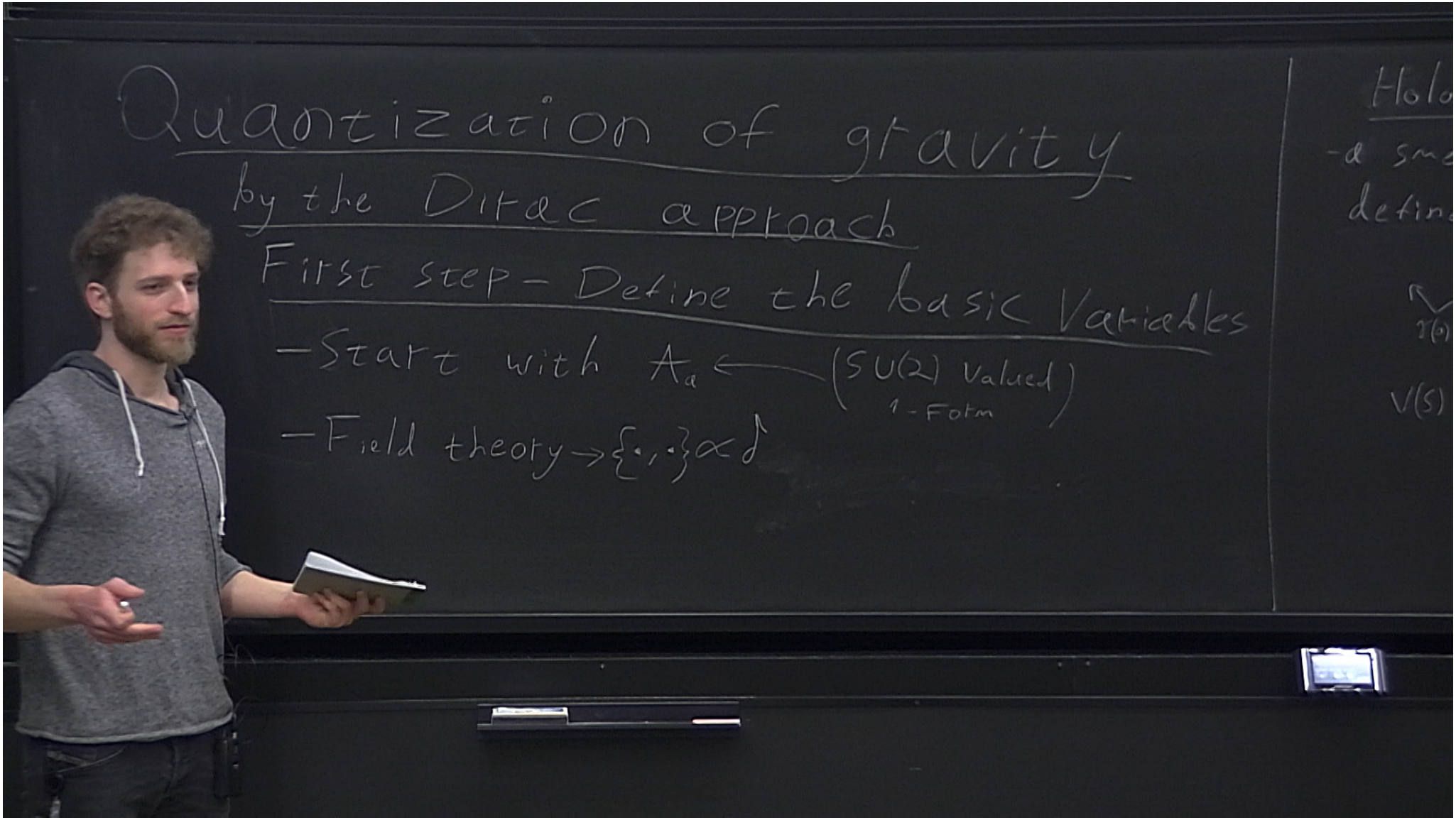
Title: PSI 2016/2017 Explorations in Quantum Gravity - Lecture 8

Date: Mar 29, 2017 10:15 AM

URL: http://pirsa.org/17030075

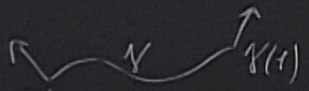
Abstract:


Quantization of gravity
by the Dirac approach
First step - Define the basic Variables
- Start with A_a ← (SU(2) Valued 1-Form)
- Field theory → {·,·}
Holo
- a sm
defin
V(S)

Holonomy

- a smearing of A_a

defined as the Parallel tr. of a v. along a curve

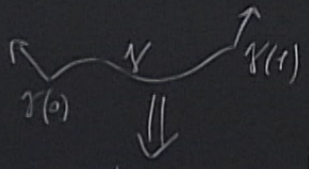


$$V(s) = h(s)\, V(0)\, h^{-1}(s)$$

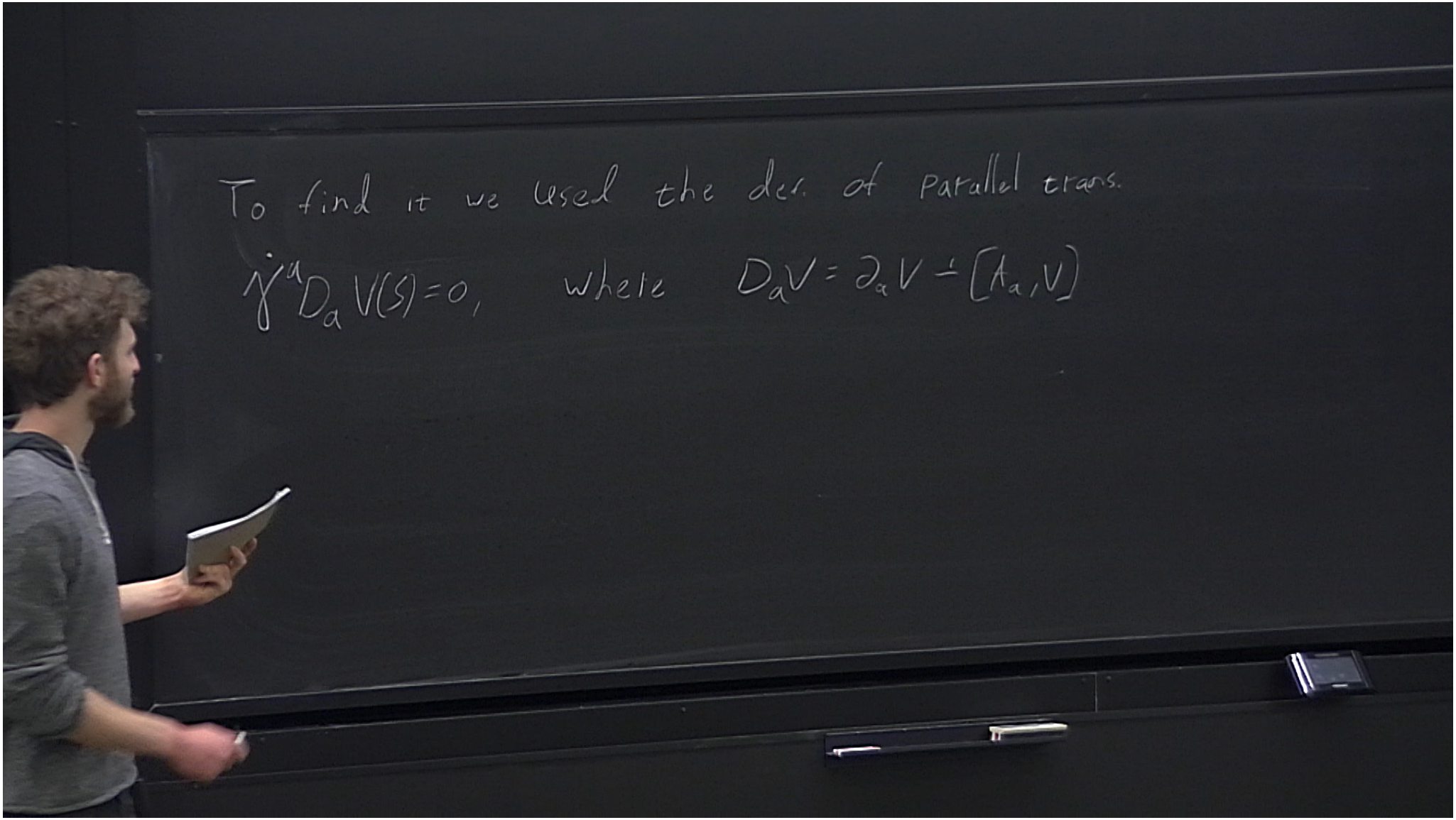
Holonomy

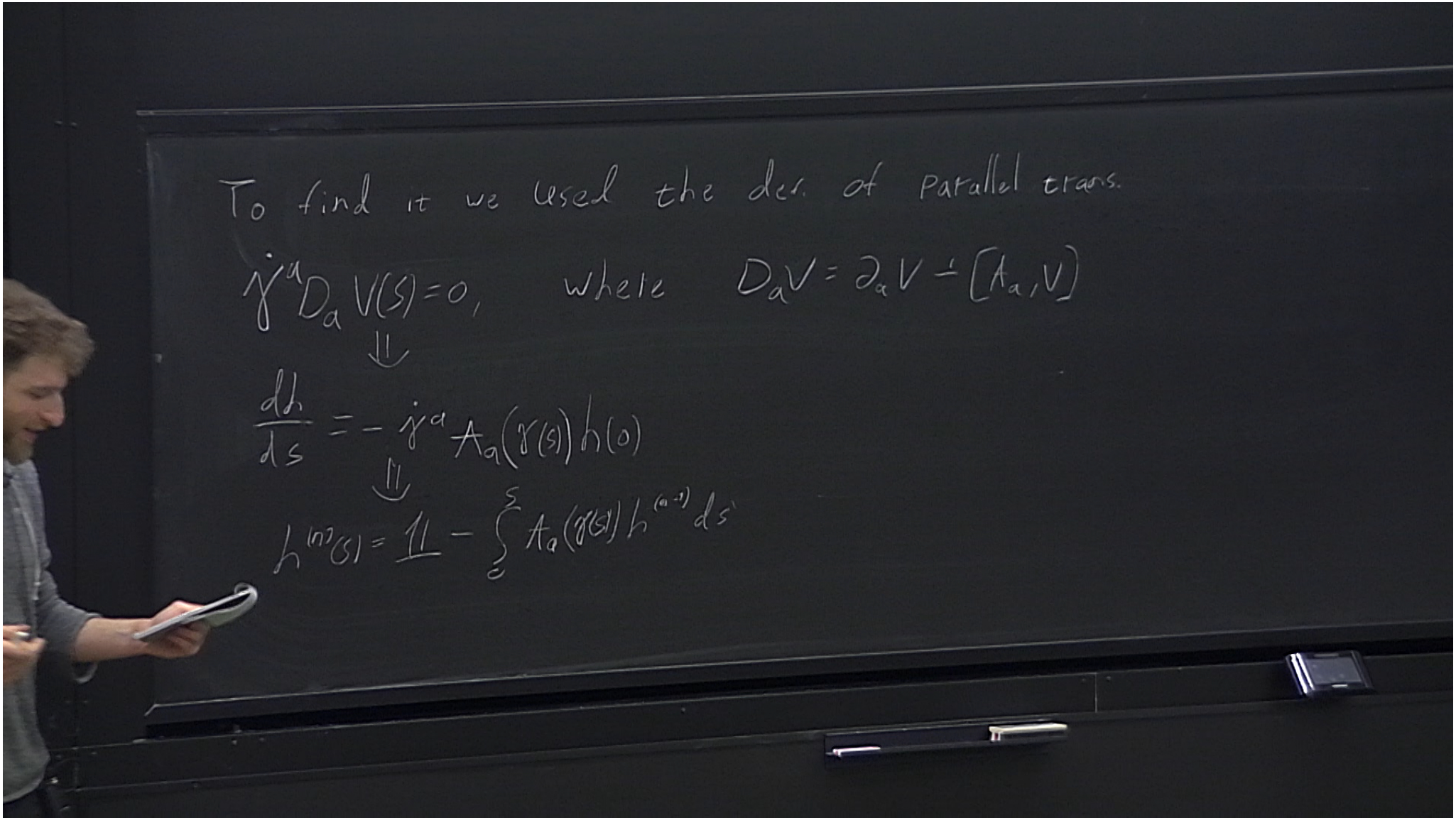
- a smearing of A_a

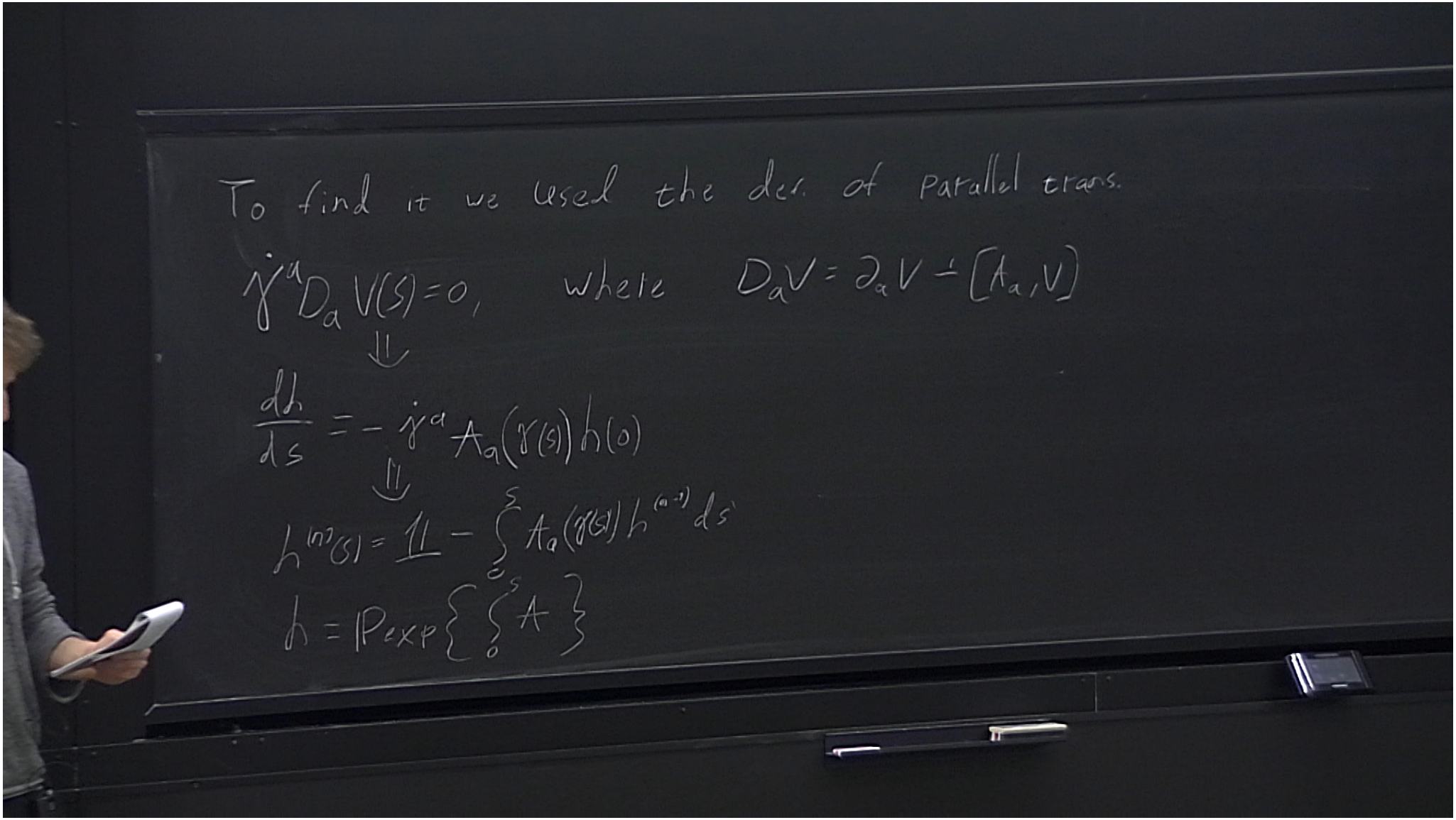
defined as the Parallel tr. of a v. along a curve



$$V(s) = h(s)\, V(0)\, h^{-1}(s)$$


To find it we used the def. of parallel trans.
$\dot{\gamma}^a D_a V(s) = 0$, where $D_a V = \partial_a V + [A_a, V]$


