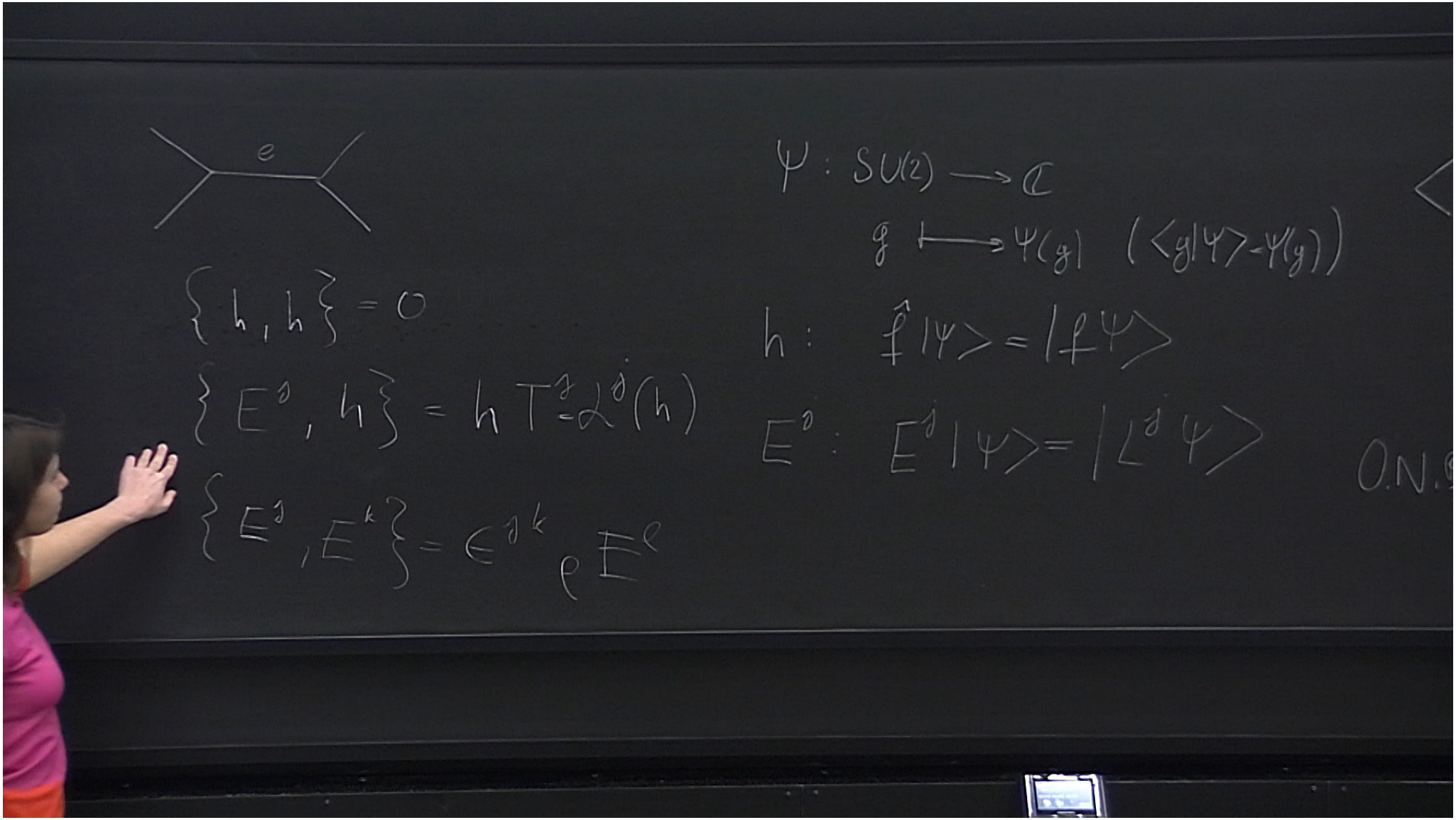
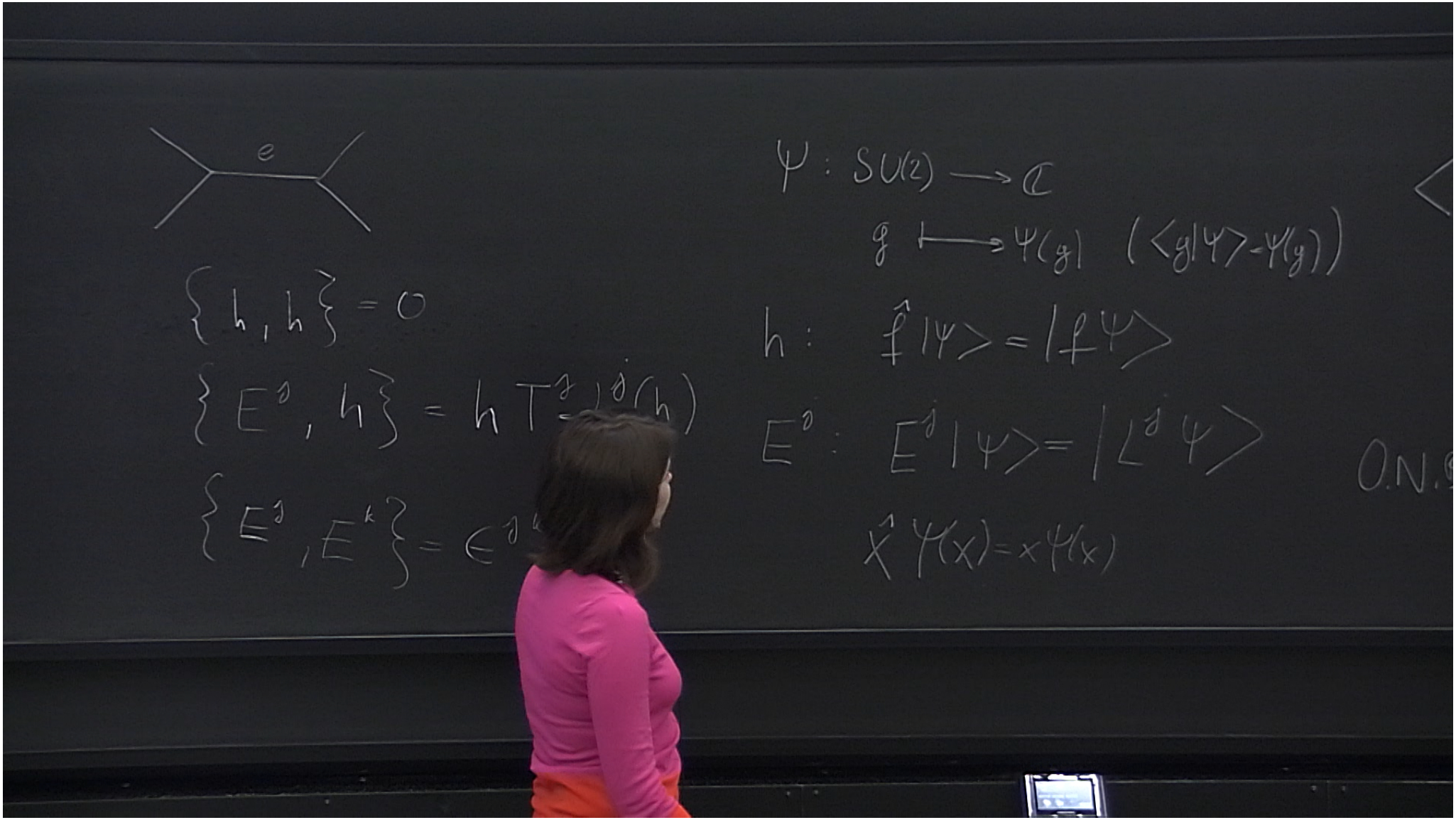
Title: PSI 2016/2017 Explorations in Quantum Gravity - Lecture 10

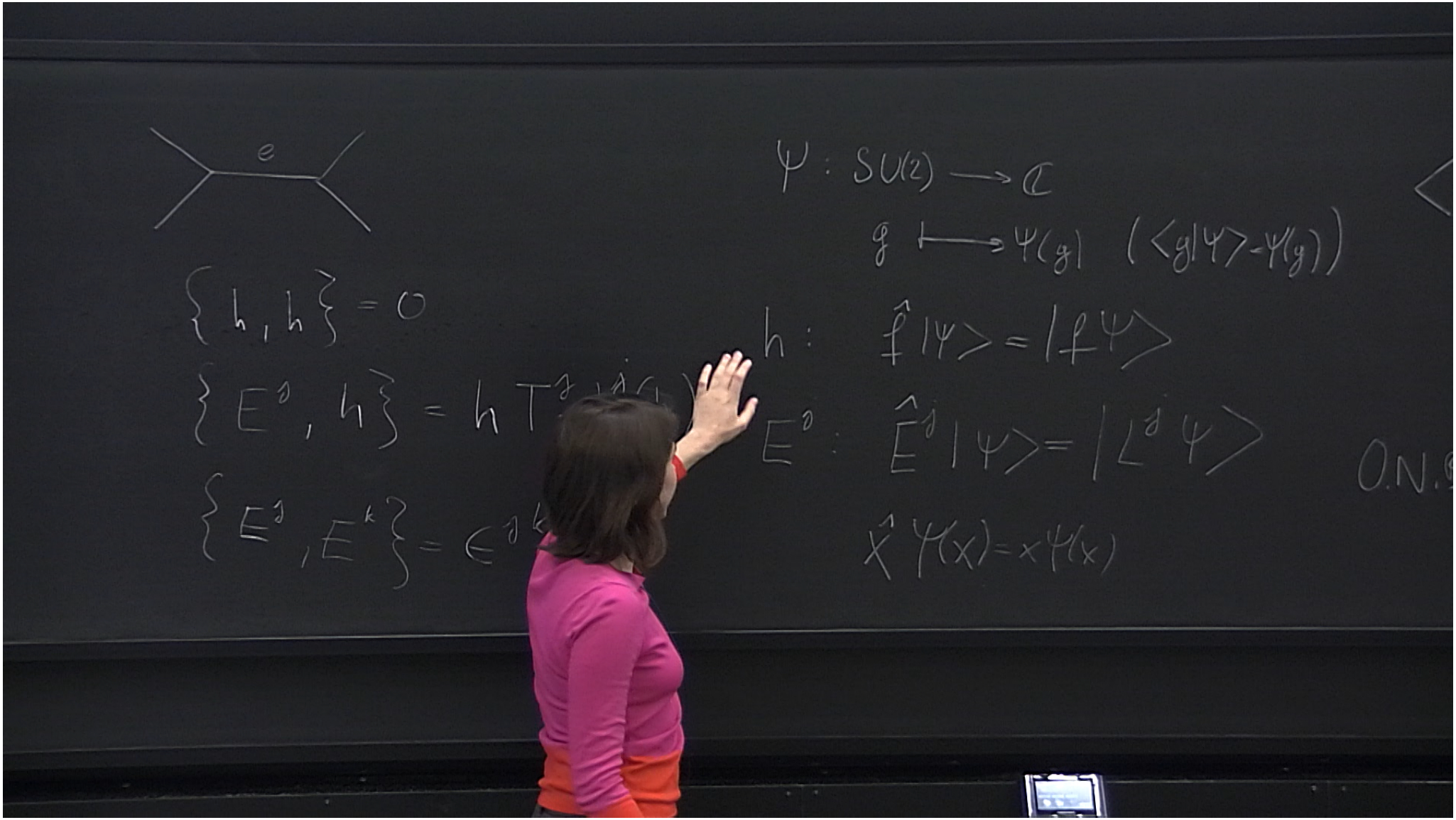
Date: Mar 31, 2017 10:15 AM

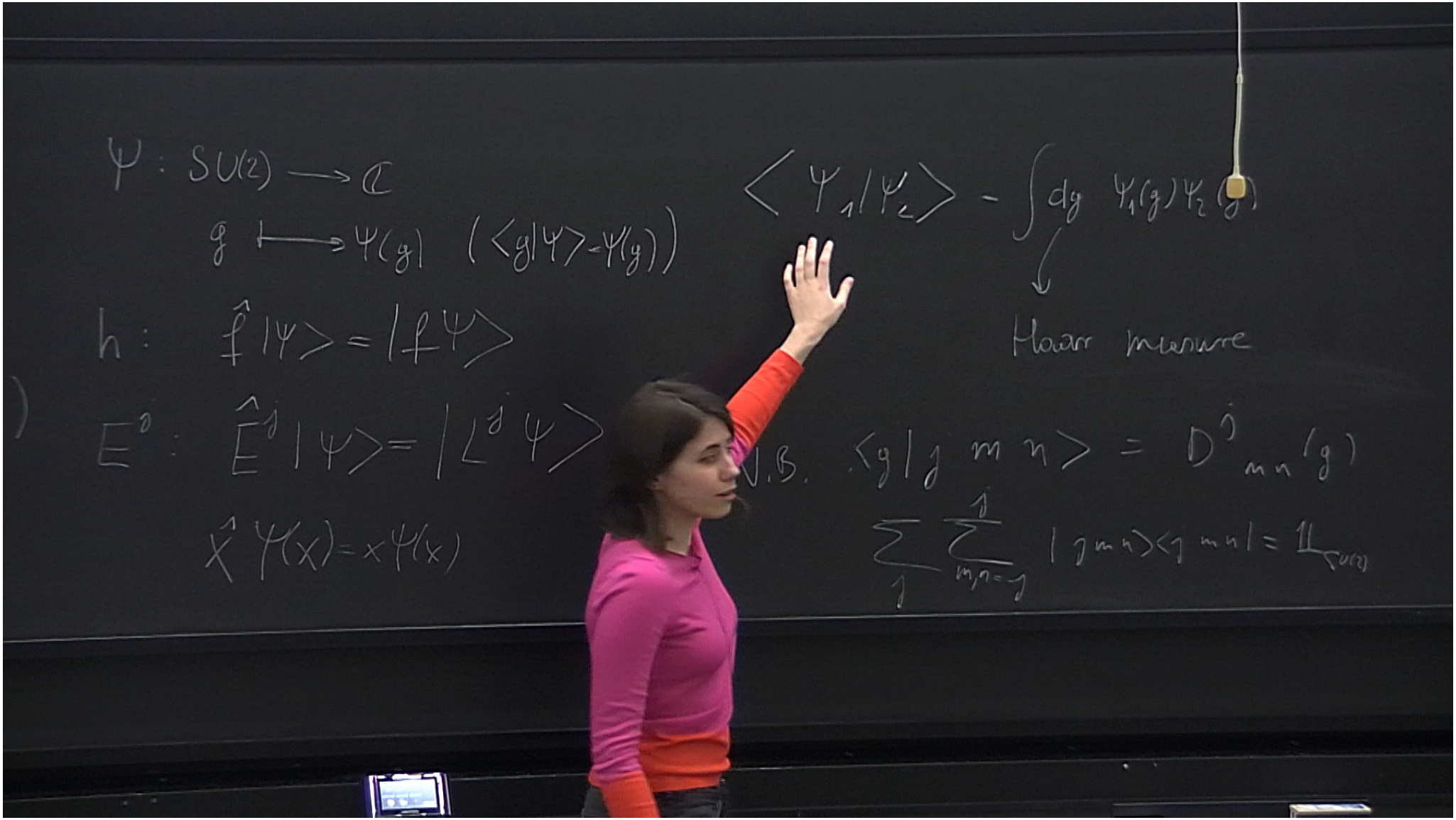
URL: http://pirsa.org/17030077

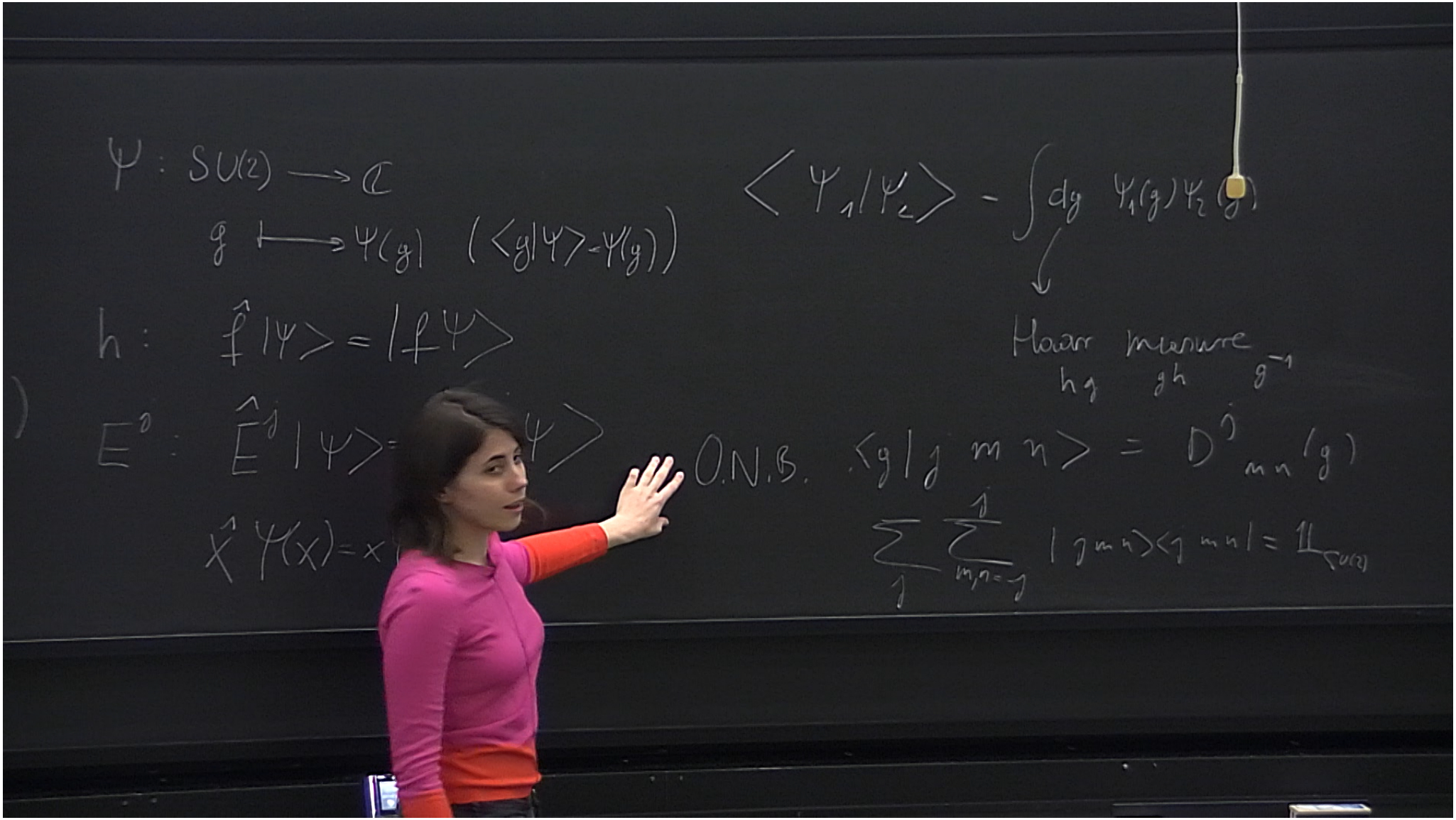
Abstract:

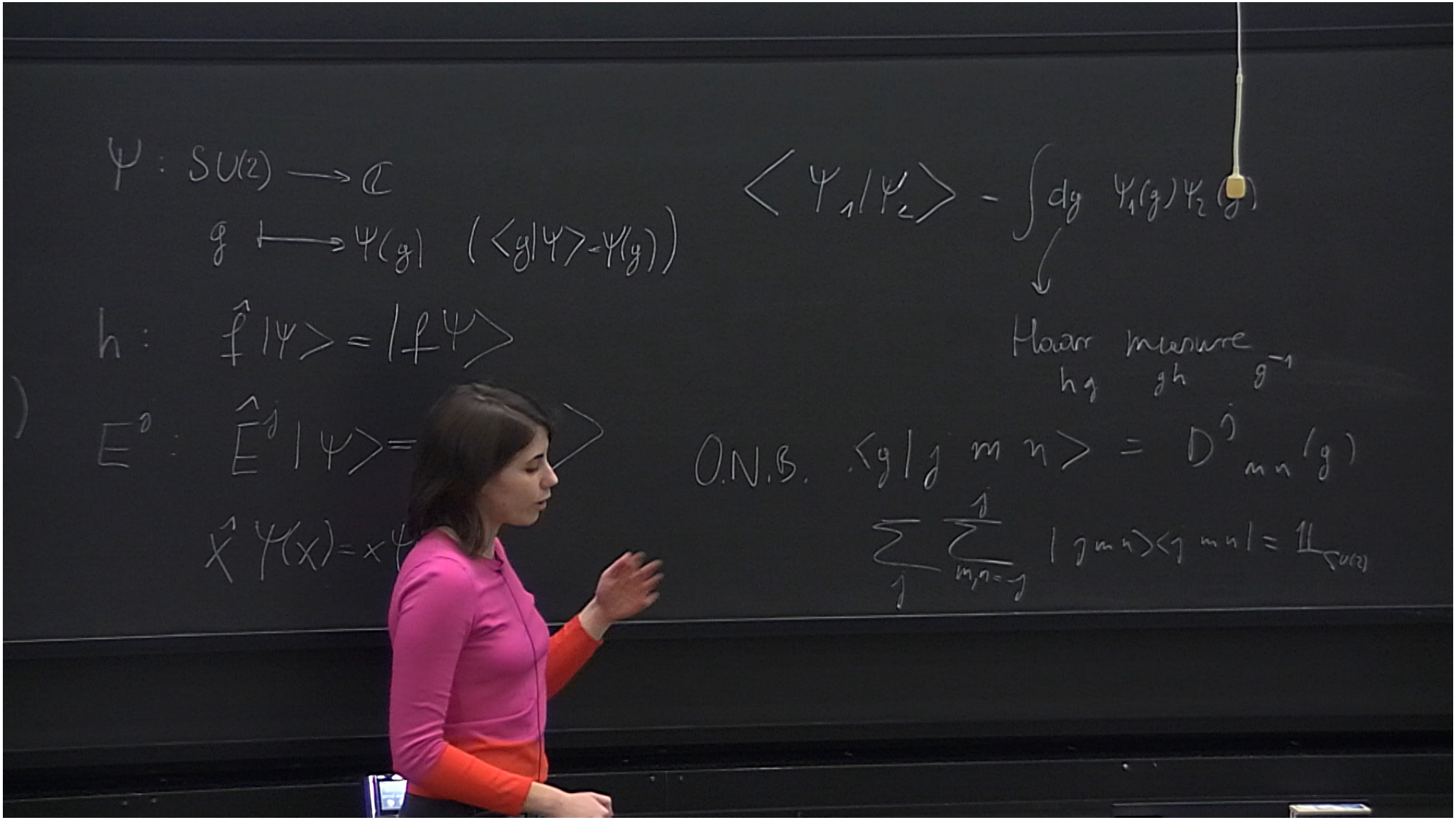

e
$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\{h, h\} = 0$
h: $\hat{f}|\Psi\rangle = |f\Psi\rangle$
$\{E^a, h\} = h T^a_{\cdot c} \alpha^c(h)$
E^a: $E^a|\Psi\rangle = |L^a\Psi\rangle$
O.N.
$\{E^j, E^k\} = \epsilon^{jk}{}_\ell E^\ell$

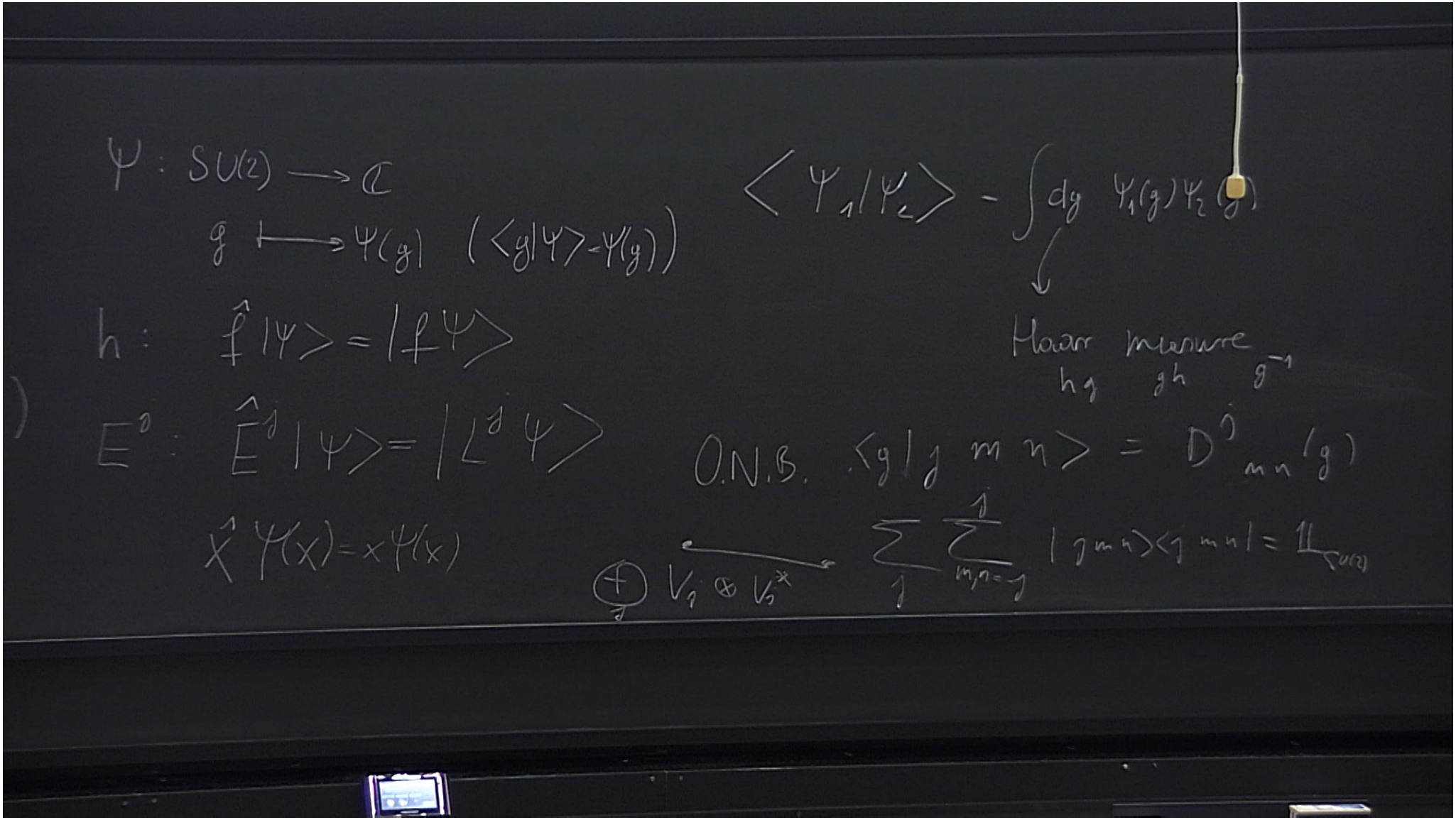

e
$\{h, h\} = 0$
$\{E^a, h\} = h T^a{}_b{}^c(h)$
$\{E^a, E^k\} = \epsilon^{a\,k}$
$\psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \psi(g)$ $(\langle g|\psi\rangle = \psi(g))$
h: $\hat{f}|\psi\rangle = |f\psi\rangle$
E^a: $E^a|\psi\rangle = |L^a\psi\rangle$
$\hat{x}\,\psi(x) = x\psi(x)$
O.N.

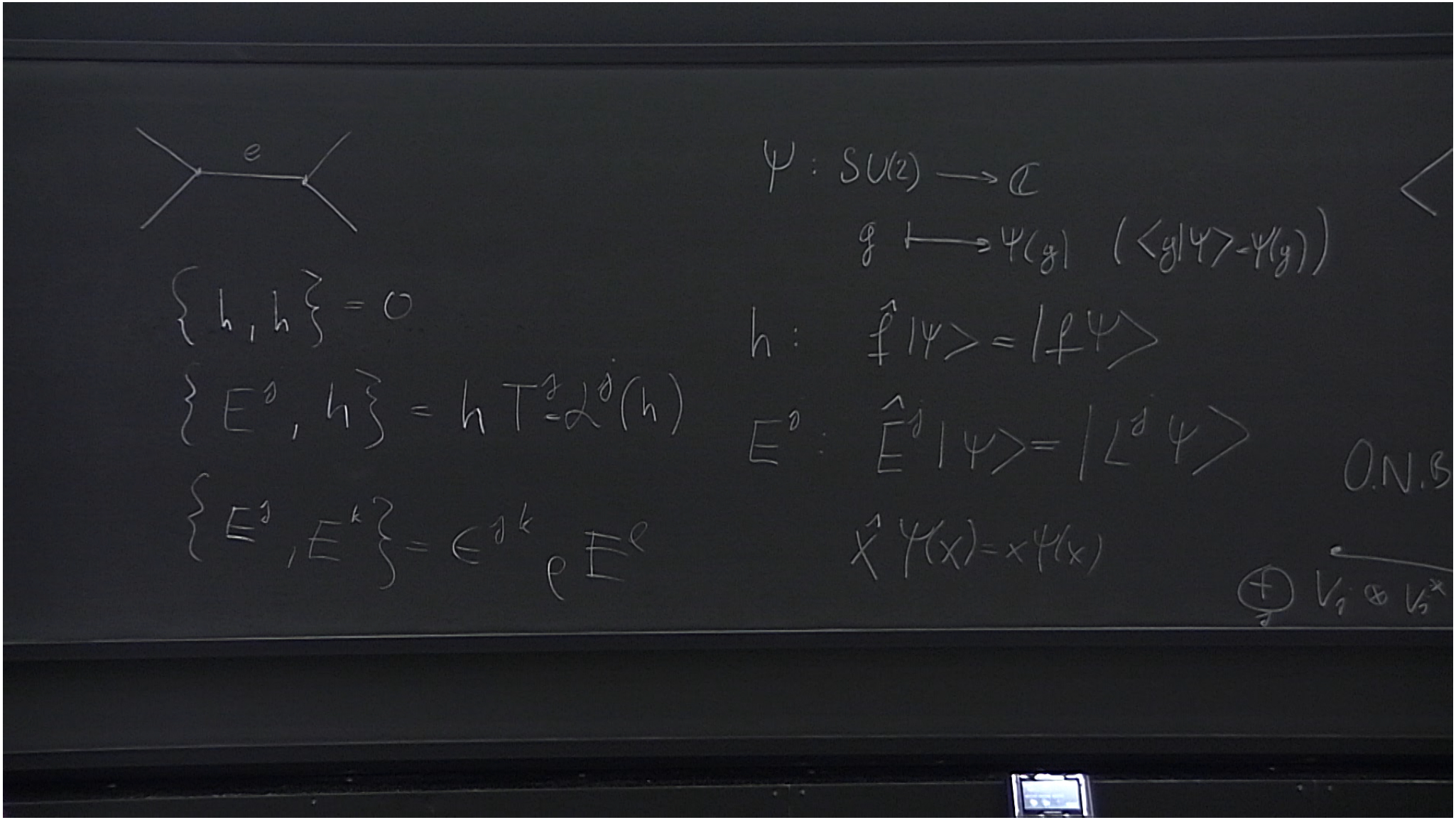

e
$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\{h, h\} = 0$
h: $\hat{f}|\Psi\rangle = |f\Psi\rangle$
$\{E^J, h\} = h T^J$
E^J: $\hat{E}^J|\Psi\rangle = |L^J\Psi\rangle$
$\{E^J, E^K\} = \epsilon^{JK}$
$\hat{x}\Psi(x) = x\Psi(x)$
O.N.

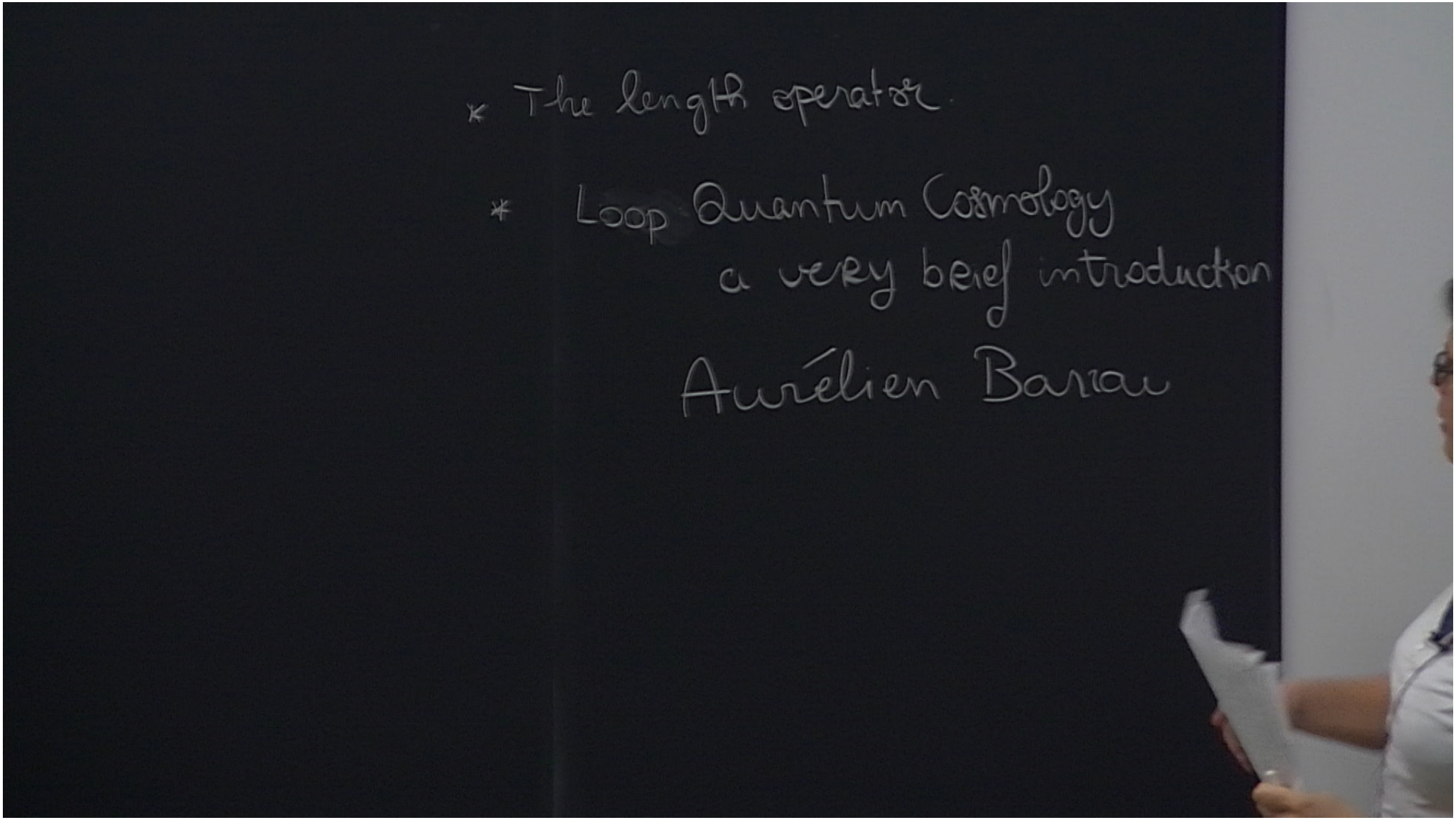

$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\langle \Psi_1 | \Psi_2 \rangle = \int dg \; \Psi_1(g) \Psi_2(g)$
Haar measure
$h: \quad \hat{f}|\Psi\rangle = |f\Psi\rangle$
$E^a: \quad \hat{E}^a|\Psi\rangle = |L^a\Psi\rangle$
N.B. $\langle g | j \; m \; n \rangle = D^j_{mn}(g)$
$\hat{x}\Psi(x) = x\Psi(x)$
$\sum_j \sum_{m,n=j} |j\,m\,n\rangle\langle j\,m\,n| = \mathbb{1}_{SU(2)}$

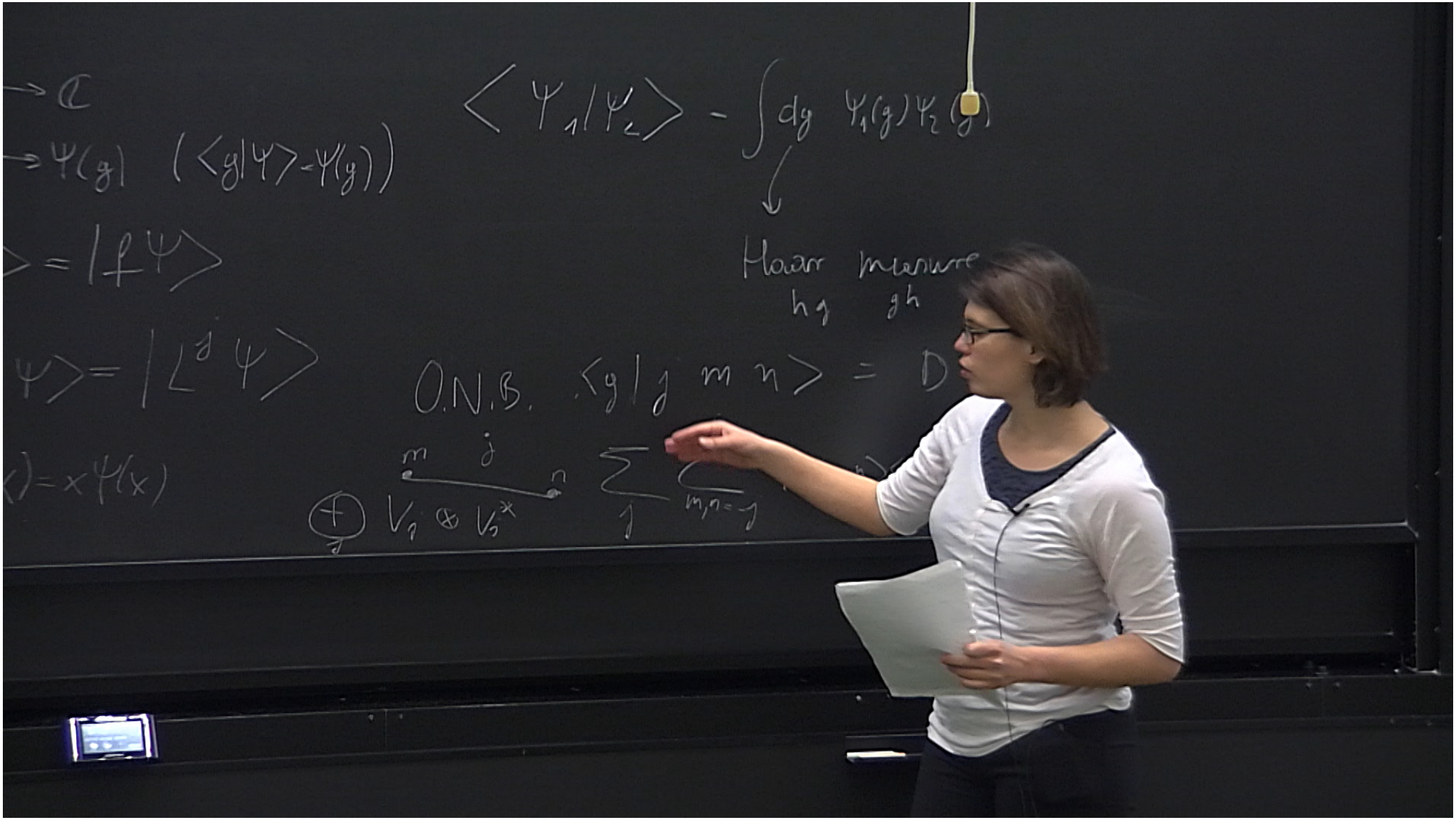

$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\langle \Psi_1 | \Psi_2 \rangle = \int dg \; \Psi_1(g) \Psi_2(g)$
Haar measure
$hg \quad gh \quad g^{-1}$
$h: \quad \hat{f}|\Psi\rangle = |f\Psi\rangle$
$E^j: \quad \hat{E}^j|\Psi\rangle = \ldots|\Psi\rangle$
O.N.B. $\langle g|j\,m\,n\rangle = D^j_{mn}(g)$
$\sum_j \sum_{m,n=-j}^{j} |j\,m\,n\rangle\langle j\,m\,n| = \mathbb{1}_{L^2(SU(2))}$
$\hat{x}\,\Psi(x) = x\,\ldots$

