation': inverts $\mathcal{M}$ from $\mathrm{Im}\,\mathcal{M}$

$\Downarrow$ AdS/CFT

dispersion relation in flat space limit of AdS

Conclusion so far:

- Look at $\mathcal{M}$ = nontrivial part of four-point correlator in Lorentzian kinematic

Caveat

Away from Regge limit (large boost),
formula has uncontrolled errors from short cut

Sigh... if only we could go beyond Regge!

2. Froissart-Gribov formula

- Regge theory:
  Flat space partial waves are analytic in spin

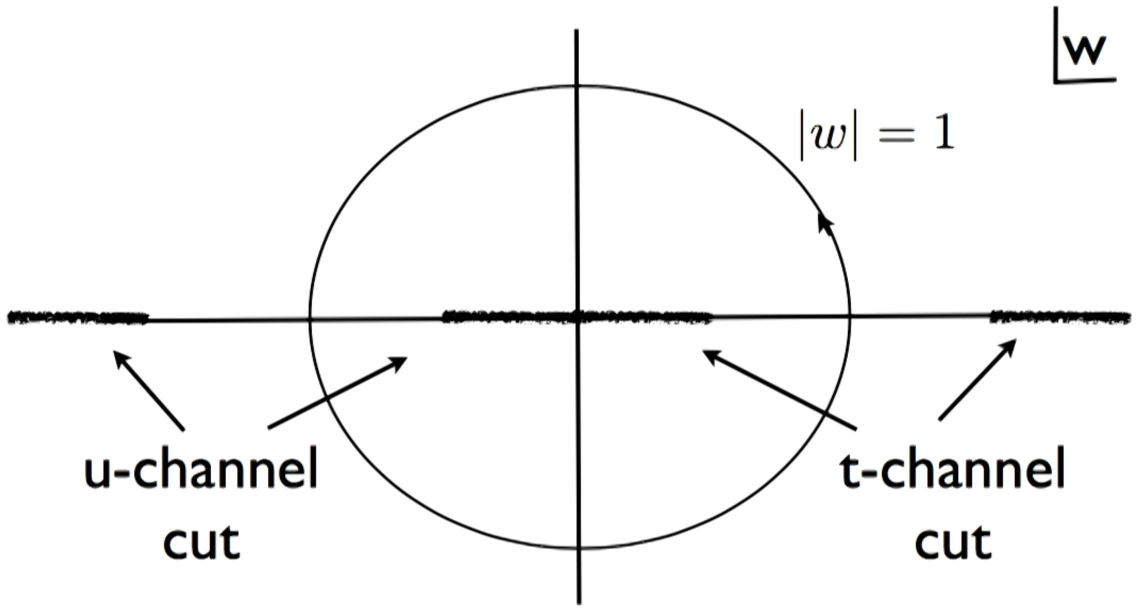
$$a_j(s) = \int_0^{2\pi} d\theta\, \cos(j\theta)\, \mathcal{M}(s, \cos\theta)$$

Why?

Use contour integral in $w = e^{i\theta}$:

$t = \frac{s-4m^2}{4}(w + 1/w - 2)$

$$a_j(s) = \oint \frac{dw}{w} w^j \mathcal{M}(s,t)$$



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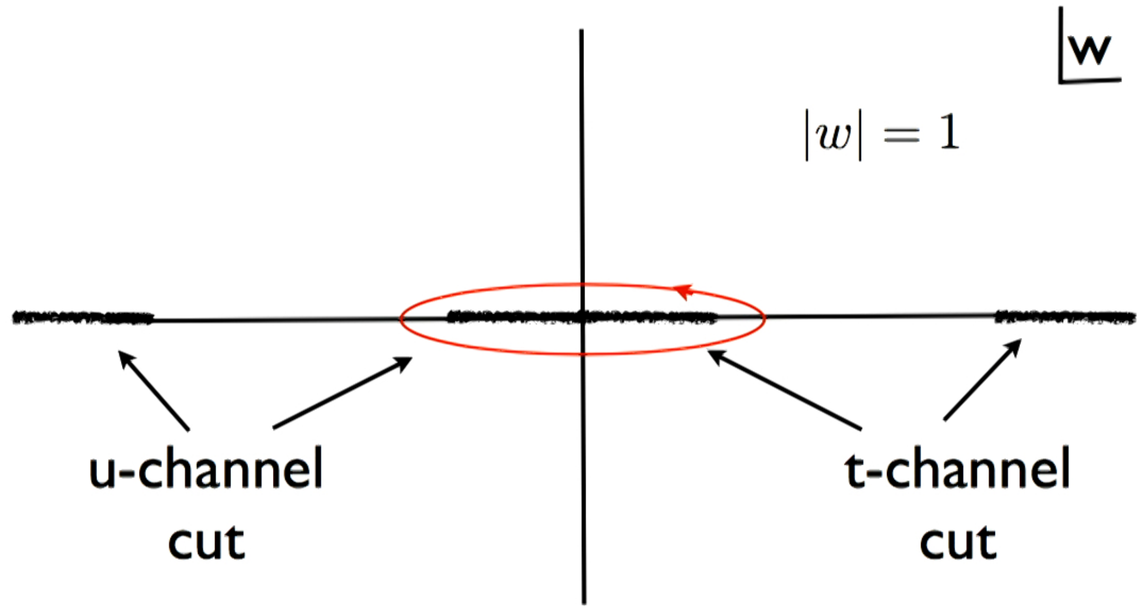

- Froissart-Gribov formula:

$$a_j(s) = a_j^t(s) + (-1)^j a_j^u(s)$$

$$a_j^t(s) = \int_{t=4m^2}^{\infty} dt\, e^{-j\eta}\, \mathrm{Im}\, \mathcal{M}$$

- Converges for $j > j_0$: analytic in j!
  (j_0 = # subtractions in dispersion relation)
- In CFT, will have $j_0 \leq 1$ by boundedness: all spins unified!

- Group theory:

Rotations compact ⇒ (half)-integer spin reps

Boosts not compact ⇒ eigenvalues continuous

Froissart-Gribov tells us how
boosts and rotations fit together

CFT derivation

Euclidean Euclidean Lorentzian

$$\sum_{j,\Delta} \longrightarrow \sum_{j} \int d\Delta \longrightarrow \int dj \int d\Delta$$

- Can't make spin continuous before first making dimensions continuous step 1:

$$G(z,\bar z) = \delta_{12}\delta_{34} + \sum_{j=0}^{\infty} \int_{d/2-i\infty}^{d/2+i\infty} \frac{d\Delta}{2\pi i}\, c(j,\Delta)\, F_{j,\Delta}(z,\bar z).$$

✓

[Costa,Goncalves&Penedones'12]
[see also: Mazac'16;
Hogervorst&van Rees '17, Gadde '17]

- Integrand requires block+shadow:

$$F_{j,\Delta} = g_{j,\Delta} + g_{j,d-\Delta}$$

[Simmons-Duffin '12]

= single-valued, needed for self-adjointness of Casimir

- Contour like a Mellin transform $g_{j,\Delta} \sim (z\bar z)^{\Delta/2}$
- Closing contour on poles, OPE reproduced if integrand has correct poles:

$$c(j,\Delta') \approx \frac{f^2_{OO\to j,\Delta}}{\Delta - \Delta'}$$

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step 2

- To extract the coefficients, just invoke orthogonality and integrate against the blocks:

$$c(j,\Delta) = \#(j,\Delta) \int d^2 z \, \mu(z,\bar{z}) \, G(z,\bar{z}) \, F_{j,\Delta}(z,\bar{z}) \, .$$

- Still just a Euclidean integral: integer spin, no $\mathrm{Im}\,\mathcal{M}$, not yet what we want

