Title: Sub-Planckian Black Holes

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Abstract: <p>The characteristics of black holes smaller than the Planck scale are addressed. These result from a modified metric that reproduces desirable aspects of a variety of disparate models in the sub-Planckian limit, while remaining Schwarzschild in the large mass limit. The self-dual nature of this solution has two interesting features: first, it naturally implies the Generalized Uncertainty Principle. Secondly, this metric exhibits an effective dimensional reduction feature, indicating that the gravitational physics of the sub-Planckian regime is effectively (1+1)-D.</p>

Sub-Planckian Black Holes

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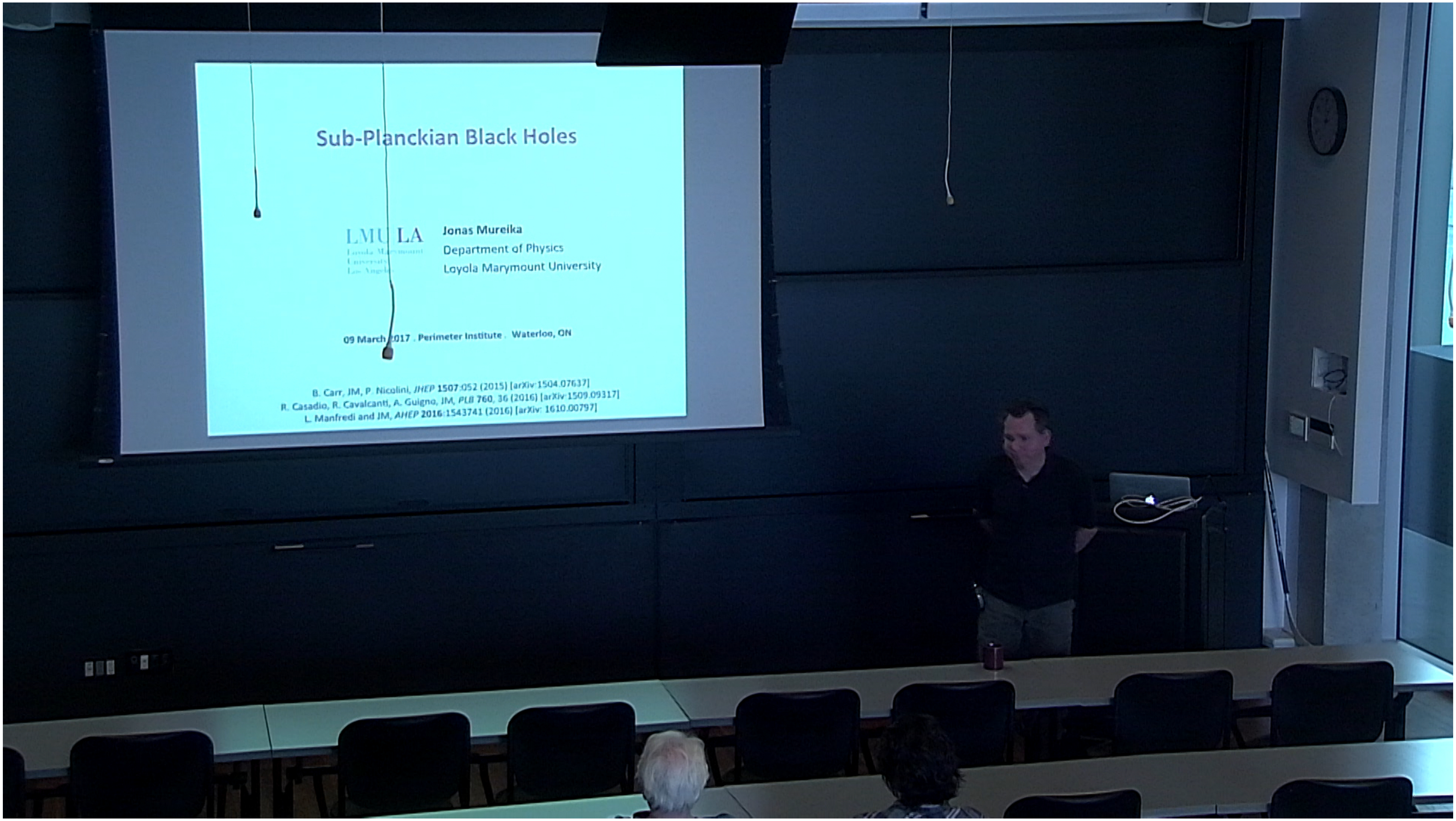
09 March 2017 . Perimeter Institute . Waterloo, ON

