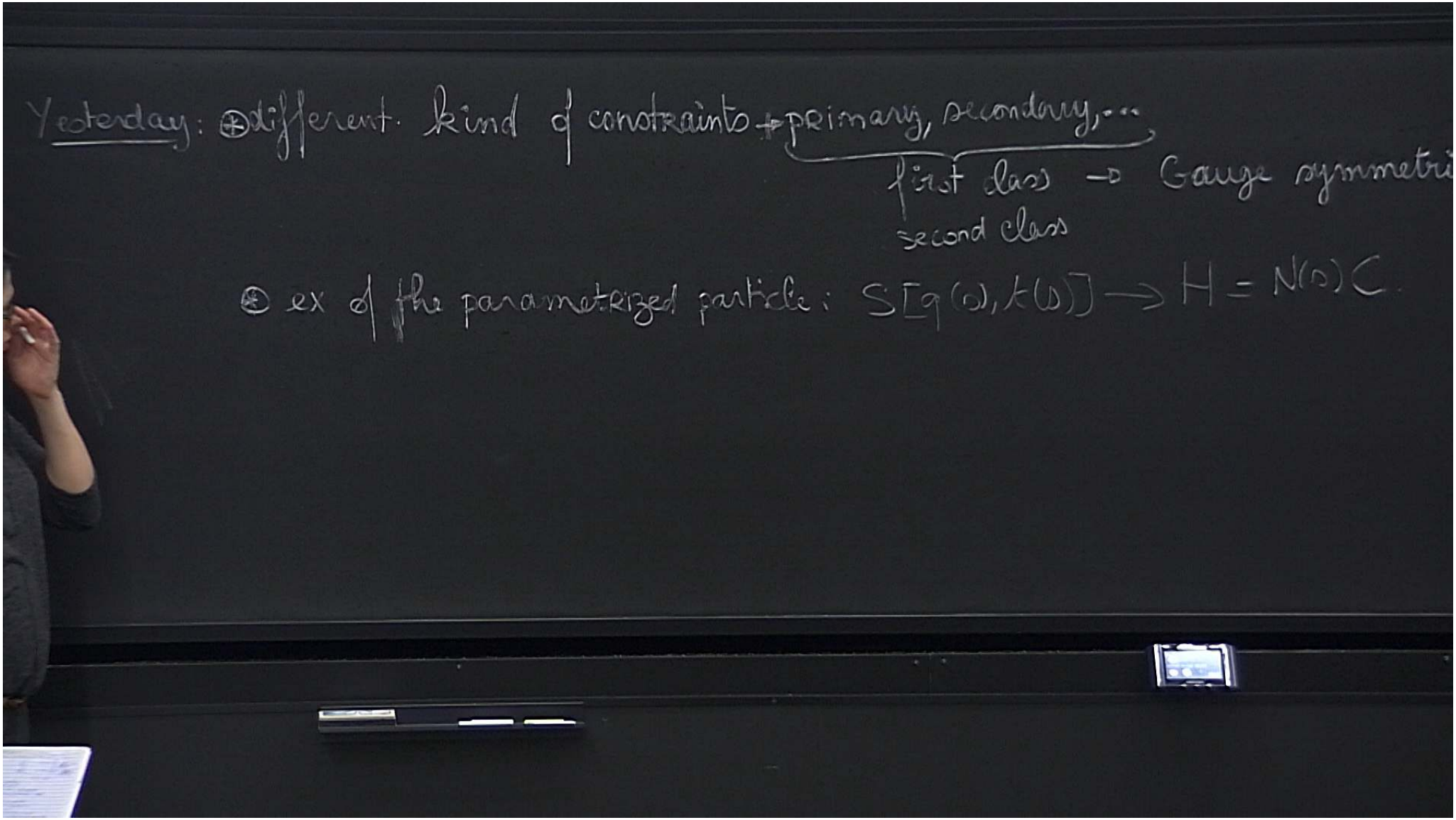
Title: PSI 2016/2017 Explorations in Quantum Gravity - Lecture 5

Date: Mar 24, 2017 10:15 AM

URL: http://pirsa.org/17030072

Abstract:


Yesterday: ⊛different. kind of constraints → primary, secondary, ...
first class → Gauge symmetri
second class
⊛ ex of the parametrized particle: S[q(σ), t(σ)] → H = N(σ)C.

Yesterday: ⊛ different kind of constraints → primary, secondary, ...

first class → Gauge symmetri

second class

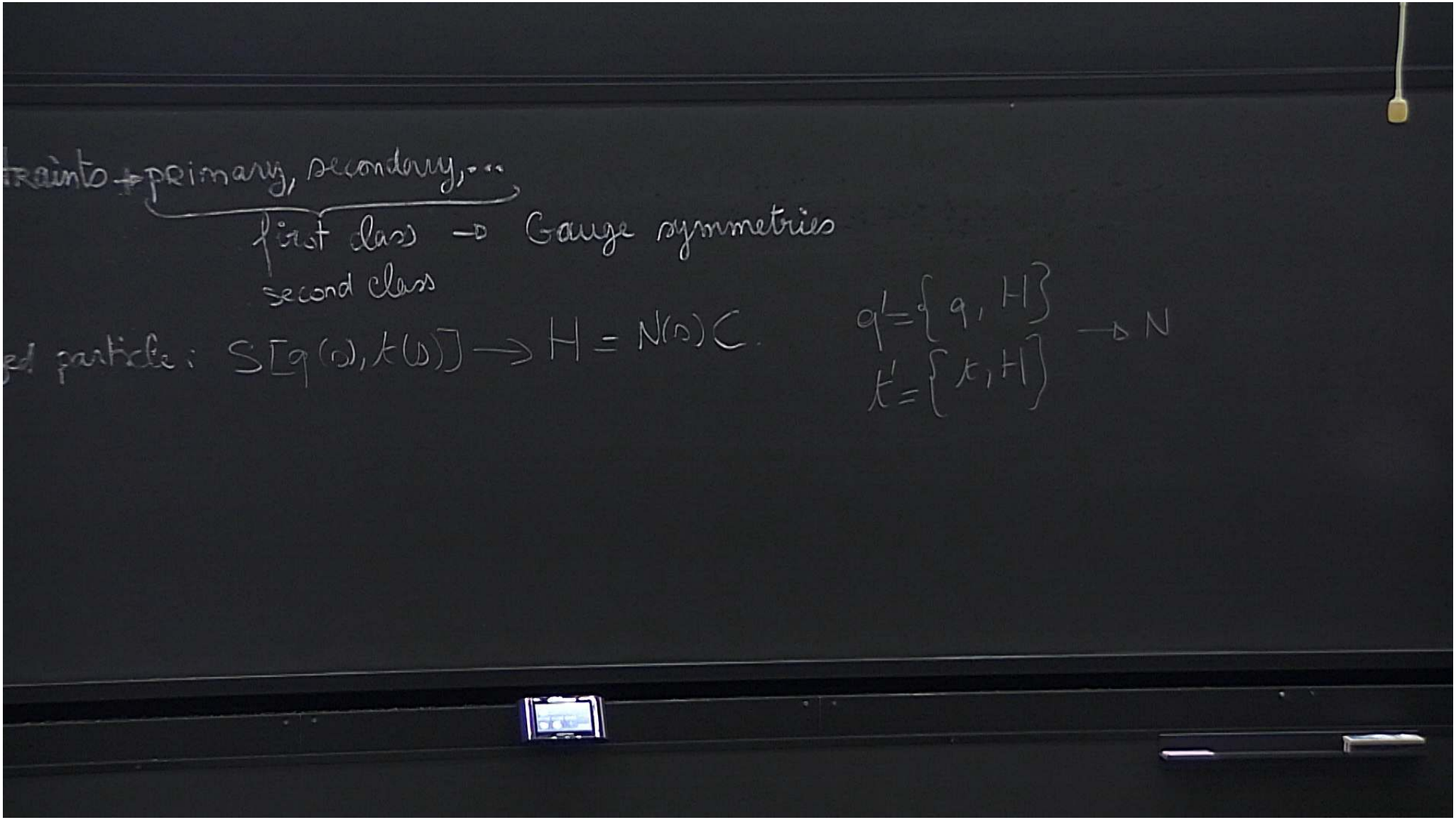
⊛ ex of the parametrized particle: $S[q(\sigma), t(\sigma)] \rightarrow H = N(\sigma)C$.

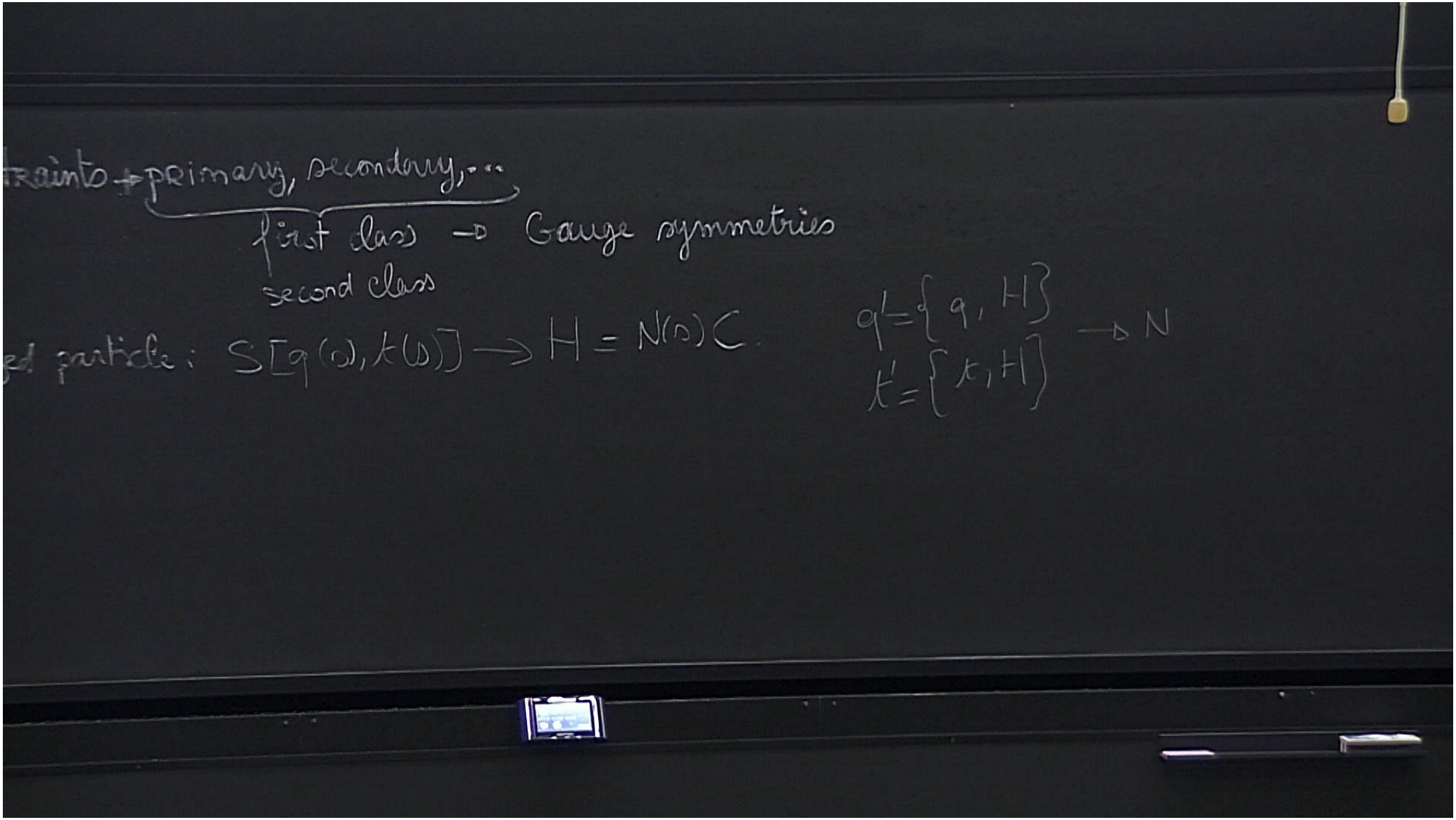
Yesterday: ⊛ different kind of constraints → primary, secondary, ...

