

Title: PSI 2016/2017 Explorations in Quantum Gravity - Lecture 1

Date: Mar 20, 2017 10:15 AM

URL: <http://pirsa.org/17030068>

Abstract:

Quantum Gravity in Waterloo



Bianca
Dittrich (PI)

Laurent
Freidel (PI)

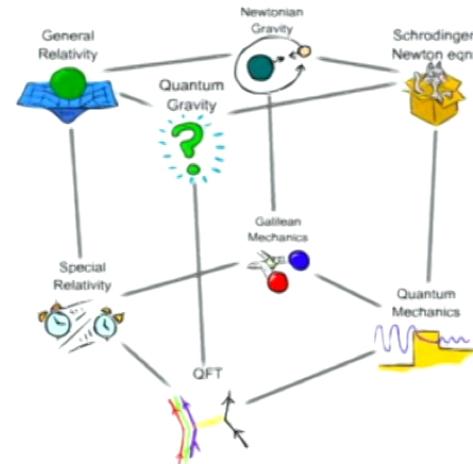
Florian
Girelli (UW)

Lee
Smolin (PI)



- What is Quantum Gravity?
- Why Quantum Gravity?

After all, atoms do fall, so the relationship between gravity and quantum is not a problem for nature,
Three roads to Quantum Gravity, Lee Smolin (2001)

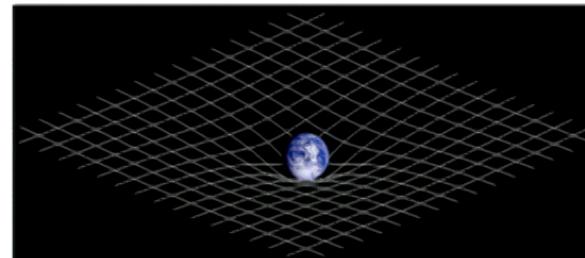
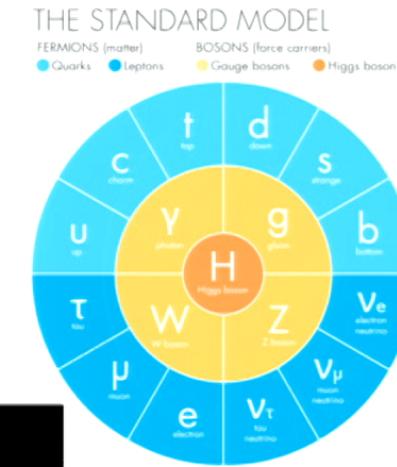
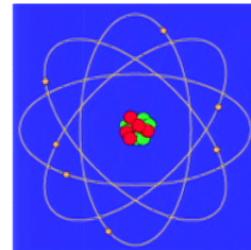


But no calls from experimentally accessible situations....

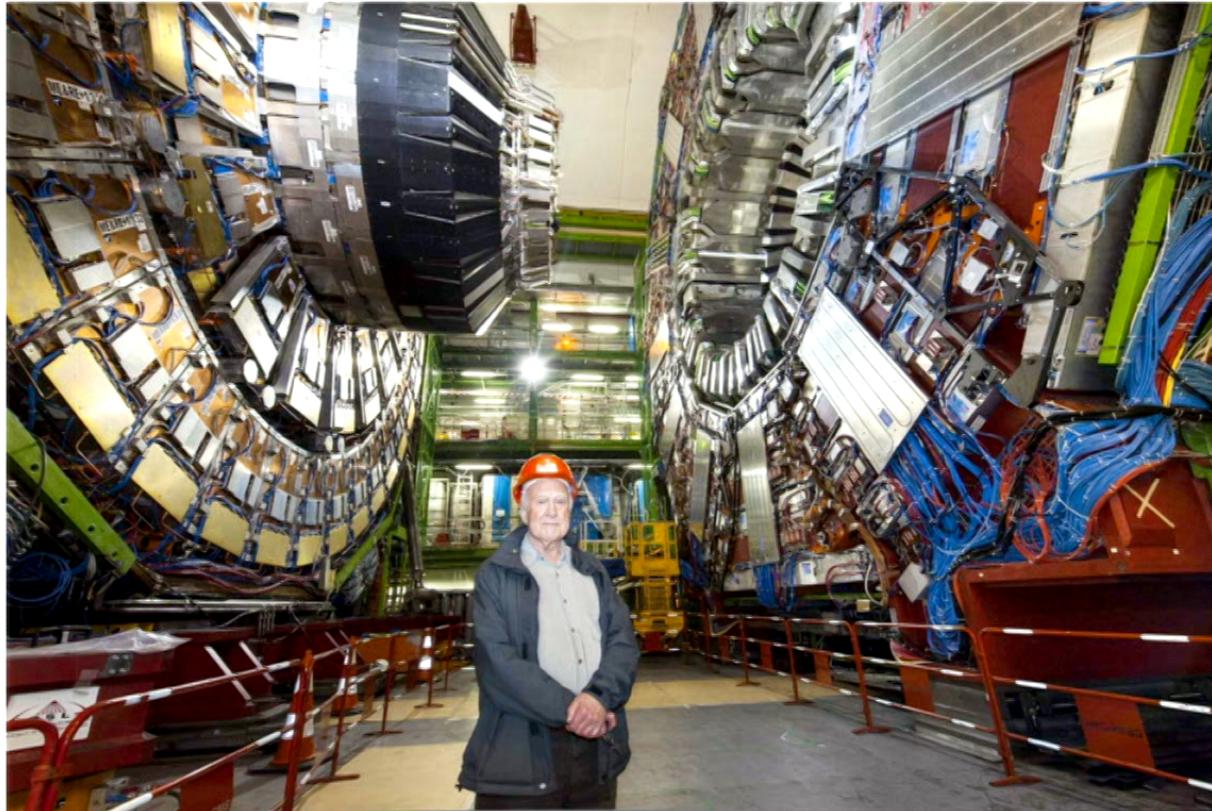
$$\frac{F_{\text{grav}}}{F_{\text{elect}}}(\text{proton - electron}) \sim 10^{-40}$$

What we know about the elementary physical world.

- Quantum Mechanics
- The $SU(3) \times SU(2) \times U(1)$ Standard Model of particle physics
- General Relativity



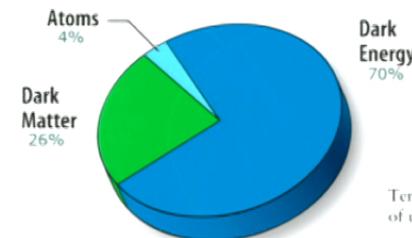
July 4th, 2012
Announcement of the observation of the Higgs
boson as predicted by the Standard Model!



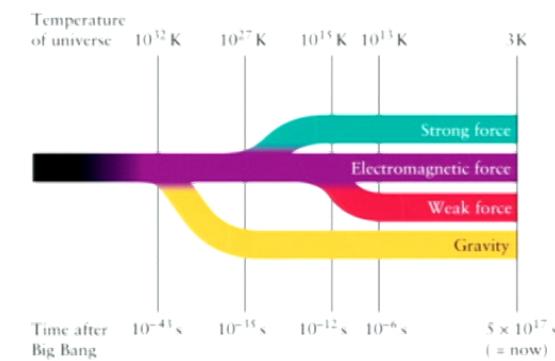
But not the final story about the elementary world...

