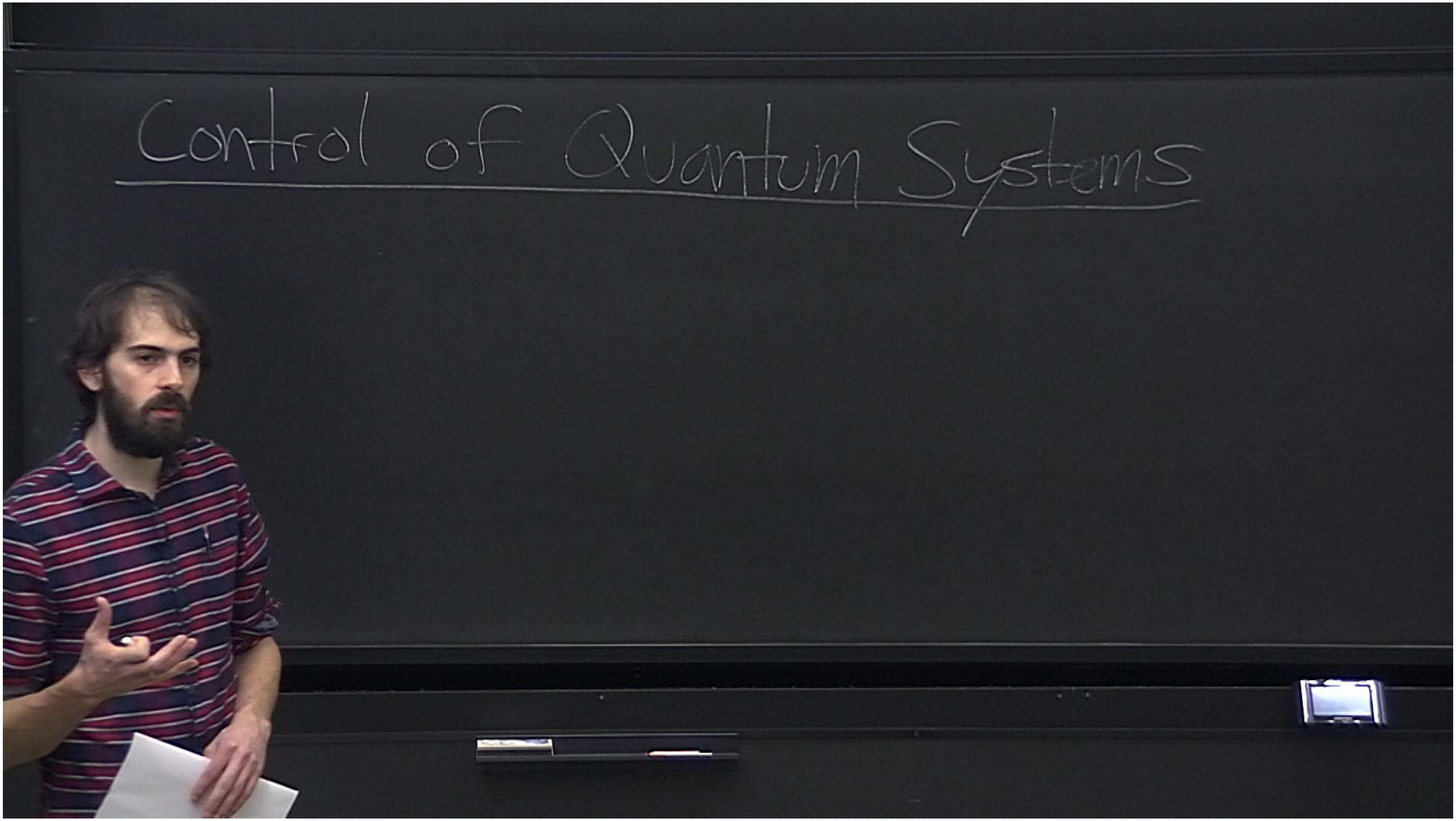
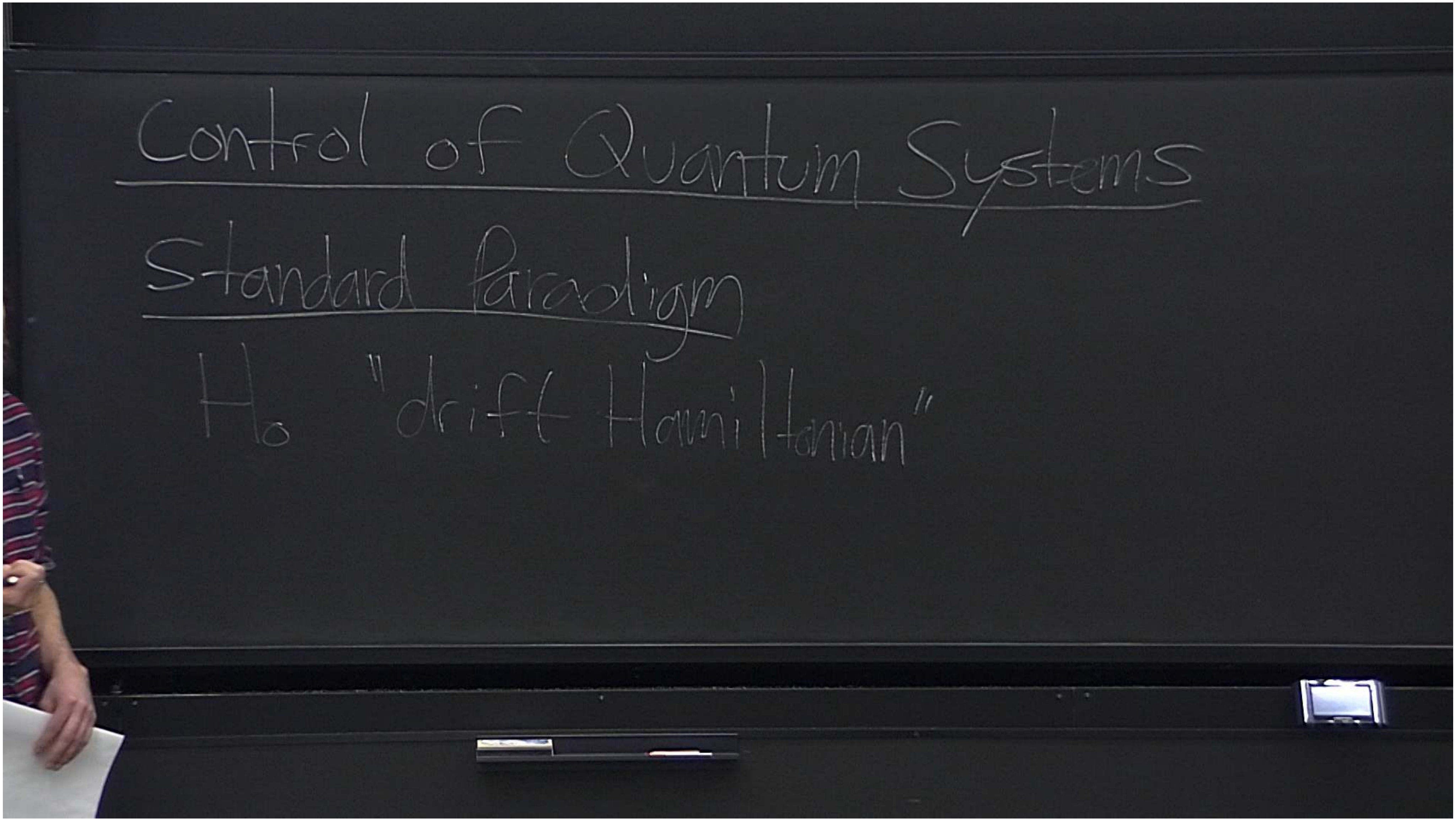
Title: PSI 2016/2017 Explorations in Quantum Information - Lecture 8

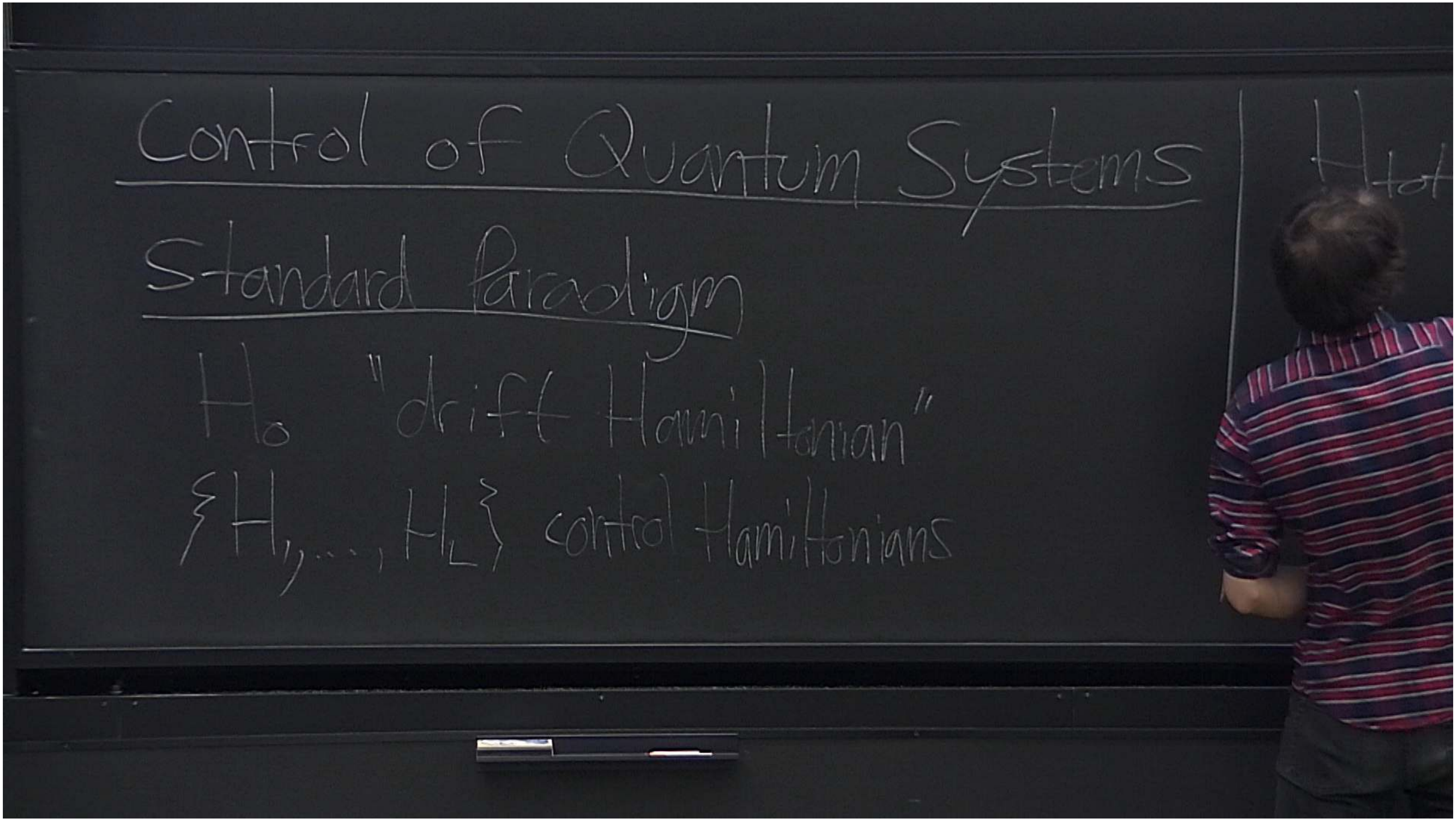
Date: Mar 29, 2017 09:00 AM

URL: http://pirsa.org/17030065

Abstract:


Control of Quantum Systems


Control of Quantum Systems
Standard Paradigm
H_0 "drift Hamiltonian"


Control of Quantum Systems
Standard Paradigm
H_0 "drift Hamiltonian"
$\{H_1, \ldots, H_L\}$ control Hamiltonians
H_{tot}

$$H_{tot}(t) = H_0 + \alpha_1(t) H_1 + \cdots + \alpha_L(t) H_L$$

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Experimenter can control $\vec{\alpha}(t) = (\alpha_1(t), \ldots, \alpha_L(t))$