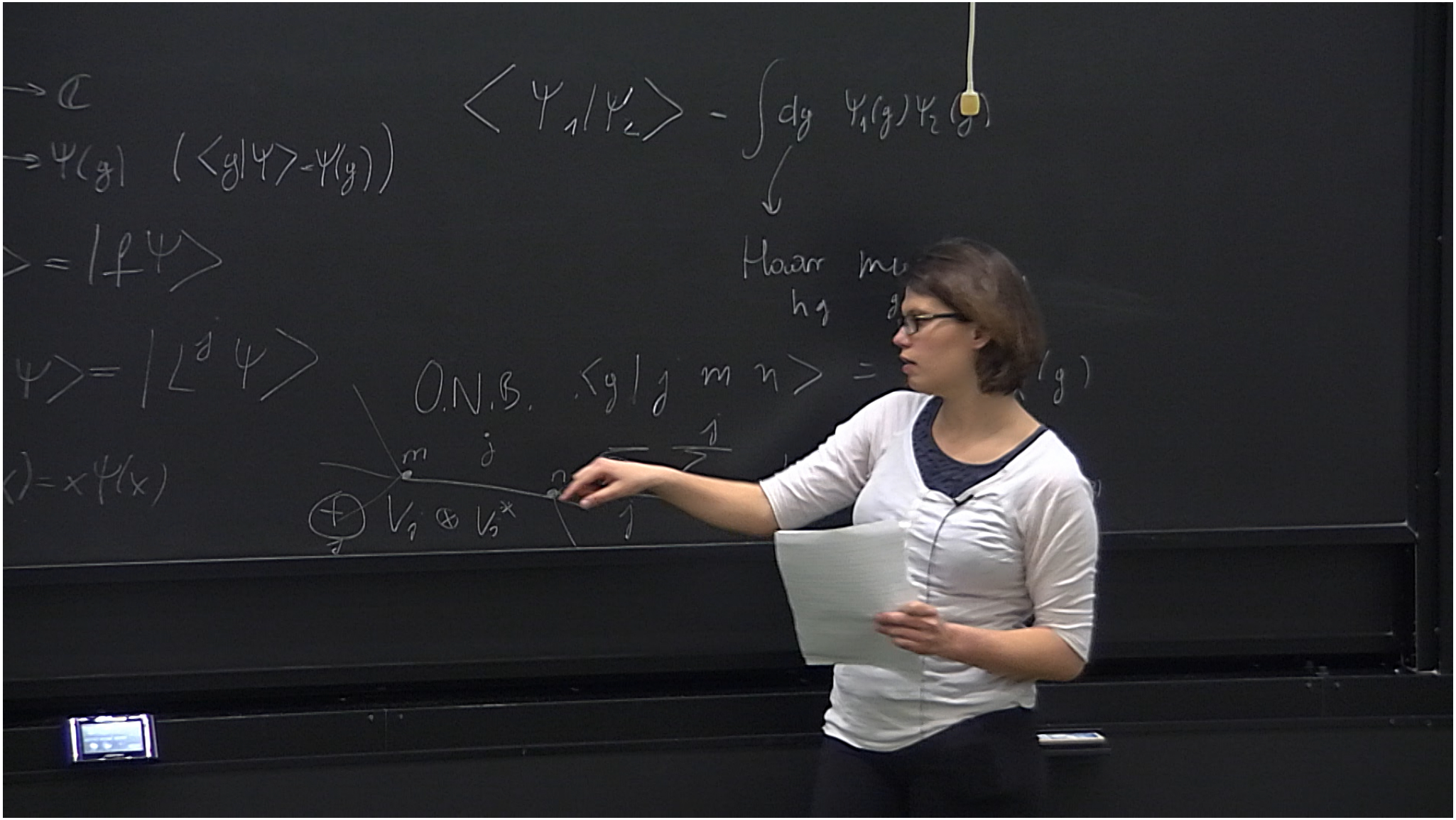

$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\langle \Psi_1 | \Psi_2 \rangle = \int dg \; \Psi_1(g) \Psi_2(g)$
Haar measure
$hg \quad gh \quad g^{-1}$
$h: \quad \hat{f}|\Psi\rangle = |f\Psi\rangle$
$E^j: \quad \hat{E}^j|\Psi\rangle =$
O.N.B. $\langle g|j\,m\,n\rangle = D^j_{mn}(g)$
$\hat{x}\,\Psi(x) = x\Psi$
$\sum_j \sum_{m,n=-j}^{j} |j\,m\,n\rangle\langle j\,m\,n| = \mathbb{1}_{SU(2)}$


$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\langle \Psi_1 | \Psi_2 \rangle = \int dg\ \Psi_1(g)\Psi_2(g)$
Haar measure
hg gh g^{-1}
$h: \hat{f}|\Psi\rangle = |f\Psi\rangle$
$E^j: \hat{E}^j|\Psi\rangle = |L^j\Psi\rangle$
O.N.B. $\langle g|j\ m\ n\rangle = D^j_{mn}(g)$
$\hat{x}\Psi(x) = x\Psi(x)$
$\bigoplus_j V_j \otimes V_j^*$
$\sum_j \sum_{m,n=j} |j\ m\ n\rangle\langle j\ m\ n| = \mathbb{1}_{SU(2)}$


e
$\{h, h\} = 0$
$\{E^j, h\} = h T^j_{\cdot} \mathcal{L}^j(h)$
$\{E^j, E^k\} = \epsilon^{jk}{}_{\ell} E^{\ell}$
$\Psi : SU(2) \longrightarrow \mathbb{C}$
$g \longmapsto \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$h : \hat{f}|\Psi\rangle = |f\Psi\rangle$
$E^j : \hat{E}^j|\Psi\rangle = |L^j\Psi\rangle$
$\hat{x}\Psi(x) = x\Psi(x)$
O.N.B
$\bigoplus_j V_j \otimes V_j^*$

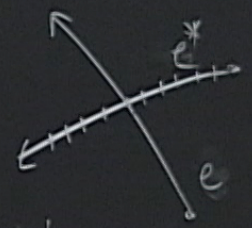

* The length operator
* Loop Quantum Cosmology
a very brief introduction
Aurélien Barrau


$\to \mathbb{C}$
$\to \Psi(g)$ $(\langle g|\Psi\rangle = \Psi(g))$
$\rangle = |f\Psi\rangle$
$\Psi\rangle = |L^g \Psi\rangle$
$(x) = x\Psi(x)$
$\langle \Psi_1 | \Psi_2 \rangle = \int dg \; \Psi_1(g) \Psi_2(g)$
Haar measure
hg gh
O.N.B. $\langle g | j \; m \; n \rangle = D$
m j n
$\bigoplus_j V_j \otimes V_j^*$
$\sum_j \sum_{m,n=j}$


$\langle \psi_1 | \psi_2 \rangle = \int dg \; \psi_1(g) \psi_2(g)$
Haar
O.N.B.


The length op
$q_{ab} = e^i_a e^i_b$
length of a curve: $\int_{\gamma^*} \delta^{ij} e^i_a e^j_b \dot{\gamma}^{*a} \dot{\gamma}^{*b} db$
$\sim \sum_{n=0}^{\infty} \sqrt{\sum_k E_{en} E}$
$\hat{l}^2_{e^*} | j,m,n)$
e^*
e