Among the open problems...

- Dark Matter



- Unification

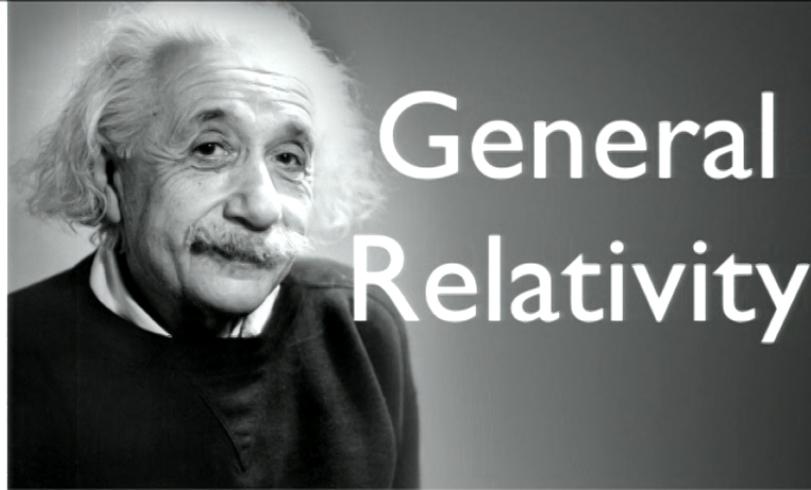


- Quantum Gravity

Quantum Gravity
=

Quantum Mechanics
+

General Relativity
???

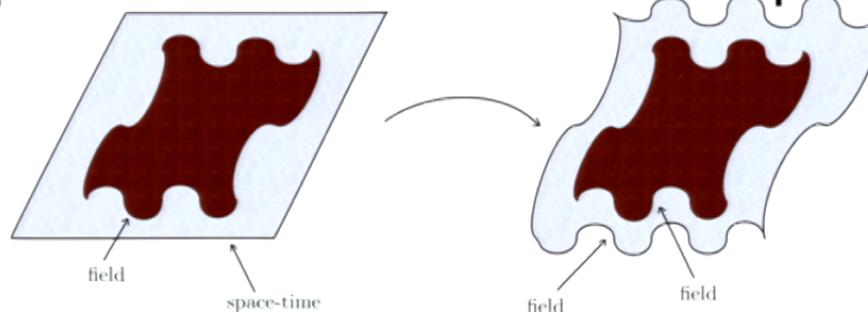


General Relativity: a theory of the gravitational field

$$S = \frac{1}{16\pi G} \int \sqrt{g} R$$

Pre-GR: Fields, particles
ON space-time.

GR: gravitational field
= space-time

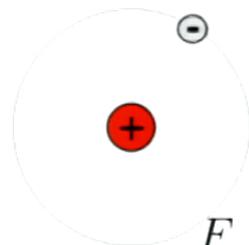


Quantum mechanics



A. PICCARD E. HENRIOT P. EHRENFELD ED. HERZEN TH. DE DONDER E. SCHRÖDINGER E. VERSCHAFFELT W. PAULI W. HEISENBERG R.H. FOWLER L. BRILLOUIN
F. DEBYE M. ENOLSEN W.L. BRAGG H.A. KRAMERS P.A.M. DIRAC A.H. COMPTON L. de BROGLIE M. BOHR N. BOHR
I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN G.H.E. GUYE C.T.R. WILSON O.W. RICHARDSON

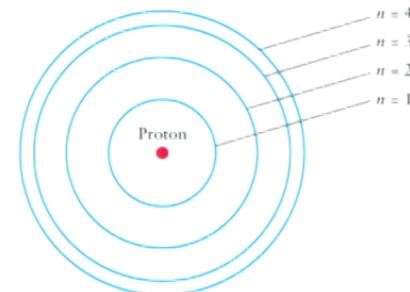
Classical



Hydrogen
atom →

$$F = \frac{q_1 q_2}{R^2}$$

Quantum



General Relativity and Quantum Mechanics?

- Quantum Mechanics: any dynamical quantity = quantized.



World formed by discrete quanta jumping over a at space-time governed by global symmetries (Poincaré).

- General Relativity: space-time is curved; everything is smooth and determinist.



QM and GR = 2 different worlds!

Planck scale?

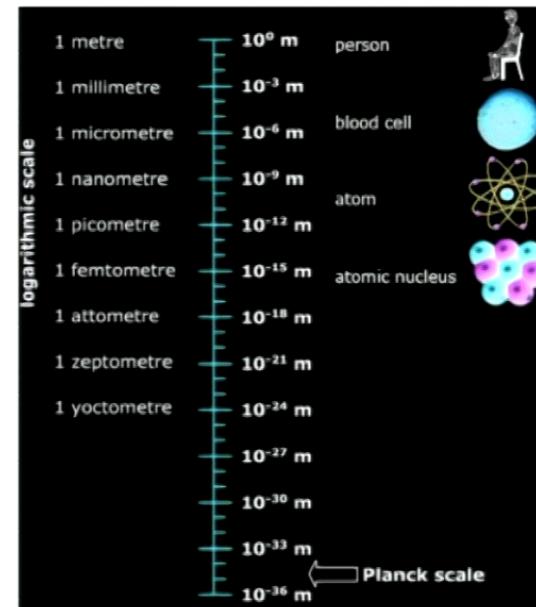
$$L_{\text{Planck}} = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35} m$$

- In QM, Compton wave length,

$$\lambda_{\min} \sim \frac{\hbar}{mc}$$

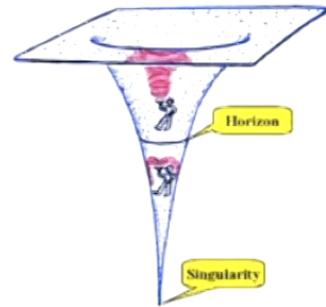
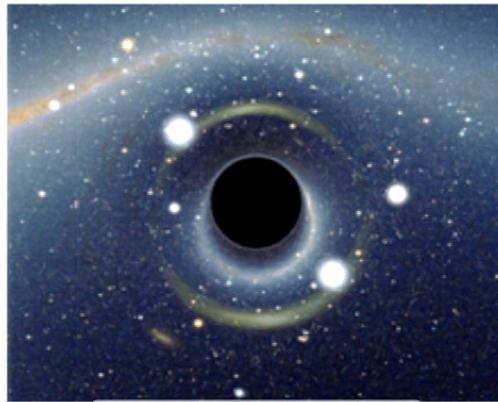
- In GR, Schwarzschild radius,