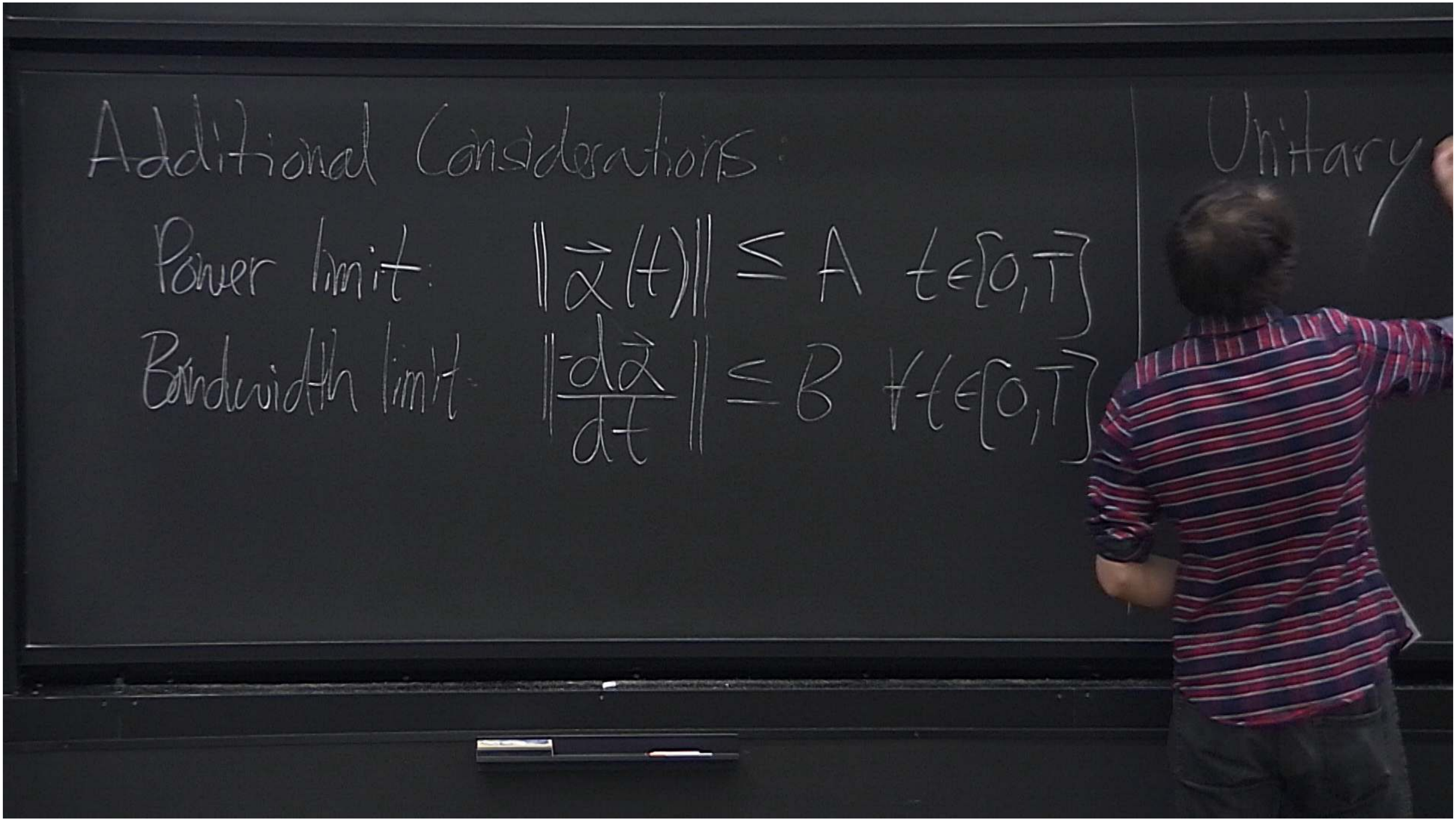
Ex) $H_0 = \frac{\delta\omega}{2}\sigma_z \quad H_1 = \frac{1}{2}\sigma_x \,, \; H_2 = \frac{1}{2}\sigma_y$

$$H_{tot}(t) = H_0 + \alpha_1(t) H_1 + \cdots + \alpha_L(t) H_L$$

Experimenter can control $\vec{\alpha}(t) = (\alpha_1(t), \ldots, \alpha_L(t))$

Ex: $H_0 = \frac{\delta\omega}{2}\sigma_z \quad H_1 = \frac{1}{2}\sigma_x, \quad H_2 = \frac{1}{2}\sigma_y$

"Quadrature Control"


Additional Considerations:
Power limit: $\|\vec{\alpha}(t)\| \leq A \quad t \in [0,T]$
Bandwidth limit: $\left\|\frac{d\vec{\alpha}}{dt}\right\| \leq B \quad \forall t \in [0,T]$
Unitary

Unitary Control

Given some $\vec{\alpha}(t)$ on $[0,T]$

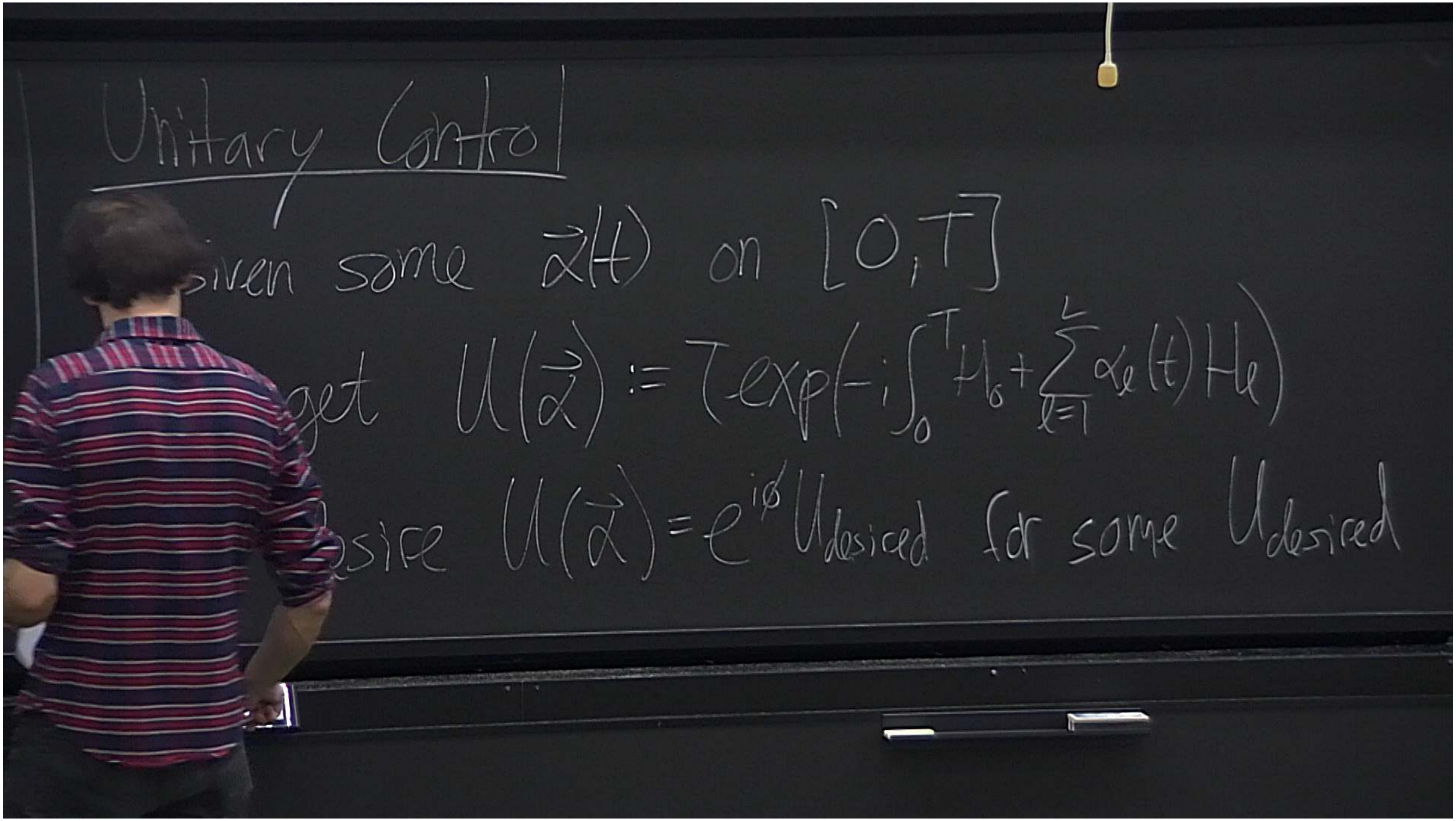
We get $U(\vec{\alpha}) := \exp\left(-i\int_0^T H_0 + \sum_{\ell=1}^{L} \alpha_\ell(t) H_\ell\right)$

Unitary Control

Given some $\vec{\alpha}(t)$ on $[0, T]$

We get $U(\vec{\alpha}) := \mathcal{T}\exp\left(-i\int_0^T H_0 + \sum_{\ell=1}^{L} \alpha_\ell(t) H_\ell\right)$

We desire $U(\vec{\alpha}) = e^{i\phi} U_{desired}$


Unitary Control
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esire $U(\vec{\alpha}) = e^{i\phi} U_{desired}$ for some $U_{desired}$

We des

Ex: $H_0 = 0 \quad H_1 = \frac{1}{2}\sigma_x \quad H_2 = \frac{1}{2}\sigma_y$

$U_{desired} = \sigma_x$ at time T.

We des

<u>Ex</u> $H_0 = 0$ $H_1 = \frac{1}{2}\sigma_x$ $H_2 = \frac{1}{2}\sigma_y$

$U_{\text{desired}} = \sigma_x$ at time T.

<u>Sol'n</u> Set $\alpha(t) = \left(\frac{\pi}{T}, 0\right)$ on $t \in [0,T]$

We desire $U(\vec{\alpha}) \in U_{\text{desired}}$ for some U_{desired}

$$U(\vec{\alpha}) = T\exp\left(-i\int_0^T 0 + \frac{\pi}{T}\frac{1}{2}\sigma_x + 0\right)$$

$$= \exp\left(-i\frac{\pi}{2}\sigma_x\right) = \sin\left(\frac{\pi}{2}\right)\mathbb{1} - i\cos\left(\frac{\pi}{2}\right)\sigma_x$$