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$\dot\gamma^a D_a V(s) = 0$, where $D_a V = \partial_a V + [A_a, V]$
$\frac{dh}{ds} = -\dot\gamma^a A_a(\gamma(s)) h(0)$
$h^{(n)}(s) = \mathbb{1} - \int_0^s A_a(\gamma(s)) h^{(n-1)} ds'$


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$h = \mathrm{P}\exp\left\{\int_0^s A\right\}$

- Basic variables: flux
- Quantization of edges

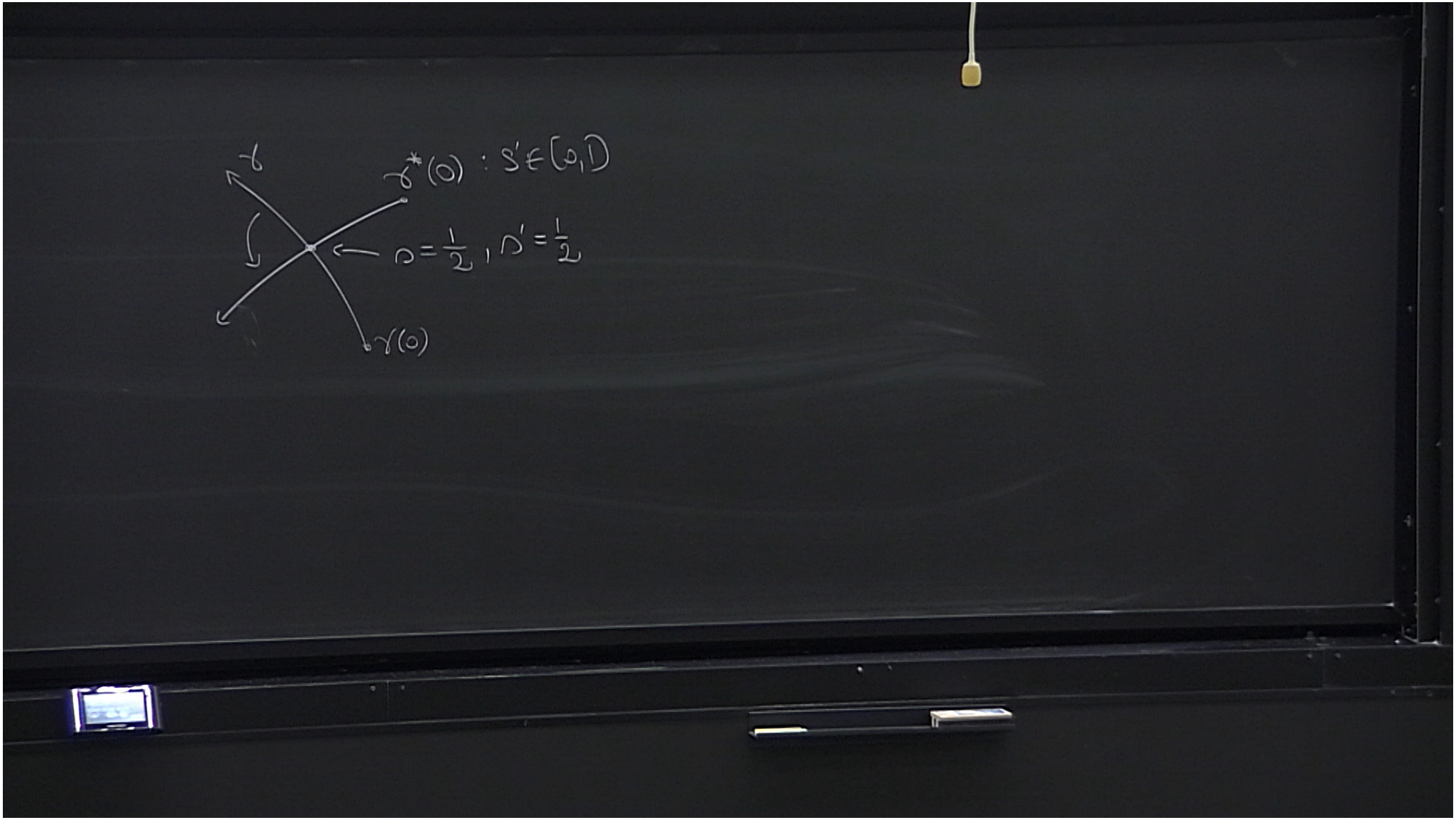
] 1st step of DIRAC program.

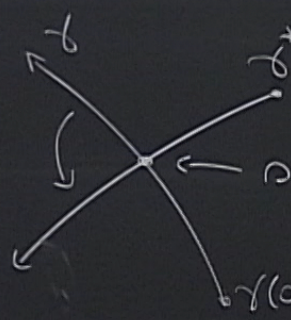
holonomy: $h_\gamma[A]$

flux $E_\gamma = \int_{\gamma^*} h_{\gamma\gamma^*(s)}\, e_a(\gamma^*(s))\, \dot{\gamma}^{*a}(s)\, (h_{\gamma\gamma^*(s)})^{-1}\, ds$

$\hookrightarrow \begin{cases} e_a = e_a^k T_k \\ e_a = \tilde{\Sigma}_{ba} E^{bd} T_d \end{cases}$

$\hookrightarrow$ //


γ
$\gamma^*(0) : s' \in [0,1)$
$s = \frac{1}{2}, s' = \frac{1}{2}$
$\gamma(0)$



- not at the end points
- transversal crossing

holonomy $h_\gamma[A]$

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$\hookrightarrow \begin{cases} e_a = e_a^k T_k \\ e_a = \tilde{\Sigma}_{ba} E^{bd} T_d \end{cases}$

$\hookrightarrow$ // transport from $\gamma^*(s)$

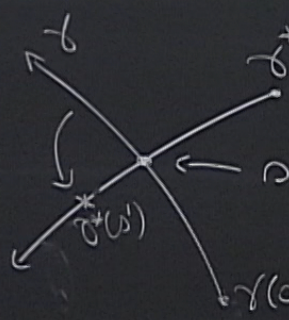
holonomy $h_\gamma[A]$

flux $E_\gamma = \int_{\gamma^*} h_{\gamma\gamma^*(s)}\, e_a(\gamma^*(s))\, \dot{\gamma}^{*a}(s)\, (h_{\gamma\gamma^*(s)})^{-1}\, ds$

$e_a = e_a^k T_k$

$e_a = \tilde{\Sigma}_{ba} E^{bj} T_j$

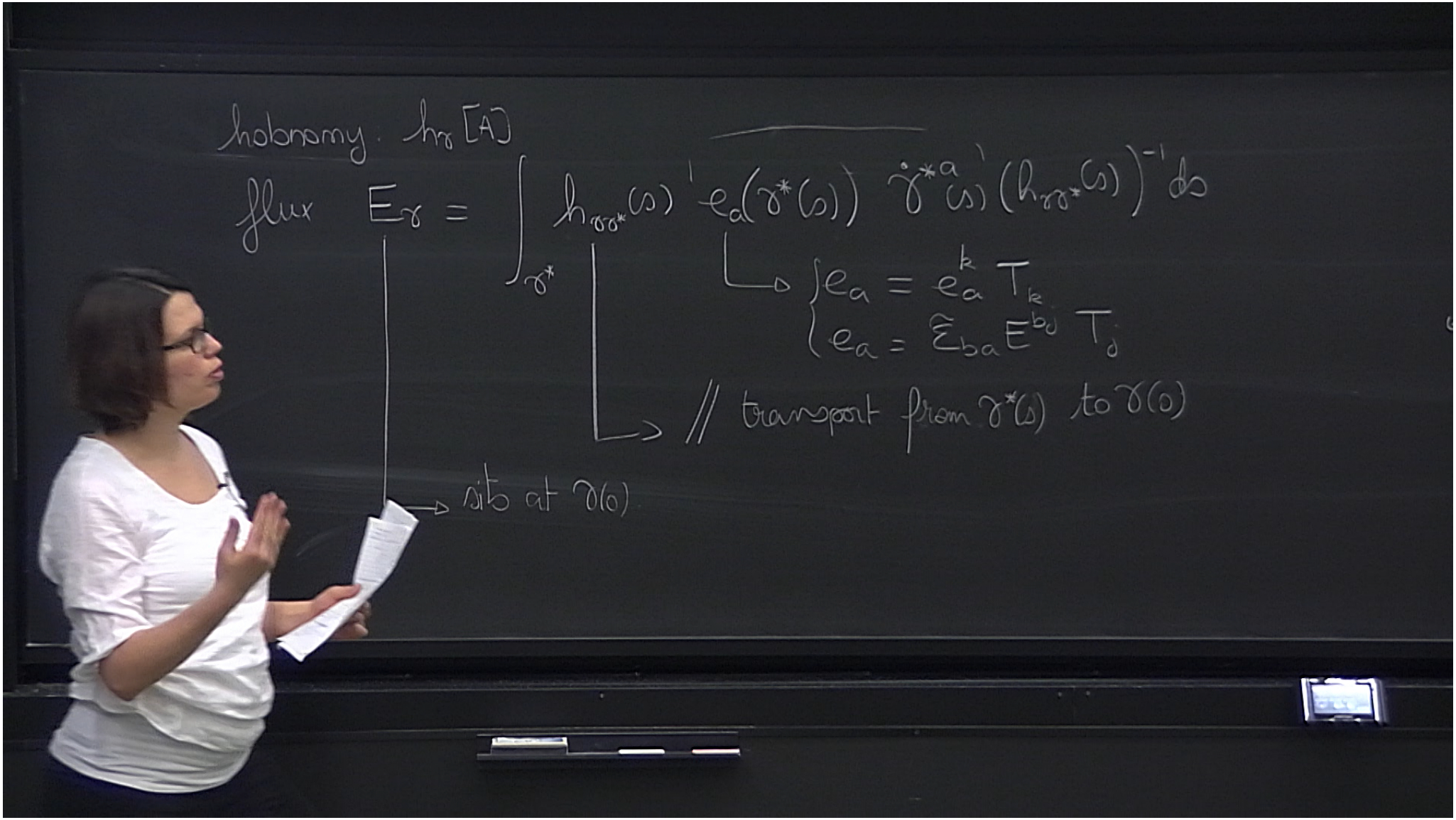
$h_{\gamma\gamma^*(s)}$ // transport from $\gamma^*(s)$ to $\gamma(0)$

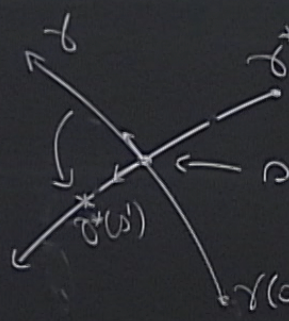


$\gamma^*(0) : s' \in (0,1)$

$s = \frac{1}{2}, s' = \frac{1}{2}$

- not at the end points
- transversal crossing


holonomy $h_\gamma[A]$
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$e_a = e_a^k T_k$
$e_a = \tilde{\Sigma}_{ba} E^{bd} T_d$
// transport from $\gamma^*(s)$ to $\gamma(0)$
sits at $\gamma(0)$



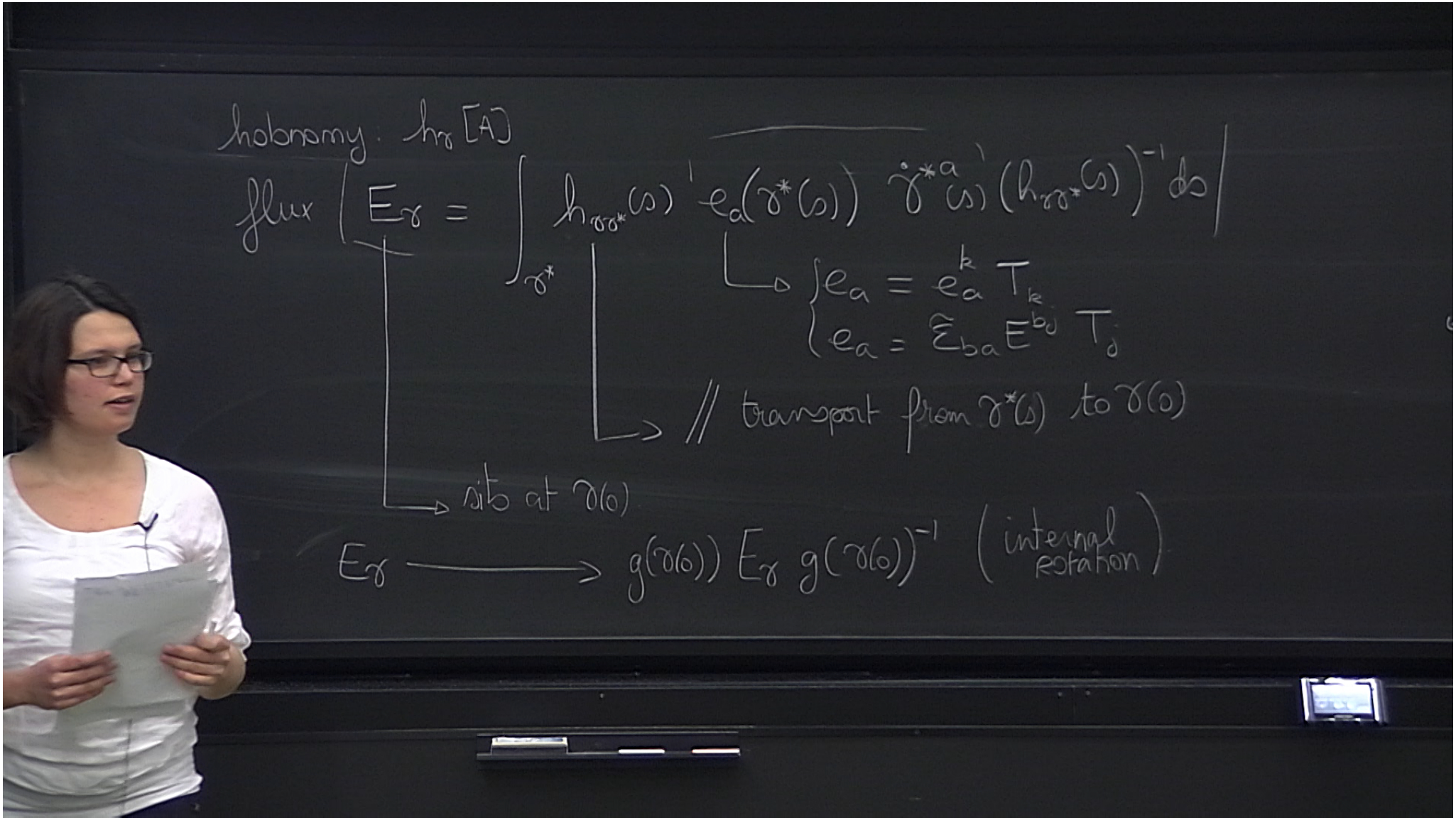
$s' \in (0,1)$

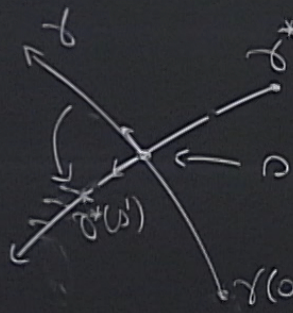
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- not at the end points
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$$\det(\dot{\gamma}^a \; \dot{\gamma}^{*b}) > 0$$

$$= \det\begin{pmatrix} \dot{\gamma}^1 & \dot{\gamma}^{*1} \\ \dot{\gamma}^2 & \dot{\gamma}^{*2} \end{pmatrix}$$


