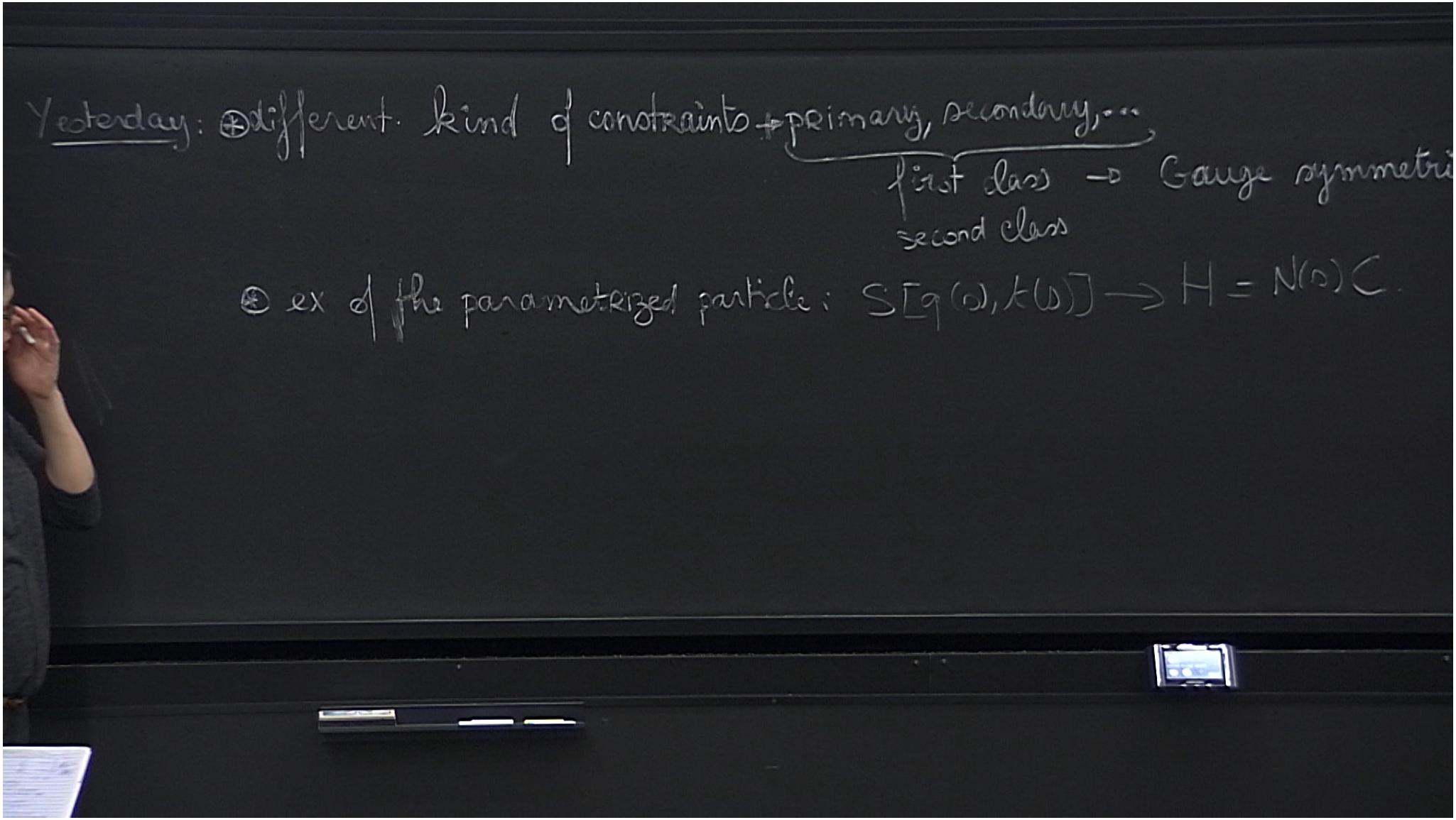
Title: PSI 2016/2017 Explorations in Quantum Gravity - Lecture 5

Date: Mar 24, 2017 10:15 AM

URL: http://pirsa.org/17030072

Abstract:


Yesterday: ⊛different. kind of constraints → primary, secondary, ...
first class → Gauge symmetri
second class
⊛ ex of the parametrized particle: S[q(σ), t(σ)] → H = N(σ)C.