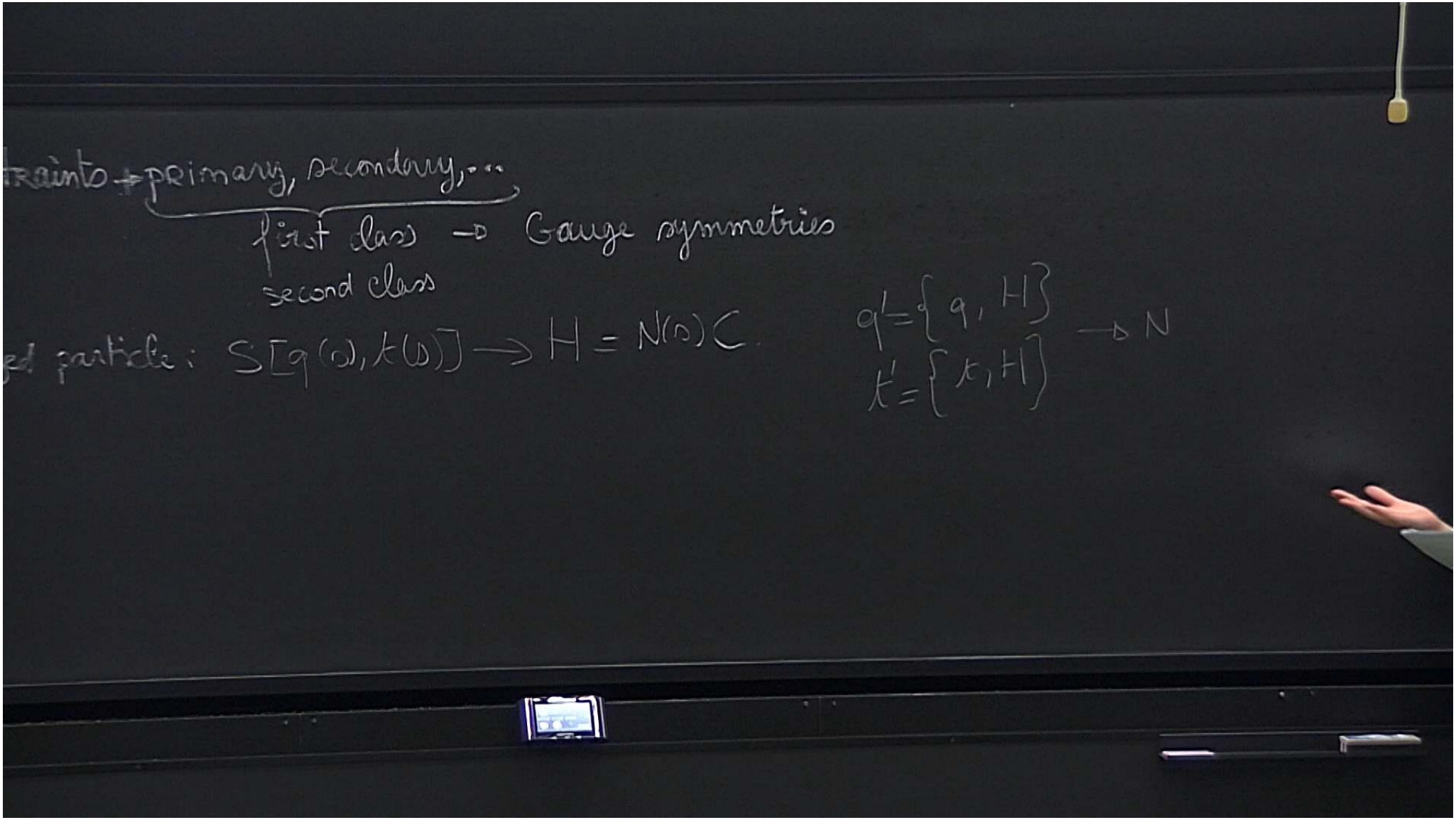
first class → Gauge symmetri

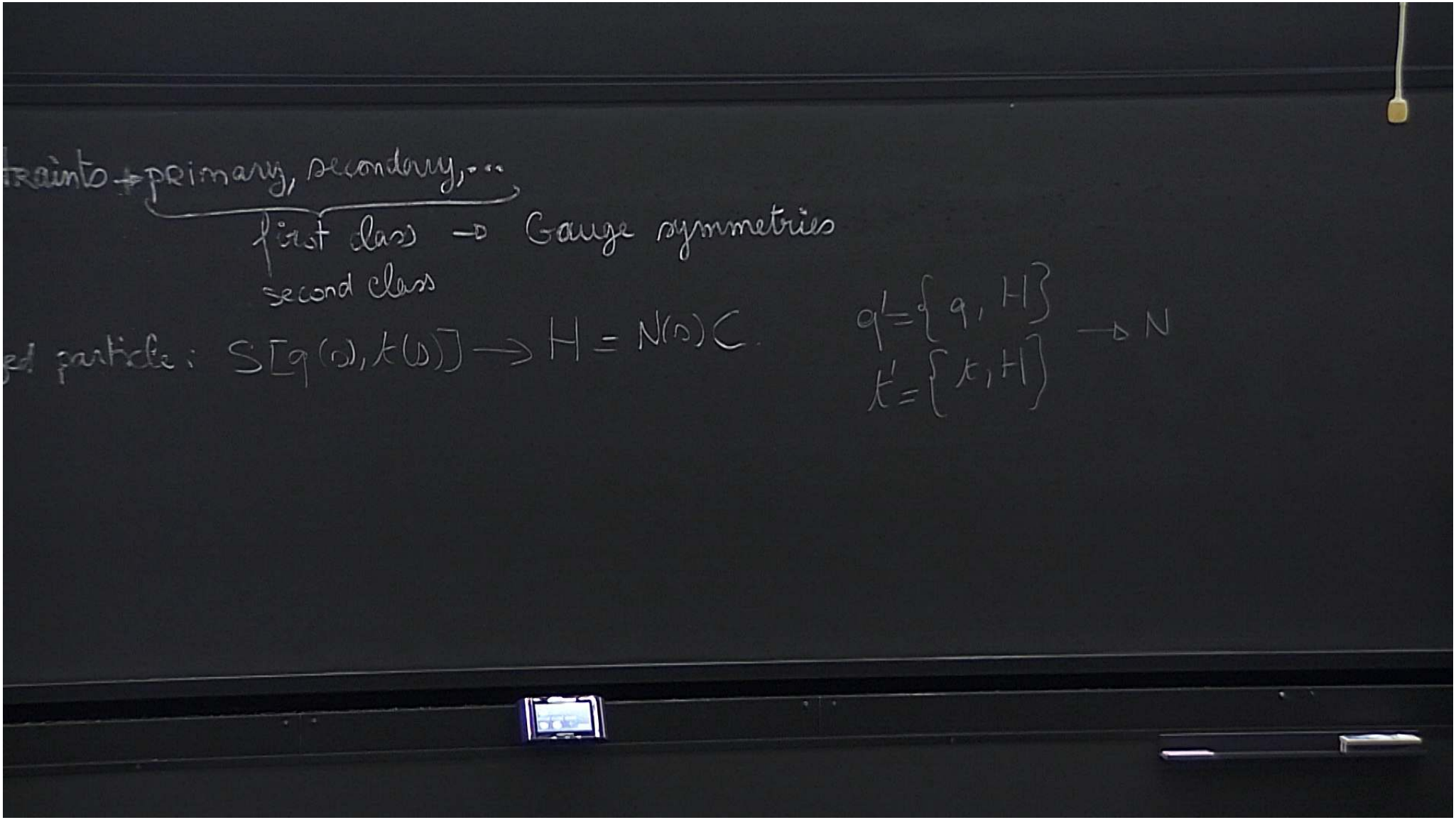
second class

⊛ ex of the parametrized particle: $S[q(\cdot), t(\cdot)] \rightarrow H = N(\cdot)C$


straints → primary, secondary, ...
first class → Gauge symmetries
second class
ed particle: $S[q(\sigma), t(\sigma)] \rightarrow H = N(\sigma)C$.
$q' = \{q, H\}$
$t' = \{t, H\}$
→ N


traints → primary, secondary, ...
first class → Gauge symmetries
second class
ed particle: S[q(t), t(t)] → H = N(t) C
q' = {q, H}
t' = {t, H}
→ N


straints → primary, secondary, ...
first class → Gauge symmetries
second class
ged particle: $S[q(\sigma), t(\sigma)] \rightarrow H = N(\sigma) C$
$q' = \{q, H\}$
$t' = \{t, H\}$
$\rightarrow N$