$$R \sim \frac{mG}{c^2}$$



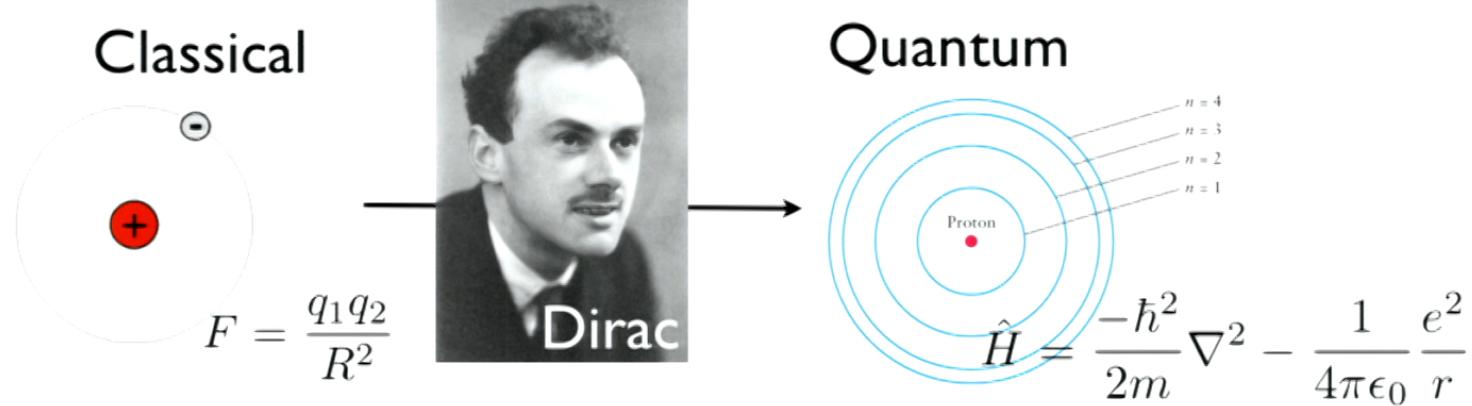
$$L_{\min} \equiv \lambda_{\min} = R \quad \Rightarrow \quad L_{\min} = L_{\text{Planck}}$$

Something very dense and very small?

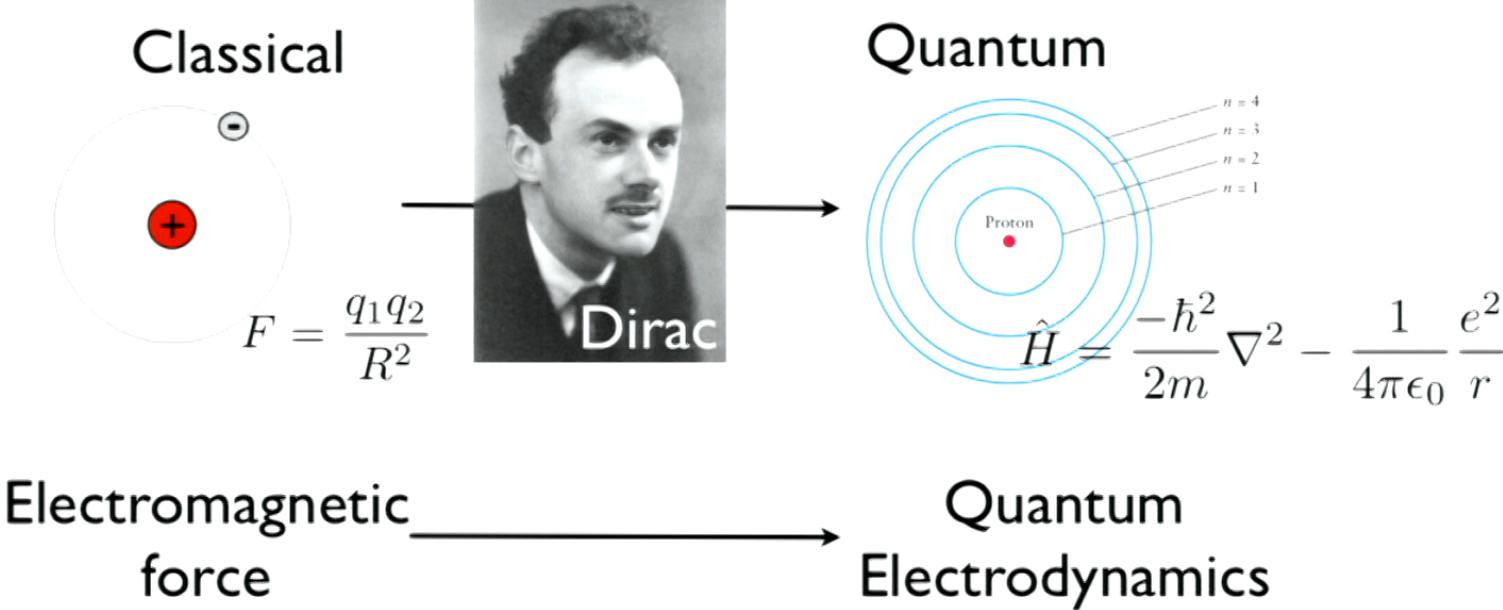


Quantization?

Quantization?



Quantization?



Quantum Gravity

- Perturbative approach... $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$

Volume 160B, number 1,2,3

PHYSICS LETTERS

3 October 1985

QUANTUM GRAVITY AT TWO LOOPS

Marc H. GOROFF¹

California Institute of Technology, Pasadena, CA 91125, USA

and

It doesn't work!

Augusto SAGNOTTI^{2,3,4}

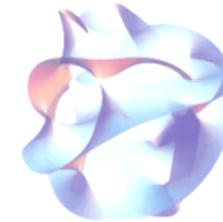
Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, USA

Received 29 April 1985

We show that the S matrix of pure Einstein gravity diverges at the two-loop order in four dimensions

Quantum Gravity

- Perturbative approach... $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$



... String theory...

- Go back to Dirac's approach (canonical quantization)...

Quantum Gravity

- Perturbative approach... $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$



... String theory...

- Go back to Dirac's approach (canonical quantization)...

→ Loop Quantum Gravity

GR (background independence; $g_{\mu\nu}$ = gravitational field) +
QM (uncertainty principle)
= Theory of Quantum Geometry?

Loop Quantum Gravity

- Einstein-Hilbert formulation (1915)

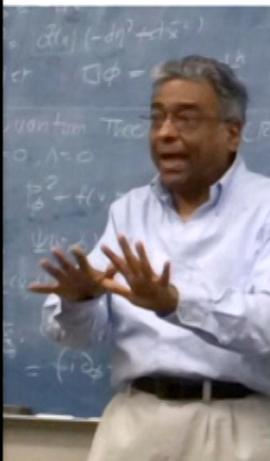
$$S = \frac{1}{16\pi G} \int d^4x \sqrt{g} R$$

- Ashtekar formulation (1986)

$$S = \frac{1}{8\pi G\gamma} \int d^4x \left(\tilde{E}_i^a \dot{A}_a^i + \underset{\sim}{N} \epsilon_{ijk} \tilde{E}_i^a \tilde{E}_j^b F_{ab}^k + \lambda^i (D_a \tilde{E}^a)^i \right)$$

with γ the Barbero-Immirzi parameter.

$$\{A_a^i(x), \tilde{E}_j^b(y)\} = 8\pi G\gamma \delta_b^a \delta_j^i \delta^3(x - y)$$