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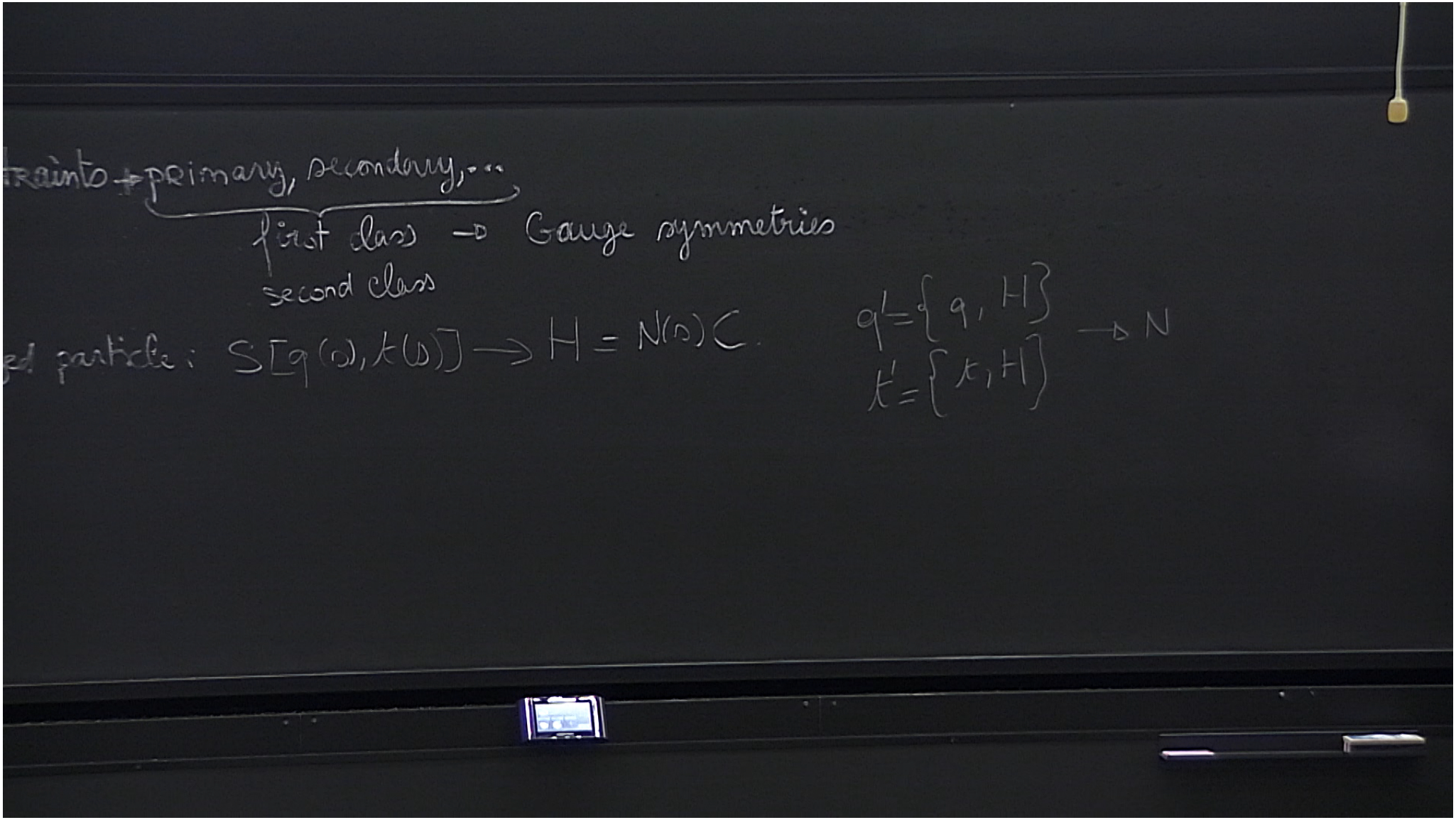
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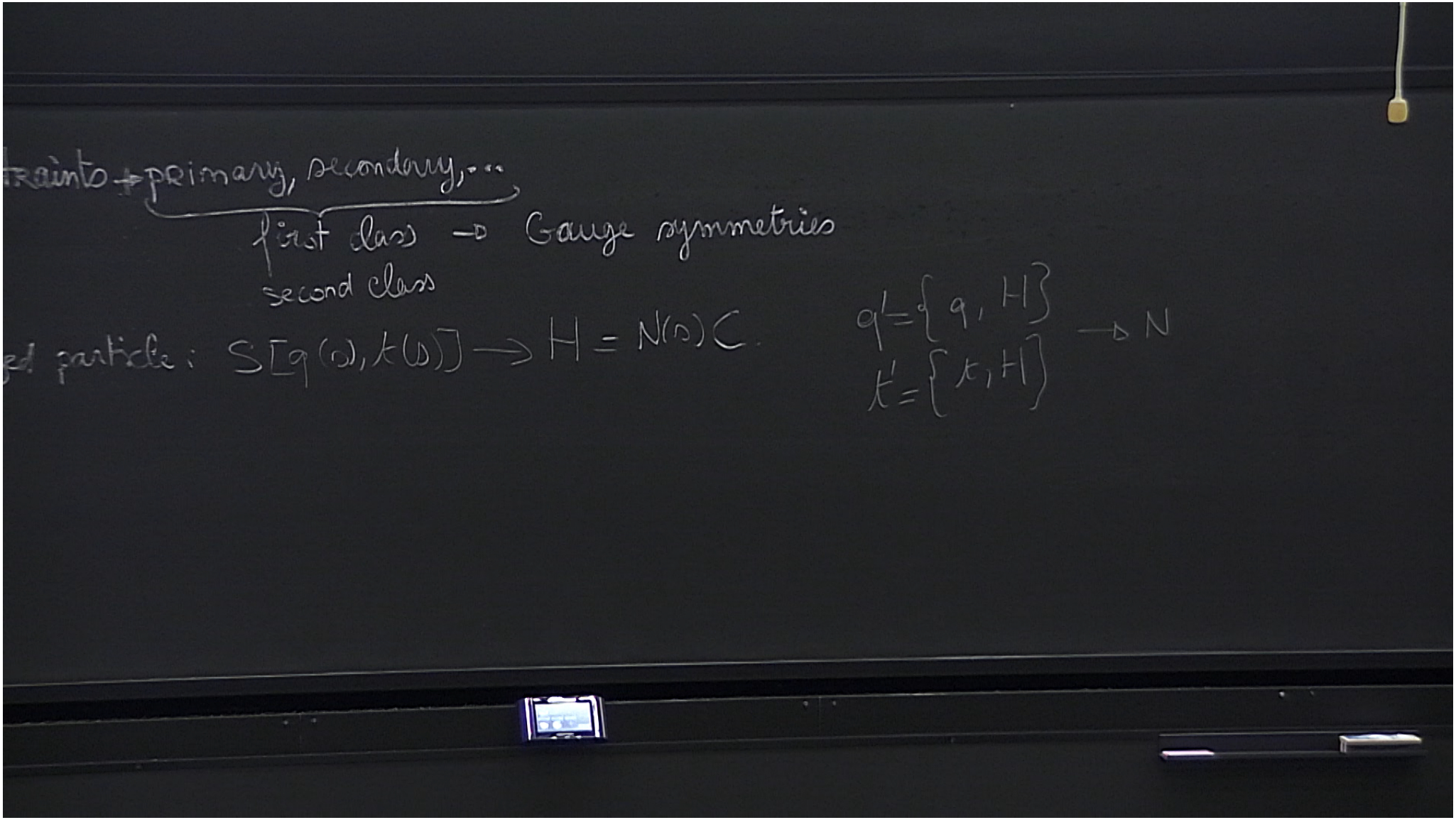
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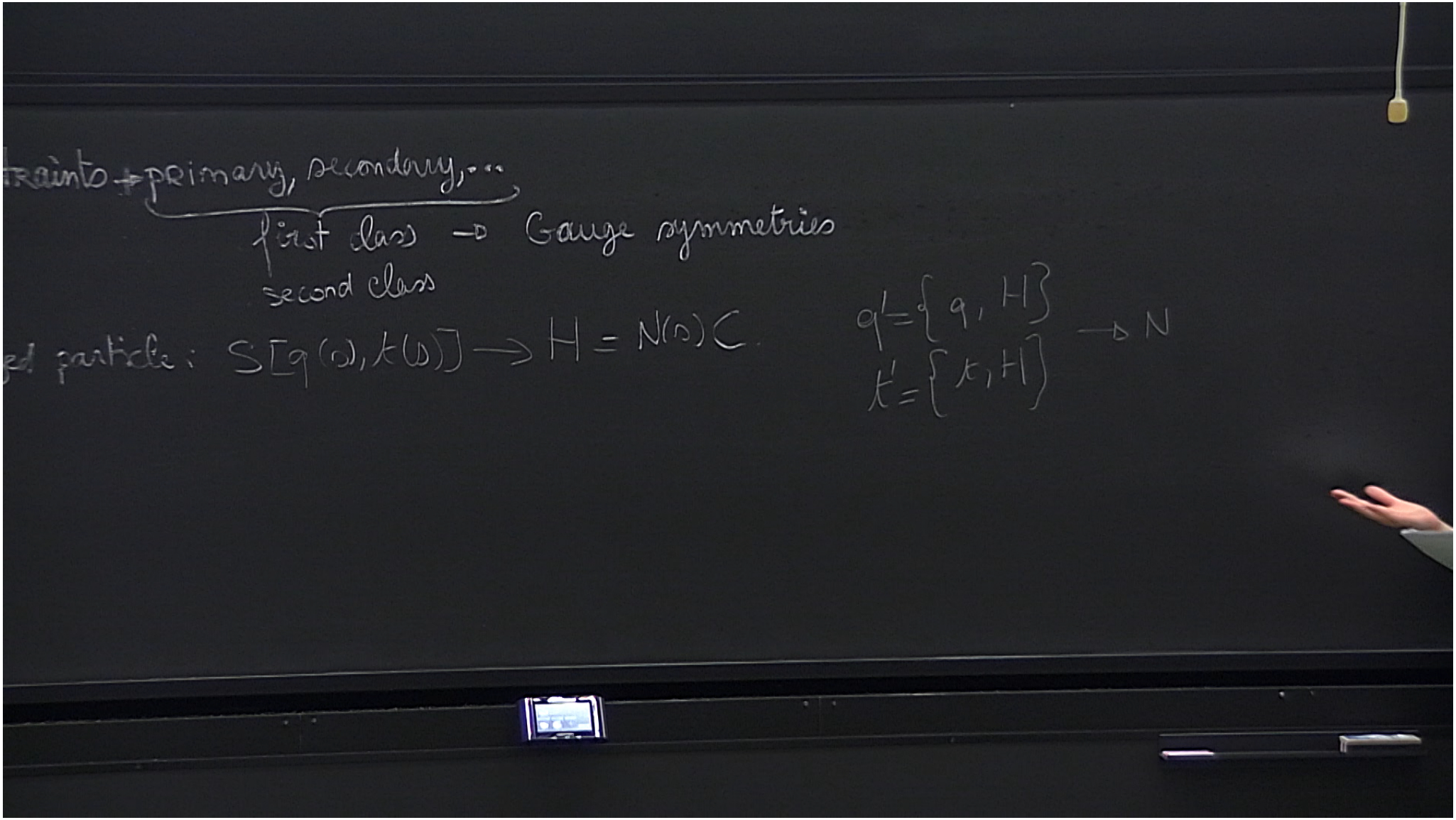
first class → Gauge symmetri