The length op

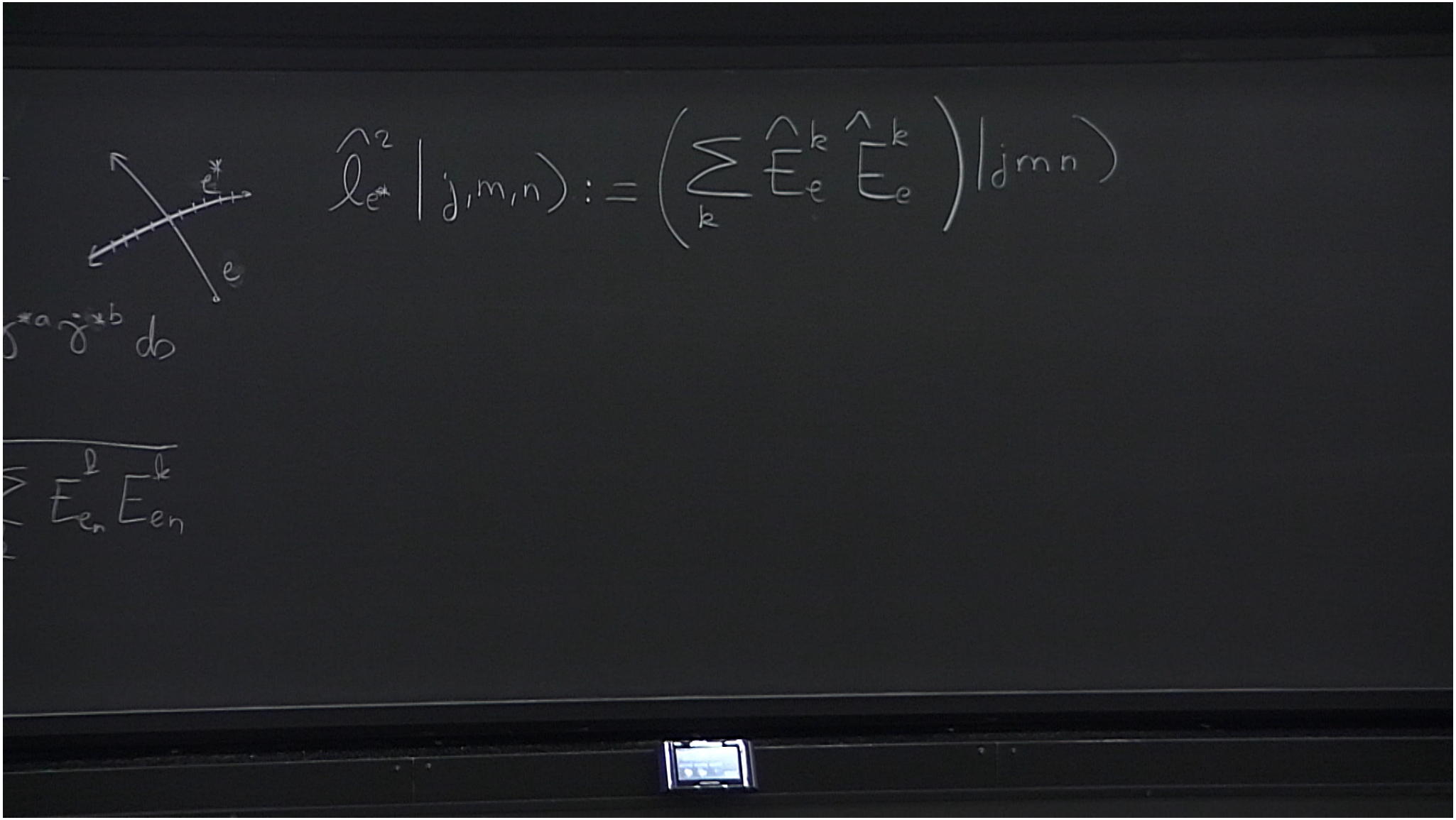


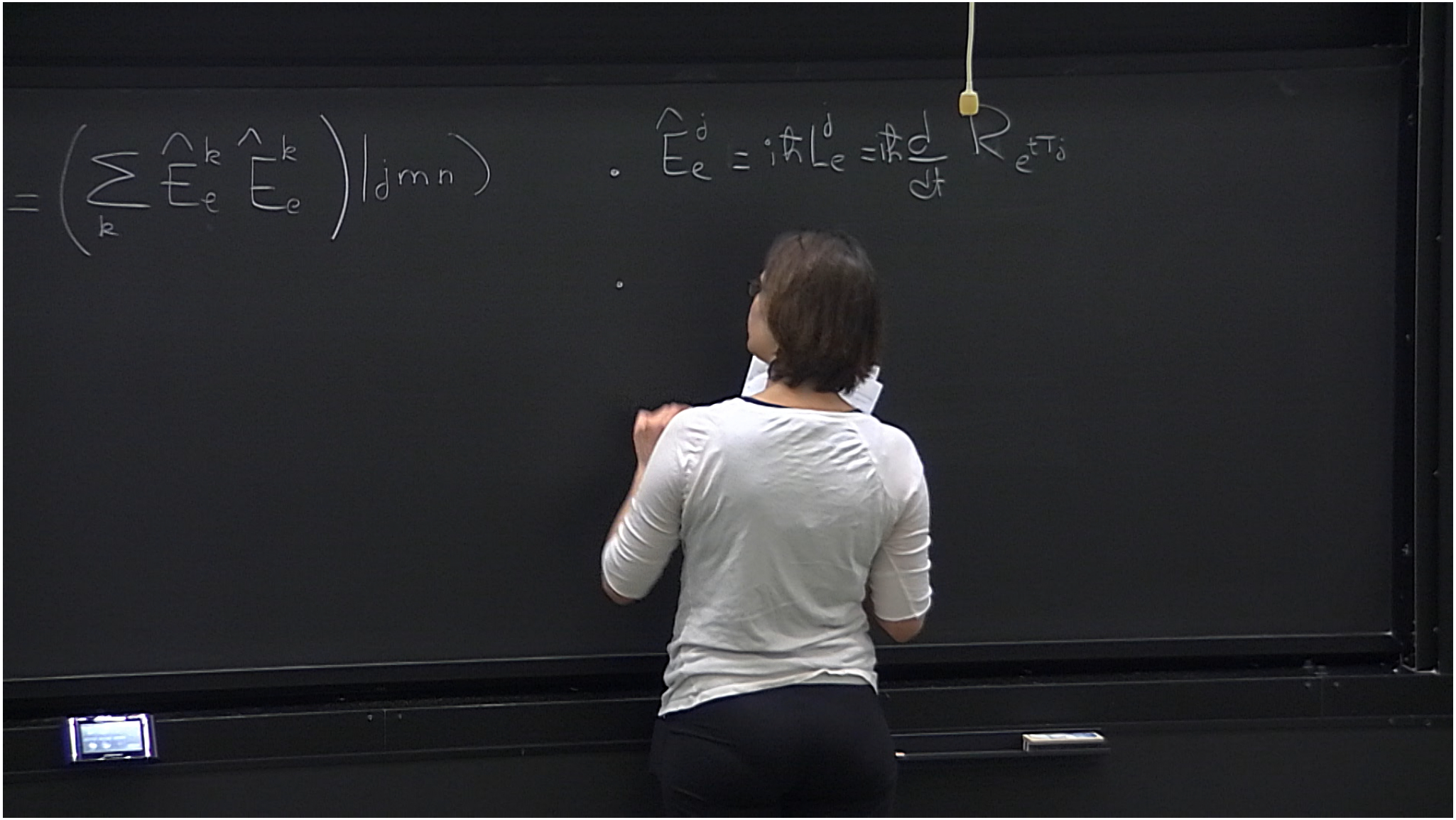
$\widehat{l^2_{e^*}} \,|\, j,m,n)$

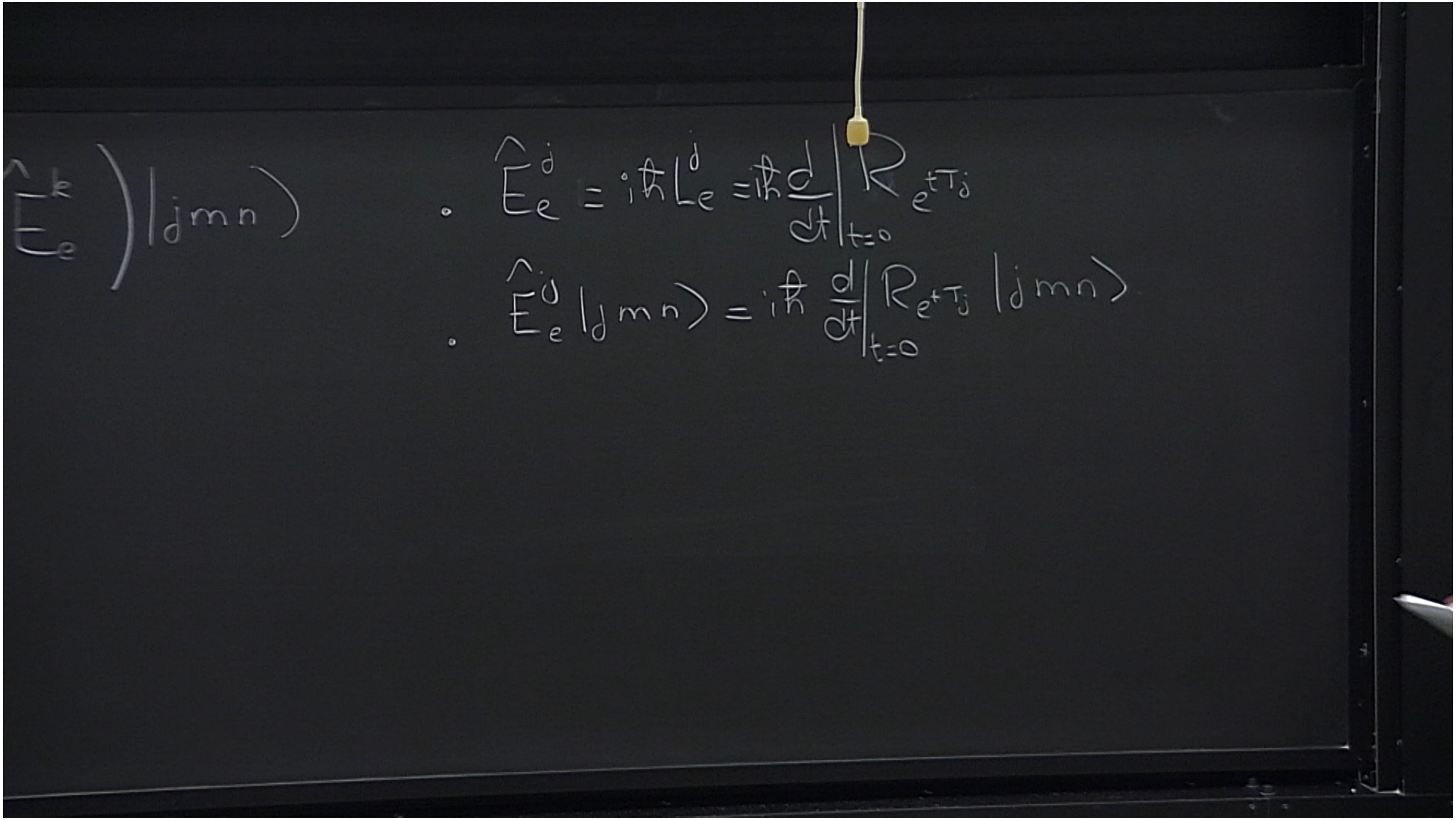
$$q_{ab} = e^i_a e^i_b$$

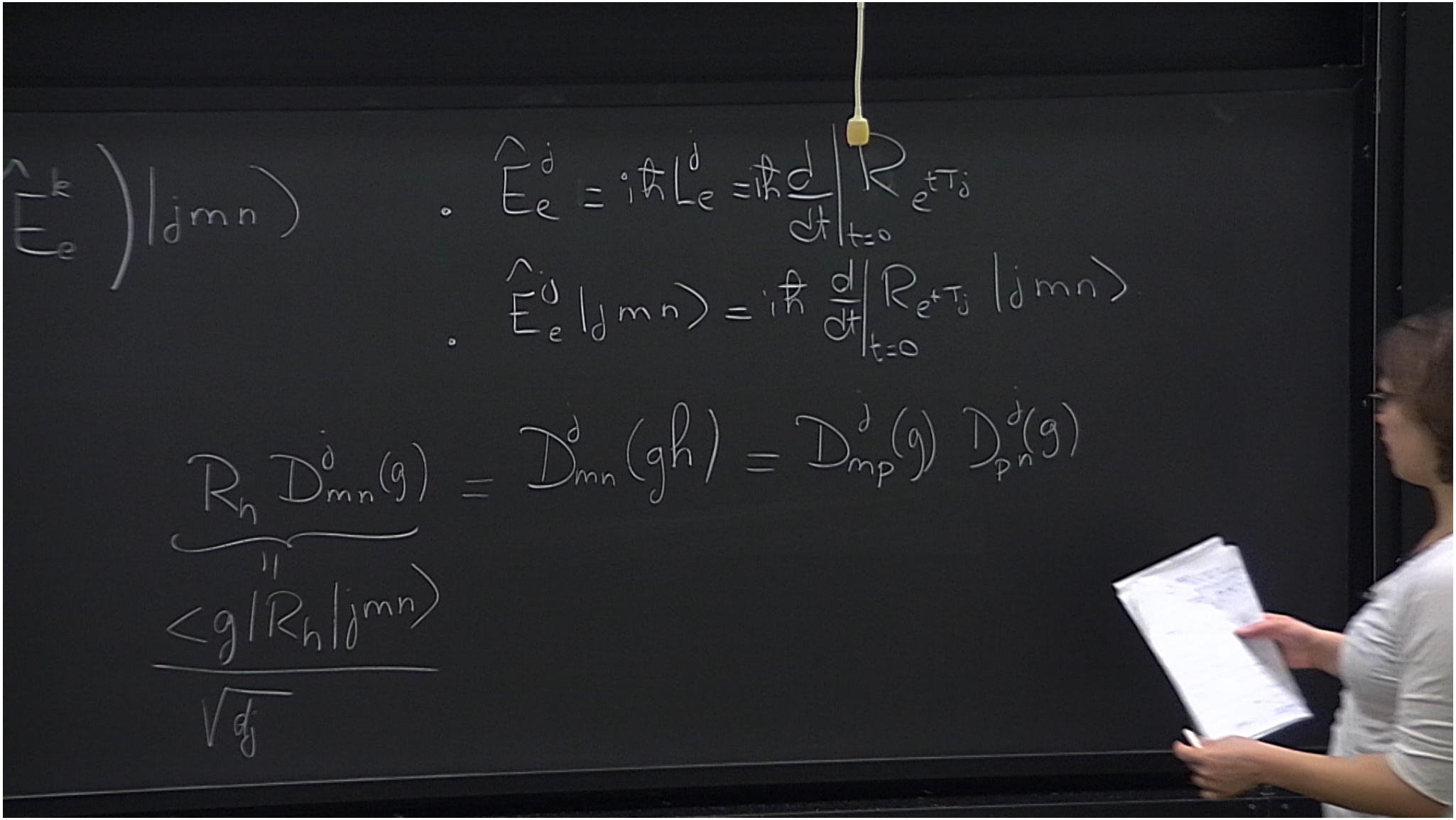
length of a curve: $\int_{\gamma^*} \delta^{ij} e^i_a e^j_b \dot{\gamma}^{*a} \dot{\gamma}^{*b} \, db$

$$\sim \sum_{n=0}^{\infty} \sqrt{\sum_k E^k_{e_n} E^k_{e_n}}$$


$\hat{l}^2_{e^*} | j,m,n\rangle := \left(\sum_k \hat{E}^k_{e^*} \hat{E}^k_e \right) | j\,m\,n \rangle$
e^*
e
$j^{*a} j^{*b}\, db$
$\sum E^l_{e_n} E^k_{e_n}$


$= \left(\sum_k \hat{E}_e^k \hat{E}_e^k \right) |jmn)$
$\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt} R_{e^t T_j}$


$\hat{E}_e^k \big) |jmn\rangle$
$\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^t T_j}$
$\hat{E}_e^j |jmn\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^t T_j} |jmn\rangle$


$\hat{E}_e^k \big) |jmn\rangle$
$\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}}$
$\hat{E}_e^j |jmn\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}} |jmn\rangle$
$R_h D^j_{mn}(g) = D^j_{mn}(gh) = D^j_{mp}(g)\, D^j_{pn}(g)$
$\frac{\langle g | R_h | jmn \rangle}{\sqrt{d_j}}$

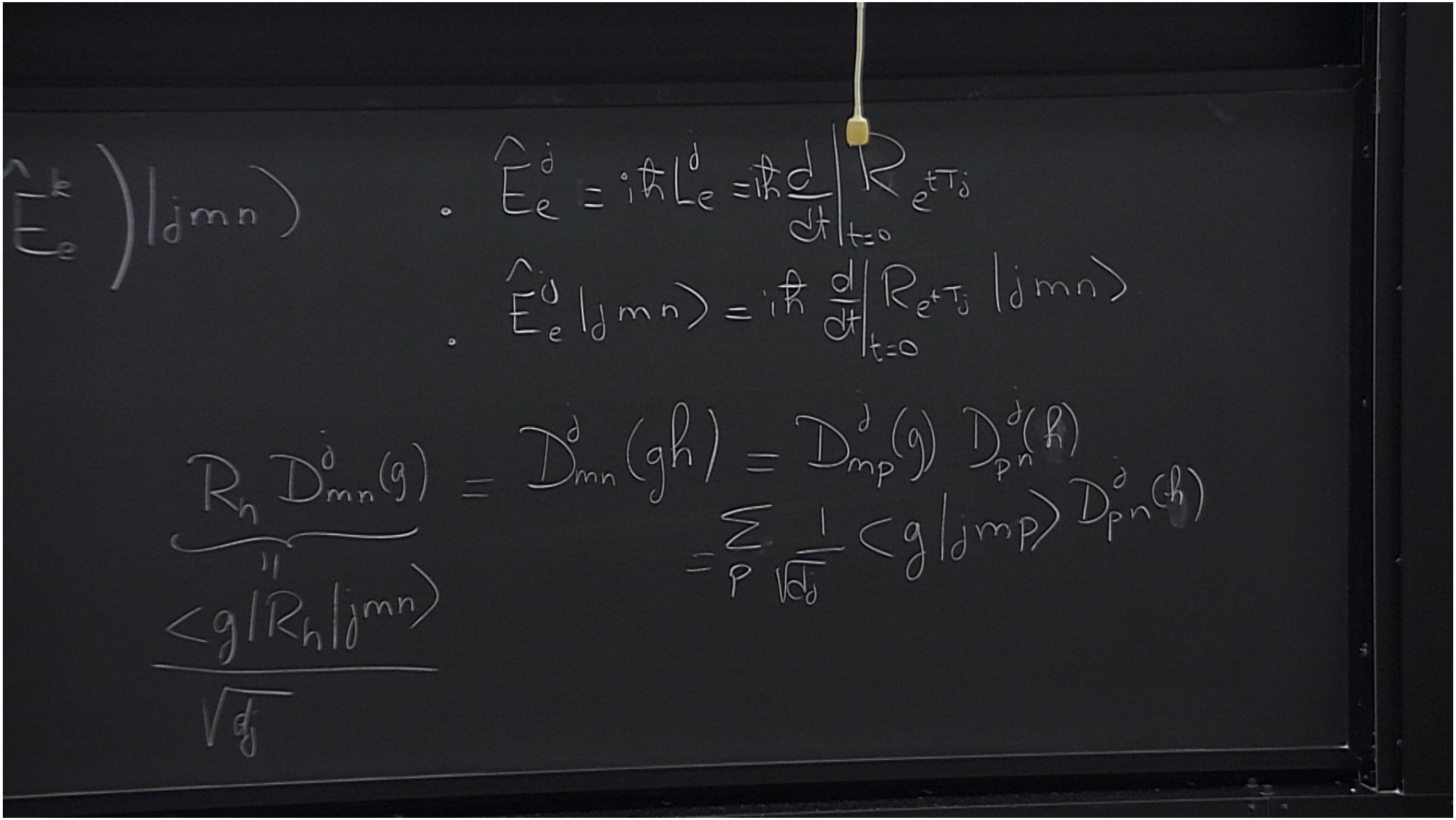
$\hat{E}_e^k \big) |jmn\rangle$

- $\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}}$
- $\hat{E}_e^j |jmn\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}} |jmn\rangle$

$$\underbrace{R_h D^j_{mn}(g)}_{\langle g | R_h | jmn \rangle} = D^j_{mn}(gh) = D^j_{mp}(g)\, D^j_{pn}(g)$$

$$= \sum_p \frac{1}{\sqrt{d_j}} \langle g | jmp \rangle D^j_{pn}(g)$$

$$\frac{\langle g | R_h | jmn \rangle}{\sqrt{d_j}}$$


$\hat{E}_e^k \big) |jmn\rangle$
$\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}}$
$\hat{E}_e^j |jmn\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}} |jmn\rangle$
$R_h D^j_{mn}(g) = D^j_{mn}(gh) = D^j_{mp}(g)\, D^j_{pn}(h)$
$= \sum_p \frac{1}{\sqrt{d_j}} \langle g|jmp\rangle D^j_{pn}(h)$
$\frac{\langle g|R_h|jmn\rangle}{\sqrt{d_j}}$

$$\left(\hat{E}_e^k\right)|jmn\rangle$$

- $\hat{E}_e^j = i\hbar L_e^j = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}}$
- $\hat{E}_e^j |jmn\rangle = i\hbar \frac{d}{dt}\Big|_{t=0} R_{e^{tT_j}} |jmn\rangle$

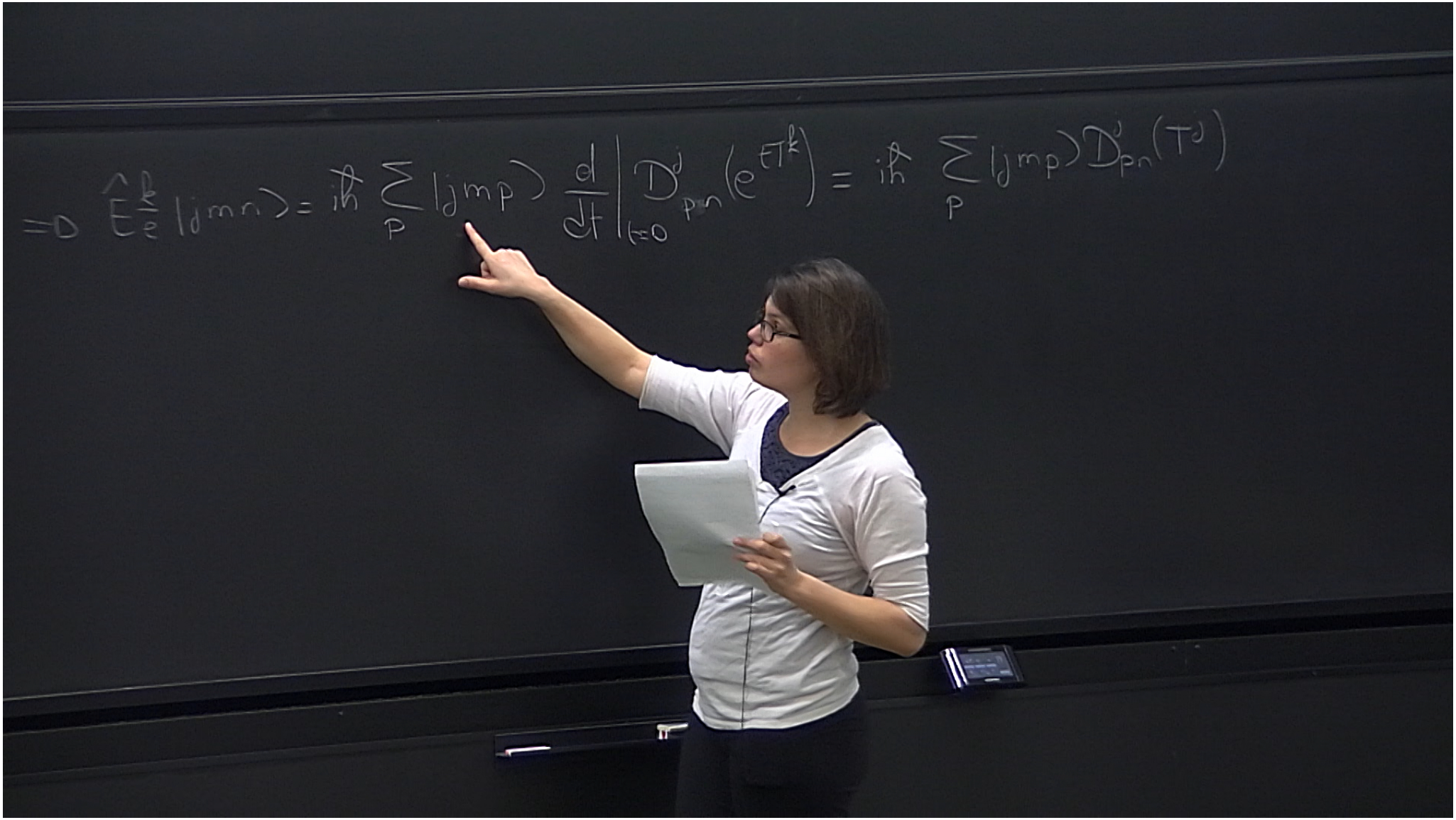
$$R_h D^j_{mn}(g) = D^j_{mn}(gh) = D^j_{mp}(g)\, D^j_{pn}(h)$$

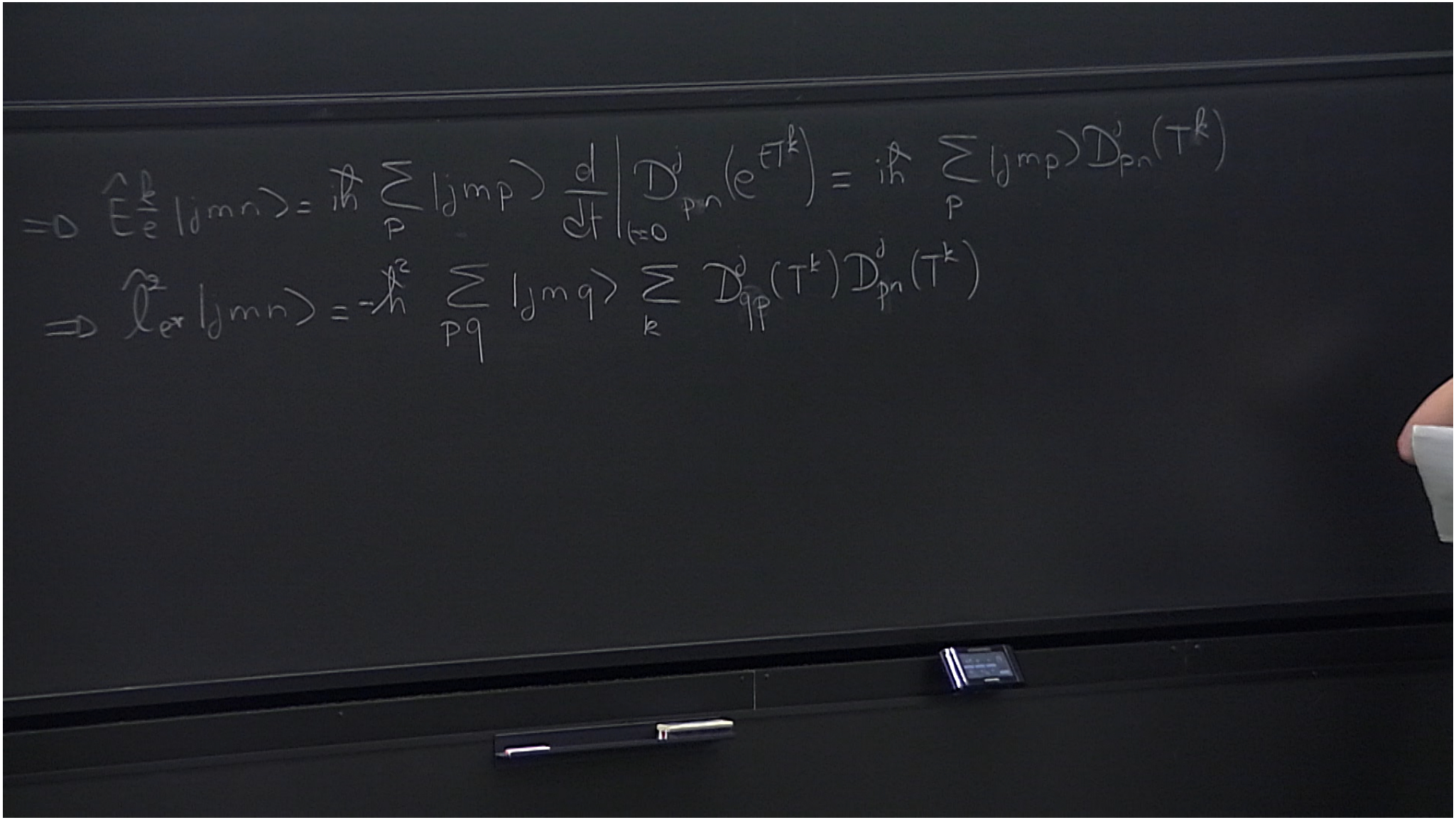
$$= \sum_p \frac{1}{\sqrt{d_j}} \langle g|jmp\rangle D^j_{pn}(h)$$

$$\underbrace{R_h D^j_{mn}(g)}_{\| \; \langle g|R_h|jmn\rangle} = \sum_p D^j_{pn}(h)|jmp\rangle$$