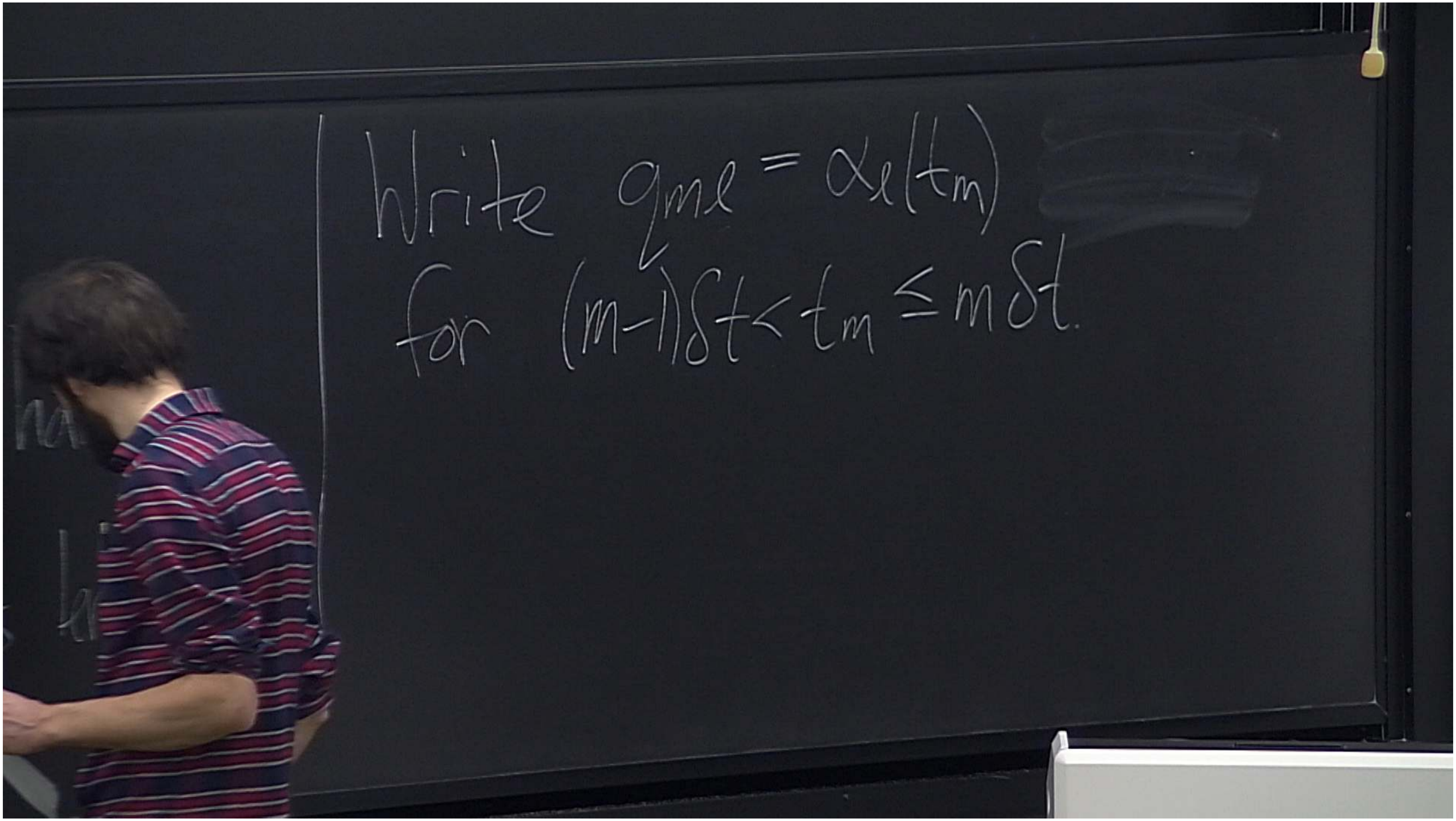
$$= -i\sigma_x$$

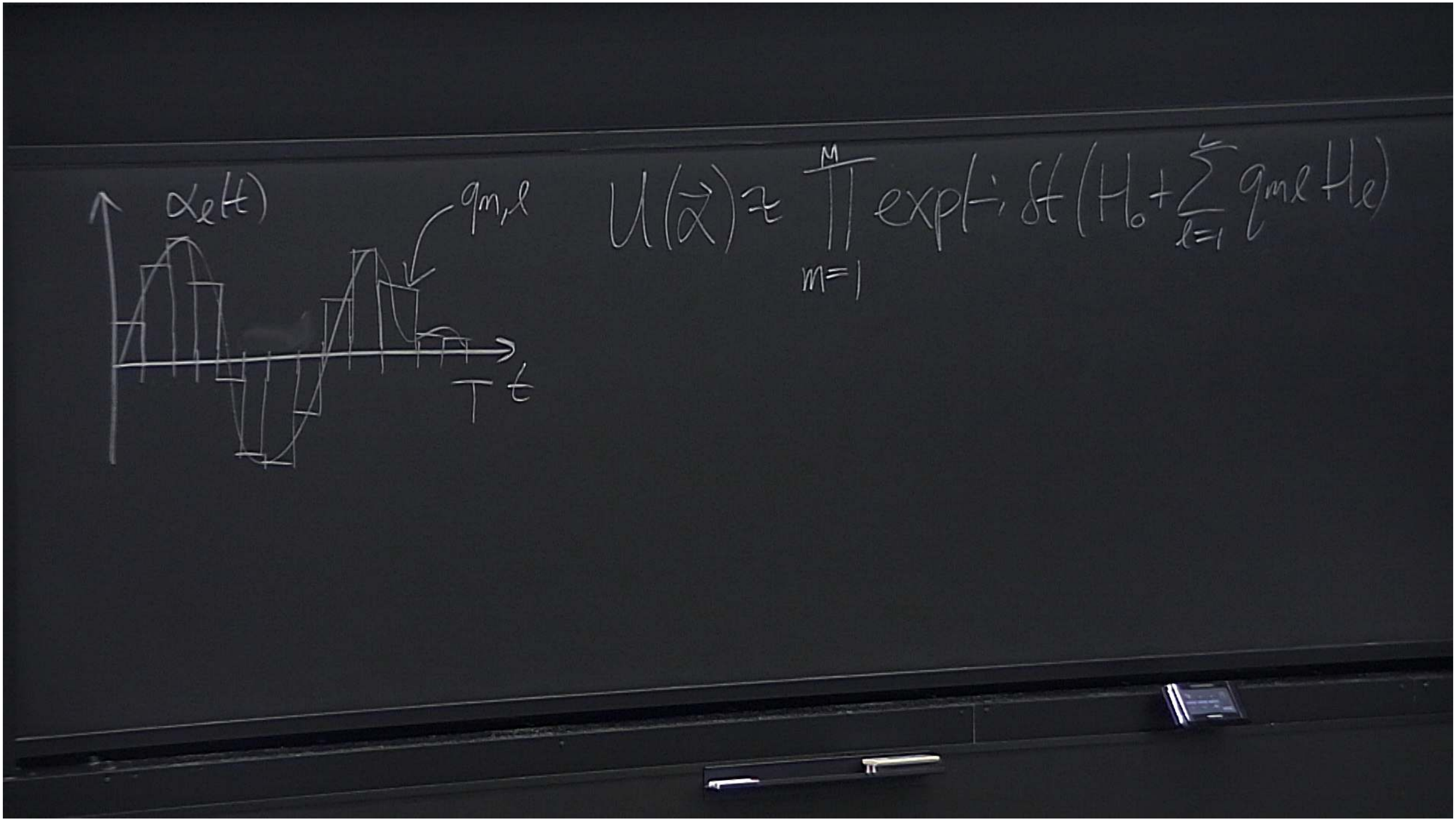
Numerical Optimal Control

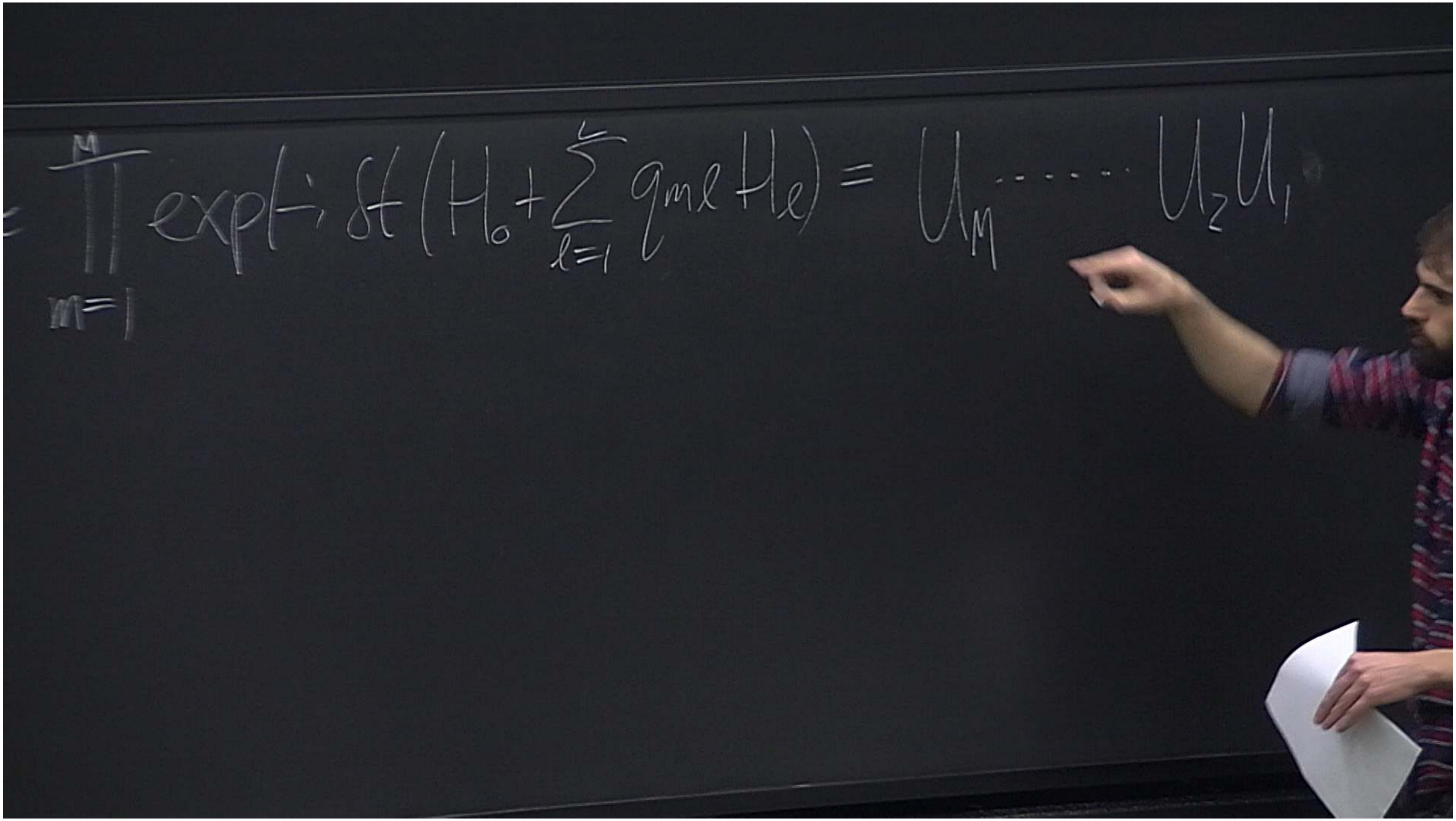
Finding $\vec{\alpha}(t)$ which results in $U_{desired}$ is hard.

Thus we resort to numerics.

Discretize time domain $[0,T]$ into M steps length δt.


Write $q_{m\ell} = \alpha_\ell(t_m)$
for $(m-1)\delta t < t_m \leq m\delta t.$


$\alpha_\ell(t)$
$q_{m,\ell}$
T
t
$U(\vec{\alpha}) = \prod_{m=1}^{M} \exp(-i\,\delta t\,(H_0 + \sum_{\ell=1}^{L} q_{m\ell} H_\ell)$


$= \prod_{m=1}^{M} \exp[-i\,\delta t\,(H_0 + \sum_{l=1}^{L} q_{ml} H_l)] = U_M \cdots\cdots U_2 U_1$

$$= \prod_{m=1}^{M} \underbrace{\exp\left(-i\,\delta t\left(H_0 + \sum_{\ell=1}^{L} q_{m\ell} H_\ell\right)\right)}_{U_m} = U_M \cdots\cdots U_2 U_1$$