traints → $\underbrace{\text{primary, secondary, ...}}$

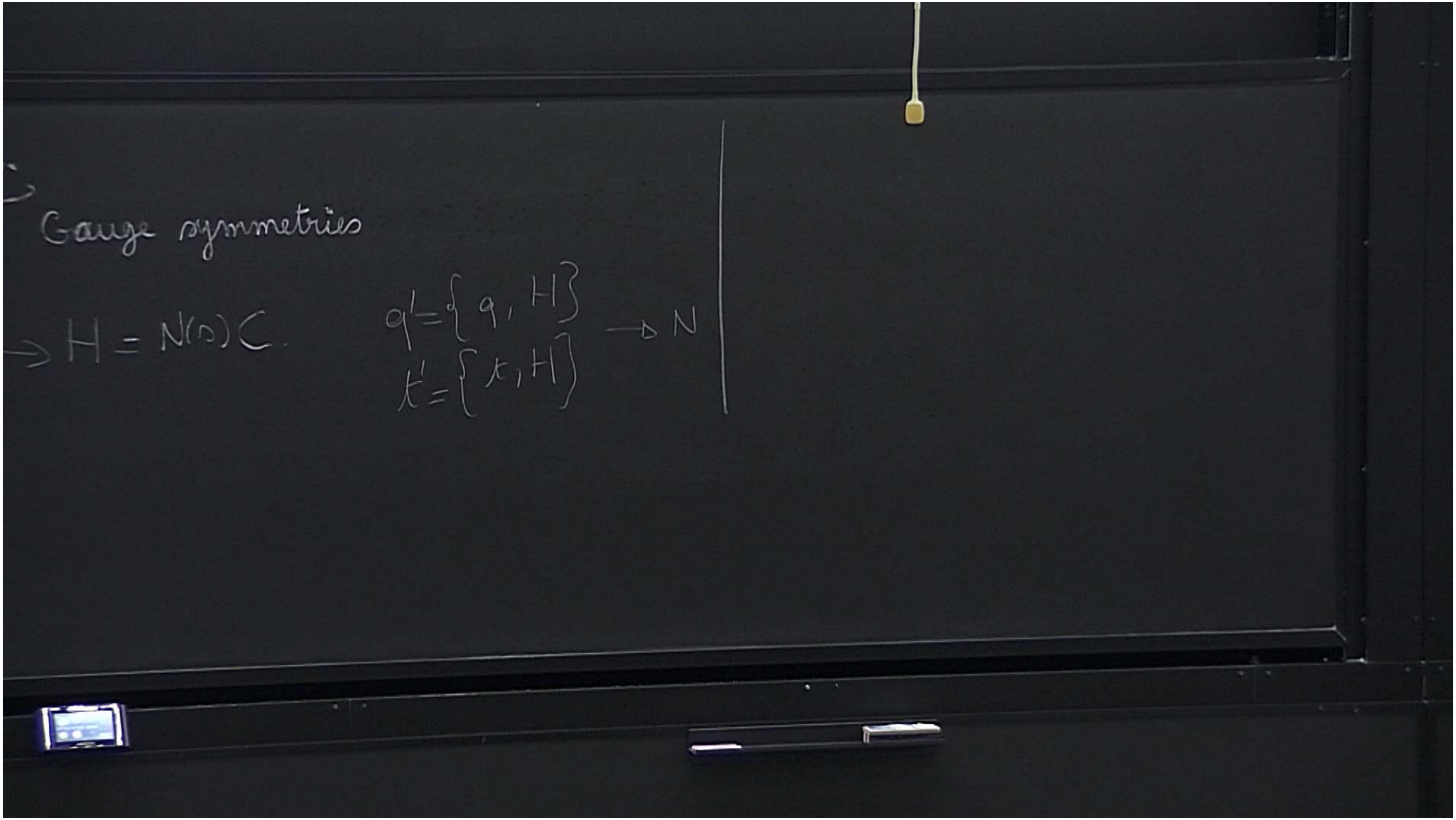
first class → Gauge symmetries

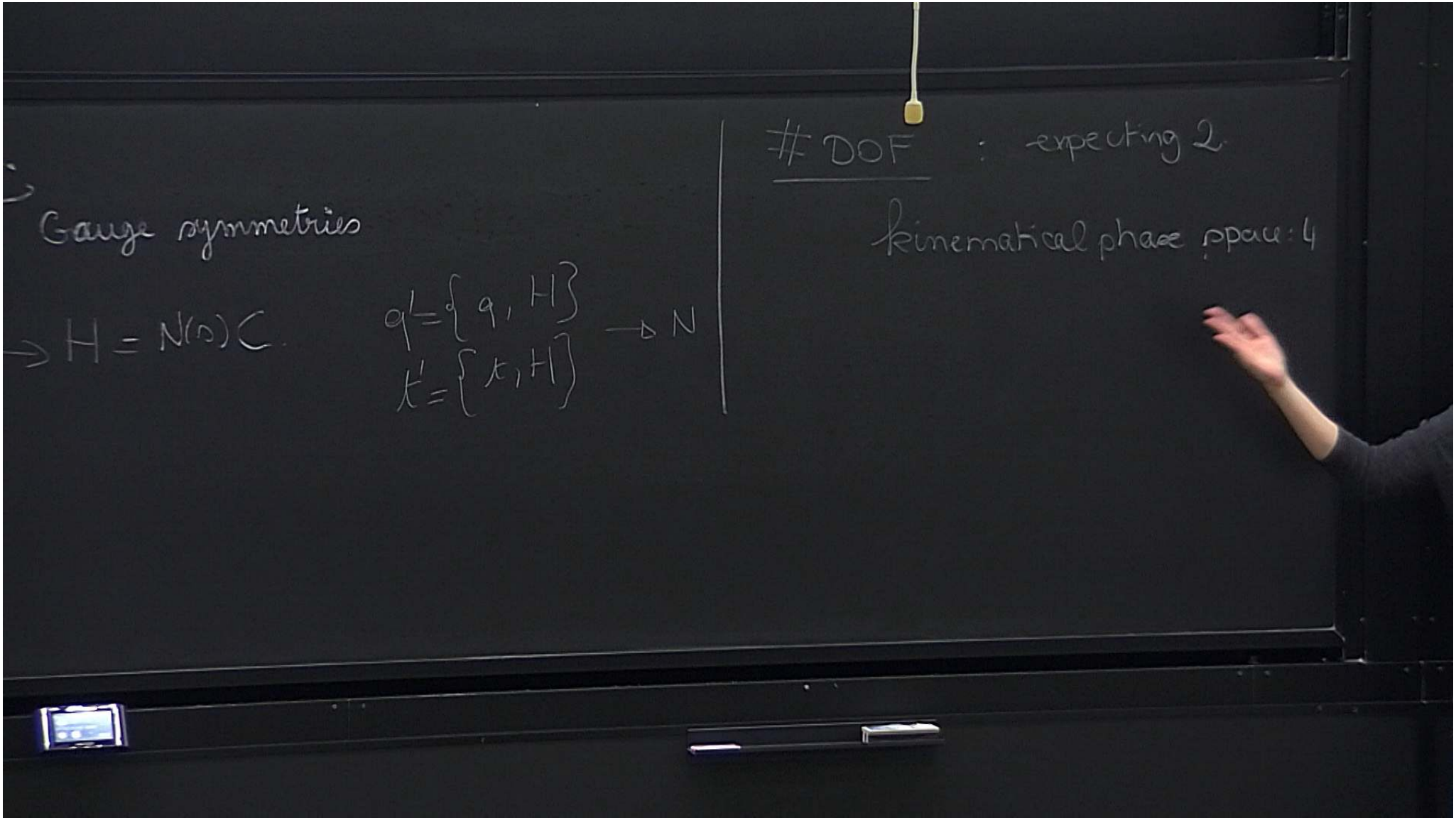
second class

ed particle: $S[q(s), t(s)] \longrightarrow H = N(s)C$

$q' = \{q, H\}$

$t' = \{t, H\}$ → N


Gauge symmetries
$\rightarrow H = N(\Omega) C$
$q' = \{q, H\}$
$t' = \{t, H\}$
$\rightarrow N$


Gauge symmetries
→ H = N(C)
q' = {q, H}
t' = {t, H}
→ N
DOF : expecting 2
kinematical phase space: 4

Gauge symmetries

$\rightarrow H = N(s)C$

$q' = \{q, H\}$
$t' = \{t, H\}$ $\rightarrow N$

DOF : expecting 2

kinematical phase space: 4

Gauge symmetries

$\rightarrow H = N(1)C$

$q' = \{q, H\}$
$t' = \{t, H\}$ $\rightarrow N$

DOF : expecting 2

kinematical phase space : 4

-1 by implementing C

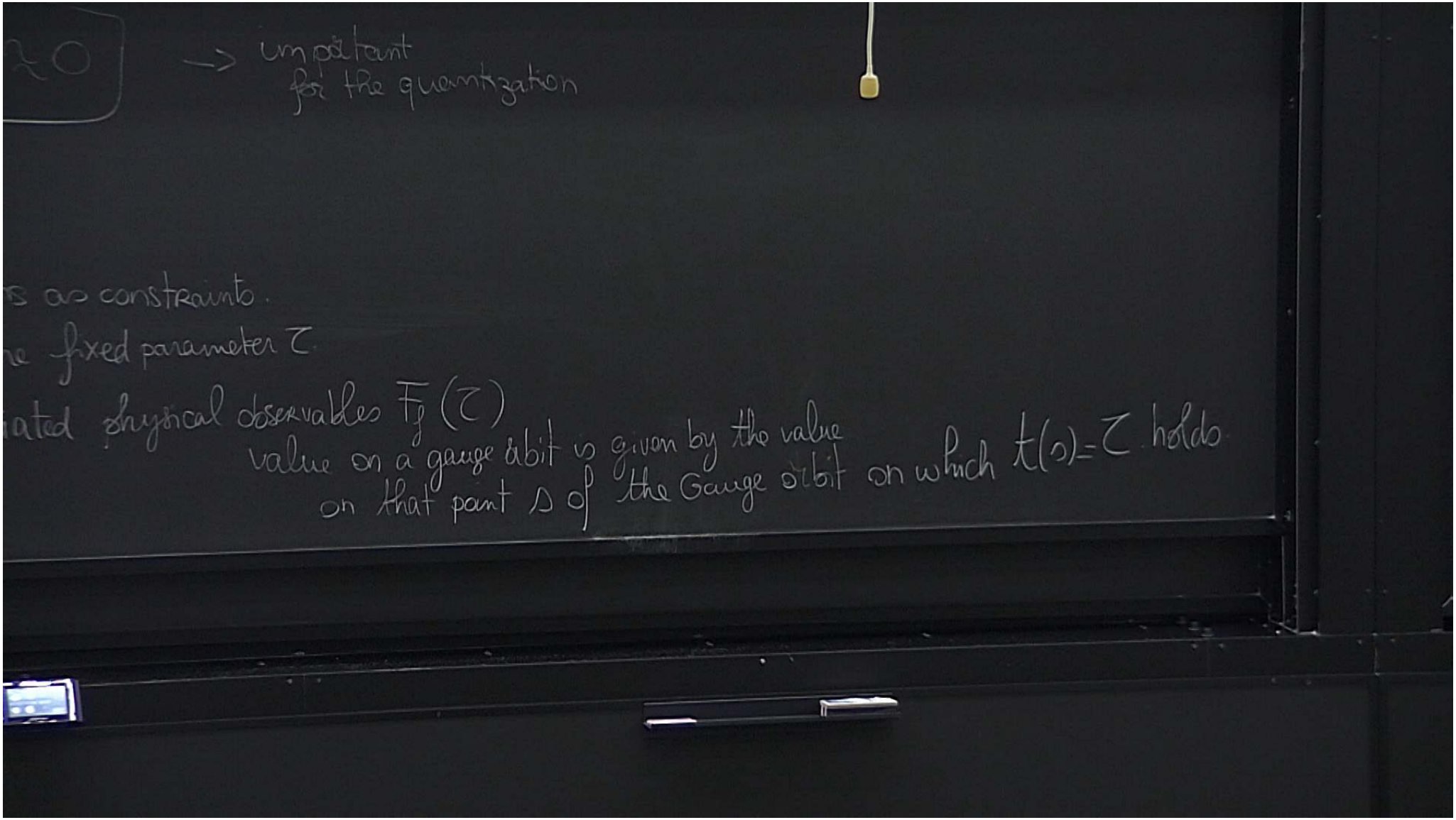
-1 by dividing by the gauge orbit.

→ 2 Dirac variables F $\{F, C\} \approx 0$
gauge inv.
Observables

→ 2 Dirac variables F gauge inv. $\{F, C\} \approx 0$ → important for the quantization

Observables

* Construction of physical observables
  - select as many gauge fixing conditions as constraints.
    $t = \tau$ for some fixed parameter τ.
  - Given a phase space function f, associated physical observables $F_f(\tau)$
    value on a gauge orbit is giv
    on that point S of th