$$\frac{\langle g|R_h|jmn\rangle}{\sqrt{d_j}}$$

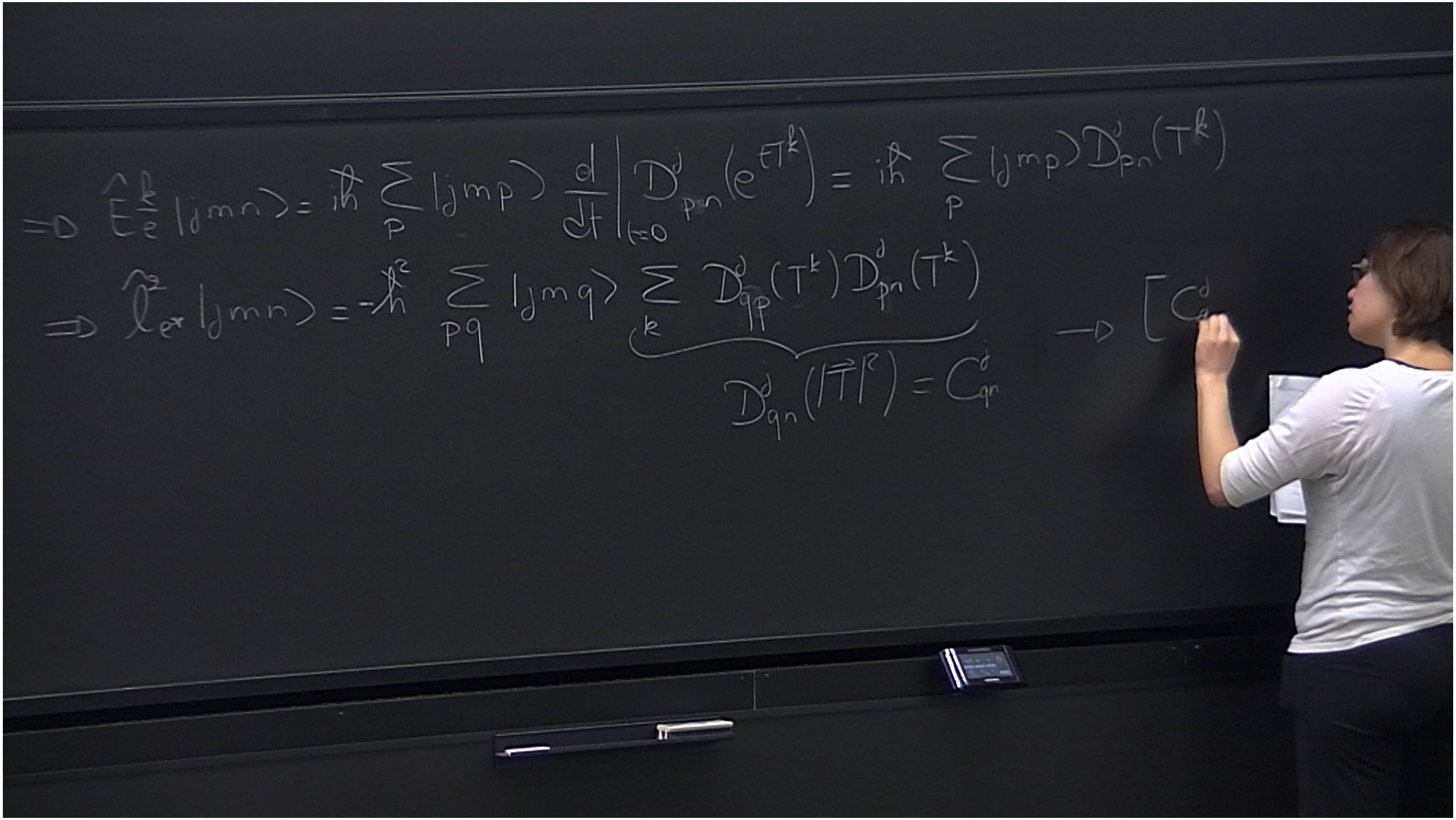
$$\Rightarrow R_h|jmn\rangle = \sum_p D^j_{pn}(h)|jmp\rangle$$

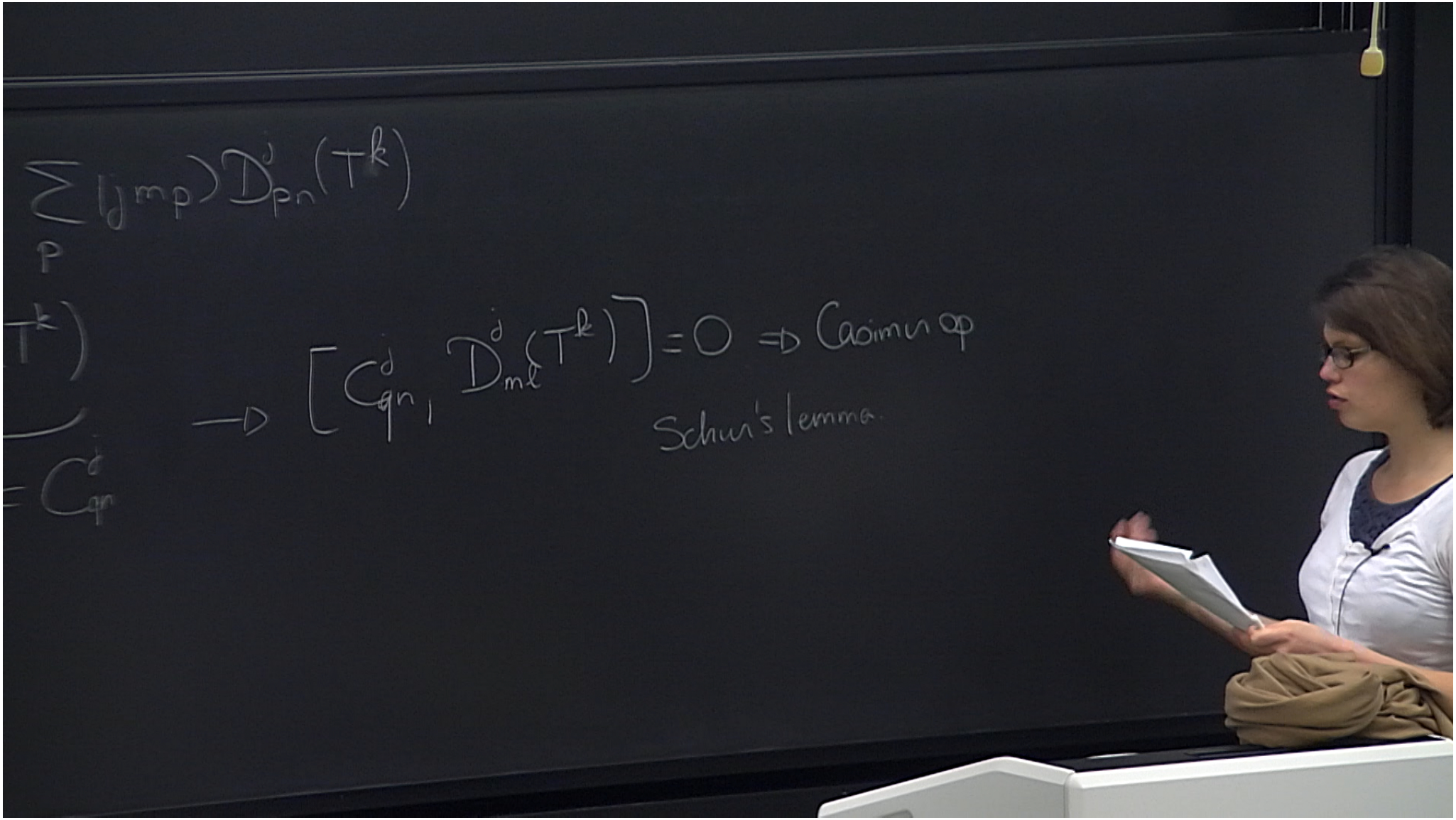

$\Rightarrow \hat{E}_e^k |jmn_i\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tTk}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^j)$

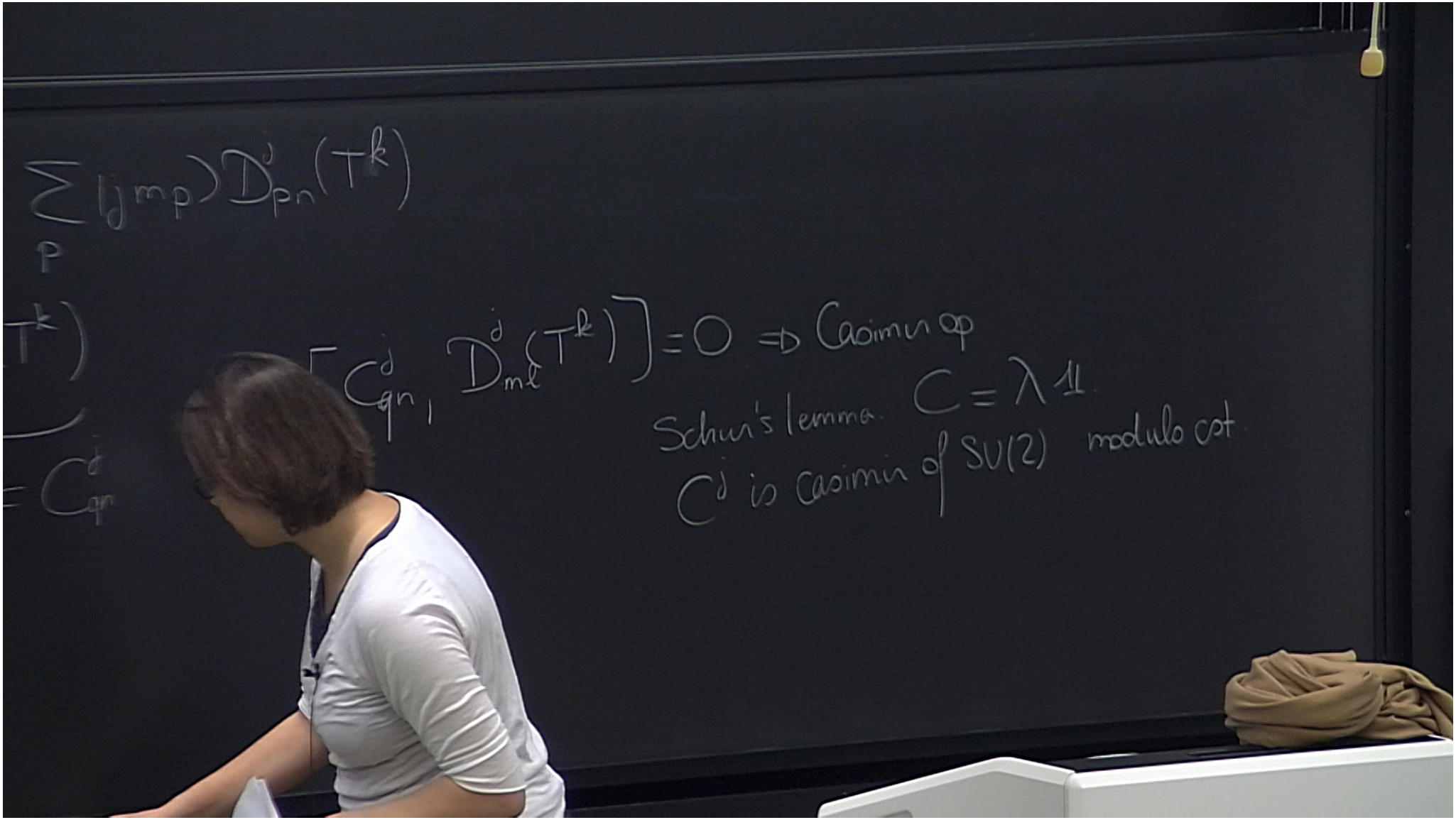

$\Rightarrow \hat{E}^k_e |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^k)$
$\Rightarrow \hat{l}^2_{er} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$

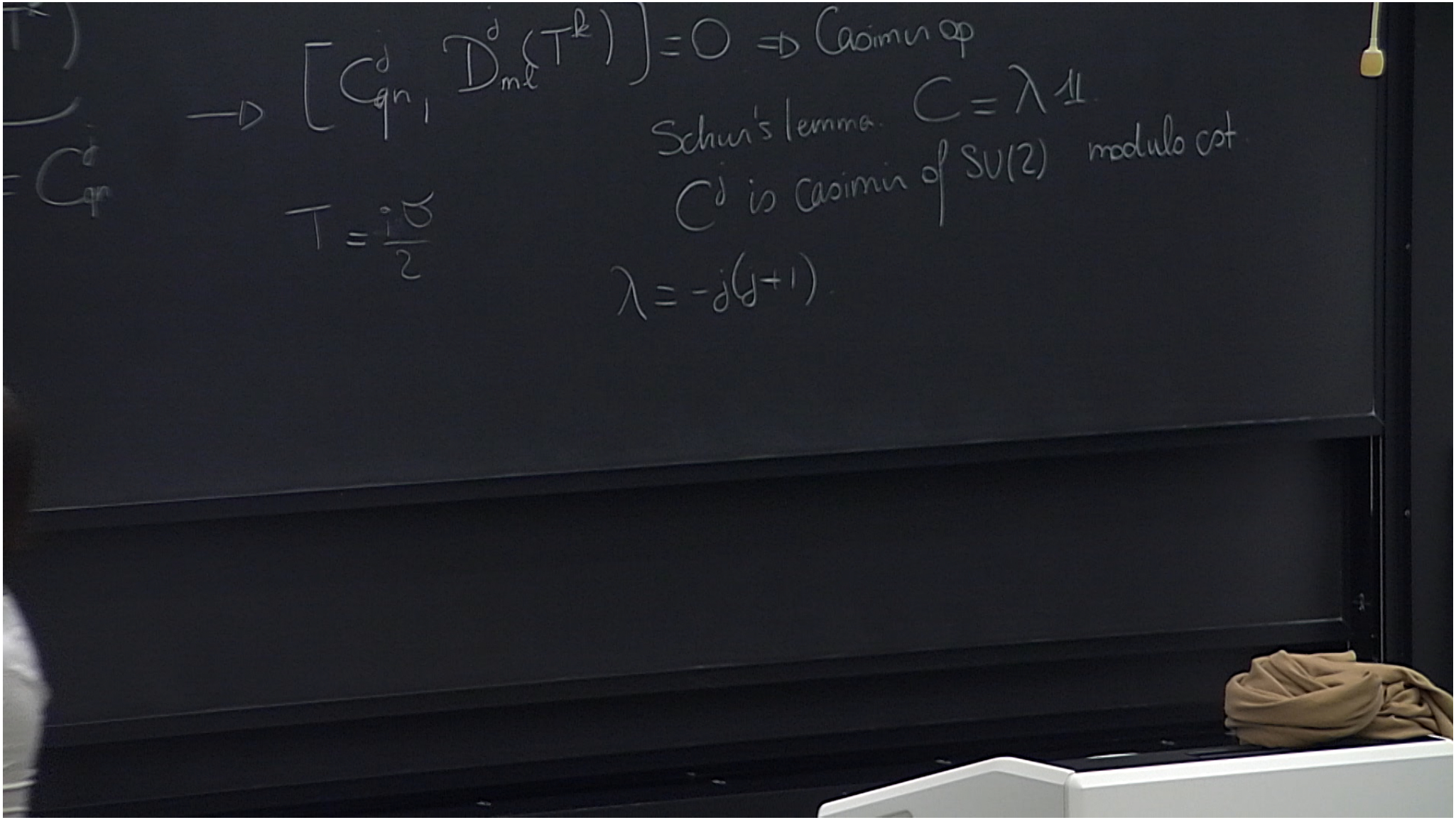
$$\Rightarrow \hat{E}^k_e |j m n\rangle = i\hbar \sum_p |j m p\rangle \left.\frac{d}{dt}\right|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |j m p\rangle D^j_{pn}(T^k)$$

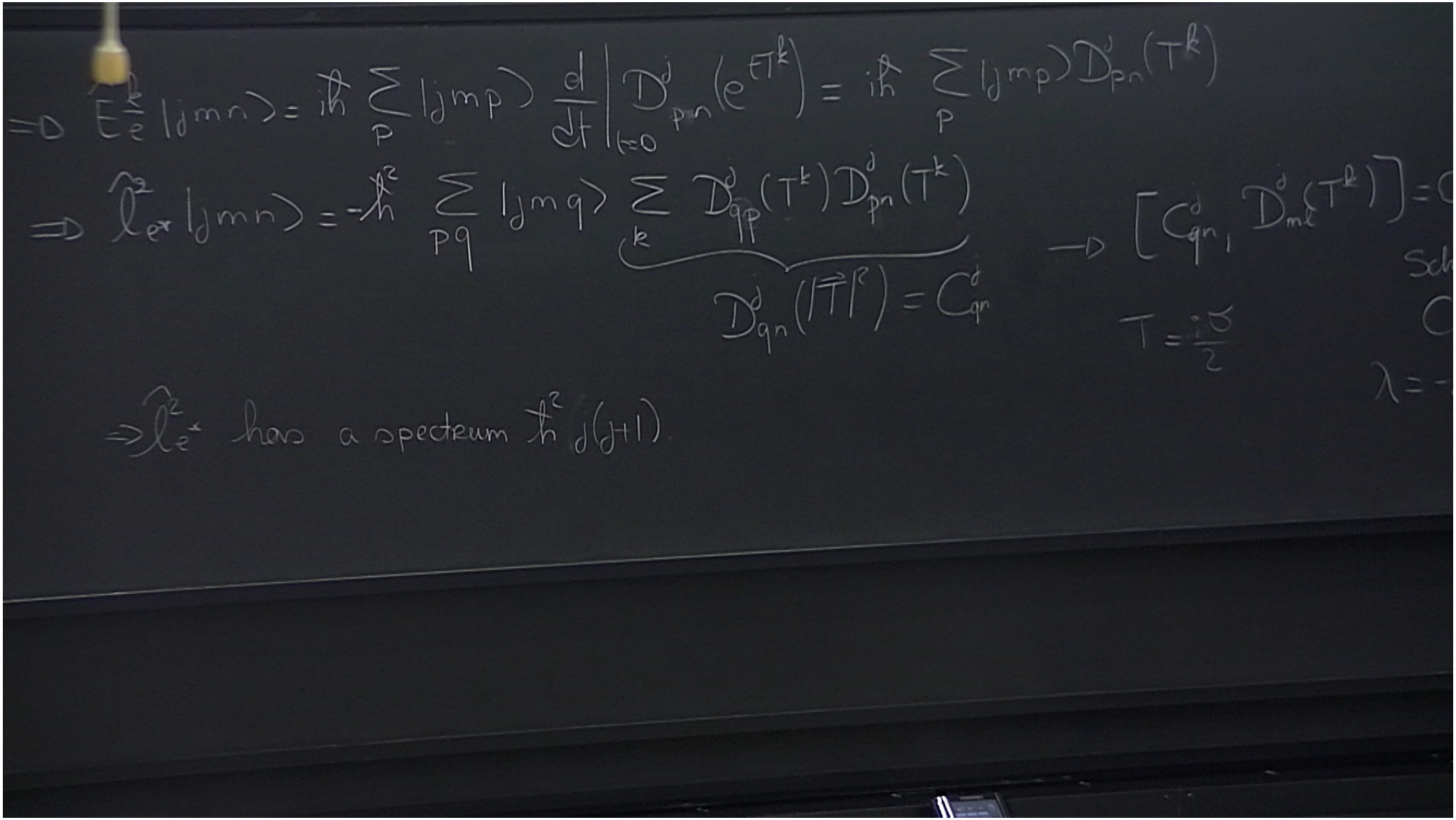
$$\Rightarrow \hat{l}^2_{er} |j m n\rangle = -\hbar^2 \sum_{pq} |j m q\rangle \underbrace{\sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)}_{D^j_{qn}(|\vec{T}|^2)}$$