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flux $E_\gamma = \int_{\gamma^*} h_{\gamma\gamma^*(s)} \dot{e}_a(\gamma^*(s)) \dot{\gamma}^{*a}(s) (h_{\gamma\gamma^*(s)})^{-1} ds$
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// transport from $\gamma^*(s)$ to $\gamma(0)$
sits at $\gamma(0)$
$E_\gamma \longrightarrow g(\gamma(0)) E_\gamma g(\gamma(0))^{-1}$ (internal rotation)



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Poisson algebra

$T_j = \frac{i}{2}\sigma_j$

→ metric on the Lie algebra $\mathrm{tr}(T_j T_k) = -\frac{1}{2}\delta_{jk}$ (negative definite)

extract component $E_j = (-2)\,\mathrm{tr}(\underbrace{E}_{E_k T^k}\, T_j)$

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so we can define the flux component

$$(E_\alpha)_j = \int (-2)\,\mathrm{tr}\left(T_j\, h_{\alpha x^a(\sigma)}\, T_k\, h_{\alpha x^a(\sigma)}^{-1}\right) \tilde{\varepsilon}_{ba}\, E^{bk}(\delta^a(\sigma))\, \dot{\delta}^a\, d\sigma$$

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Poisson algebra

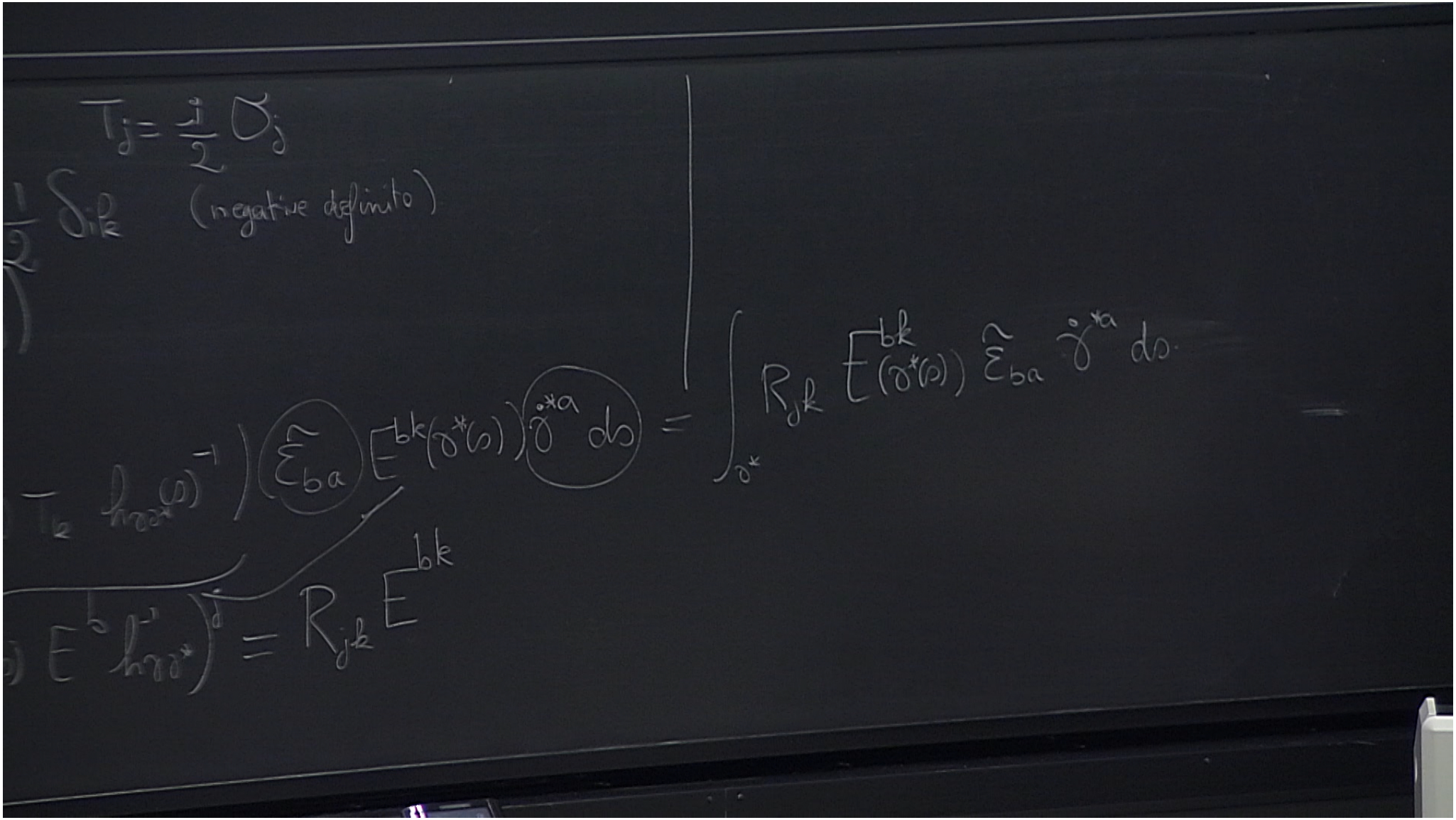
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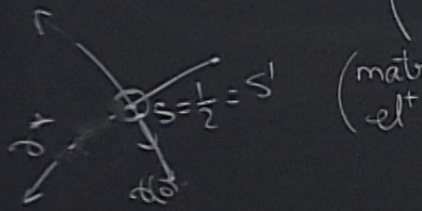
so we can define the flux component

$$(E_\sigma)_j = \int \underbrace{(-2)\,\mathrm{tr}\left(T_j\, h_{\sigma\sigma'}(\sigma)\, T_k\, h_{\sigma\sigma'}(\sigma)^{-1}\right)}_{\left(h_{\sigma\sigma'}(\sigma)\, E^b\, h^{-1}_{\sigma\sigma'}\right)^j = R_{jk} E^{bk}} \tilde{\varepsilon}_{ba}\, E^{bk}(\sigma'(\sigma))\, \partial^{*a}\, d\sigma = \int_{\sigma^*} R_{jk} E^{bk}$$


$T_j = \frac{i}{2} \sigma_j$
$\frac{1}{2} \delta_{ik}$ (negative definite)
$\tilde{\varepsilon}_{ba} E^{bk}(\sigma^*(\sigma)) \dot{\sigma}^{*a} d\sigma = \int_{\sigma^*} R_{jk} E^{bk}(\sigma^*(\sigma)) \tilde{\varepsilon}_{ba} \dot{\sigma}^{*a} d\sigma$
$= R_{jk} E^{bk}$

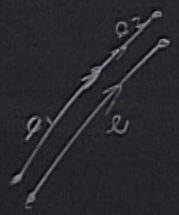

$\{(E_\gamma)_{\gamma'}, h_\gamma\}$
(matrix el$^+$)
$h_\gamma = h_\gamma$
$s=\frac{1}{2}=s'$
γ'
γ
α_2
α_1
e

$\{(E_\gamma)_0, h_\gamma\}$

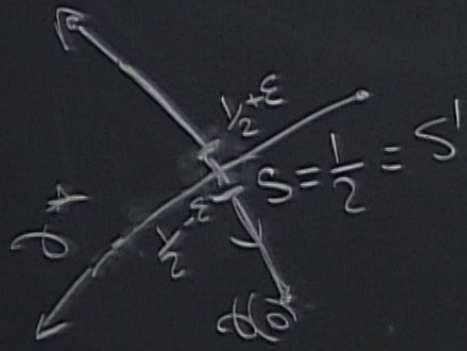


(matrix el^t)

$h_\gamma = h_\gamma\left(1, \frac{1}{2}+\frac{\varepsilon}{2}\right)\ h_\gamma\left(\frac{1}{2}+\frac{\varepsilon}{2}, \frac{1}{2}-\frac{\varepsilon}{2}\right)\ h_\gamma\left(\frac{1}{2}-\frac{\varepsilon}{2}, 0\right)$

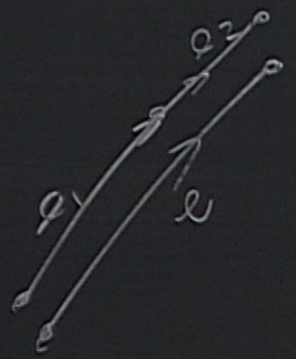


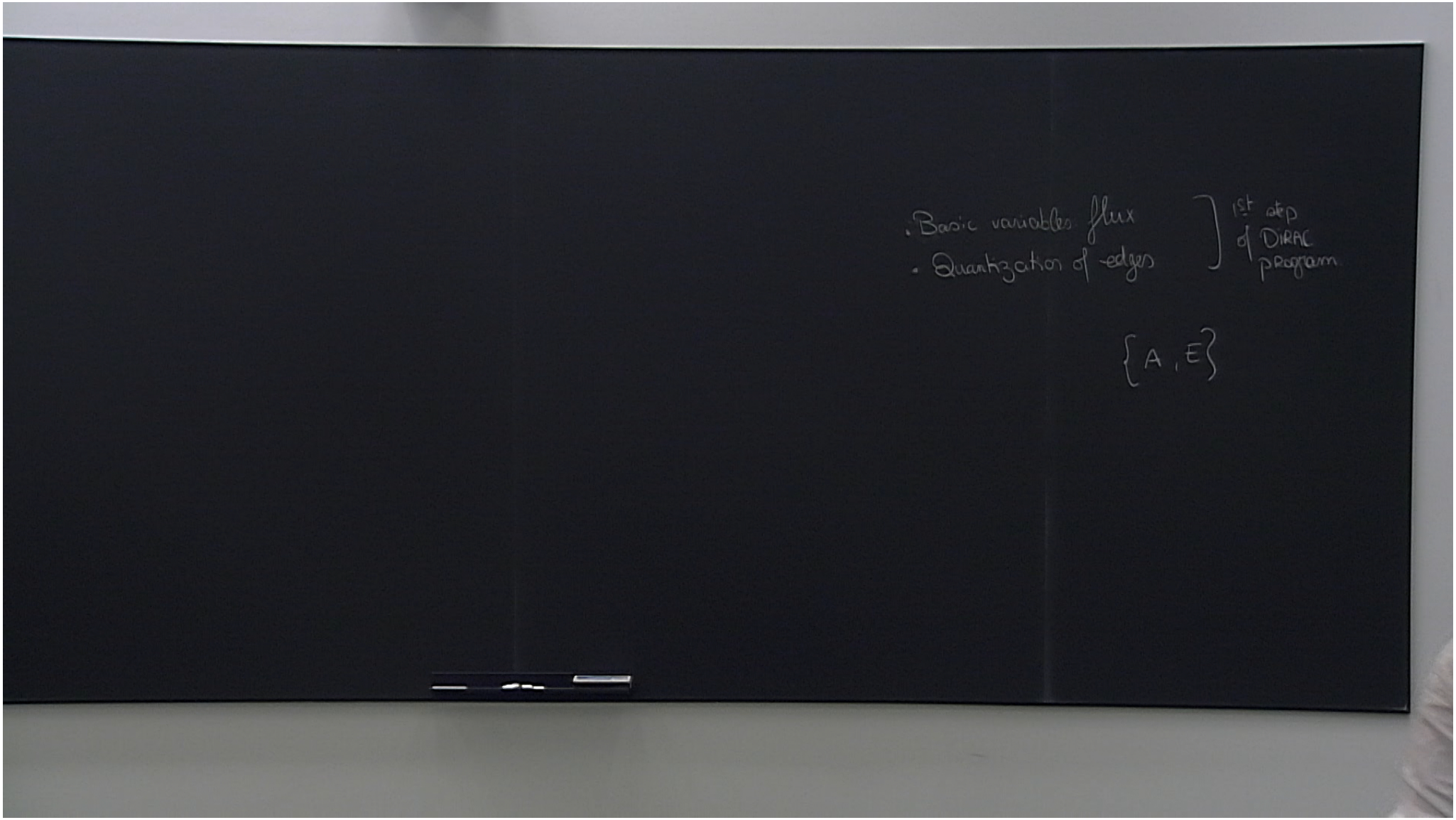
$\{(E_\gamma)_{ij}, h_\gamma\}$

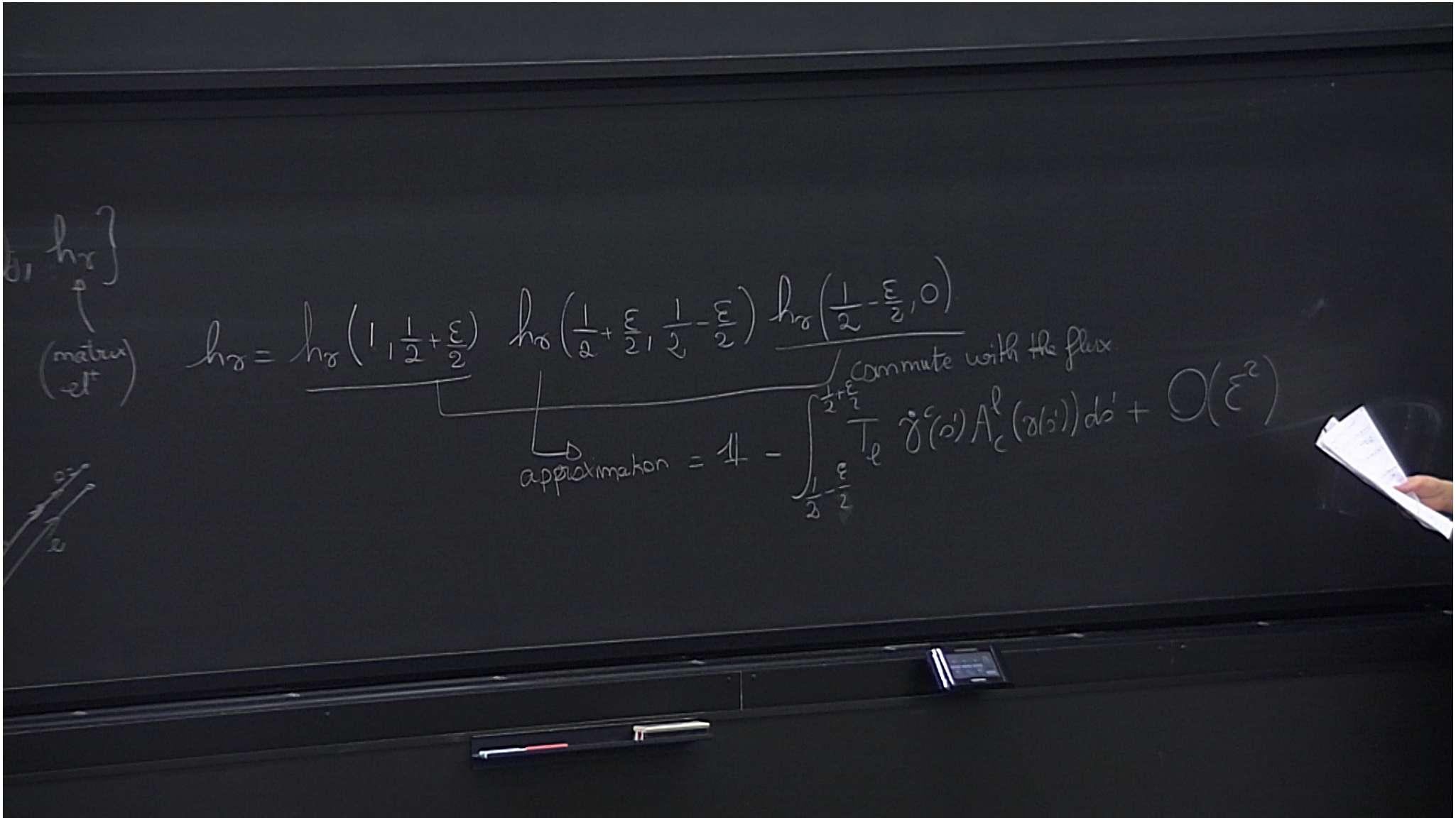


(matrix el$^+$)

