

Title: PSI 2016/2017 Explorations in Quantum Information - Lecture 9

Date: Mar 30, 2017 09:00 AM

URL: <http://pirsa.org/17030066>

Abstract:

Efficient Characterization of Q. Gates with

The protocol Suppose $\{U_1, \dots, U_n\}$ is a

Write $U_n(g) \equiv U_n g U_n^\dagger$. Write $\tilde{U}_n$ to denote

For simplicity, use gate independent error model ;

with Randomized Benchmarking

$\{U_n\}$ is a finite group which is also a 2-design.

to denote the (imperfect) physical implementation of U_n .

model; $\tilde{U}_n = \Lambda \circ U_n$. Λ is error map

- RB:
- 1) Pick $i_1, \dots, i_k \in \{1, \dots, N\}$ iid uniformly
 - 2) Obtain $i_{k+1} \in \{1, \dots, N\}$ s.t. $U_{i_{k+1}} \circ U_{i_k} \circ \dots \circ U_{i_1} = I$
 - 3) Experimentally measure the expectation value

$$Q_i^K = \text{Tr} \left[E_0 \hat{U}_{i_{k+1}} \hat{U}_{i_k} \circ \dots \circ \hat{U}_{i_1} (f_0) \right]$$

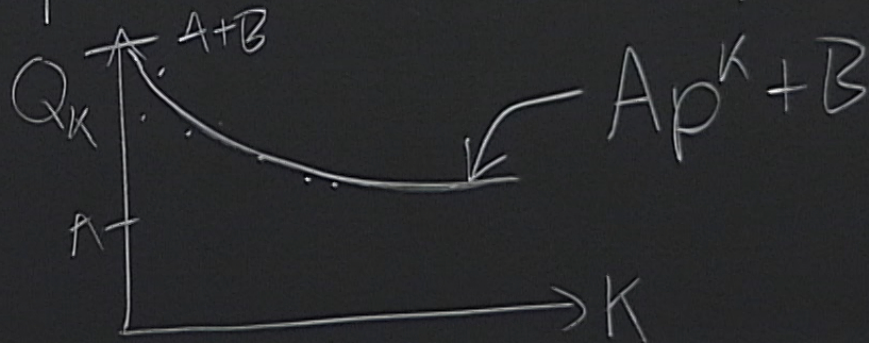
Ideally $E_0 = \mathcal{F} = |0\rangle\langle 0|$

... $u_i = \underline{I}$
value

Repeat (1), (2), (3) L times.

$$\text{Let } Q_k = \frac{1}{L} \sum_{l=1}^L Q_{i_l}^k$$

Repeat all this for multiple K .



A and B depend only on γ_0 E_0

$$\overline{F_g}(\Lambda) = p + \frac{1-p}{d}$$

Important properties

- SPAM free
- Scalable in number of qubits
- No exponential overhead

Def'n The avg gate fidelity of Λ is

$$\overline{F}_g(\Lambda) = \int \langle \psi | \Lambda(|\psi\rangle\langle\psi|) | \psi \rangle d\psi$$

Note $\overline{F}_g(\Lambda) \in [\frac{1}{d}, 1]$ and $\overline{F}_g(\Lambda) = 1$ iff $\Lambda = \mathbb{I}$

We have $\overline{F}_g(\Lambda) = \int \langle \psi | U^\dagger \Lambda(U|\psi\rangle\langle\psi|U^\dagger) U | \psi \rangle dU$

$L \xrightarrow{\quad} K$

gate fidelity of Λ is

$$\int \Lambda(|\psi\rangle\langle\psi|) |\psi\rangle d\psi$$

$$U(g) = U_g U^\dagger$$

$[\frac{1}{d}, \mathbb{I}]$ and $\overline{F}_g(\Lambda) = 1$ iff $\Lambda = \mathbb{I}$

$$\Lambda = \int \langle \psi | U^\dagger \Lambda(U |\psi\rangle\langle\psi| U^\dagger) U | \psi \rangle d\psi$$

$$= \langle \psi | \left(\int U^\dagger \Lambda U (|\psi\rangle\langle\psi|) dU \right) | \psi \rangle \quad \text{any } |\psi\rangle$$

$$= \langle \psi | \Lambda_{\text{avg}} (|\psi\rangle\langle\psi|) | \psi \rangle$$

where $\Lambda_{\text{avg}}(g) := \int U^\dagger \Lambda U (g) dU$

Fact $\Lambda_{\text{avg}}(g) = pg + (1-p)\frac{I}{d}$ for some p

$$[\Lambda_{\text{avg}} = W^\dagger \Lambda_{\text{avg}} W \text{ any unitary } W]$$

We get
$$\begin{aligned} F_g(\Lambda) &= \langle \psi | \Lambda_{\text{avg}} (|\psi\rangle\langle\psi|) | \psi \rangle \\ &= \langle \psi | (p|\psi\rangle\langle\psi| + (1-p)\frac{\mathbb{I}}{d}) | \psi \rangle \\ &= p + \frac{1-p}{d} \end{aligned}$$

Def'n A unitary 2 -design is a set $\{U_1, \dots, U_N\}$ of unitaries such that

$$\int f(U) dU = \frac{1}{N} \sum_{n=1}^N f(U_n)$$

when f is a polynomial of deg 2 in U
and $U^\dagger$.