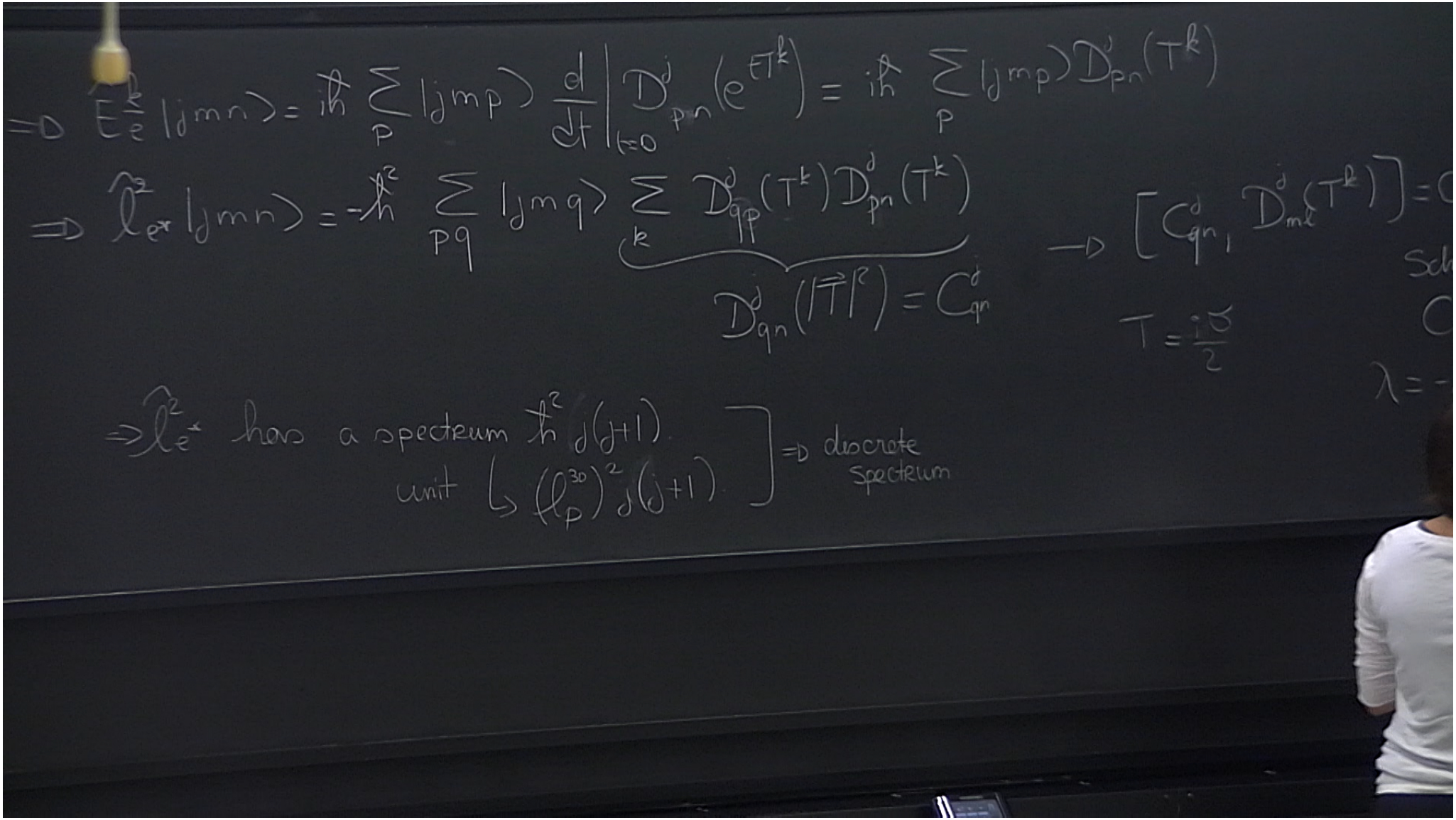

$\Rightarrow \hat{E}^k_e |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^k)$
$\Rightarrow \hat{l}^2_{er} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
$\rightarrow [C^j_{qn}$

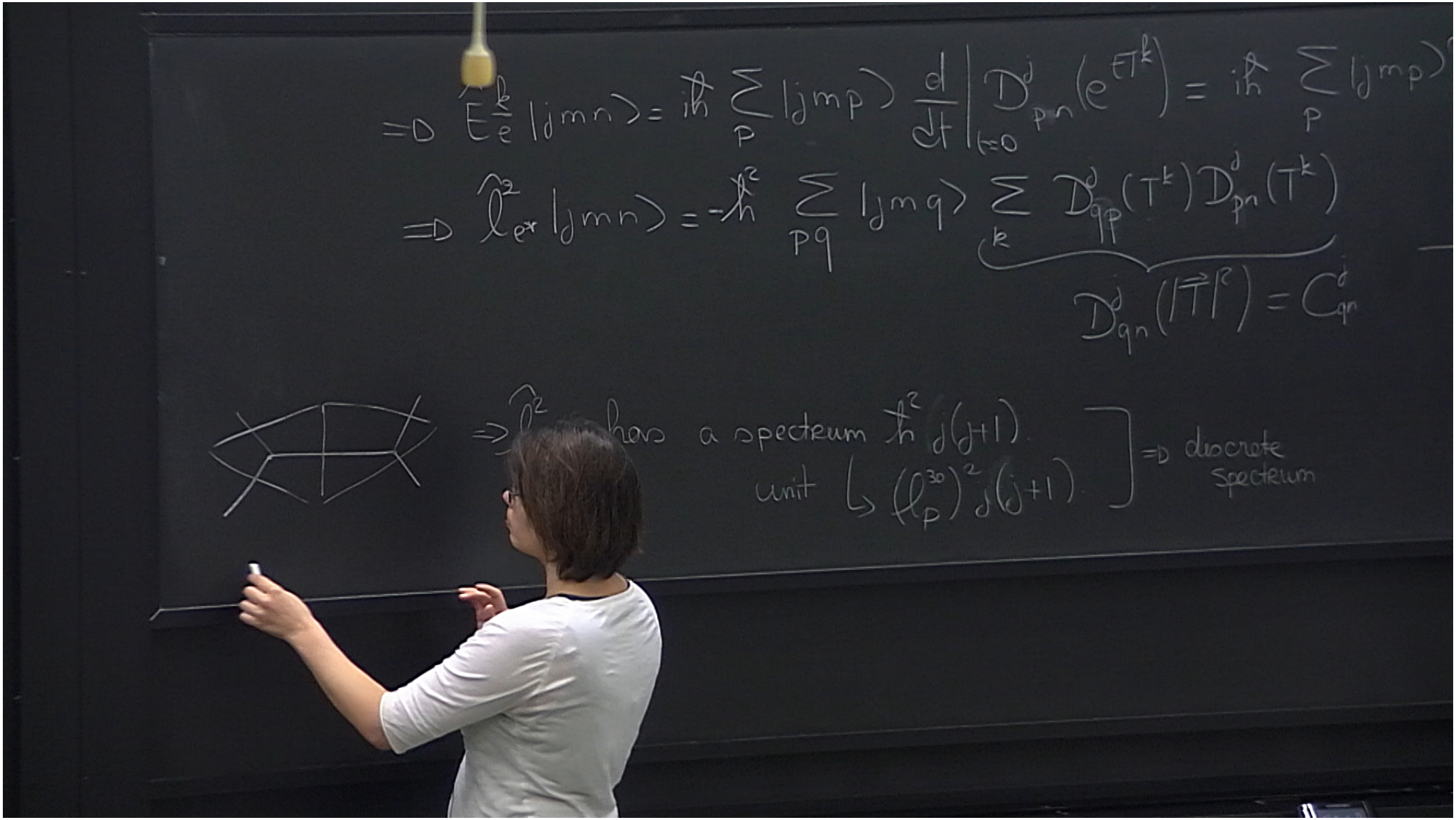

$\sum_P |ijmp\rangle D^j_{pn}(T^k)$
$T^k)$
$= C^d_{qn}$
$\rightarrow [C^d_{qn}, D^d_{ml}(T^k)] = 0 \Rightarrow$ Casimir op
Schur's lemma.

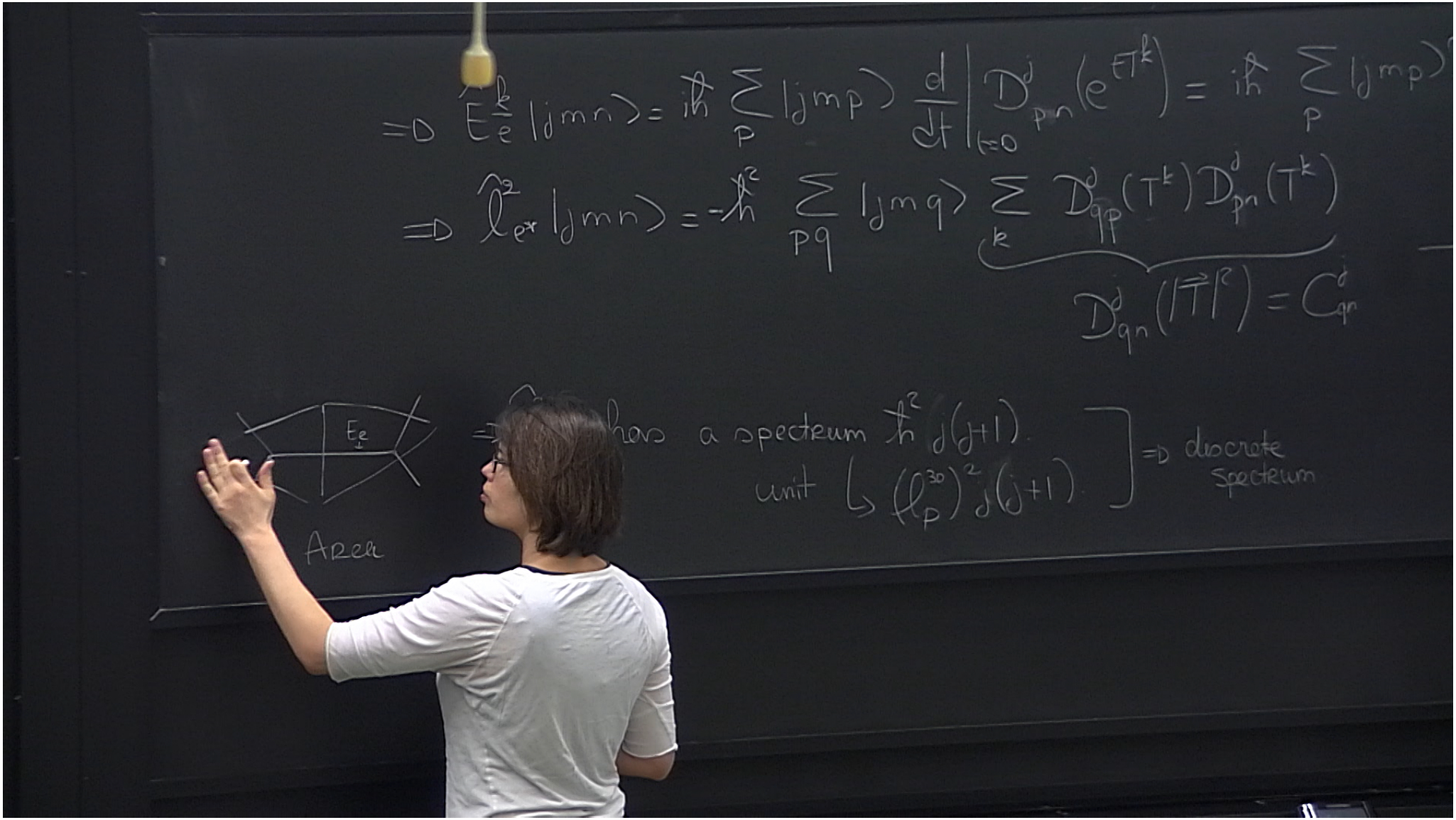

$\sum_p |ijmp\rangle D^j_{pn}(T^k)$
$= C^j_{qr}$
$\left[C^j_{qn}, D^j_{ml}(T^k) \right] = 0 \Rightarrow$ Casimir op
Schur's lemma $C = \lambda \mathbb{1}$
C^j is Casimir of SU(2) modulo cst

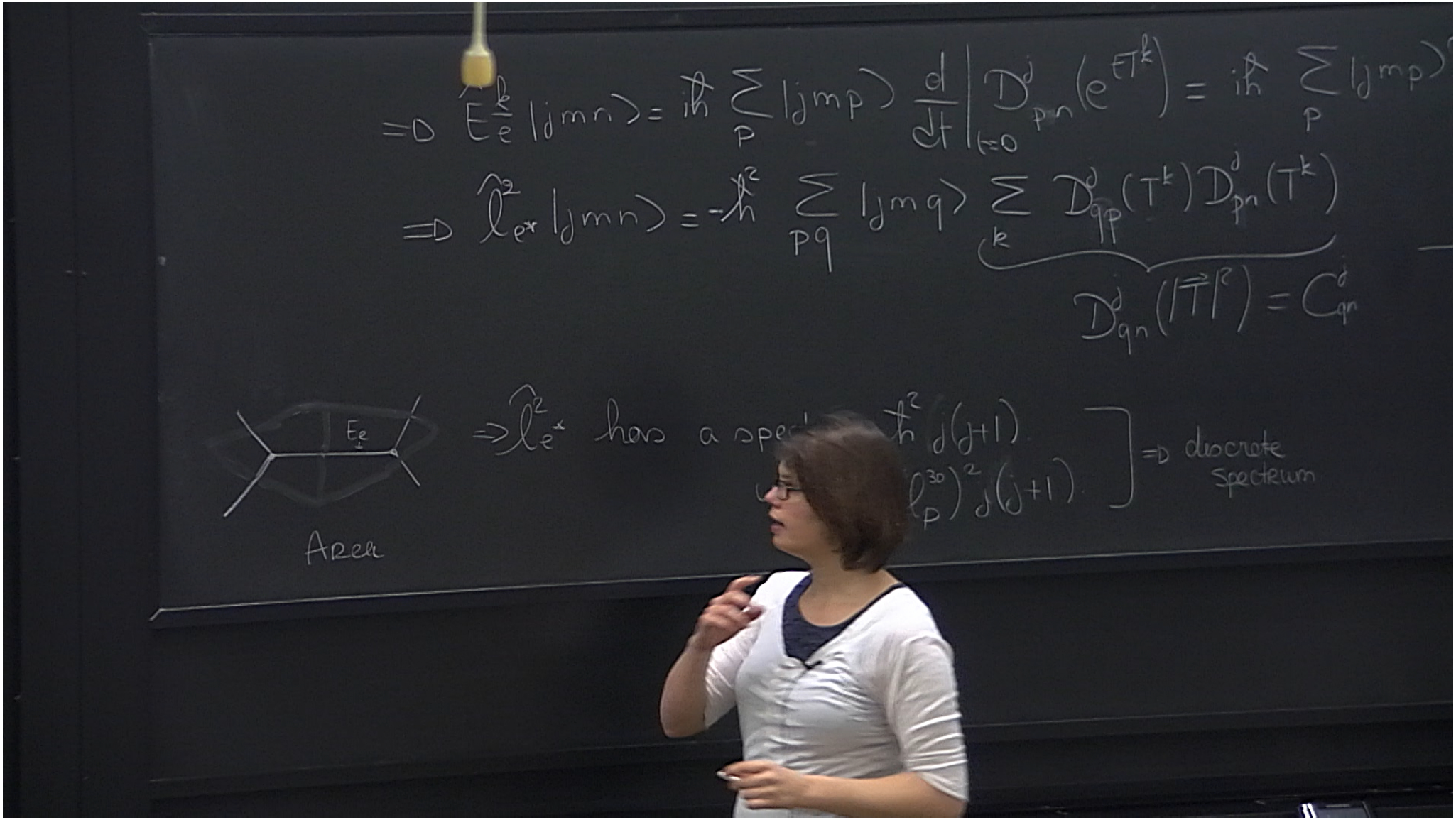

$\rightarrow \left[C^d_{qn}, D^d_{ml}(T^k)\right] = 0 \Rightarrow$ Casimir op
Schur's lemma $C = \lambda \mathbb{1}$
C^d is Casimir of SU(2) modulo cst
$T = \frac{i\sigma}{2}$
$\lambda = -j(j+1)$

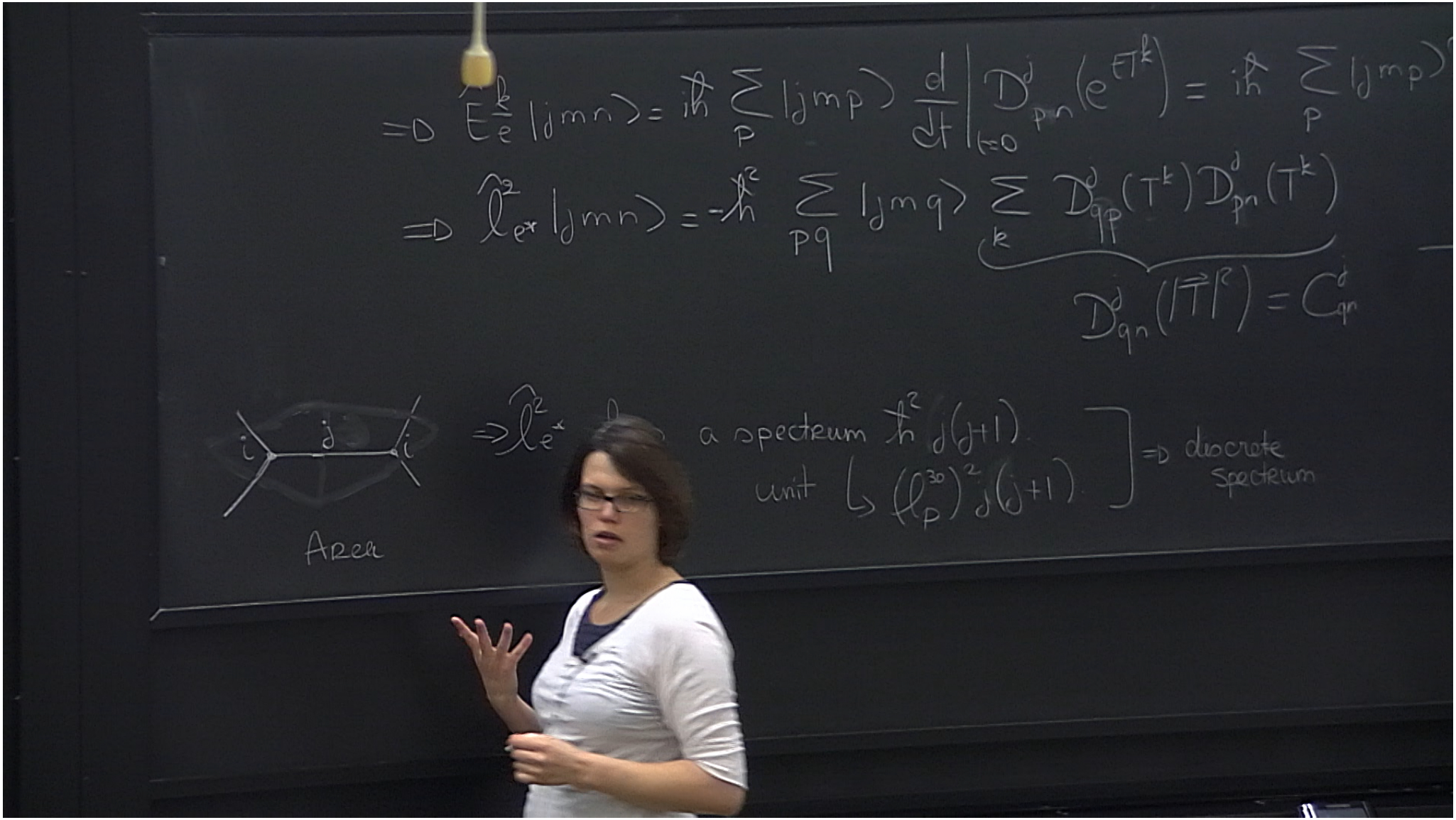

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^k)$
$\Rightarrow \hat{l}^2_{ex} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
$\rightarrow [C^j_{qn}, D^j_{mt}(T^k)] = 0$
$T = \frac{i\sigma}{2}$
$\Rightarrow \hat{l}^2_{ex}$ has a spectrum $\hbar^2 j(j+1)$

