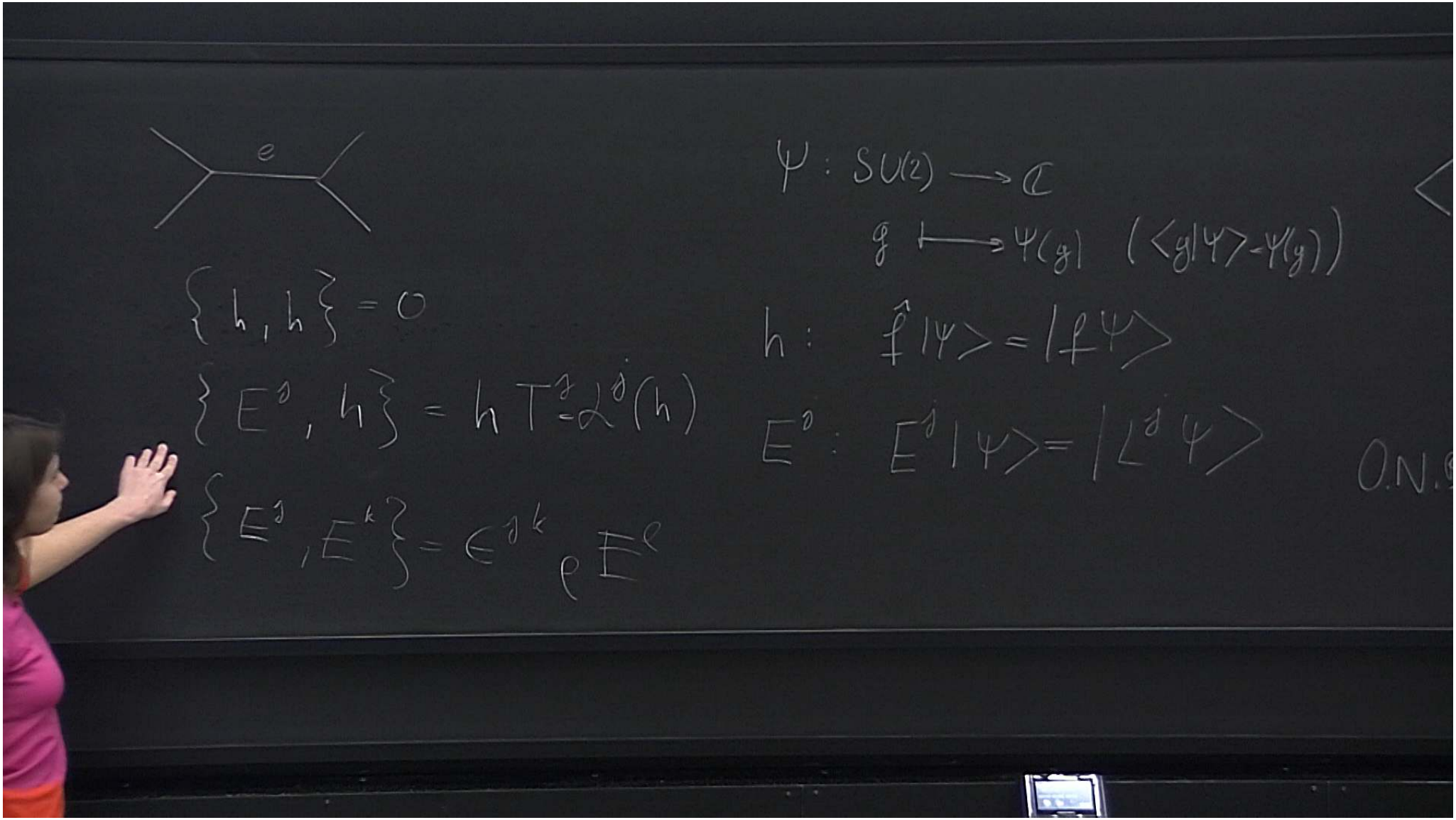
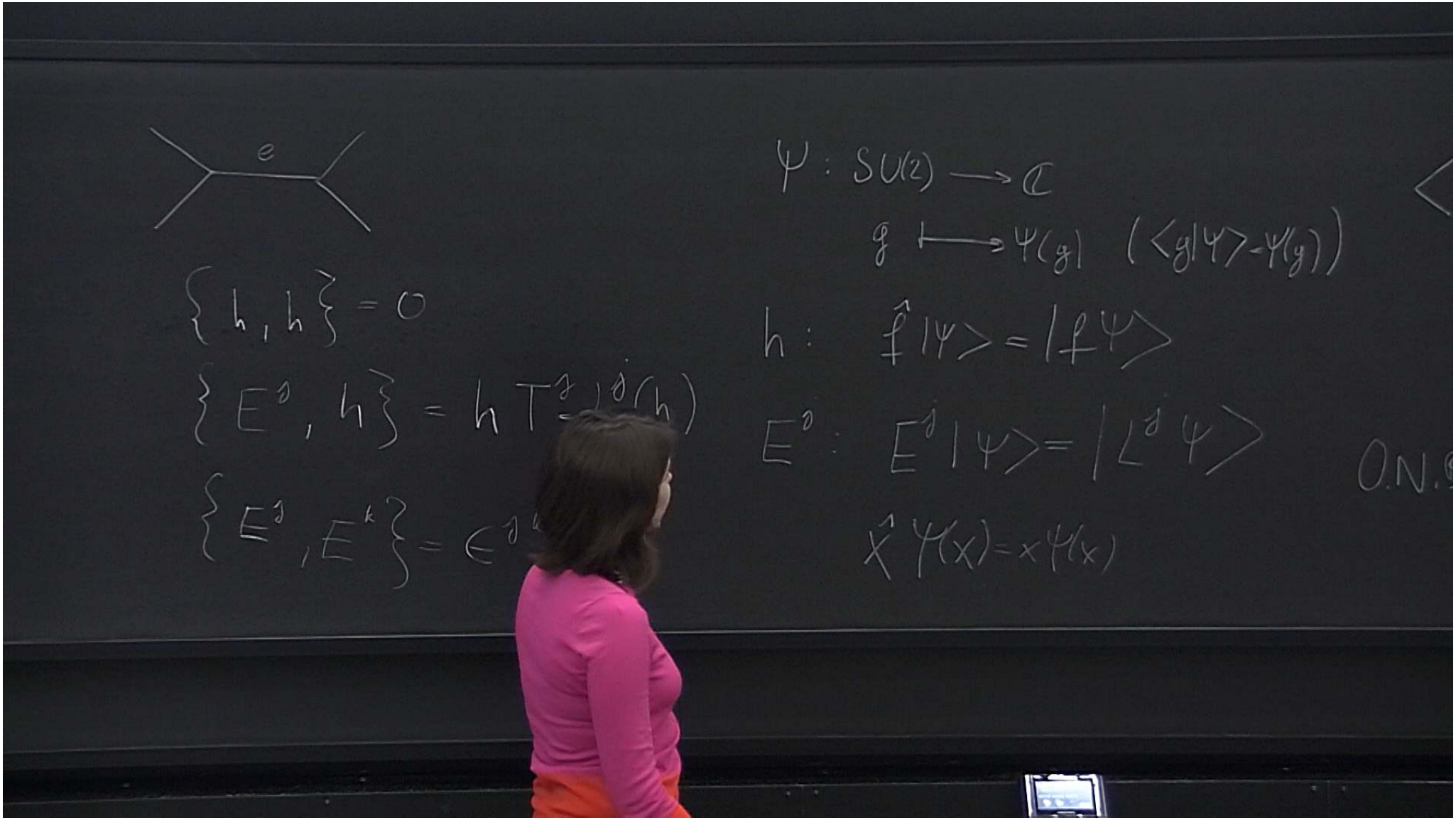
Title: PSI 2016/2017 Explorations in Quantum Gravity - Lecture 10

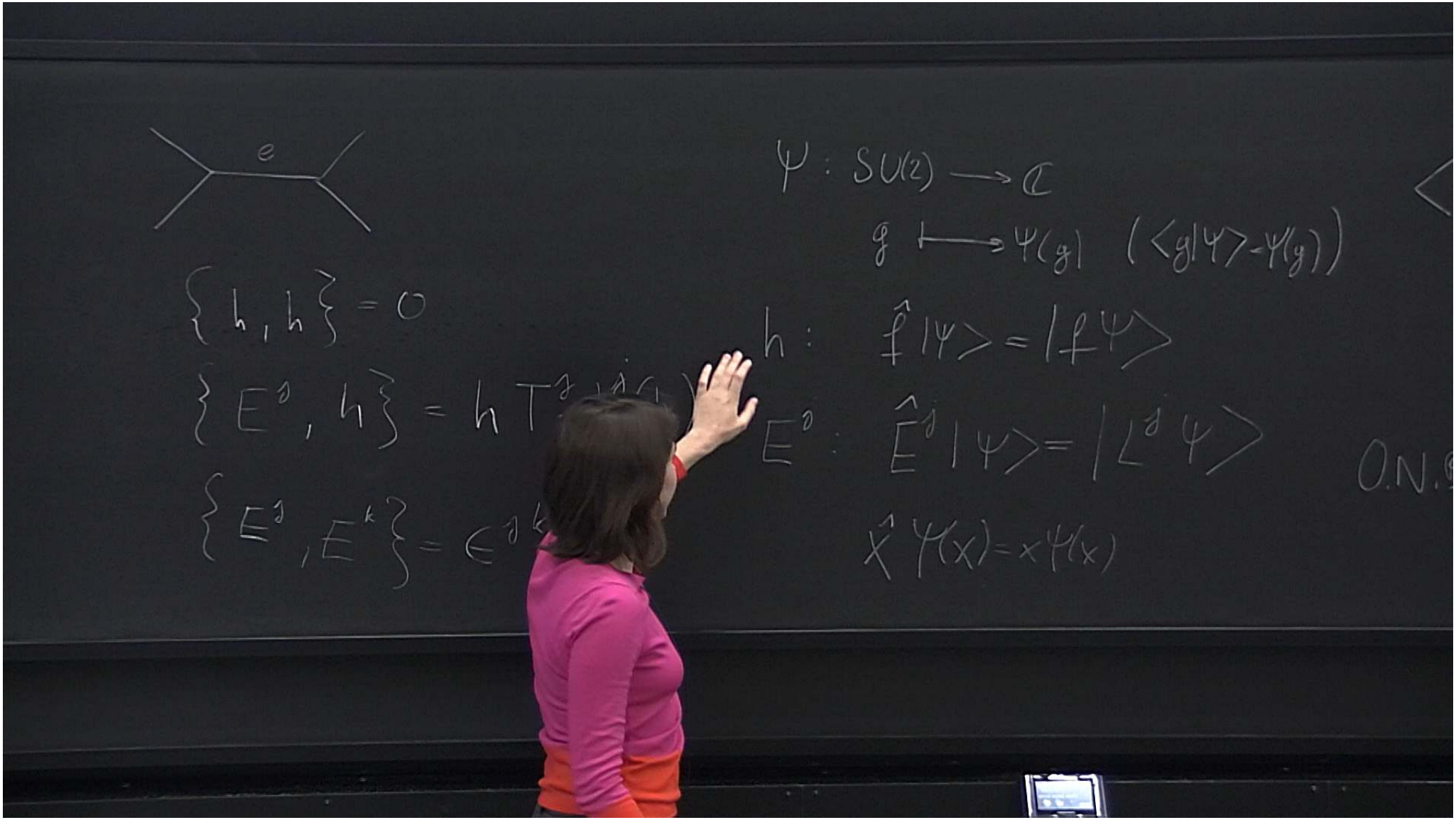
Date: Mar 31, 2017 10:15 AM

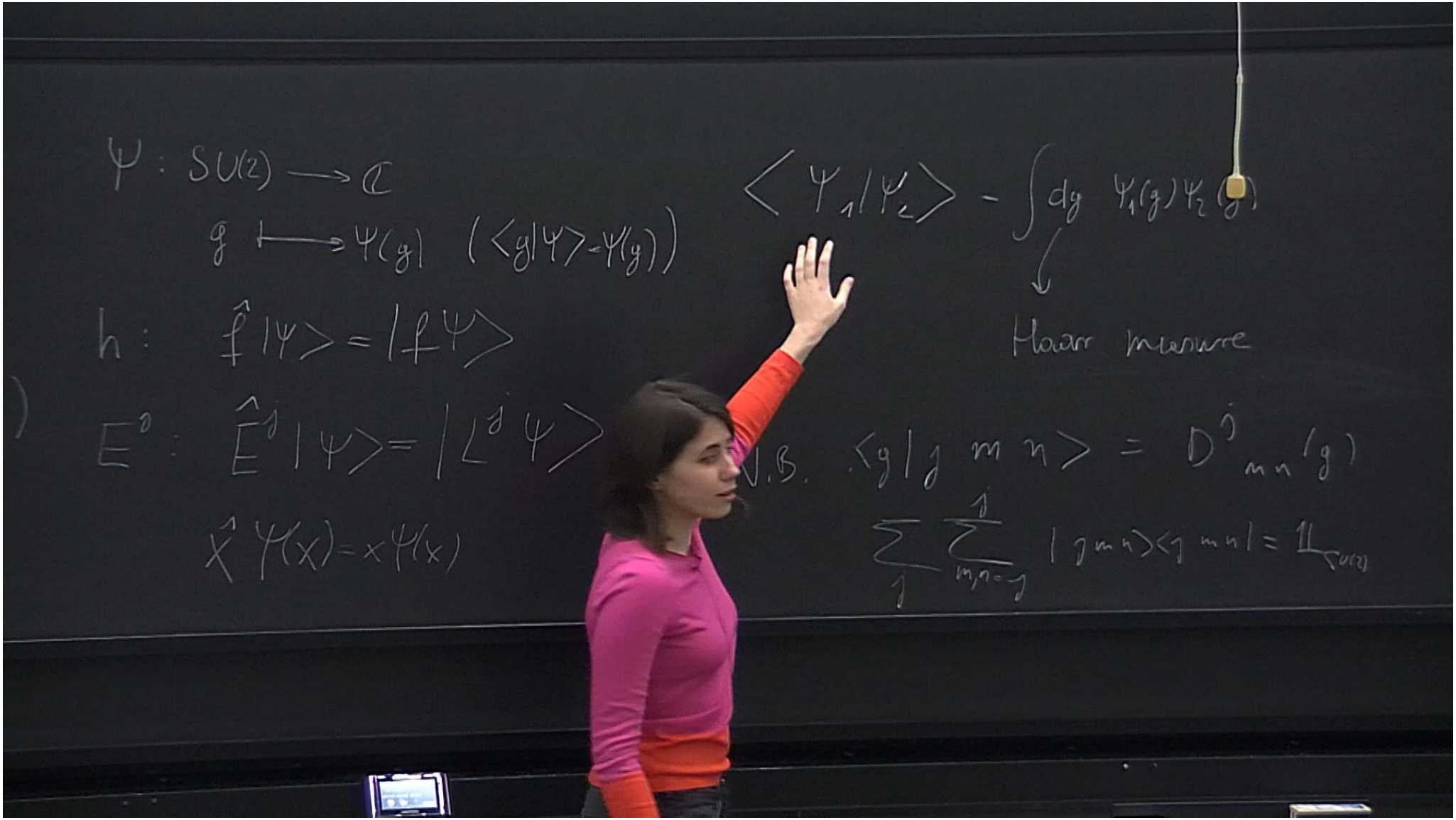
URL: http://pirsa.org/17030077

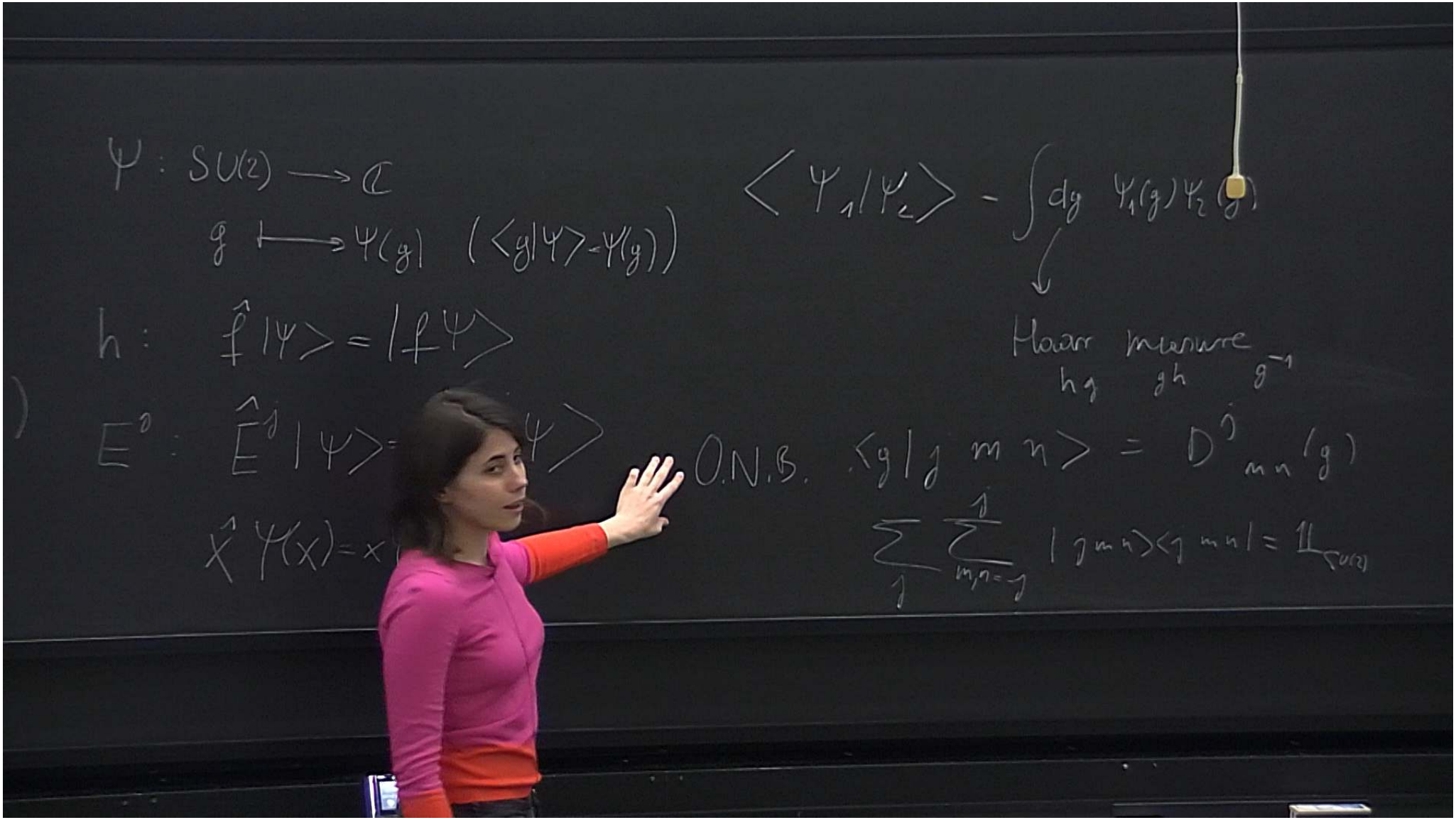
Abstract:

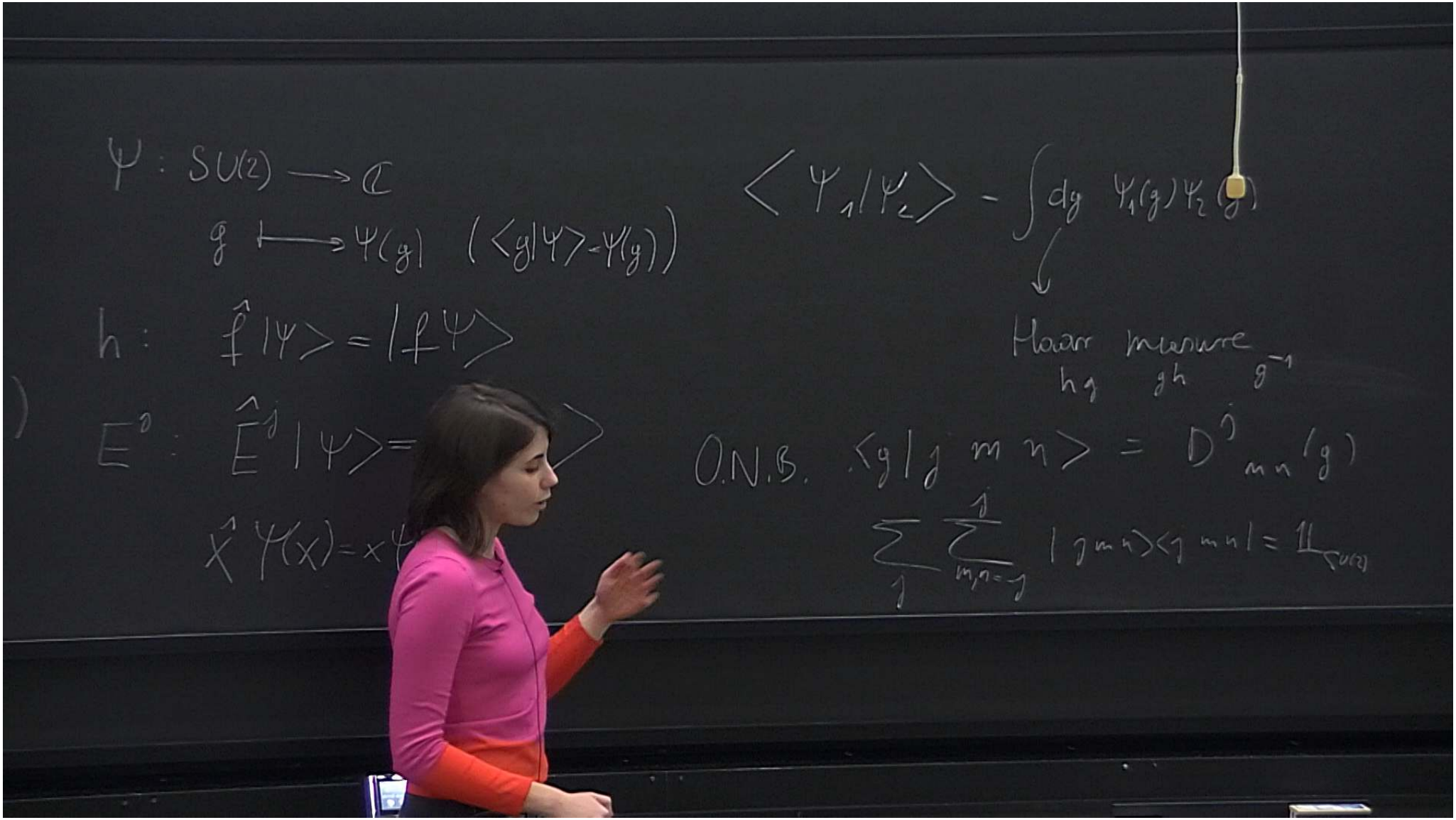

e
$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\{h, h\} = 0$
h: $\hat{f}|\Psi\rangle = |f\Psi\rangle$
$\{E^a, h\} = h T^a = L^a(h)$
E^a: $E^a|\Psi\rangle = |L^a\Psi\rangle$
O.N.
$\{E^j, E^k\} = \epsilon^{jk}{}_\ell E^\ell$


e
{h, h} = 0
{E^j, h} = h T^j = L^j(h)
{E^j, E^k} = ε^j
Ψ : SU(2) → ℂ
g ↦ Ψ(g) (⟨g|Ψ⟩ = Ψ(g))
h : f̂|Ψ⟩ = |fΨ⟩
E^j : E^j|Ψ⟩ = |L^j Ψ⟩
x̂ Ψ(x) = x Ψ(x)
O.N.


e
$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\{h, h\} = 0$
h : $\hat{f}|\Psi\rangle = |f\Psi\rangle$
$\{E^a, h\} = h T^a$
E^a : $\hat{E}^a|\Psi\rangle = |L^a\Psi\rangle$
$\{E^j, E^k\} = \epsilon^{jk}$
$\hat{x}\,\Psi(x) = x\Psi(x)$
O.N.


$\psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \psi(g)$ $(\langle g|\psi\rangle = \psi(g))$
$\langle \psi_1 | \psi_2 \rangle = \int dg\ \psi_1(g)\psi_2(g)$
Haar measure
$h : \hat{f}|\psi\rangle = |f\psi\rangle$
$E^j : \hat{E}^j|\psi\rangle = |L^j\psi\rangle$
N.B. $\langle g | j\ m\ n\rangle = D^j_{mn}(g)$
$\hat{x}\,\psi(x) = x\psi(x)$
$\sum_j \sum_{m,n=-j}^{j} |j\ m\ n\rangle\langle j\ m\ n| = \mathbb{1}_{L^2(SU(2))}$


$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\langle \Psi_1 | \Psi_2 \rangle = \int dg \; \Psi_1(g) \Psi_2(g)$
Haar measure
hg gh g^{-1}
$h: \quad \hat{f}|\Psi\rangle = |f\Psi\rangle$
$E^j: \quad \hat{E}^j|\Psi\rangle = |\Psi\rangle$
O.N.B. $\langle g | j \; m \; n \rangle = D^j_{mn}(g)$
$\sum_j \sum_{m,n=-j}^{j} |j\,m\,n\rangle\langle j\,m\,n| = \mathbb{1}_{L^2(SU(2))}$


$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\langle \Psi_1 | \Psi_2 \rangle = \int dg \; \Psi_1(g) \Psi_2(g)$
Haar measure
$hg \quad gh \quad g^{-1}$
$h: \quad \hat{f}|\Psi\rangle = |f\Psi\rangle$
$E^j: \quad \hat{E}^j|\Psi\rangle =$
O.N.B. $\langle g | j \; m \; n \rangle = D^j_{mn}(g)$
$\hat{x}\Psi(x) = x\Psi$
$\sum_j \sum_{m,n=j}^{j} |j\,m\,n\rangle\langle j\,m\,n| = \mathbb{1}_{SU(2)}$

$$\Psi : SU(2) \longrightarrow \mathbb{C}$$

$$g \longmapsto \Psi(g) \quad (\langle g|\Psi\rangle = \Psi(g))$$

$$h : \quad \hat{f}|\Psi\rangle = |f\Psi\rangle$$

$$E^j : \quad \hat{E}^j|\Psi\rangle = |L^j\Psi\rangle$$

$$\hat{X}\Psi(x) = x\Psi(x)$$

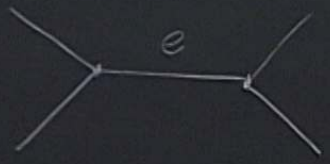
$$\langle \Psi_1|\Psi_2\rangle = \int dg \; \Psi_1(g)\Psi_2(g)$$

Haar measure

$hg \quad gh \quad g^{-1}$

O.N.B. $\langle g|j\,m\,n\rangle = D^j{}_{mn}(g)$

$$\bigoplus_j V_j \otimes V_j^* \longleftarrow \sum_j \sum_{m,n=-j}^{j} |j\,m\,n\rangle\langle j\,m\,n| = \mathbb{1}_{SU(2)}$$



$\{h, h\} = 0$

$\{E^j, h\} = h T^j = L^j(h)$

$\{E^j, E^k\} = \epsilon^{jk}{}_\ell E^\ell$

$\Psi : SU(2) \longrightarrow \mathbb{C}$

$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$

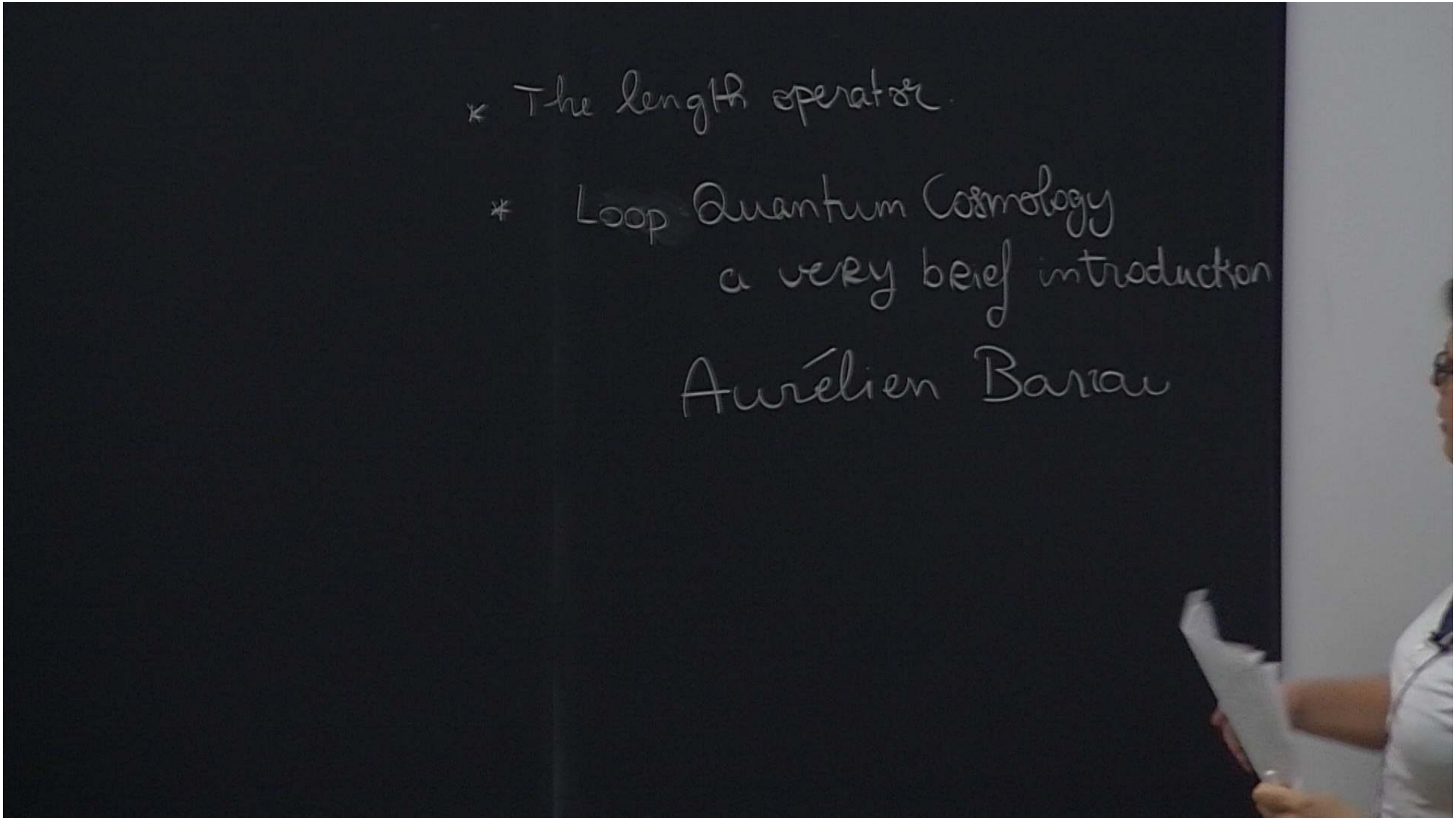
h: $\hat{f}|\Psi\rangle = |f\Psi\rangle$

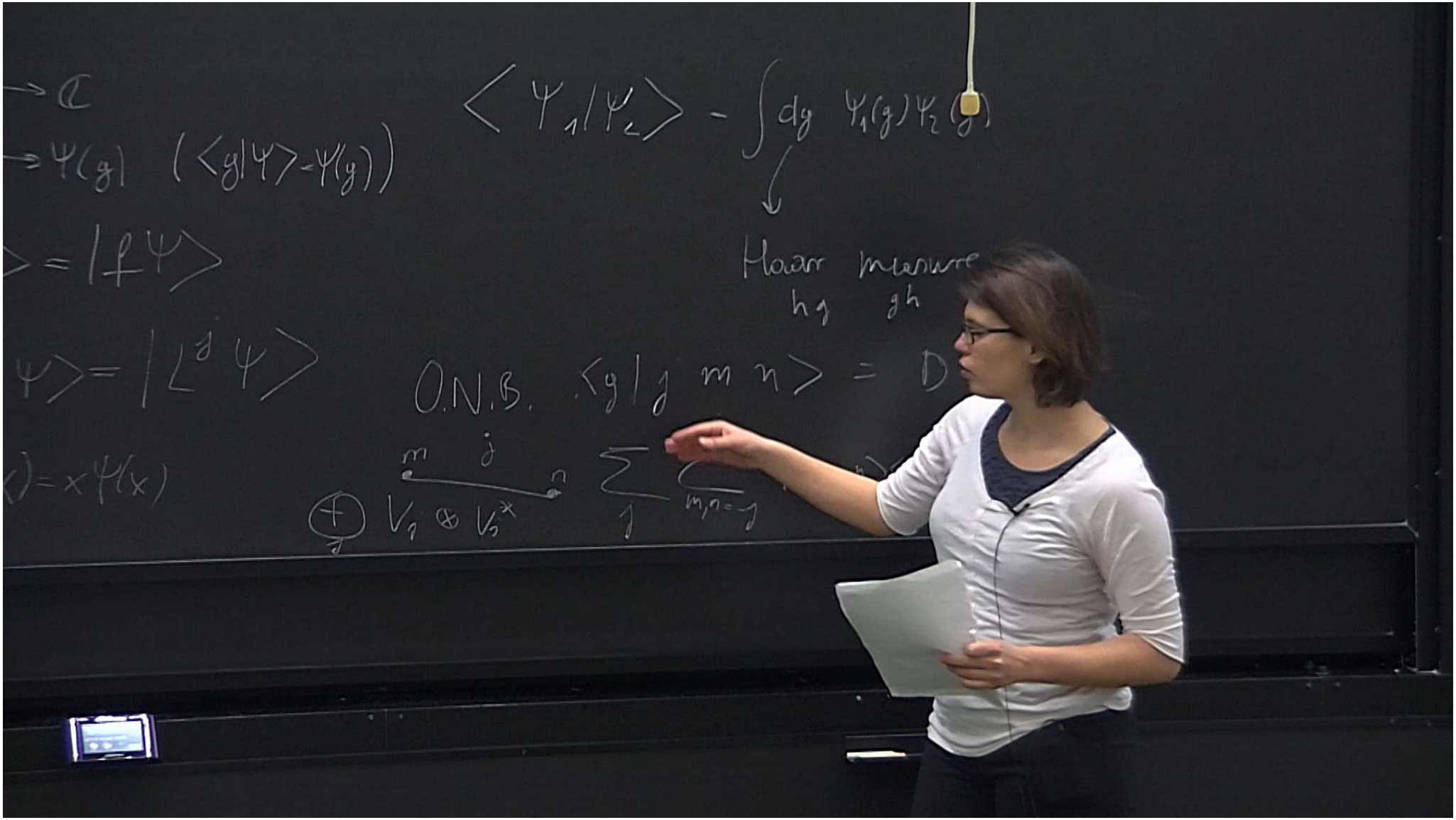
E^j: $\hat{E}^j|\Psi\rangle = |L^j\Psi\rangle$