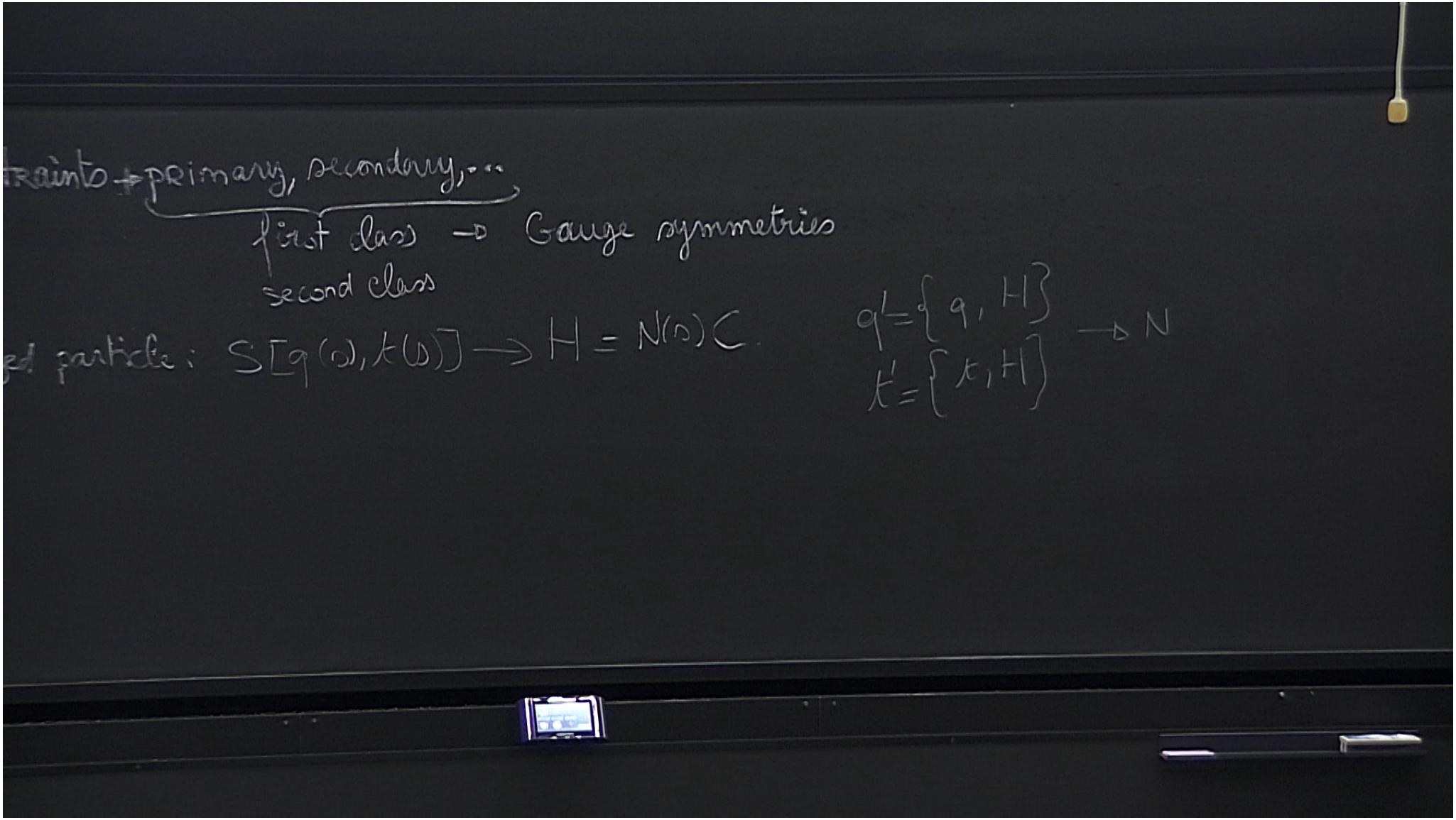
second class

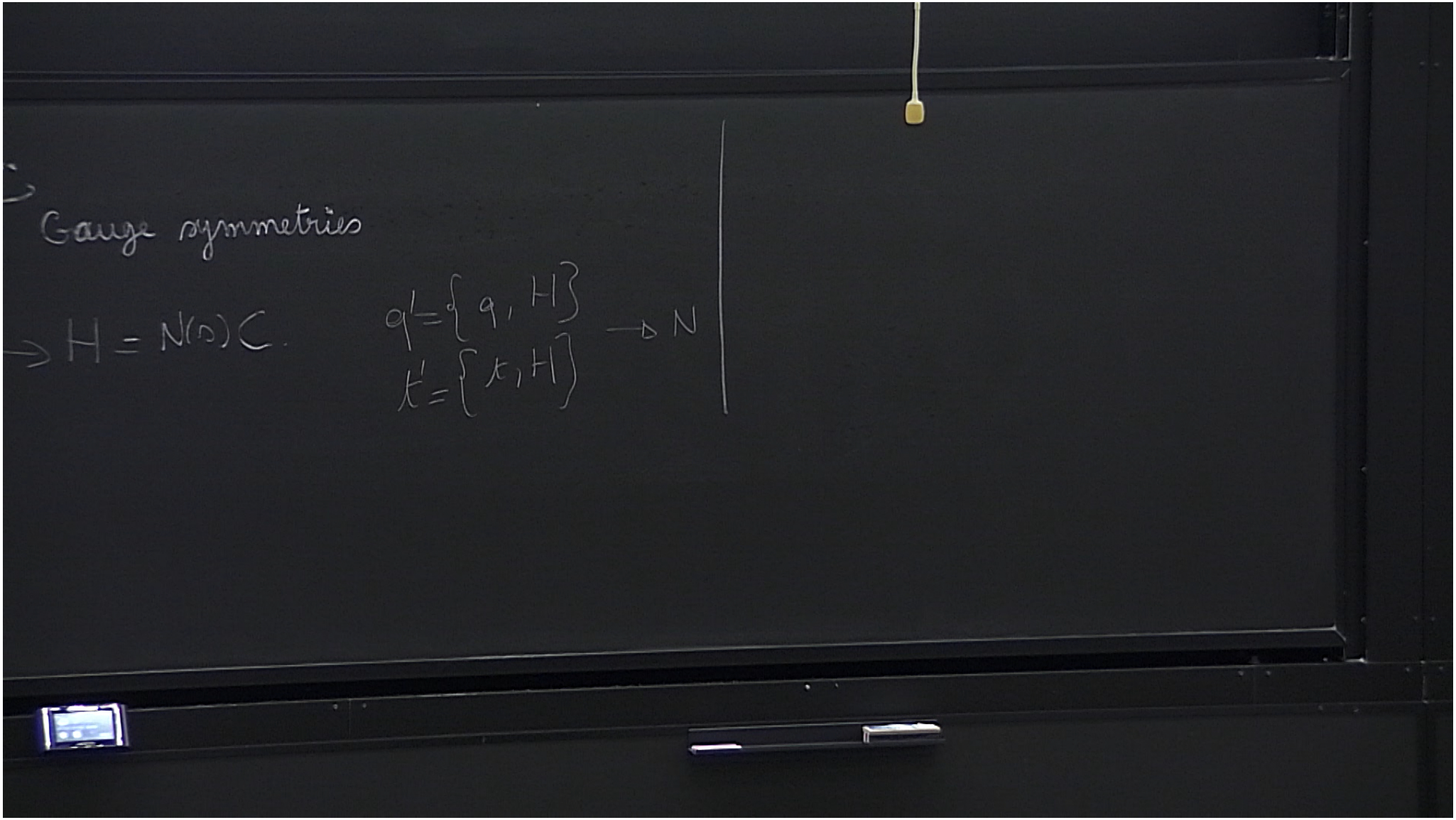
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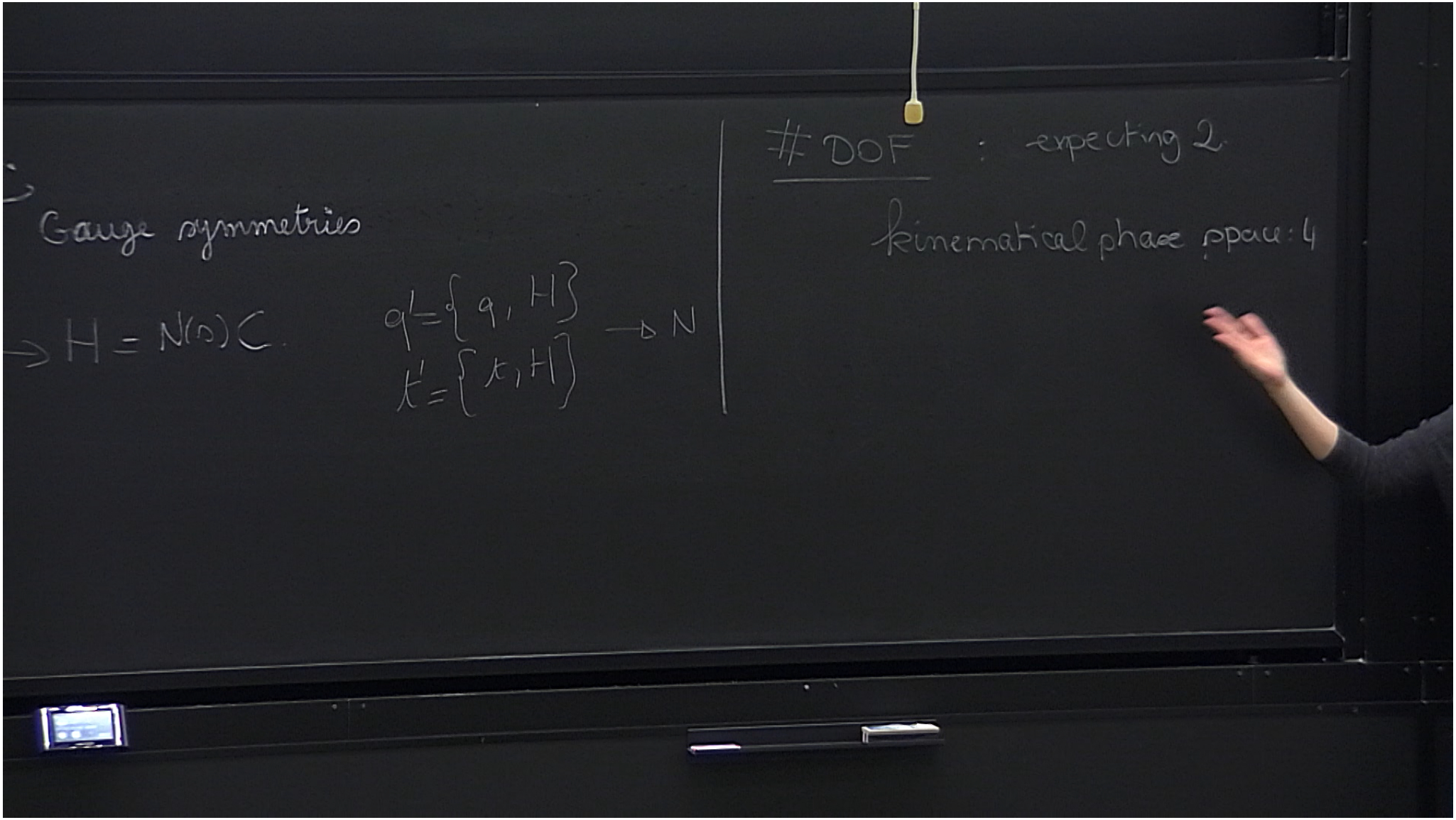

straints → primary, secondary, ...
first class → Gauge symmetries
second class
ged particle: S[q(σ), t(σ)] → H = N(σ)C
q' = {q, H}
t' = {t, H}
→ N


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Gauge symmetries
→ H = N(D) C.
q' = {q, H}
t' = {t, H}
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Gauge symmetries
$\rightarrow H = N(\Delta) C$
$q' = \{q, H\}$
$t' = \{t, H\}$
$\rightarrow N$
DOF : expecting 2
kinematical phase space: 4