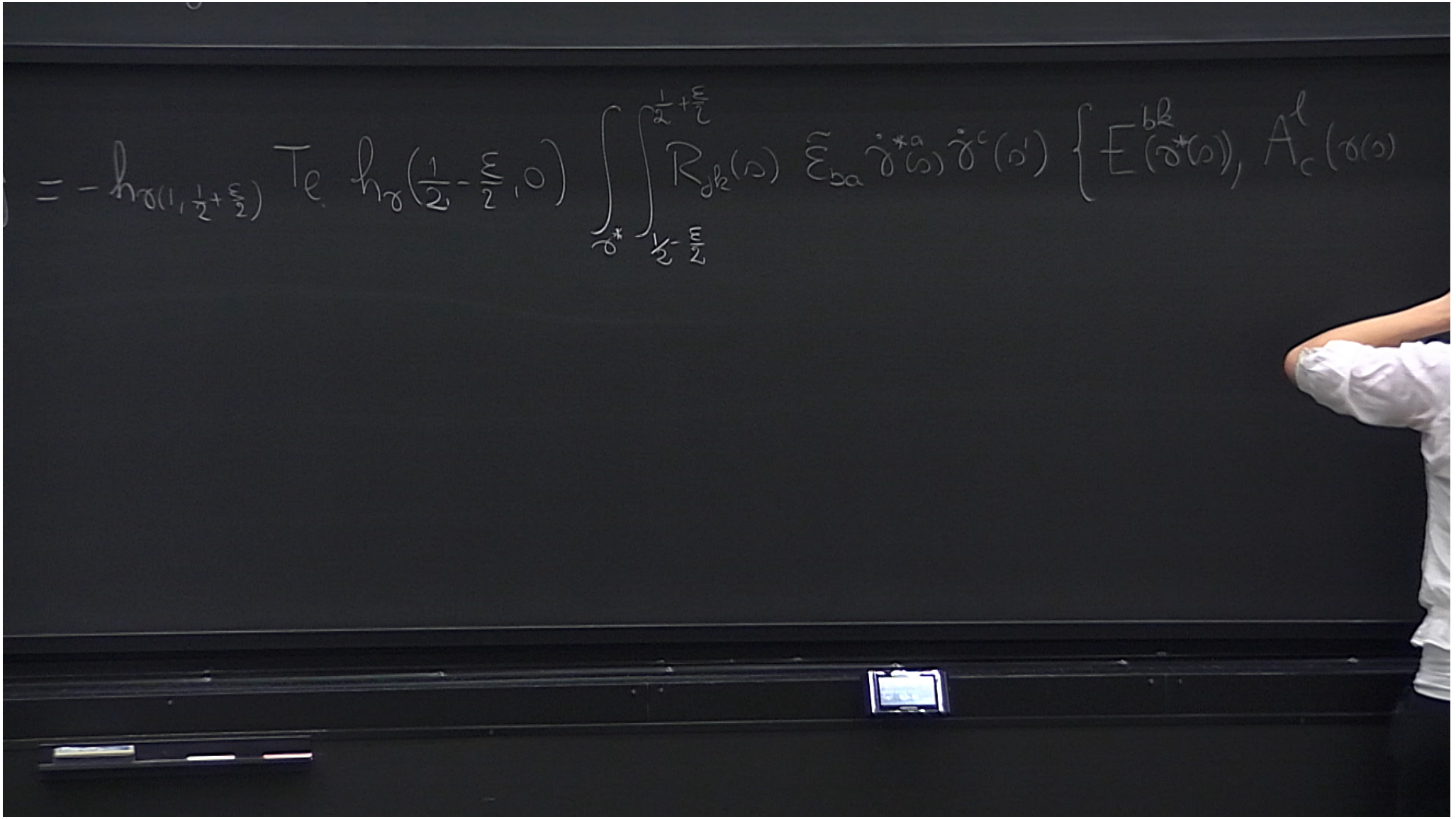
$h_\gamma = h_\gamma\left(1, \frac{1}{2}+\frac{\varepsilon}{2}\right)$

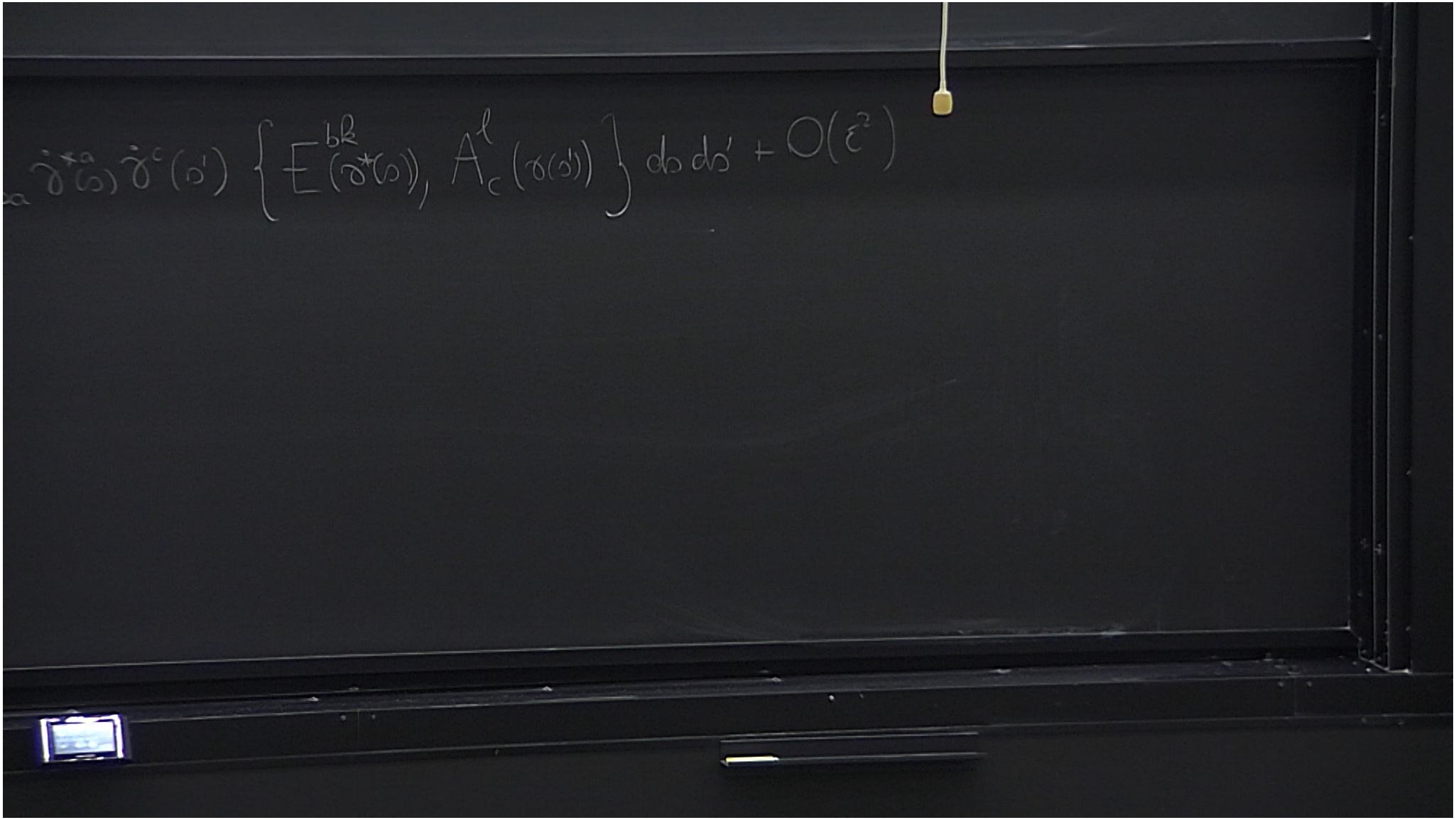


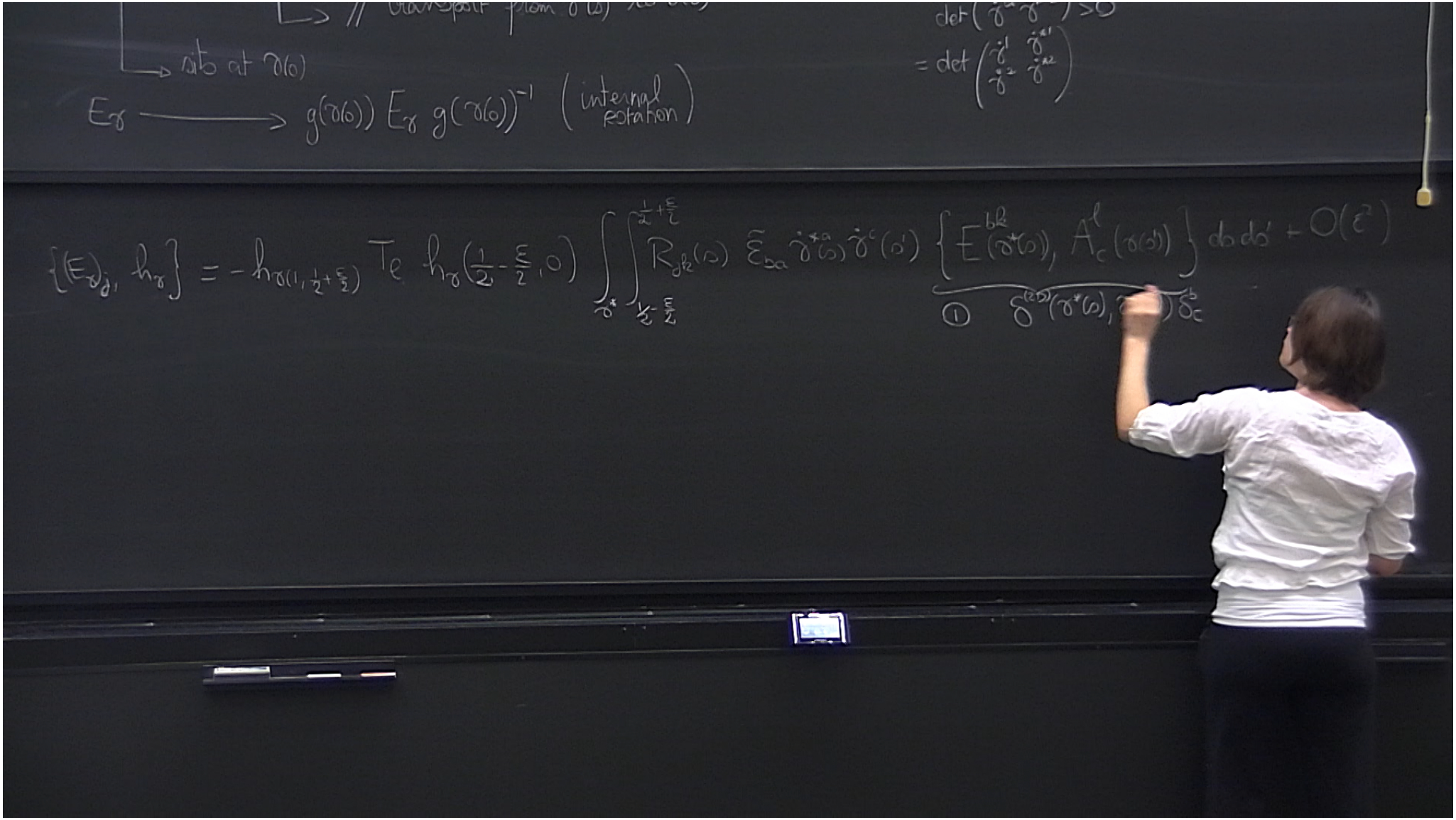

• Basic variables: flux
• Quantization of edges
1st step of DIRAC program
{A, E}

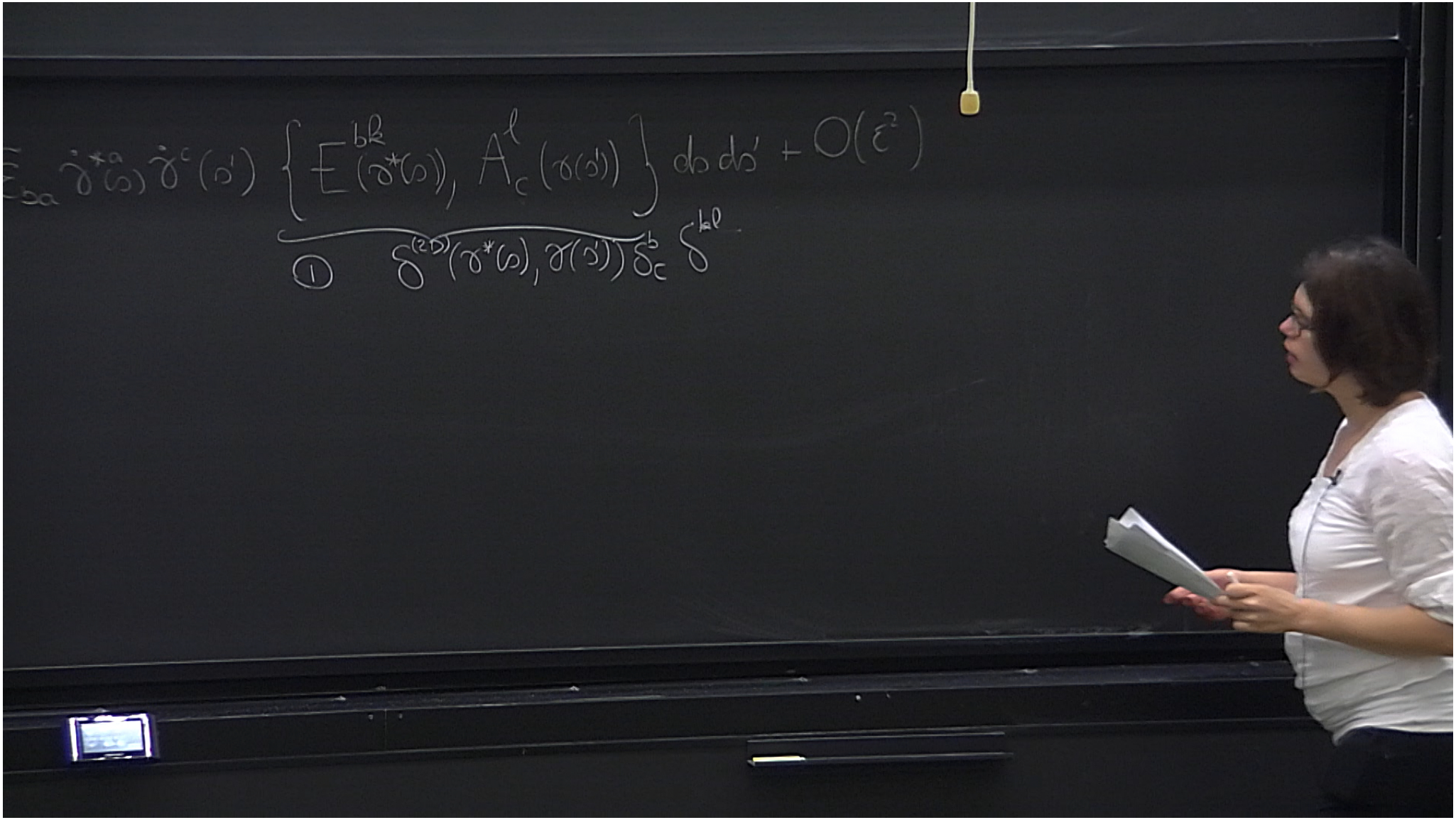

$h_\gamma = h_\gamma\left(1, \frac{1}{2}+\frac{\varepsilon}{2}\right) h_\gamma\left(\frac{1}{2}+\frac{\varepsilon}{2}, \frac{1}{2}-\frac{\varepsilon}{2}\right) h_\gamma\left(\frac{1}{2}-\frac{\varepsilon}{2}, 0\right)$
commute with the flux
approximation $= \mathbb{1} - \int_{\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} T_\ell \, \dot\gamma^c(\sigma') A_c^\ell(\gamma(\sigma')) \, d\sigma' + O(\varepsilon^2)$