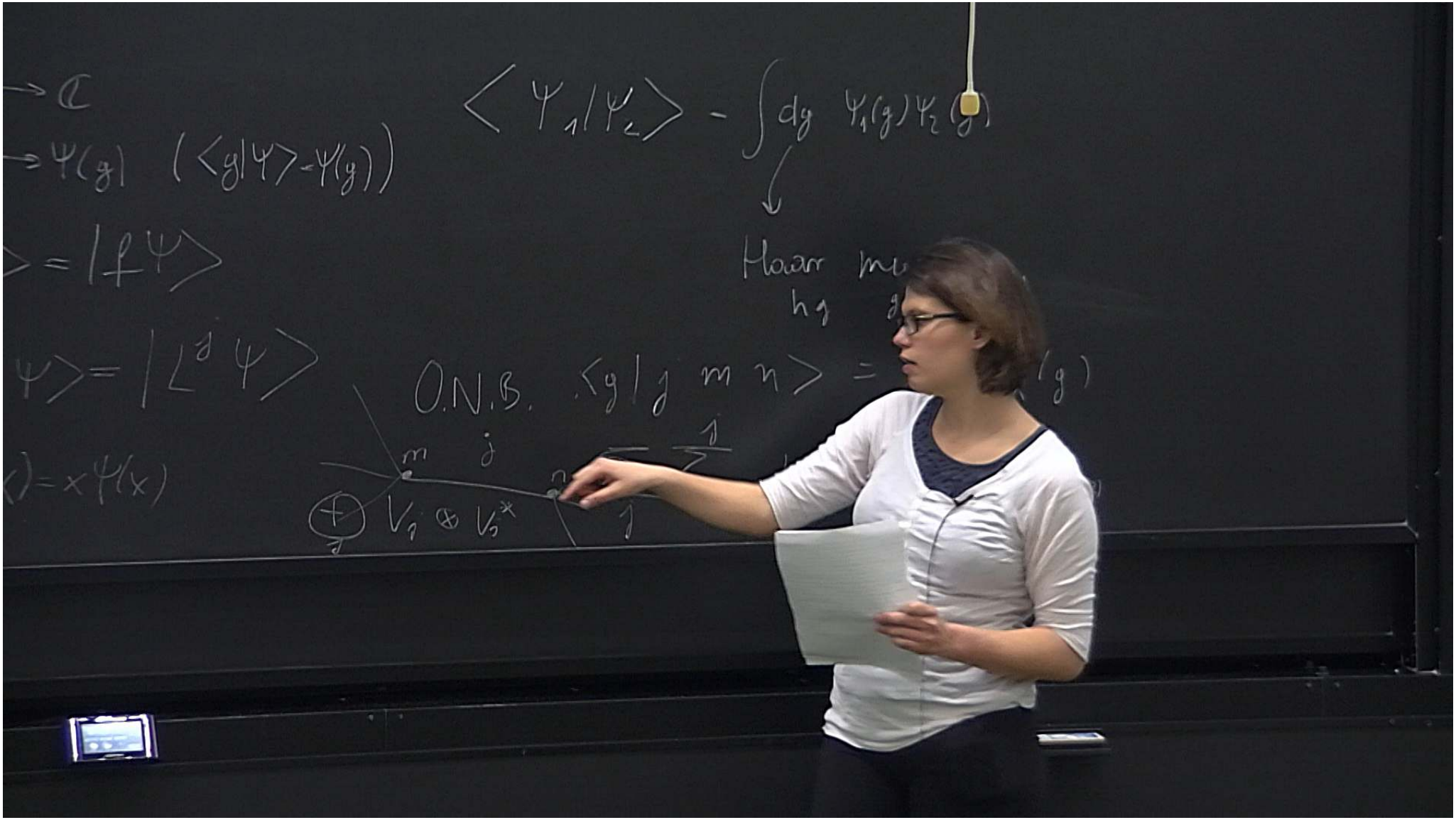
$\hat{x}\Psi(x) = x\Psi(x)$

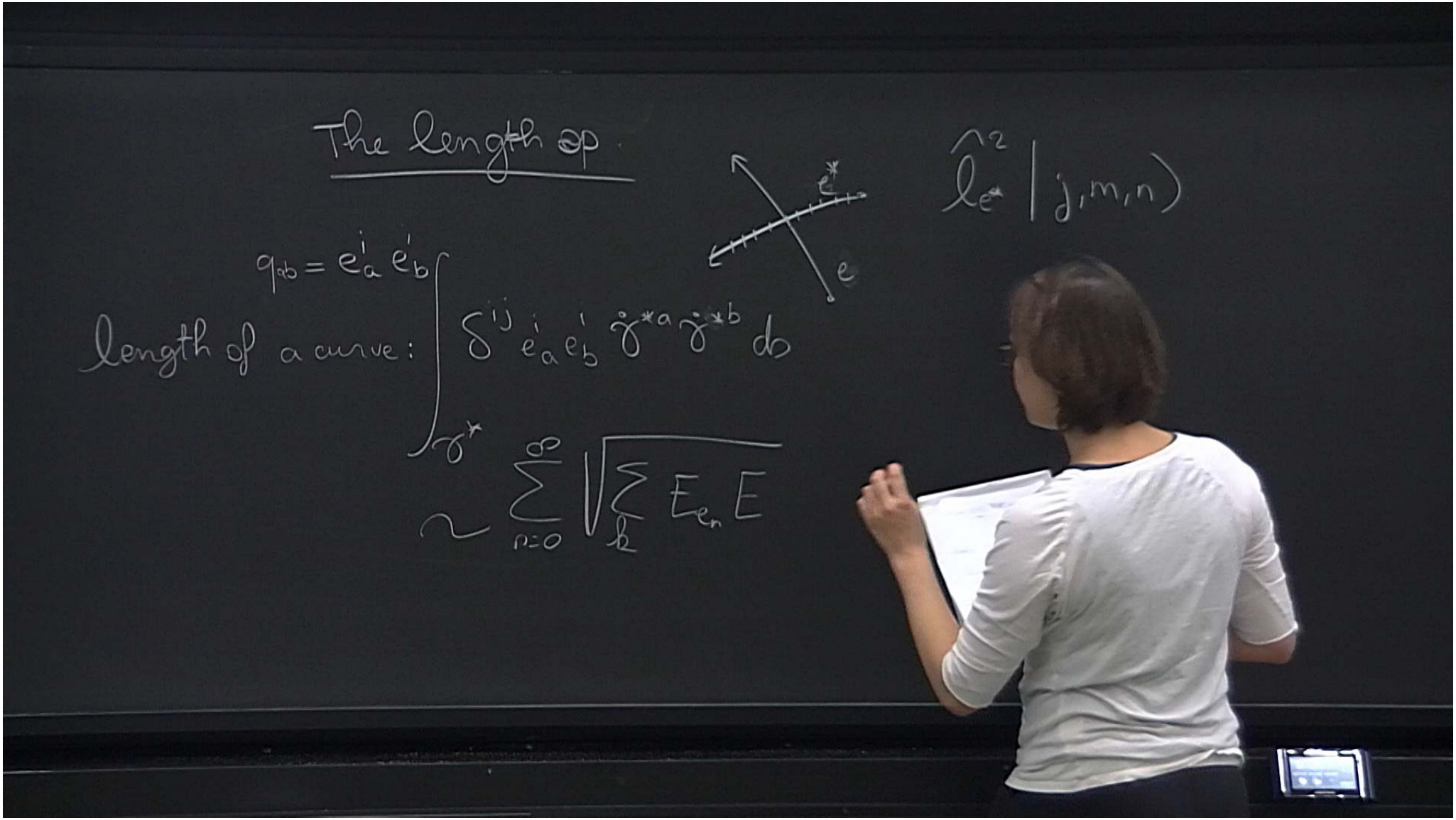
O.N.B

$\bigoplus_j V_j \otimes V_j^*$


* The length operator
* Loop Quantum Cosmology
a very brief introduction
Aurélien Barrau


$\to \mathbb{C}$
$\to \psi(g)$ $(\langle g|\psi\rangle = \psi(g))$
$\rangle = |f\psi\rangle$
$\psi\rangle = |L^g\psi\rangle$
$(x) = x\psi(x)$
$\langle \psi_1 | \psi_2 \rangle = \int dg\ \psi_1(g)\psi_2(g)$
Haar measure
hg gh
O.N.B. $\langle g | j\ m\ n \rangle = D$
m j n
$\bigoplus_j V_j \otimes V_j^*$
$\sum_j$ $m,n=j$


$\langle \psi_1 | \psi_2 \rangle = \int dg \; \psi_1(g) \psi_2(g)$
Haar
O.N.B.

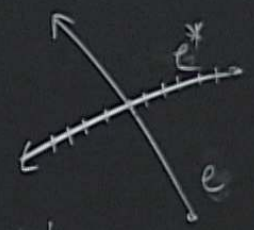

The length op
$q_{ab} = e^i_a e^i_b$
length of a curve: $\int_{\gamma^*} \delta^{ij} e^i_a e^j_b \dot{\gamma}^{*a} \dot{\gamma}^{*b} db$
$\sim \sum_{n=0}^{\infty} \sqrt{\sum_k E_{en} E}$
$\hat{l}^2_{e^*} | j,m,n)$
e^*
e

The length op

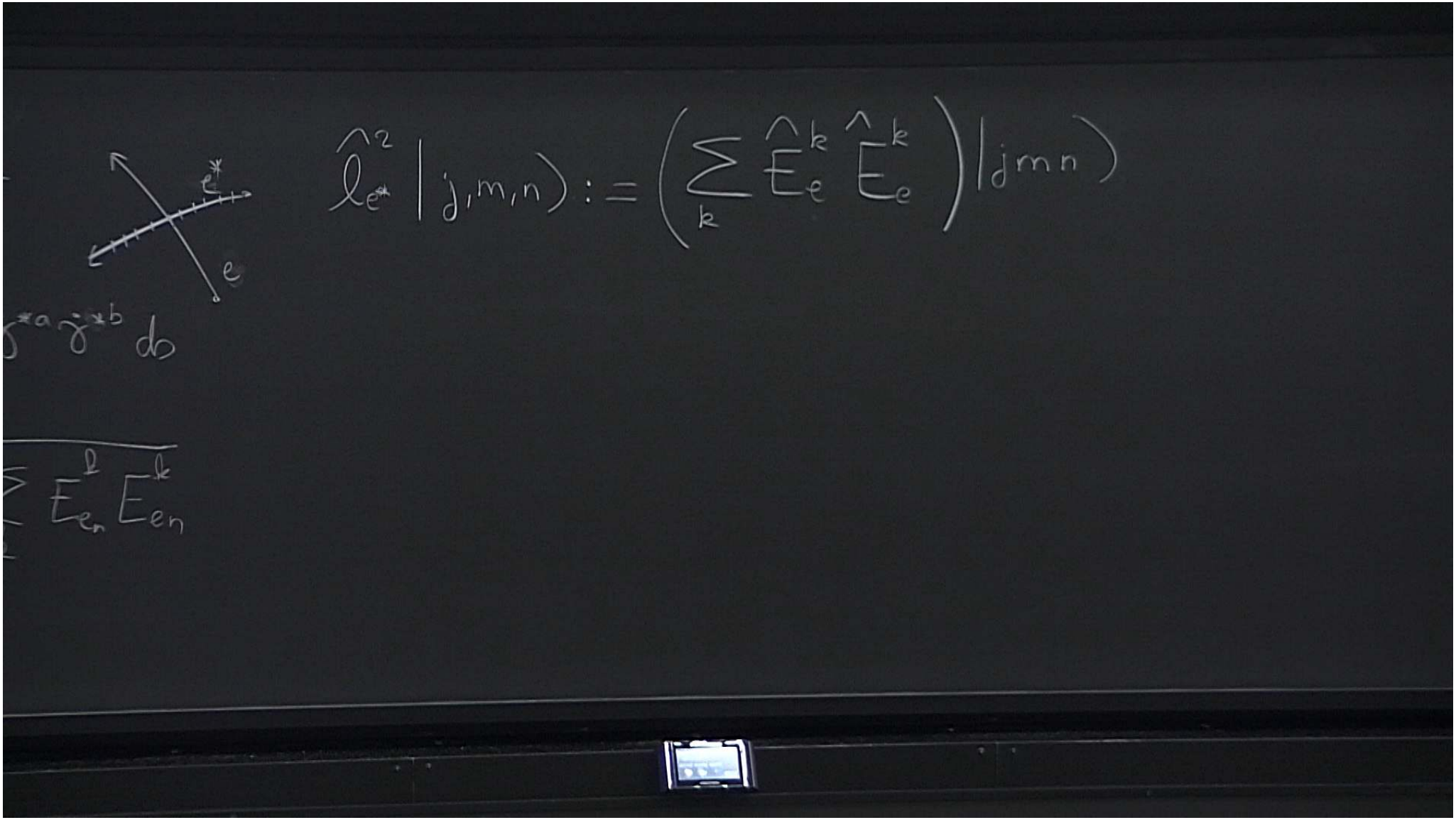
$q_{ab} = e^i_a e^i_b$

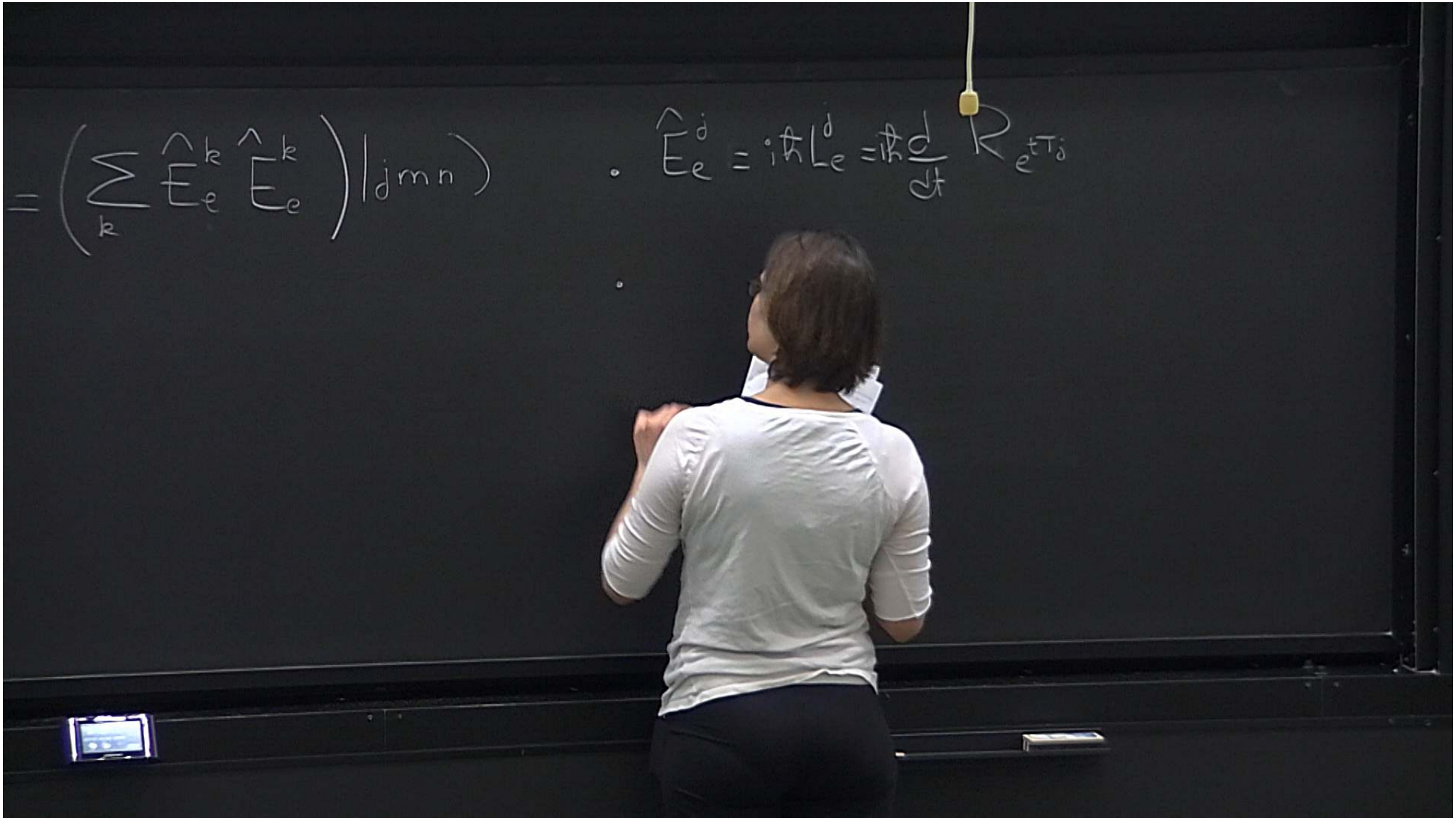
length of a curve: $\int_{\gamma^*} \delta^{ij} e^i_a e^j_b \dot{\gamma}^{*a} \dot{\gamma}^{*b} \, db$

