Title: PSI 2016/2017 Explorations in Quantum Gravity - Lecture 7

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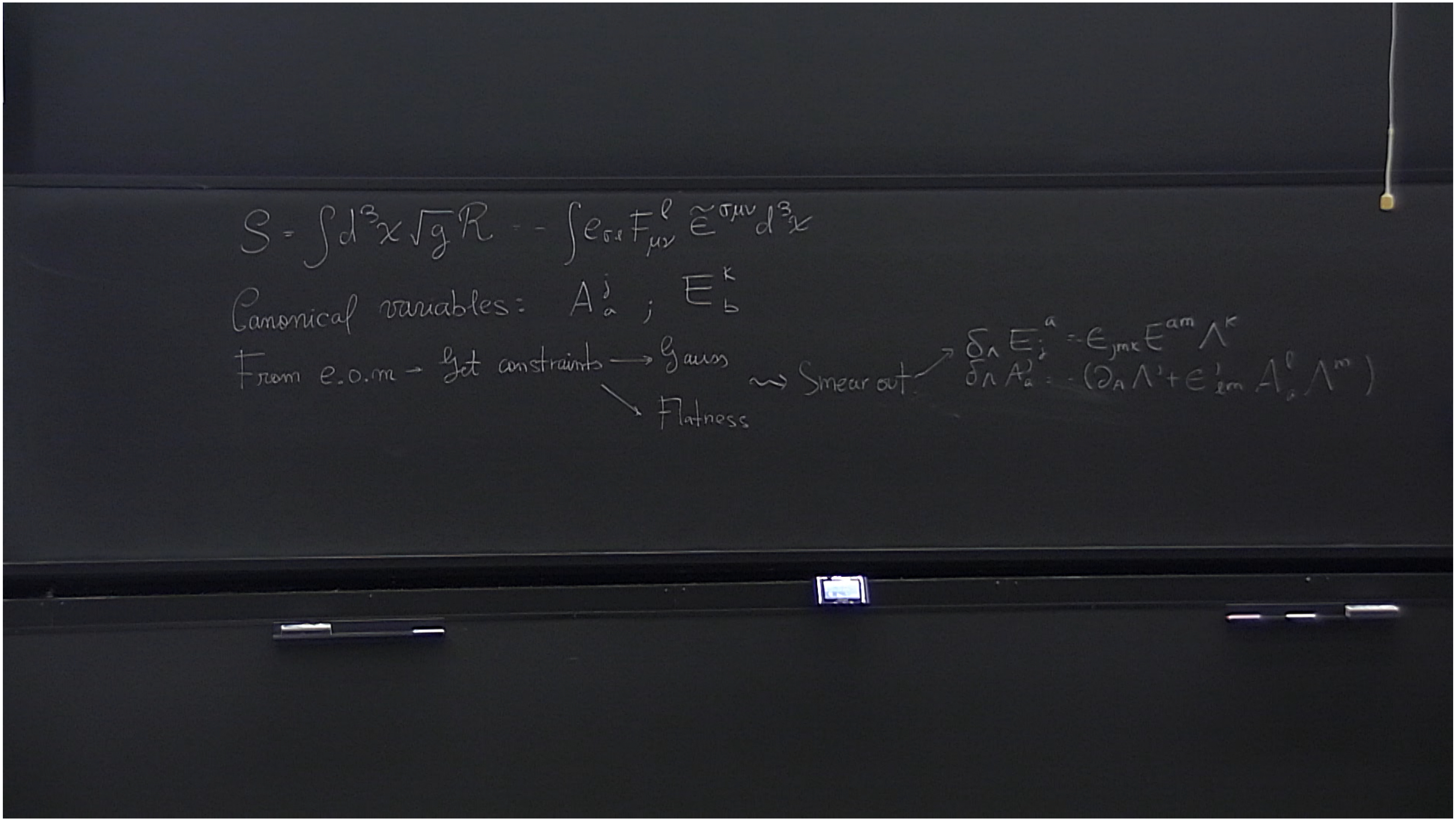
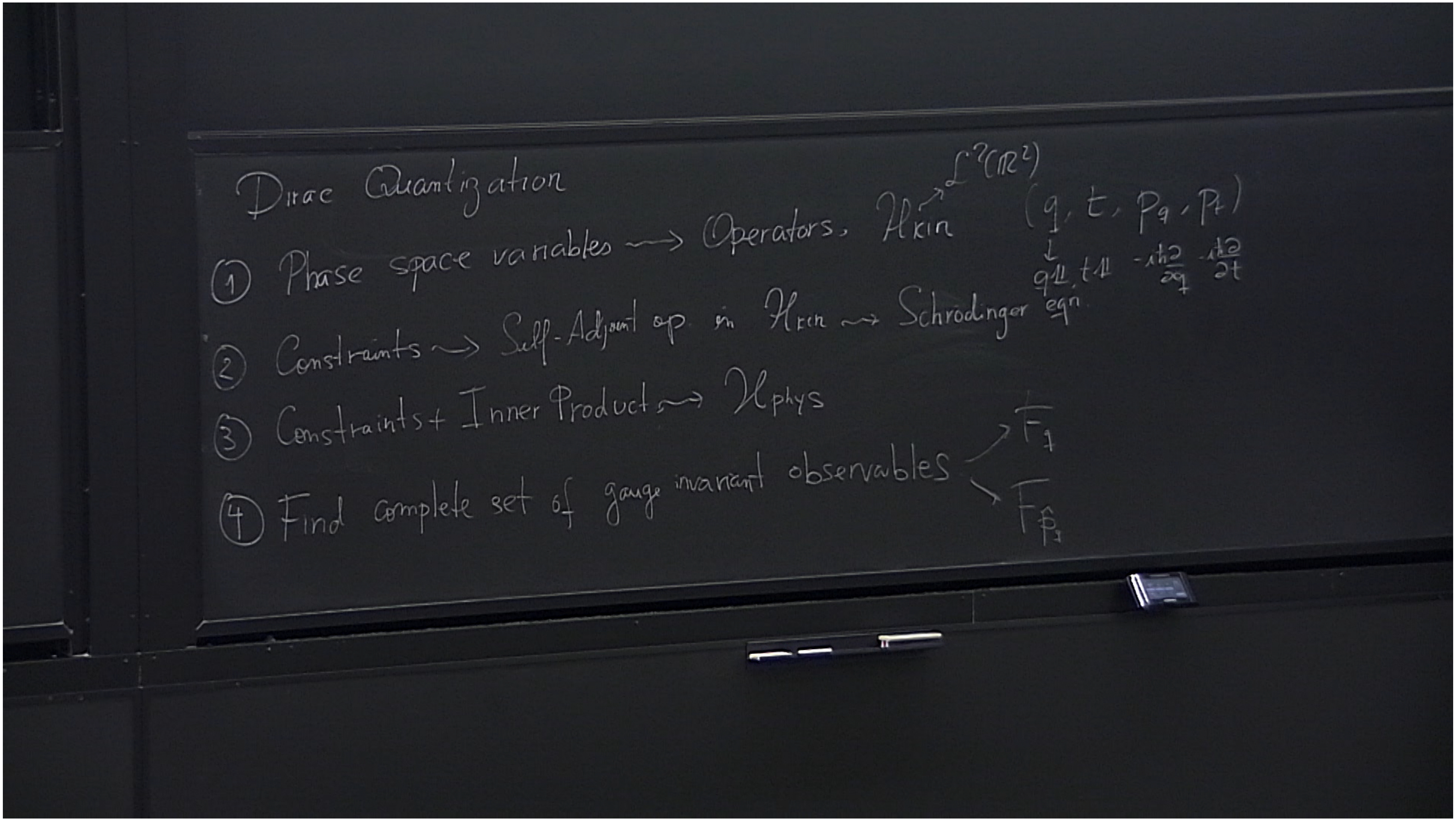
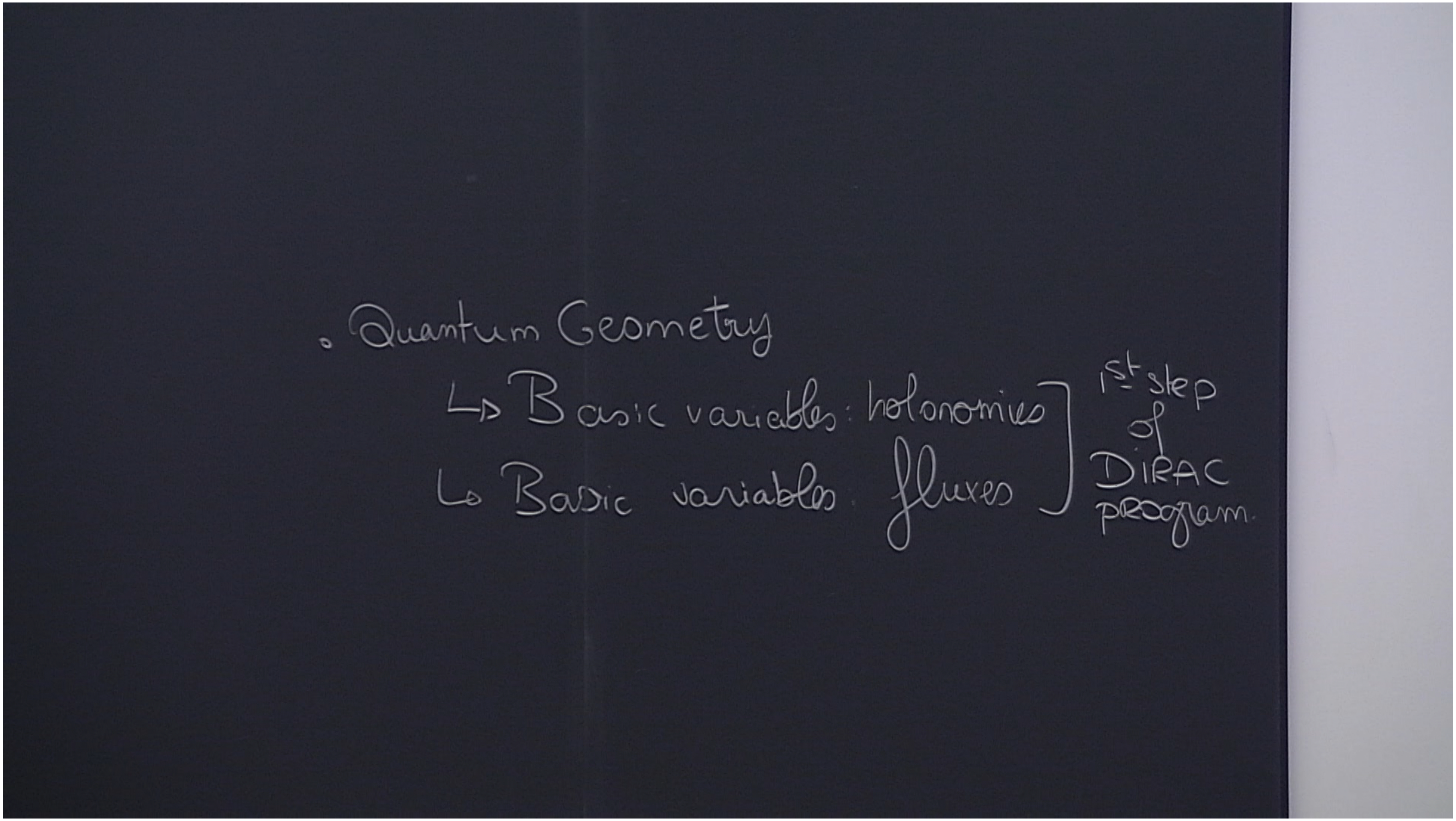
URL: http://pirsa.org/17030074