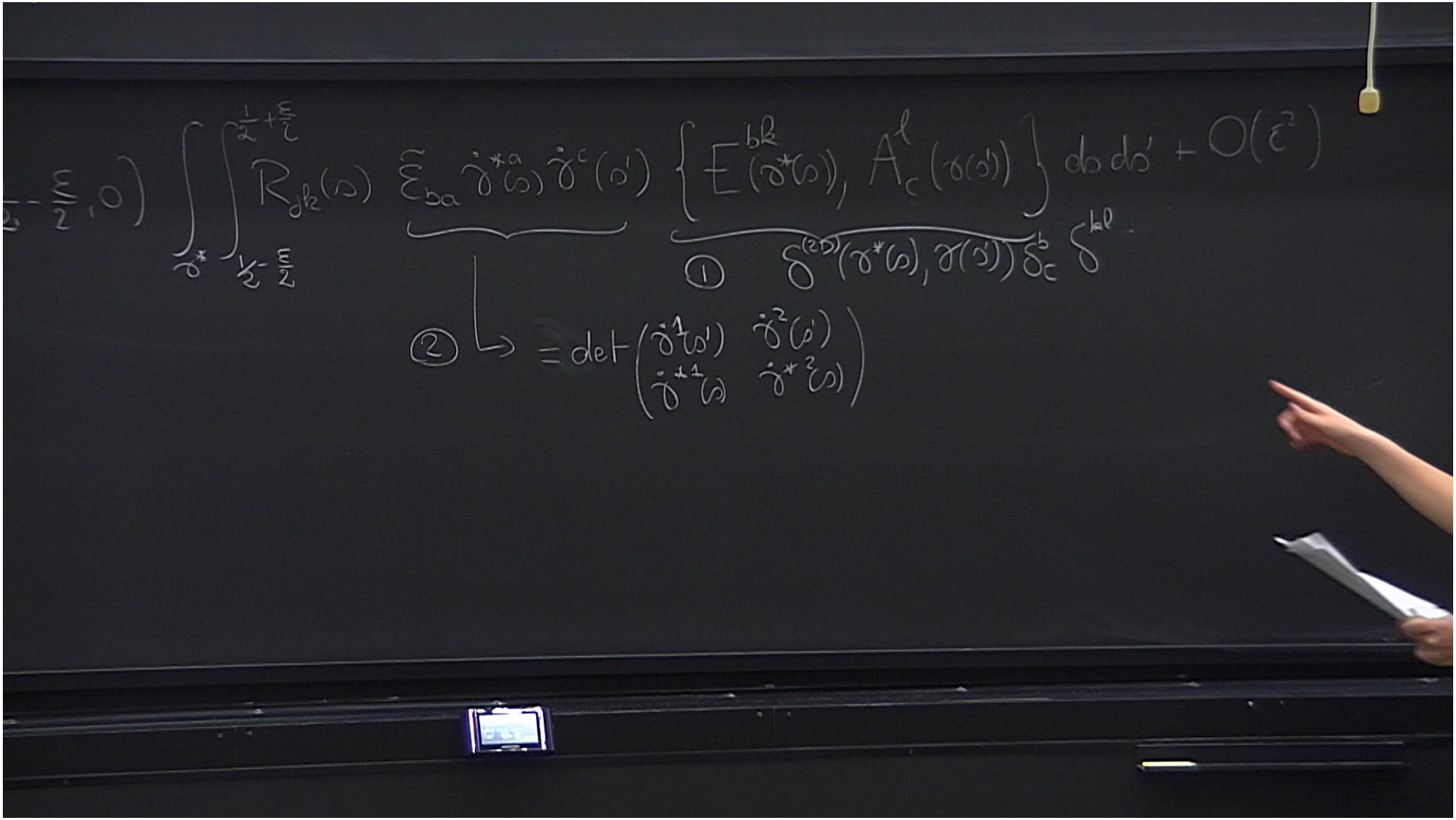
$$\{(E_\gamma)_j, h_\gamma\} = -h_{\sigma(1,\frac{1}{2}+\frac{\varepsilon}{2})}\, Te\, h_\gamma\left(\frac{1}{2}-\frac{\varepsilon}{2},0\right) \int_{\gamma^*}\int_{\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} R$$

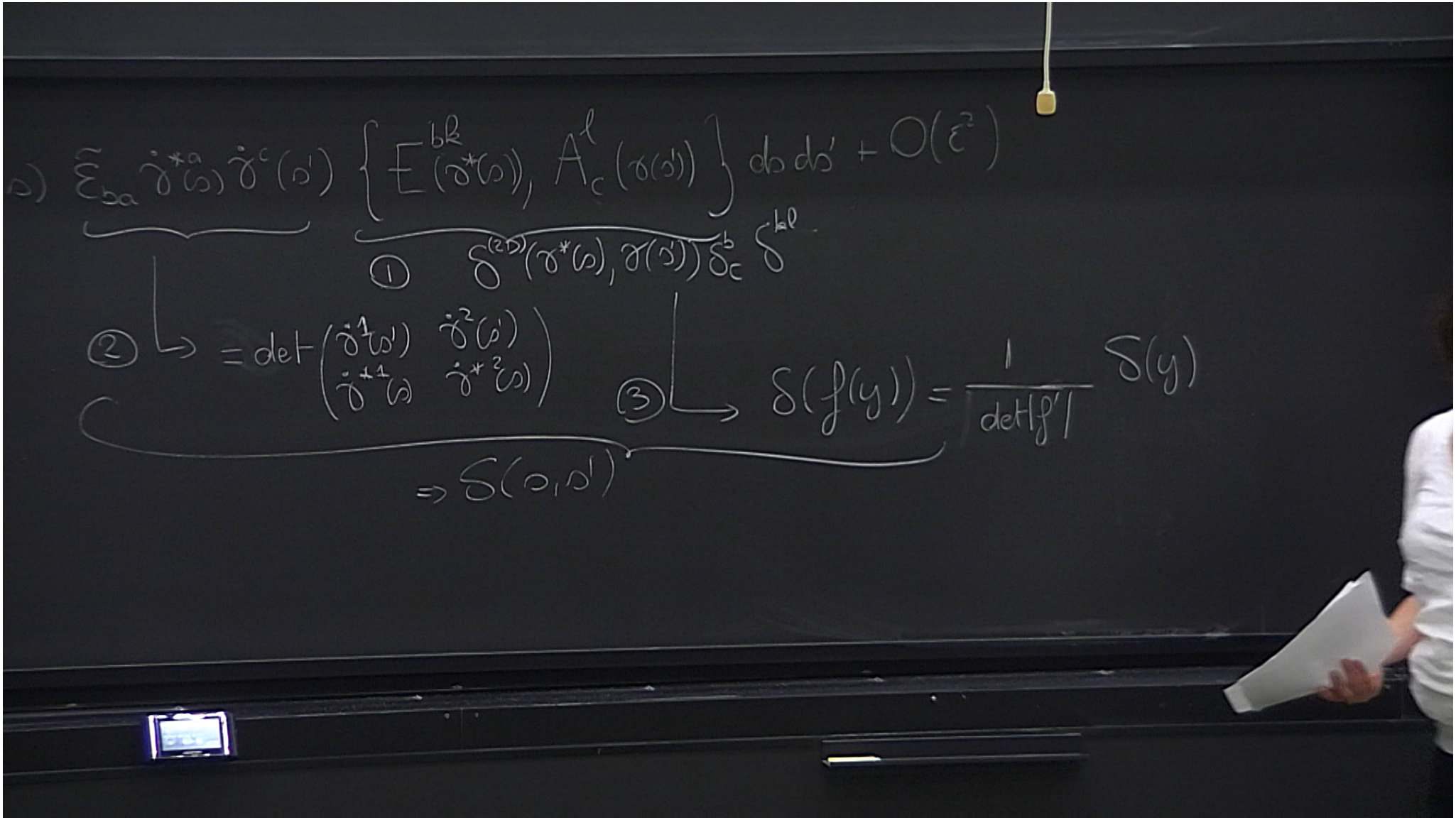

$= -h_{\gamma(1,\frac{1}{2}+\frac{\varepsilon}{2})}\, \mathrm{Te}\, h_{\gamma}\left(\frac{1}{2}-\frac{\varepsilon}{2},0\right) \int_{\gamma^*} \int_{\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} R_{jk}(s)\, \tilde{\varepsilon}_{ba}\, \dot{\gamma}^{*a}(s)\, \dot{\gamma}^{c}(s') \left\{ E^{bk}(\gamma^*(s)),\, A^{l}_{c}(\gamma(s)\right.$

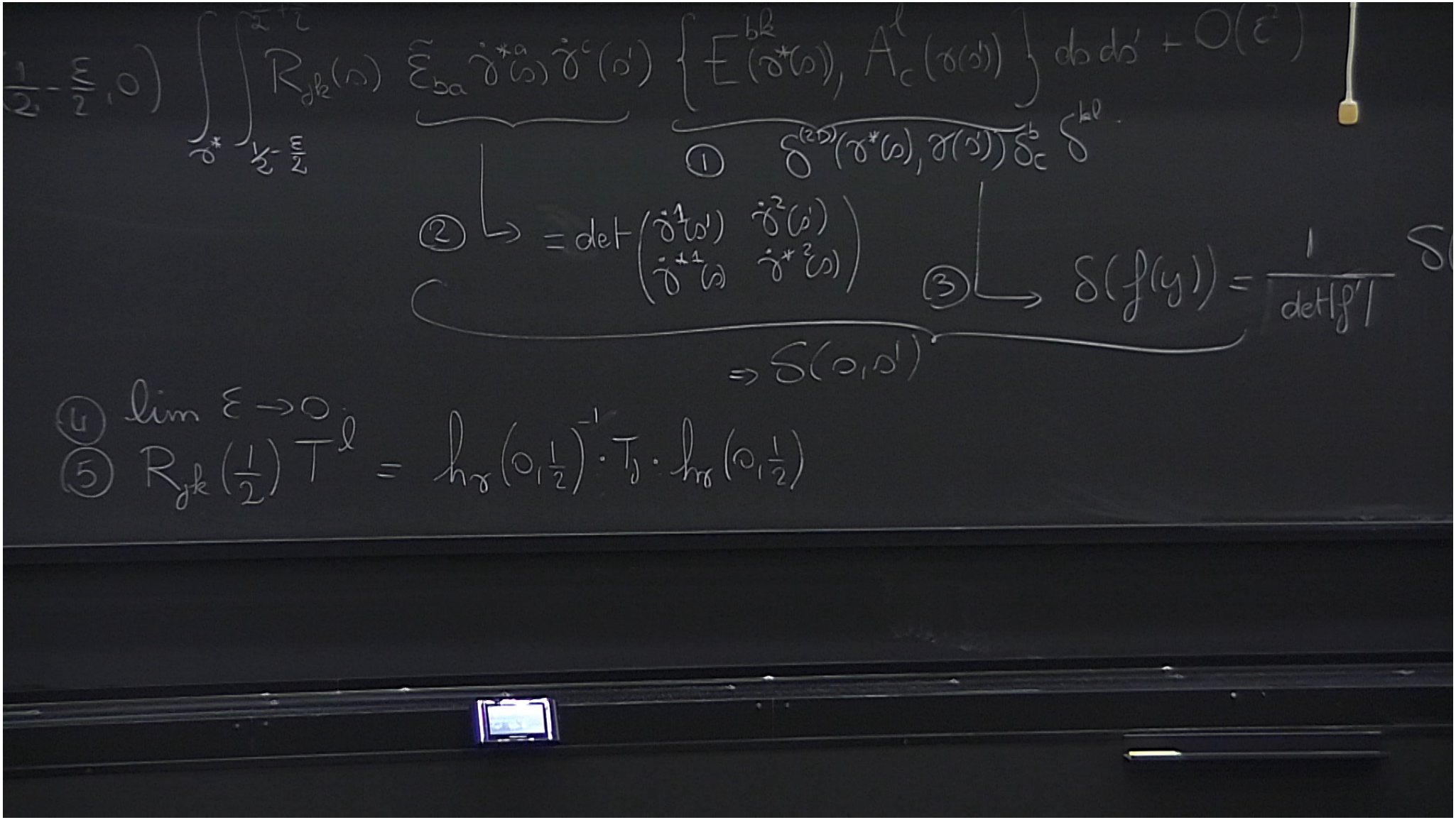

$\dot{\sigma}^{*a}(\sigma)\,\dot{\sigma}^{c}(\sigma')\left\{E^{bk}(\sigma^{*}(\sigma)),\ A^{l}_{c}(\sigma(\sigma'))\right\}d\sigma\,d\sigma' + O(\varepsilon^{2})$

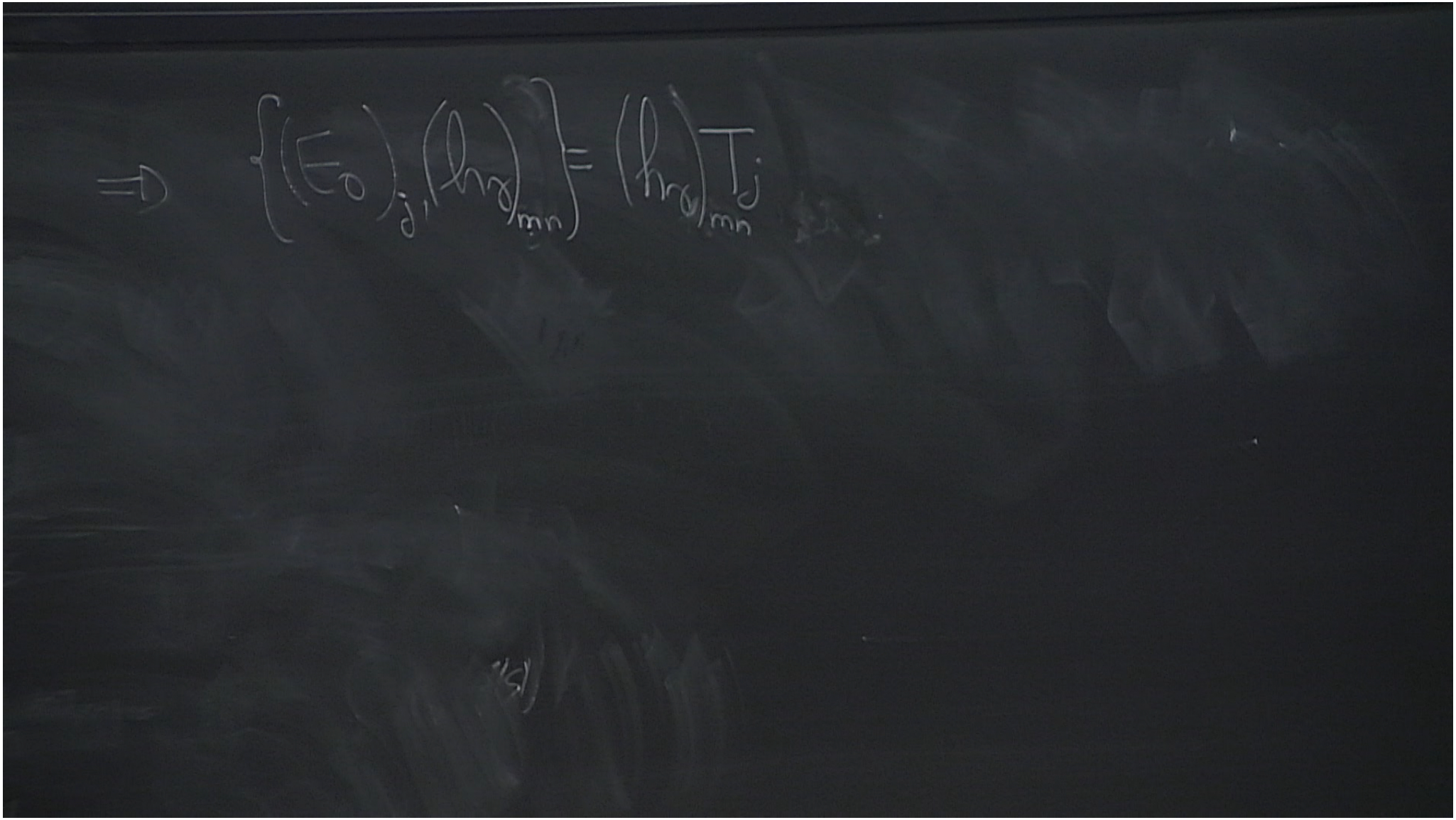

sits at γ(0)
$E_\gamma \longrightarrow g(\gamma(0))\, E_\gamma\, g(\gamma(0))^{-1}$ (internal rotation)
$\det(\dot\gamma^a \dot\gamma^b) > 0$
$= \det\begin{pmatrix} \dot\gamma^1 & \dot\gamma^{s'1} \\ \dot\gamma^2 & \dot\gamma^{s'2} \end{pmatrix}$
$\{(E_\gamma)_j^a, h_\gamma\} = -h_{\gamma(1,\frac{1}{2}+\frac{\varepsilon}{2})}\, \mathrm{Te}\, h_\gamma(\frac{1}{2}-\frac{\varepsilon}{2},0) \int_{\gamma^*}\int_{\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} R_{jkl}(s)\, \tilde\varepsilon_{ba}\, \dot\gamma^{*a}(s)\, \dot\gamma^c(s')\, \{E^{bk}(\gamma^*(s)), A_c^l(\gamma(s'))\}\, ds\, ds' + O(\varepsilon^2)$
① $\delta^{(2)}(\gamma^*(s), \gamma(s'))\, \delta_c^b$