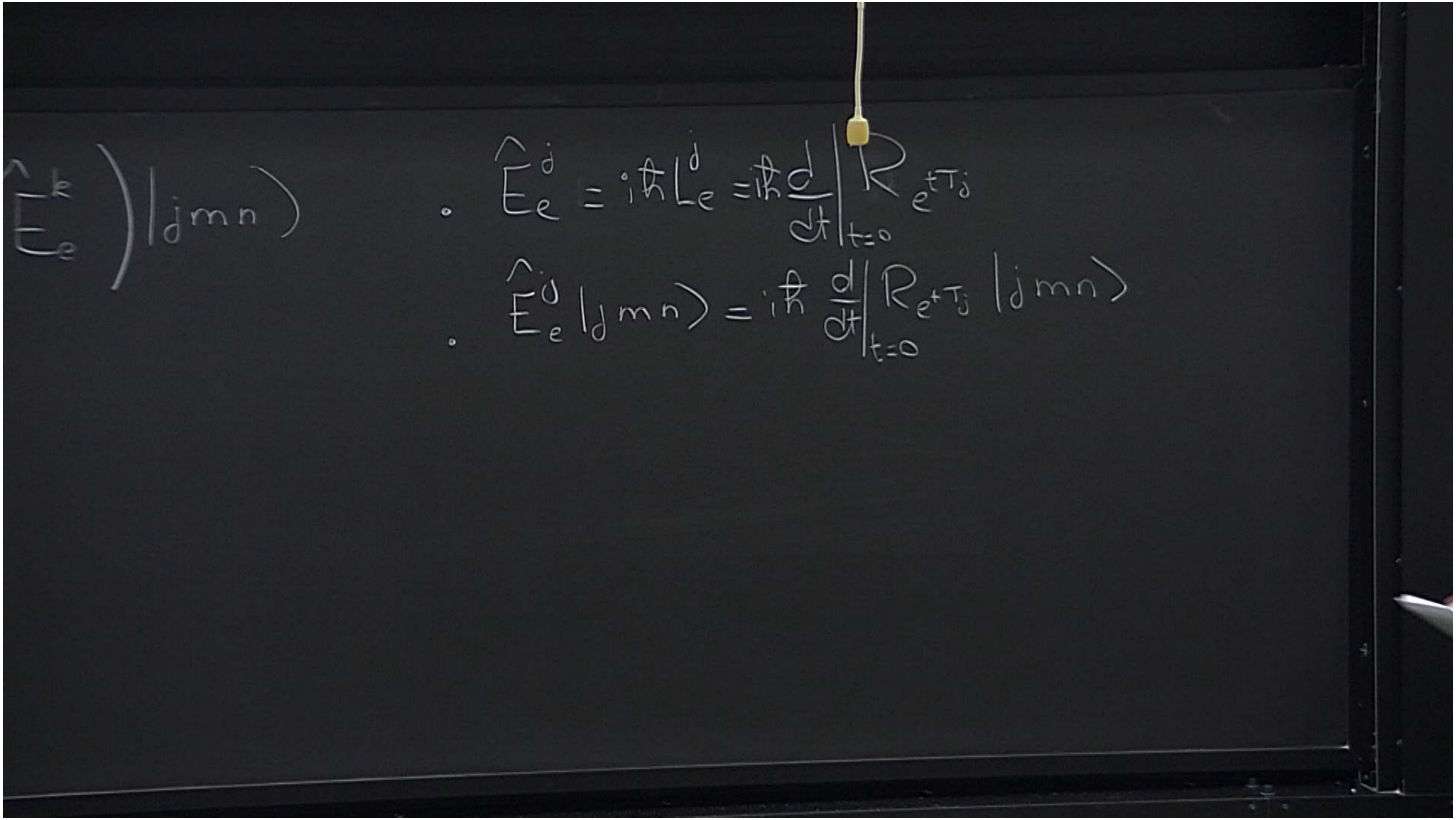
$\sim \sum_{n=0}^{\infty} \sqrt{\sum_k E^k_{e_n} E^k_{e_n}}$

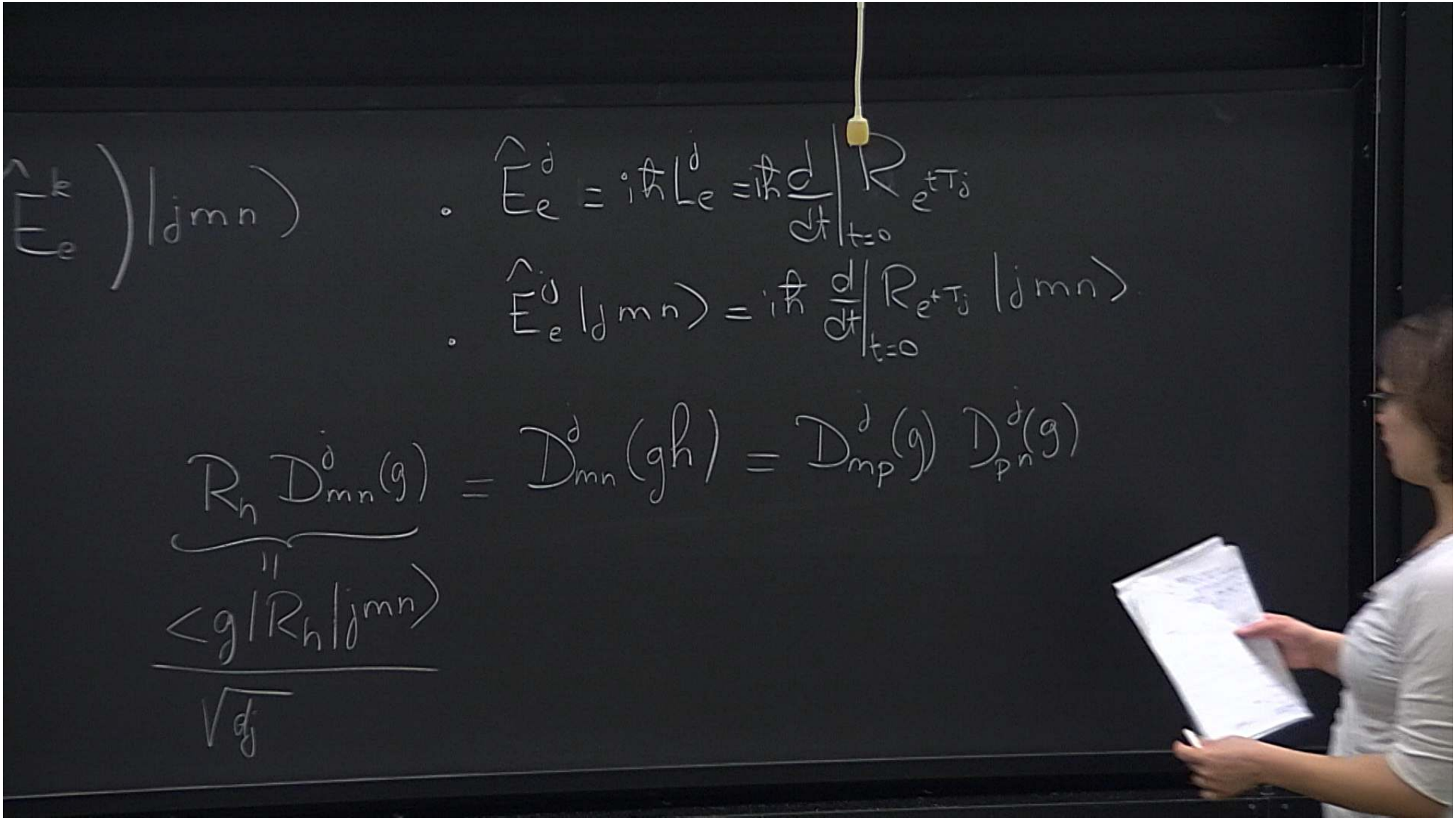


$\hat{l}^2_{e^*} | j,m,n)$


$\hat{l}^2_{e^*} \,|\, j,m,n\rangle := \left(\sum_k \hat{E}^k_e \hat{E}^k_e \right) |j\,m\,n\rangle$
e^*
e
$\gamma^{*a}\gamma^{*b}\,db$
$\sum E^l_{e_n} E^k_{e_n}$


$= \left(\sum_k \hat{E}_e^k \hat{E}_e^k \right) |jmn)$
$\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt} R_{e^t T_j}$


$\hat{E}_e^k)|jmn\rangle$
$\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}}$
$\hat{E}_e^j |jmn\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}} |jmn\rangle$


$\hat{E}_e^k \big) |jmn\rangle$
$\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}}$
$\hat{E}_e^j |jmn\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}} |jmn\rangle$
$R_h D^j_{mn}(g) = D^j_{mn}(gh) = D^j_{mp}(g)\, D^j_{pn}(h)$
$\frac{\langle g | R_h | jmn \rangle}{\sqrt{d_j}}$

$\hat{E}_e^k \big) |j m n\rangle$

- $\hat{E}_e^j = i\hbar L_e^j = i\hbar \left.\frac{d}{dt}\right|_{t=0} R_{e^{tT_j}}$
- $\hat{E}_e^j |j m n\rangle = i\hbar \left.\frac{d}{dt}\right|_{t=0} R_{e^{tT_j}} |j m n\rangle$

$$\underbrace{R_h D^j_{mn}(g)}_{\| \; \langle g | R_h | j m n \rangle} = D^j_{mn}(gh) = D^j_{mp}(g)\, D^j_{pn}(g)$$

$$= \sum_p \frac{1}{\sqrt{d_j}} \langle g | j m p \rangle D^j_{pn}(g)$$

$$\frac{\langle g | R_h | j m n \rangle}{\sqrt{d_j}}$$

$$\hat{E}_e^k \Big) |jmn\rangle$$

- $\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}}$
- $\hat{E}_e^j |jmn\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}} |jmn\rangle$

$$\underbrace{R_h D^j_{mn}(g)}_{\substack{\parallel \\ \frac{\langle g|R_h|jmn\rangle}{\sqrt{d_j}}}} = D^j_{mn}(gh) = D^j_{mp}(g)\, D^j_{pn}(h)$$

$$= \sum_p \frac{1}{\sqrt{d_j}} \langle g|jmp\rangle D^j_{pn}(h)$$

$\hat{E}_e^k \big) |jmn\rangle$

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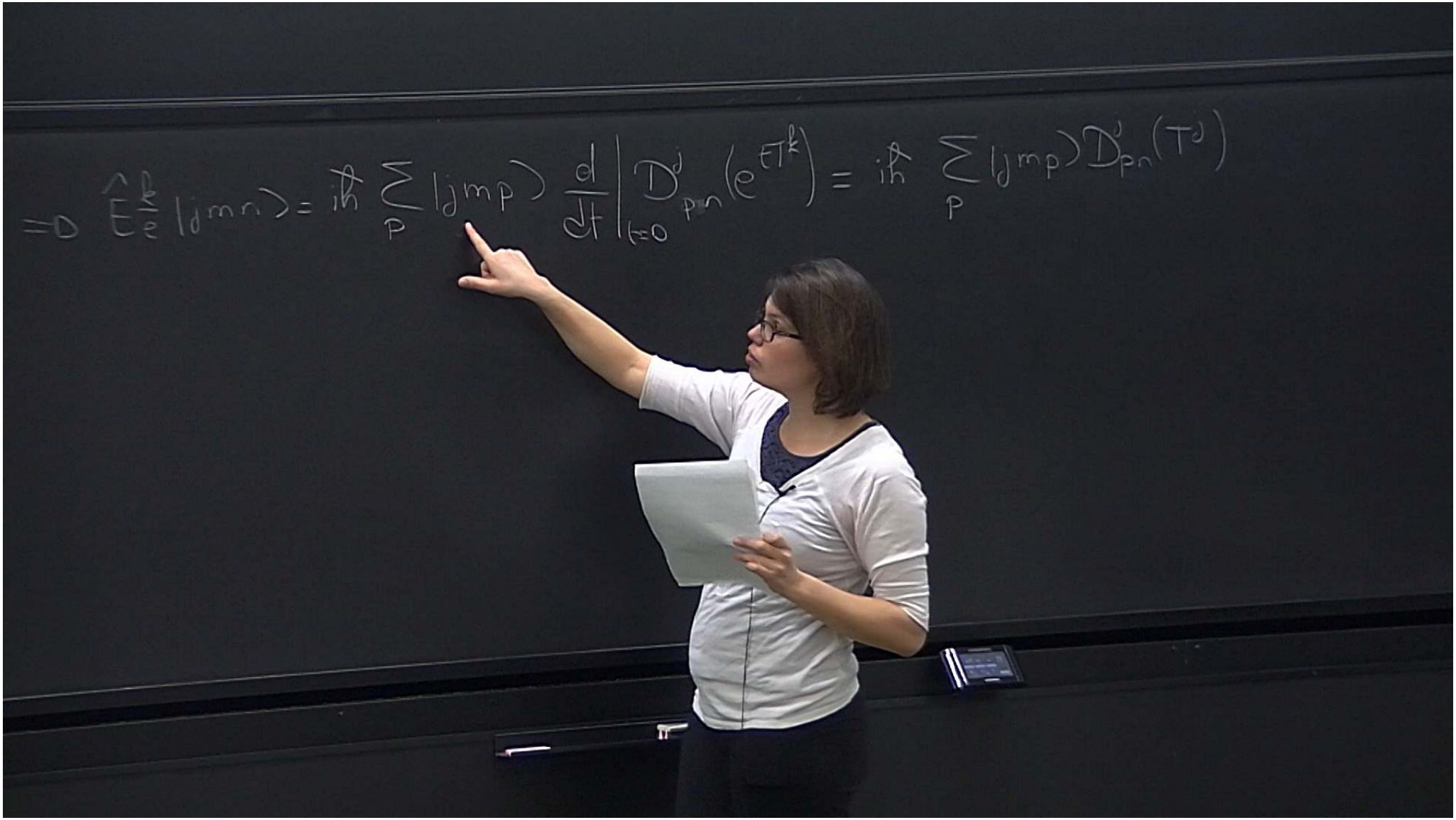
$$R_h D^j_{mn}(g) = D^j_{mn}(gh) = D^j_{mp}(g)\, D^j_{pn}(h)$$

$$\underbrace{R_h D^j_{mn}(g)}_{\langle g | R_h | jmn\rangle} = \sum_p \frac{1}{\sqrt{d_j}} \langle g | jmp\rangle D^j_{pn}(h)$$

$$\frac{\langle g | R_h | jmn\rangle}{\sqrt{d_j}}$$

$$\Rightarrow R_h |jmn\rangle = \sum_p D^j_{pn}(h) |jmp\rangle$$

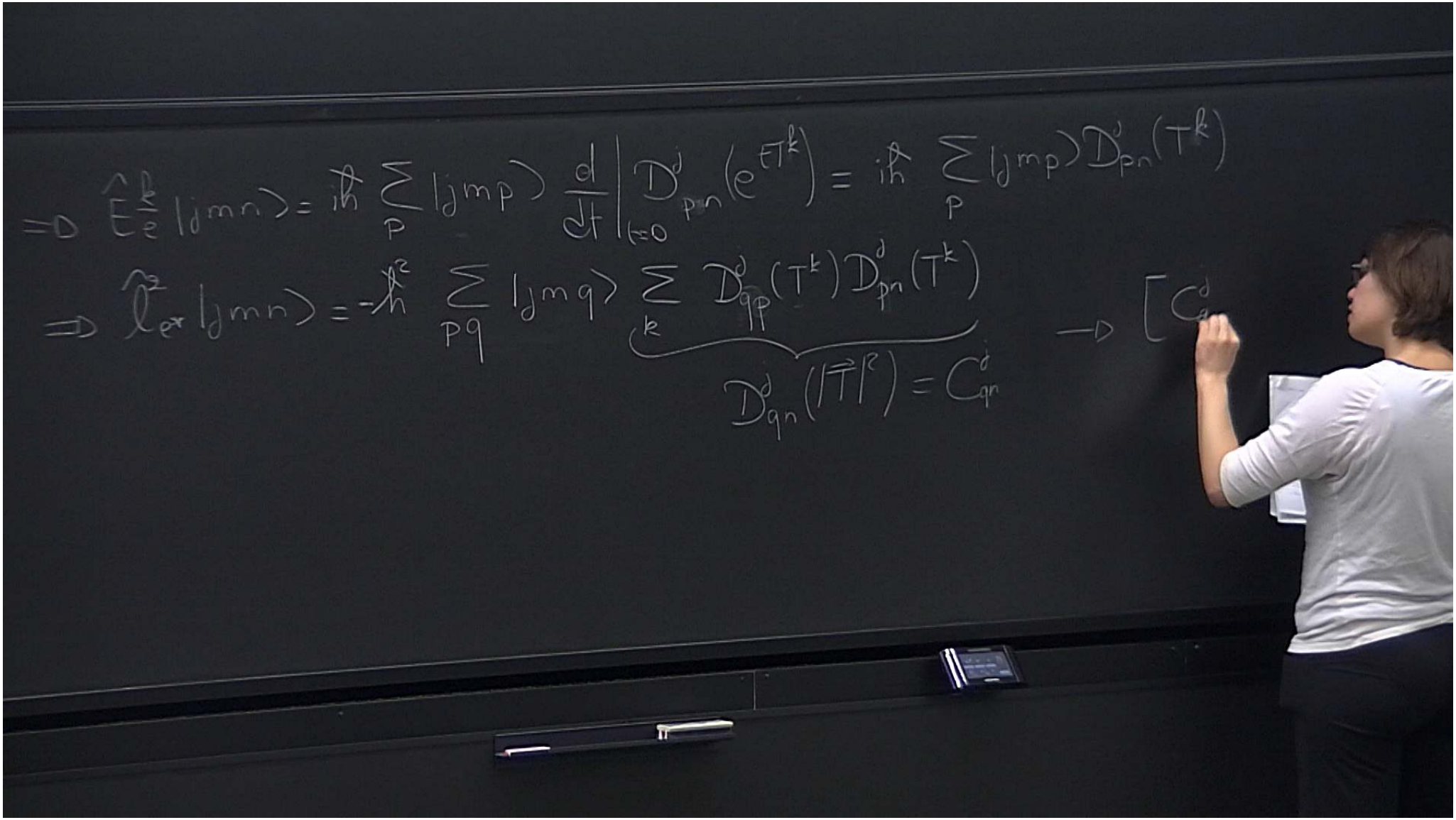

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tTk}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^j)$