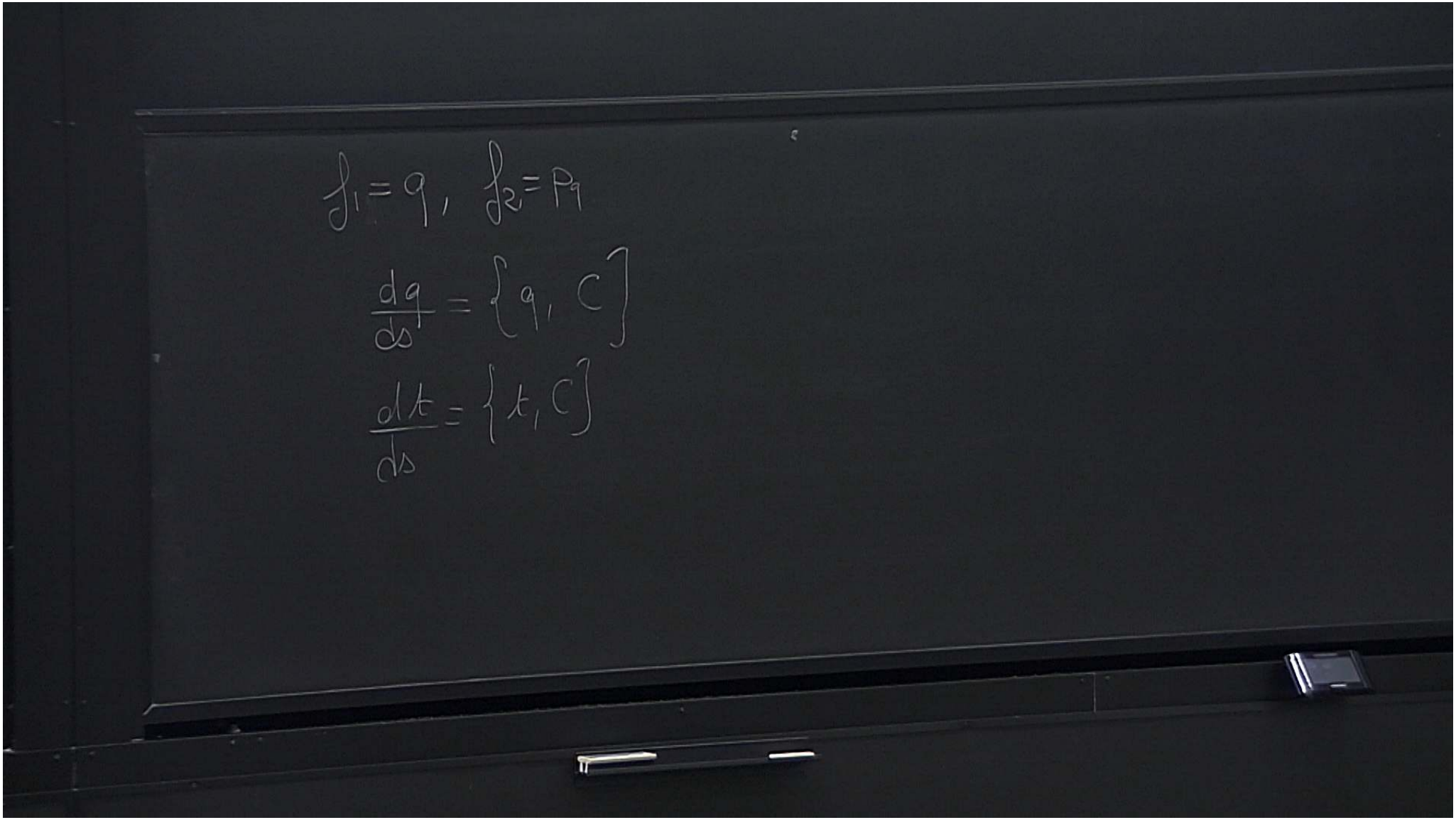
≈ 0 → important for the quantization

...s as constraints.

...e fixed parameter τ.

...iated physical observables $F_f(\tau)$

value on a gauge orbit is given by the value on that point s of the Gauge orbit on which $t(s) = \tau$ holds.


$f_1 = q, \quad f_2 = p_q$
$\frac{dq}{ds} = \{q, C\}$
$\frac{dt}{ds} = \{t, C\}$

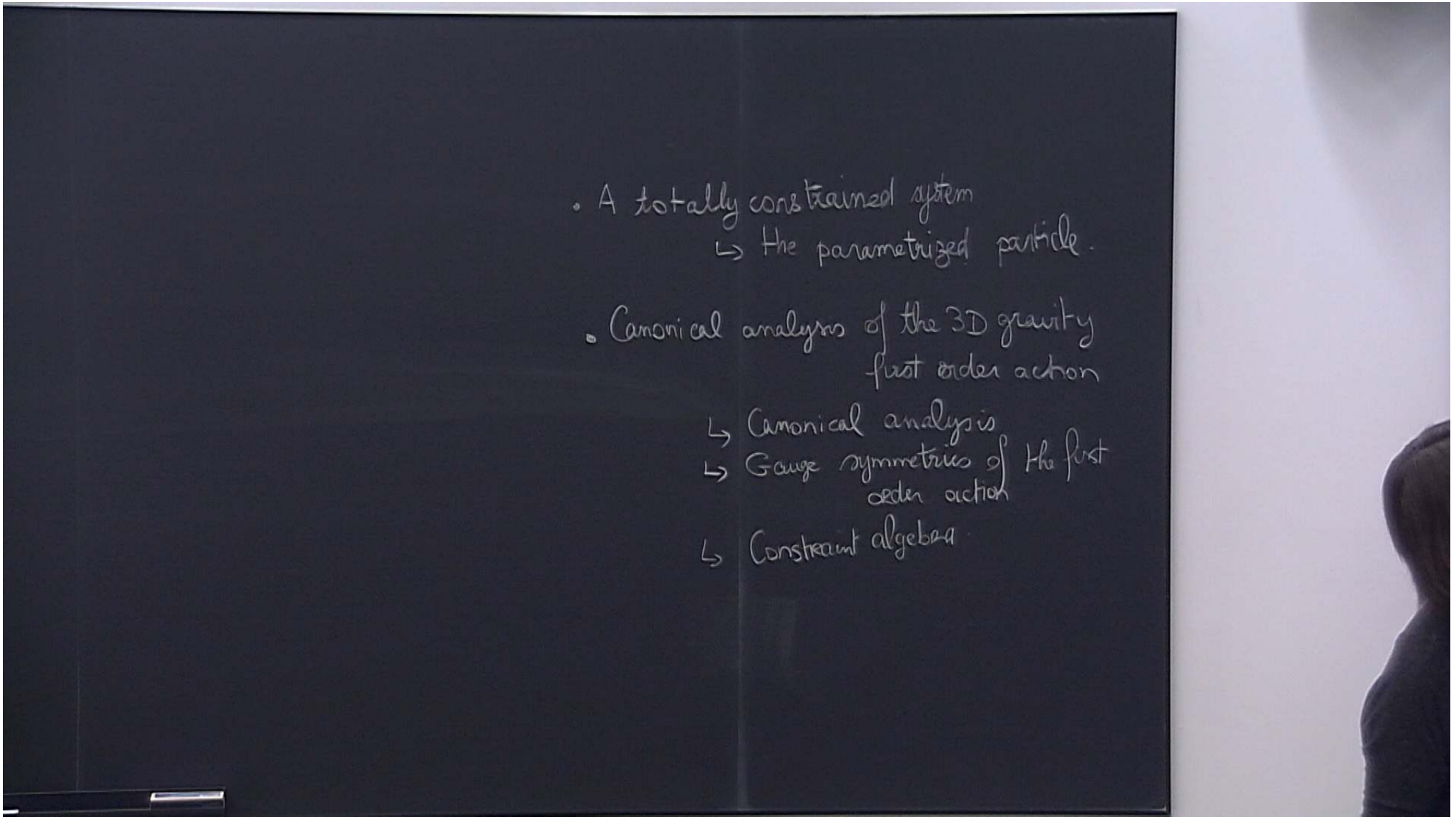
$f_1 = q, \quad f_2 = p_q$

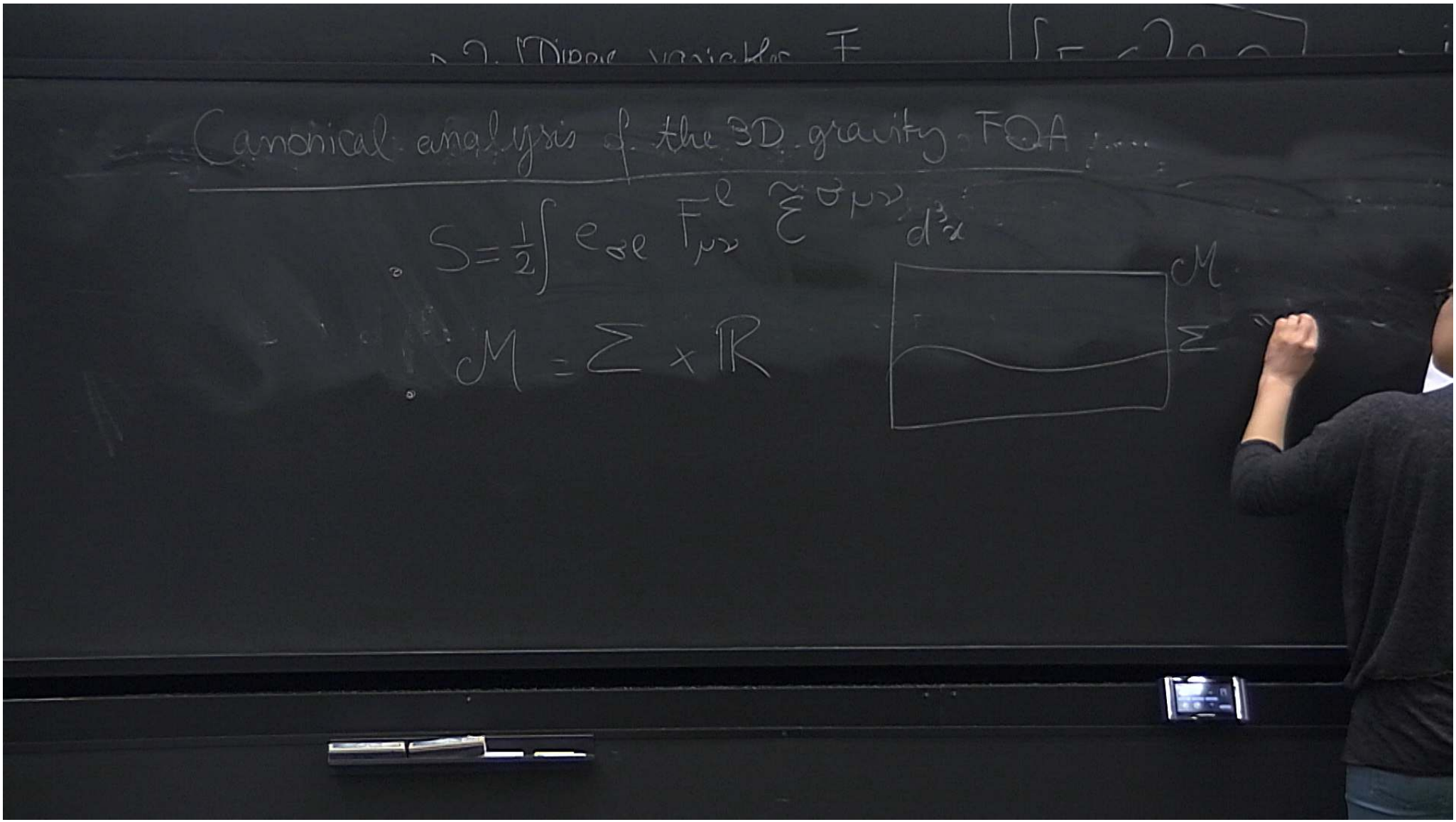
$\frac{dq}{ds} = \{q, C\}$ $\qquad$ $\frac{dp_q}{ds} =$

$\frac{dt}{ds} = \{t, C\}$ $\qquad$ $\frac{dp_t}{ds} =$

$\hookrightarrow t(s) = \tau$

$\hookrightarrow q(s) \longrightarrow F_q^\tau$: value of q at the point of the gauge orbit where t takes the value τ.