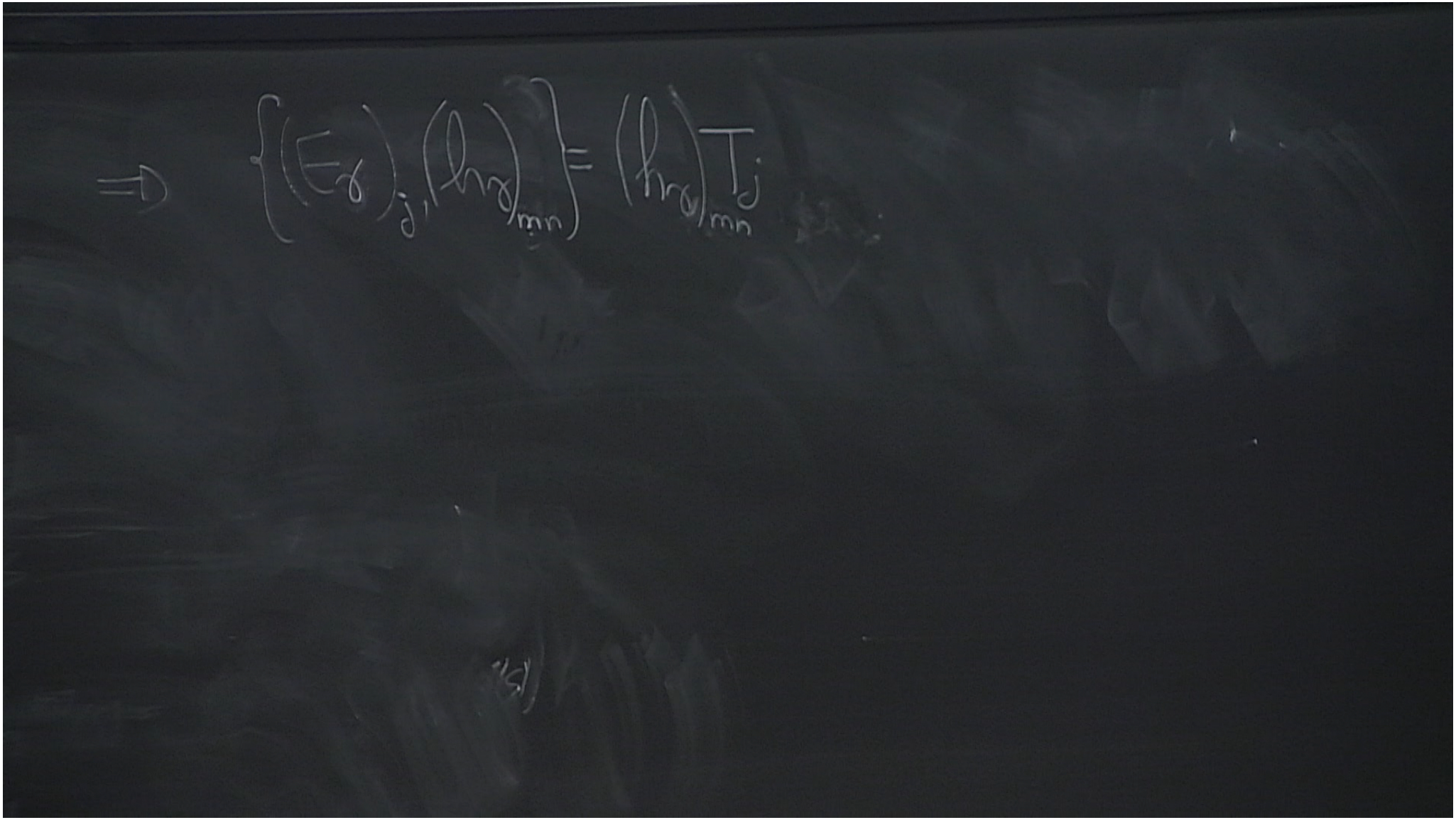

$\varepsilon_{ba}\, \dot{\sigma}^{*a}(\sigma)\, \dot{\sigma}^{c}(\sigma') \left\{ E^{bk}(\sigma^*(\sigma)),\ A^{l}_{c}(\sigma(\sigma')) \right\} d\sigma\, d\sigma' + O(\varepsilon^2)$
$\underbrace{\qquad\qquad}_{①}\ \delta^{(2D)}(\sigma^*(\sigma), \sigma(\sigma'))\, \delta^{b}_{c}\, \delta^{kl}.$

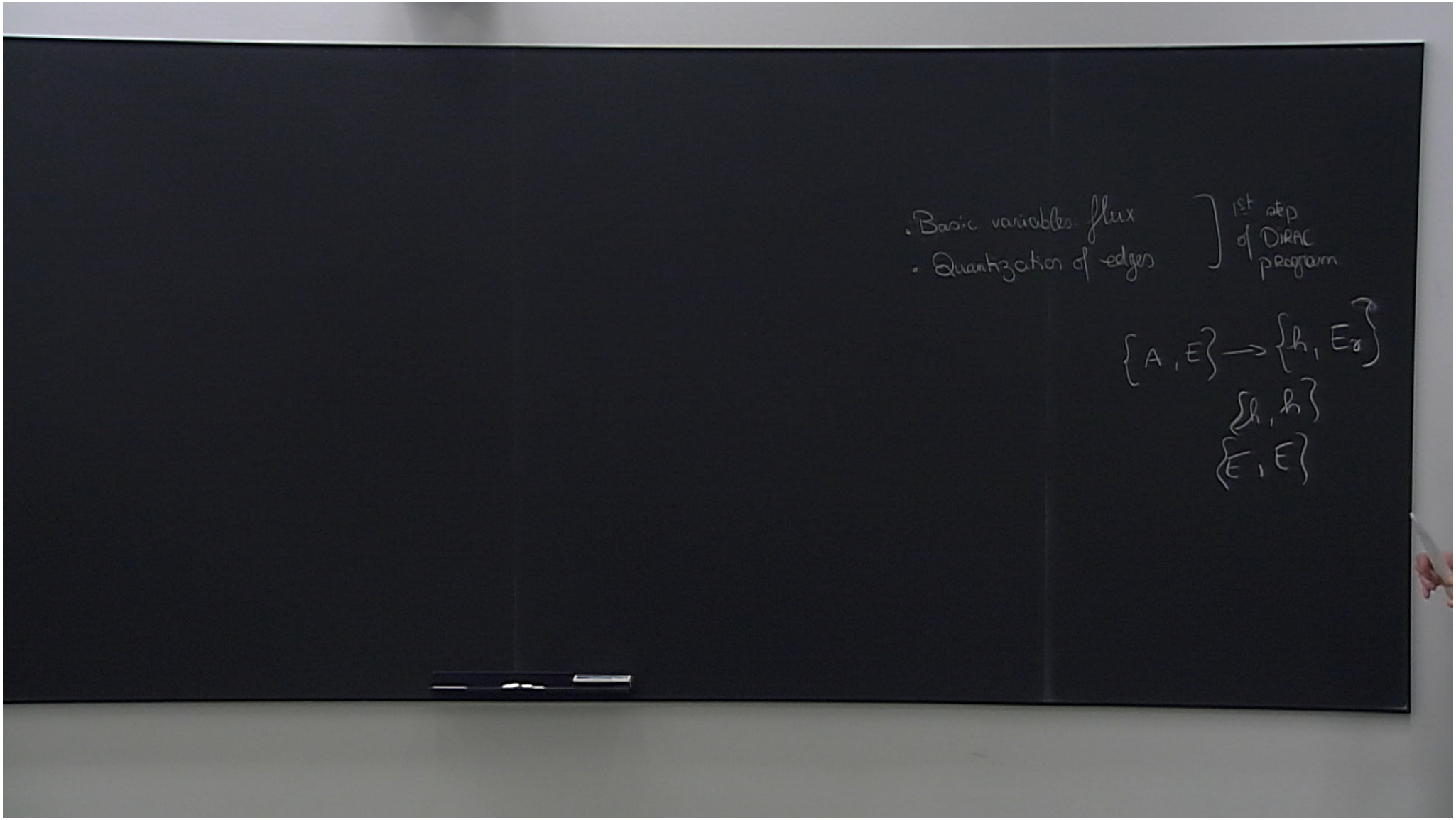

$\frac{\xi}{2}, 0)$
$\int_{\gamma^*} \int_{\frac{1}{2}-\frac{\xi}{2}}^{\frac{1}{2}+\frac{\xi}{2}} R_{jk}(\sigma)\, \tilde{\varepsilon}_{ba}\, \dot{\gamma}^{*a}(\sigma)\, \dot{\gamma}^{c}(\sigma')$
$\{ E^{bk}(\gamma^*(\sigma)),\ A^{l}_{c}(\gamma(\sigma')) \}$
$d\sigma\, d\sigma' + O(\varepsilon^2)$
① $\delta^{(2D)}(\gamma^*(\sigma), \gamma(\sigma'))\, \delta^{b}_{c}\, \delta^{kl}$
② $= \det \begin{pmatrix} \dot{\gamma}^1(\sigma') & \dot{\gamma}^2(\sigma') \\ \dot{\gamma}^{*1}(\sigma) & \dot{\gamma}^{*2}(\sigma) \end{pmatrix}$

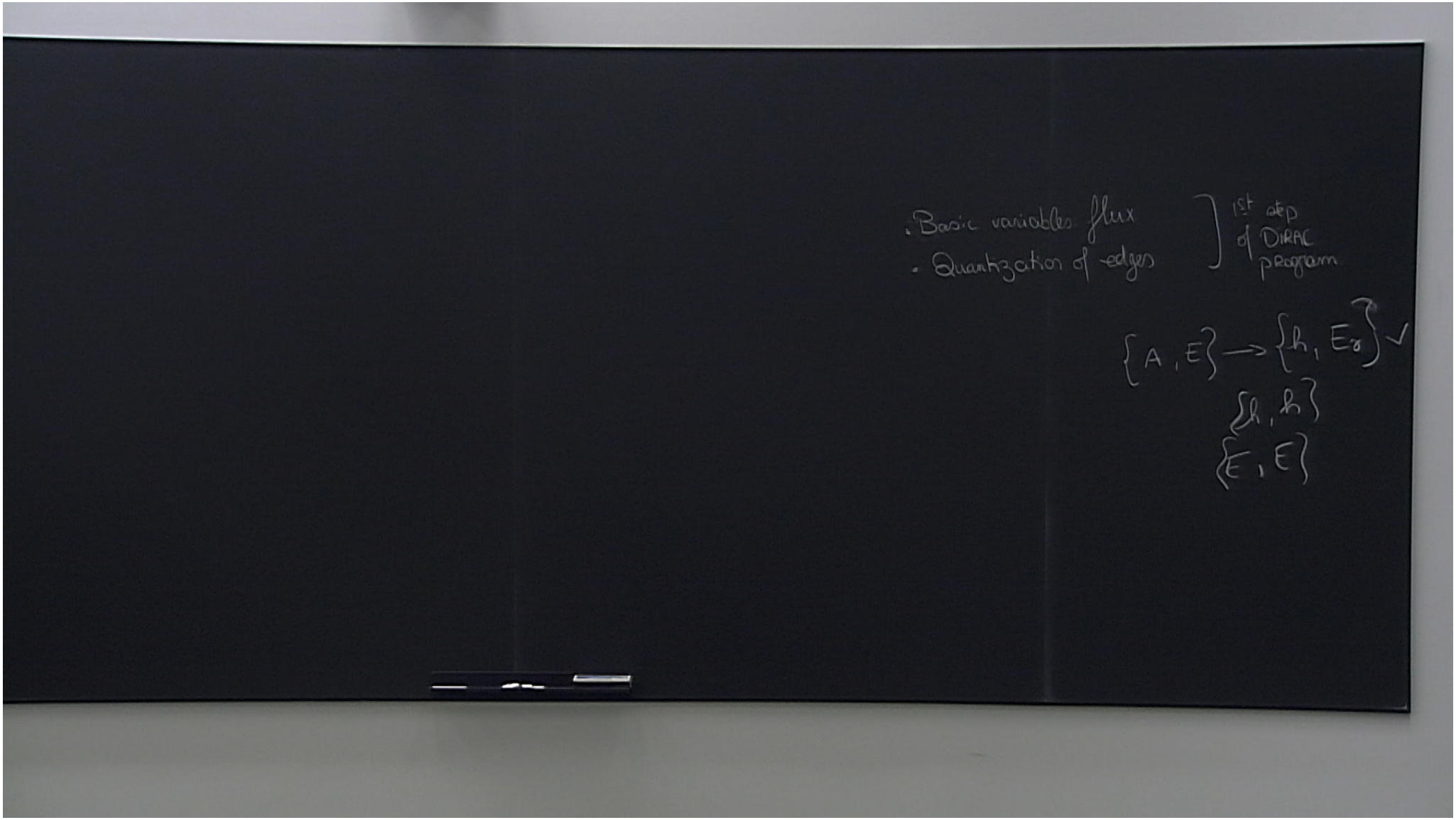

$\tilde{\xi}_{ba}\,\dot{\gamma}^{*a}(\sigma)\,\dot{\gamma}^{c}(\sigma')\left\{E^{bk}(\gamma^*(\sigma)),\, A^{l}_{c}(\gamma(\sigma'))\right\} d\sigma\, d\sigma' + O(\tilde{\epsilon}^2)$
① $\delta^{(2D)}(\gamma^*(\sigma), \gamma(\sigma'))\,\delta^{b}_{c}\,\delta^{kl}$
② $= \det\begin{pmatrix} \dot{\gamma}^1(\sigma) & \dot{\gamma}^2(\sigma) \\ \dot{\gamma}^{*1}(\sigma) & \dot{\gamma}^{*2}(\sigma) \end{pmatrix}$
③ $\delta(f(y)) = \frac{1}{|\det f'|}\,\delta(y)$
$\Rightarrow \delta(\sigma, \sigma')$