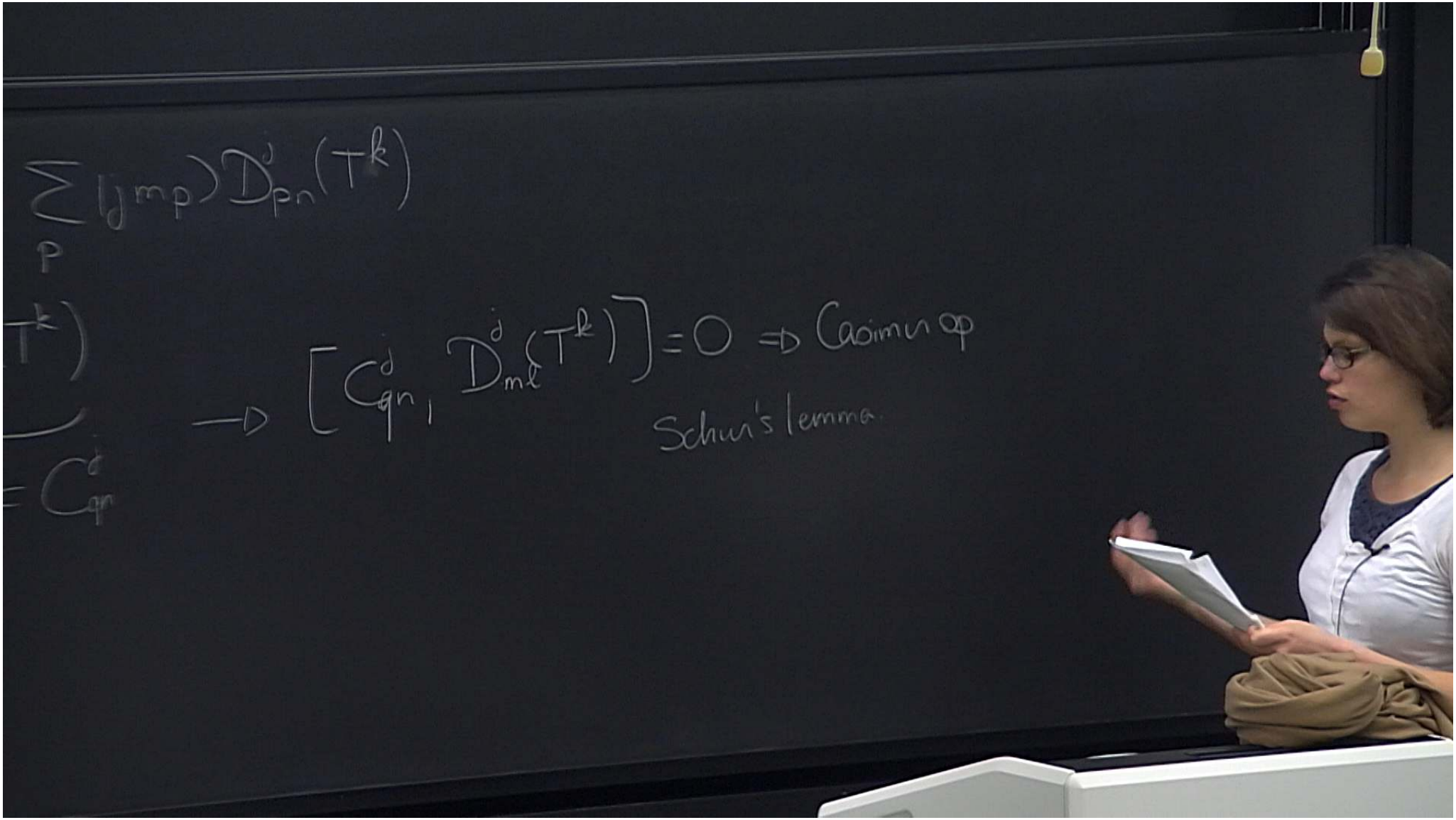
$$\Rightarrow \hat{E}^k_e |jmn\rangle = i\hbar \sum_p |jmp\rangle \left.\frac{d}{dt}\right|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^k)$$

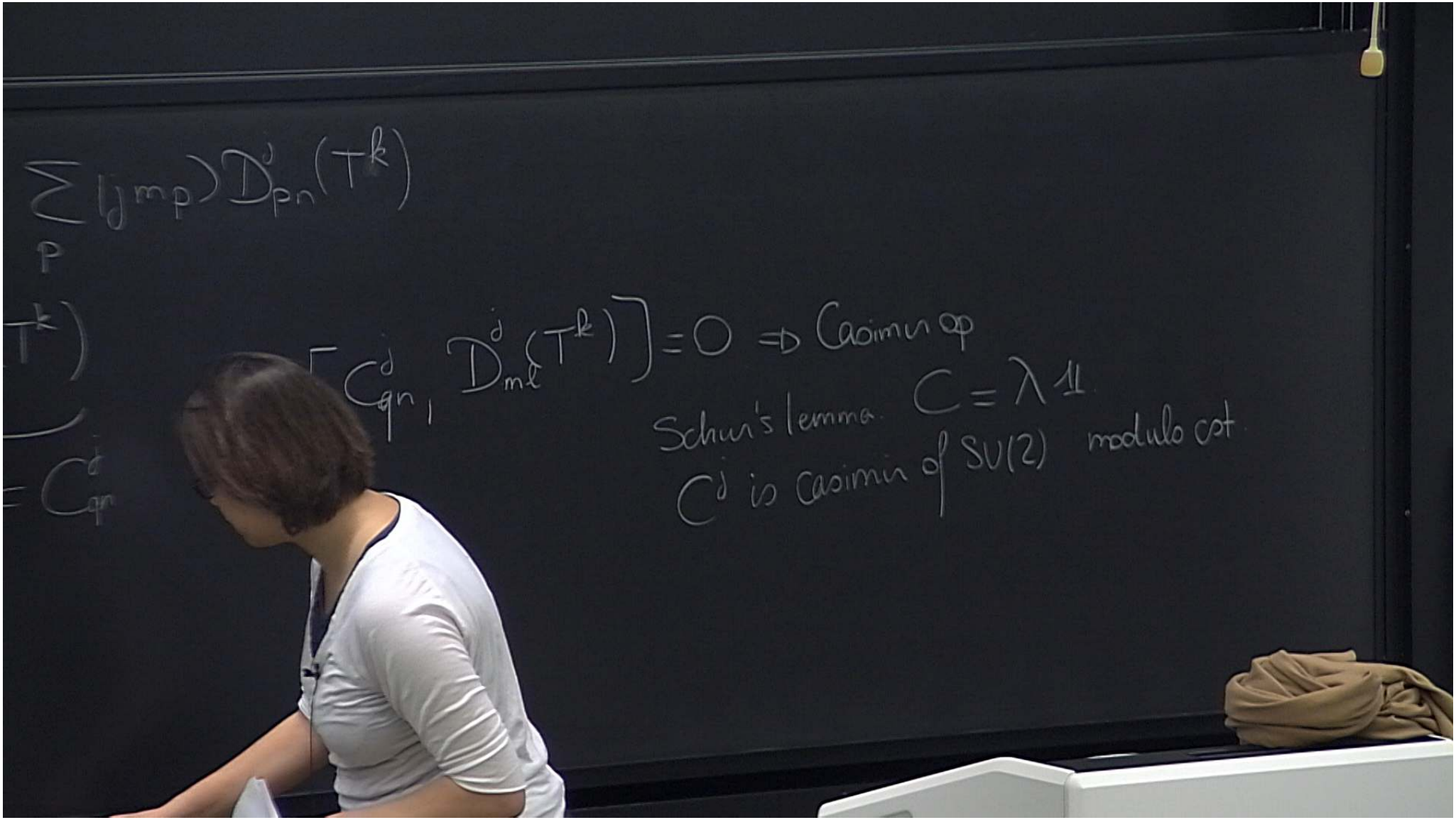
$$\Rightarrow \hat{l}^2_{er} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$$

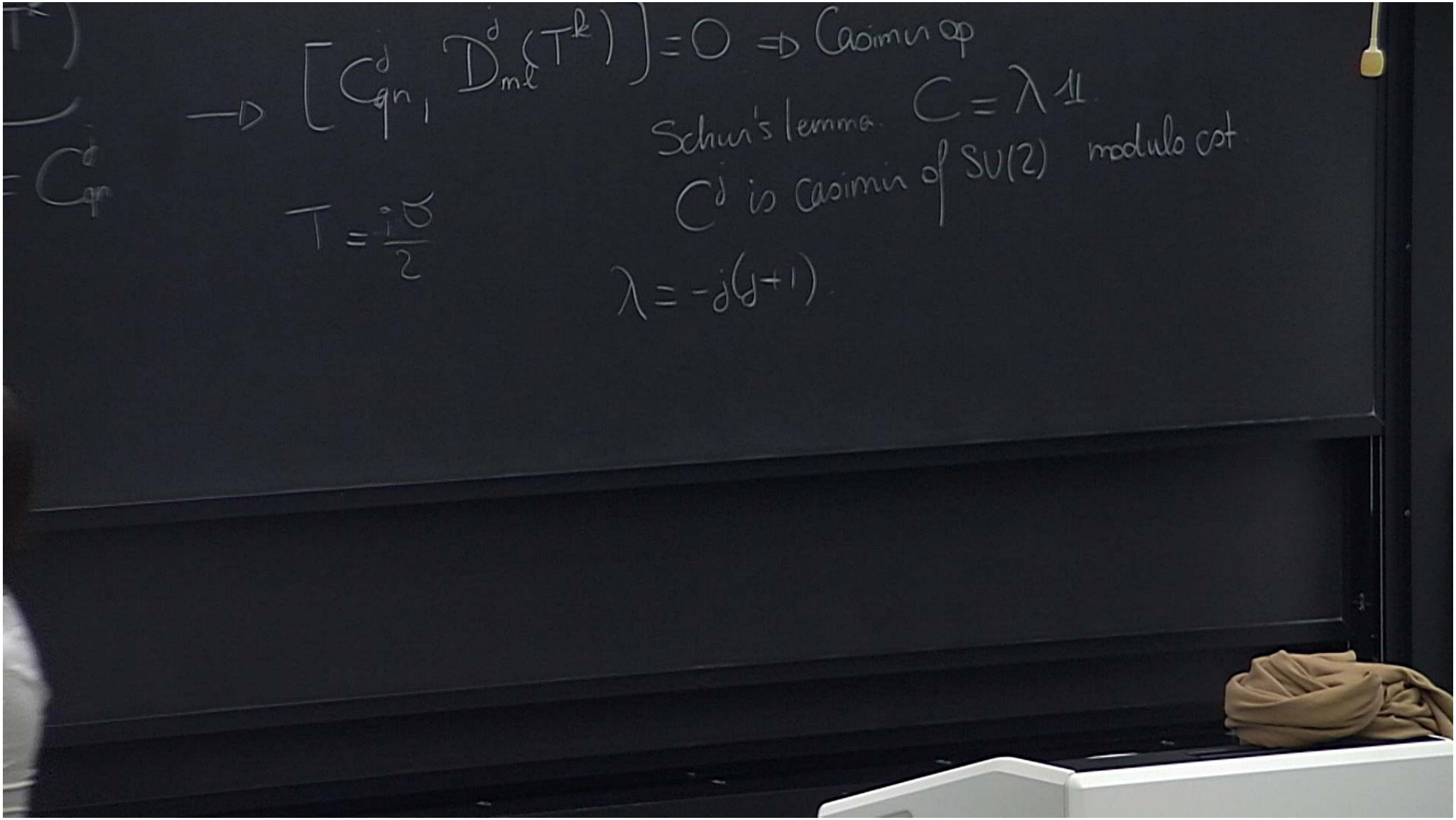
$$\Rightarrow \hat{E}^{k}_{e} |jmn\rangle = i\hbar \sum_{p} |jmp\rangle \left.\frac{d}{dt}\right|_{t=0} D^{j}_{pn}\left(e^{tT^k}\right) = i\hbar \sum_{p} |jmp\rangle D^{j}_{pn}(T^k)$$

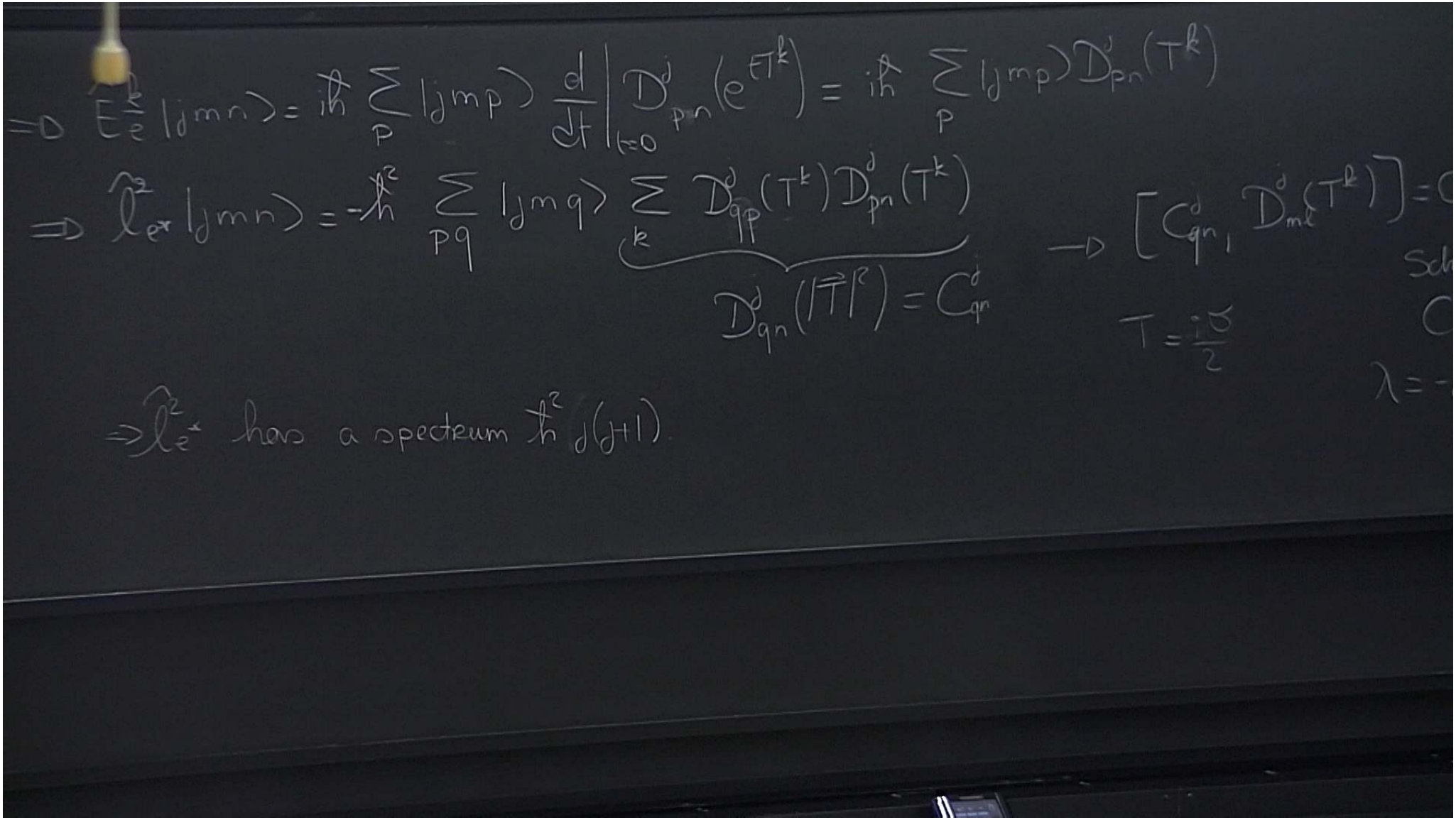
$$\Rightarrow \hat{l}^{2}_{er} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \underbrace{\sum_{k} D^{j}_{qp}(T^k) D^{j}_{pn}(T^k)}_{D^{j}_{qn}(|\vec{T}|^2)}$$

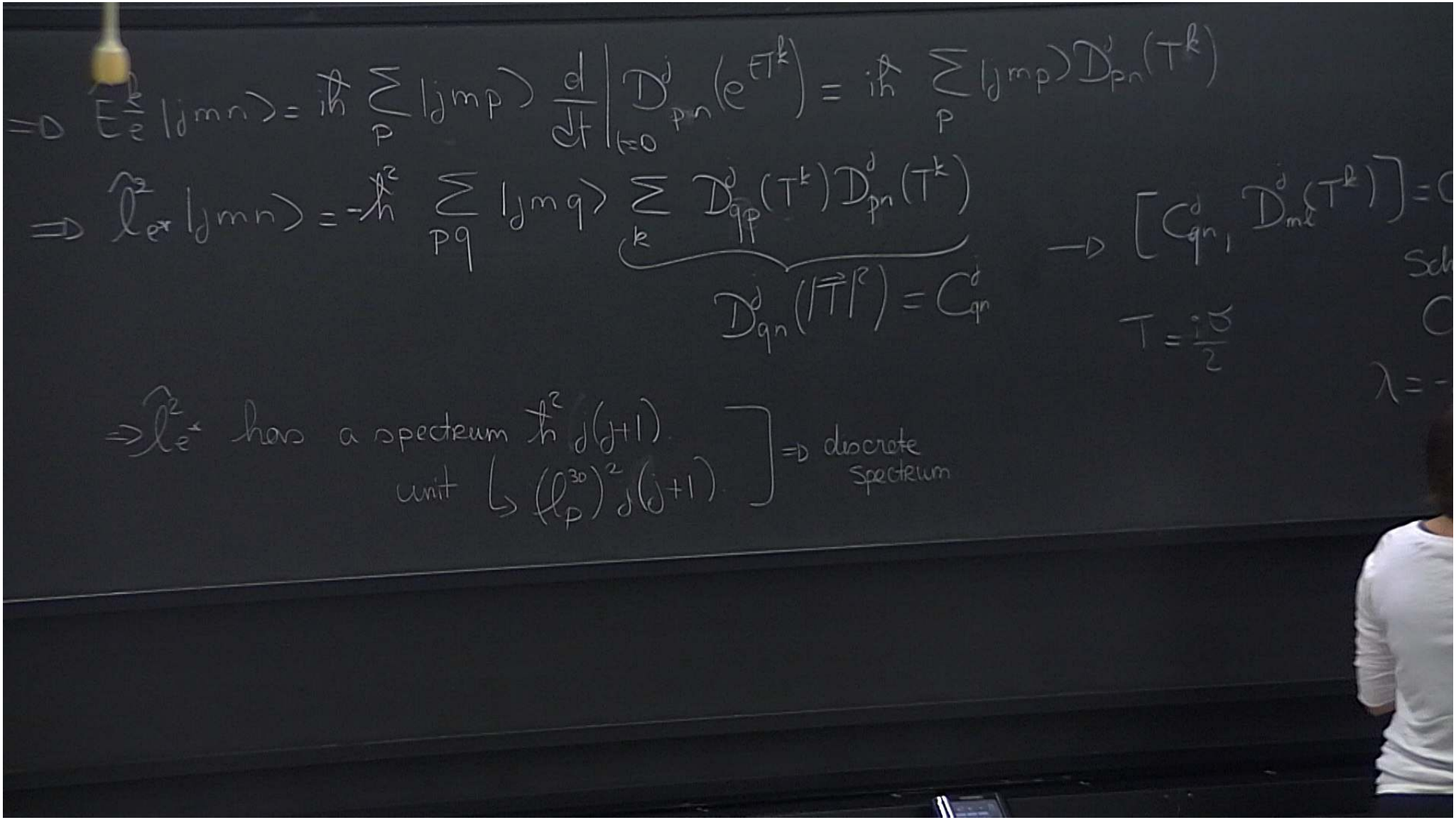

$\Rightarrow \hat{E}^k_e |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^k)$
$\Rightarrow \hat{l}^2_{er} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \underbrace{\sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)}_{D^j_{qn}(|\vec{T}|^2) = C^j_{qn}}$
$\rightarrow [C^j_$

