Title: Loop-Corrected Virasoro Symmetry of 4D Quantum Gravity

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Abstract: <p>Recently a boundary energy-momentum tensor Tzz has been constructed from the soft graviton operator for any 4D quantum theory of gravity in asymptotically flat space. Up to an "anomaly" which is one-loop exact, Tzz generates a Virasoro action on the 2D celestial sphere at null infinity. Here we show by explicit construction that the effects of the IR divergent part of the anomaly can be eliminated by a one-loop renormalization that shifts Tzz.</p>

Loop-corrected Virasoro Symmetry of 4D Quantum Gravity

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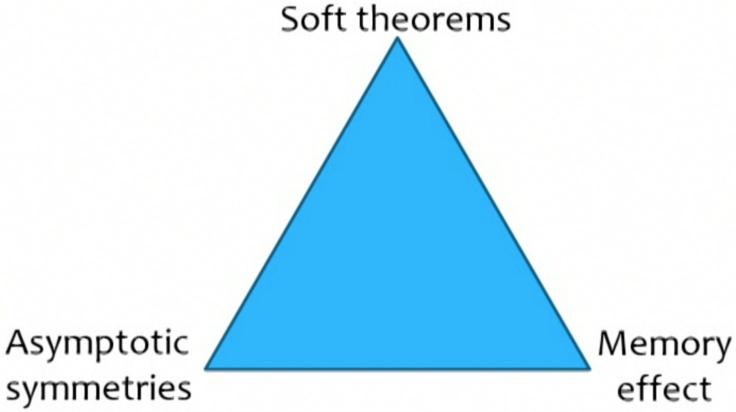
03/28/17

Perimeter Institute Seminar

Collaboration

* Main focus on 1701.00496, done in collaboration with D. Kapec, A.-M. Raclariu, & A. Strominger
* Focuses on the soft theorem/asymptotic symmetry side of the triangle that Strominger introduced



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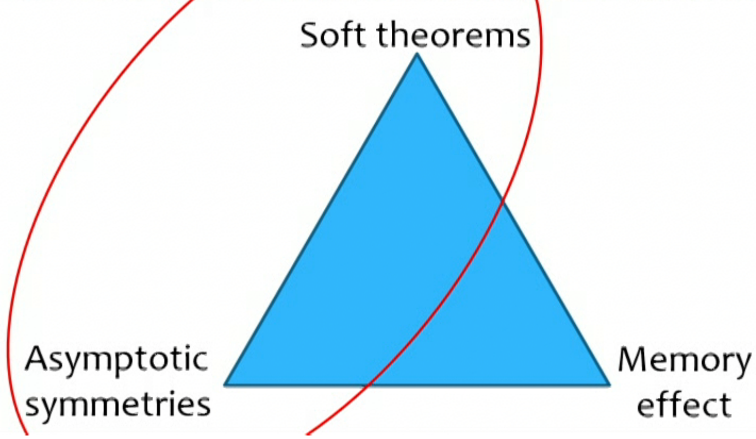


History

* Relevant papers
  * 1404.4091 (F. Cachazo & A. Strominger)
  * 1406.3312 (D. Kapec, V. Lysov, S. Pasterski, & A. Strominger)
  * 1609.00282 (D. Kapec, P. Mitra, A.-M. Raclariu, & A. Strominger)

Goal of presentation

* **Subleading soft graviton theorem** – Conjectured and shown to be true at tree-level by F. Cachazo and A. Strominger in 2014.
* **2D stress tensor for 4D gravity** – Correlation functions in 4D Minkowski gravity can be recast as correlators of a 2D CFT living on a sphere at null infinity.
* We would like to derive the **one-loop correction** to the **2D stress tensor** using the **one-loop correction** to the **subleading soft graviton theorem.**