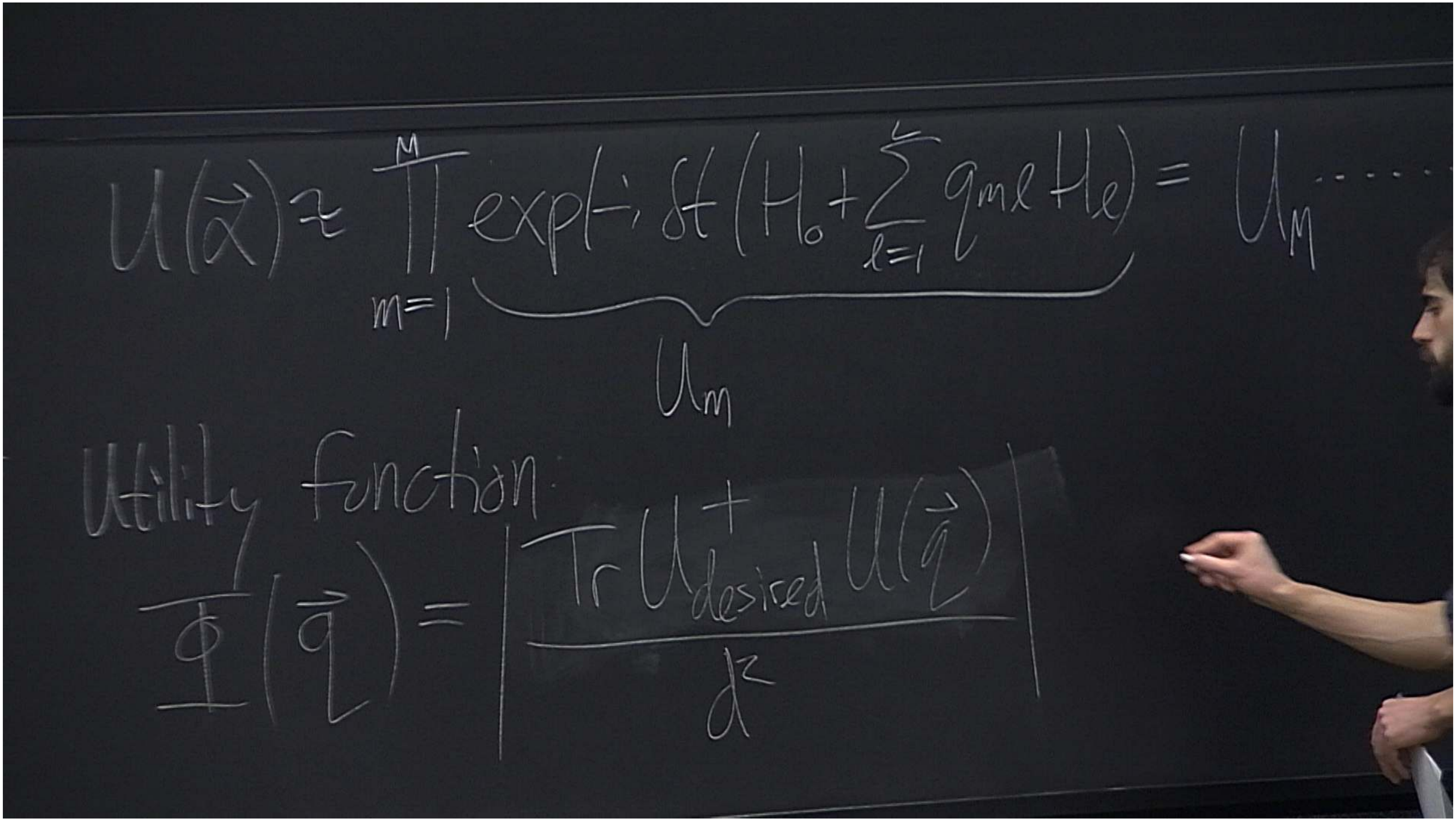
$q_{m,l}$

$$U(\vec{\alpha}) \approx \prod_{m=1}^{M} \underbrace{\exp\left(-i\,\delta t\left(H_0 + \sum_{l=1}^{L} q_{ml} H_l\right)\right)}_{U_m} = U_M \cdots$$

$\overrightarrow{\quad}$ T t

Utility function:

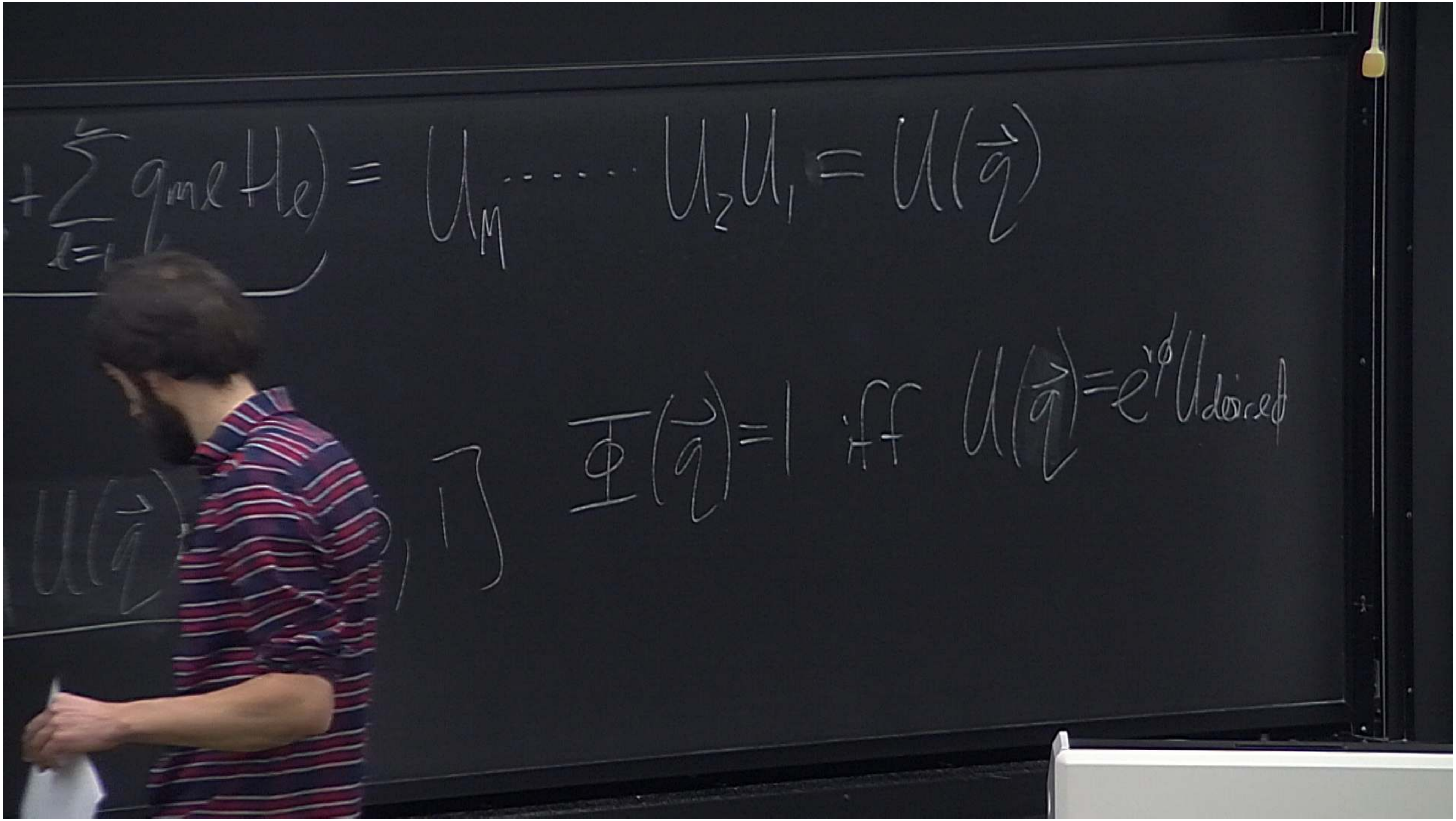
$$\Phi(\vec{q}) =$$

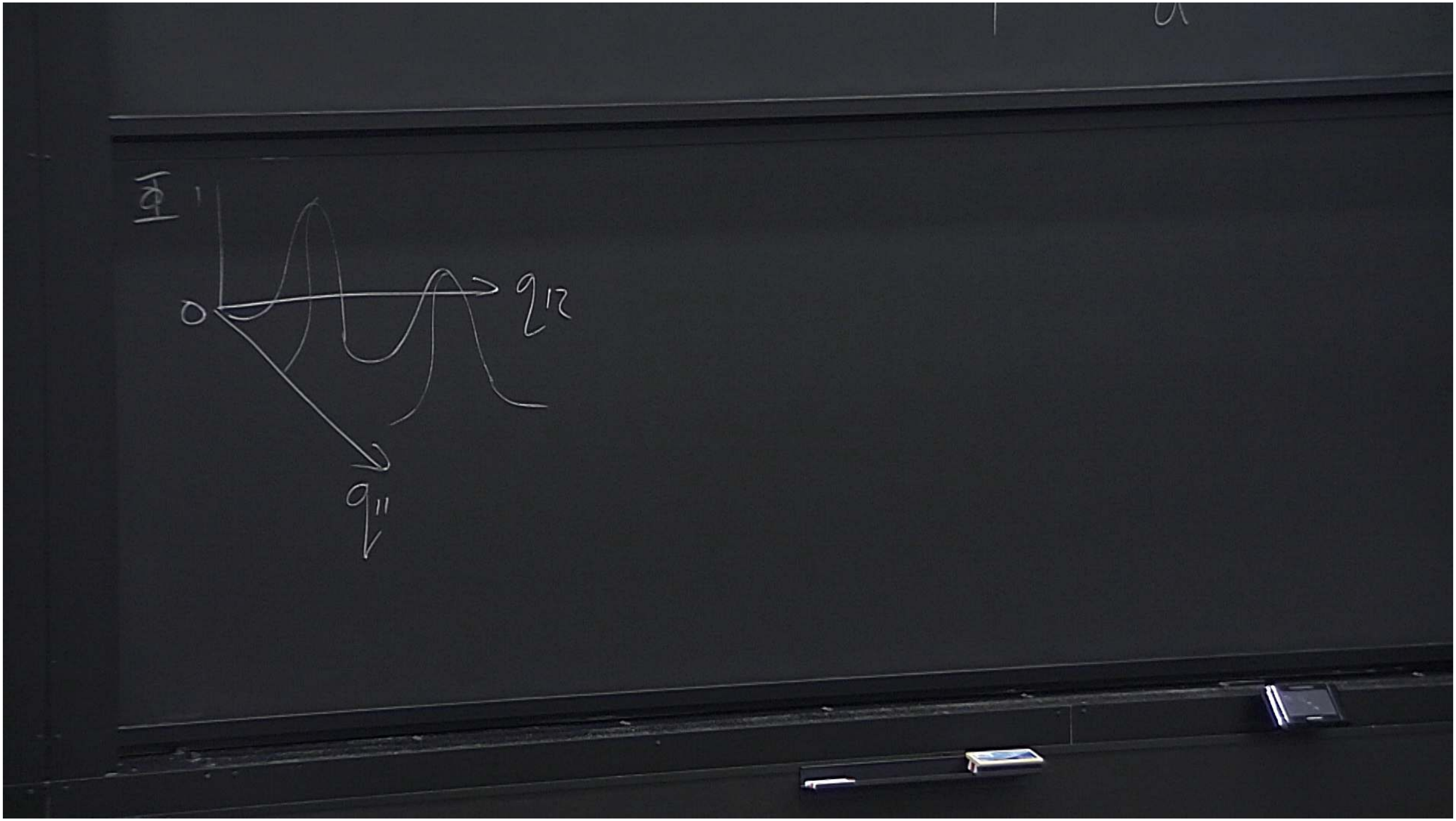

$U(\vec{\alpha}) \approx \prod_{m=1}^{M} \underbrace{\exp(-i\,\delta t\,(H_0 + \sum_{l=1}^{L} q_{ml} H_l)}_{U_m} = U_M \cdots$
Utility function:
$\Phi(\vec{q}) = \left| \frac{\mathrm{Tr}\, U^{\dagger}_{desired}\, U(\vec{q})}{d^2} \right|$

$$U(\vec{\alpha}) = \prod_{m=1}^{M} \underbrace{\exp[-i\,\delta t\,(H_0 + \sum_{l=1}^{L} q_{ml} H_l)]}_{U_m} = U_M \cdots\cdots U_2 U_1$$