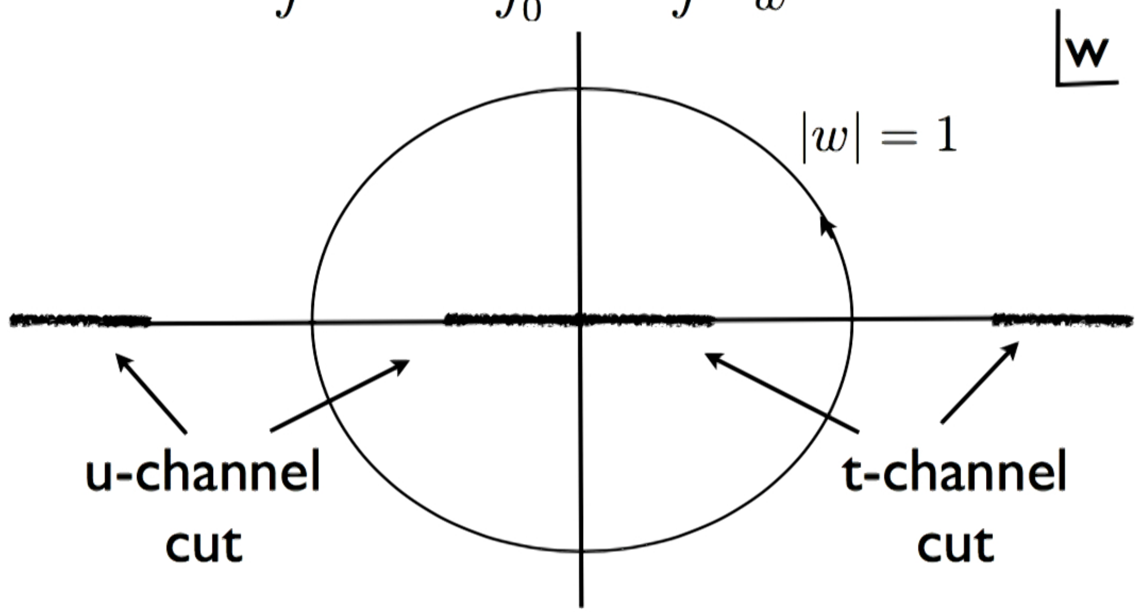
step 3

- Contour deformation. Use clever variables

[Hogervorst&Rychkov '13]

$$z = \frac{4\rho}{(1+\rho)^2} \qquad w = \sqrt{\rho/\bar{\rho}} = e^{i\theta}$$

$$\int d^2 z \to \int_0^1 d|\rho| \oint \frac{dw}{w}$$



- What can we use?
- 8 solutions to conformal Casimirs diff eqs.: (quadratic and quartic)

$$g^{\text{pure}}_{j,\Delta}(z,\bar{z}) \sim z^{\frac{\Delta-j}{2}}\,\bar{z}^{\frac{\Delta+j}{2}} \qquad (0 \ll z \ll \bar{z} \ll 1)$$

- Solutions related by symmetries:

$$j \longleftrightarrow 2-d-j, \qquad \Delta \longleftrightarrow d-\Delta, \qquad \Delta \longleftrightarrow 1-j\,.$$

- Only 2 are nice (convergent) in Regge limit:

$$g^{\text{pure}}_{\Delta+1-d,j+d-1}, \quad g^{\text{pure}}_{1-\Delta,j+d-1} \sim (z\bar{z})^{j/2}$$

- Would like to split 'block + shadow' into bits that are nice in individual Regge limits:

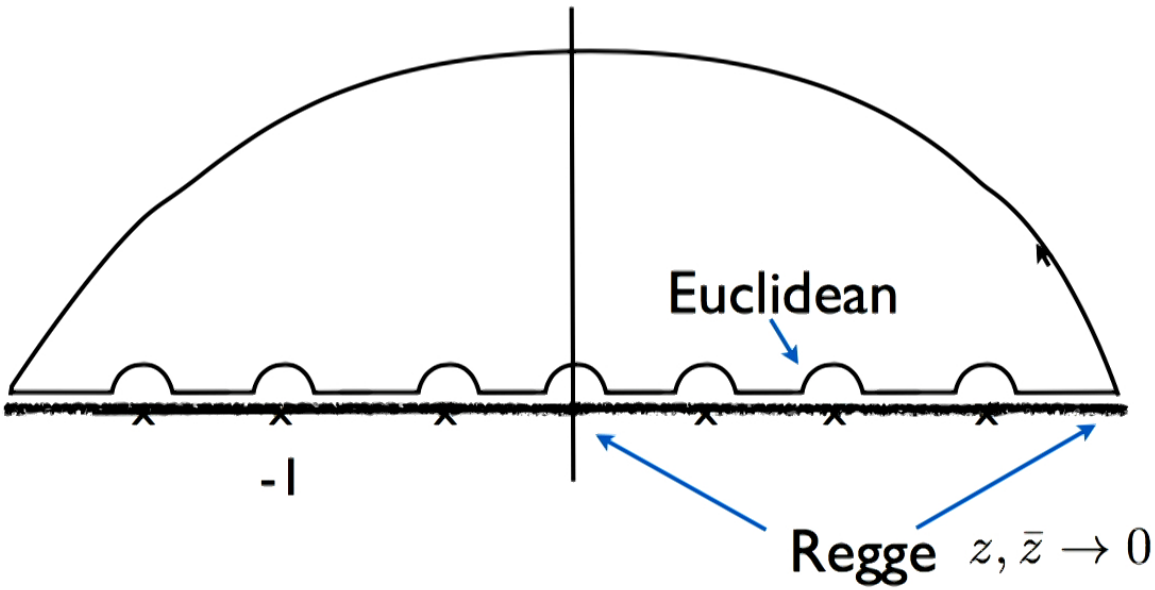
$$F_{j,\Delta}(z,\bar{z}) = F^{(+)}_{j,\Delta} + F^{(-)}_{j,\Delta}$$

$\sim w^{j}$ $\quad (w \to 0)$

$\sim w^{-j}$ $\quad (w \to \infty)$

similar to: $\quad 2\cos(j\theta) = e^{ij\theta} + e^{-ij\theta}$

- This imposes conditions on different sheets:



(all 8 solutions mix under continuation:)

$$g^{\text{pure}}_{j,\Delta}(z,\bar{z})^{\circlearrowleft} = g^{\text{pure}}_{j,\Delta}(z,\bar{z}) \left[1 - 2i\frac{e^{-i\pi(a+b)}}{\sin(\pi\beta)} \sin(\pi(\beta/2+a))\sin(\pi(\beta/2+b))\right] - g^{\text{pure}}_{1-\Delta,1-j}(z,\bar{z})\, 2\pi i \frac{e^{-i\pi(a+b)}\Gamma(\Delta+j-1)\Gamma(\Delta+j)}{\Gamma(\frac{\Delta+j}{2}-a)\Gamma(\frac{\Delta+j}{2}+a)\Gamma(\frac{\Delta+j}{2}-b)\Gamma(\frac{\Delta+j}{2}+b)}\,.$$

4 parameters, 8 constraints,
fingers crossed…

Result: Froissart-Gribov formula

$$c(J,\Delta) = c^t(J,\Delta) + (-1)^J c^u(J,\Delta)$$

$$\boxed{\begin{aligned} &c^t(J,\Delta) = \\ &\frac{\kappa_{J+\Delta}}{4}\int_0^1 dz d\bar{z}\, \mu(z,\bar{z})\, G_{\Delta+1-d,J+d-1}(z,\bar{z})\, \mathrm{dDisc}\left[G(z,\bar{z})\right] \end{aligned}}$$

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Result: Froissart-Gribov formula

$$c(J,\Delta) = \int_{\Diamond} [\text{Inverse block}] \times [\text{dDisc}\, G]$$

s-channel
OPE coefficients

convergent
t-channel sum

Result: Froissart-Gribov formula

$$c(J,\Delta) = \int_\diamond \ [\text{Inverse block}] \times [\text{dDisc}\, G]$$

s-channel
OPE coefficients

block with
j and Δ
exchanged

convergent
t-channel sum

converges for j>1 (Regge limit bounds)

A (boring) test: 2D Ising

$$G(\rho,\bar\rho) = \left|\frac{1}{(1-\rho^2)^{1/4}}\right|^2 + \left|\frac{\sqrt{\rho}}{(1-\rho^2)^{1/4}}\right|^2$$

- Double discontinuity:

$$\frac{1-\frac{1}{\sqrt{2}}(\sqrt{\rho}+\sqrt{\bar\rho})+\sqrt{\rho\bar\rho}}{(1-\rho^2)^{1/4}(1-\bar\rho^2)^{1/4}} > 0$$