B. Carr, JM, P. Nicolini, *JHEP* **1507**:052 (2015) [arXiv:1504.07637]
R. Casadio, R. Cavalcanti, A. Guigno, JM, *PLB* **760**, 36 (2016) [arXiv:1509.09317]
L. Manfredi and JM, *AHEP* **2016:**1543741 (2016) [arXiv: 1610.00797]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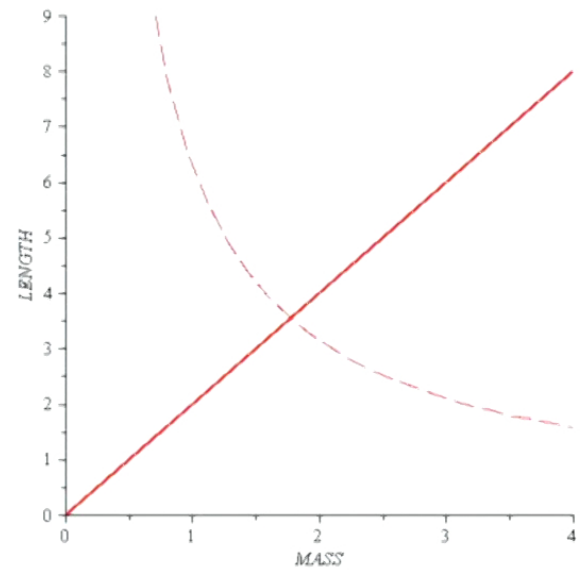
Characteristic Scales of Nature

The large- and short-scale characteristics of Nature are defined by different and (apparently) disconnected theories and length scales



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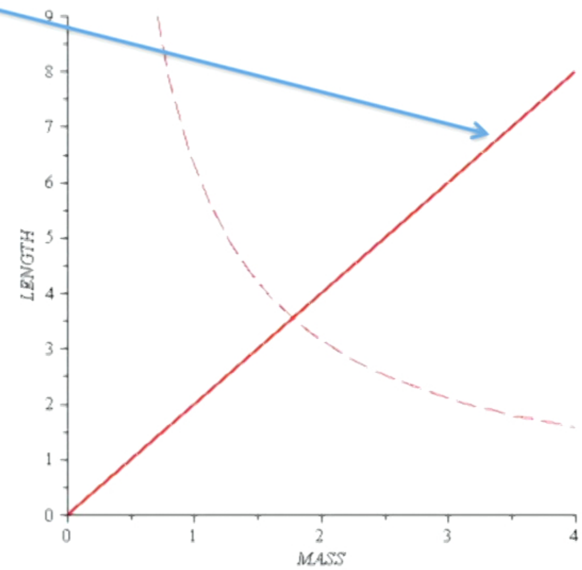
Large: General Relativity