Gauge symmetries

$\rightarrow H = N(\mathcal{D})C$

$q' = \{q, H\}$
$t' = \{t, H\}$ $\longrightarrow N$

DOF : expecting 2

kinematical phase space: 4

Gauge symmetries

$\rightarrow H = N(\mathcal{D})\,C$

$q' = \{q, H\}$

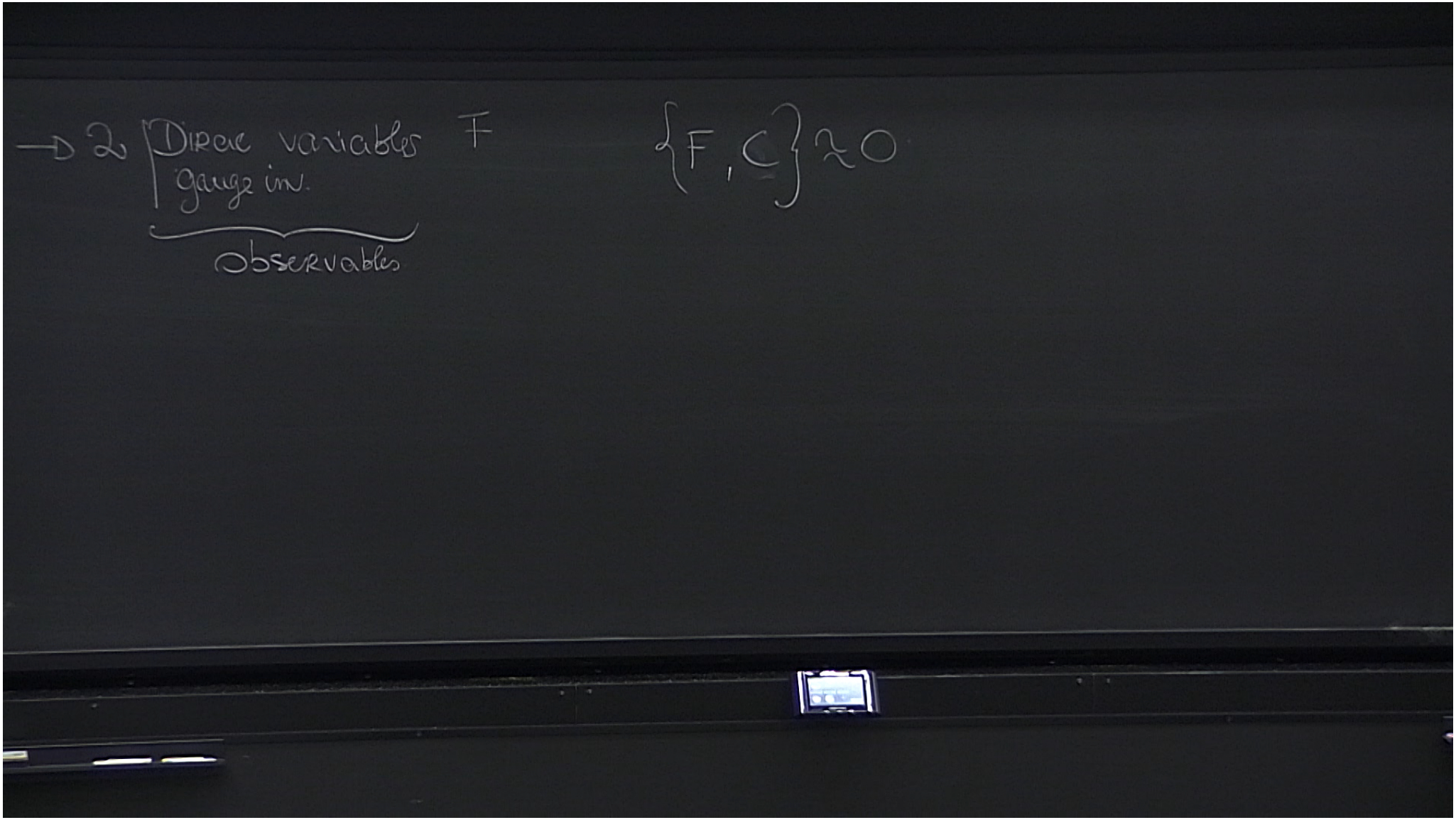
$t' = \{t, H\}$ $\rightarrow N$

DOF : expecting 2

kinematical phase space: 4

-1 by implementing C

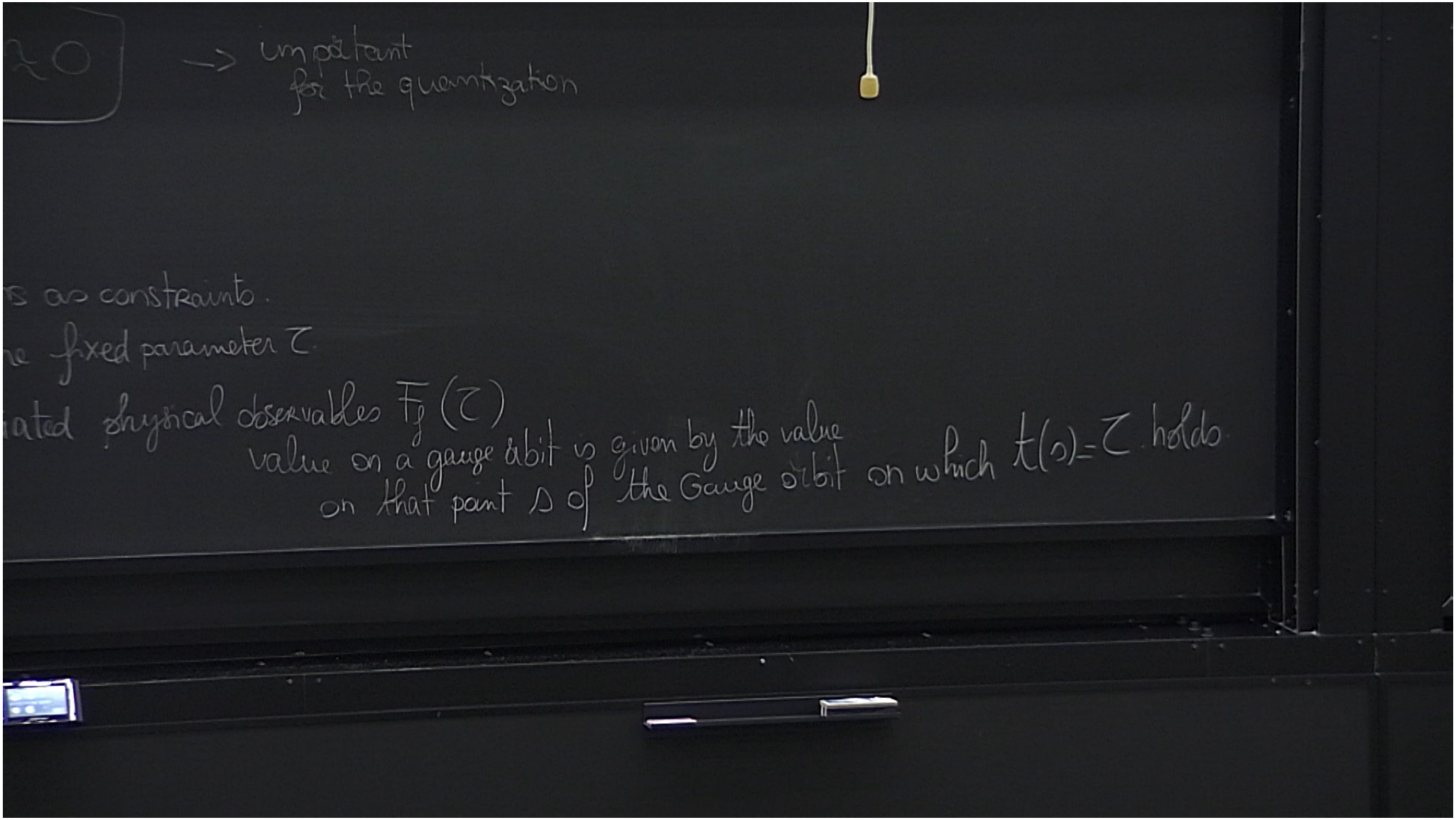
-1 by dividing by the gauge orbit.

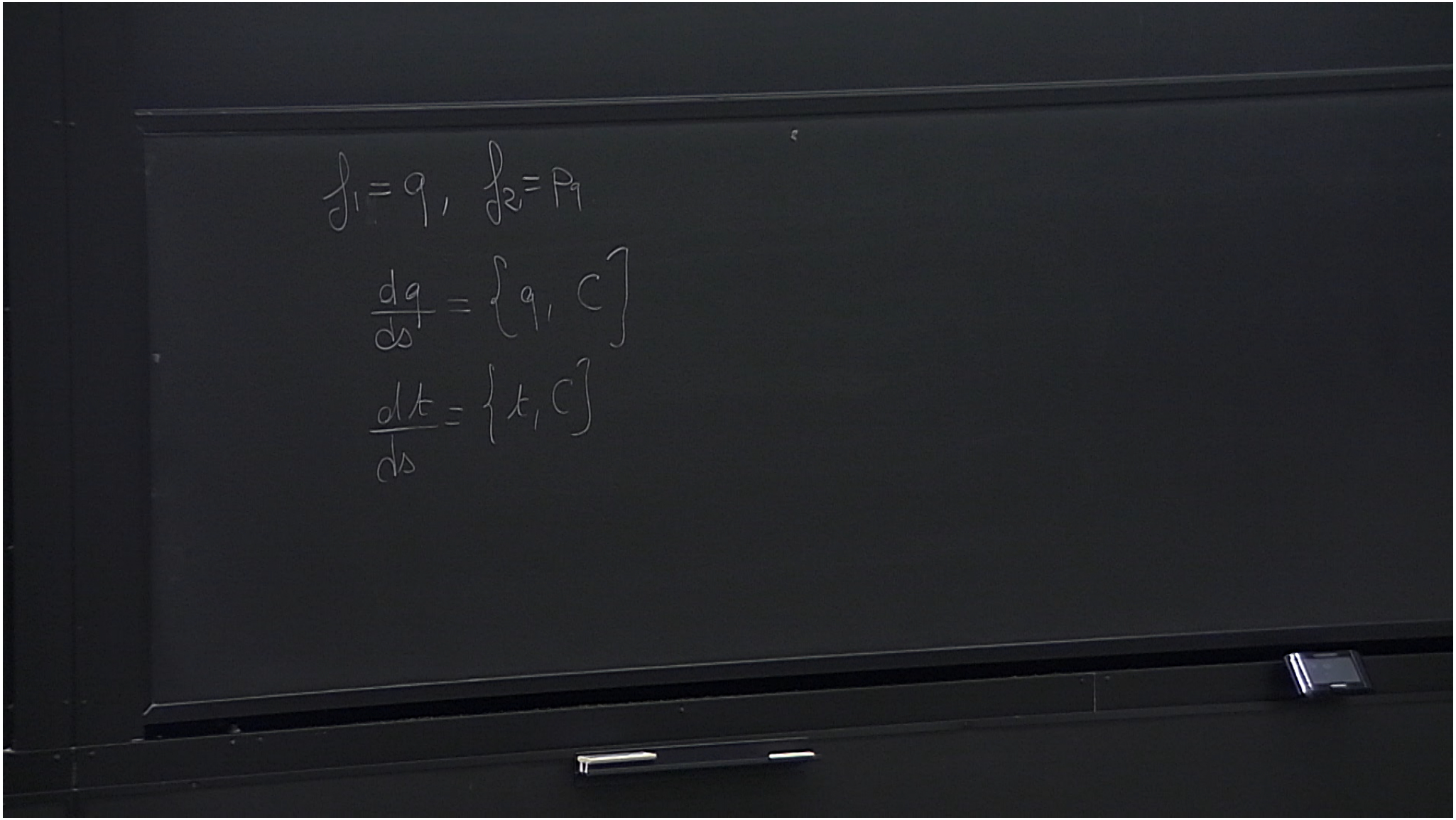

→ 2 Dirac variables F
gauge inv.
Observables
{F, C} ≈ 0

→ 2 Dirac variables F gauge inv. $\{F, C\} \approx 0$ → important for the quantization

Observables

* Construction of physical observables
  - select as many gauge fixing conditions as constraints.
    $t = \tau$ for some fixed parameter τ
  - Given a phase space function f, associated physical observables $F_f(\tau)$
    value on a gauge orbit is giv
    on that point S of th


-> important
for the quantization
s as constraints.
e fixed parameter τ.
ated physical observables $F_f(\tau)$
value on a gauge orbit is given by the value
on that point s of the Gauge orbit on which $t(s)=\tau$ holds