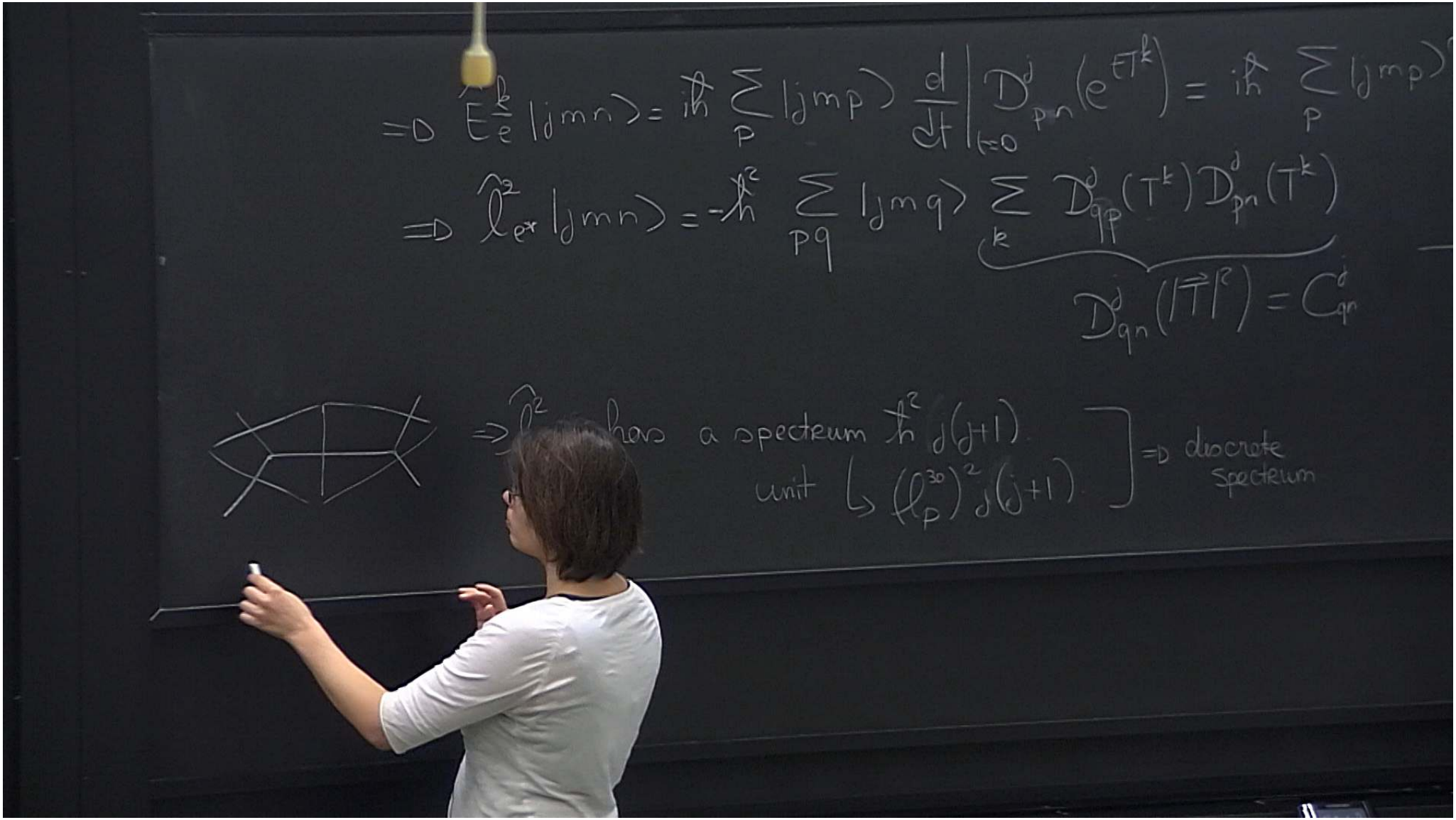

$\sum_p |ijmp\rangle D^j_{pn}(T^k)$
$T^k)$
$= C^d_{qr}$
$\rightarrow [C^d_{qn}, D^d_{ml}(T^k)] = 0 \Rightarrow$ Casimir op
Schur's lemma.

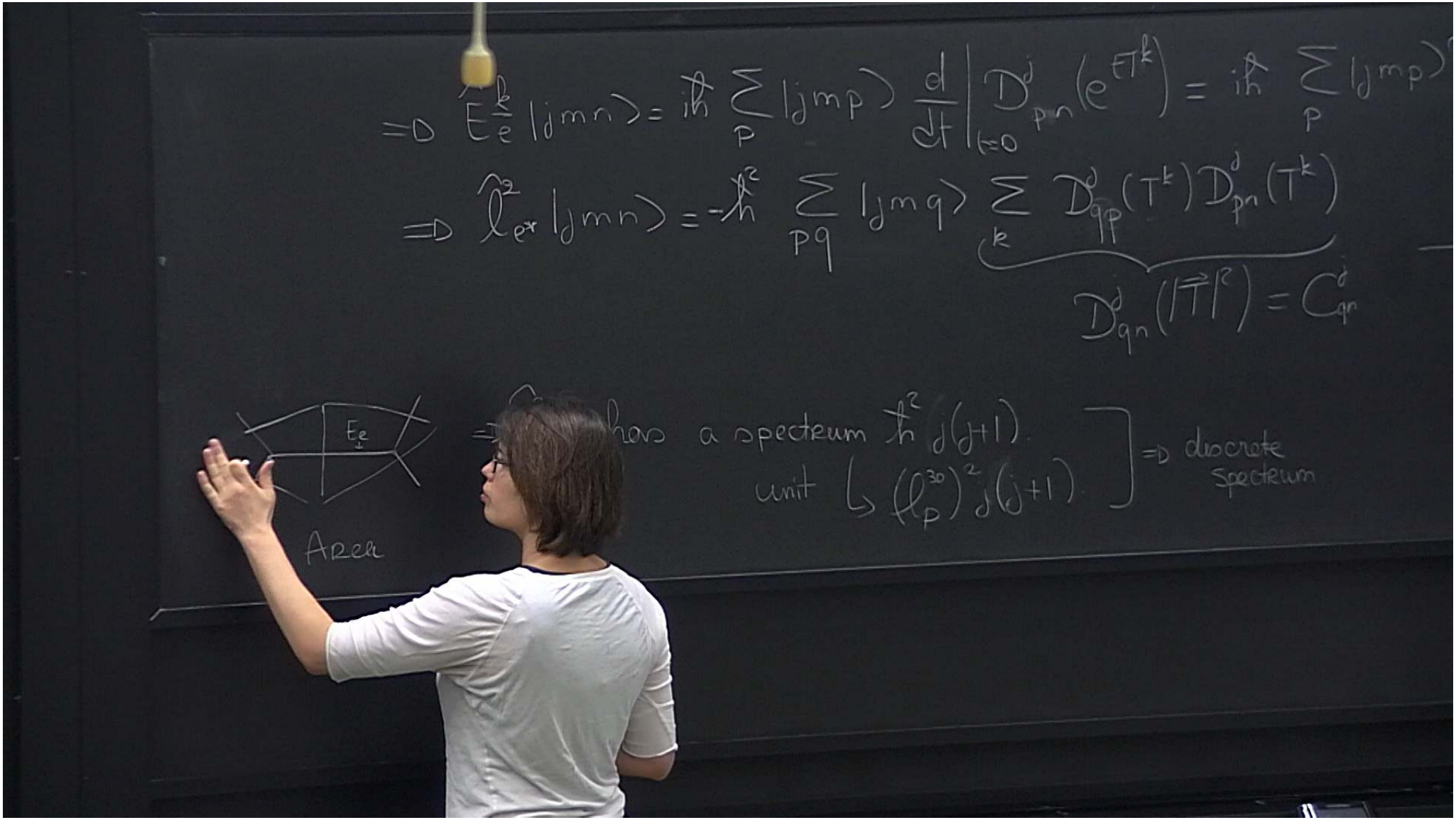

$\sum_p |jmp\rangle D^j_{pn}(T^k)$
$= C^j_{qn}$
$\left[C^j_{qn}, D^j_{ml}(T^k) \right] = 0 \Rightarrow$ Casimir op
Schur's lemma $C = \lambda \mathbb{1}$
C^j is Casimir of SU(2) modulo cst

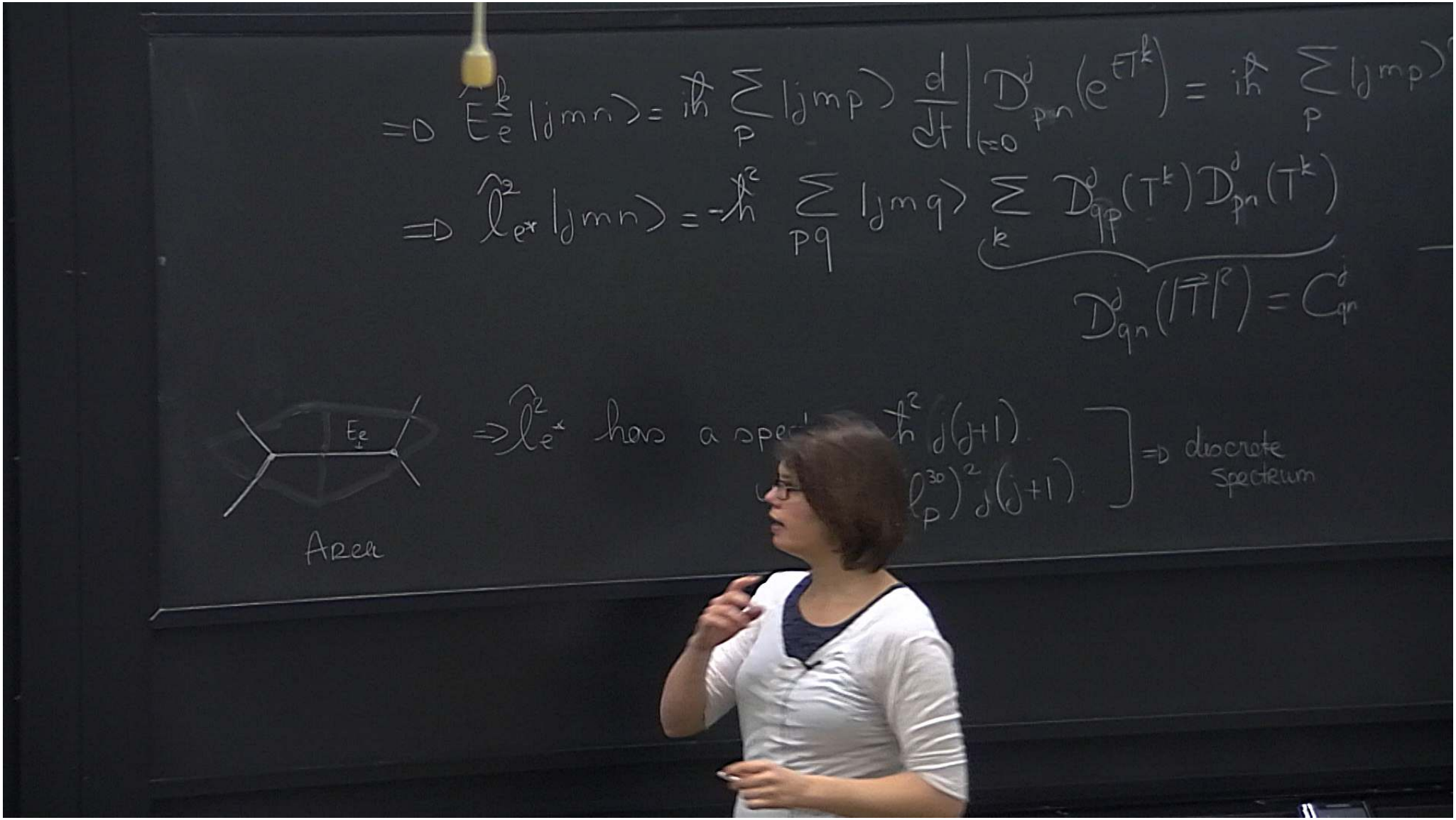

$\rightarrow [C^d_{qn}, D^d_{ml}(T^k)] = 0 \Rightarrow$ Casimir op
Schur's lemma $C = \lambda \mathbb{1}$
C^d is Casimir of SU(2) modulo cst
$T = \frac{i\sigma}{2}$
$\lambda = -d(d+1)$

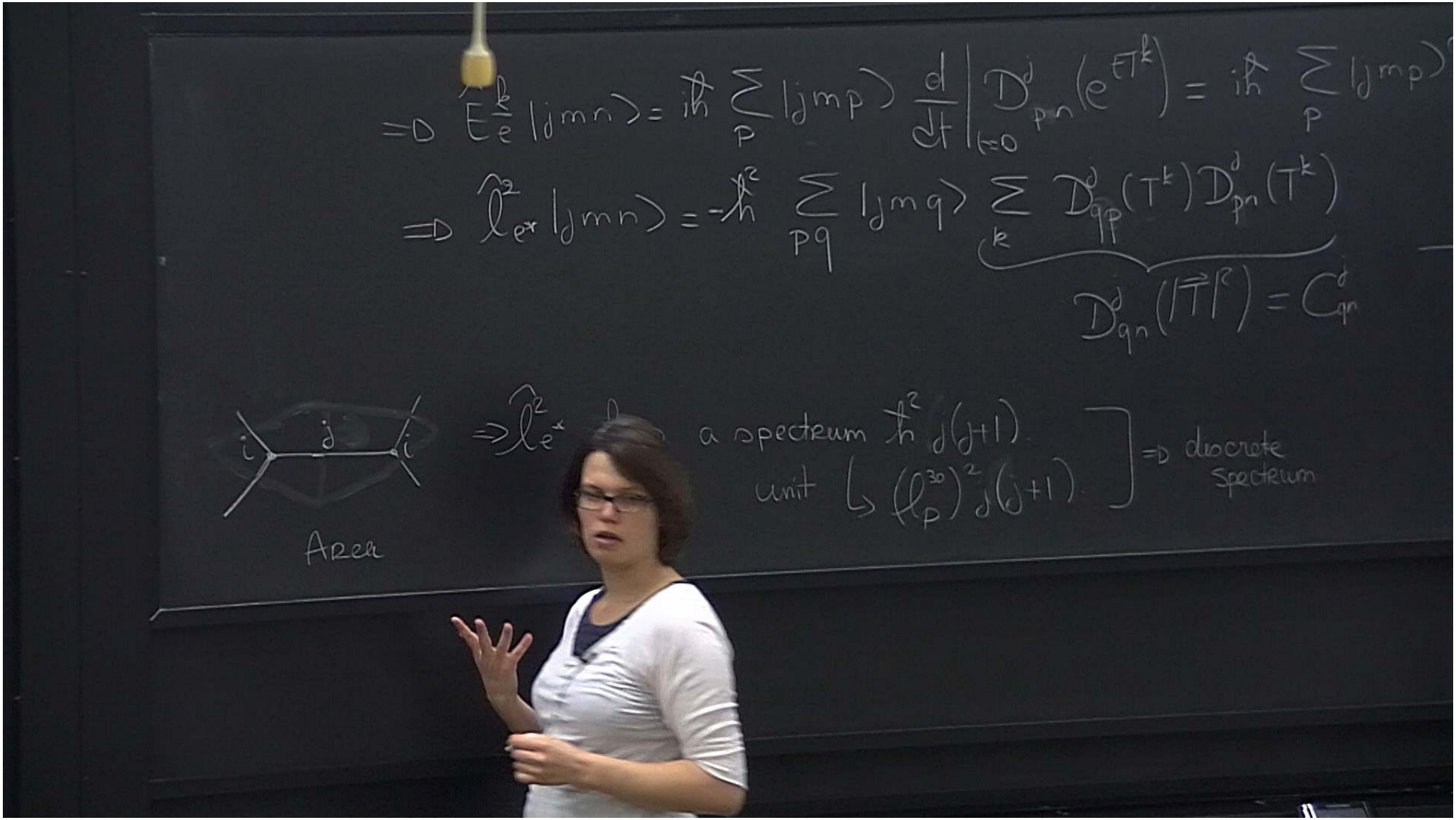

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^k)$
$\Rightarrow \hat{l}^2_{ex} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
$\rightarrow [C^j_{qn}, D^j_{mt}(T^k)] = 0$
$T = \frac{i\sigma}{2}$
$\Rightarrow \hat{l}^2_{ex}$ has a spectrum $\hbar^2 j(j+1)$