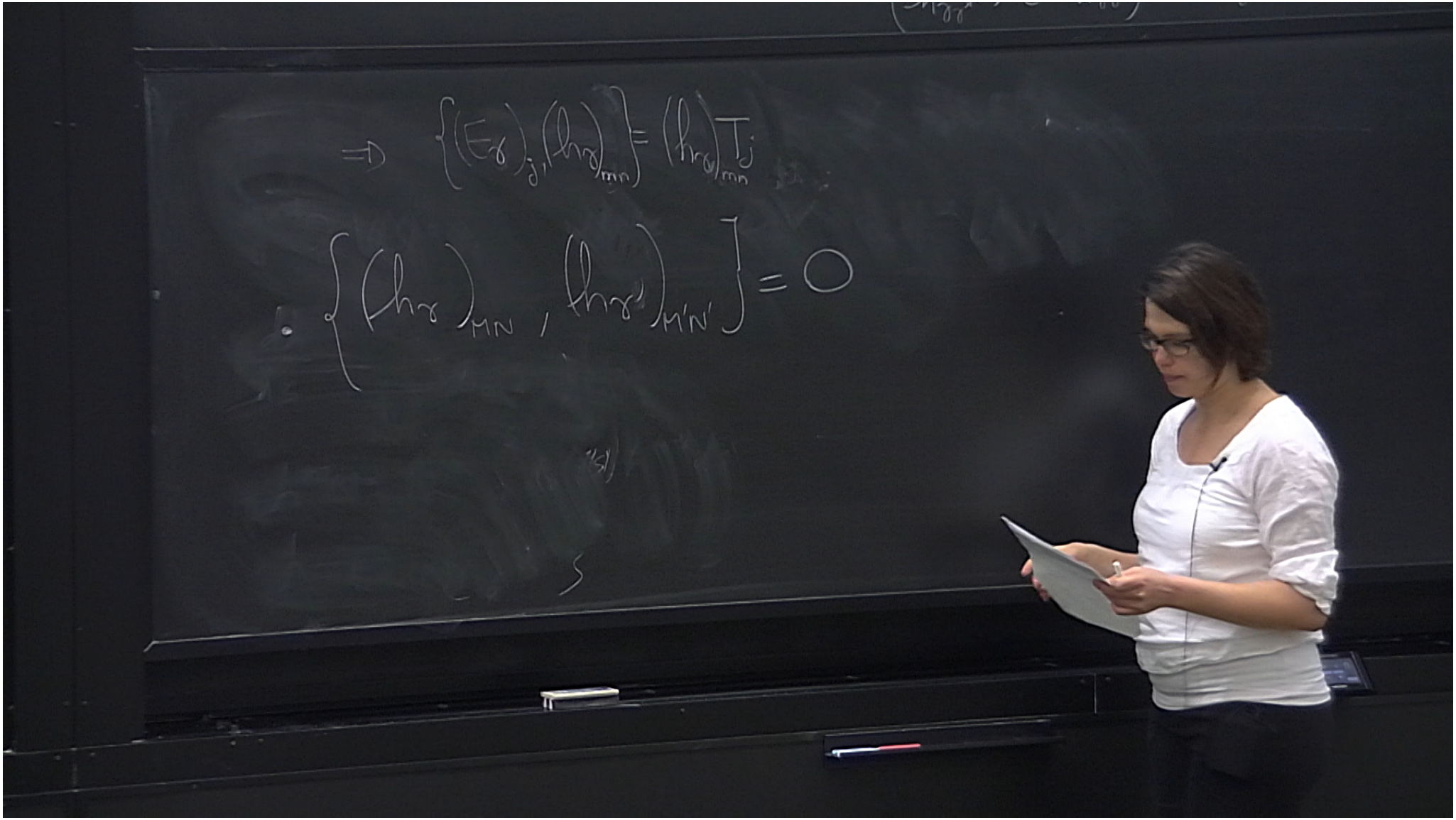

$\left(\frac{1}{2}-\frac{\varepsilon}{2},0\right)\int_{\gamma^*}\int_{\frac{1}{2}-\frac{\varepsilon}{2}}^{\frac{1}{2}+\frac{\varepsilon}{2}} R_{jk}(\sigma)\,\tilde{\varepsilon}_{ba}\,\dot{\gamma}^{*a}(\sigma)\,\dot{\gamma}^{c}(\sigma')\,\{E^{bk}(\gamma^*(\sigma)),A^{l}_{c}(\gamma(\sigma'))\}\,d\sigma\,d\sigma' + O(\varepsilon^2)$
① $\delta^{(2D)}(\gamma^*(\sigma),\gamma(\sigma'))\,\delta^{b}_{c}\,\delta^{kl}$
② $= \det\begin{pmatrix}\dot{\gamma}^1(\sigma') & \dot{\gamma}^2(\sigma')\\ \dot{\gamma}^{*1}(\sigma) & \dot{\gamma}^{*2}(\sigma)\end{pmatrix}$
③ $\delta(f(y)) = \frac{1}{|\det f'|}$
$\Rightarrow \delta(\sigma,\sigma')$
④ $\lim \varepsilon \to 0$
⑤ $R_{jk}\left(\frac{1}{2}\right)T^{l} = h_\gamma(0,\frac{1}{2})^{-1}\cdot T_j \cdot h_\gamma(0,\frac{1}{2})$

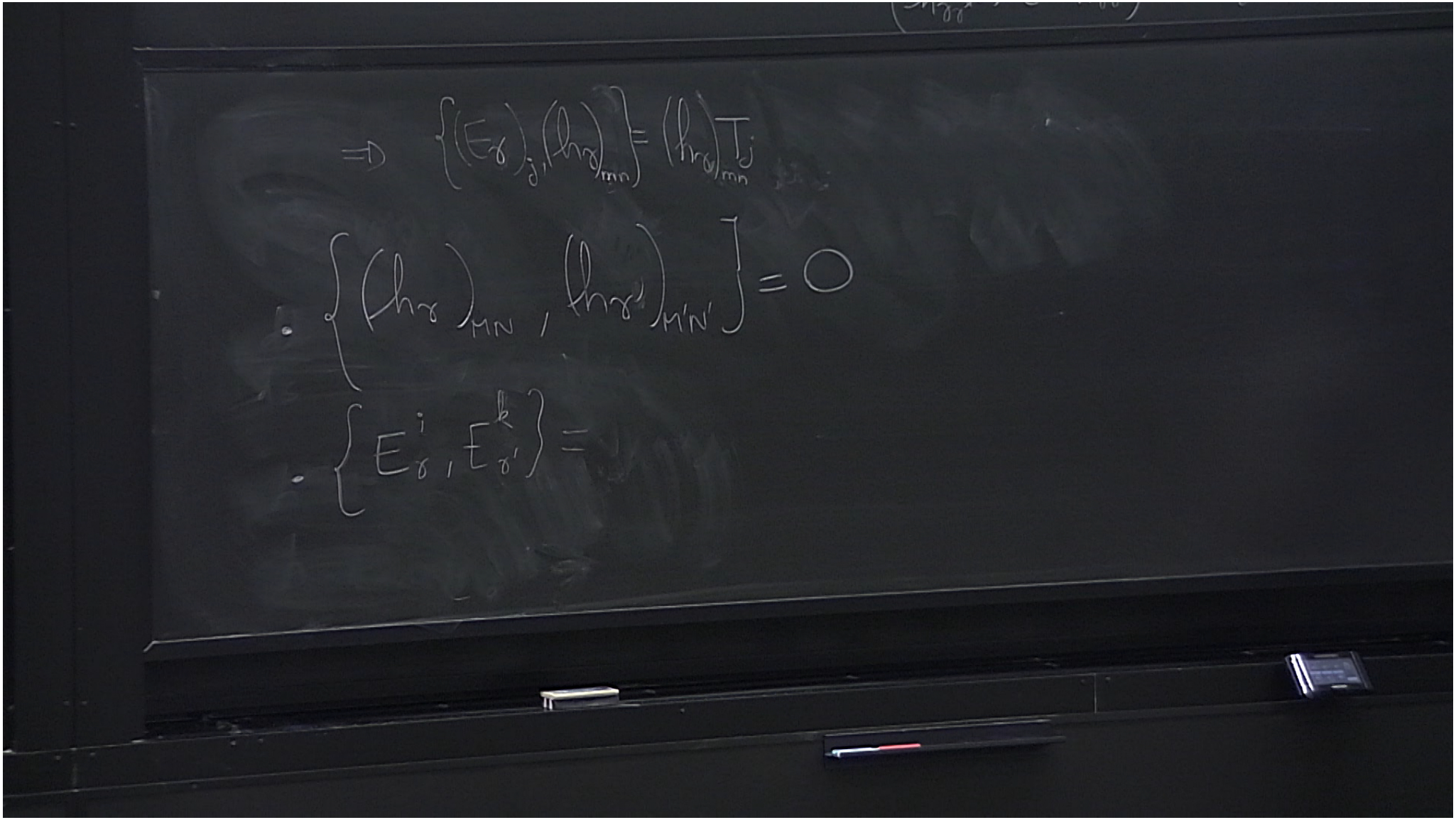

$\Rightarrow \{(E_0)_{jj'}(h_\alpha)_{mn}\} = (h_\alpha)_{mn} T_j$

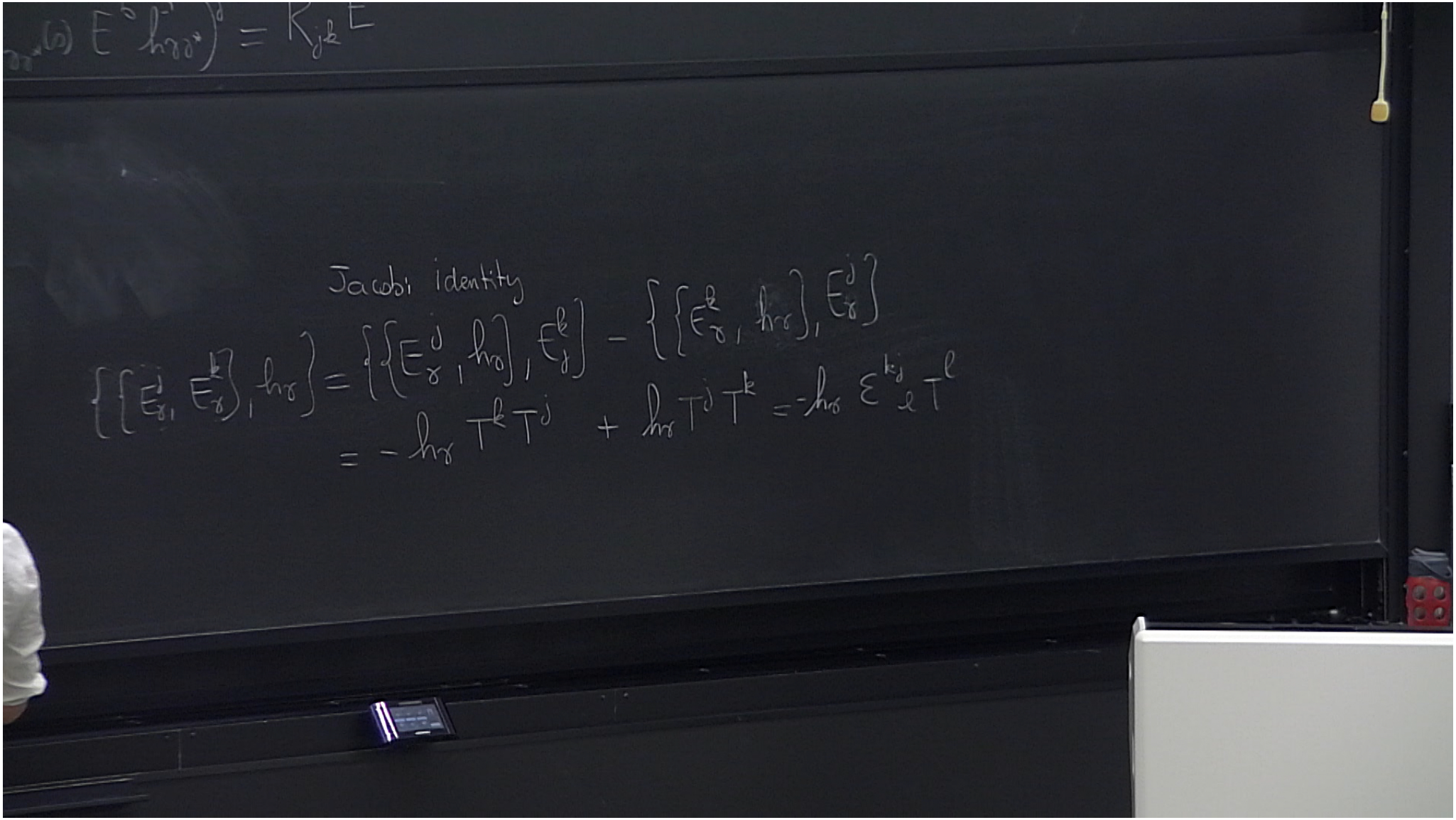

$\Rightarrow \{(\epsilon_\alpha)_{jj'}, (h_\alpha)_{mn}\} = (h_\alpha)_{mn}^{T_j}$

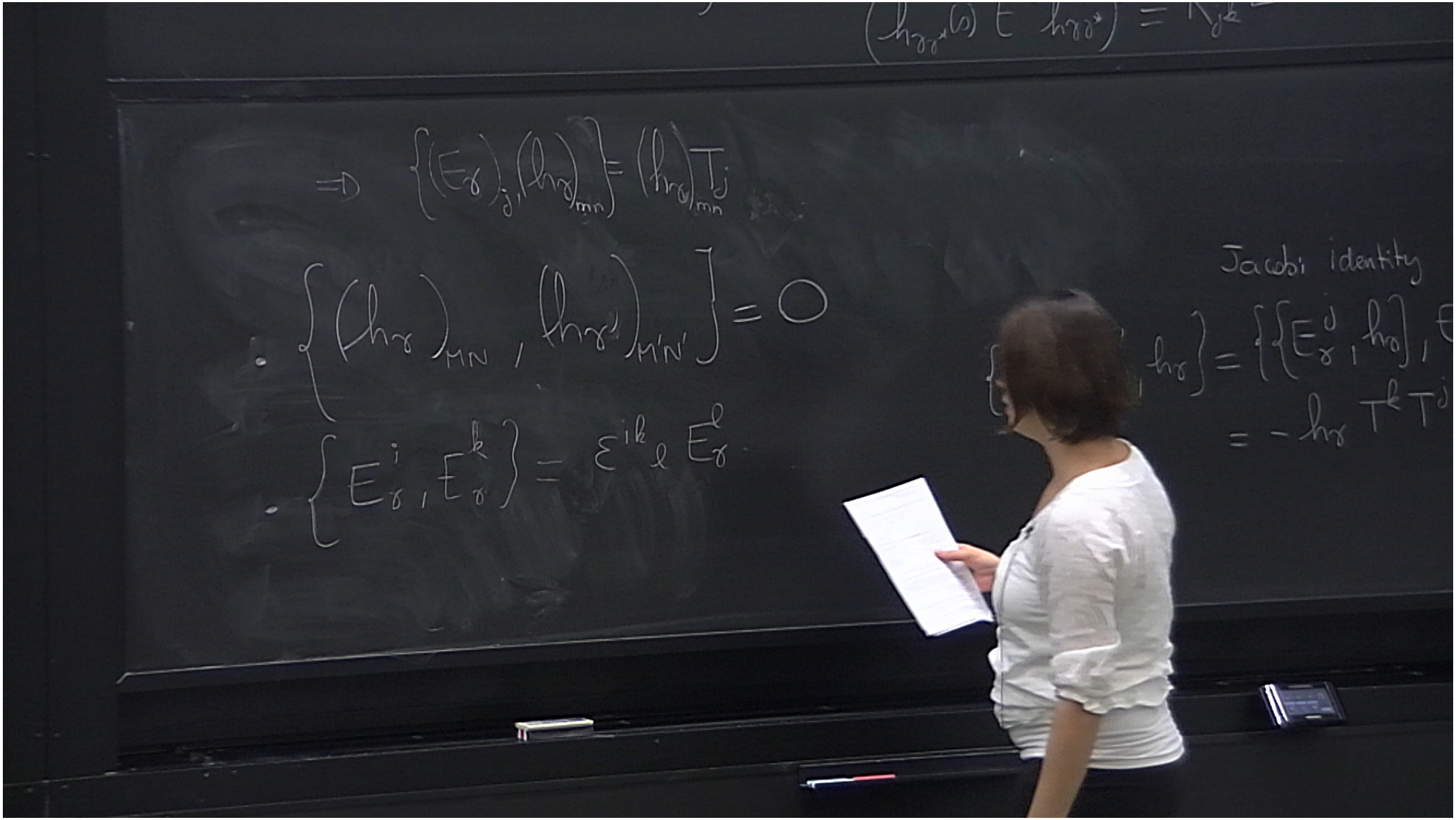

• Basic variables: flux
• Quantization of edges
1st step of DIRAC program
{A, E} → {h, E_σ}
{h, h}
{E, E}