Abstract:

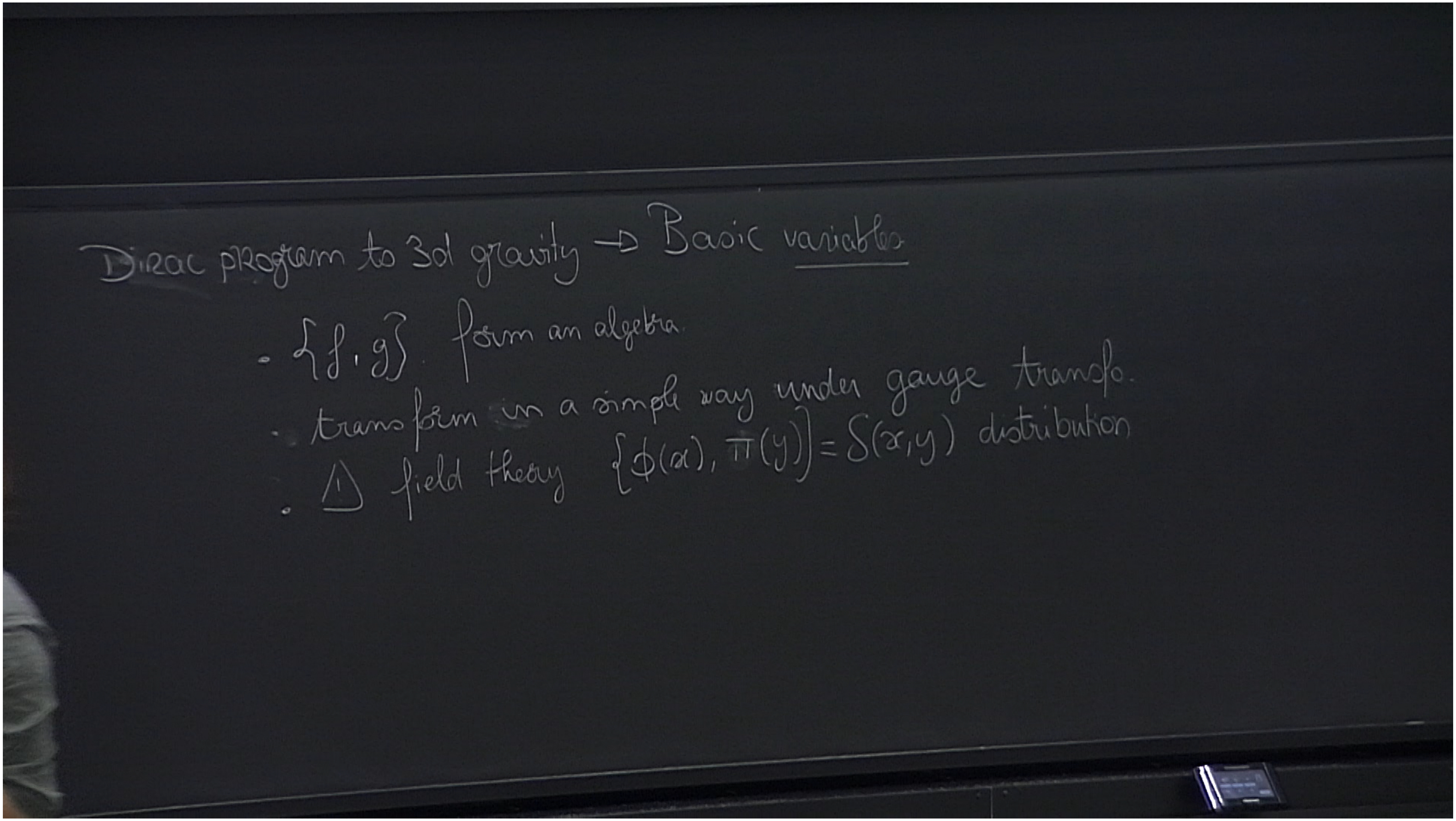


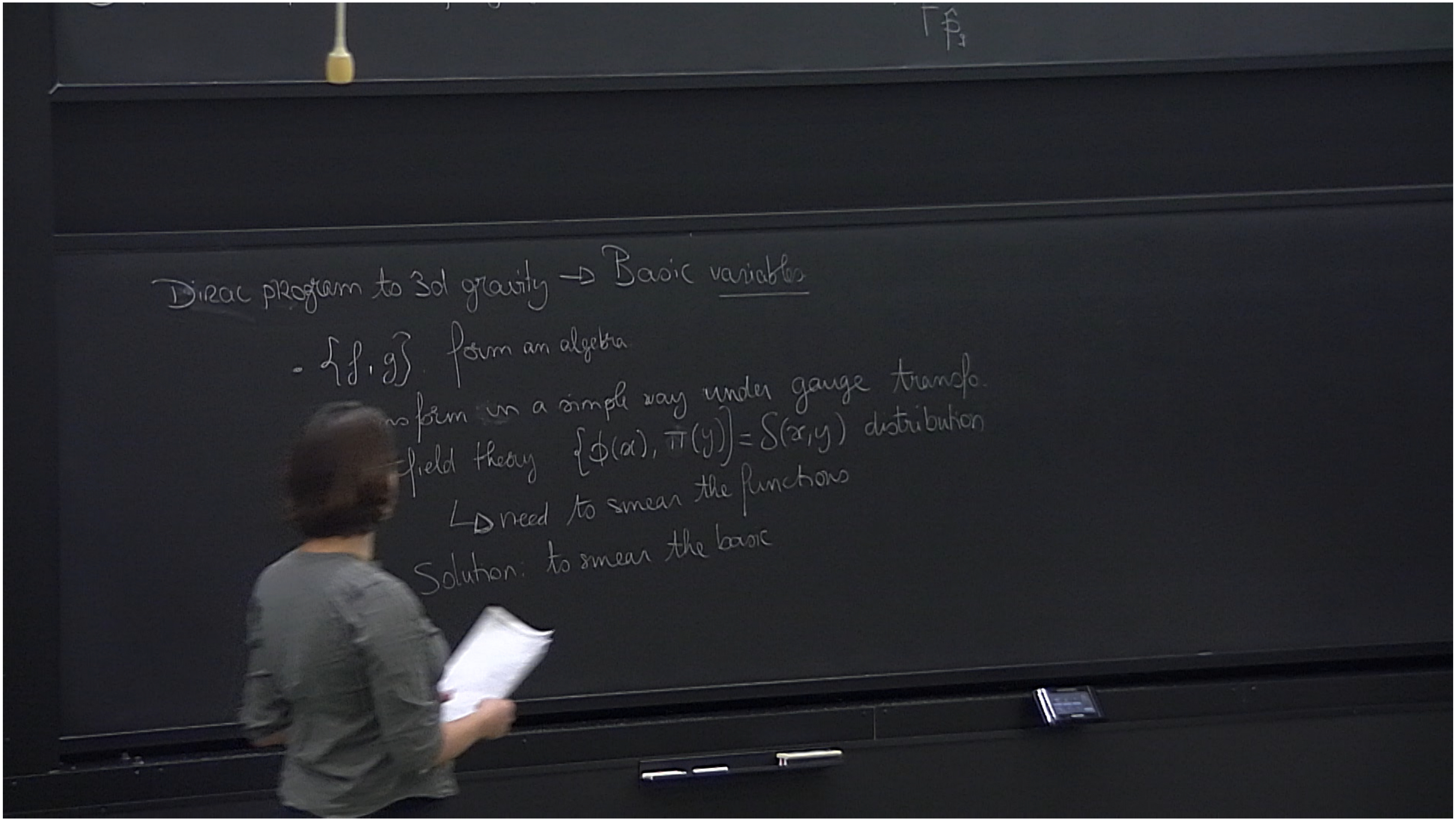
$S = \int d^3x \sqrt{g} R = -\int e_{\sigma\ell} F^{\ell}_{\mu\nu} \tilde{\epsilon}^{\sigma\mu\nu} d^3x$
Canonical variables: A^j_a ; E^k_b
From e.o.m → Get constraints → Gauss
Flatness
⇝ Smear out
$\delta_\Lambda E^a_j = -\epsilon_{jmk} E^{am} \Lambda^k$
$\delta_\Lambda A^j_a = -(\partial_A \Lambda^j + \epsilon^j_{\ell m} A^\ell_a \Lambda^m)$


Dirac Quantization
① Phase space variables ⟶ Operators, $\mathcal{H}_{kin}$ → $L^2(\mathbb{R}^2)$
(q, t, p_q, p_t) → $q\mathbb{1}, t\mathbb{1}, -i\hbar\frac{\partial}{\partial q}, -i\hbar\frac{\partial}{\partial t}$
② Constraints ⟶ Self-Adjoint op. in $\mathcal{H}_{kin}$ ⟶ Schrodinger eqn.
③ Constraints + Inner Product ⟶ $\mathcal{H}_{phys}$
④ Find complete set of gauge invariant observables → F_q, $F_{\hat{p}_q}$



Quantum Geometry
↳ Basic variables: holonomies
↳ Basic variables: fluxes
1st step of DIRAC program.