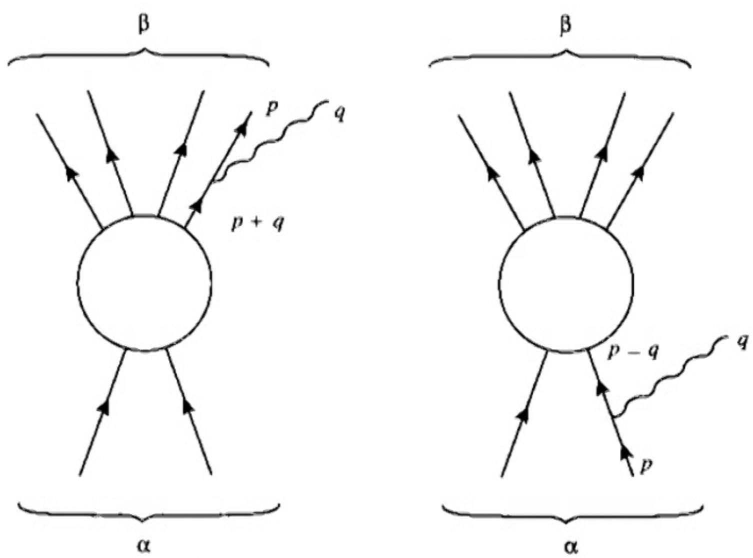
Outline of presentation

* Review tree-level leading and subleading soft graviton theorem
* Construct tree-level 2D stress tensor at null infinity
* Review one-loop IR-divergent correction to subleading soft graviton theorem
* Derive one-loop correction to 2D stress tensor

Leading soft graviton theorem

* We look at massless gravity since only massless particles reach $\mathcal{I}^+$.
* Leading soft graviton theorem relates the matrix elements of a Feynman diagram with an external soft graviton insertion to that of the same diagram without an external soft graviton.

Leading soft graviton theorem



From Weinberg, *The Quantum Theory of Fields, Vol. 1*

* Two ways to insert a soft graviton
* In the diagrams we take $q \rightarrow 0$.

Leading soft graviton theorem

* In equation form, assuming all particles are outgoing for simplicity,

$$\lim_{q\to 0}\mathcal{M}^{\pm}_{n+1}(q)=\lim_{q\to 0}\frac{\kappa}{2}\sum_{k=1}^{n}\frac{p_k^\mu p_k^\nu \varepsilon^{\pm}_{\mu\nu}(q)}{p_k\cdot q}\mathcal{M}_n$$

$$\kappa\equiv\sqrt{32\pi G}$$

* This is exact to all loop orders.

Subleading term

* Leading soft factor is O(1/q). What about O(1) term?
* The subleading soft graviton theorem was conjectured by Cachazo and Strominger in 2014.
* Shown to be universal for tree-level scattering

Soft graviton theorem

* At tree-level, we have

$$\lim_{q\to 0}\mathcal{M}^{\pm}_{n+1}(q)=\left[S_n^{(0)\pm}+S_n^{(1)\pm}+\mathcal{O}(q)\right]\mathcal{M}_n$$

$$S_n^{(0)\pm}=\frac{\kappa}{2}\sum_{k=1}^{n}\frac{p_k^\mu p_k^\nu \varepsilon^{\pm}_{\mu\nu}(q)}{p_k\cdot q},\quad S_n^{(1)\pm}=-\frac{i\kappa}{2}\sum_{k=1}^{n}\frac{\varepsilon^{\pm}_{\mu\nu}(q)p_k^\mu q_\lambda}{p_k\cdot q}\mathcal{J}_k^{\lambda\nu}$$

* The subleading term has a one-loop exact correction.
* Gauge invariance implies momentum and angular momentum conservation.

Asymptotic symmetries

* Symmetries are one of the most useful tools in theoretical physics.
* One way to think of certain symmetries is in terms of a Ward identity, i.e. their charges commute with the S-matrix.
* Asymptotic symmetries are symmetries that act on the physical states in a nontrivial manner.

Asymptotic symmetries

* In many cases, asymptotic symmetries are local in spacetime coordinates.
* Sometimes called "large gauge symmetries" to distinguish from trivial gauge symmetries
* This definition of large gauge symmetries allows for topologically trivial symmetries as well.
* Large gauge symmetry physical iff it gives rise to nontrivial Ward identity

Remarkable equivalence

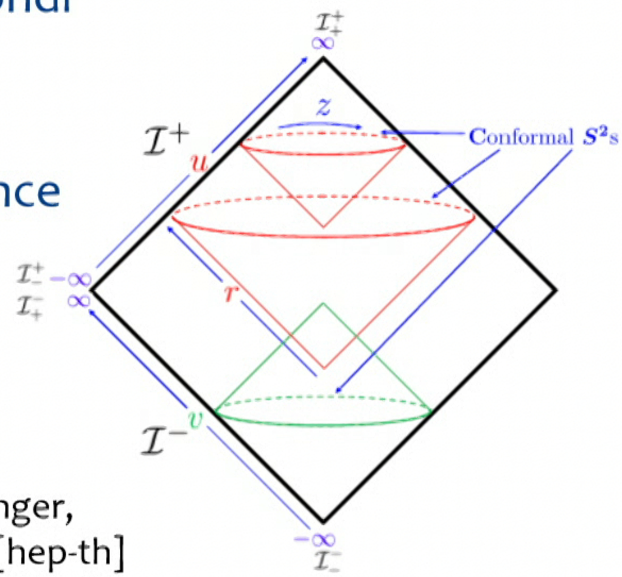
* Certainly some asymptotic symmetries are physical!
* Strong link established between many asymptotic symmetries and soft theorems – soft theorems are just Ward identities for asymptotic symmetries
* Examples:
  * Leading soft photon theorem = Ward identity for asymptotic symmetry in QED (TH, Mitra, Porfyriadis, Strominger)
  * Leading soft graviton theorem = Ward identity for supertranslations (TH, Mitra, Lysov, Strominger)