• Basic variables: flux
• Quantization of edges
1st step of DIRAC program
{A, E} → {h, E_a}
{h, h}
{E, E}

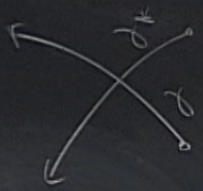

$\Rightarrow \{(\epsilon_\alpha)_{ij}, (h_\alpha)_{mn}\} = (h_\alpha)_{mn} T_j$
$\therefore \{(h_\alpha)_{MN}, (h_{\alpha'})_{M'N'}\} = 0$


$\Rightarrow \{(E_\alpha)_j, (h_\alpha)_{mn}\} = (h_\alpha)_{mn} T_j$
$\{(h_\alpha)_{MN}, (h_{\alpha'})_{M'N'}\} = 0$
$\{E^i_\alpha, E^k_{\alpha'}\} =$


Jacobi identity
$\{\{E^j_\gamma, E^k_\gamma\}, h_\sigma\} = \{\{E^j_\gamma, h_\sigma\}, E^k_\gamma\} - \{\{E^k_\gamma, h_\sigma\}, E^j_\gamma\}$
$= -h_\sigma T^k T^j + h_\sigma T^j T^k = -h_\sigma \varepsilon^{kj}{}_l T^l$


$\Rightarrow \{(E_\gamma)_j, (h_\gamma)_{mn}\} = (h_\gamma)_{mn} T_j$
$\{(h_\gamma)_{MN}, (h_{\gamma'})_{M'N'}\} = 0$
$\{E_\gamma^i, E_\gamma^k\} = \varepsilon^{ik}{}_\ell E_\gamma^\ell$
Jacobi identity
$= -h_\gamma T^k T^j$

For one γ



$$\begin{cases} \{(h_\gamma)_{MN}, (h_{\gamma'})_{M'N'}\} = 0 \\ \{E^j_\gamma, h_\gamma\} = h_\gamma T^j \\ \{E^j_\gamma, E^k_\gamma\} = \varepsilon_{jk}{}^l E^l_\gamma \end{cases}$$

closed algebra.

For one γ



$$\begin{cases} \{(h_\gamma)_{MN}, (h_\gamma)_{M'N'}\} = 0 \\ \{E^J_\gamma, h_\gamma\} = h_\gamma T^J \\ \{E^J_\gamma, E^k_\gamma\} = \varepsilon_{Jk}{}^l E^l_\gamma \end{cases}$$

closed algebra.

$T^* SU(2)$

– phase space –

For one γ

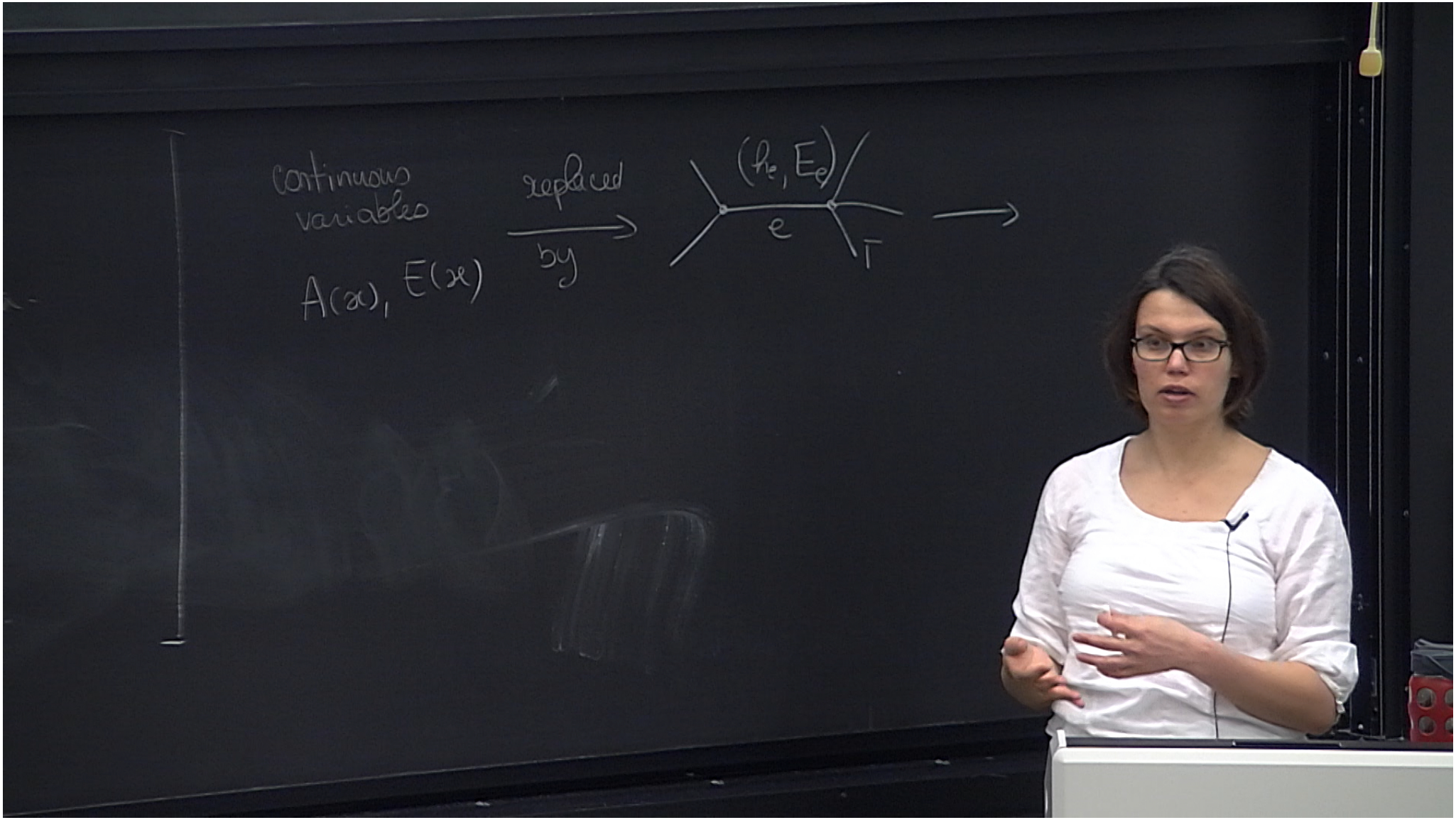


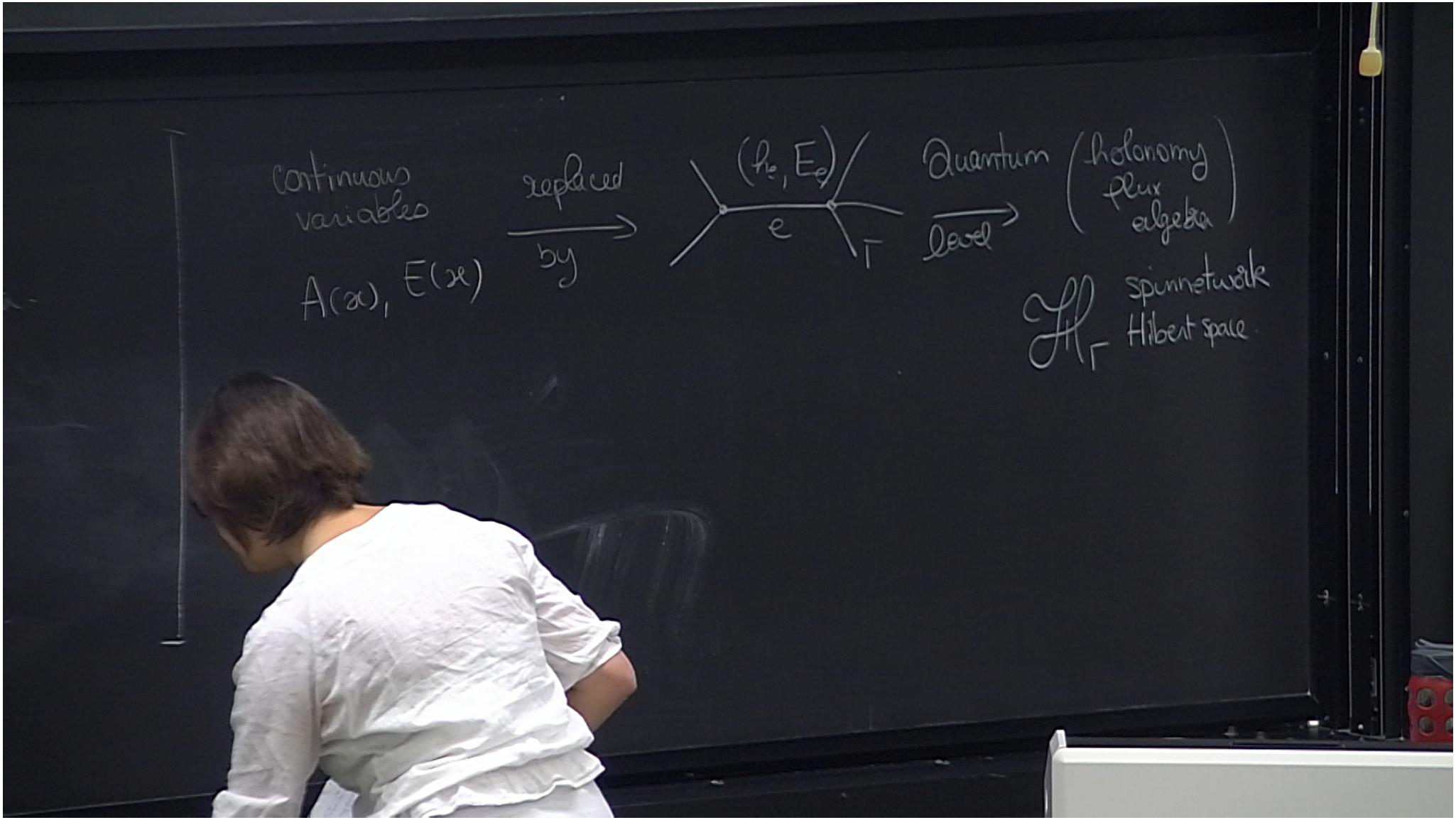
$$\begin{cases} \{(h_\gamma)_{MN}, (h_\gamma)_{M'N'}\} = 0 \\ \{E^j_\gamma, h_\gamma\} = h_\gamma T^j \\ \{E^j_\gamma, E^k_\gamma\} = \varepsilon_{jk}{}^l E^l_\gamma \end{cases}$$

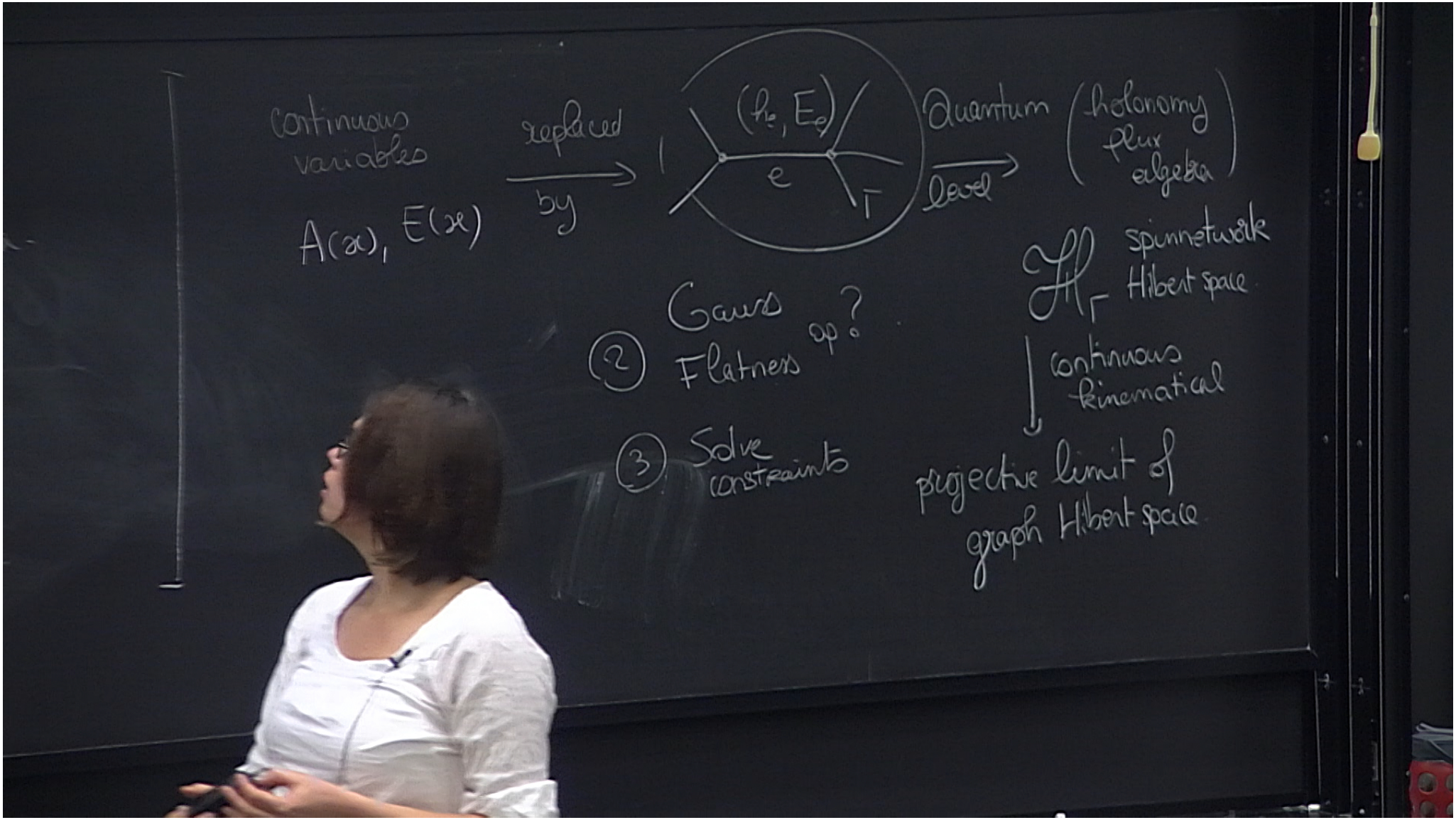
closed algebra.

$T^* SU(2)$

– phase space associated with one edge.


continuous variables
A(x), E(x)
replaced by
(k_e, E_e)
e
T


Continuous variables
A(x), E(x)
replaced by
(h_e, E_e)
e
Γ
Quantum level
holonomy flux algebra
H_Γ
spinnetwork Hilbert space


continuous variables
A(x), E(x)
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e
v
Quantum level
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Gauss
Flatness
Solve constraints
spinnetwork Hilbert space
continuous kinematical
projective limit of graph Hilbert space