Dirac program to 3d gravity → Basic variables
· {f, g} form an algebra
· transform in a simple way under gauge transfo.
· field theory {φ(x), π(y)} = δ(x,y) distribution


Dirac program to 3d gravity → Basic variables
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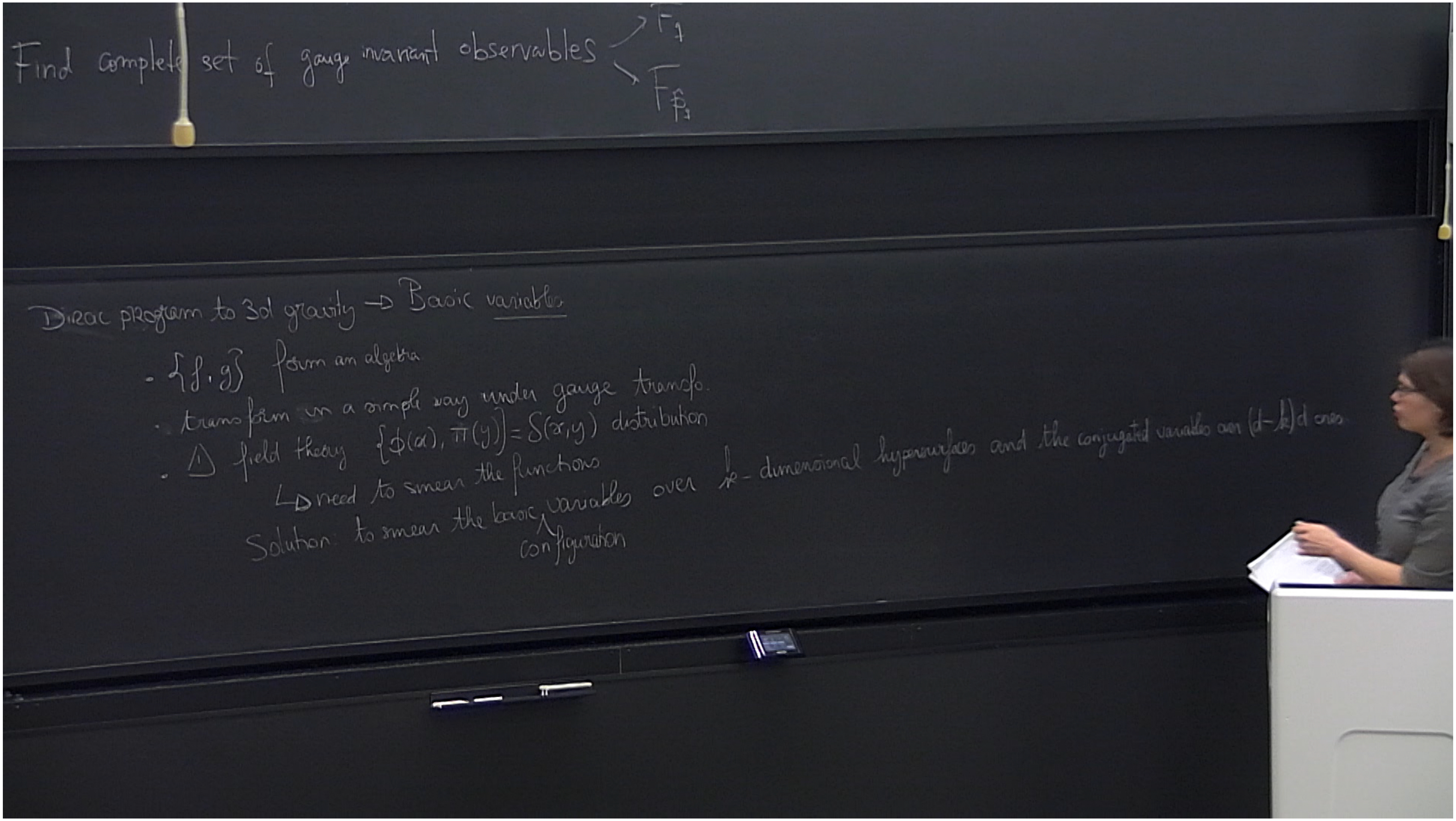
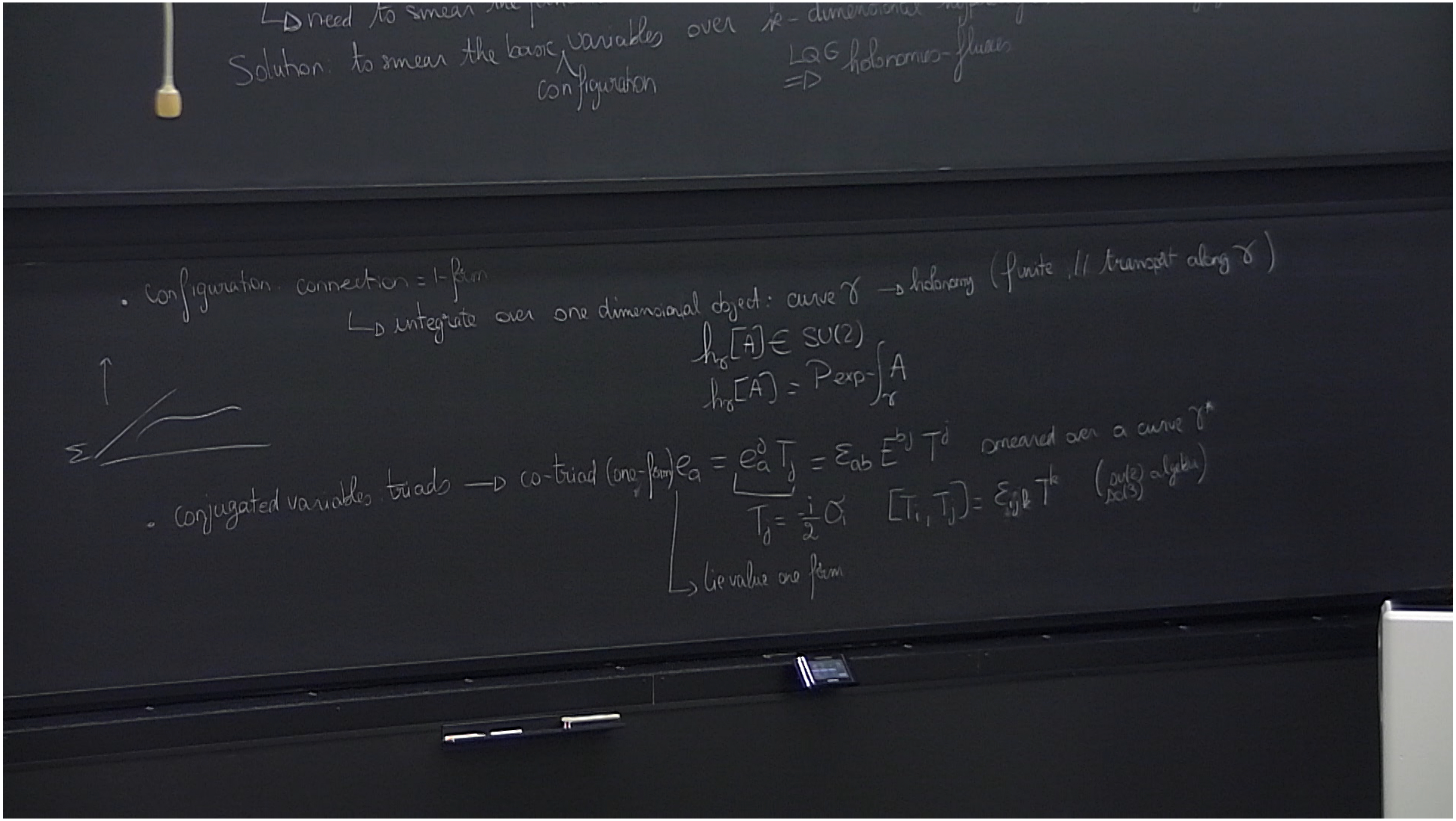
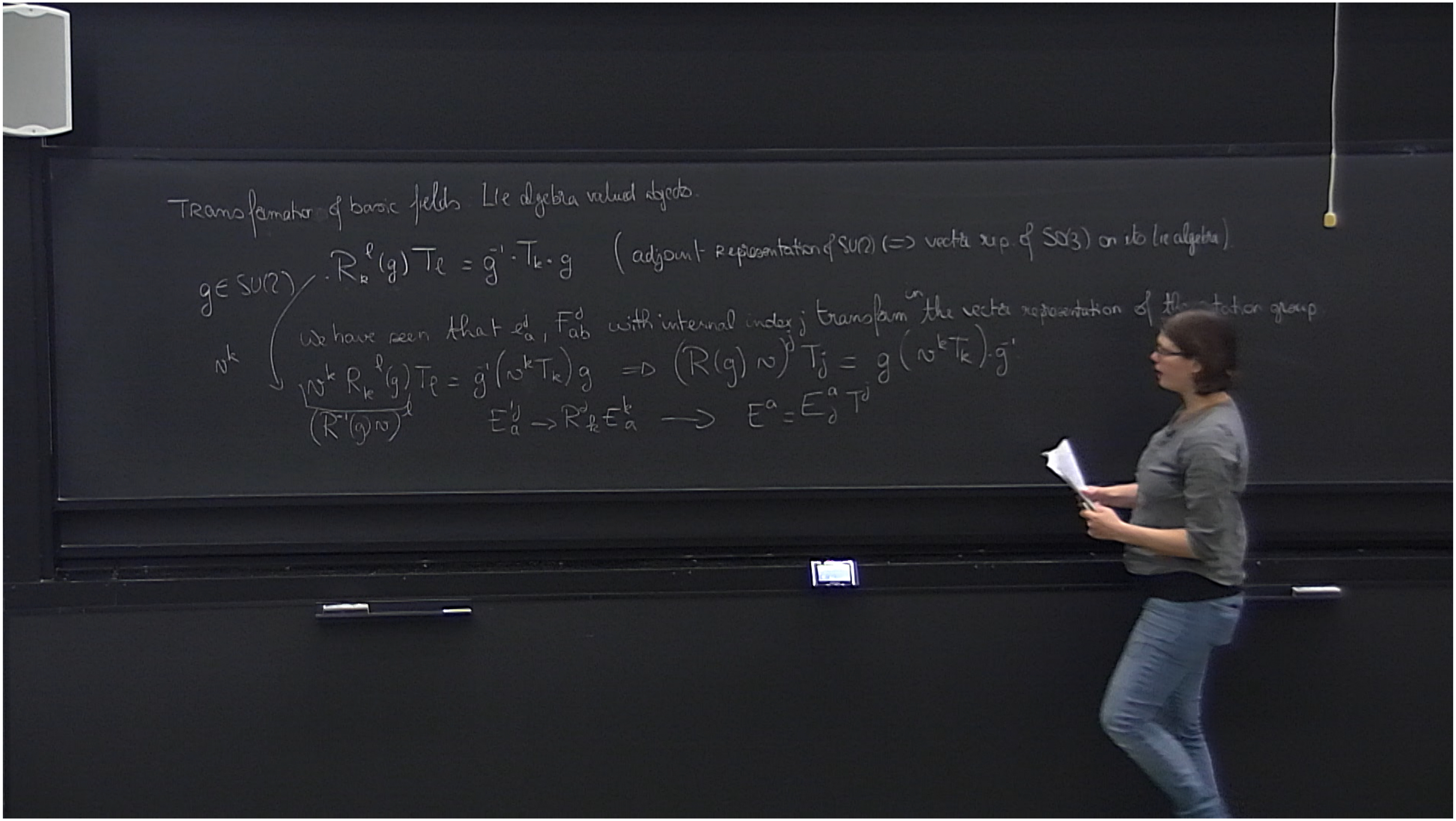
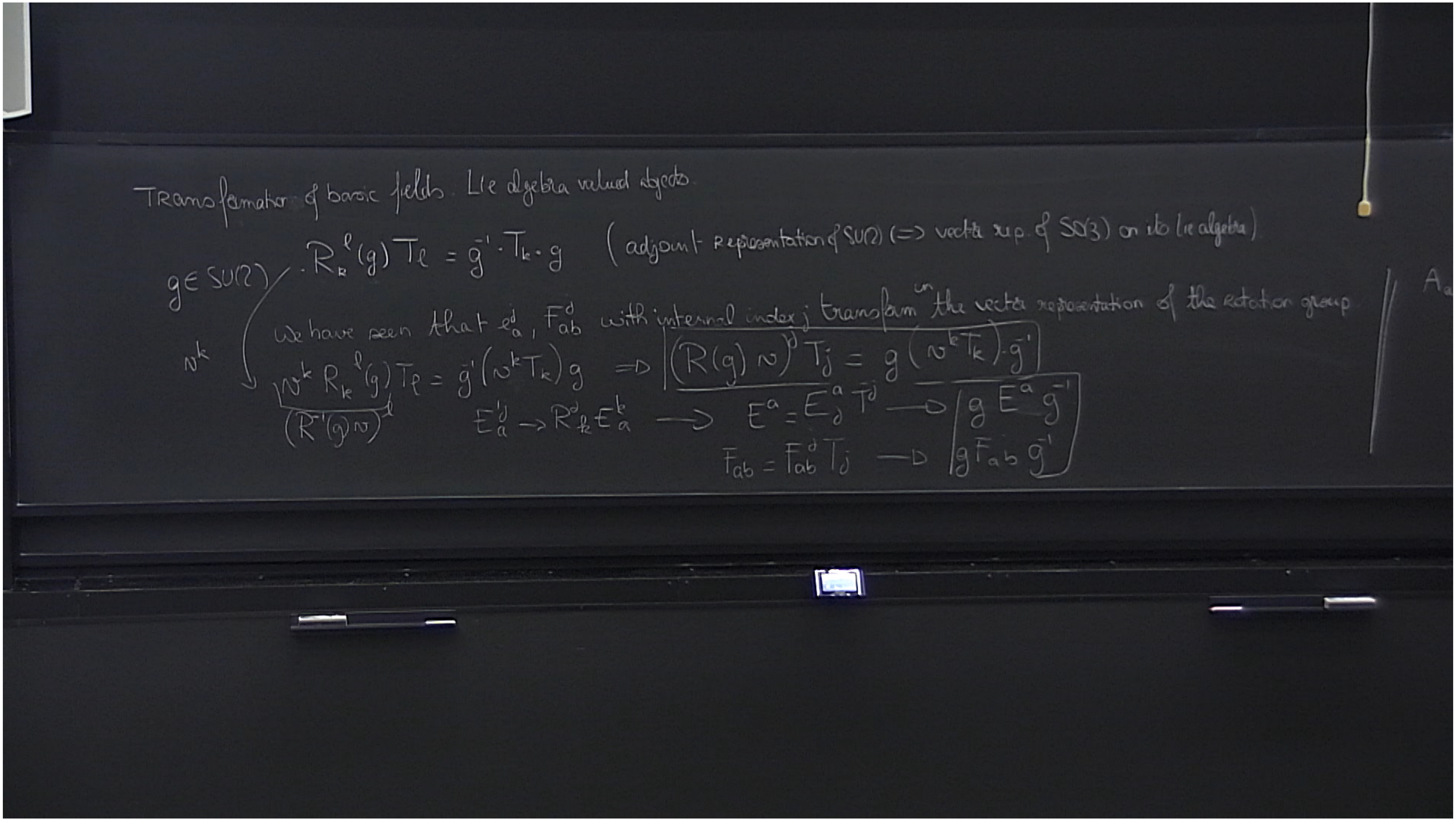
need to smear the functions
Solution: to smear the basic