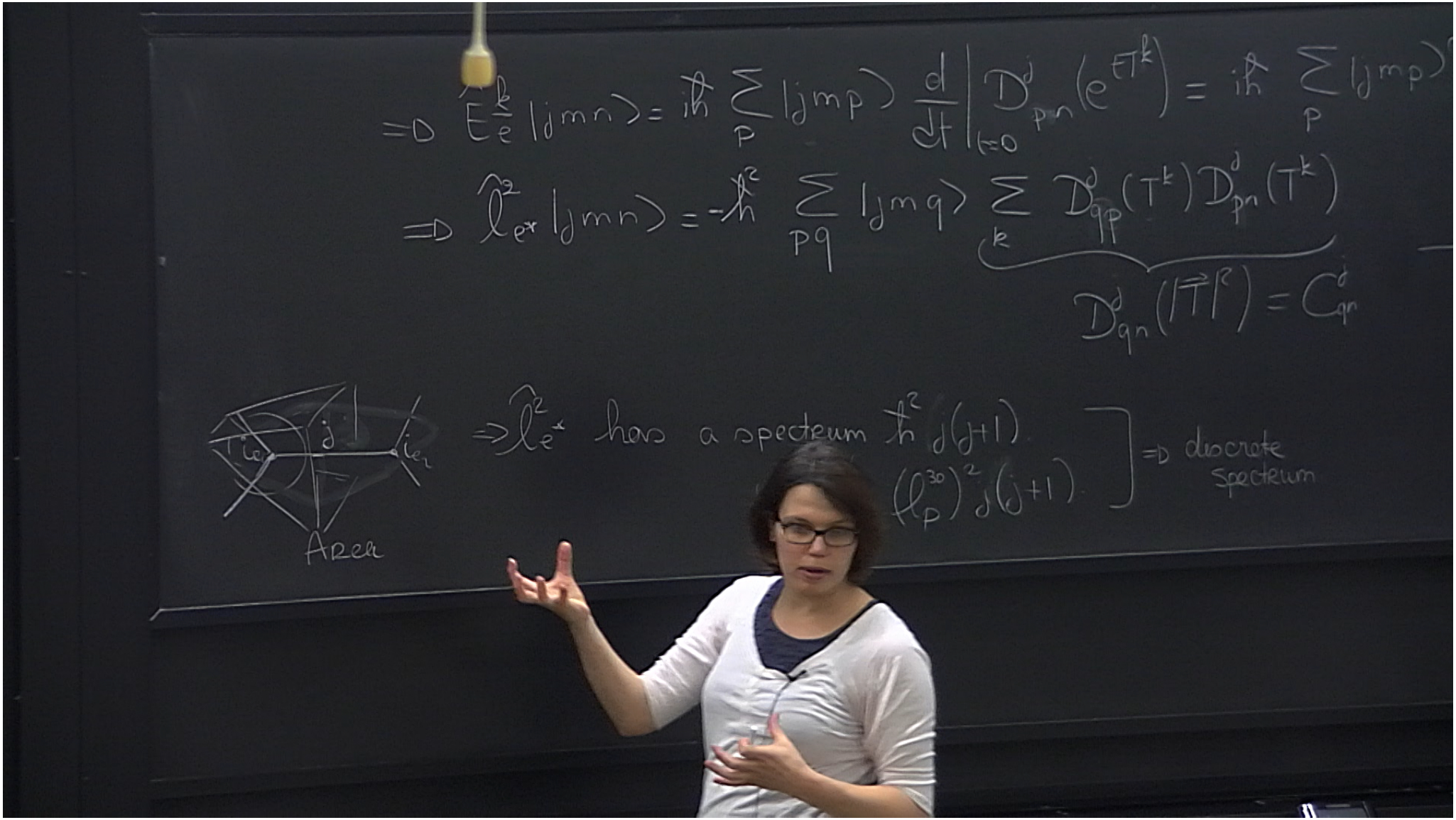

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle D^j_{pn}(T^k)$
$\Rightarrow \hat{l}^2_{ex} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
$\rightarrow [C^j_{qn}, D^j_{mt}(T^k)] = 0$
$T = \frac{i\sigma}{2}$
$\Rightarrow \hat{l}^2_{ex}$ has a spectrum $\hbar^2 j(j+1)$
unit $\hookrightarrow (l_p^{3D})^2 j(j+1)$
$\Rightarrow$ discrete spectrum

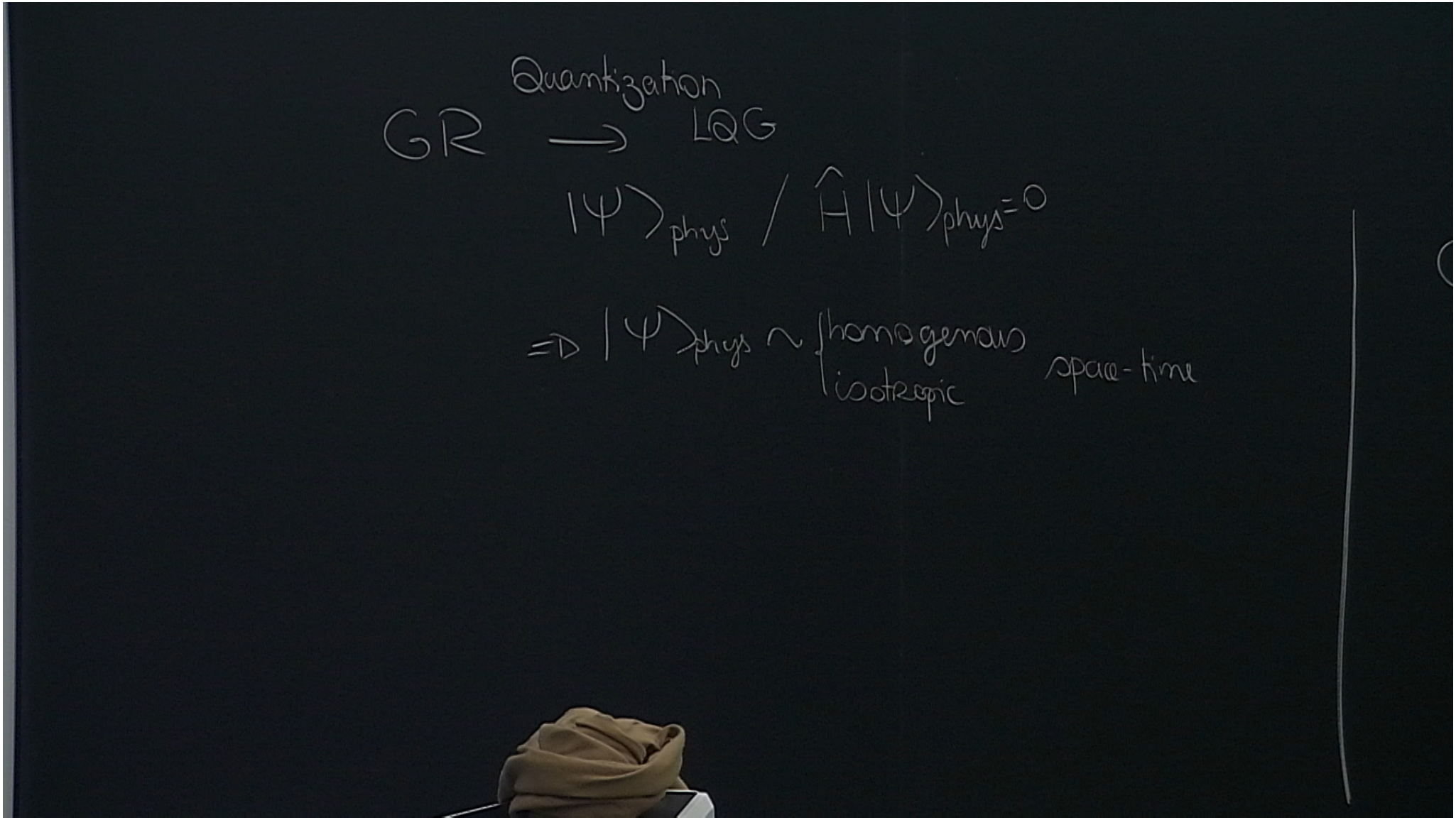

$\Rightarrow \hat{E}^k_e |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_{ext} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
$\Rightarrow \hat{l}^2$ has a spectrum $\hbar^2 j(j+1)$
unit $\hookrightarrow (l_P^{3D})^2 j(j+1)$
$\Rightarrow$ discrete spectrum

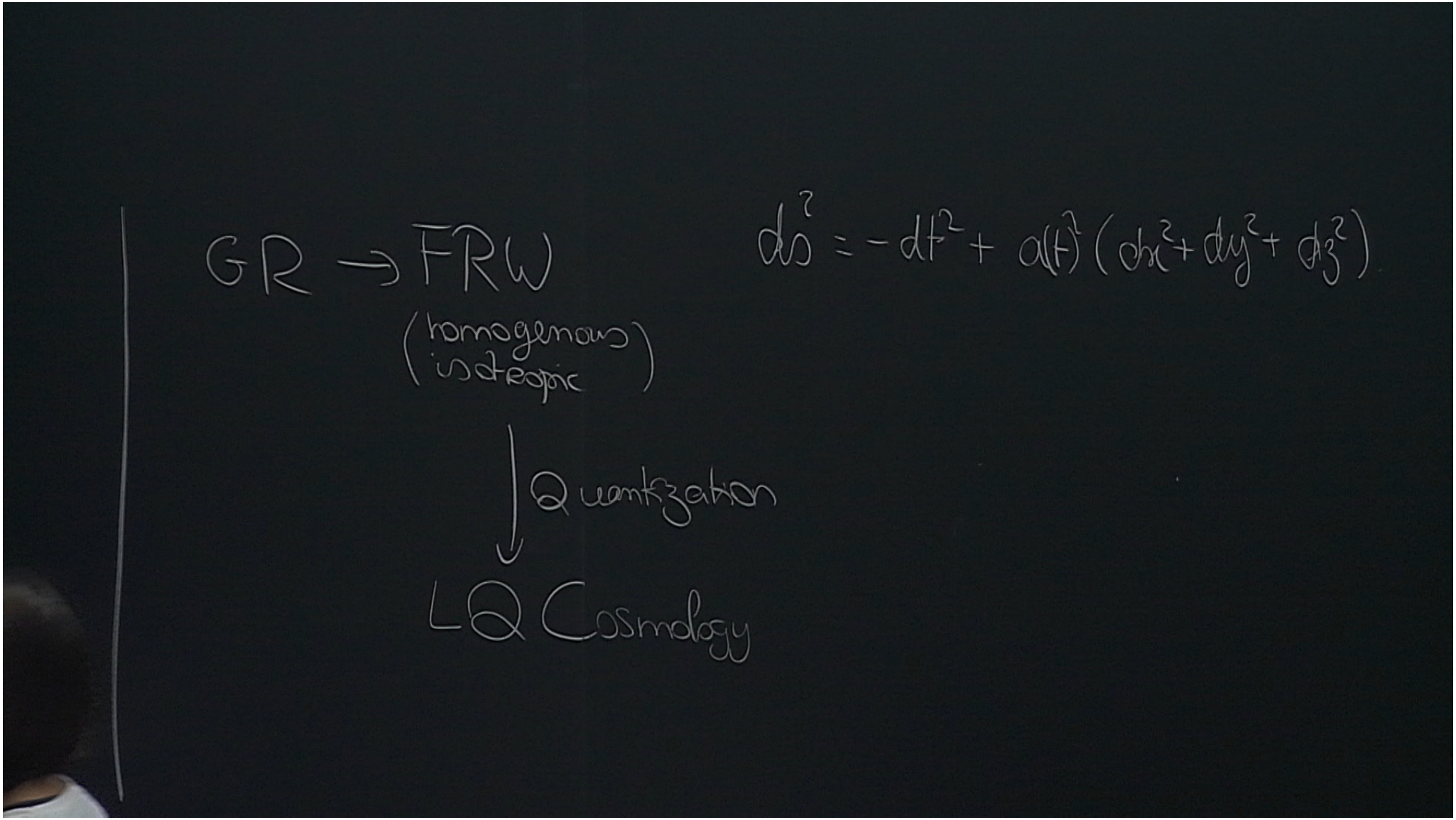

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_{ext} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
E_e
Area
$\Rightarrow$ ... have a spectrum $\hbar^2 j(j+1)$
unit $\hookrightarrow (l_P^{30})^2 j(j+1)$] $\Rightarrow$ discrete spectrum

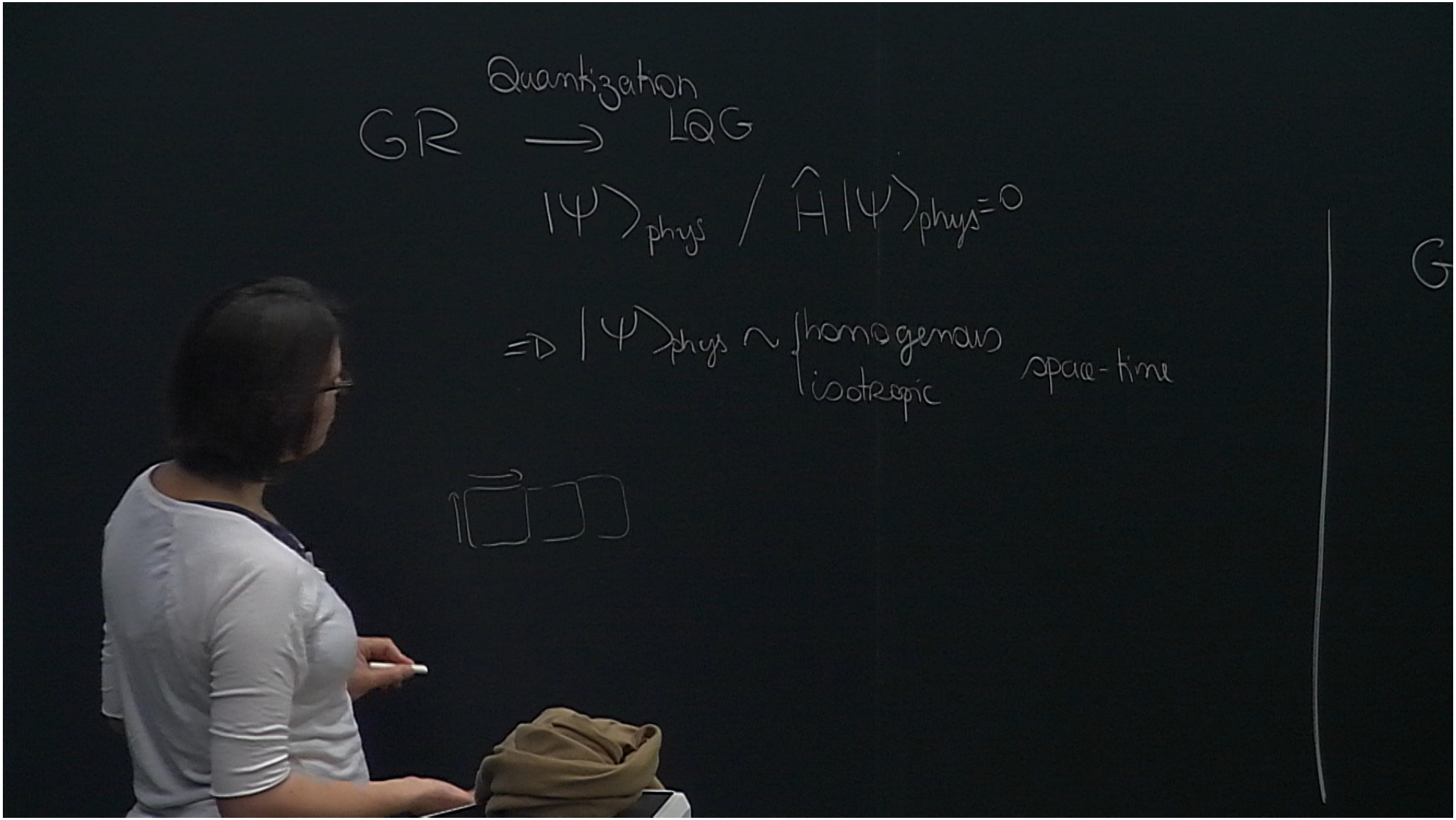

$\Rightarrow \hat{E}_e^k |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_{ext} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
E_e
Area
$\Rightarrow \hat{l}^2_{ext}$ have a spec
$\hbar^2 j(j+1)$
$(l_p^{3D})^2 j(j+1)$
$\Rightarrow$ discrete spectrum