$f_1 = q, \quad f_2 = p_q$
$\frac{dq}{ds} = \{q, C\}$
$\frac{dt}{ds} = \{t, C\}$

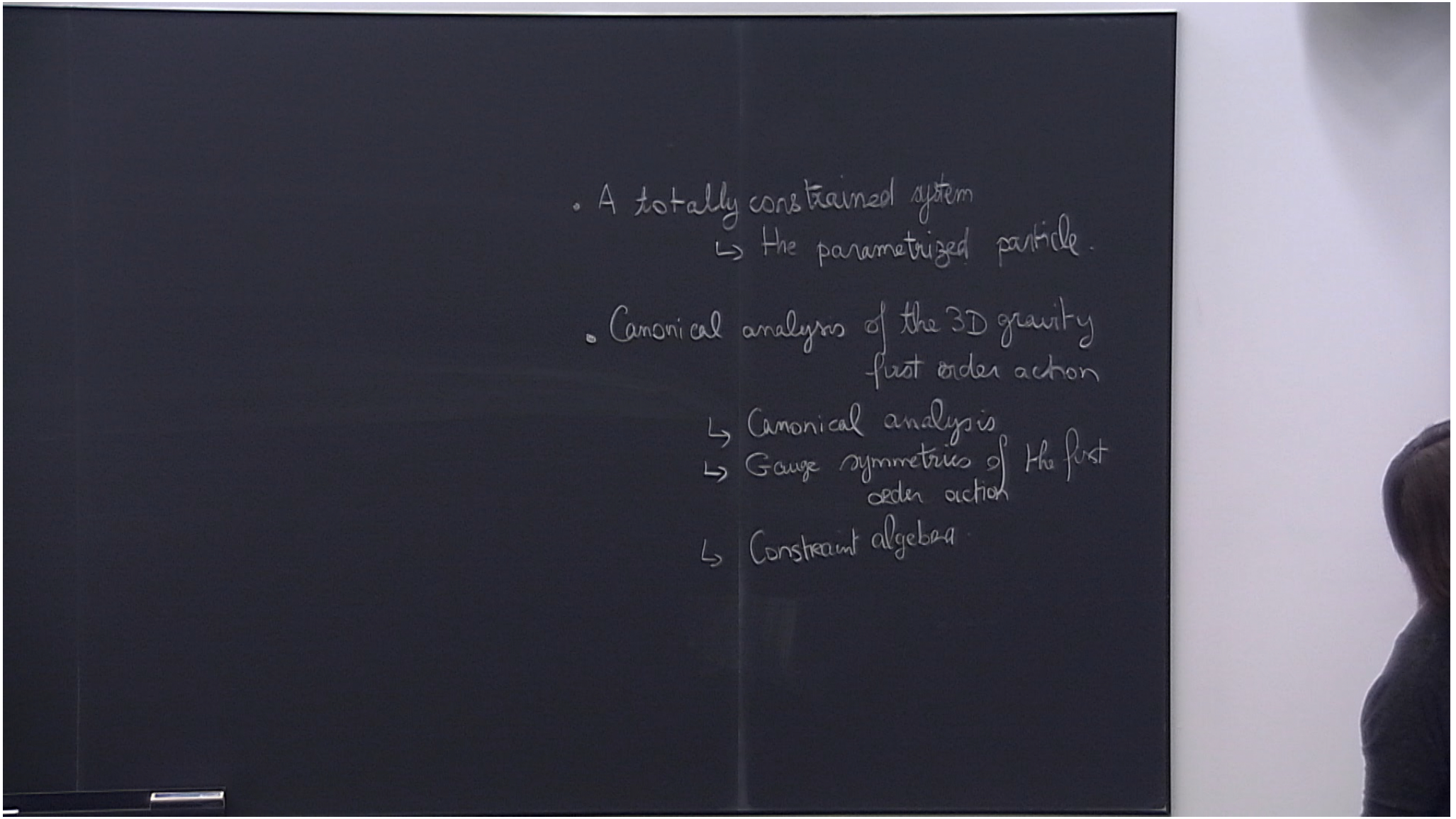
$f_1 = q, \quad f_2 = p_q$

$\frac{dq}{ds} = \{q, C\}$ $\qquad$ $\frac{dp_q}{ds} =$

$\frac{dt}{ds} = \{t, C\}$ $\qquad$ $\frac{dp_t}{ds} =$

$\hookrightarrow t(s) = \tau$

$\hookrightarrow q(0) \longrightarrow F_q^\tau$: value of q at the point of the gauge orbit where t takes the value τ.


• A totally constrained system
↳ the parametrized particle.
• Canonical analysis of the 3D gravity first order action
↳ Canonical analysis
↳ Gauge symmetries of the first order action
↳ Constraint algebra

2. Dirac variables F

Canonical analysis of the 3D gravity FOA ....

$$S = \frac{1}{2}\int e_{\sigma e} F^{e}_{\mu\nu} \tilde{\varepsilon}^{\sigma\mu\nu} d^3x$$

$$\mathcal{M} = \Sigma \times \mathbb{R}$$



2. Dirac variables F

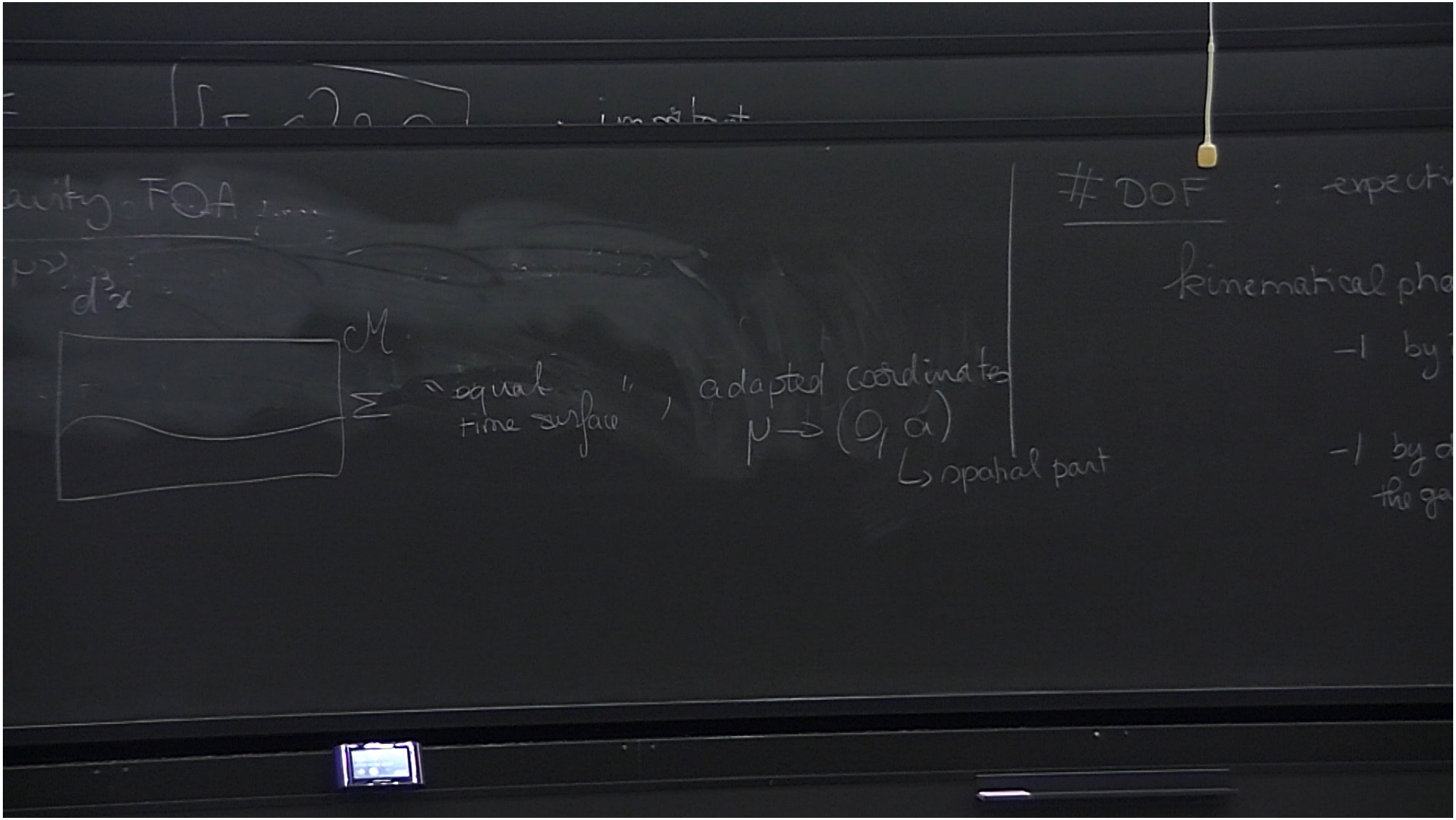
Canonical analysis of the 3D gravity FOA :

$$S = \frac{1}{2}\int e_{ae}\, F^{e}_{\mu\nu}\, \tilde{\varepsilon}^{a\mu\nu}\, d^3x$$