Find complete set of gauge invariant observables → F_τ → F_{P_τ}
Dirac program to 3d gravity → Basic variables
- {f, g} form an algebra
- transform in a simple way under gauge transfo.
- ⚠ field theory {φ(x), π(y)} = δ(x,y) distribution
↳ need to smear the functions
Solution: to smear the basic configuration variables over k-dimensional hypersurfaces and the conjugated variables over (d−k)d ones


↳ need to smear
Solution: to smear the basic variables over
configuration
LQG holonomies-fluxes
⇒
• configuration: connection = 1-form
↳ integrate over one dimensional object: curve γ → holonomy (finite // transport along γ)
h_γ[A] ∈ SU(2)
h_γ[A] = P exp -∫_γ A
Σ
• conjugated variables: triads → co-triad (one-form) e_a = e_a^j T_j = ε_ab E^bj T^j smeared over a curve γ*
T_j = -i/2 σ_i   [T_i, T_j] = ε_ijk T^k   (su(2) so(3) algebra)
↳ Lie value one form


Transformation of basic fields. Lie algebra valued objects.
$g \in SU(2)$
$R_k^{\ l}(g)\, T_l = g^{-1} \cdot T_k \cdot g$
(adjoint representation of SU(2) (=> vector rep. of SO(3)) on its Lie algebra)
We have seen that e_a^j, F_{ab}^j with internal index j transform in the vector representation of the rotation group.
v^k
$v^k R_k^{\ l}(g)\, T_l = g^{-1} (v^k T_k)\, g \Rightarrow (R(g)\, v)^j\, T_j = g\, (v^k T_k) \cdot g^{-1}$
$(R^{-1}(g)\, v)^l$
$E_a^j \rightarrow R^j_{\ k} E_a^k \longrightarrow E^a = E^a_j T^j$


Transformation of basic fields. Lie algebra valued objects
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$g \in SU(2)$
v^k
We have seen that e^j_a, F^j_{ab} with internal index j transform in the vector representation of the rotation group
$v^k R_k{}^l(g) T_l = g^{-1}(v^k T_k) g \Rightarrow (R(g) v)^j T_j = g (v^k T_k) g^{-1}$
$(R^{-1}(g) v)^l$
$E^j_a \to R^j{}_k E^k_a \to E^a = E^a_j T^j \to g E^a g^{-1}$
$F_{ab} = F^j_{ab} T_j \to g F_{ab} g^{-1}$
A_a

v^k

We have seen that e^j_a, F^j_{ab} with internal index j transform the vector representation of the rotation group

$$v^k R_k{}^l(g) T_l = g^{-1}(v^k T_k) g \Rightarrow (R(g)\, v)^j T_j = g\,(v^k T_k)\, g^{-1}$$

$$(R^{-1}(g)\, v)^l$$

$$E^j_a \to R^j{}_k E^k_a \longrightarrow E^a = E^a_j T^j \longrightarrow g\, E^a g^{-1}$$

$$F_{ab} = F^j_{ab} T_j \longrightarrow g F_{ab} g^{-1}$$

$$\to g A_a g^{-1} + g \partial_a g^{-1}$$

Holonomies: Given an internal vector $V = V^j T_j$ at a point x

V at x, V'; $\gamma(s)$, $s \in [0,1]$ (tangent vector $\dot{\gamma}(s) = \frac{d}{ds}\gamma(s)$)

$\Rightarrow$ parallel transport along the curve γ_s

V transforms in the adjoint rep

So we define the // transport of V by $V(s) = h(s)\, V(0)\, h(s)^{-1}$ $h(s)$ holonomy from $\gamma(0) = x$ to $\gamma(s)$