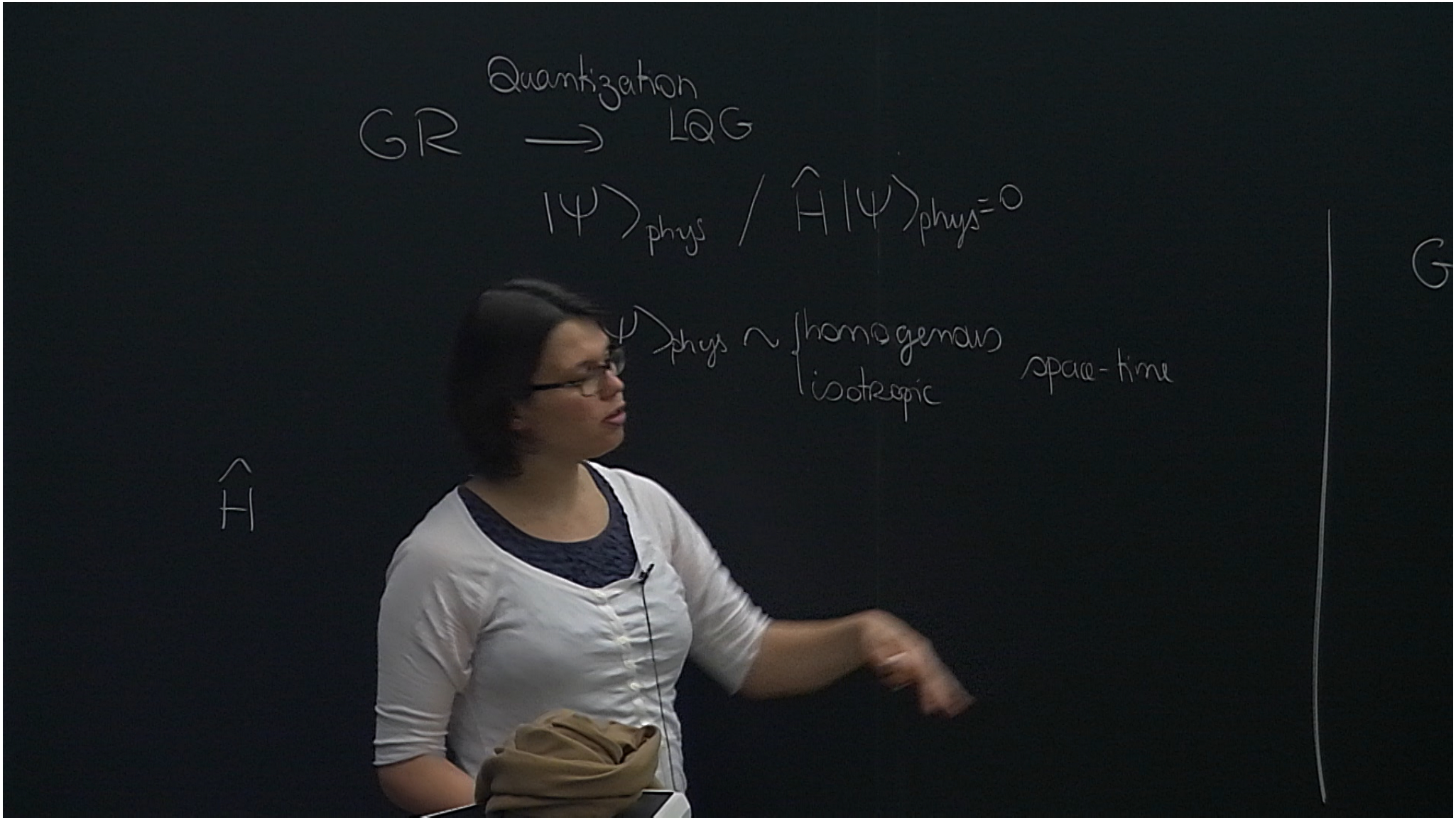

$\Rightarrow \hat{E}^k_e |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_e |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
i
j
i
Area
$\Rightarrow \hat{l}^2_e$
a spectrum $\hbar^2 j(j+1)$
unit $\hookrightarrow (\ell_P^{30})^2 j(j+1)$
$\Rightarrow$ discrete spectrum

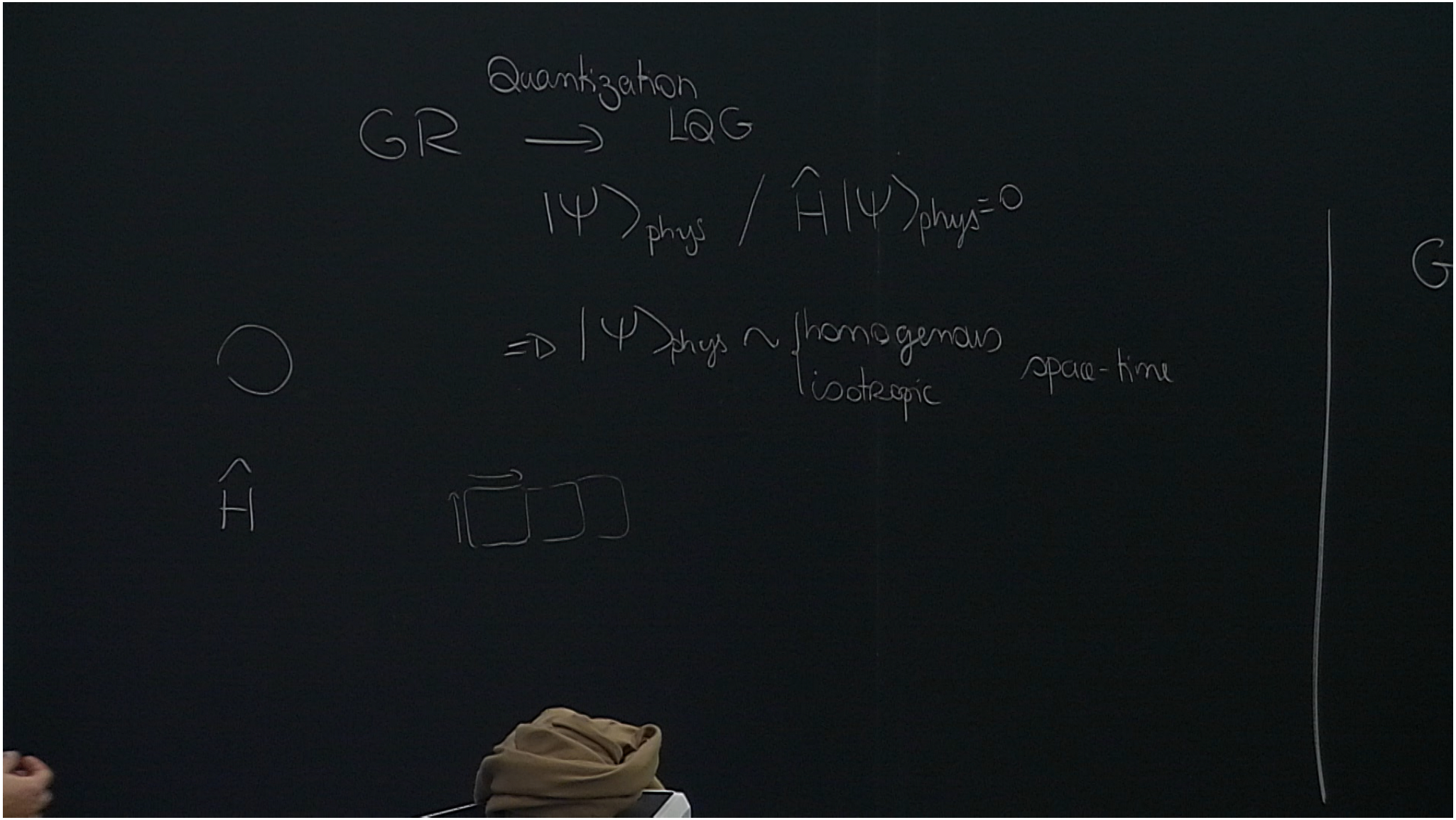

$\Rightarrow \hat{E}^k_e |jmn\rangle = i\hbar \sum_p |jmp\rangle \frac{d}{dt}\Big|_{t=0} D^j_{pn}(e^{tT^k}) = i\hbar \sum_p |jmp\rangle$
$\Rightarrow \hat{l}^2_{ext} |jmn\rangle = -\hbar^2 \sum_{pq} |jmq\rangle \sum_k D^j_{qp}(T^k) D^j_{pn}(T^k)$
$D^j_{qn}(|\vec{T}|^2) = C^j_{qn}$
$\Rightarrow \hat{l}^2_{ext}$ have a spectrum $\hbar^2 j(j+1)$
$(l_P^{30})^2 j(j+1)$
$\Rightarrow$ discrete spectrum
Area

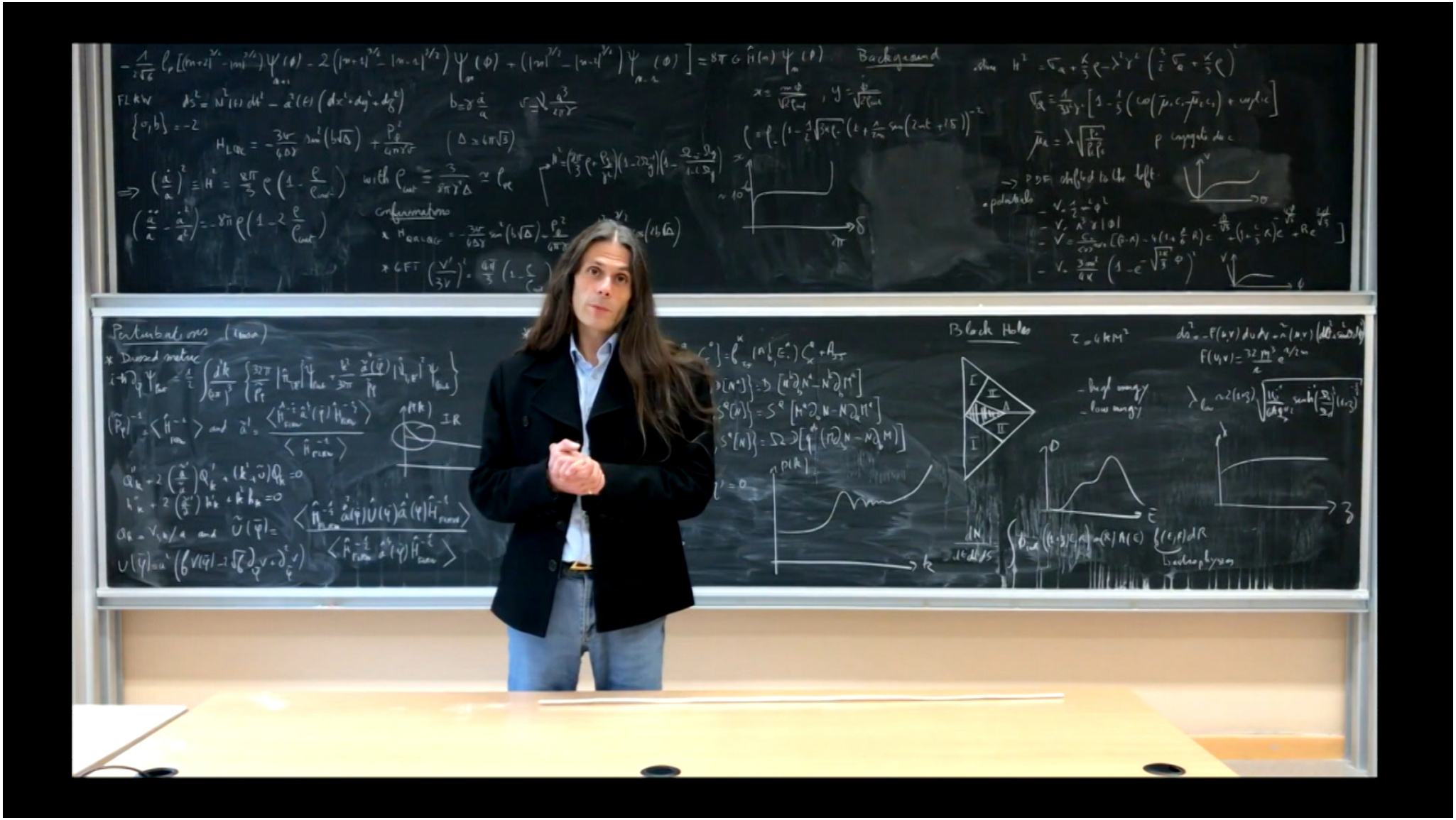

Quantization
GR ⟶ LQG
$|\Psi\rangle_{phys}$ / $\hat{H}|\Psi\rangle_{phys}=0$
$\Rightarrow |\Psi\rangle_{phys} \sim$ homogeneous isotropic space-time

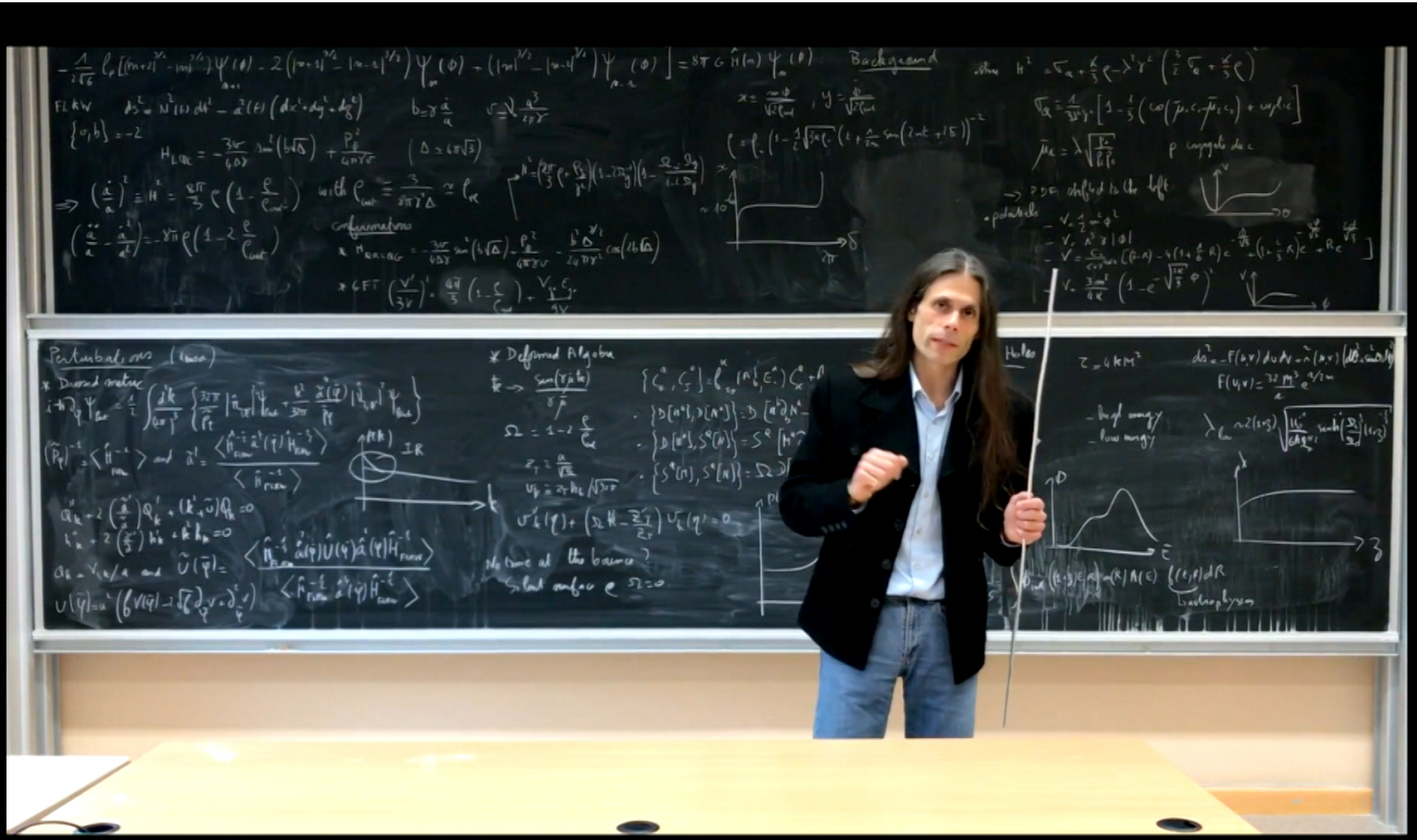

GR → FRW
(homogenous isotropic)
$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$
Quantization
LQC Cosmology

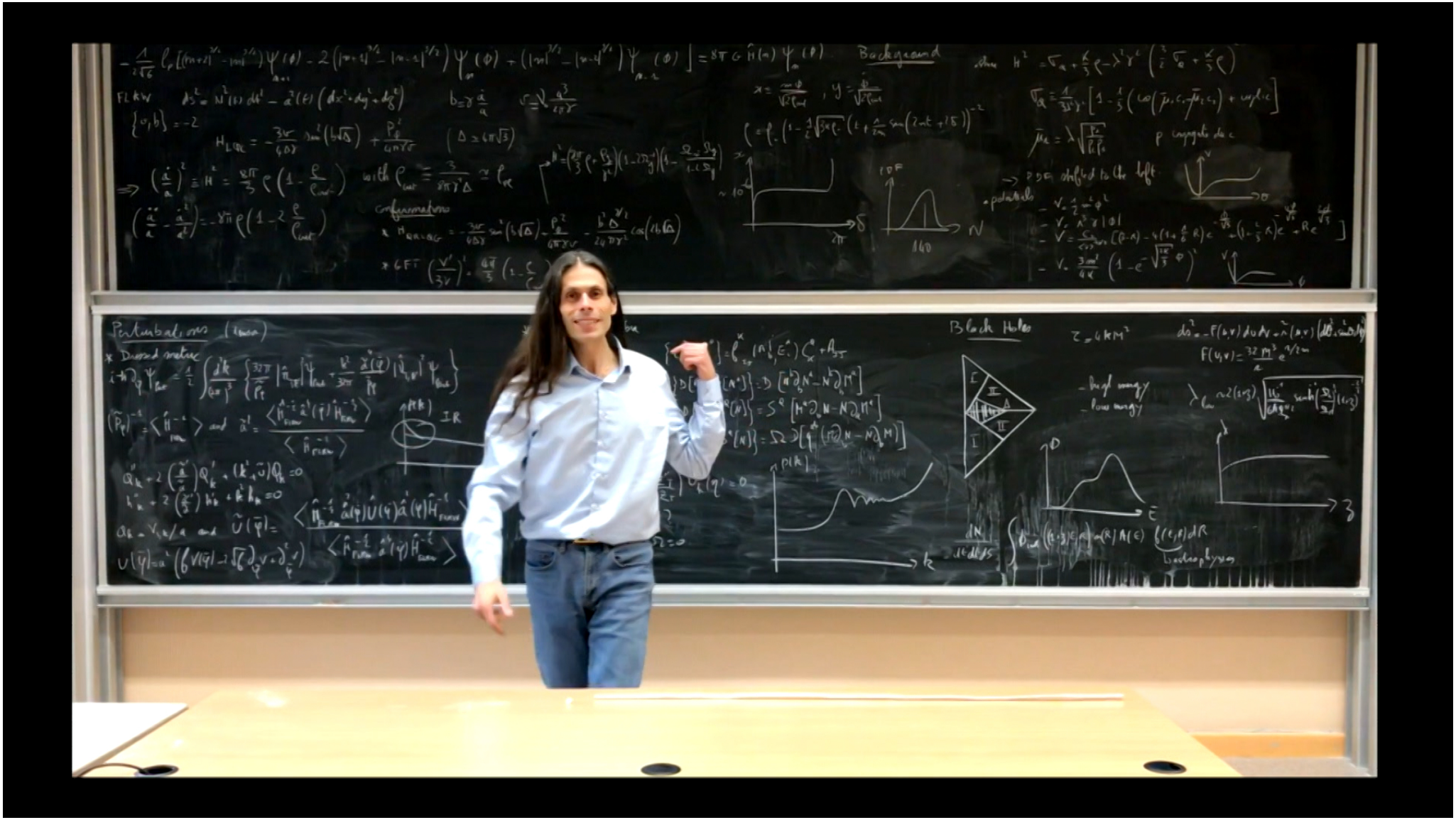

Quantization
GR → LQG
$|\psi\rangle_{phys}$ / $\hat{A}|\psi\rangle_{phys}=0$
$\Rightarrow |\psi\rangle_{phys} \sim$ {homogenous isotropic space-time
G


Quantization
GR → LQG
$|\psi\rangle_{phys}$ / $\hat{H}|\psi\rangle_{phys}=0$
$\psi\rangle_{phys}$ ~ homogenous isotropic space-time
$\hat{H}$
G


Quantization
GR → LQG
$|\psi\rangle_{phys}$ / $\hat{H}|\psi\rangle_{phys}=0$
$\Rightarrow |\psi\rangle_{phys} \sim$ {homogenous isotropic space-time
$\hat{H}$
G


Background
FLRW
Perturbations
Bloch Holes


Background
FLRW
Perturbations
Deformed Algebra
Holes
No time at the bounce?
Singularity
Page 42/43


Background
FLRW
Perturbations
Bloch Hades
PDF
Boltzmann hierarchy