Remarkable equivalence

* This equivalence is remarkable!
  * Soft theorems were first studied in QED in 1937 by Bloch and Nordsieck, later by Low et al in 1958, then later extended by Weinberg to gravity in 1965.
  * Asymptotic symmetries appeared in the work of BMS in 1962, where they deduced the symmetry group for asymptotically flat spacetimes.
* One uses perturbative Feynman diagrams, the other uses asymptotic structures at null infinity.

Switching coordinate systems

* Easier to work in retarded Bondi coordinates
* We choose our asymptotic boundary to be $\mathcal{I}^+$ and $\mathcal{I}^-$. Hence we only focus on massless particles.



From A. Strominger, arXiv:1312.2229 [hep-th]

Correlators on asymptotic S^2

- We can think of any 4D scattering process as a 2D correlation function on the asymptotic S^2 at null infinity.

$$\mathcal{M}_n = \langle \mathcal{O}_1(\omega_1, z_1, \bar{z}_1) \cdots \mathcal{O}_n(\omega_n, z_n, \bar{z}_n) \rangle$$

- 4D Lorentz group SL(2,C) is manifested as the global conformal group on the S^2.

$$z \to z' = \frac{az+b}{cz+d}, \quad ad - bc = 1$$

Superrotations

* SL(2,C) is the globally-defined symmetry group acting on correlators on S^2.
* Can extend it to include the full Virasoro symmetry if we allow for local poles (Banks; Barnich, Troessaert)
* This extended symmetry is generated by the "superrotations."
* Tree-level subleading soft graviton theorem was shown to imply Ward identity from superrotations.

CFT_2 on asymptotic S^2

* Virasoro symmetry on asymptotic S^2 suggests 4D scattering amplitudes can be thought of as CFT_2 correlators
* Hints at a holographic formulation of flat space, the original motivation for this subject
* Would like to write down a stress tensor for this CFT_2
* Note this CFT may not be a unitary CFT_2

Soft graviton insertion

* We can recast our soft theorem in terms of an operator insertion.
* In Bondi coordinates, the tree-level leading and subleading soft factors are

$$S_n^{(0)-} = -\frac{\kappa}{2\omega}(1+z\bar{z})\sum_{k=1}^{n}\frac{\omega_k(z-z_k)}{(\bar{z}-\bar{z}_k)(1+z_k\bar{z}_k)}$$

$$S_n^{(1)-} = \frac{\kappa}{2}\sum_{k=1}^{n}\frac{(z-z_k)^2}{\bar{z}-\bar{z}_k}\left[\frac{2h_k}{z-z_k} - \Gamma^{z_k}_{z_k z_k}h_k - \partial_{z_k} + |s_k|\Omega_{z_k}\right]$$

Conformal weights

Spin connection

Soft graviton insertion

* Define the operators

$$N_{zz}^{(0)} = -\frac{\kappa}{8\pi}\hat{\varepsilon}_{zz} \lim_{\omega\to 0}\left[\omega a_{+}^{\text{out}}(\omega\hat{x}) + \omega a_{-}^{\text{out}}(\omega\hat{x})^{\dagger}\right]$$

$$N_{zz}^{(1)} = \frac{i\kappa}{8\pi}\hat{\varepsilon}_{zz} \lim_{\omega\to 0}\left(1+\omega\partial_{\omega}\right)\left[a_{+}^{\text{out}}(\omega\hat{x}) - a_{-}^{\text{out}}(\omega\hat{x})^{\dagger}\right]$$

* They satisfy at tree-level

$$\langle\,\text{out}\,|N_{\bar{z}\bar{z}}^{(0)}\mathcal{S}|\,\text{in}\,\rangle = -\frac{\kappa}{8\pi}\hat{\varepsilon}_{\bar{z}\bar{z}} \lim_{\omega\to 0}\omega S_{n}^{(0)-}\langle\,\text{out}\,|\mathcal{S}|\,\text{in}\,\rangle$$

$$\langle\,\text{out}\,|N_{\bar{z}\bar{z}}^{(1)}\mathcal{S}|\,\text{in}\,\rangle = \frac{i\kappa}{8\pi}\hat{\varepsilon}_{\bar{z}\bar{z}} S_{n}^{(1)-}\langle\,\text{out}\,|\mathcal{S}|\,\text{in}\,\rangle$$