$$r_g = \frac{2GM}{c^2} \longrightarrow M$$



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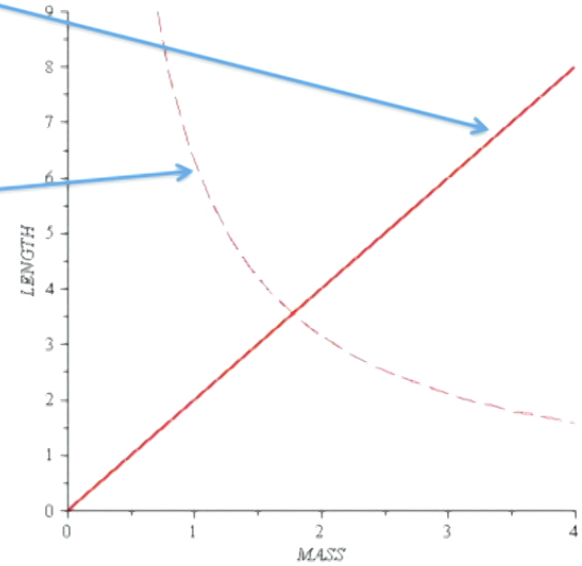
Large: General Relativity

$$r_g = \frac{2GM}{c^2} \longrightarrow M$$

Short: Quantum Mechanics

$$\lambda_C = \frac{\hbar}{Mc} \rightarrow \frac{1}{M}$$



Planck Scale Criticality

The point $\lambda_C = r_g$ is a critical point

$$\lambda_C = r_g \implies M_{\min} = \frac{1}{\sqrt{G}} = M_{\rm Pl}$$

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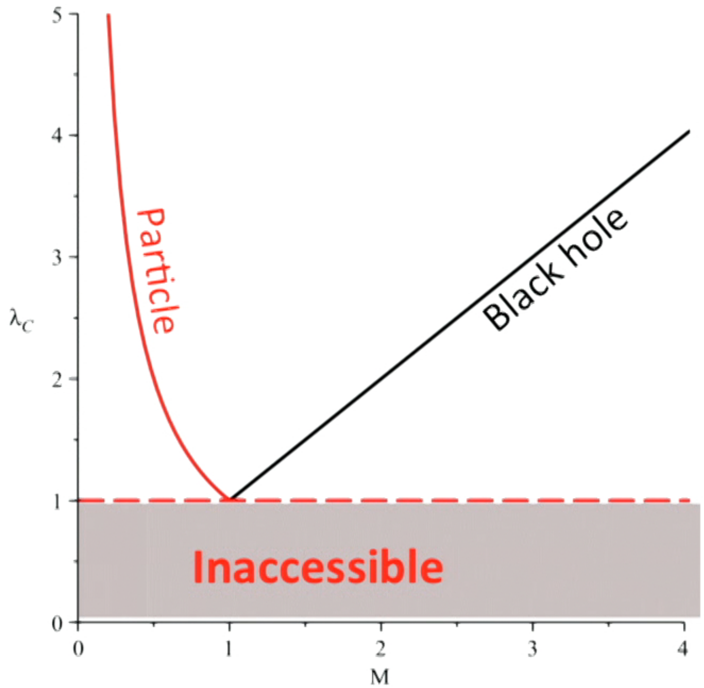