VOLUME 57, NUMBER 18

PHYSICAL REVIEW LETTERS

3 NOVEMBER 1986

New Variables for Classical and Quantum Gravity

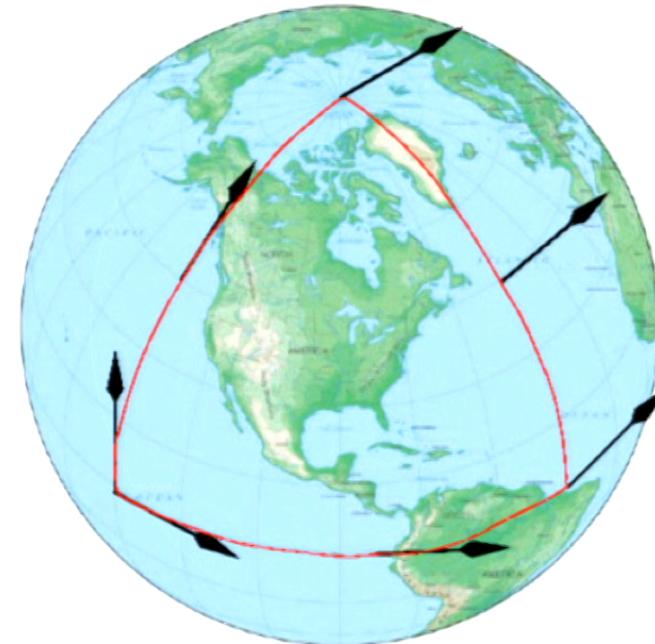
Abhay Ashtekar

Physics Department, Syracuse University, Syracuse, New York 13244, and Institute for Theoretical Physics,
University of California, Santa Barbara, Santa Barbara, California 93106
(Received 18 December 1985; revised manuscript received 29 August 1986)

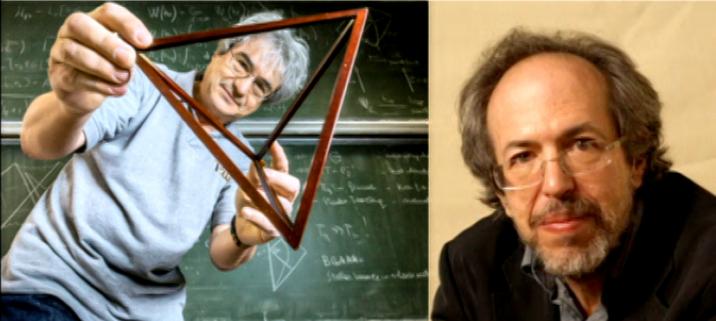
A Hamiltonian formulation of general relativity based on certain spinorial variables is introduced. These variables simplify the constraints of general relativity considerably and enable one to imbed the constraint surface in the phase space of Einstein's theory into that of Yang-Mills theory. The imbedding suggests new ways of attacking a number of problems in both classical and quantum gravity. Some illustrative applications are discussed.

Loop Quantum Gravity

- First step: Ashtekar variables.
- Second step: the loops... to measure the curvature of space-time.



Spin network states



PHYSICAL REVIEW D

VOLUME 52, NUMBER 10

15 NOVEMBER 1995

Spin networks and quantum gravity

Carlo Rovelli*

Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

Lee Smolin†

Center for Gravitational Physics and Geometry, Department of Physics, Pennsylvania State University,

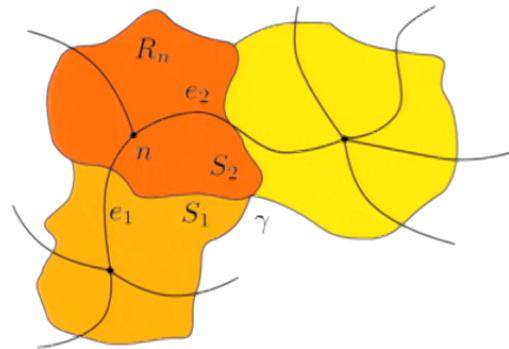
University Park, Pennsylvania 16802-6360

and School of Natural Science, Institute for Advanced Study, Princeton, New Jersey 08540

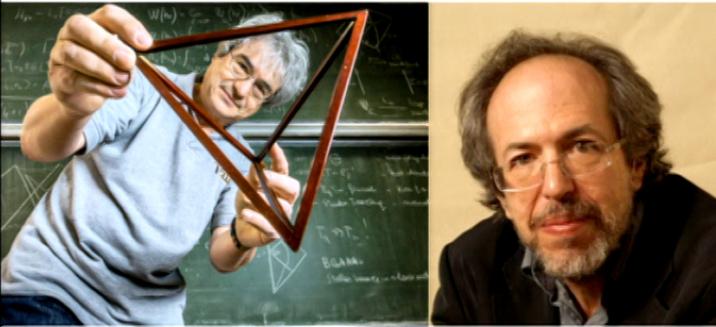
(Received 5 May 1995)

We introduce a new basis on the state space of nonperturbative quantum gravity. The states of this basis are linearly independent, are well defined in both the loop representation and the connection representation, and are labeled by a generalization of Penrose's spin networks. The new basis fully reduces the spinor identities [SU(2) Mandelstam identities] and simplifies calculations in nonperturbative quantum gravity. In particular, it allows a simple expression for the exact solutions of the Hamiltonian constraint (Wheeler-DeWitt equation) that have been discovered in the loop representation. The states in this basis diagonalize operators that represent the three-geometry of space, such as the area and the volume of arbitrary surfaces and regions, and therefore provide a discrete picture of quantum geometry at the Planck scale.

PACS number(s): 04.60.Ds, 75.25.+z



Spin network states



Nuclear Physics B 442 (1995) 593-619

NUCLEAR
PHYSICS B

Discreteness of area and volume in quantum gravity

Carlo Rovelli^{a,1}, Lee Smolin^{b,2}

^a Department of Physics, University of Pittsburgh, Pittsburgh, PA 15260, USA

^b Center for Gravitational Physics and Geometry, Department of Physics, Pennsylvania State University, University Park, PA 16802-6360, USA

Received 2 November 1994; accepted 20 March 1995

Abstract

We study the operator that corresponds to the measurement of volume, in non-perturbative quantum gravity, and we compute its spectrum. The operator is constructed in the loop representation, via a regularization procedure; it is finite, background independent, and diffeomorphism-invariant, and therefore well defined on the space of diffeomorphism invariant states (knot states). We find that the spectrum of the volume of any physical region is discrete. A family of eigenstates are in one to one correspondence with the spin networks, which were introduced by Penrose in a different context. We compute the corresponding component of the spectrum, and exhibit the eigenvalues explicitly. The other eigenstates are related to a generalization of the spin networks, and their eigenvalues can be computed by diagonalizing finite dimensional matrices. Furthermore, we show that the eigenstates of the volume diagonalize also the area operator. We argue that the spectra of volume and area determined here can be considered as predictions of the loop-representation formulation of quantum gravity on the outcomes of (hypothetical) Planck-scale sensitive measurements of the geometry of space.