$$\mathcal{M} = \Sigma \times \mathbb{R}$$



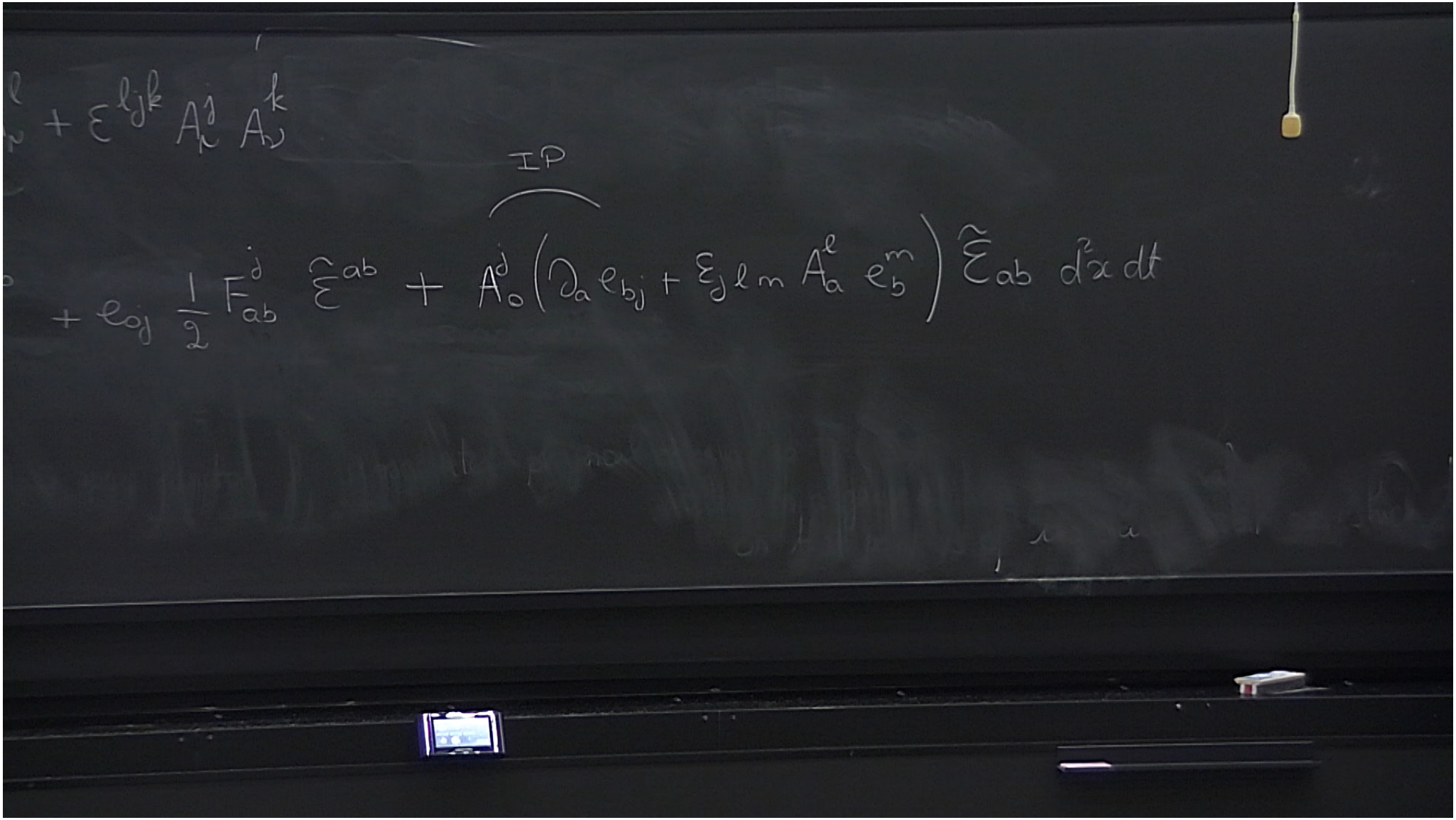
= equal time surface

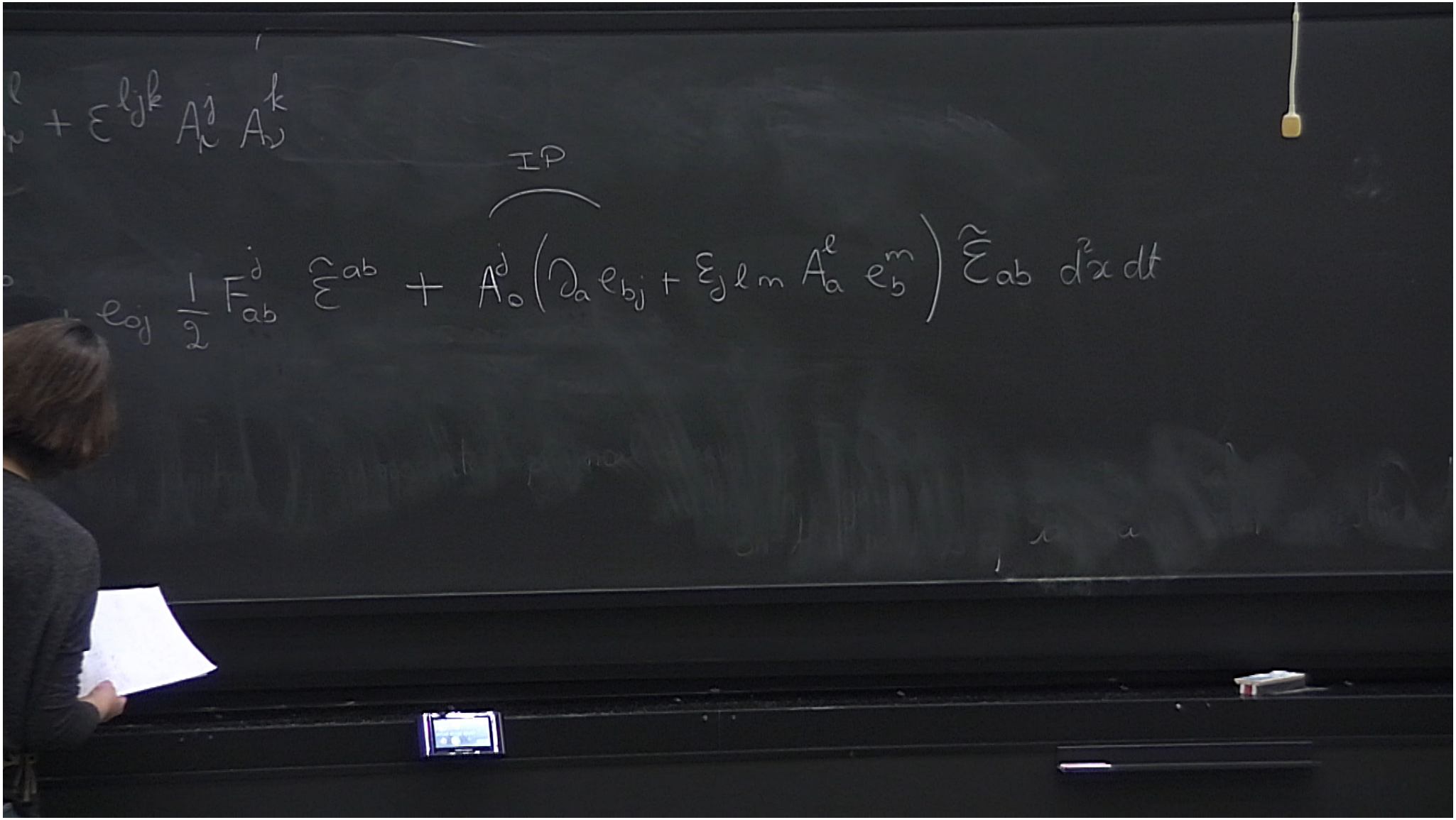

#DOF
kinematical
M
Σ
"equal time surface"
adapted coordinates
μ → (0, a)
↳ spatial part

$$F^{l}_{\mu\nu} = \partial_\mu A^{l}_{\nu} - \partial_\nu A^{l}_{\mu} + \varepsilon^{ljk} A^{j}_{\mu} A^{k}_{\nu}$$

$$F^{l}_{oa} = - F^{l}_{ao}$$

$$S = \int e^{d}_{b}\, \partial_o A_{aj}\, \tilde{\mathcal{E}}^{ab} + e_{oj}\, \frac{1}{2} F^{j}_{ab}\, \tilde{\mathcal{E}}^{ab} + \overbrace{A^{d}_{o} \left(\partial_a e_{bj} + \varepsilon_{jlm} A^{l}_{a} e^{m}_{b} \right)}^{IP} \tilde{\mathcal{E}}_{a}$$


$+ \varepsilon^{ijk} A_\mu^j A_\nu^k$
IP
$+ e_{oj} \frac{1}{2} F_{ab}^j \tilde{\varepsilon}^{ab} + A_o^j \left(\partial_a e_{bj} + \varepsilon_{jlm} A_a^l e_b^m \right) \tilde{\tilde{\varepsilon}}_{ab} \, d^3x \, dt$


$+\varepsilon^{ljk} A^j_\nu A^k_\nu$
IP
$e_{oj} \frac{1}{2} F^j_{ab} \tilde{\varepsilon}^{ab} + A^j_o \left(\partial_a e_{bj} + \varepsilon_{jlm} A^l_a e^m_b \right) \tilde{\tilde{\varepsilon}}_{ab} \, d^2x \, dt$

$$F^l_{\mu\nu} = \partial_\mu A^l_\nu - \partial_\nu A^l_\mu + \varepsilon^{ljk} A^j_\mu A^k_\nu$$

$$F^l_{oa} = -F^l_{ao}$$

$$= \tilde{\varepsilon}^{oab}$$

$$S = \int e^d_b \, \partial_o A_{aj} \, \tilde{\varepsilon}^{ab} + e_{oj} \frac{1}{2} F^j_{ab} \, \tilde{\varepsilon}^{ab} + \overbrace{A^d_o \left(\partial_a e_{bj} + \varepsilon_{jlm} \right.}^{IP}$$

other Poisson brac

Canonical variables

$$\{ A^d_a(x), \underbrace{\tilde{\varepsilon}^{bc} e^k_c}_{E^k_b(y)} \} = \delta^{dk} \delta_{ab} \delta(x,y) \, ,$$

densitized triad

$$F^l_{\mu\nu} = \partial_\mu A^l_\nu - \partial_\nu A^l_\mu + \varepsilon^{ljk} A^j_\mu A^k_\nu$$

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IP

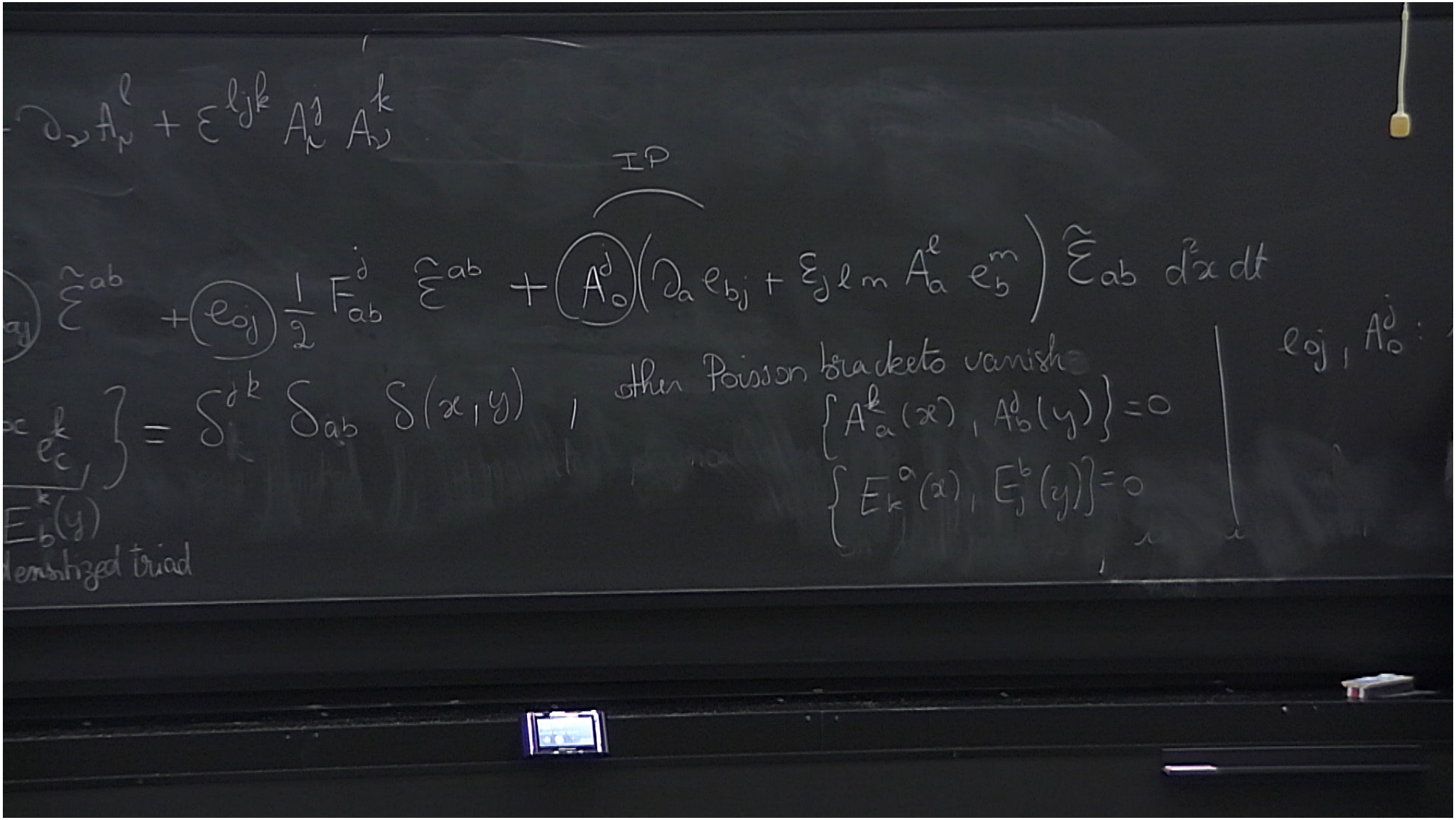
$$S = \int e^d_b \, \partial_o A_{aj} \, \tilde{\varepsilon}^{ab} + e_{oj} \frac{1}{2} F^j_{ab} \, \tilde{\varepsilon}^{ab} + A^d_o \left(\partial_a e_{bj} + \varepsilon_{jlm} A^l_a e^m_b \right) \tilde{\varepsilon}_{ab} \, d$$

$$\{ A^d_a(x), \underbrace{\tilde{\varepsilon}^{bc} e^k_c}_{E^k_b(y)} \} = \delta^{dk} \delta_{ab} \delta(x,y) \, ,$$

densitized triad

other Poisson brackets vanish

$$\{ A^k_a(x), A^l_b(y) \}$$


IP
other Poisson brackets vanish
densitized triad

$$\left(\partial_a e_{bj} + \varepsilon_{jlm} A_a^l e_b^m\right) \tilde{\varepsilon}_{ab}\, d^3x\, dt$$