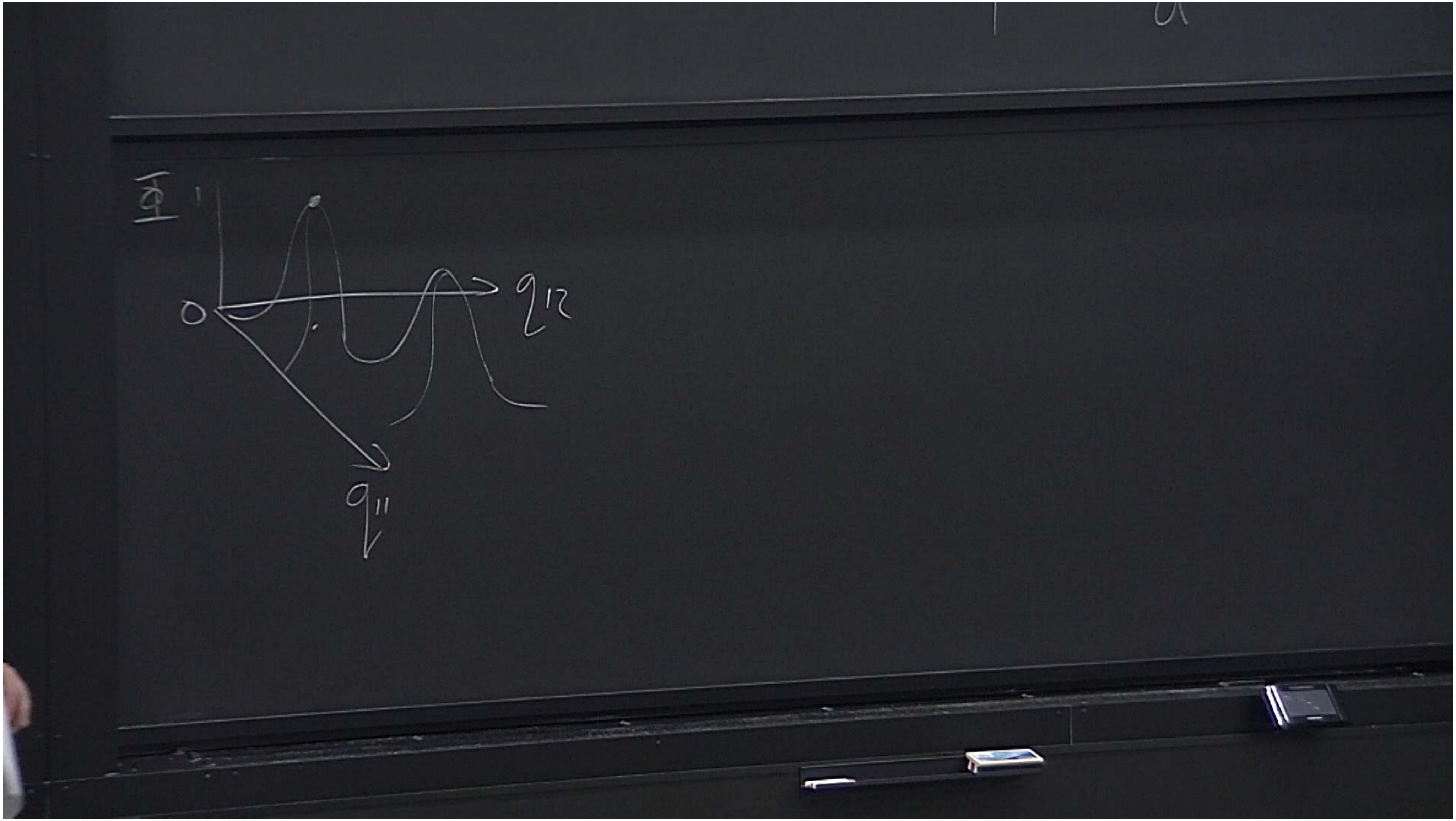
Utility function:

$$\Phi(\vec{q}) = \left| \frac{\mathrm{Tr}\, U^{\dagger}_{desired}\, U(\vec{q})}{d^2} \right| \in [0,1] \qquad \Phi(\vec{q}) = 1$$

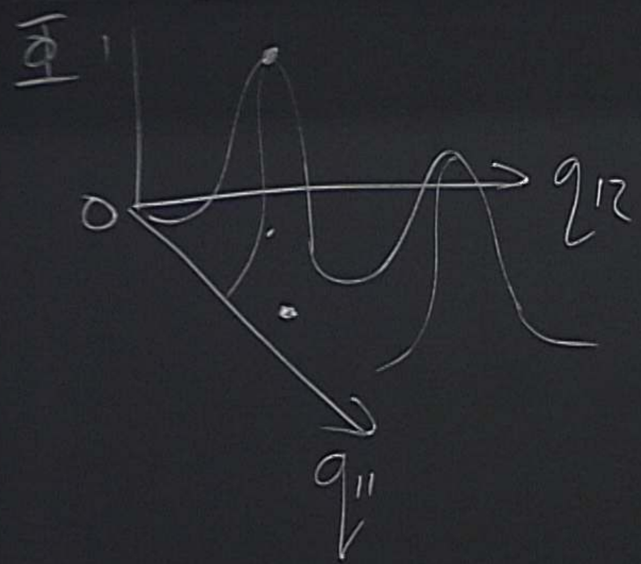

$+\sum_{\ell=1}^{L} q_{m\ell} H_\ell) = U_M \cdots U_2 U_1 = U(\vec{q})$
$\Phi(\vec{q}) = 1$ iff $U(\vec{q}) = e^{i\varphi} U_{desired}$