PHYSICAL REVIEW D

VOLUME 52, NUMBER 10

15 NOVEMBER 1995

Spin networks and quantum gravity

Carlo Rovelli*

Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

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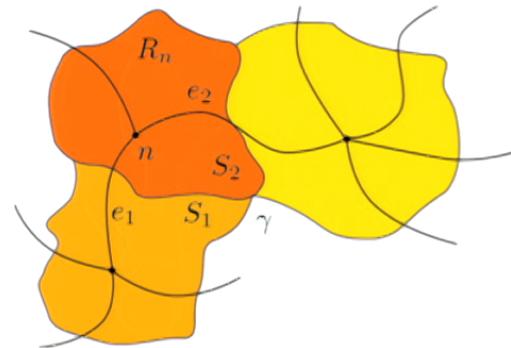
University Park, Pennsylvania 16802-6360

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(Received 5 May 1995)

We introduce a new basis on the state space of nonperturbative quantum gravity. The states of this basis are linearly independent, are well defined in both the loop representation and the connection representation, and are labeled by a generalization of Penrose's spin networks. The new basis fully reduces the spinor identities [SU(2) Mandelstam identities] and simplifies calculations in nonperturbative quantum gravity. In particular, it allows a simple expression for the exact solutions of the Hamiltonian constraint (Wheeler-DeWitt equation) that have been discovered in the loop representation. The states in this basis diagonalize operators that represent the three-geometry of space, such as the areas and the volume of arbitrary surfaces and regions, and therefore provide a discrete picture of quantum geometry at the Planck scale.

PACS number(s): 04.60.Ds, 75.25.+z



$$A|\psi\rangle = 8\pi\gamma L_{\text{Planck}}^2 |\psi\rangle$$

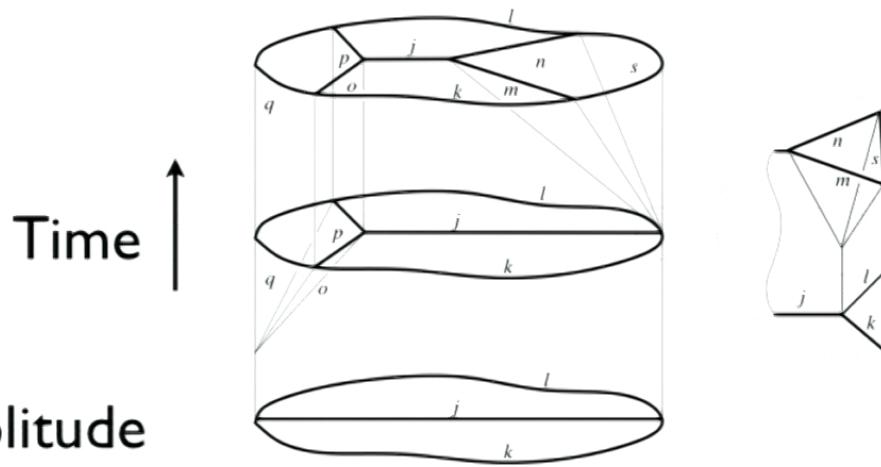
$$\text{with } L_{\text{Planck}} = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35} m$$

$$A|\Psi\rangle = (-) \sqrt{\delta_e(j_e^+)} |\Psi\rangle.$$

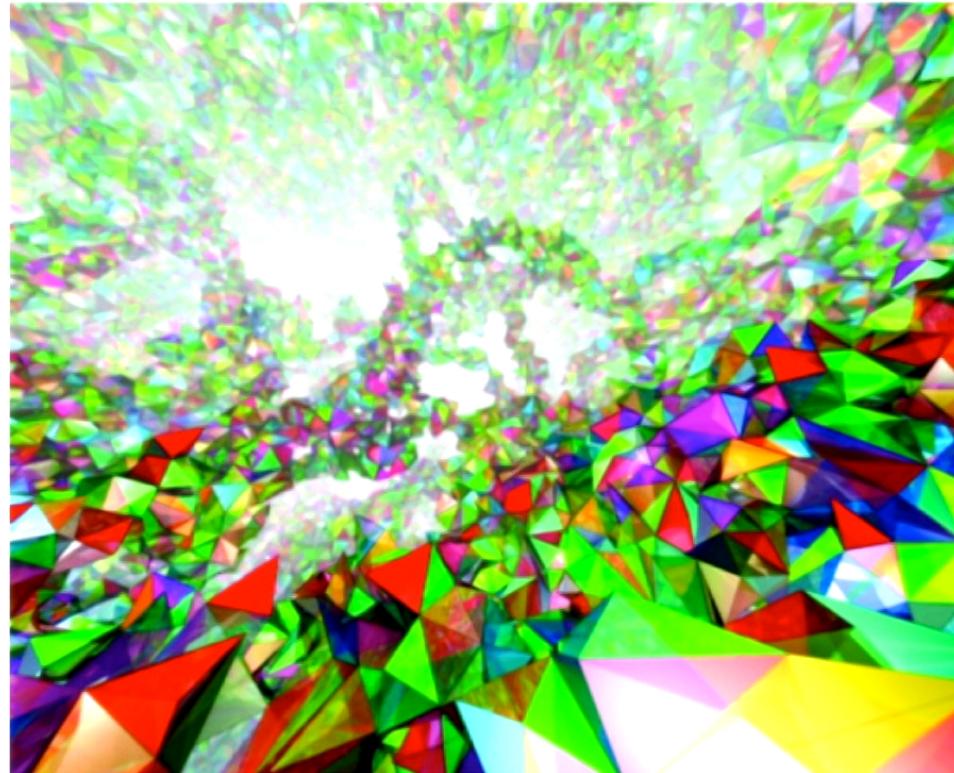
And the dynamics?

- Canonical approach: solve the Hamiltonian constraint...
- Spin Foam framework: the path integral à la Feynman...

Time ↑
Transition amplitude
between spin network states
of Loop Quantum Gravity



Loop Quantum Gravity/ Spinfoam



In this course...

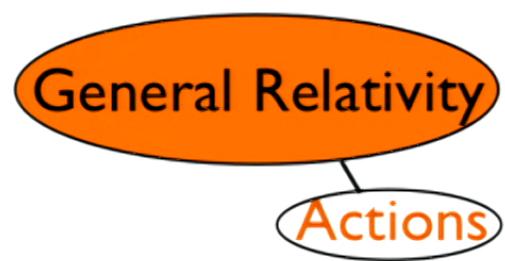
We will focus on **3D Euclidean gravity** with a **zero cosmological constant** and on the **Loop Quantum Gravity** (LQG) approach.

- Actions for gravity
- Canonical formulation of constrained systems and gauge symmetries
- Canonical analysis of the 3D gravity first order action
- Dirac program for the parametrized particle
- Quantum Geometry
- The path integral representation: spinfoams (SF)

Quantum Gravity!