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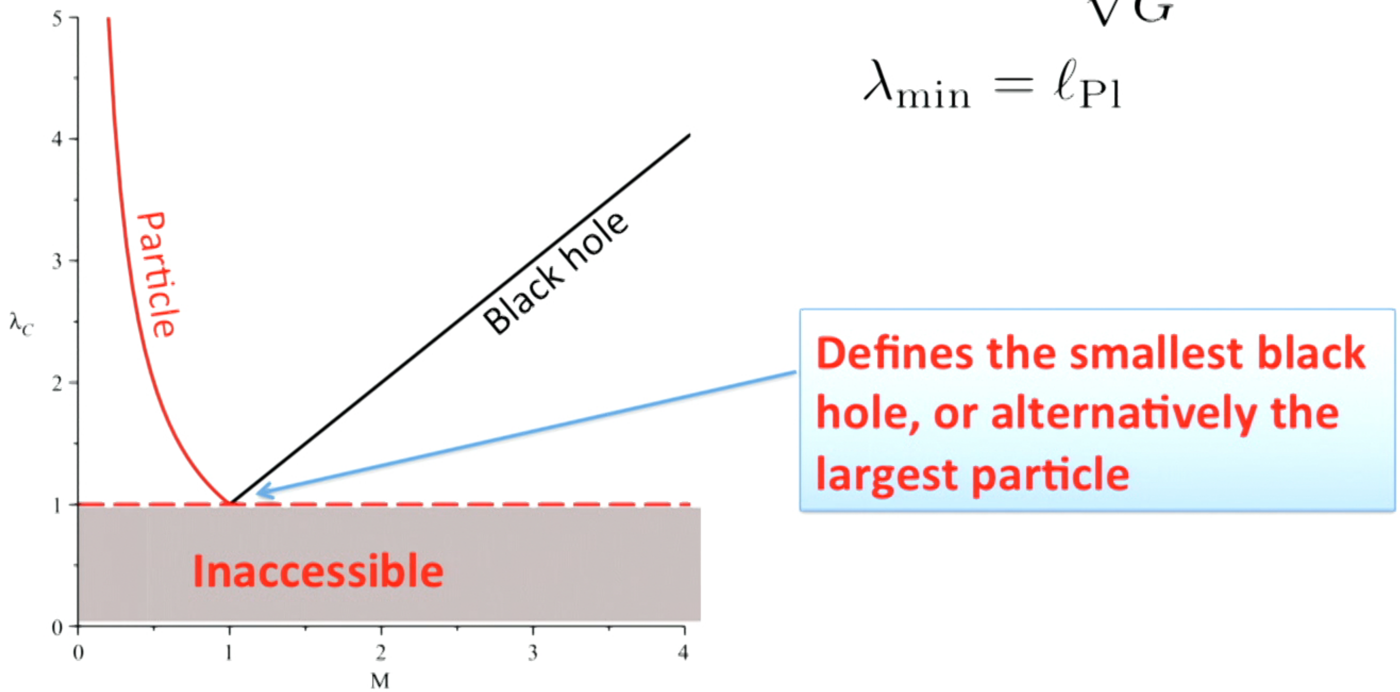

The Generalized Uncertainty Principle

A momentum-dependent modification of the HUP

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + \beta \Delta p^2\right)$$

[Veneziano, Europhys. Lett. **2**, 199 (1986); Amati, Ciafaloni, Veneziano, PLB **197**, 81 (1987); Amati, Ciafaloni, Veneziano, PLB **216**, 41 (1989)]

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Can be thought of as adding a ***gravitational*** correction to the standard quantum uncertainty that "connects" scale diagram:

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Can be thought of as adding a ***gravitational*** correction to the standard quantum uncertainty that "connects" scale diagram:

$$\Delta x_Q \sim \frac{\hbar}{2\Delta p} \quad , \quad \Delta x_G \sim 2G\Delta p$$

$$\Delta x = \Delta x_Q + \Delta x_G \quad \Longrightarrow \quad \Delta x \sim \Delta p + \frac{1}{\Delta p}$$

Characterized by a ***momentum / mass duality***

Black Holes and the GUP

QG effects become important for BHs at scales where GUP is relevant

1. Hot unstable remnants [Adler and Chen, Nucl.Phys.Proc.Sup. **124** (2003) 103]
   - Minimum sized BH with *maximum* temperature
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3. Cold stable remnants [Isi, JRM, Nicolini, JHEP **1311**:139 (2013)]
   - Exact solution to deformed stress-energy tensor
   - Minimum sized BH; thermodynamically-stable remnants

$$ds^2 = -\left(1 - \frac{2GM}{r}\gamma\left(2; \frac{r}{\sqrt{\beta}}\right)\right) dt^2 + \frac{dr^2}{1 - \frac{2GM}{r}\gamma\left(2; \frac{r}{\sqrt{\beta}}\right)} + r^2 d\Omega^2$$

GUP and Sub-Planckian Black Holes

[Carr, JM, Nicolini, JHEP 07:052 (2015)]

Can ***black holes*** exist *below* the Planck scale? $M_{\rm BH} < M_{\rm Pl}$

An approach to including the GUP in general relativity is to emphasize the ***duality*** in the black hole mass

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$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1 + \beta \Delta p^2\right)$$

$$\longrightarrow \quad \Delta x \sim \frac{1}{\Delta p} + \Delta p$$

$$\longrightarrow \quad \Delta x_G \sim \frac{1}{M_{\rm bh}} + M_{\rm bh}$$

What Is the Mass in the GUP?