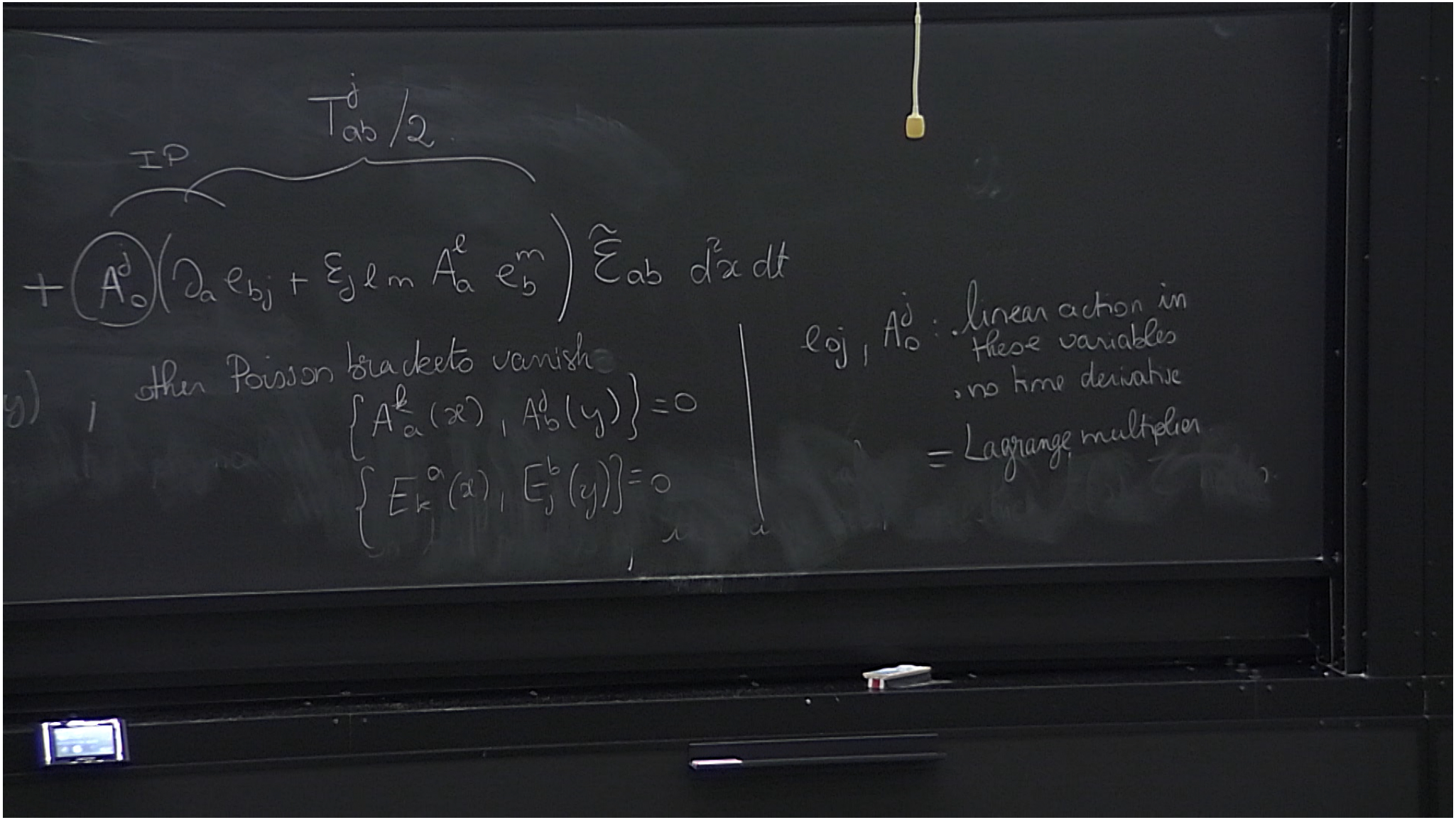
then Poisson brackets vanish

$$\{A_a^k(x), A_b^d(y)\} = 0$$

$$\{E_k^a(x), E_j^b(y)\} = 0$$

e_{oj}, A_o^d : linear action in these variables, no time derivative

= Lagrange multiplier

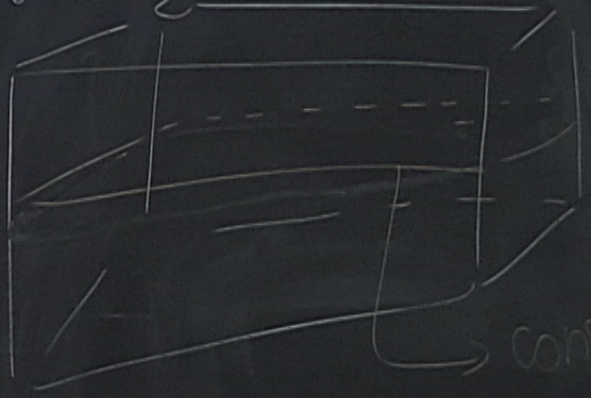

$T^j_{ab}/2$
IP
$+ A^d_0 \left(\partial_a e_{bj} + \varepsilon_{jlm} A^l_a e^m_b \right) \tilde{\varepsilon}_{ab} \, d^2x \, dt$
other Poisson brackets vanish
$\{ A^k_a(x), A^d_b(y) \} = 0$
$\{ E^a_k(x), E^b_j(y) \} = 0$
e_{oj}, A^j_0 : linear action in these variables, no time derivative
= Lagrange multiplier

EOM

No time derivative = constraints

$\mathcal{G}^i = \frac{1}{2} F^i_{ab} \tilde{\epsilon}^{ab} = 0 = \tilde{\epsilon}_{ab} \left(\partial_a A^i_b + \frac{1}{2} \epsilon^i_{jk} A^j_a A^k_b \right)$ → Flatness constraint

$\mathcal{C}^i = \frac{1}{2} T^i_{ab} \tilde{\epsilon}^{ab} = 0 = \partial_a E^a_i + \epsilon_{jmk} A^j_a E^{am}$ → Gauss constraint