- Integral against 2d (global) blocks: factorize

$$c_{j,\Delta} = f_0(j+\Delta)f_0(j+2-\Delta) - \tfrac{1}{2}f_{1/4}(j+\Delta)f_0(j+2-\Delta) + \ldots$$

$$f_p(\alpha) = 2^{a-3+2p}\frac{\Gamma(\frac{7}{4})\Gamma(p+\frac{\alpha-2}{4})}{\Gamma(p+\frac{\alpha+5}{4})}\,{}_3F_2\left(\tfrac{1}{2},\tfrac{\alpha}{2},p+\tfrac{\alpha-2}{4};\tfrac{a+1}{2},p+\tfrac{\alpha+5}{4};1\right). \qquad \text{(B.6)}$$

- Residues at all poles do match global OPE!*

$$C_{j,\Delta} = -K_{j,\Delta}\mathrm{Res}_{\Delta'=\Delta}c(j,\Delta')$$

$$C_{0,1} = \frac{1}{4}, \quad C_{2,2} = \frac{1}{64}, \quad C_{4,4} = \frac{9}{40960}, \quad C_{0,4} = \frac{1}{4096}$$
$$C_{4,5} = \frac{1}{65536}, \quad C_{6,6} = \frac{35}{3670016} \quad C_{2,6} = \frac{9}{2621440}, \quad C_{6,7} = \frac{1}{1310720}, \dots$$

* Including (predicted) spurious poles for $\Delta - j - d = 0, 1, 2 \dots$

$$\frac{(-1)^{m+1}\Gamma(1+a+\frac{m}{2})\Gamma(1+b+\frac{m}{2})}{m!(m+1)!\Gamma(a-\frac{m}{2})\Gamma(b-\frac{m}{2})}\times$$
$$\times K_{j+1+m,j+d-1}c(j+1+m,j+d-1)$$

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**And: never trust Mathematica's Residue on 3F2........

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Large spin expansions

large spin in s-channel $\leftarrow$ low twist in t-channel

- Usual story: double-light-cone limit $(z, \bar{z}) \to (0, 1)$

- non-analytic behaviour in $(1 - \bar{z})$ needs large spin:

$$\sum_j \frac{1}{j^\alpha} F_j(\bar{z}) = (1 - \bar{z})^{\alpha/2} + \text{regular}$$

$\Rightarrow$ Solve OPE in asymptotic series in 1/j

[Komargodski&Zhiboedov,
Fitzpatrick,Kaplan,Poland&Simmons-Duffin,
Alday&Bissi&...,
,Kaviraj,Sen,Sinha&...,
Alday,Bissi,Perlmutter&Aharony,...]

- What about inversion formula?

$$c(j,\Delta) \sim \int_0^1 dz d\bar{z}\, z^{j-\Delta} \bar{z}^{j+\Delta} F_{j+\Delta}(\bar{z}) \mathrm{dDisc} G(z,\bar{z})$$

- Recall, OPE data encoded in Δ-poles: $z \to 0$

if $G(z,\bar{z}) \to z^{\tau} G_{\tau}(\bar{z})$,

$$c(j,\Delta) = \frac{1}{j-\Delta-\tau} \times \int_0^1 d\bar{z}\, \bar{z}^{j+\Delta} F_{j+\Delta}(\bar{z}) \mathrm{dDisc} G_{\tau}(\bar{z})$$

- Large j+Δ and low twist push to (0,1) corner

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- Large j+Δ and low twist push to (0,1) corner

- Analytic result for collinear integral of power:

$$I^{a,b}_{\tau'}(\bar h) \equiv \int_0^1 \frac{d\bar z}{\bar z^2}(1-\bar z)^{a+b}\kappa_{\bar h} k_{\bar h}(\bar z)\,\mathrm{dDisc}\left[\left(\frac{1-\bar z}{\bar z}\right)^{\frac{\tau'}{2}-b}(\bar z)^{-b}\right] \quad (4.7)$$

$$= \frac{1}{\Gamma(-\frac{\tau'}{2}-a)\Gamma(-\frac{\tau'}{2}+b)} \times \frac{\Gamma(\bar h-a)\Gamma(\bar h+b)}{\Gamma(2\bar h-1)} \times \frac{\Gamma(\bar h-\frac{\tau'}{2}-1)}{\Gamma(\bar h+\frac{\tau'}{2}+1)}.$$