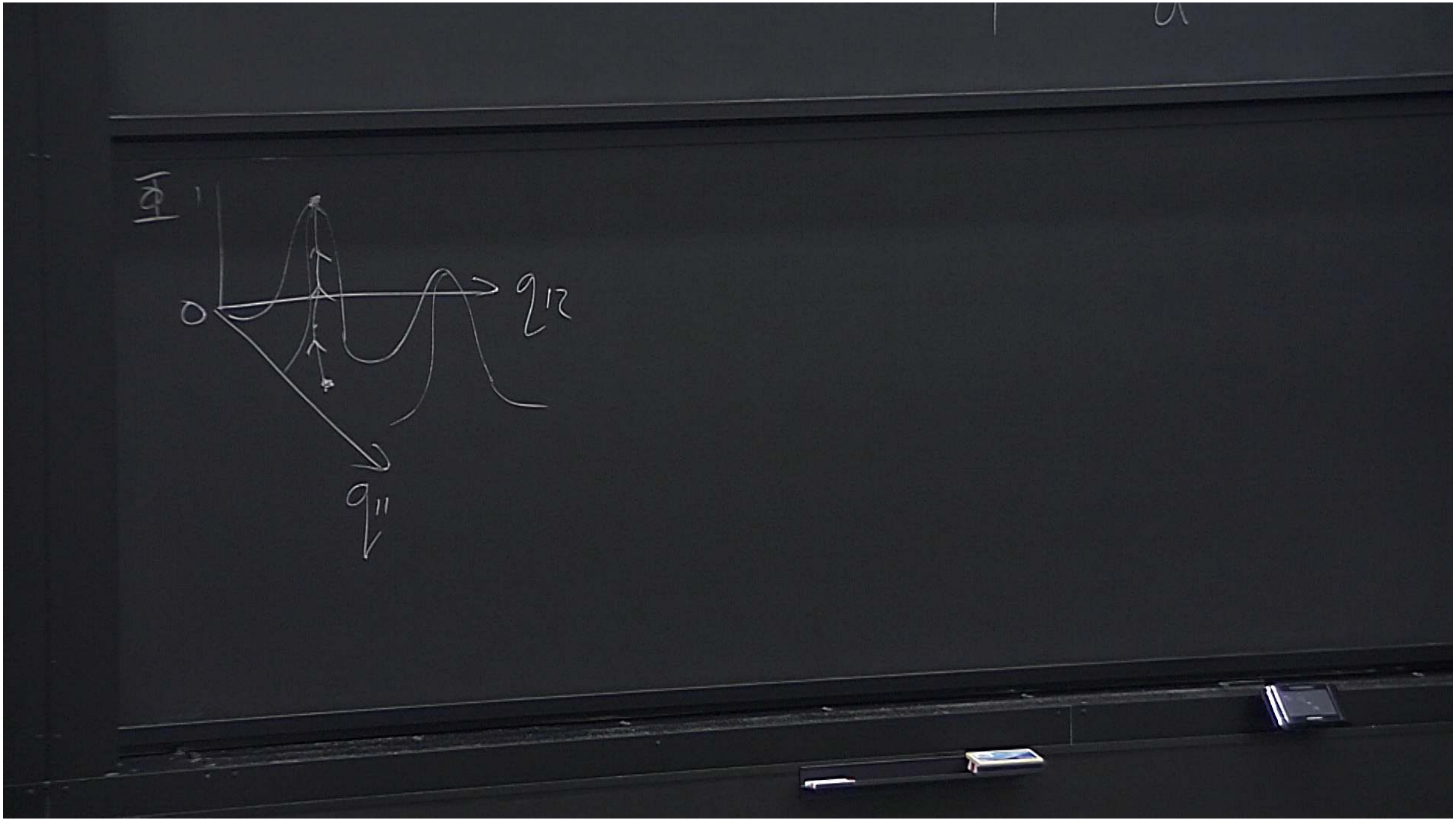
Φ
0
q^{12}
q^{11}



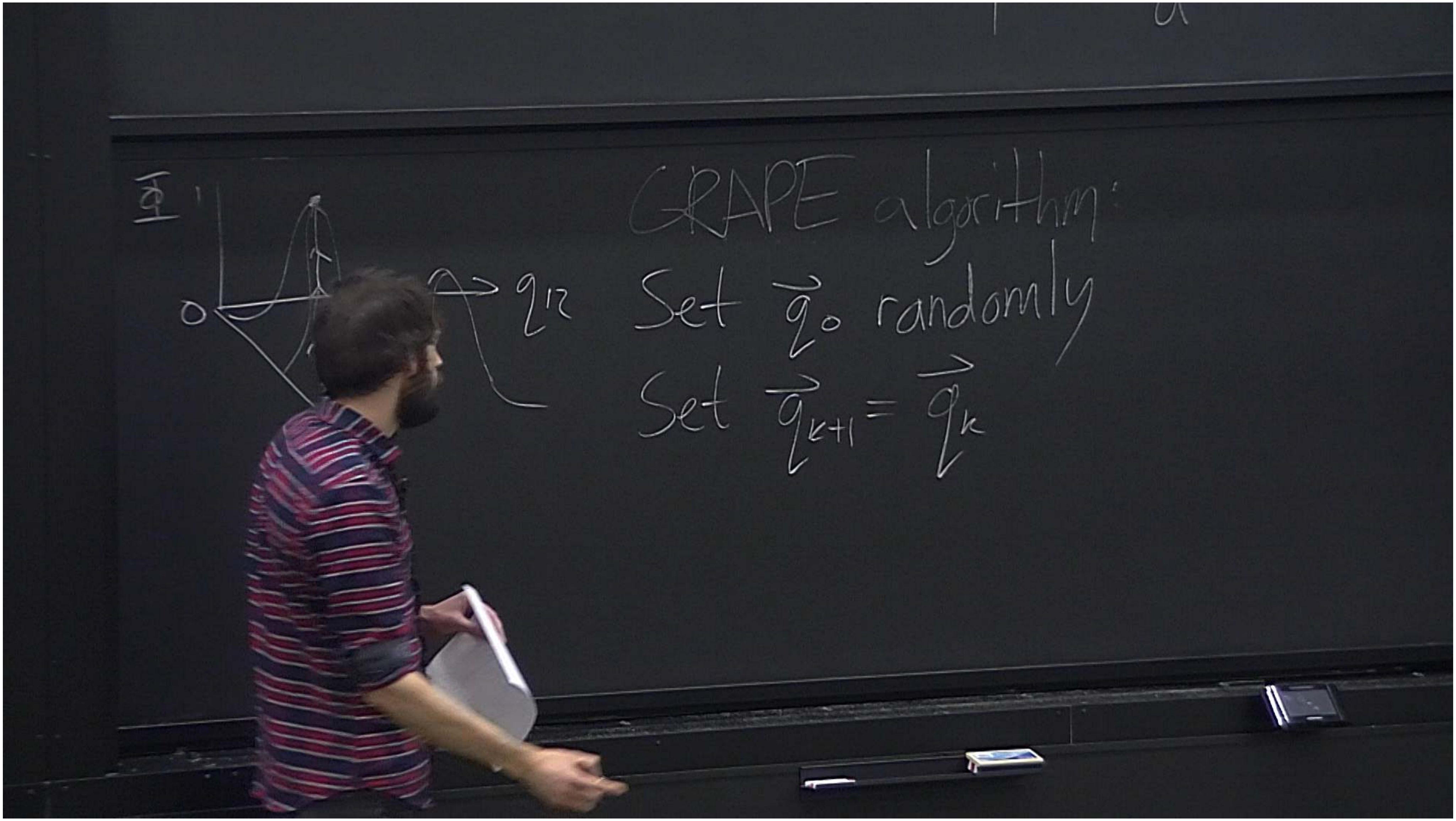
Φ
0
q12
q11

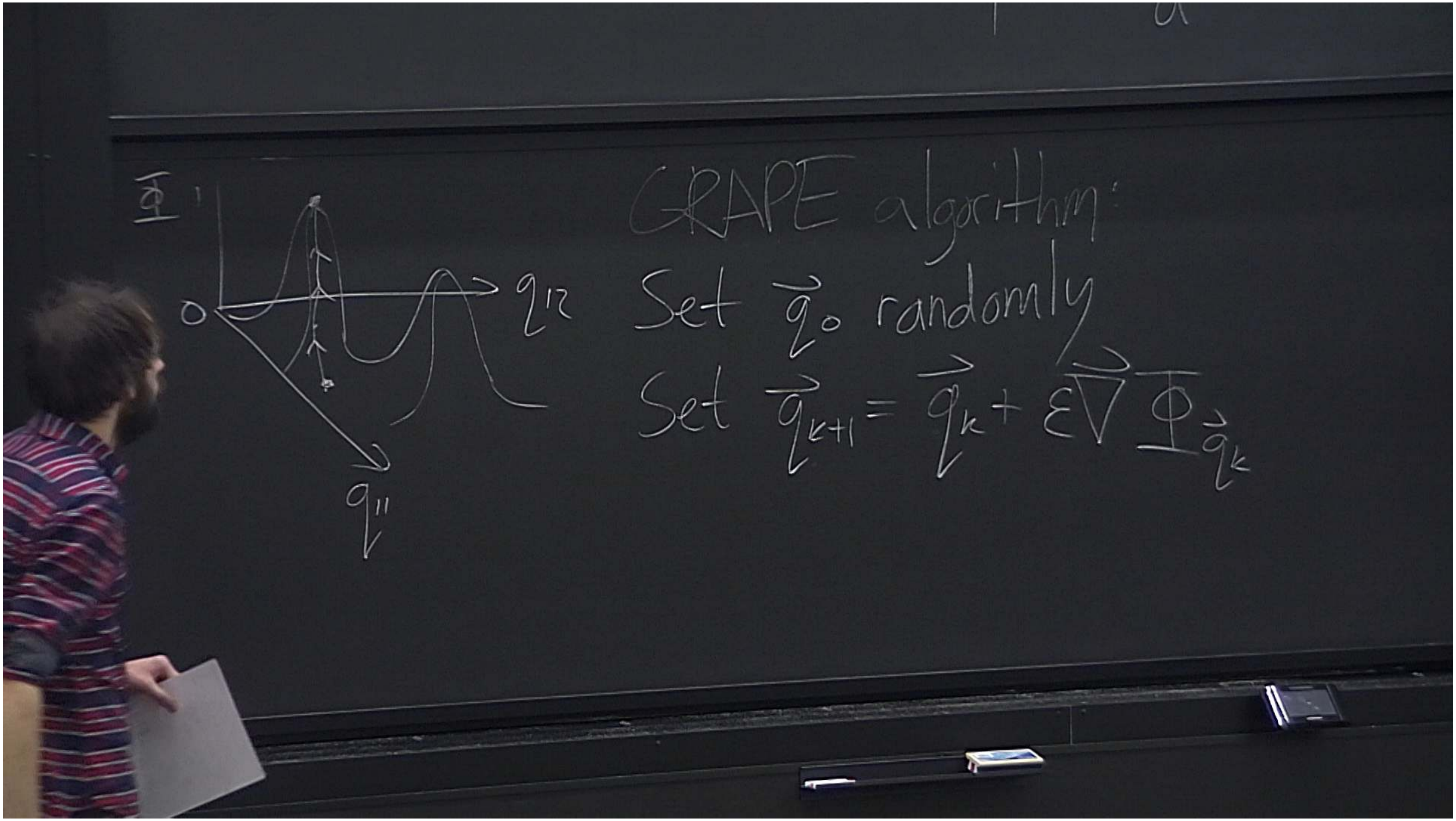


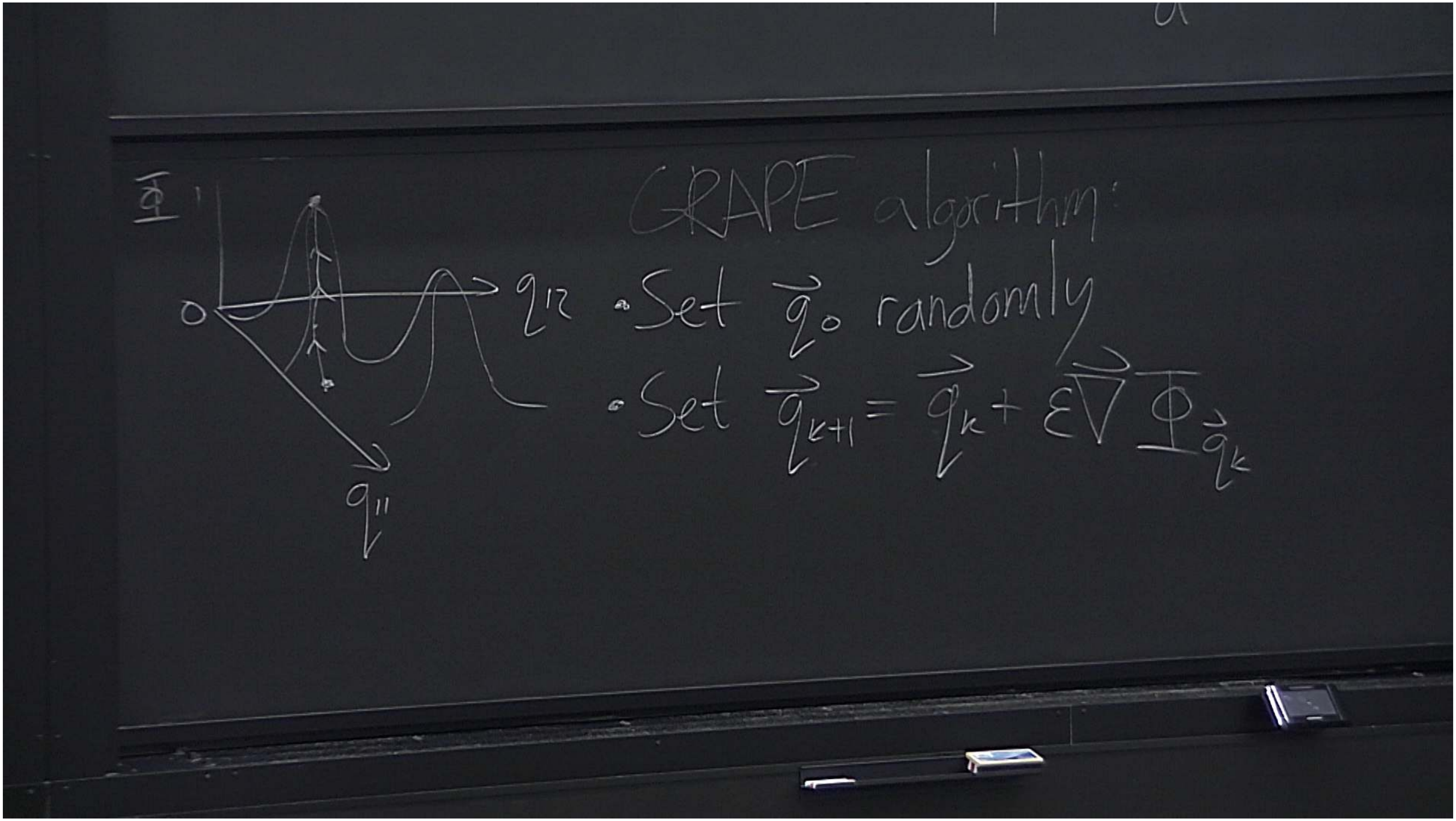
Φ
1
O
q^{12}
q^{11}



Φ
0
q^{12}
q^{11}


GRAPE algorithm:
Set $\vec{q}_0$ randomly
Set $\vec{q}_{k+1} = \vec{q}_k$
Φ
q_{12}
0


GRAPE algorithm:
Set $\vec{q}_0$ randomly
Set $\vec{q}_{k+1} = \vec{q}_k + \varepsilon \vec{\nabla} \Phi_{\vec{q}_k}$
Φ
0
q^{12}
q^{11}


GRAPE algorithm:
• Set $\vec{q}_0$ randomly
• Set $\vec{q}_{k+1} = \vec{q}_k + \varepsilon \vec{\nabla} \Phi_{\vec{q}_k}$
Φ
0
q^{12}
q^{11}

$$\vec{\nabla}\Phi = \left(\frac{\partial \Phi}{\partial q_{ml}}\right)_{m=1,\ l=1}^{M \quad L}$$

For any m, l we can write

Sol'n Set $\alpha(t) = \left(\frac{\pi}{T}, 0\right)$ on $t \in [0, T]$

$$\Phi(\vec{q}) = \frac{|\mathrm{Tr}\, U_d^{\dagger} U(\vec{q})|^2}{d^2} = \frac{1}{d^2} \Big| \mathrm{Tr}\, \underbrace{U_d^{\dagger} U_M U_{M-1} + \cdots + U_{m+1}}_{P_m^{\dagger}} \underbrace{U_m + \cdots + U_2 U_1}_{X_m} \Big|^2$$

$$= \frac{1}{d^2} \left(\mathrm{Tr}\, P_m^{\dagger} X_m\right) \left(\mathrm{Tr}\, X_m^{\dagger} P_m\right)$$

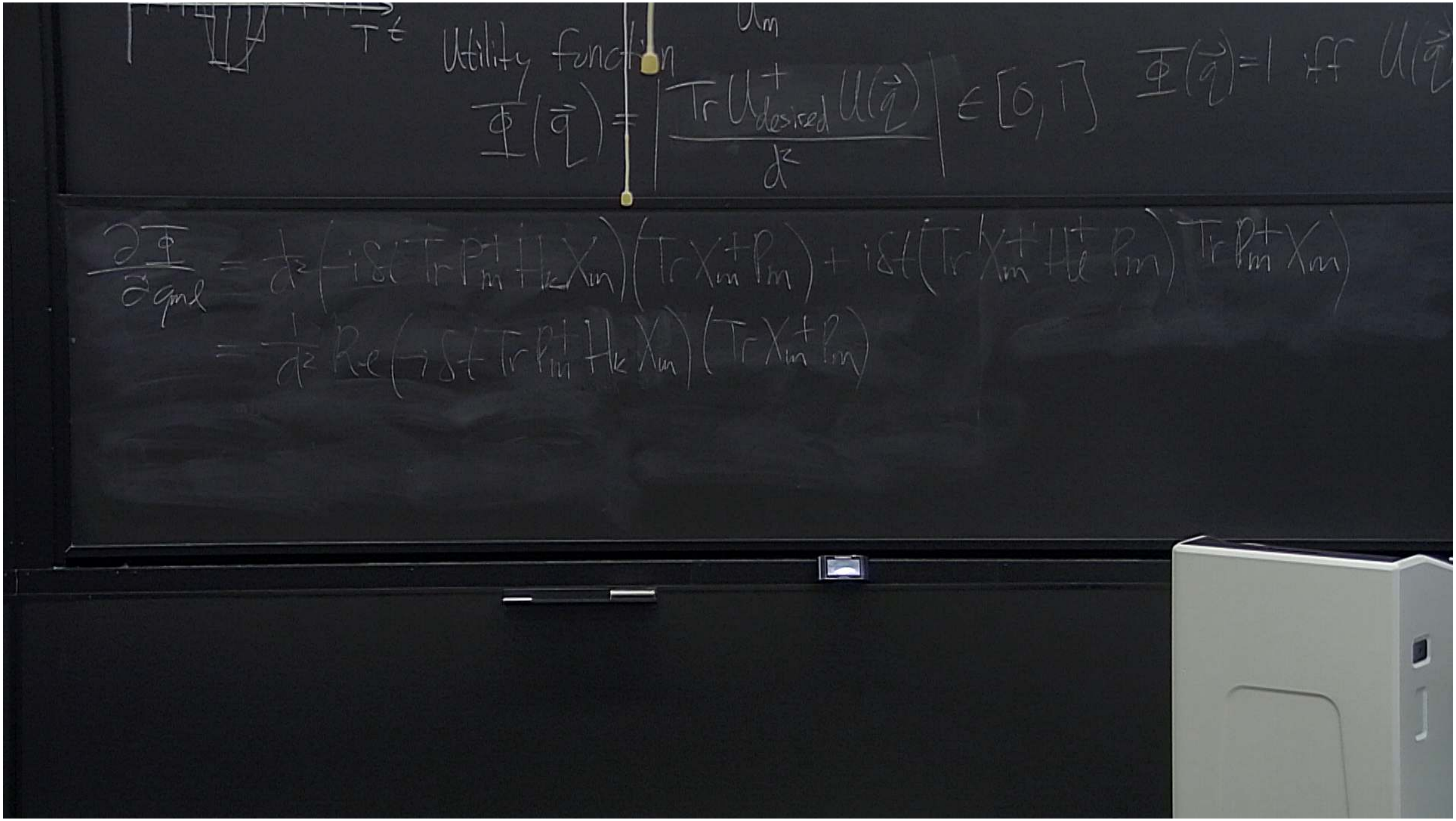
$$\frac{\partial P_m}{\partial q_{m\ell}} = 0 \qquad \frac{\partial X_m}{\partial q_{m\ell}} = \frac{\partial U_m}{\partial q_{m\ell}} U_{m-1} \cdots U_1$$