[Carr, JM, Nicolini, JHEP 07:052 (2015)]

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Quantum: $M \sim \sqrt{s}$

GR: $M = M_{\rm ADM}$

M in classical Schwarzschild is ADM (Komar):

$$M \equiv \frac{1}{4\pi G} \int_{\partial\Sigma} d^2x \sqrt{\gamma^{(2)}}\, n_\mu \sigma_\nu \nabla^\mu K^\nu$$

$M < M_{Pl}$ determined from approximate stress-energy interior to R_C

$$M \equiv \int_\Sigma d^3x \sqrt{\gamma}\, n_\mu K_\nu T^{\mu\nu} \simeq -4\pi \int_0^{R_C} dr\, r^2 {T_0}^0$$

GUP and Sub-Planckian Black Holes

[Carr, JM, Nicolini, JHEP 07:052 (2015)]

Proposal: $M = -4\pi \int_0^{\ell_\mathrm{P}} dr\, r^2\, T_0{}^0$ — Unspecified quantum gravity stress-energy

Gives new *M* (duality): $M \longrightarrow M\left(1 + \frac{\beta}{2}\frac{M_{\mathrm{Pl}}^2}{M^2}\right)$

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Metric is:

$$ds^2 = F(r)dt^2 - F(r)^{-1}dr^2 - r^2 d\Omega^2$$

$$F(r) = 1 - \frac{2}{M_{\rm Pl}^2}\frac{M}{r}\left(1 + \frac{\beta}{2}\frac{M_{\rm Pl}^2}{M^2}\right)$$

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Planck mass is now critical point for which...

$$M \gg M_{\rm Pl} \implies F(r) \sim 1 - \frac{M}{r}$$

$$M \ll M_{\rm Pl} \implies F(r) \sim 1 - \frac{1}{Mr}$$

Black Hole Characteristics: Horizon

$$F(r_H) = 0$$

$$r_H = \frac{2}{M_{\rm Pl}^2}\left(\frac{M^2 + \frac{\beta}{2}M_{\rm Pl}^2}{M}\right)$$

Black Hole Characteristics: Horizon

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$$\Downarrow$$

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$$M \gg M_{\rm Pl} \quad \Longrightarrow \quad r_H \approx \frac{2M}{M_{\rm Pl}^2}$$

$$M \sim M_{\rm Pl} \quad \Longrightarrow \quad r_H \sim \frac{2+\beta}{M_{\rm Pl}}$$