→ the transformation under local gauge transfo. $g(x) \in SU(2)$ of the holonomy

$$h(s) \longrightarrow g(\gamma(s))\, h(s)\, g(\gamma(0))^{-1}$$

$$\partial_a v^i = \partial_a v^i + \epsilon_{ijk} A_a^j v^k$$

Covariant derivative with the motivation to // transport objects

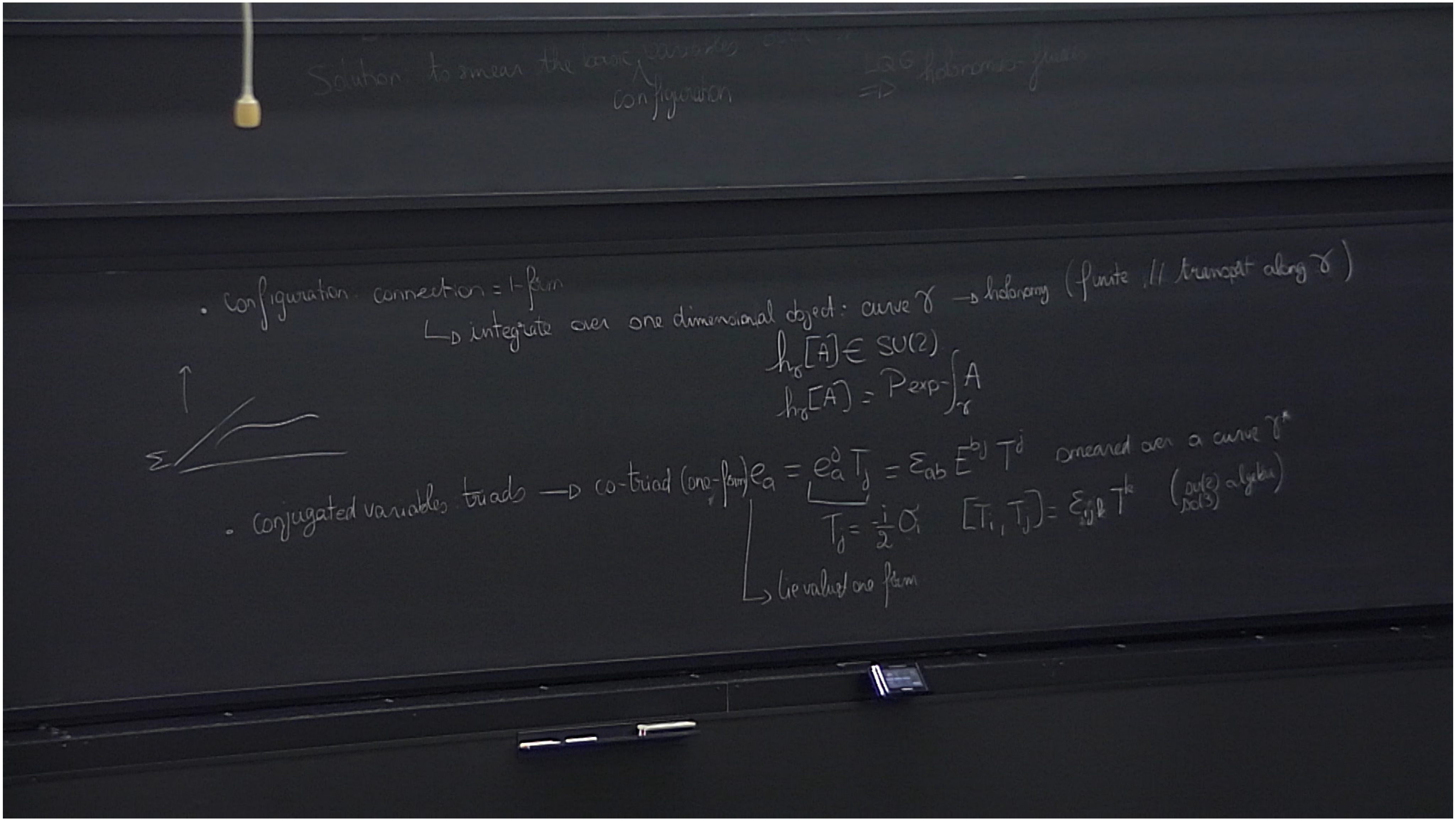
$$D_a V = \partial_a V + [A_a, V]$$

$$\left([A_a^i T_i, V^j T_j] = A_a^i V^j [T_i, T_j]\right)$$

(V: Lie algebra valued scalar)

Solution: to smear the basic variables (configuration) ⇒ LQG holonomies-fluxes

• configuration: connection = 1-form ... one dimensional object: curve γ → holonomy (finite // transport along γ