Ex Clifford group is unitary 2-design
In $d=2$, the group generated by $T = \begin{pmatrix} 1 & \\ & i \end{pmatrix}$
and $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, get $|\text{Clif}_2| = 24$

⇒ Clifford group is unitary 2^d -dim

In $d=2$, the group generated by $T = \begin{pmatrix} 1 & \\ & i \end{pmatrix}$
and $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, get $|\text{Clif}_2| = 24$

$$\Lambda_{\text{avg}}(f) = \frac{1}{N} \sum_{n=1}^N U_n^\dagger \circ f \circ U_n$$

and U :

Back to protocol

For $\vec{i} = (i_1, \dots, i_{k+1})$ Let $\tilde{U}_{\vec{i}} = \tilde{U}_{i_{k+1}} \circ \tilde{U}_{i_k} \circ \dots \circ \tilde{U}_{i_1}$

$$\begin{aligned} \tilde{U}_{\vec{i}} &= \Lambda \circ U_{i_{k+1}} \circ \Lambda \circ U_{i_k} \circ \dots \circ \Lambda \circ U_{i_2} \circ \Lambda \circ U_{i_1} \quad j_1 = i_1 \\ &= \Lambda \circ U_{i_{k+1}} \circ \dots \circ \Lambda \circ \underbrace{U_{i_2} \circ U_{j_1} \circ U_{j_1}^\dagger}_{U_{j_2}} \circ \Lambda \circ U_{j_1} \end{aligned}$$

Ideally $L_0 \cong 10/101$

$$= \Lambda \circ U_{i_{k+1}} \circ \dots \circ \Lambda \circ U_{i_3} \circ U_{i_2} \circ U_{i_2}^+ \circ \Lambda \circ U_{i_2} \circ U_{i_1}^+ \circ \Lambda$$

$$= \dots = \Lambda \circ U_{i_{k+1}} \circ U_{i_k} \circ U_{i_k}^+ \circ \Lambda \circ U_{i_k} \circ U_{i_{k-1}} \circ \Lambda \circ U_{i_{k-1}}$$

$\xrightarrow{\quad \text{II} \quad U_{i_3} \quad}$

$$U_{i_{k+1}} \circ U_{i_k} \circ \dots \circ U_{i_1} = \text{II}$$

$$\begin{aligned}
 & \circ \Lambda \circ \underbrace{U_{j,3} \circ U_{j,2} \circ U_{j,2}^+}_{U'_{j,3}} \circ \underbrace{\Lambda \circ U_{j,2} \circ U_{j,1}^+}_{U'_{j,1}} \circ \Lambda \circ U_{j,1} \\
 & \xrightarrow{\text{II}} U'_{j,3} \\
 & U_{j,k} \circ \underbrace{U_{j,k}^+ \circ \Lambda \circ U_{j,k}}_{U'_{j,k}} \circ \underbrace{U_{j,k-1} \circ \Lambda \circ U_{j,k-1}}_{U'_{j,k-1}} \circ \dots \circ \underbrace{U_{j,1}^+ \circ \Lambda \circ U_{j,1}}_{U'_{j,1}} \\
 & \underbrace{U_{j,k} \circ \dots \circ U_{j,1}}_{U_j} = \text{II}
 \end{aligned}$$

Identity 20 to 10/101

$$U_{i_{k+1}} \circ \dots \circ \Lambda \circ \underbrace{U_{i_3} \circ U_{j_2}}_{U'_{j_3}} \circ \underbrace{U_{j_2}^+ \circ \Lambda \circ U_{j_2}}_{U'_{j_2}} \circ \underbrace{U_{j_1}^+ \circ \Lambda \circ U_{j_1}}_{U'_{j_1}}$$

$$\Lambda \circ U_{i_{k+1}} \circ U_{j_k} \circ \underbrace{U_{j_k}^+ \circ \Lambda \circ U_{j_k}}_{U'_{j_k}} \circ \underbrace{U_{j_{k-1}} \circ \Lambda \circ U_{j_{k-1}}}_{U'_{j_{k-1}}} \circ \dots \circ$$