$\underline{\text{Sol'n}}$ Set $\alpha(t) = \left(\frac{\pi}{T}, 0\right)$ on $t \in [0, T]$

$$\Phi(\vec{q}) = \frac{|\mathrm{Tr}\, U_d^\dagger U(\vec{q})|^2}{d^2} = \frac{1}{d^2}\Big|\mathrm{Tr}\, \underbrace{U_d^\dagger U_M U_{M-1} \cdots U_{m+1}}_{P_m^\dagger} \underbrace{U_m \cdots U_2 U_1}_{X_m}\Big|^2$$

$$= \frac{1}{d^2}\left(\mathrm{Tr}\, P_m^\dagger X_m\right)\left(\mathrm{Tr}\, X_m^\dagger P_m\right)$$

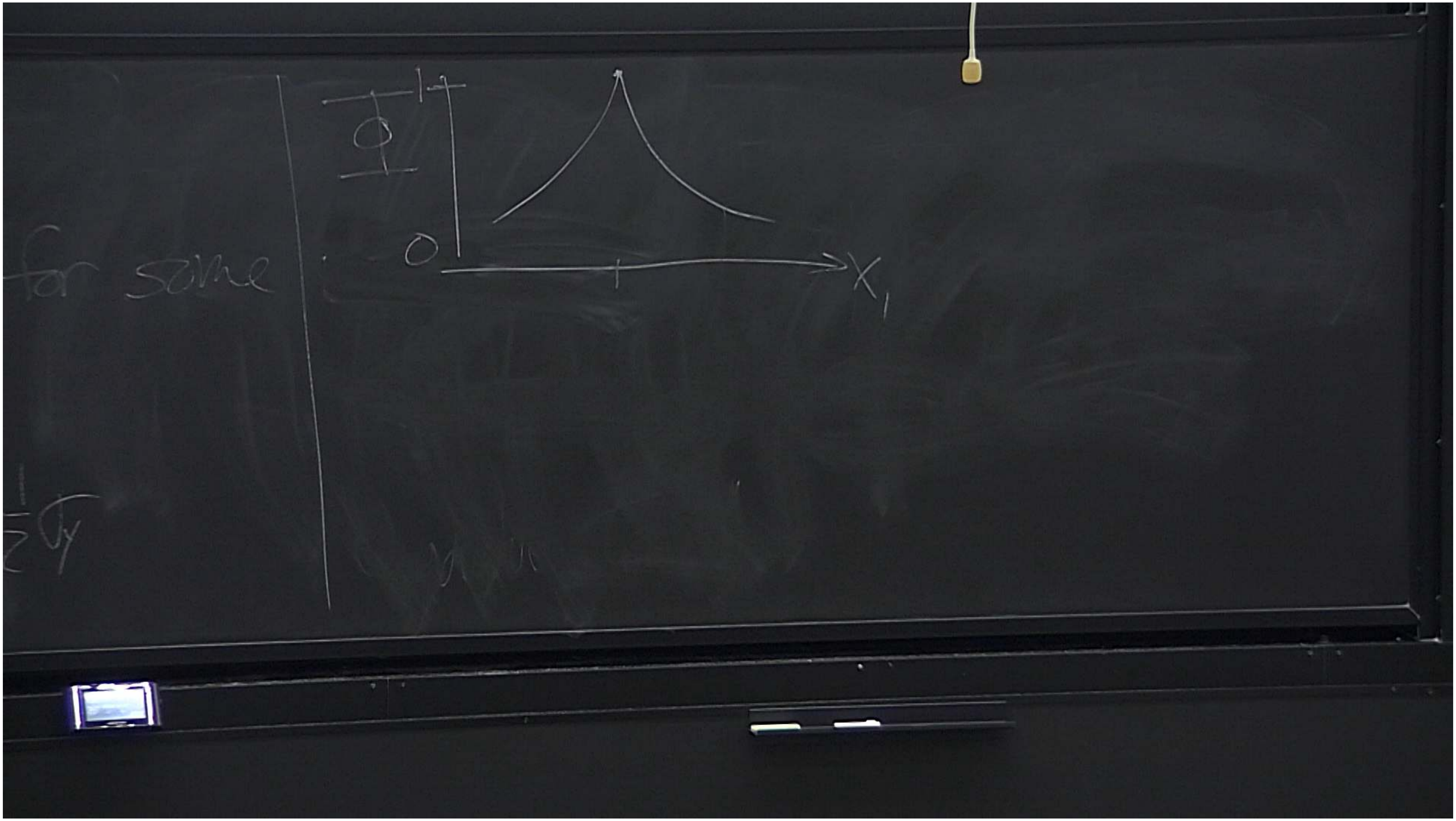
$$\frac{\partial P_m}{\partial q_{ml}} = 0 \qquad \frac{\partial X_m}{\partial q_{ml}} = \frac{\partial U_m}{\partial q_{ml}} U_{m-1} \cdots U_1 = -i\delta t\, H_l U_m U_{m-1} \cdots U_1 = \underline{-i\delta t\, H_l X_m}$$

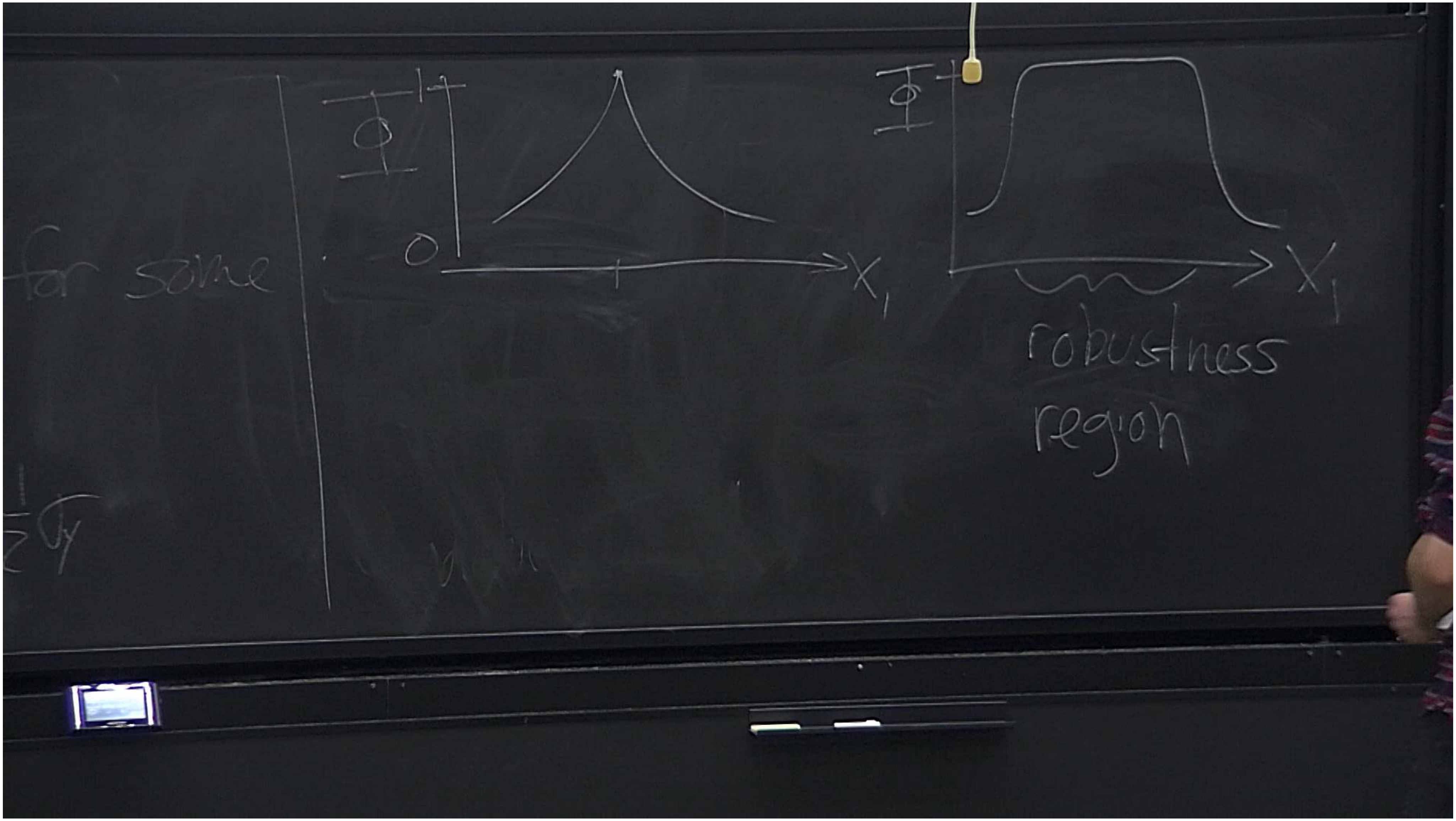

U_m
Utility function
$\Phi(\vec{q}) = \left| \frac{\mathrm{Tr}\, U^\dagger_{desired} U(\vec{q})}{d^2} \right| \in [0,1]$
$\Phi(\vec{q}) = 1$ iff $U(\vec{q}$
$\frac{\partial \Phi}{\partial q_{ml}} = \frac{1}{d^2}\left(-i\delta t\, \mathrm{Tr}\, P_m^\dagger H_k X_m\right)\left(\mathrm{Tr}\, X_m^\dagger P_m\right) + i\delta t\left(\mathrm{Tr}\, X_m^\dagger H_l^\dagger P_m\right) \mathrm{Tr}\, P_m^\dagger X_m)$
$= \frac{1}{d^2} \mathrm{Re}\left(-i\delta t\, \mathrm{Tr}\, P_m^\dagger H_k X_m\right)\left(\mathrm{Tr}\, X_m^\dagger P_m\right)$

Robustness of Control

Suppose $H_\ell = H_\ell(\vec{x})$ $\ell = 0, \dots, L$ for some vector of parameters $\vec{x}$

Ex $\vec{x} = (\delta\omega)$ $H_0 = \delta\omega \frac{\sigma_z}{2}$ $H_1 = \frac{1}{2}\sigma_x$ $H_2 = \frac{1}{2}\sigma_y$