• A totally constrained system
↳ the parametrized particle.
• Canonical analysis of the 3D gravity first order action
↳ Canonical analysis
↳ Gauge symmetries of the first order action
↳ Constraint algebra


Canonical analysis of the 3D gravity FOA ....
$S = \frac{1}{2}\int e_{\sigma\ell} F^{\ell}_{\mu\nu} \tilde{\varepsilon}^{\sigma\mu\nu} d^3x$
$\mathcal{M} = \Sigma \times \mathbb{R}$
$\mathcal{M}$
Σ

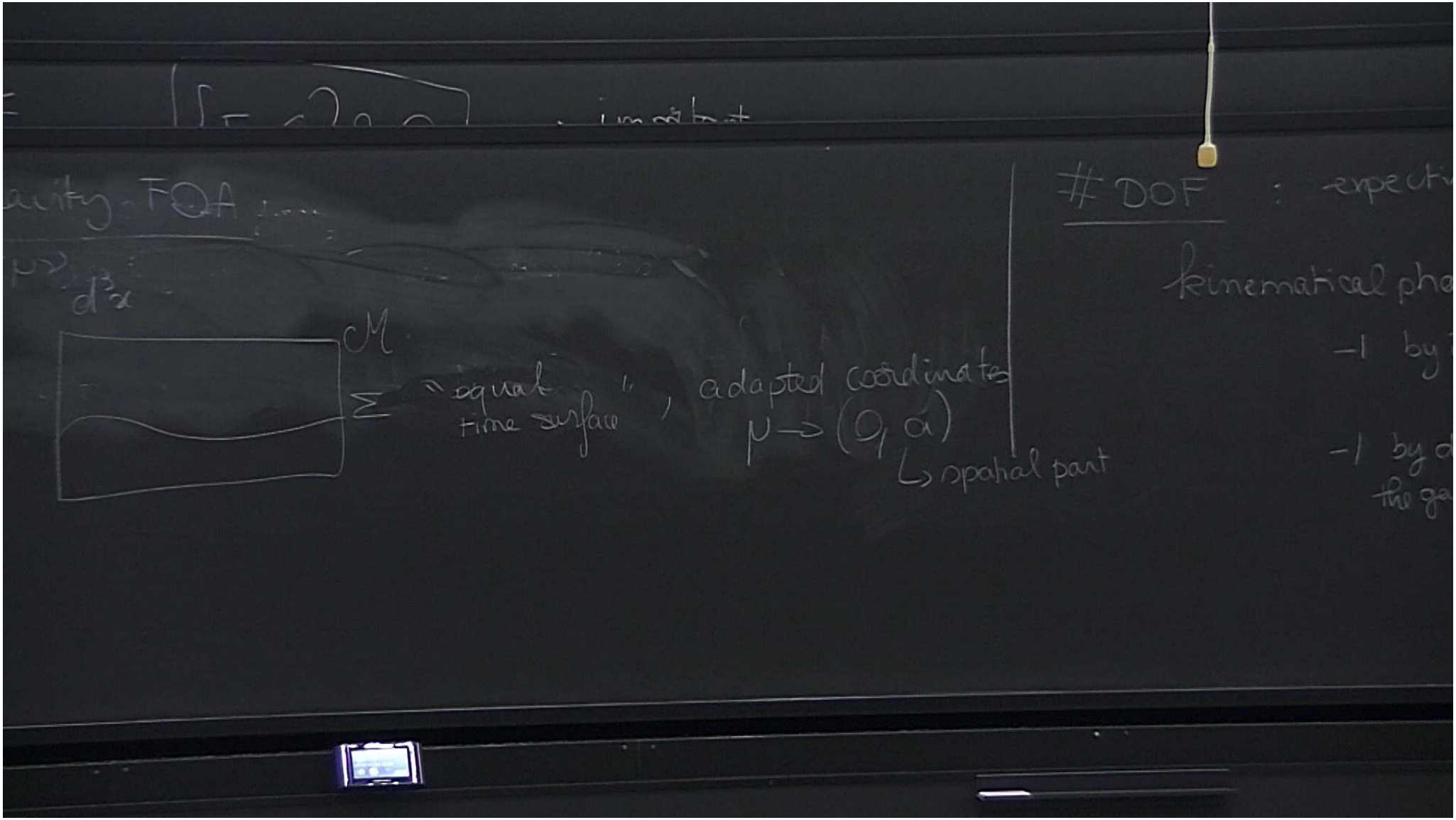
2. Dirac variables F

Canonical analysis of the 3D gravity FOA :

$$S = \frac{1}{2}\int e_{ae}\, F^{e}_{\mu\nu}\, \tilde{\varepsilon}^{a\mu\nu}\, d^3x$$

$$\mathcal{M} = \Sigma \times \mathbb{R}$$

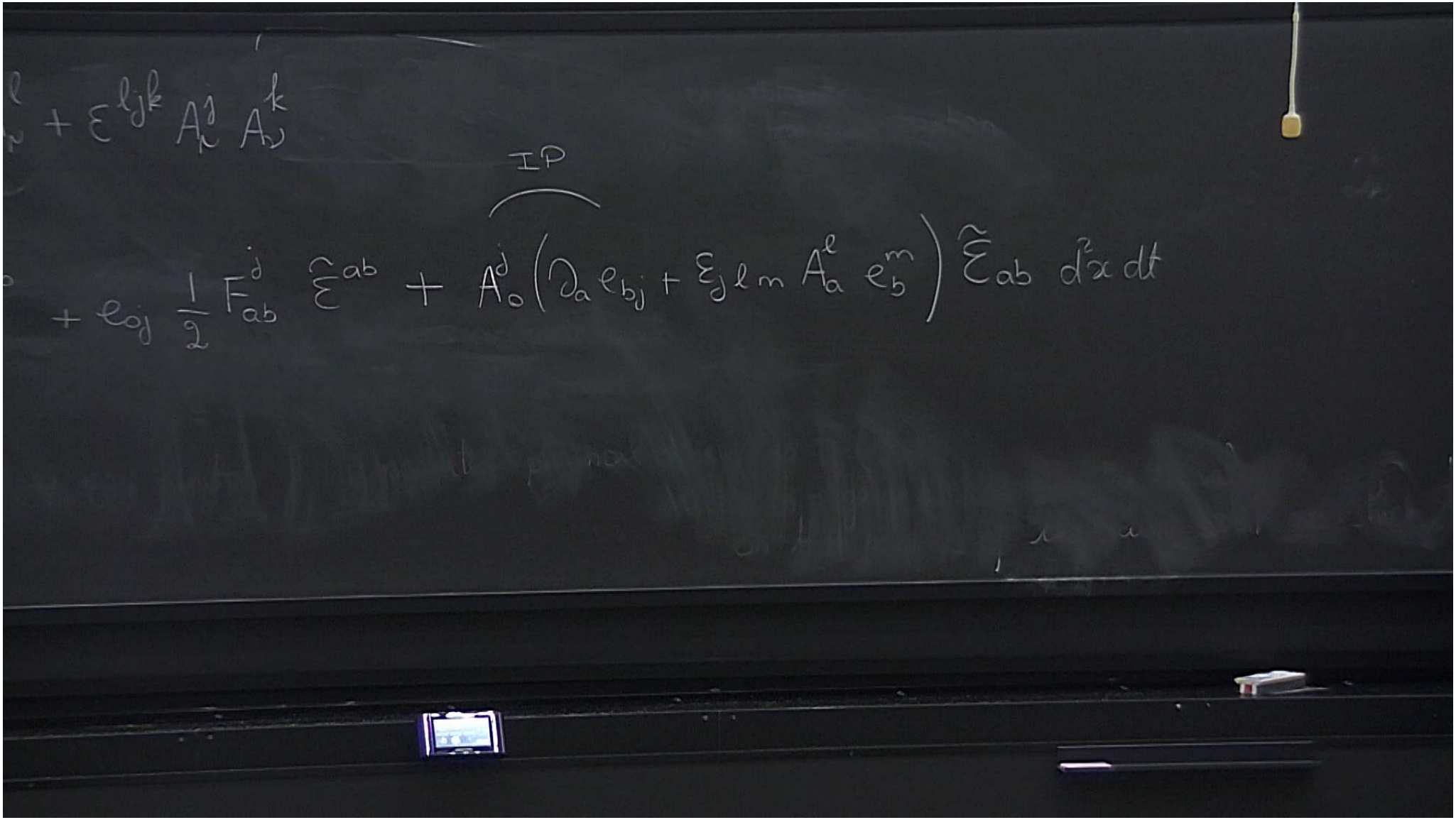
= equal time surface

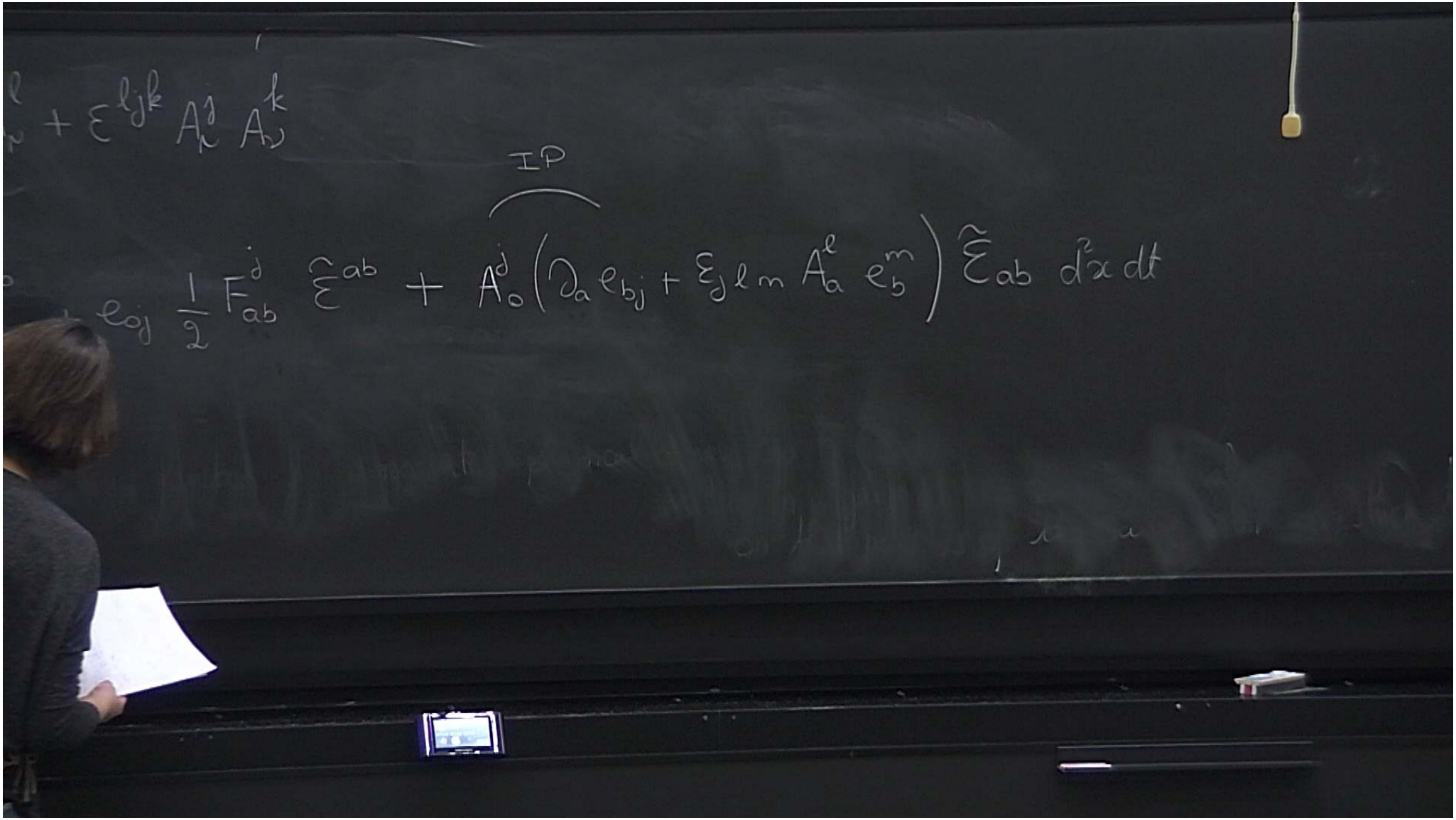

#DOF
kinematical
M
Σ
"equal time surface"
adapted coordinates
μ → (0, a)
↳ spatial part

$$F^l_{\mu\nu} = \partial_\mu A^l_\nu - \partial_\nu A^l_\mu + \varepsilon^{ljk} A^j_\mu A^k_\nu$$

$$F^l_{oa} = -F^l_{ao}$$

$$S = \int e^d_b \partial_o A_{aj} \tilde{E}^{ab} + e_{oj} \frac{1}{2} F^j_{ab} \tilde{E}^{ab} + \overbrace{A^d_o \left(\partial_a e_{bj} + \varepsilon_{jlm} A^l_a e^m_b \right)}^{IP} \tilde{E}^{ab}$$