2D stress tensor

* Define

$$T_{zz} \equiv \frac{4i}{\kappa^2} \int d^2 w \frac{\gamma^{w\bar{w}}}{z-w} D_w^3 N_{\bar{w}\bar{w}}^{(1)}$$

* It follows at tree-level

$$\langle \text{out}|T_{zz}\mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n} \left[\frac{h_k}{(z-z_k)^2} + \frac{h_k}{z-z_k}\Gamma^{z_k}_{z_k z_k} + \frac{1}{z-z_k}\left(\partial_{z_k} - |s_k|\Omega_{z_k}\right)\right] \langle \text{out}|\mathcal{S}|\text{in}\rangle$$

which is the precise form of a CFT stress tensor correlator on a curved background.

Soft charge

* Using the stress tensor we can construct the charge via the contour integral

$$T_{\mathcal{C}}[Y] = \oint_{\mathcal{C}} \frac{dz}{2\pi i} Y^z T_{zz}$$

* This generates local conformal transformations

$$\langle T_{\mathcal{C}}[Y] \mathcal{O}_1 \cdots \mathcal{O}_n \rangle = \sum_{k \in \mathcal{C}} \left[D_{z_k} Y^{z_k} h_{z_k} + Y^{z_k} \left(\partial_{z_k} - |s_k| \Omega_{z_k} \right) \right] \langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle$$

Moving beyond tree-level

* There are no corrections to the leading soft factor, but there is an IR-divergent one-loop exact correction to the subleading soft factor.
* Three options
  * The stress tensor becomes anomalous.
  * The stress tensor needs to be renormalized.
  * It doesn't make sense to work with IR-divergent quantities.
* We give evidence in support of option 2.

One-loop scattering amplitude

* The loop expansion of the scattering amplitude is

$$\mathcal{M}_n = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} \kappa^{2\ell}$$

* The one-loop IR-divergence is given by (Bern, Davies, Nohle)

$$\mathcal{M}_n^{(1)}\Big|_{\text{div}} = \frac{\sigma_n}{\epsilon} \mathcal{M}_n^{(0)}$$

where

$$\sigma_n \equiv -\frac{1}{4(4\pi)^2} \sum_{i,j=1}^{n} (p_i \cdot p_j) \log \frac{\mu^2}{-2p_i \cdot p_j}$$

One-loop correction to soft theorem

* We may take the tree-level soft limit on both sides to get

$$\mathcal{M}^{(1)-}_{n+1}\Big|_{\text{div}} \xrightarrow{q\to 0} \left(S^{(0)-}_n + S^{(1)-}_n\right)\mathcal{M}^{(1)}_n\Big|_{\text{div}} + \frac{\sigma'_{n+1}}{\epsilon} S^{(0)-}_n \mathcal{M}^{(0)}_n - \frac{1}{\epsilon}\left(S^{(1)-}_n \sigma_n\right)\mathcal{M}^{(0)}_n$$

where

$$\sigma'_{n+1} \equiv -\frac{1}{2(4\pi)^2}\sum_{i=1}^{n}(p_i \cdot q)\log\frac{\mu^2}{-2p_i \cdot q}$$

One-loop correction to T_{zz}

* Recall the tree-level subleading soft theorem

$$N^{(1)}_{\bar{z}\bar{z}} = \frac{i\kappa}{8\pi}\hat{\varepsilon}_{\bar{z}\bar{z}} \lim_{\omega\to 0} (1+\omega\partial_\omega)\left[a^{\text{out}}_{-}(\omega\hat{x}) - a^{\text{out}}_{+}(\omega\hat{x})^{\dagger}\right]$$

$$\langle \text{out} | N^{(1)}_{\bar{z}\bar{z}} \mathcal{S} | \text{in} \rangle = \frac{i\kappa}{8\pi}\hat{\varepsilon}_{\bar{z}\bar{z}} S^{(1)-}_{n} \langle \text{out} | \mathcal{S} | \text{in} \rangle$$