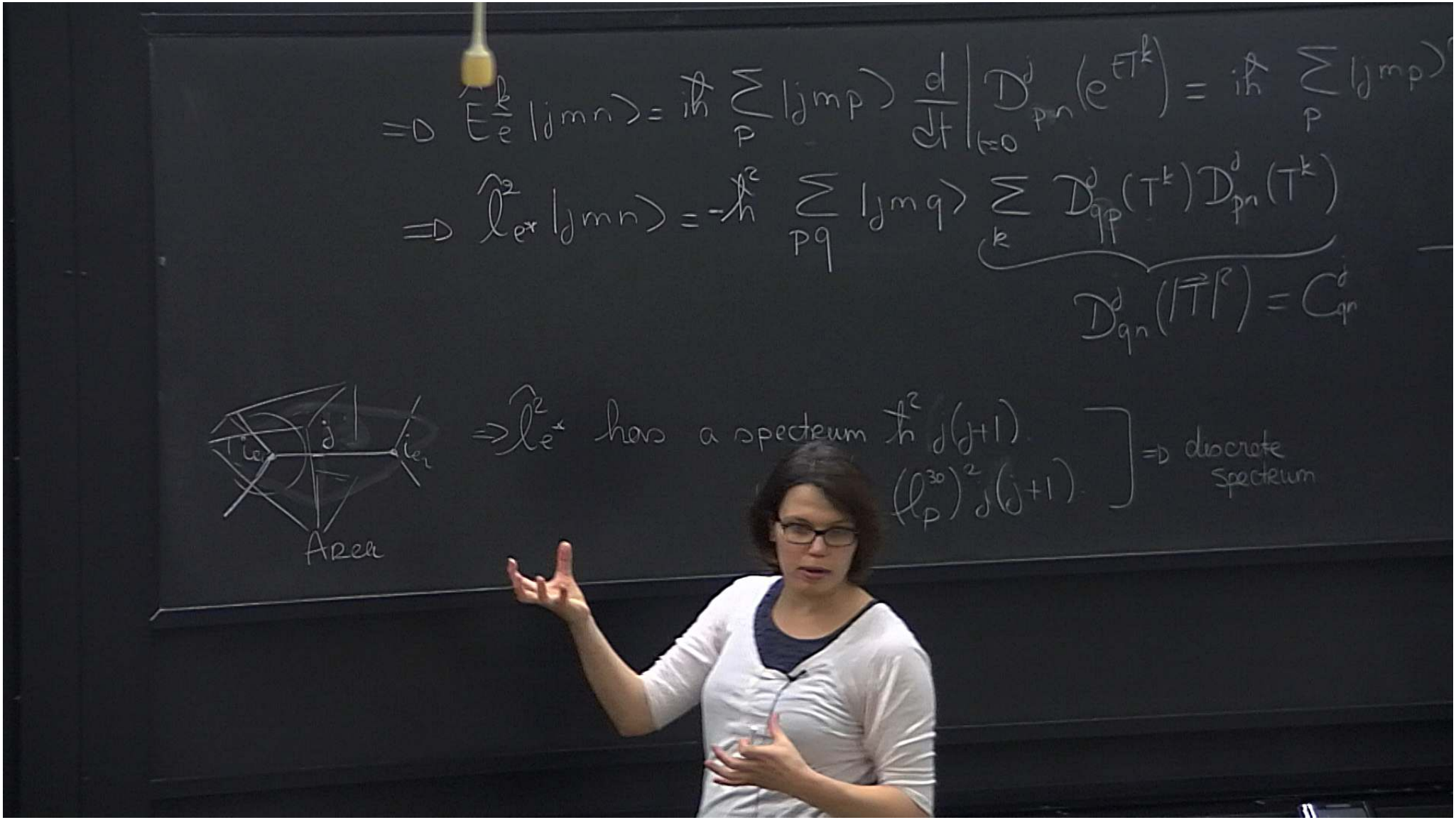

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^k)$
$\Rightarrow \hat{l}^2_{e^k} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
$\rightarrow [C^j_{qn}, D^j_{mt}(T^k)] = 0$
$T = \frac{i\sigma}{2}$
$\Rightarrow \hat{l}^2_{e^k}$ have a spectrum $\hbar^2 j(j+1)$
unit $\hookrightarrow (l_p^{3D})^2 j(j+1)$ $\Rightarrow$ discrete spectrum

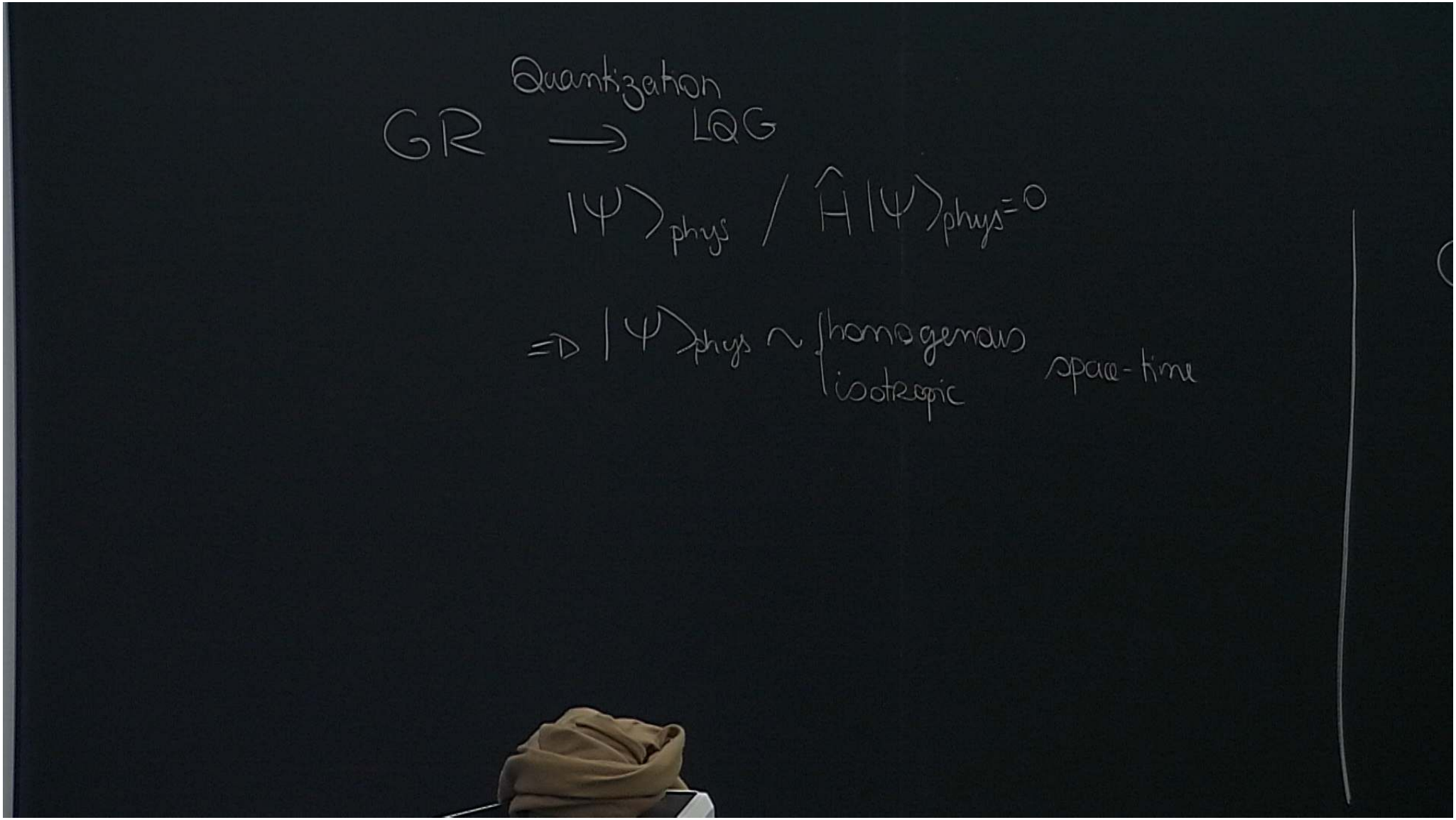

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{ET^k}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_{ext} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
$\Rightarrow \hat{l}^2$ has a spectrum $\hbar^2 j(j+1)$
unit $\hookrightarrow (l_P^{30})^2 j(j+1)$
$\Rightarrow$ discrete spectrum

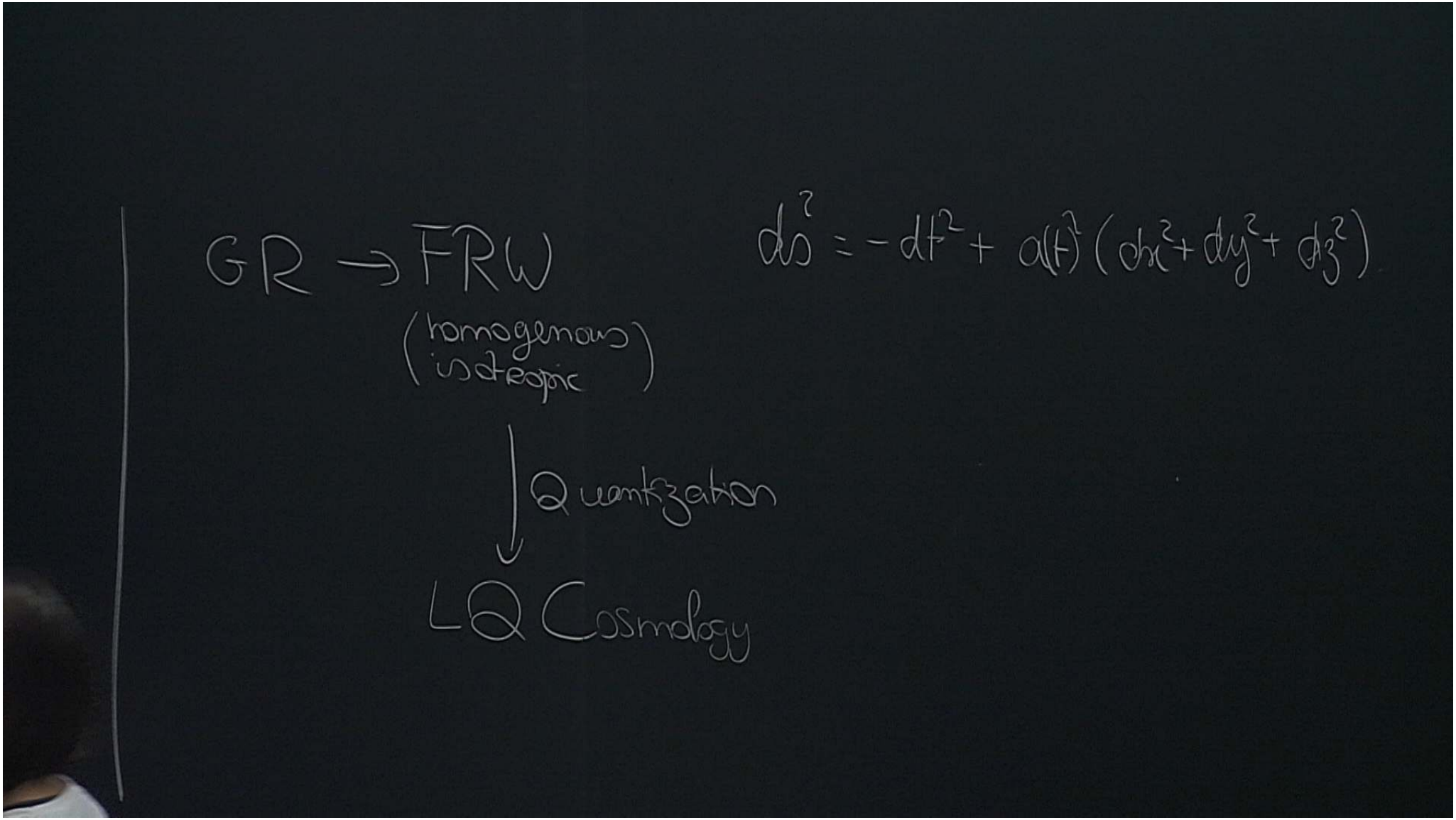

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{-iTk}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_{ext} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \underbrace{\sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)}$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
E_e
Area
have a spectrum $\hbar^2 j(j+1)$
unit $\hookrightarrow (l_p^{30})^2 j(j+1)$
$\Rightarrow$ discrete spectrum

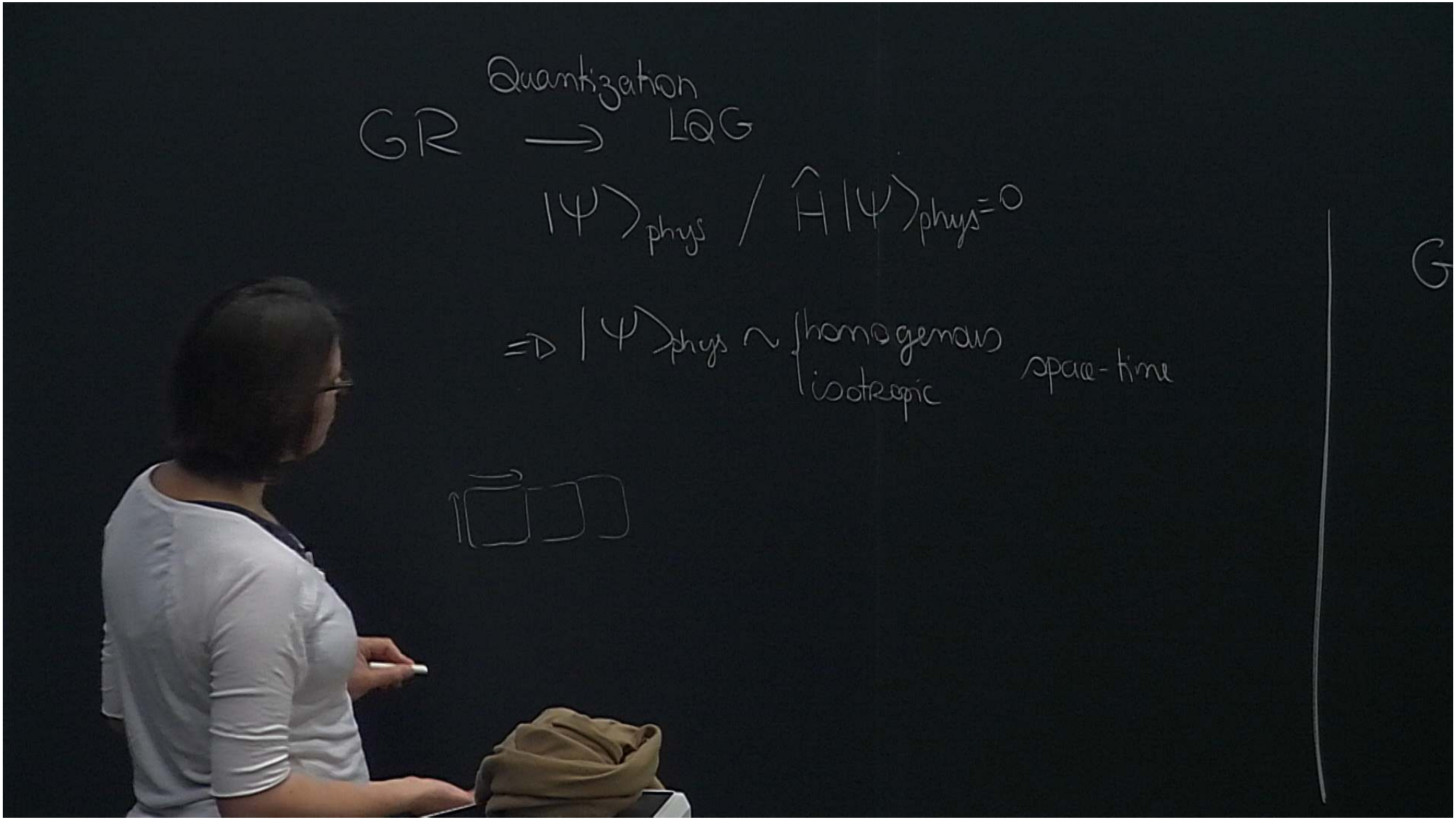

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_{e^*} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
E_e
Area
$\Rightarrow \hat{l}^2_{e^*}$ have a spe
$\hbar^2 j(j+1)$
$(l_p^{30})^2 j(j+1)$
$\Rightarrow$ discrete spectrum

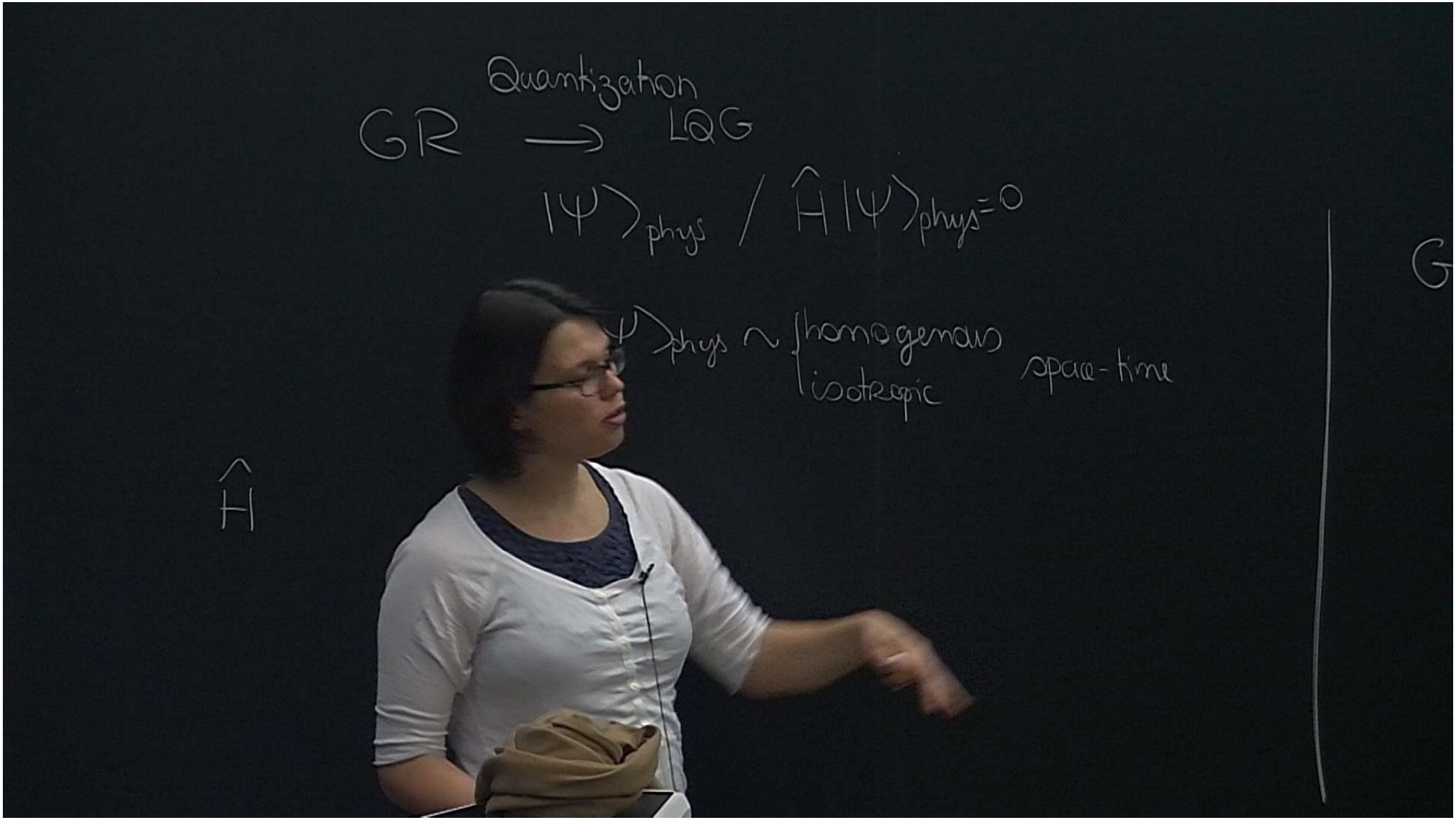

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_{e*} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
i
j
i
Area
$\Rightarrow \hat{l}^2_{e*}$
a spectrum $\hbar^2 j(j+1)$
unit $\hookrightarrow (\ell_P^{30})^2 j(j+1)$
$\Rightarrow$ discrete spectrum