$+ \varepsilon^{ljk} A^j_\lambda A^k_\nu$
IP
$+ e_{oj} \frac{1}{2} F^j_{ab} \tilde{\varepsilon}^{ab} + A^j_o \left(\partial_a e_{bj} + \varepsilon_{jlm} A^l_a e^m_b \right) \tilde{\tilde{\varepsilon}}_{ab} \, d^2x \, dt$


$+ \varepsilon^{ljk} A^j_\mu A^k_\nu$
IP
$+ e_{0j} \frac{1}{2} F^j_{ab} \tilde{\varepsilon}^{ab} + A^j_0 \left(\partial_a e_{bj} + \varepsilon_{jlm} A^l_a e^m_b \right) \tilde{\varepsilon}_{ab} \, d^2x \, dt$

$$F^l_{\mu\nu} = \partial_\mu A^l_\nu - \partial_\nu A^l_\mu + \varepsilon^{ljk} A^j_\mu A^k_\nu$$

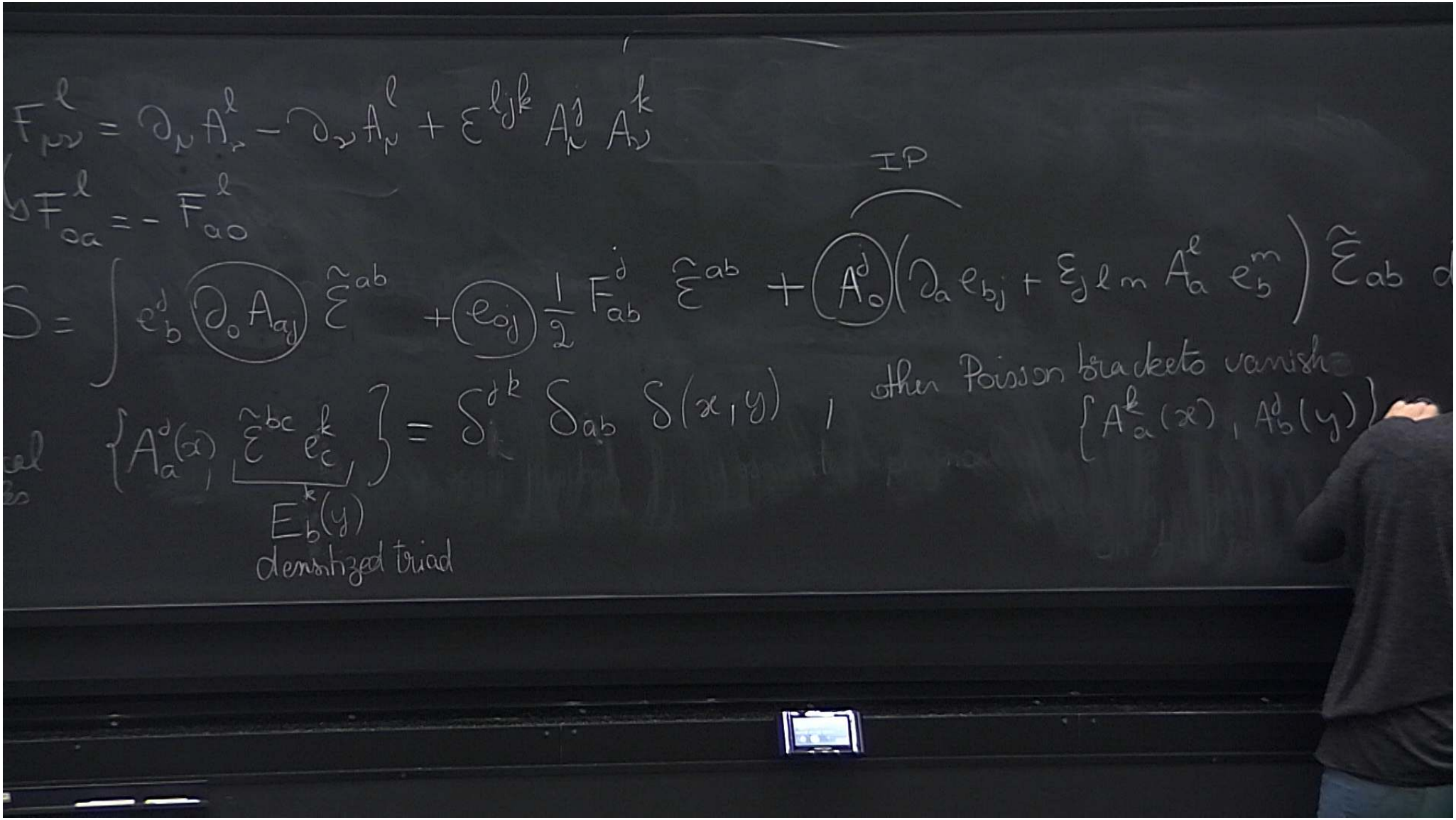
$$F^l_{oa} = -F^l_{ao}$$

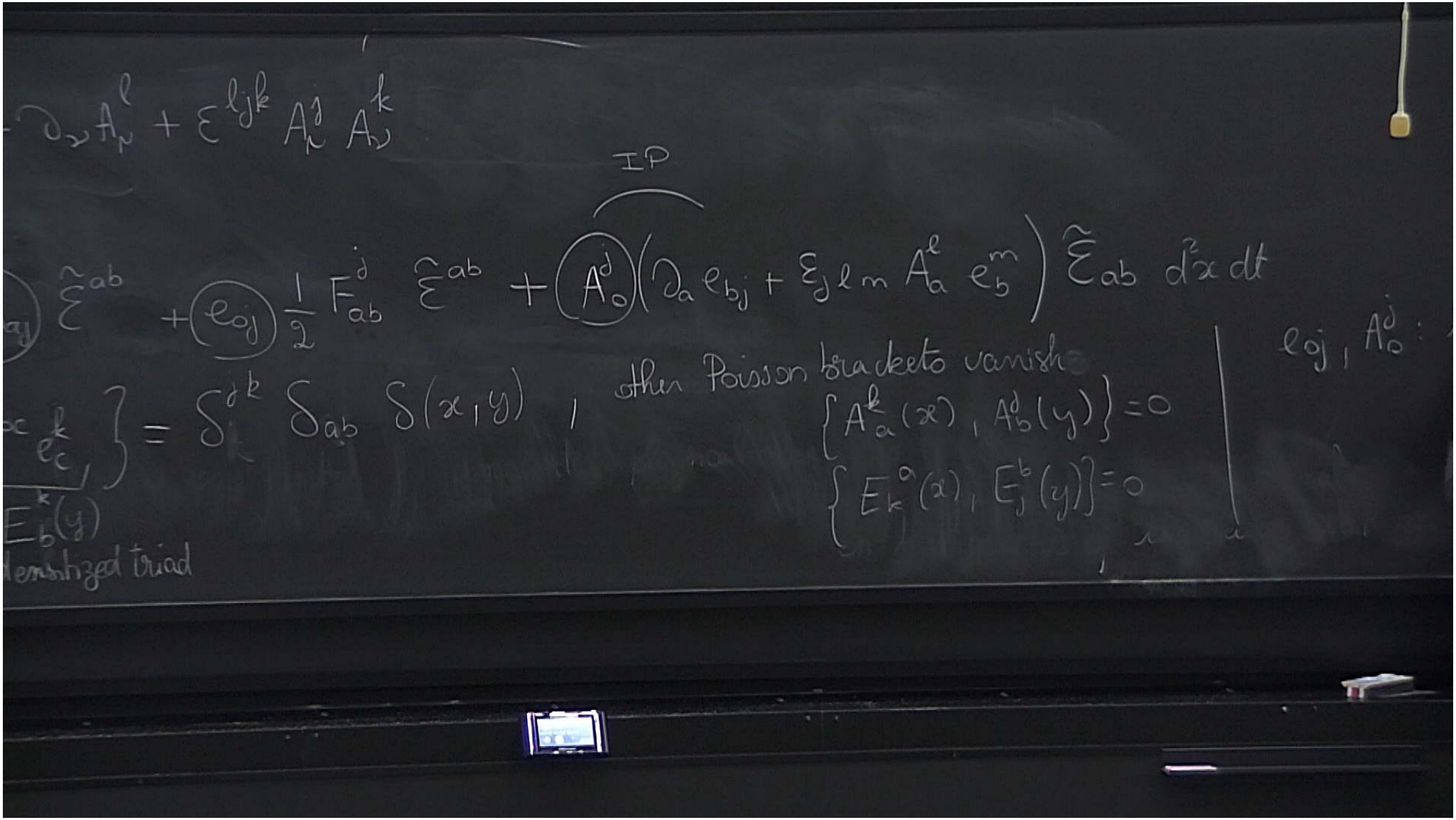
$$S = \int e^d_b \, \partial_o A_{aj} \, \tilde{\varepsilon}^{ab} + e_{oj} \frac{1}{2} F^j_{ab} \, \tilde{\varepsilon}^{ab} + \overbrace{A^d_o \left(\partial_a e_{bj} + \varepsilon_{jlm} \right.}^{IP}$$

$= \tilde{\varepsilon}^{oab}$

Canonical variables $\{ A^d_a(x), \underbrace{\tilde{\varepsilon}^{bc} e^k_c}_{E^k_b(y)} \} = \delta^{dk} \delta_{ab} \delta(x,y)$, other Poisson brac

$E^k_b(y)$ densitized triad


$F^l_{\mu\nu} = \partial_\mu A^l_\nu - \partial_\nu A^l_\mu + \varepsilon^{ljk} A^j_\mu A^k_\nu$
$F^l_{oa} = - F^l_{ao}$
IP
$S = \int e^d_b \partial_o A_{aj} \tilde{\varepsilon}^{ab} + e_{oj} \frac{1}{2} F^j_{ab} \tilde{\varepsilon}^{ab} + A^d_o (\partial_a e_{bj} + \varepsilon_{jlm} A^l_a e^m_b) \tilde{\varepsilon}_{ab}$
$\{A^d_a(x), \tilde{\varepsilon}^{bc} e^k_c\} = \delta^{dk} \delta_{ab} \delta(x,y)$, other Poisson brackets vanish
$\{A^k_a(x), A^l_b(y)\}$
$E^k_b(y)$
densitized triad



$$\ldots \partial_\nu A_\mu^l + \varepsilon^{ljk} A_\mu^j A_\nu^k$$

$$\ldots_{aj}\, \tilde{\varepsilon}^{ab} + e_{0j} \frac{1}{2} F_{ab}^j\, \tilde{\varepsilon}^{ab} + \overbrace{A_0^j \left(\partial_a e_{bj} + \varepsilon_{jlm} A_a^l e_b^m\right)}^{IP} \tilde{\varepsilon}_{ab}\, d^2x\, dt$$

$$\ldots e_c^k \Big\} = \delta^{jk}\, \delta_{ab}\, \delta(x,y)\,,$$

$E_b^k(y)$

densitized triad

other Poisson brackets vanish

$$\{A_a^k(x), A_b^j(y)\} = 0$$

$$\{E_k^a(x), E_j^b(y)\} = 0$$

e_{0j}, A_0^j :

$$\left(\partial_a e_{bj} + \varepsilon_{jlm} A_a^l e_b^m\right) \tilde{\tilde{\varepsilon}}_{ab}\, d^3x\, dt$$