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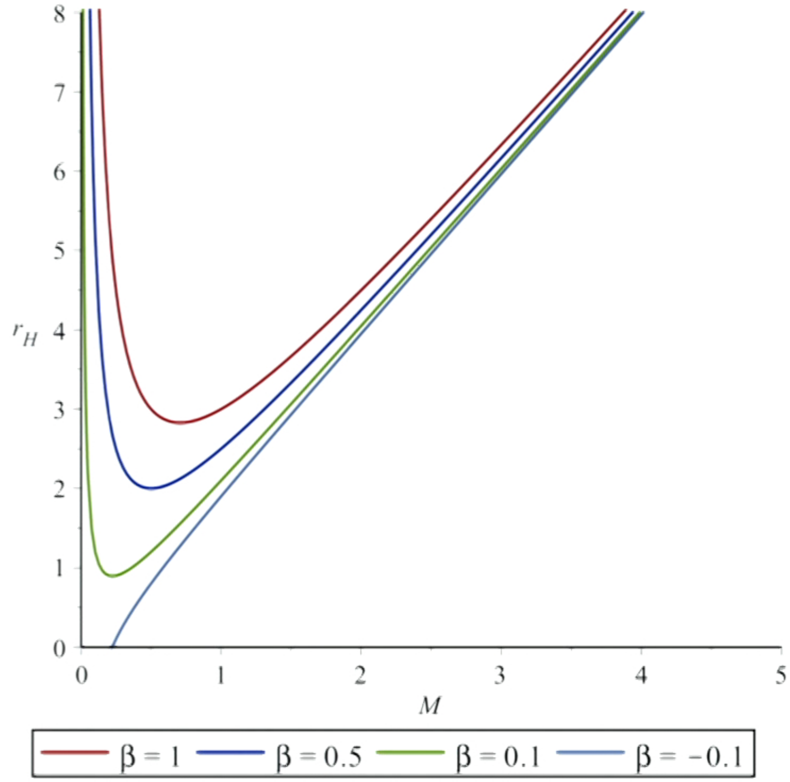
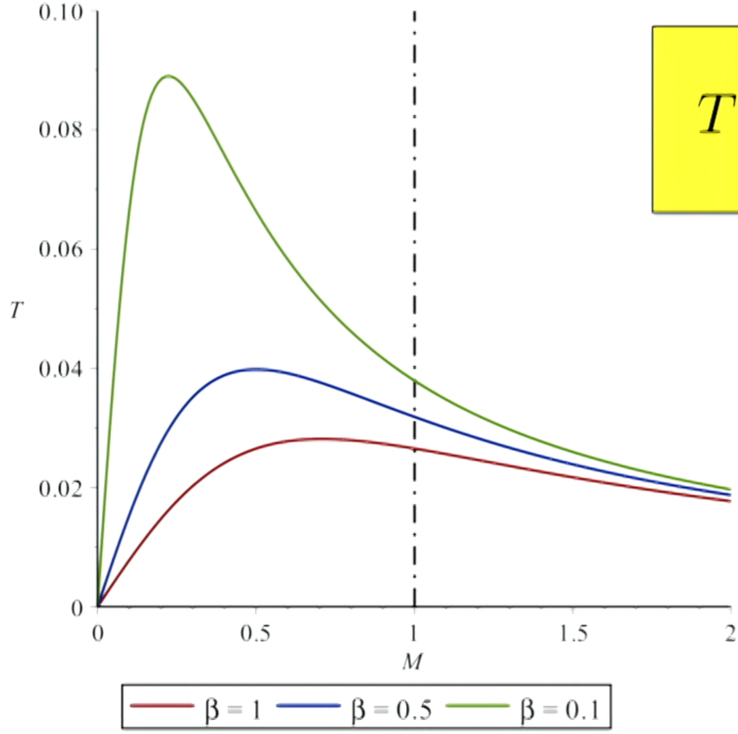


Black Hole Characteristics: Temperature

From surface gravity: $T = \dfrac{\kappa}{2\pi} \quad , \quad \kappa = \dfrac{1}{2}\dfrac{dF}{dr}(r = r_H)$