$$\underbrace{U_{i_{k+1}} \circ U_{i_k} \circ \dots \circ U_{i_1}}_{U_{j_k}} = \mathbb{I}$$

$$U_{jk}$$

$$\text{Set } I_k = \{ \vec{i} = (i_1, \dots, i_{k-1}) \mid U_{i_{k-1}} \circ \dots \circ U_{i_1} = I \}$$

$$\begin{aligned} \tilde{U}_k &:= \frac{1}{|I_k|} \sum_{i \in I_k} \tilde{U}_{\vec{i}} = \frac{1}{|I_k|} \sum_{i \in I_k} \Lambda \circ U_{j_k}^+ \circ \Lambda \circ U_{j_k} \dots \circ U_{j_1} \\ &= \Lambda \circ \left(\frac{1}{N} \sum_{j_k=1}^N U_{j_k}^+ \circ \Lambda \circ U_{j_k} \right) \circ \dots \circ \left(\frac{1}{N} \right) \end{aligned}$$

u_{jk}

$$= (i_1, \dots, i_{k+1}) \mid u_{i_{k+1}} \circ \dots \circ u_{i_1} = \mathbb{I} \} \quad \text{Notice } |I_k| = N^k$$

$$\begin{aligned} \vec{1} &= \frac{1}{|I_k|} \sum_{i \in I_k} \Lambda \circ u_{j_k}^+ \circ \Lambda \circ u_{j_k} \dots \circ u_{j_1}^+ \circ \Lambda \circ u_{j_1} \\ &= \Lambda \circ \left(\frac{1}{N} \sum_{j_k=1}^N u_{j_k}^+ \circ \Lambda \circ u_{j_k} \right) \circ \dots \circ \left(\frac{1}{N} \sum_{j_1=1}^N u_{j_1}^+ \circ \Lambda \circ u_{j_1} \right) \end{aligned}$$

U_{jk}

$\vec{i} = (i_1, \dots, i_{k-1}) \mid U_{i_{k-1}} \circ \dots \circ U_{i_1} = \mathbb{I} \}$ Notice $|I_k| = N^k$

$$\begin{aligned}\tilde{U}_i &= \frac{1}{|I_k|} \sum_{i \in I_k} \Lambda \circ U_{j_k}^+ \circ \Lambda \circ U_{j_k} \dots \circ U_{j_1}^+ \circ \Lambda \circ U_{j_1} \\ &= \Lambda \circ \left(\frac{1}{N} \sum_{jk=1}^N U_{j_k}^+ \circ \Lambda \circ U_{j_k} \right) \circ \dots \circ \left(\frac{1}{N} \sum_{j1=1}^N U_{j_1}^+ \circ \Lambda \circ U_{j_1} \right) \\ &= \Lambda \circ \Lambda_{\text{avg}}^k\end{aligned}$$

$$|I_k| = N^k \quad \Lambda_{\text{avg}}(p) = pg + (1-p)\frac{\Pi}{d}$$

$$\Lambda_{\text{avg}}^2(p) = \Lambda_{\text{avg}}\left(pg + (1-p)\frac{\Pi}{d}\right)$$

$$= p\Lambda_{\text{avg}}(p) + (1-p)\Lambda_{\text{avg}}\left(\frac{\Pi}{d}\right)$$

$$= p\left(pg + (1-p)\frac{\Pi}{d}\right) + (1-p)\frac{\Pi}{d}$$

$$= p^2g + (1-p^2)\frac{\Pi}{d}$$

$$\Lambda_{\text{avg}}^K(\rho) = p^K \rho + (1-p^K) \frac{\mathbb{I}}{d}.$$

$$Q_K = \frac{1}{L} \sum_{\ell=1}^L Q_{i\ell}^K = \frac{1}{L} \sum_{\ell=1}^L \text{Tr} E_0 \tilde{U}_i^{\sim}(\rho_0)$$

$$= \text{Tr} \left[E_0 \frac{1}{L} \sum_{\ell=1}^L \tilde{U}_i^{\sim}(\rho_0) \right]$$

$$\approx \text{Tr} \left[E_0 \frac{1}{|I_K|} \sum_{i \in I_K} \tilde{U}_i^{\sim}(\rho_0) \right]$$

$$= \text{Tr} E_0 \Lambda_0 \Lambda_{\text{avg}}^k(g_0)$$

$$= \text{Tr} E_0 \Lambda \left(p^k g_0 + (1-p^k) \frac{\mathbb{I}}{d} \right)$$

$$= \underbrace{\left(\text{Tr} E_0 \left(\Lambda g_0 - \frac{\mathbb{I}}{d} \right) \right)}_{\equiv A} p^k + \underbrace{\text{Tr} E_0 \Lambda \left(\frac{\mathbb{I}}{d} \right)}_{\equiv B}$$