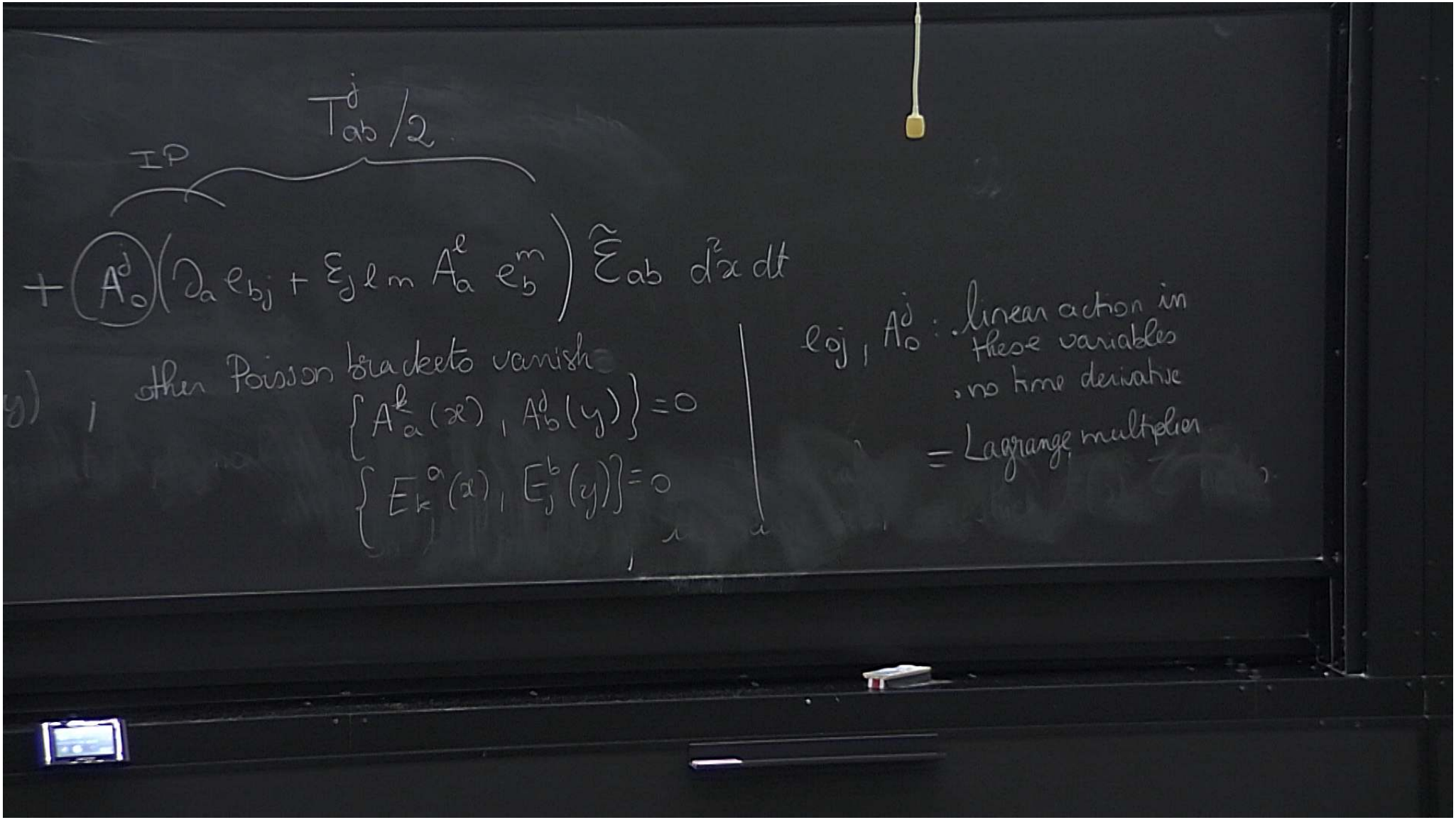
Then Poisson brackets vanish

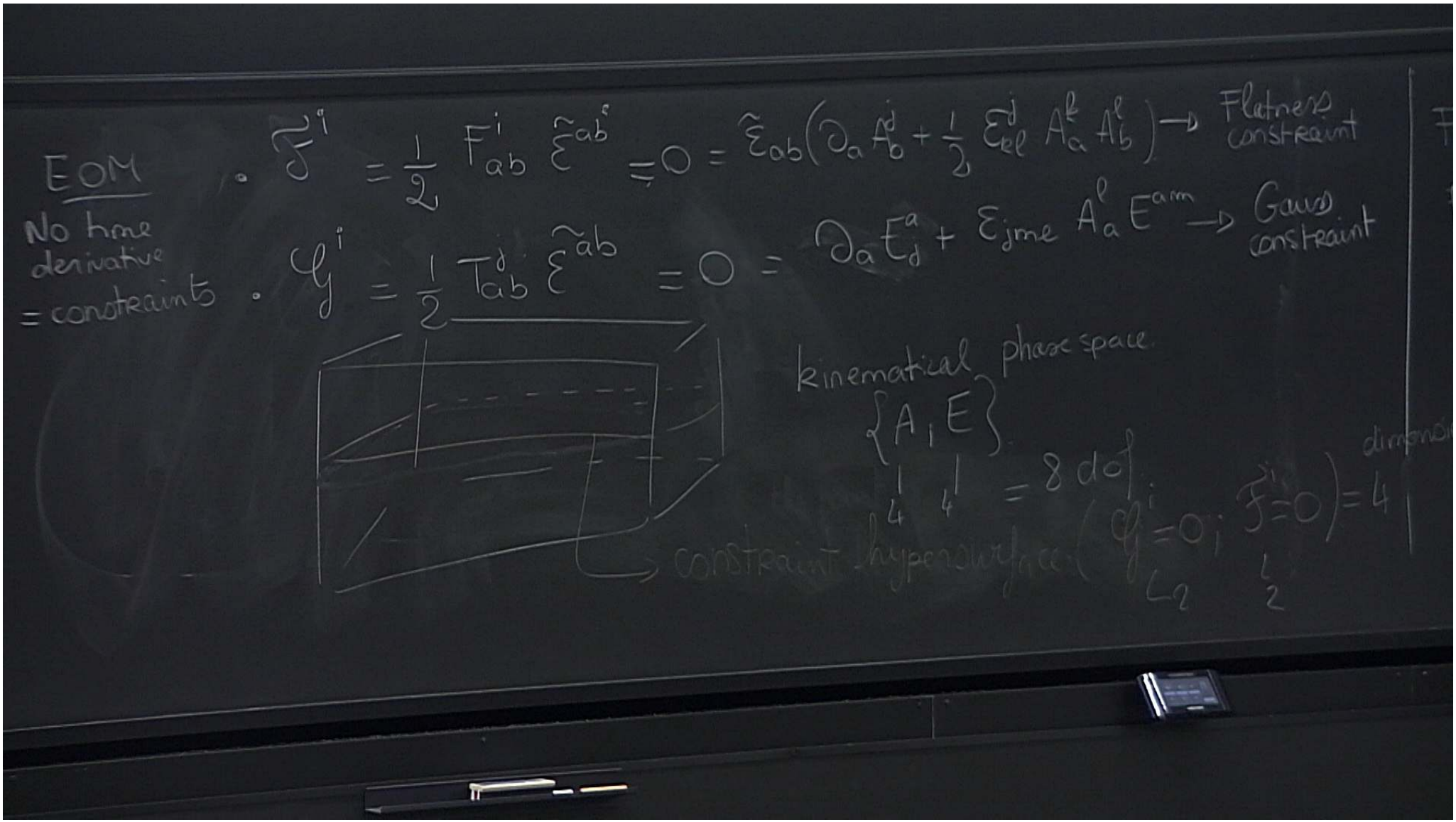
$$\left\{A_a^k(x), A_b^l(y)\right\} = 0$$

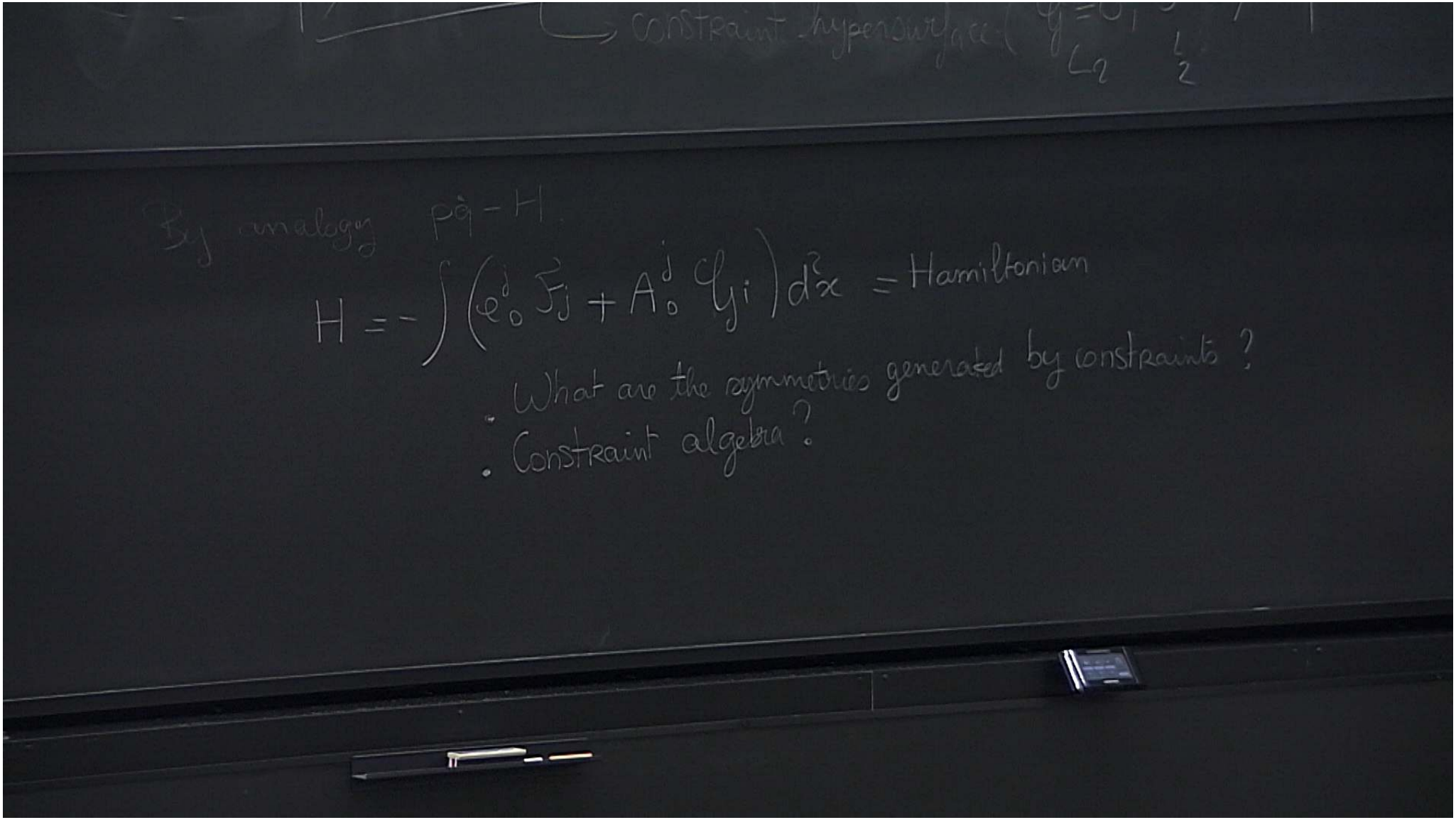
$$\left\{E_k^a(x), E_j^b(y)\right\} = 0$$