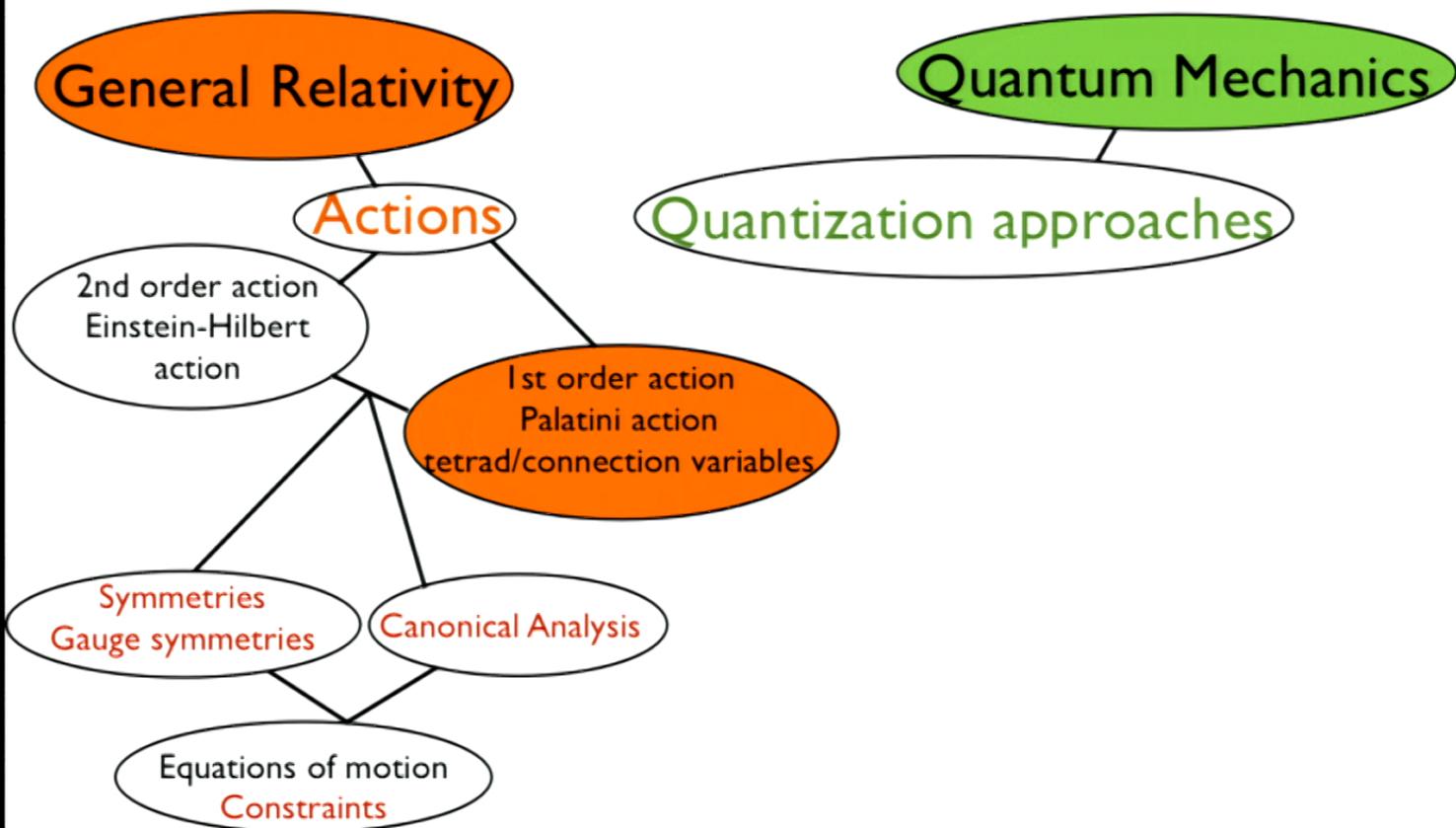
General Relativity

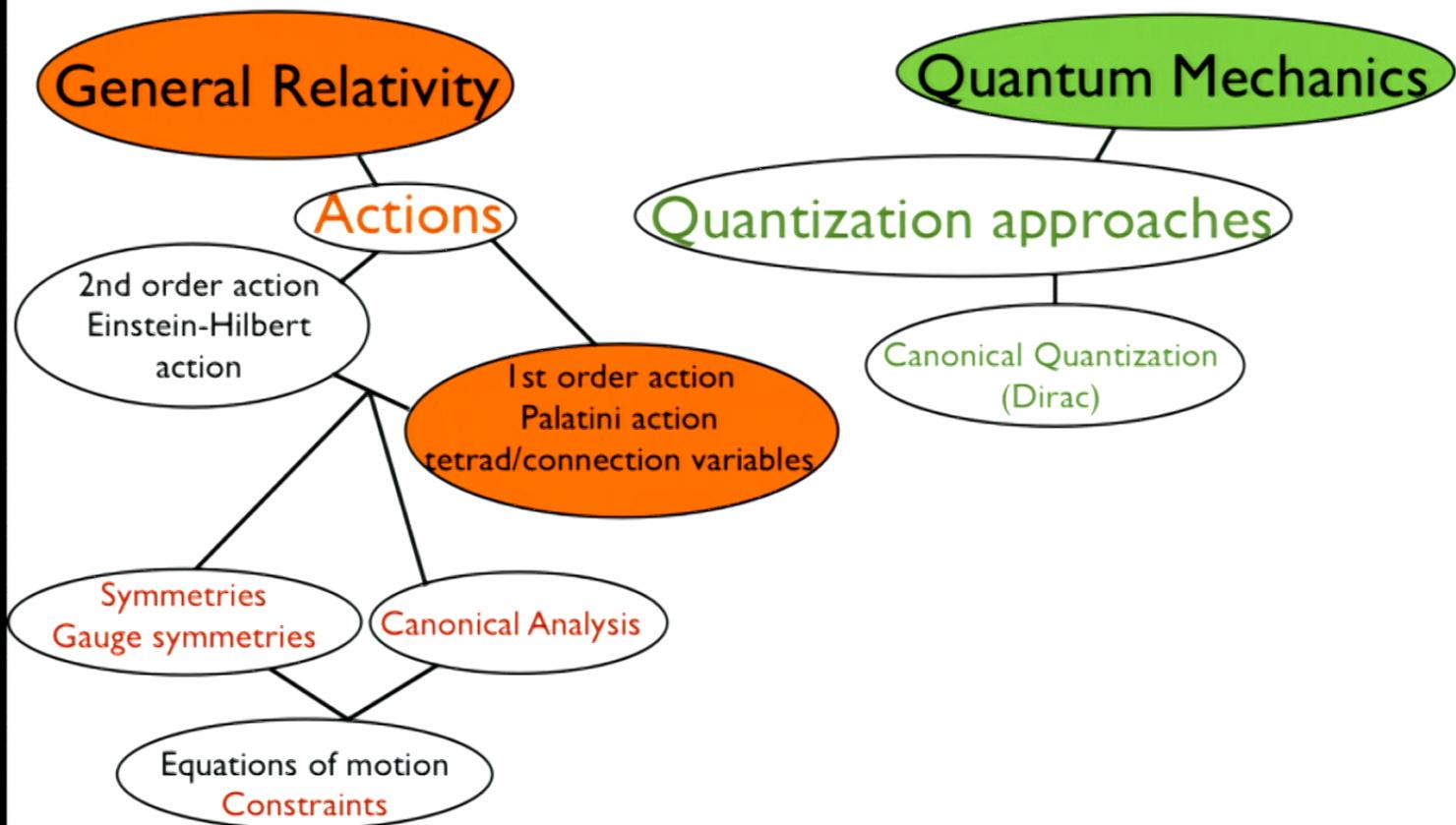
Quantum Mechanics



Quantum Mechanics







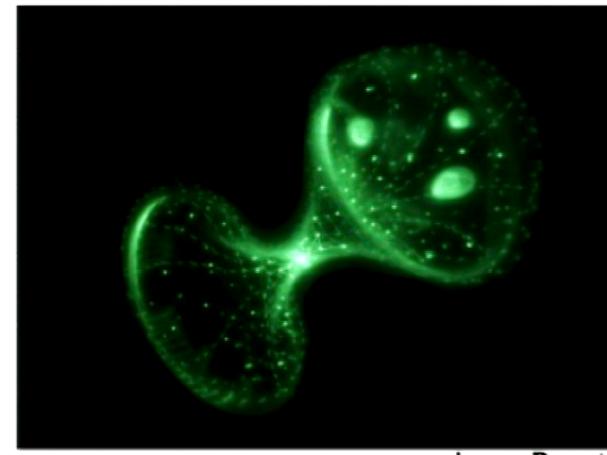
And what about...