$$T = \frac{M_{\rm Pl}^2}{8\pi M(1 + \beta M_{\rm Pl}^2/2M^2)}$$

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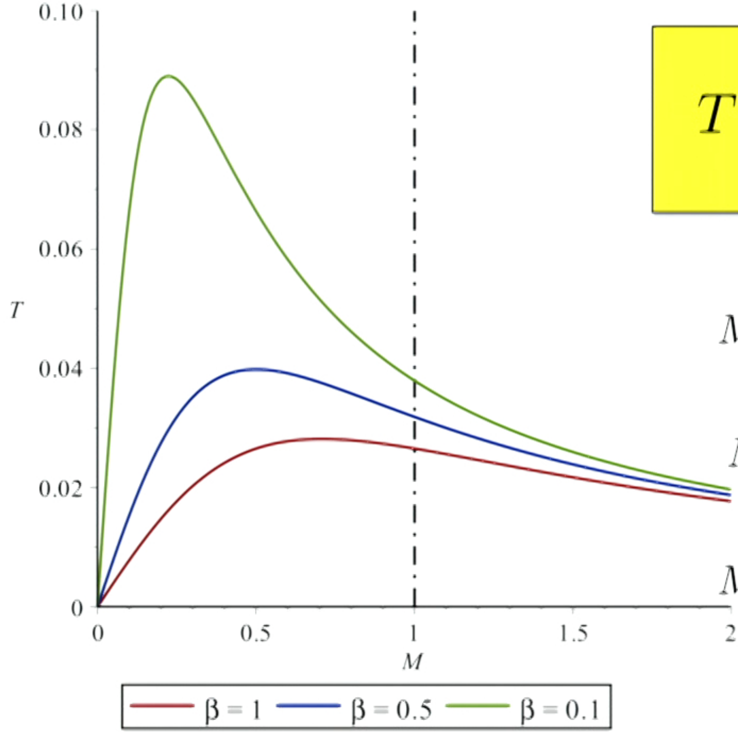

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$$M \gg M_{\text{Pl}} \Longrightarrow T \approx \frac{M_{\text{Pl}}^2}{8\pi M}\left[1 - \beta\left(\frac{M_{\text{Pl}}}{M}\right)^2\right]$$

$$M \approx M_{\text{Pl}} \Longrightarrow T \approx \frac{M_{\text{Pl}}}{8\pi(1 + \beta/2)}$$

$$M \ll M_{\text{Pl}} \Longrightarrow T \approx \frac{M}{4\pi\beta}\left[1 - \frac{1}{\beta}\left(\frac{M}{M_{\text{Pl}}}\right)^2\right]$$

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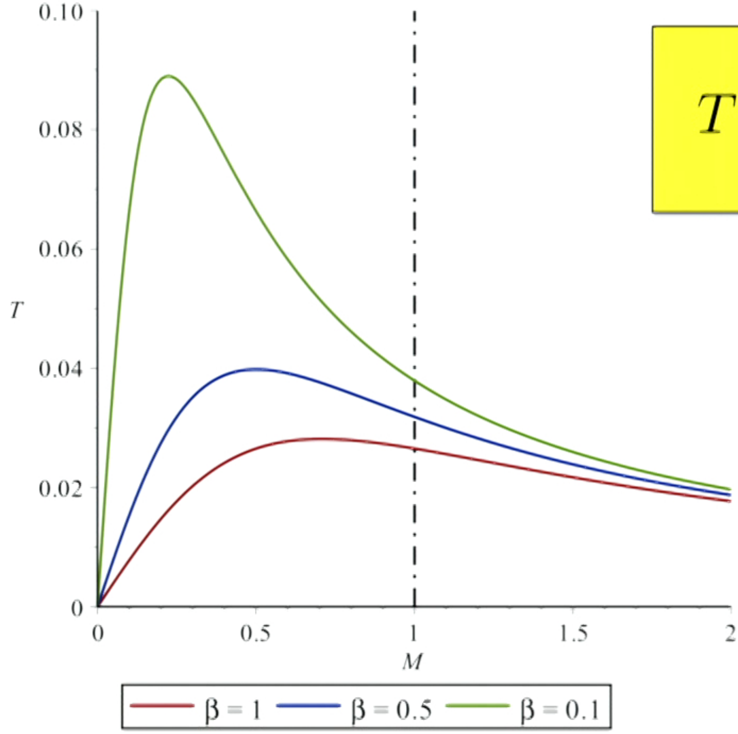