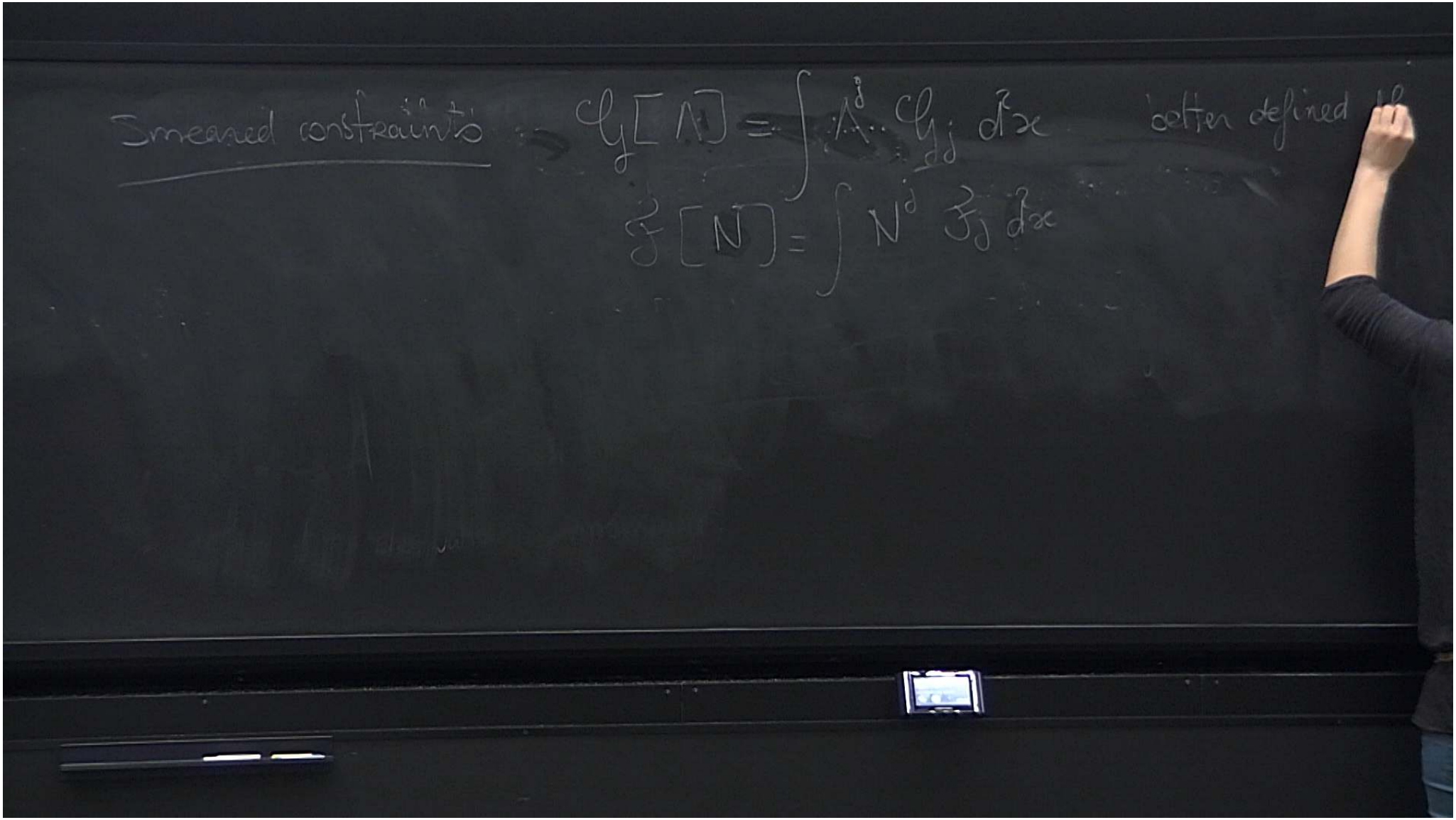
e_{oj}, A_o^j : linear action in these variables, no time derivative

= Lagrange multiplier


$T^j_{ab}/2$
IP
$+ A^d_0 \left(\partial_a e_{bj} + \varepsilon_{jlm} A^l_a e^m_b \right) \tilde{\varepsilon}_{ab} \, d^3x \, dt$
other Poisson brackets vanish
$\{ A^k_a(x), A^d_b(y) \} = 0$
$\{ E^a_k(x), E^b_j(y) \} = 0$
e_{oj}, A^j_0 : linear action in these variables, no time derivative
= Lagrange multiplier


EOM
No time derivative = constraints
$\mathcal{F}^i = \frac{1}{2} F^i_{ab} \tilde{\varepsilon}^{ab} = 0 = \tilde{\varepsilon}_{ab}(\partial_a A^j_b + \frac{1}{2} \varepsilon^j_{kl} A^k_a A^l_b) \to$ Flatness constraint
$\mathcal{G}^i = \frac{1}{2} T^j_{ab} \tilde{\varepsilon}^{ab} = 0 = \partial_a E^a_j + \varepsilon_{jmk} A^l_a E^{am} \to$ Gauss constraint
kinematical phase space
$\{A, E\}$
4 4 = 8 dof
constraint hypersurface $(\mathcal{G}^i = 0; \mathcal{F}^i = 0) = 4$
2 2


constraint hypersurface
By analogy pq̇ − H
$H = -\int \left(e^j_0 \mathcal{F}_j + A^j_0 \mathcal{G}_{ji} \right) d^3x$ = Hamiltonian
What are the symmetries generated by constraints ?
Constraint algebra ?


Smeared constraints
$\mathcal{C}_j[\Lambda] = \int \Lambda^j \mathcal{C}_j \, d^3x$
often defined
$\mathcal{F}[N] = \int N^j \mathcal{F}_j \, d^3x$

$$\Rightarrow \mathcal{G}[\Lambda] = \int \Lambda^j \mathcal{G}_j \, d^2x$$

better defined than the pointwise constraints

$$\mathcal{F}[N] = \int N^j \mathcal{F}_j \, d^2x$$

defined for all choices of Λ, N.

$\mathcal{F}[N] = \int N^{\cdots} \mathcal{F}_j \, dx$

pretation of the constraints

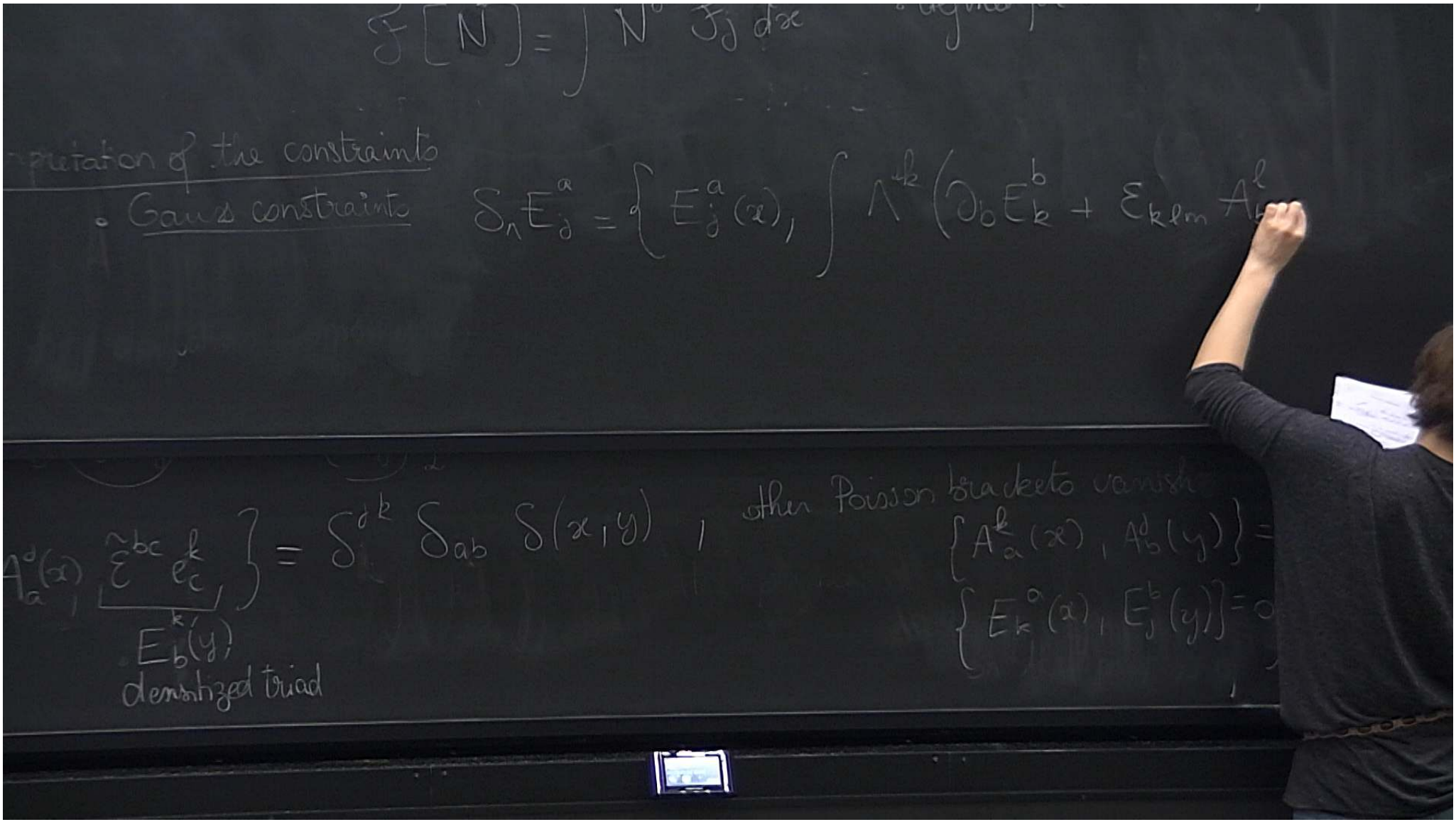
- Gauss constraint $\quad \delta_\Lambda E^a_j = \{ E^a_j(x), \int \Lambda^k (\partial_b E^b_k + \varepsilon_{klm} A^l_{\cdots}$

$A^a_a(x), \underbrace{\tilde{\varepsilon}^{bc} e^k_c}_{E^k_b(y)} \} = \delta^{ak} \delta_{ab} \delta(x,y)$, other Poisson brackets vanish

densitized triad

$\{ A^k_a(x), A^b_b(y) \} =$

$\{ E^a_k(x), E^b_j(y) \} = 0$



d^2x better defined than the pointwise constraints

$\int d^2x$ defined for all choices of Λ, N.

$$(x), \int \Lambda^k \left(\partial_b E^b_k + \varepsilon_{klm} A^l_b E^{bm}\right)(y)\, d^3y$$

$$\varepsilon_{jmk} E^{am} \Lambda^k(x)$$

internal rotation (infinitesimal

$$e'^k_\mu - e^k_\mu = \delta e^k_\mu$$

Co-triad

$$e'^k_\mu = R^k{}_j e^j_\mu$$

$$R = e^{-\varepsilon \vec{\Lambda} \cdot \vec{J}}$$

SO(3) generators

$$R_{ij} = \delta_{ij} + \varepsilon\, \varepsilon_{ijk} \Lambda^k$$

other Poisson brackets vanish

$$\{A^k_a(x), A^j_b(y)\} = 0$$

$$\{E^a_k(x), E^b_j(y)\} = 0$$

e_{0j}, A^j_0 linear ... these variables

no time derivative

= Lagrange multiplier