- the Bing Bang singularity?

Luca Pozzi

And what about...

- the Bing Bang singularity?
- Big Bounce predicted by Loop Quantum Cosmology!



Luca Pozzi

$$(\dot{a}/a)^2 = 8\pi G\rho \longrightarrow (\dot{a}/a)^2 = 8\pi G\rho(1 - \rho/\rho_{\text{sup}})$$

Loop Quantum Cosmology and predictions



More details next week by Aurélien
Barrau (IN2P3, Grenoble, France)!
see also Tutorial 3

And what about...

- Black Holes?



Now!

Eugenio Bianchi (Penn State, USA)

Black holes & Quantum Gravity

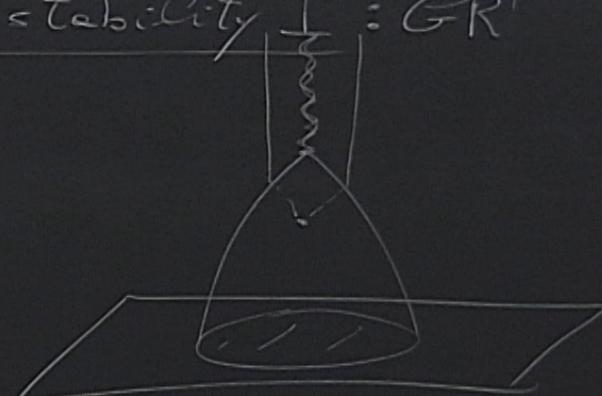
- BHs are hot.
- Two puzzles
- BHs & LQG

Black holes & Quantum Gravity

- BHs are hot.
- Two puzzles
- BHs & LQG

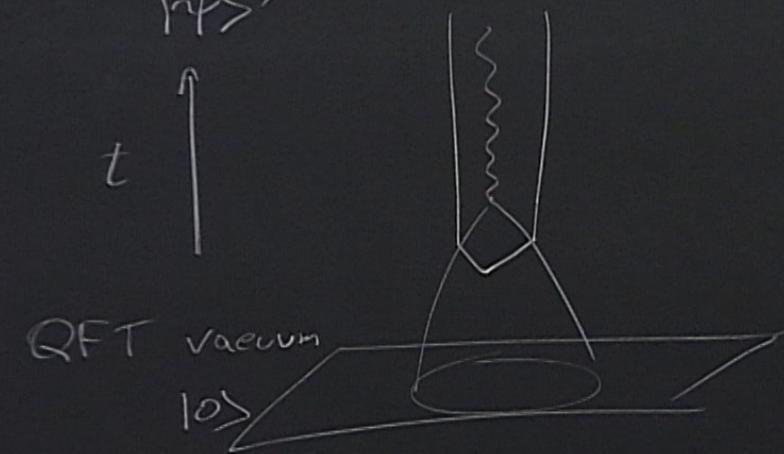
Instability $I = GR$

$t \uparrow$



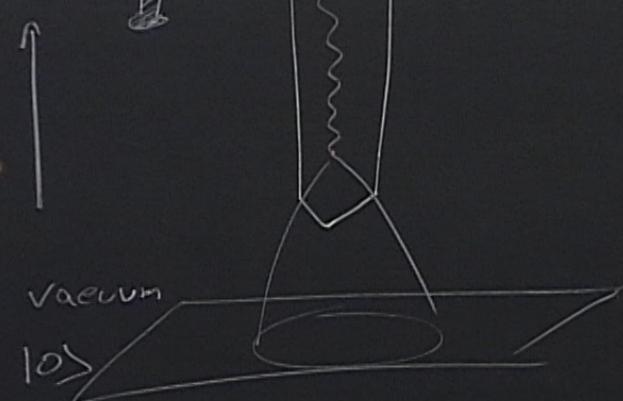
Instability: GR + QFT

Hawking - Evaporation '74



Instability: GR + QFT

$$r_P > \frac{1}{M}$$



FT vacuum

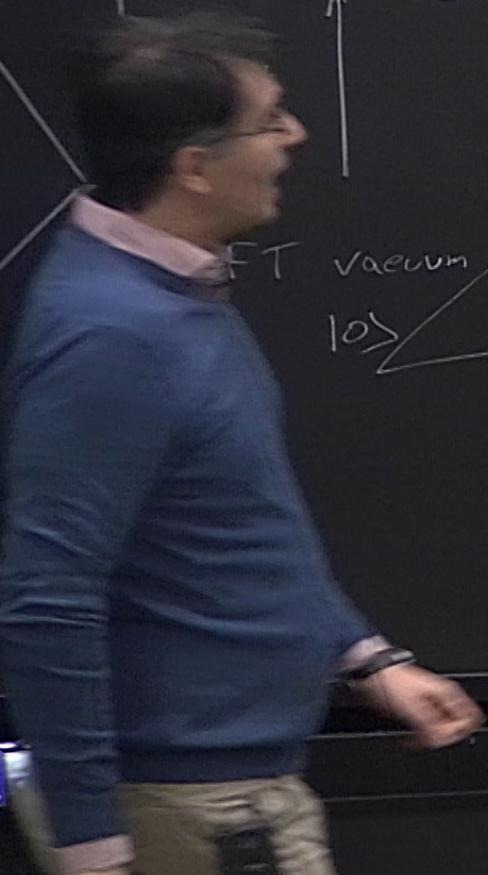
$|0\rangle$

Hawking Evaporation '74

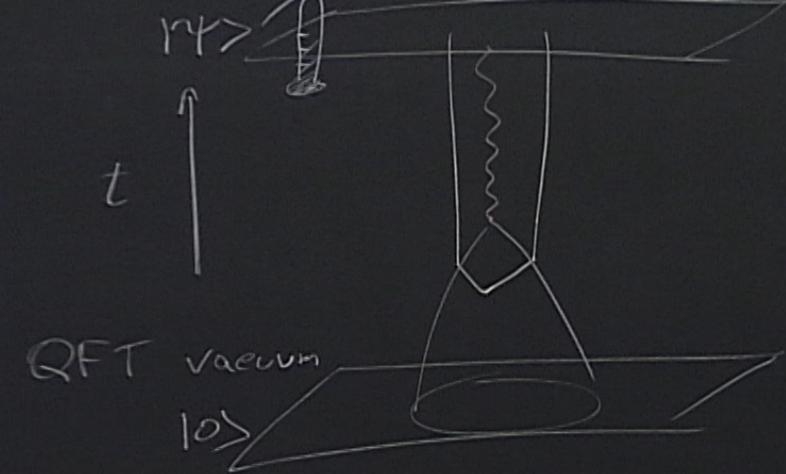
Temperature

for $M_\odot \Rightarrow T \sim 10^{-8} K$

$$T_H = \frac{\hbar}{8\pi GM}$$



Instability: GR + QFT



Hawking Evaporation '74

Temperature

$$\text{for } M_\odot \Rightarrow T \sim 10^{-8} \text{ K}$$

$$T_H = \frac{\hbar}{8\pi GM}$$

Cosmology

$$A_S = \frac{G \hbar}{8\pi^2 \epsilon_x} \frac{H_*^2}{\epsilon_x}$$

Entropy (Bekenstein & Hawking)