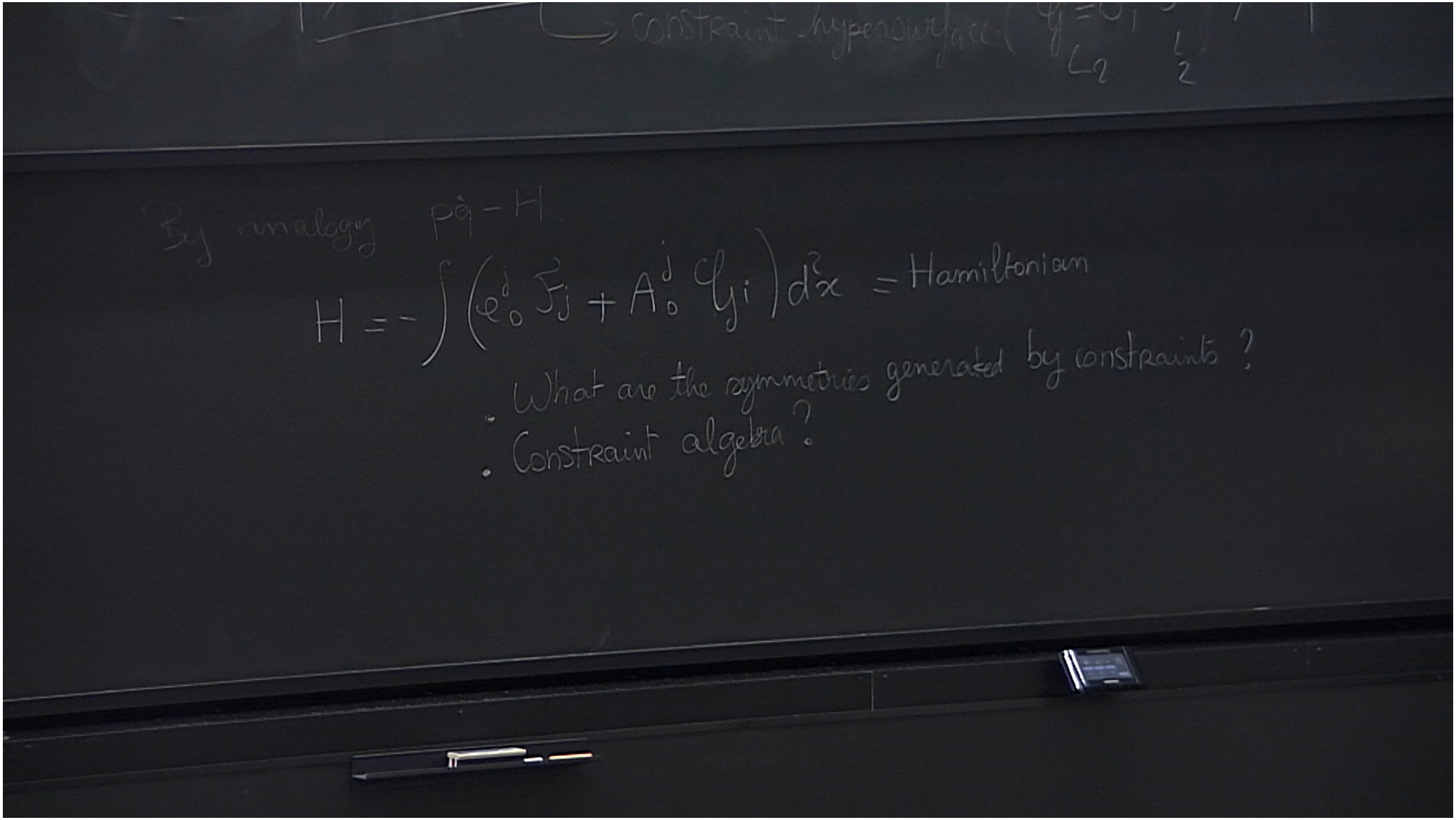
kinematical phase space

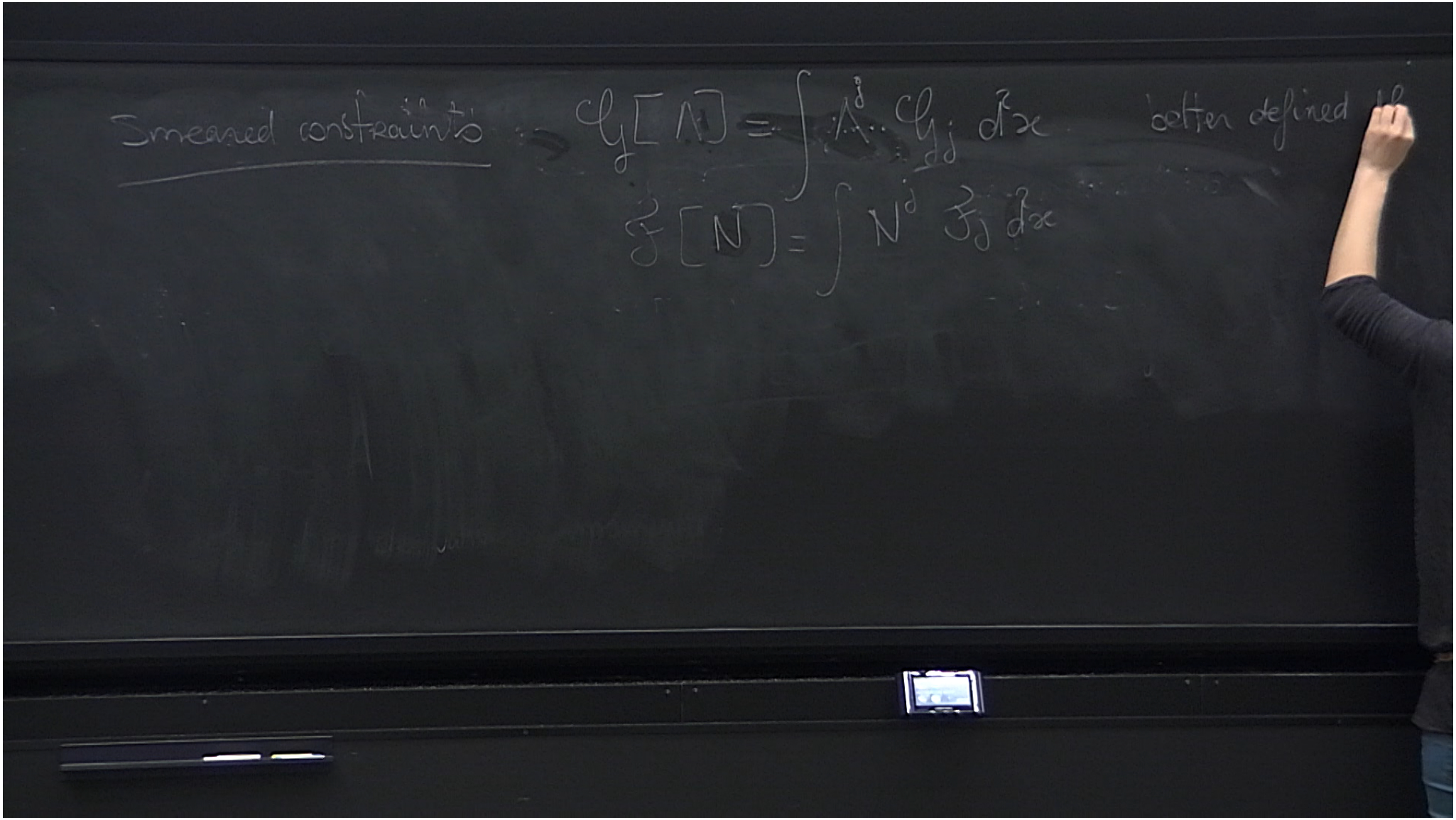
$\{A, E\}$

$4 \quad 4 \quad = 8$ dof

constraint hypersurface $(\mathcal{C}^i = 0; \mathcal{F}^i = 0)$

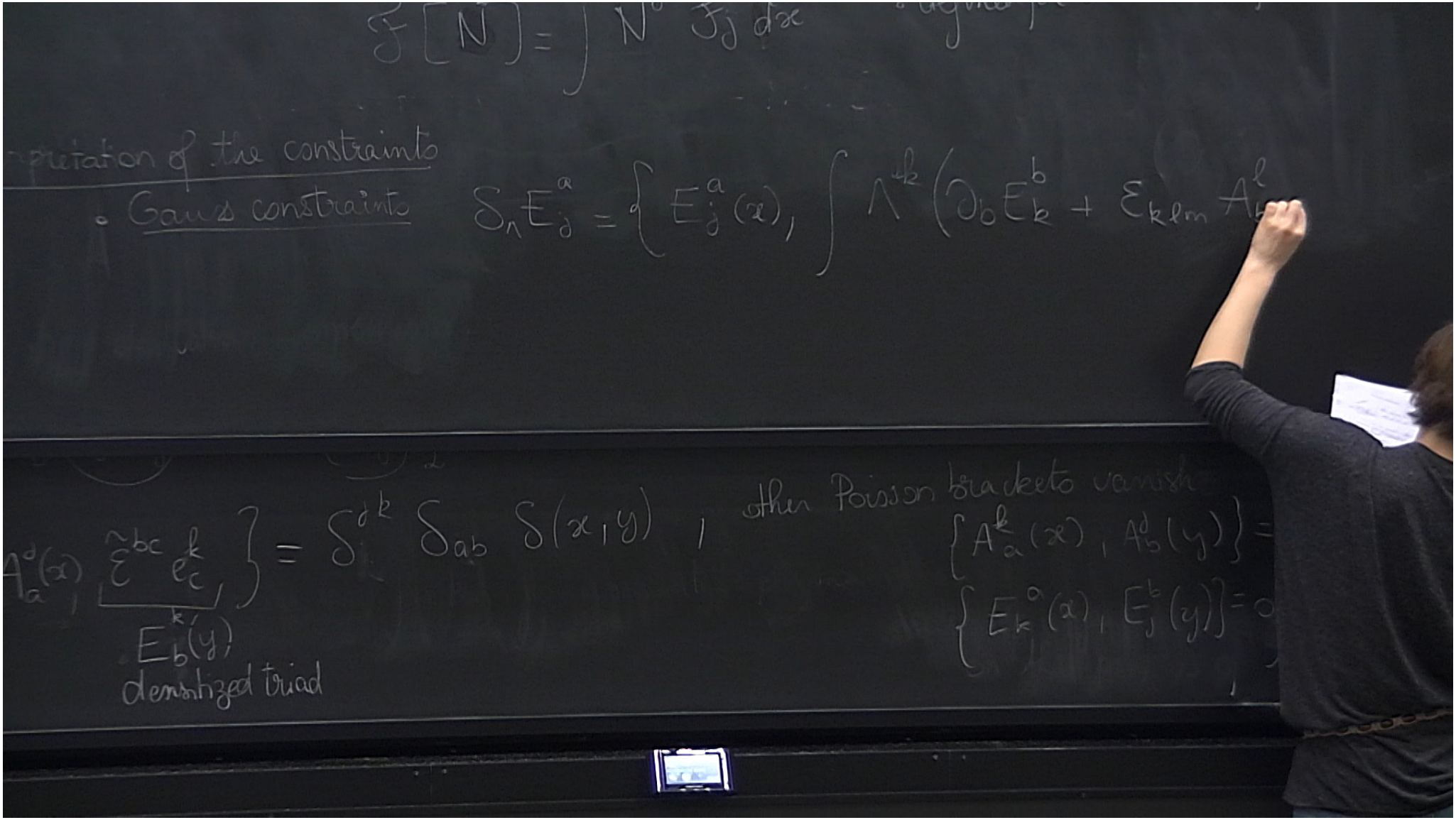
dimension $= 4$


constraint hypersurface
By analogy pq̇ − H
$H = -\int \left(e_0^j \mathcal{F}_j + A_0^j \mathcal{G}_j \right) d^3x$ = Hamiltonian
What are the symmetries generated by constraints ?
Constraint algebra ?


Smeared constraints
$\mathcal{C}_j[\Lambda] = \int \Lambda^j \mathcal{C}_j \, d^3x$
$\mathcal{F}[N] = \int N^j \mathcal{F}_j \, d^3x$
often defined

$\Rightarrow \mathcal{C}_\chi[\Lambda] = \int \Lambda^j \mathcal{C}_{\chi j}\, d^3x$ · better defined than the pointwise constraints

$\vec{\mathcal{J}}[N] = \int N^j \vec{\mathcal{J}}_j\, d^3x$ · defined for all choices of Λ, N.


$\mathcal{F}[N] = \int N \mathcal{F}_j \, dx$
Gauss constraints
$\delta_\Lambda E^a_j = \{ E^a_j(x), \int \Lambda^k (\partial_b E^b_k + \varepsilon_{klm} A^l_b$
$A^d_a(x), \underbrace{\tilde{\epsilon}^{bc} e^k_c}_{E^k_b(y)} \} = \delta^{dk} \delta_{ab} \delta(x,y)$
densitized triad
other Poisson brackets vanish
$\{ A^k_a(x), A^b_b(y) \} =$
$\{ E^a_k(x), E^b_j(y) \} = 0$

$\int dx$ often defined than the pointwise constraints

$\int \partial x$. defined for all choices of Λ, N.

$$e^{\prime k}_\mu - e^b_\mu = \delta e^k_\mu$$

Co-triad

$$e^{\prime k}_\mu = R^k_{\ d}\, e^d_\nu$$

$$R = e^{-\varepsilon \vec{\Lambda} \cdot \vec{J}}$$

J: SO(3) generators

$$R_{ij} = \delta_{ij} + \varepsilon\, \varepsilon_{ijk} \Lambda^k$$

$$(x), \int \Lambda^k \left(\partial_b E^b_k + \varepsilon_{klm} A^l_b E^{bm} \right)(y)\, d^3y$$

$\varepsilon_{jmk} E^{am} \Lambda^k(x)$ internal rotation (infinitesimal

then Poisson brackets vanish

$$\{ A^k_a(x), A^d_b(y) \} = 0$$

$$\{ E^a_k(x), E^b_j(y) \} = 0$$

e_{oj}, A_o these variables, no time derivative

= Lagrange multiplier