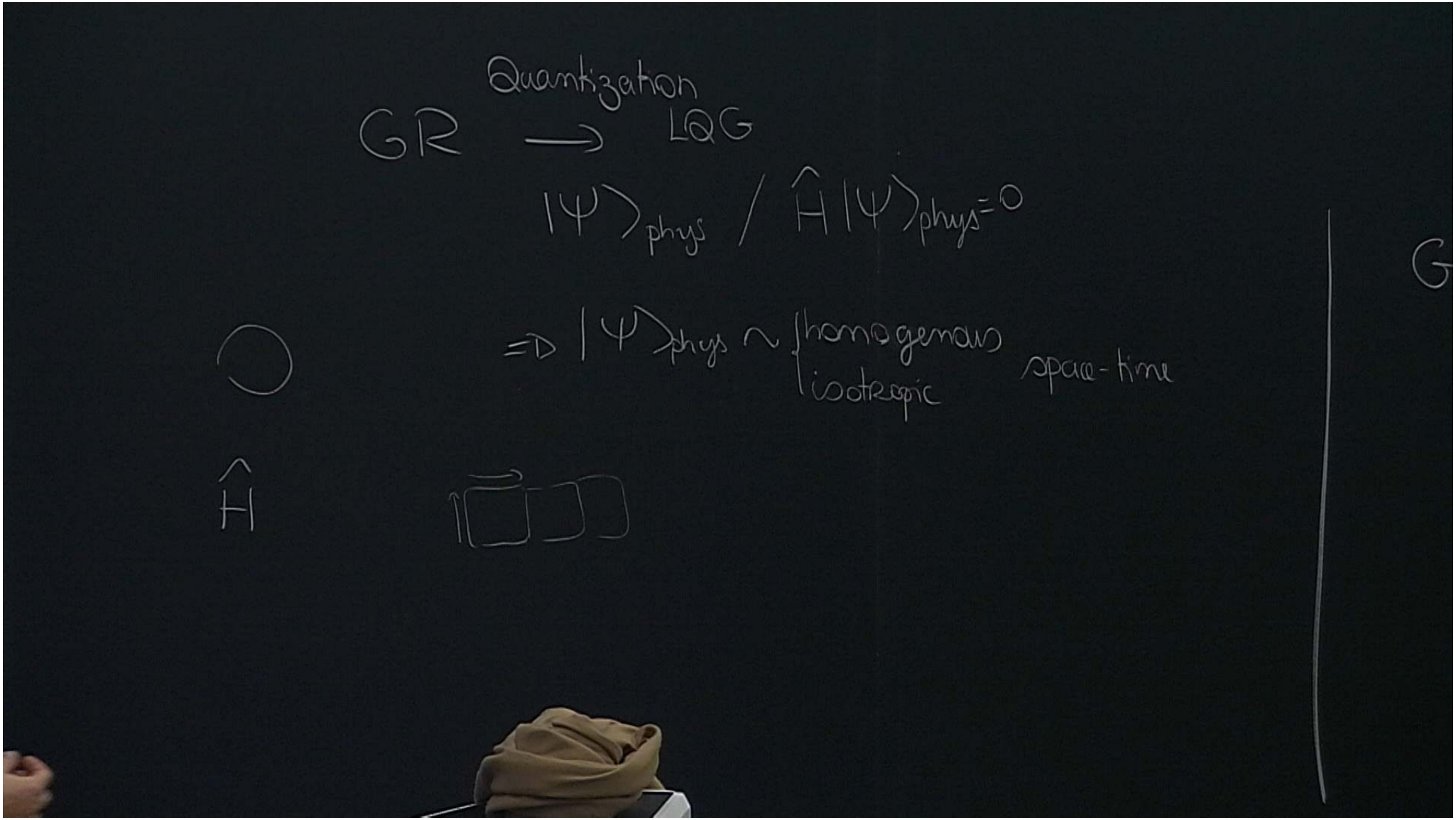

$\Rightarrow \hat{E}^k_e |jmn\rangle = i\hbar \sum_p |jmp\rangle \left.\frac{d}{dt}\right|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_{ext} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \underbrace{\sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)}$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
$\Rightarrow \hat{l}^2_{ext}$ have a spectrum $\hbar^2 j(j+1)$
$(l_P^{30})^2 j(j+1)$
$\Rightarrow$ discrete spectrum
j
Area


Quantization
GR ⟶ LQG
$|\Psi\rangle_{phys}$ / $\hat{H}|\Psi\rangle_{phys}=0$
$\Rightarrow |\Psi\rangle_{phys} \sim$ homogenous isotropic space-time


GR → FRW
(homogenous isotropic)
$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$
Quantization
LQC Cosmology


Quantization
GR → LQG
$|\psi\rangle_{phys}$ / $\hat{A}|\psi\rangle_{phys}=0$
$\Rightarrow |\psi\rangle_{phys} \sim$ {homogenous isotropic space-time
G


Quantization
GR → LQG
$|\psi\rangle_{phys}$ / $\hat{H}|\psi\rangle_{phys}=0$
$\psi\rangle_{phys}$ ~ {homogenous isotropic space-time
$\hat{H}$
G


Quantization
GR → LQG
$|\psi\rangle_{phys}$ / $\hat{H}|\psi\rangle_{phys}=0$
$\Rightarrow |\psi\rangle_{phys} \sim$ {homogenous isotropic
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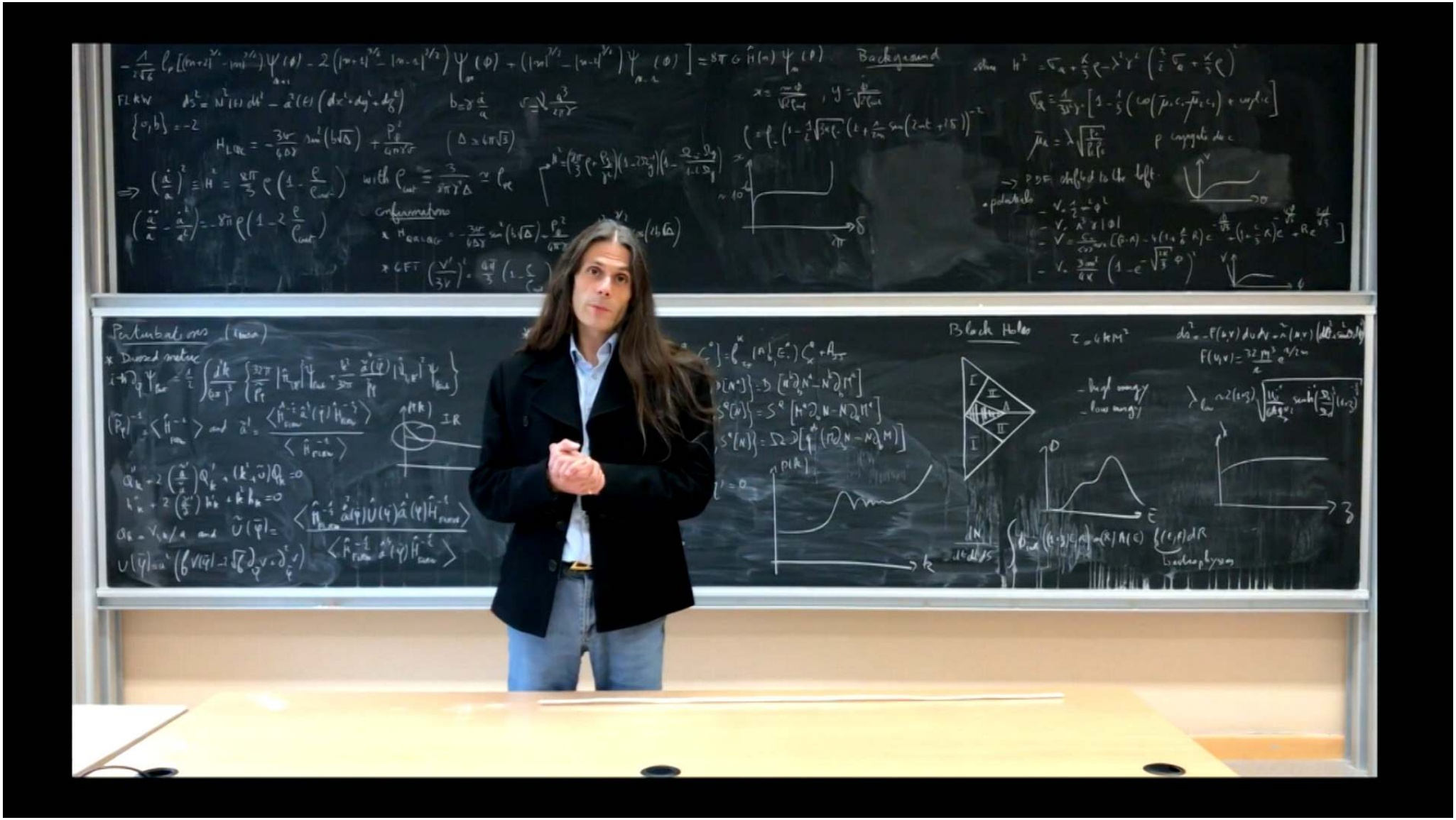
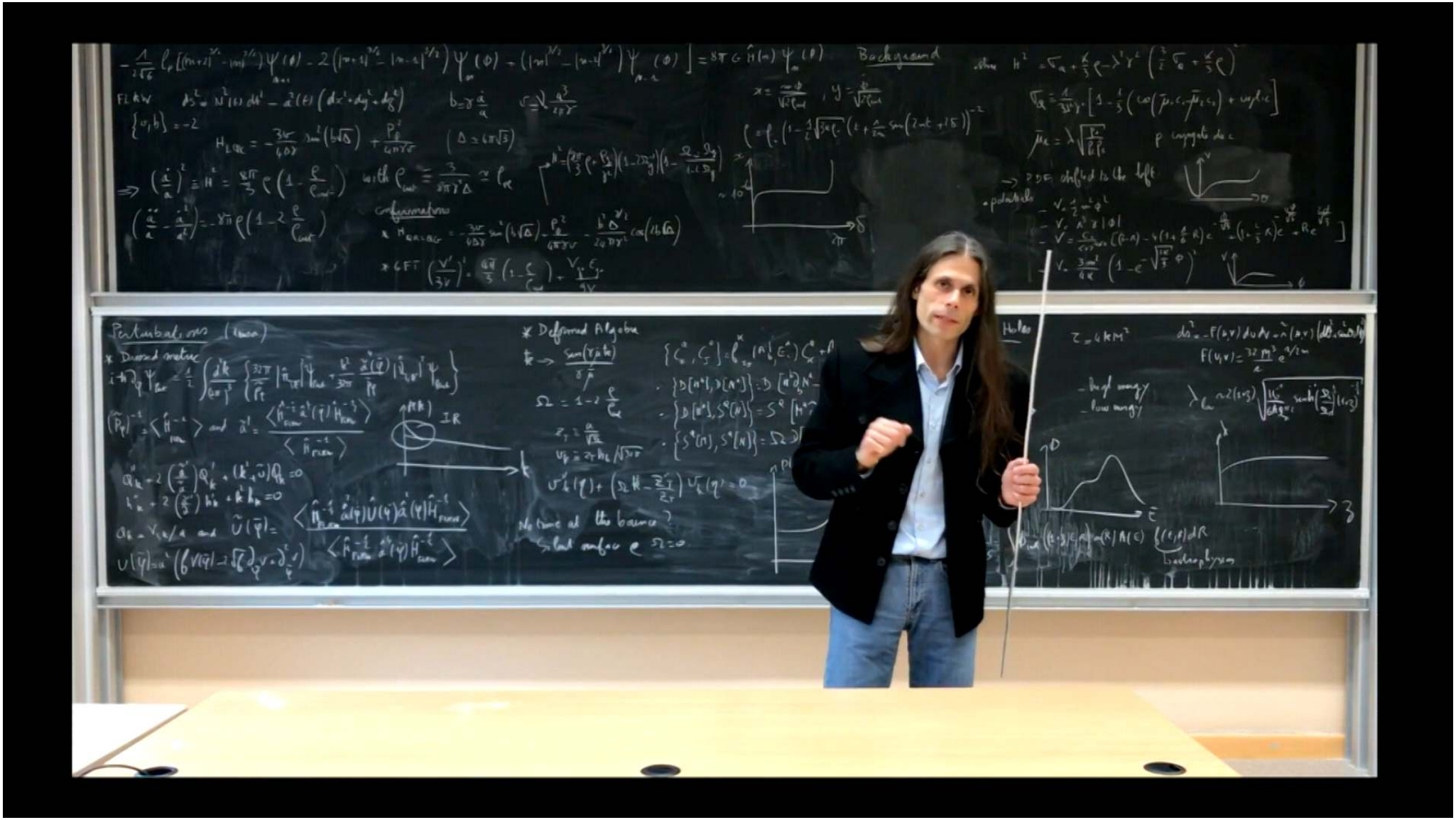
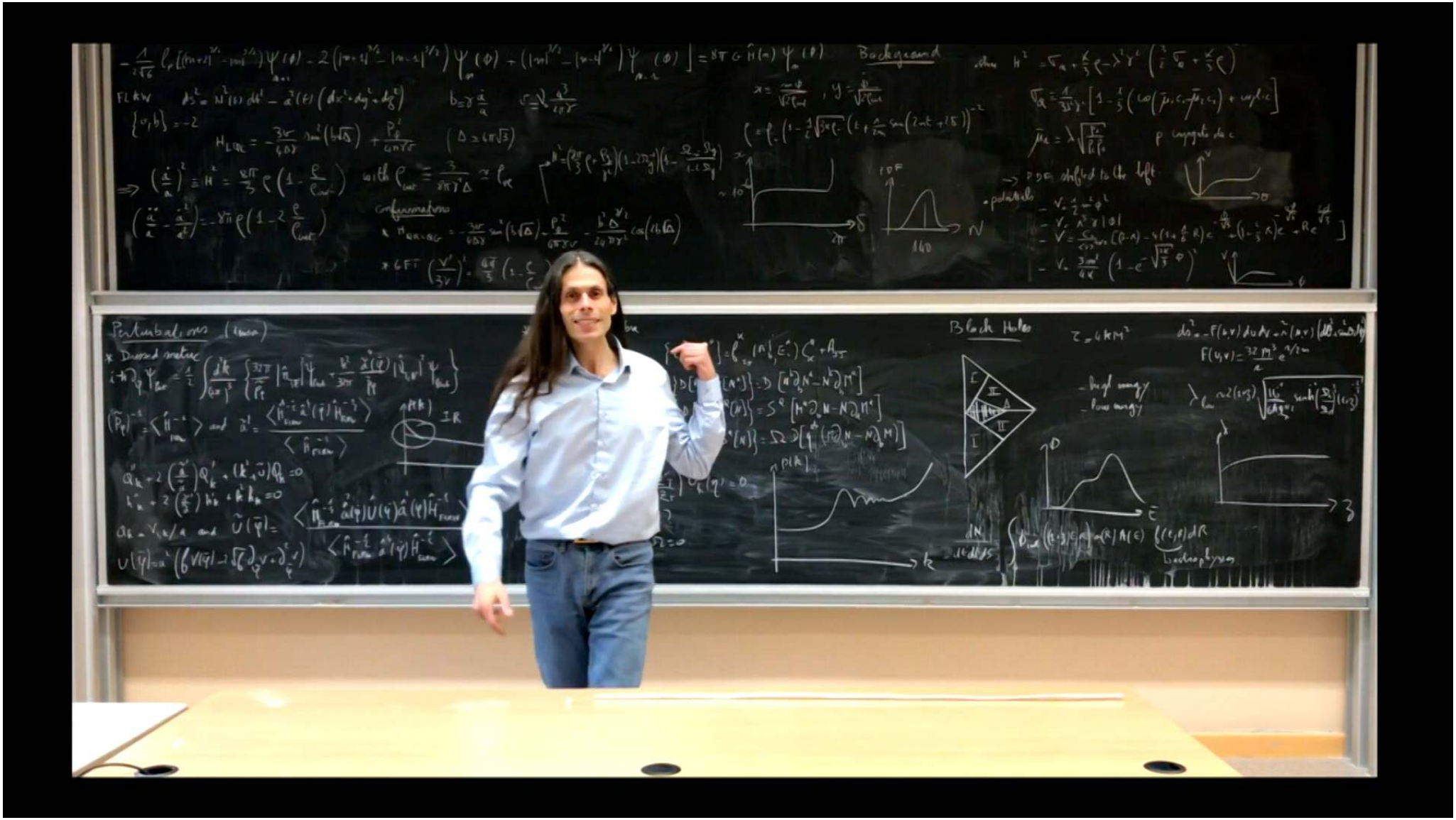
G


Perturbations
Bloch Holes


Background
Perturbations
Deformed Algebra
Holes


Background
Perturbations
Black Holes