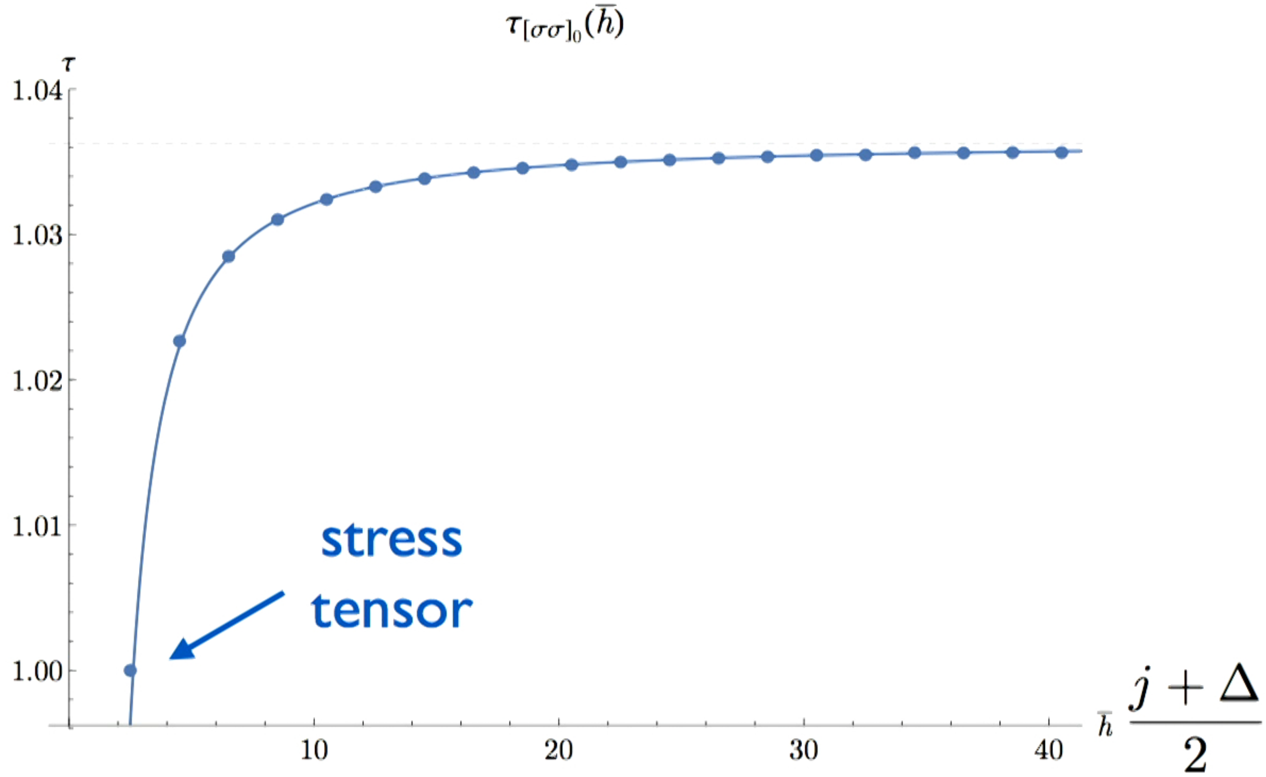
$$\sim 1/\bar h^{\tau'} \qquad (\bar h = \tfrac{j+\Delta}{2})$$

- Earlier results reproduced by: 'expand cross-channel OPE in $\frac{1-\bar z}{\bar z}$ and integrate termwise using (4.7)'

- Conceptually, no need to expand like this now
(=why earlier expansions were asymptotic)

[Alday&Zhiboedov '15; Simmons-Duffin '16]

Asymptotic series in 3D Ising



[Plot from Simmons-Duffin '16;
see Alday&Zhiboedov '15]

What's new:

- Asymptotic expansion $\Rightarrow$ convergent sum

  (no need to expand in $(1-\bar{z})/\bar{z}$)

- Control over individual spins, not only averages over many spins ('no stick-out')

- Can try to bound errors?