Solution to smear the basic variables
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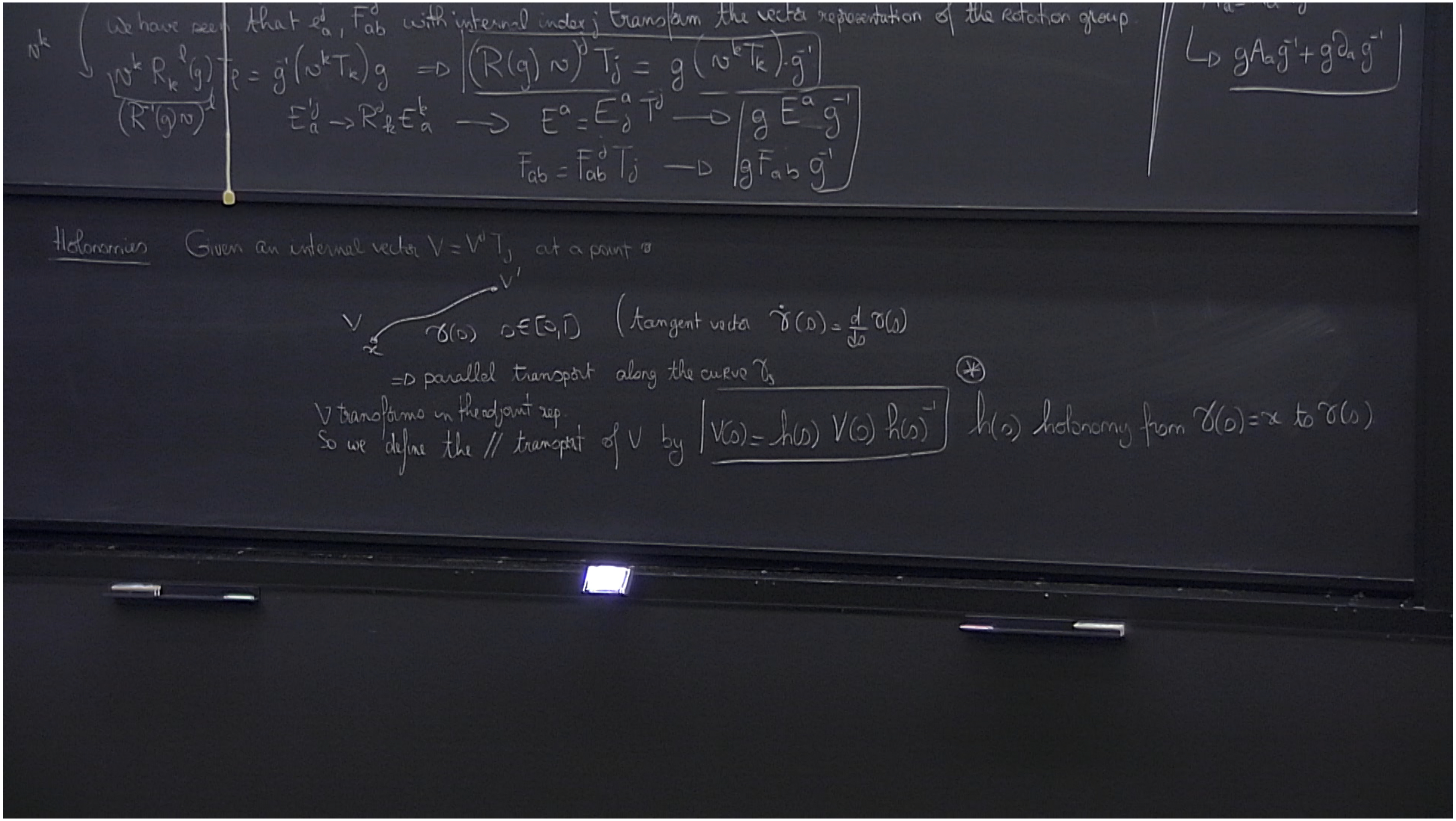
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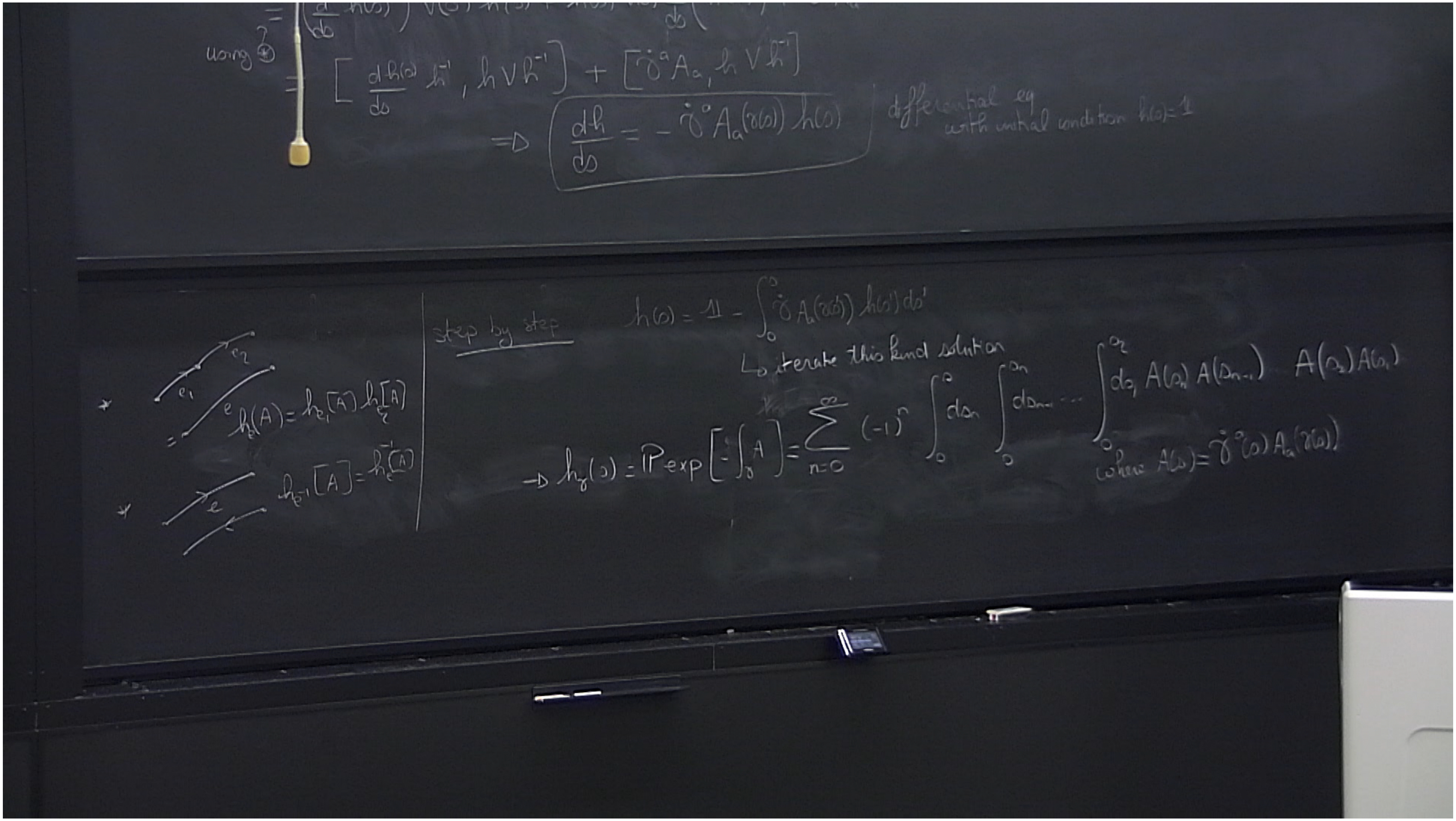
v^k R_k^l(g) T_l = g^-1 (v^k T_k) g =D (R(g) v)^j T_j = g (v^k T_k) g^-1
(R^-1(g) v)^l
E^j_a -> R^j_k E^k_a --> E^a = E^a_j T^j --D g E^a g^-1
F_ab = F^j_ab T_j --D g F_ab g^-1
L_D gA_a g^-1 + g ∂_a g^-1
Holonomies Given an internal vector V = V^j T_j at a point x
V'
V
x
γ(s) s ∈ [0,1] (tangent vector γ̇(s) = d/ds γ(s)
=D parallel transport along the curve γ_s
V transforms in the adjoint rep.
So we define the // transport of V by V(s) = h(s) V(0) h(s)^-1
h(s) holonomy from γ(0) = x to γ(s)


using ⊛
differential eq
with initial condition h(0) = 1
step by step
iterate this kind solution
where A(s) = γ̇^a(s) A_a(γ(s))