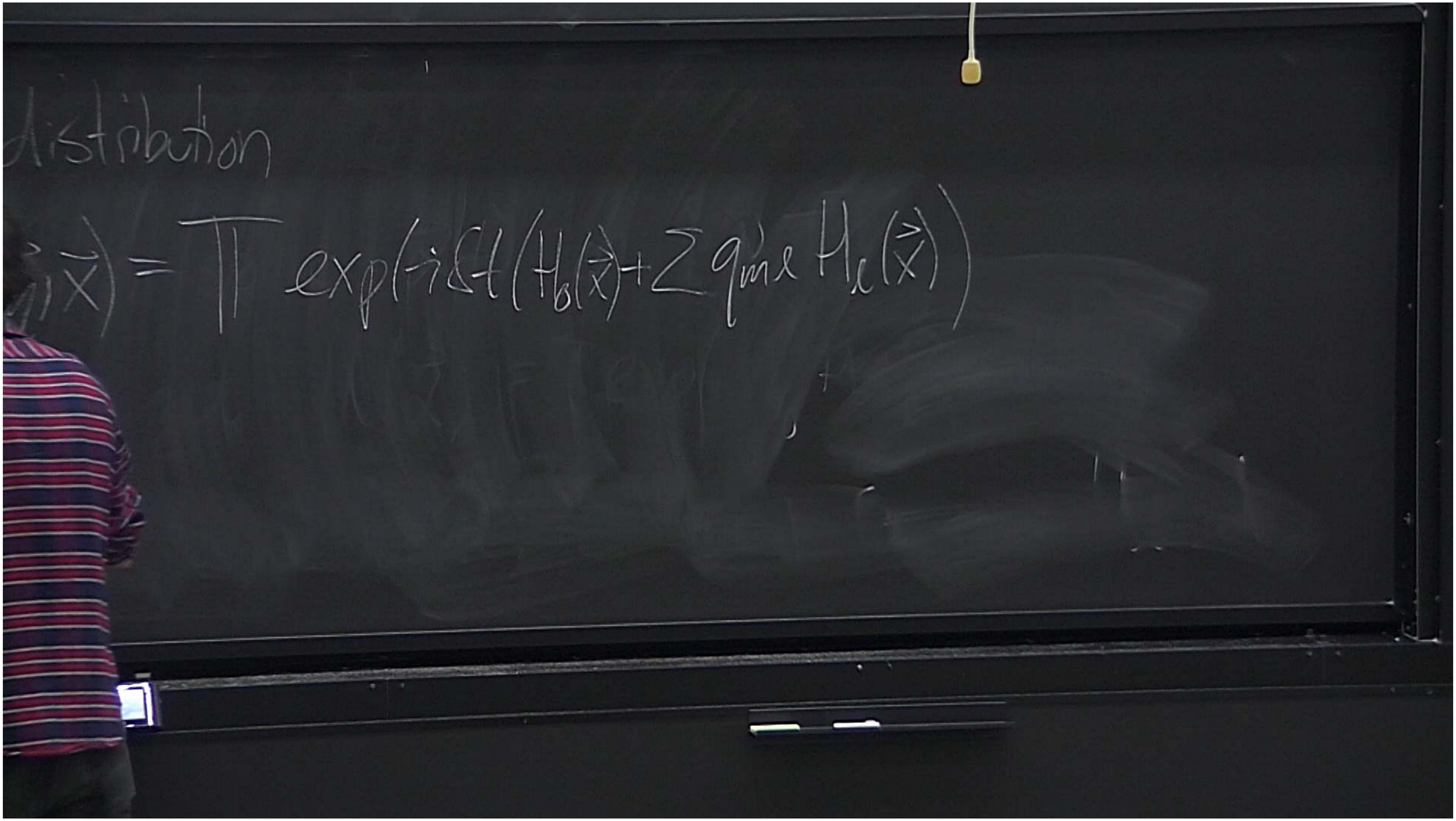

for some
0
1
x1


for some
Φ
0
x_1
robustness
region

Encode our belief about the physical distribution of $\vec{x}$ as $\Pr(\vec{x})$.

$$U(\vec{q},\vec{x}) = \mathbb{T}\, \exp$$

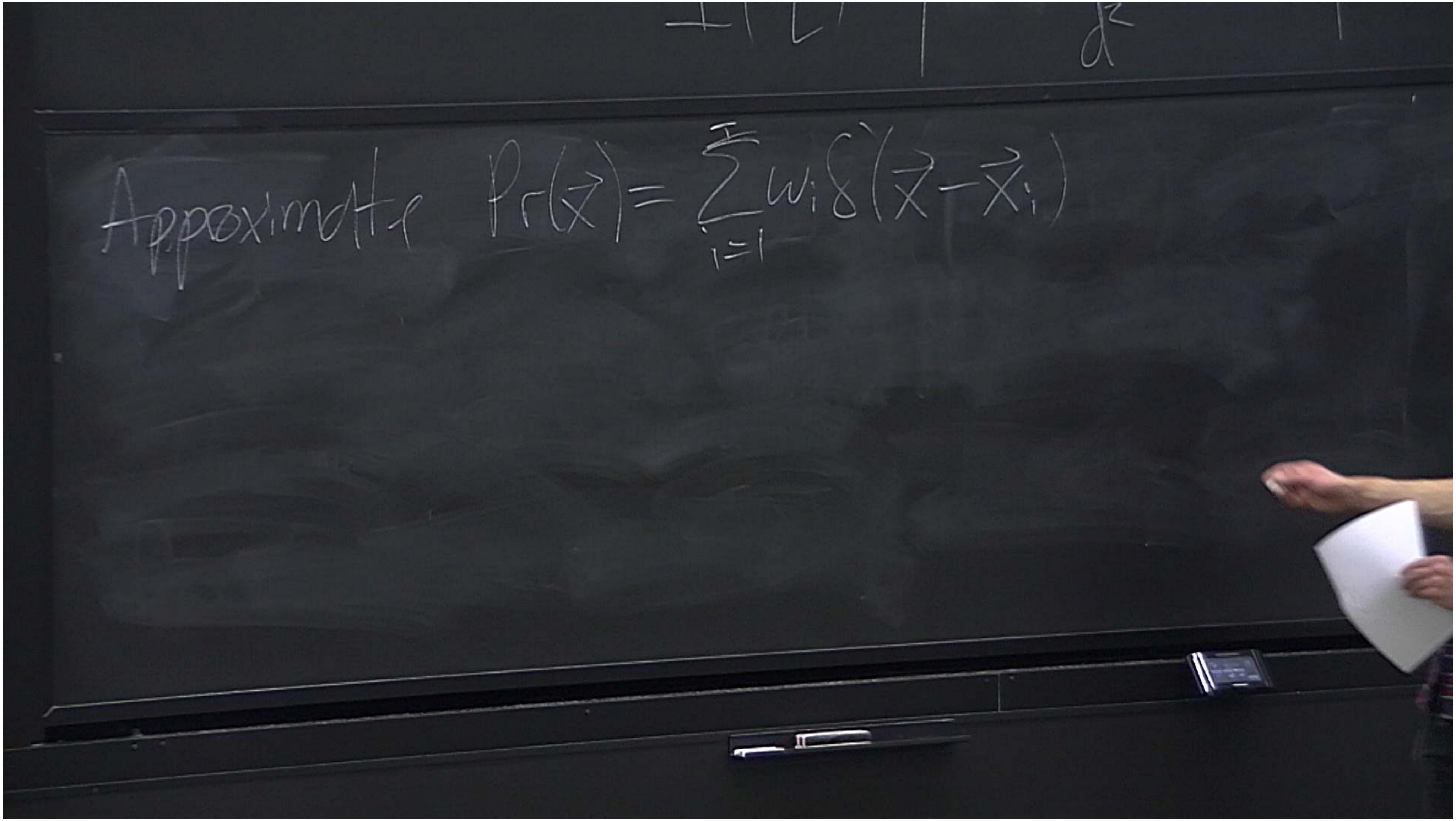
Set $\Phi(\vec{q},\vec{x}) = \dfrac{\left|\mathrm{Tr}\, U_d^{\dagger} U(\vec{q},\vec{x})\right|}{d^2}$


distribution
$\vec{x}) = \prod \exp(-i\delta t(H_0(\vec{x}) + \sum q_m^i \ell H_\ell(\vec{x})))$

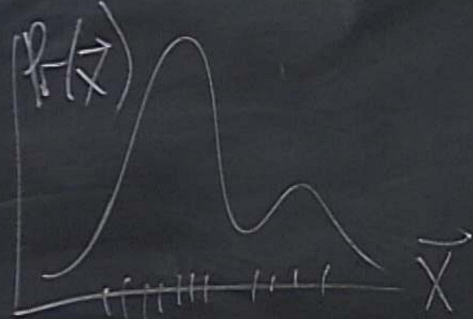
sical distribution

$$U(\vec{q}, \vec{x}) = \prod \exp\left(-i\delta t\left(H_0(\vec{x}) + \sum q_{m\ell} H_\ell(\vec{x})\right)\right)$$

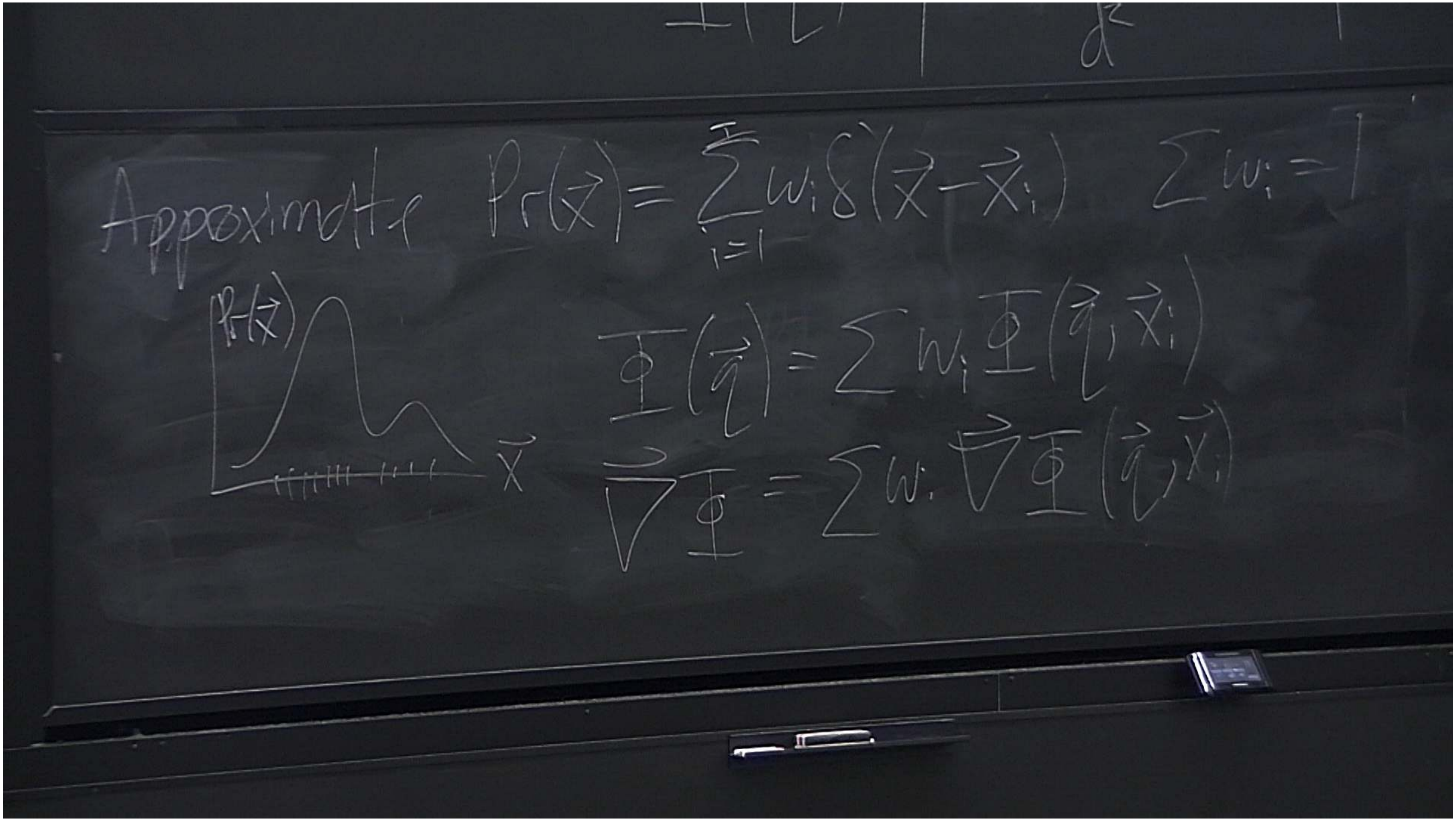
Set $\Phi'(\vec{q}) = \mathbb{E}_{\vec{x}}\left[\Phi(\vec{q}, \vec{x})\right]$


Approximate $Pr(\vec{x}) = \sum_{i=1}^{I} w_i \delta(\vec{x} - \vec{x}_i)$

Approximate $\Pr(\vec{x}) = \sum_{i=1}^{I} w_i \delta(\vec{x}-\vec{x}_i)$ $\quad \sum w_i = 1$



$$\Phi(\vec{q}) = \sum w_i \Phi(\vec{q}, \vec{x}_i)$$


Approximate $Pr(\vec{x}) = \sum_{i=1}^{I} w_i \delta(\vec{x} - \vec{x}_i)$ $\sum w_i = 1$
$Pr(\vec{x})$
$\vec{x}$
$\Phi(\vec{q}) = \sum w_i \Phi(\vec{q}, \vec{x}_i)$
$\vec{\nabla}\Phi = \sum w_i \vec{\nabla}\Phi(\vec{q}, \vec{x}_i)$