Contact Witten diagrams

- Plug in a heavy single-trace in t-channel:

$$c(j, \tfrac{d}{2} + i\nu) \sim \int_0^1 d(z\bar{z})(z\bar{z})^{j/2-1} e^{-(\sqrt{z}+\sqrt{\bar{z}})\Delta_{\text{gap}}} \times \int d(z/\bar{z})(\ldots)$$

$$\propto \frac{1}{(\Delta_{\text{gap}}^2)^{j-1}}$$

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- Can further bound the heavy contribution:
  -integrand 'dDisc G' positive and locally bounded
  -stress tensor case (j=2) known

$$\left|c(j, \tfrac{d}{2} + i\nu)_{\text{heavy}}\right| \leq \frac{1}{c_T} \frac{\#}{(\Delta_{\text{gap}}^2)^{j-2}}$$

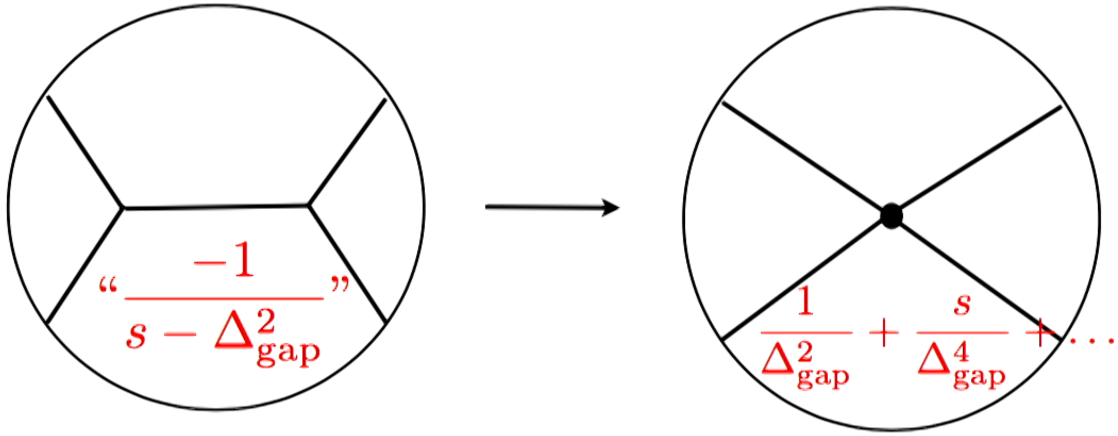
- What was known:
  Solutions to crossing symmetry in a large-N CFT with large gap = Witten diagrams

  [Heemskerk,Penedones, Polchinski& Sully '09]

  For given light spectrum, solutions are ambiguous by contact interactions

- What we learn:
  analyticity in spin (or Regge limit) singles out a unique solution with bounded errors

  $< 1/(\Delta^2_{\rm gap})^{j-2}$



- AdS/CFT expectation: EFT coefficients suppressed by dimension: $(\partial^{2k})\phi^4$ down by $1/\Delta_{\text{gap}}^{2k}$
- What we proved: down by spin $1/\Delta_{\text{gap}}^{2(j-2)}$
- Same as ‘causality bound’ conjectured recently (equivalent to existence of dispersion relation in the bulk) [Maldacena,Simmons-Duffin&Zhiboedov ’15]

Spin versus dimension

- Some sporadic few-derivative interactions remain unconstrained
- Consider an AdS interaction with flat-space limit:

$$stu$$

- This has spin two in the Regge limit in all channels:

$$stu = st(s+t) \sim s^2 \equiv s^j \qquad (s \to \infty, t \text{ fixed})$$

- All interactions with more derivative, however, must have small coefficients

Conclusion

- Novel formula:

$$\underbrace{c(j,\Delta)}_{\text{s-channel}} \equiv \int_0^1 dz d\bar{z}\, g_{\Delta,j}\, \underbrace{\mathrm{dDisc}\, G}_{\text{t-channel}}$$

- Valid in any unitary $\mathrm{CFT_D}$. Regge behavior ensures convergence; bounds derivative interactions in AdS

- Outlook:
  -Numerical bootstrap: bound errors using convergent 1/j expansion?
  -Loops in AdS?
  -Higher points?

& much more!