* The IR-divergent one-loop correction:

$$\langle \text{out} | N^{(1)}_{\bar{z}\bar{z}} \mathcal{S} | \text{in} \rangle \Big|_{\text{div}} = \frac{i\kappa^3}{8\pi}\hat{\varepsilon}_{\bar{z}\bar{z}} \lim_{\omega\to 0} (1+\omega\partial_\omega)\left(\frac{\sigma'_{n+1}}{\epsilon} S^{(0)-}_{n} - \frac{1}{\epsilon}\left(S^{(1)-}_{n}\sigma_n\right)\right) \langle \text{out} | \mathcal{S} | \text{in} \rangle$$

One-loop correction to T_{zz}

* Recall T_{zz} previously defined:

$$T_{zz} \equiv \frac{4i}{\kappa^2} \int d^2w \frac{\gamma^{w\bar{w}}}{z-w} D_w^3 N_{\bar{w}\bar{w}}^{(1)}$$

* There is now a one-loop correction to the matrix element

$$\langle \text{out}|\Delta T_{zz}\mathcal{S}|\text{in}\rangle$$
$$= -\frac{\kappa}{2\pi\epsilon} \int d^2w \, \frac{\gamma^{w\bar{w}}}{z-w} D_w^3 \left[\hat{\varepsilon}_{\bar{w}\bar{w}} \lim_{\omega\to 0}(1+\omega\partial_\omega)\left(\sigma'_{n+1}S_n^{(0)-} - (S_n^{(1)-}\sigma_n)\right)\right] \langle \text{out}|\mathcal{S}|\text{in}\rangle$$

One-loop correction to T_{zz}

- Surprisingly, we can write the correction term as

$$\Delta T_{zz} = -\frac{2}{\pi\kappa^2\epsilon}\int d^2w\,\frac{\gamma^{w\bar{w}}}{z-w}\left(2N^{(0)}_{ww}D_wN^{(0)}_{\bar{w}\bar{w}} + D_w\left(N^{(0)}_{ww}N^{(0)}_{\bar{w}\bar{w}}\right)\right)$$

- Recall the correction originally involved

$$\sigma'_{n+1} \equiv -\frac{1}{2(4\pi)^2}\sum_{i=1}^{n}(p_i\cdot q)\log\frac{\mu^2}{-2p_i\cdot q}$$

- All logs and dependence on renormalization scale μ dropped out!

Renormalizing T_{zz}

* Correction factor only involves tree-level local operators $N_{ww}^{(0)}$ and its complex conjugate
* Define

$$\tilde{T}_{zz} = T_{zz} - \Delta T_{zz}$$

* This eliminates the one-loop correction, and we have, assuming no IR-finite correction,

$$\langle \text{out}|\tilde{T}_{zz}\mathcal{S}|\text{in}\rangle = \sum_{k=1}^{n}\left[\frac{h_k}{(z-z_k)^2} + \frac{h_k}{z-z_k}\Gamma^{z_k}_{z_k z_k} + \frac{1}{z-z_k}\left(\partial_{z_k} - |s_k|\Omega_{z_k}\right)\right]\langle \text{out}|\mathcal{S}|\text{in}\rangle$$

Renormalized stress tensor

IR-finite correction?

* We have only focused on the IR-divergent correction.
* There should in principle also be an IR-finite correction to the scattering amplitude:

$$\mathcal{M}_n^{(1)} = \mathcal{M}_n^{(1)}\Big|_{\text{div}} + \mathcal{M}_n^{(1)}\Big|_{\text{fin}}$$

IR-finite correction?

* Including the possible IR-finite correction, the all-loop soft graviton theorem has the form

$$\mathcal{M}_{n+1}^{-} \xrightarrow{q\to 0} \left[S_n^{(0)-} + S_n^{(1)-} + \kappa^2 \left(\frac{\sigma'_{n+1}}{\epsilon} S_n^{(0)-} - \frac{1}{\epsilon} \left(S_n^{(1)-} \sigma_n \right) + \Delta_{\text{fin}} S_n^{(1)-} \right) \right] \mathcal{M}_n$$

* Little is known about the finite correction, and in all explicitly checked cases it is zero. (He, Huang, Wen)

Summary

* Determined the tree-level 2D stress tensor T_{zz} living on the asymptotic S^2
* Computed the one-loop IR-divergent correction to the subleading soft theorem
* Translated this to a one-loop IR-divergent correction to the stress tensor T_{zz}
* Renormalized T_{zz} by subtracting the correction, which can be written as an integral over local operators

Open questions

* Is there an IR-finite one-loop correction? Is it universal?
* Can we work out the problem in the Faddeev-Kulish basis, where everything is IR-finite?
* What about multiple insertions of T_{zz}?

Thank you for listening!