$$dQ = T dS$$

$$dM = T$$

Entropy (Bekenstein & Hawking)

$$dQ = T dS$$

$$\text{Area} = 4\pi (2G\Gamma)^2$$

$$dM = T_+ dS$$

$$\Rightarrow dS = \frac{8\pi G M dM}{\hbar} = d\left(\frac{\text{Area}}{4G\hbar}\right)$$

$$S_{BH} = \frac{\text{Area}}{4G\hbar}$$

Entropy (Bekenstein & Hawking)

$$dQ = T dS$$

$$\text{Area} = 4\pi (2G\Gamma)^2$$

$$dM = T_+ dS$$

$$\Rightarrow dS = \frac{8\pi G M dM}{\hbar} = d\left(\frac{\text{Area}}{4G\hbar}\right)$$

$$S_{BH} = \frac{\text{Area}}{4G\hbar}$$

Puzzle 1



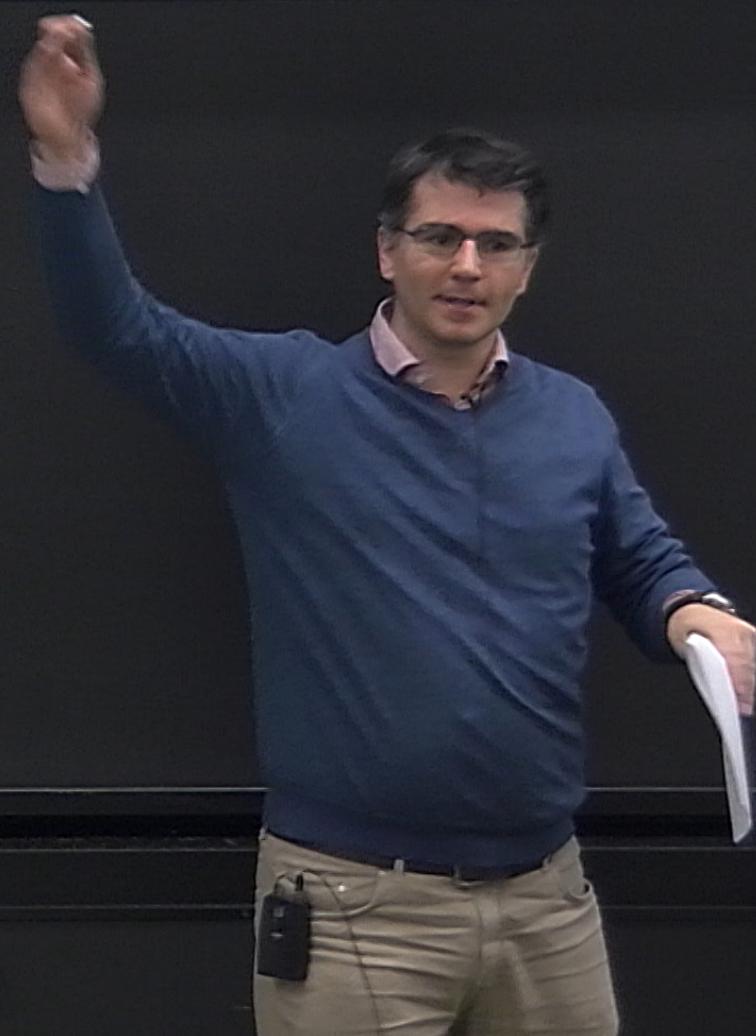
$$(2GM)^2$$

Puzzle 1 this huge
Stat Mech origin of $\sqrt{S_{\text{BH}}}$

$$S = \log(\dim \mathcal{H})$$

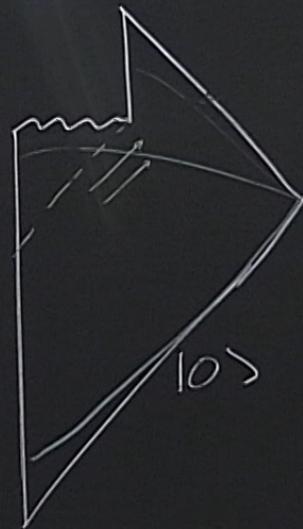
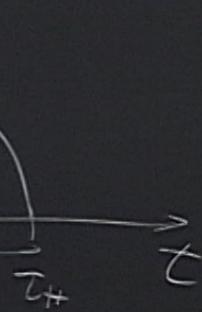
$$\Rightarrow \dim \mathcal{H} \sim \exp\left(\frac{M^2}{m_p^2}\right)$$

Energy conservation



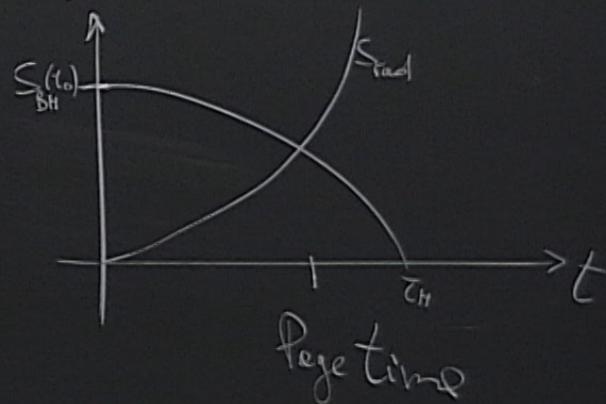
$$-F = -\frac{\hbar}{M^2}$$

Time-scale
 $\tau_H \sim \frac{M_0^3}{\hbar}$



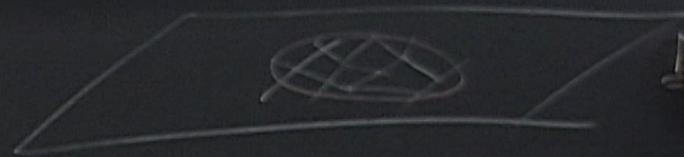
Puzzle 2 : information loss

- initial state : pure
- final state : mixed ?



BH & LQG

Puzzle 1. Entropy



BH & LQG

Puzzle 1. Entropy



Puzzle 2. Curvature is bounded in LQG

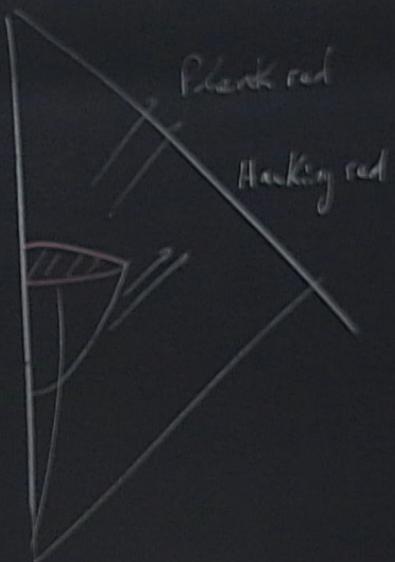
BH & LQG

Puzzle 1. Entropy



Puzzle 2. Curvature is bounded in LQG

Instability III : QR



H M

RG