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$$M \sim M_{\rm Pl} \implies T \sim \frac{M_{\rm Pl}}{8\pi(1 + \beta/2}$$

$$M \ll M_{\rm Pl} \implies T \approx \frac{M}{4\pi\beta}$$

Black Hole Characteristics: Entropy

$$S = \int_{M_0}^{M} \frac{dM'}{T(M')} \sim 4\pi \left(\frac{M^2}{M_{\rm Pl}^2} - \frac{M_0^2}{M_{\rm Pl}^2} + \beta \log \frac{M}{M_0} \right)$$

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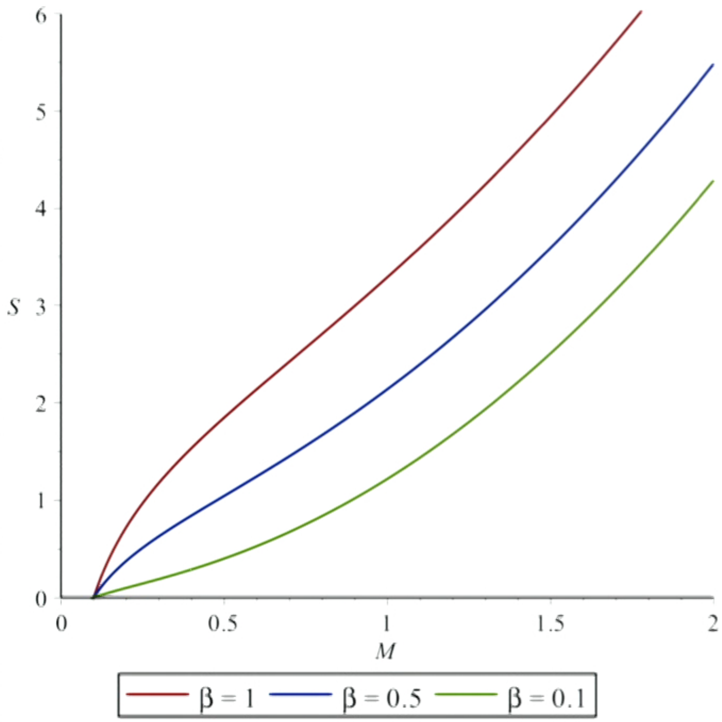

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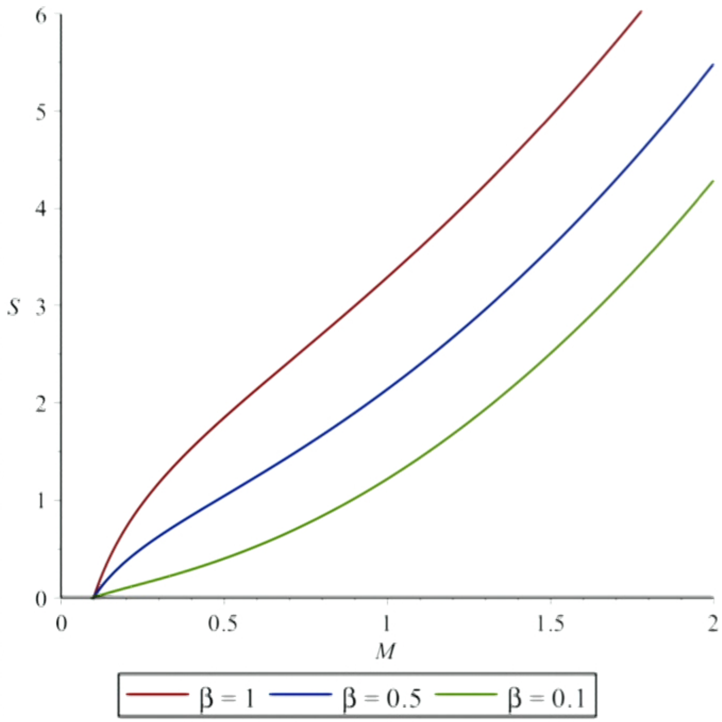

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I've seen these quantities before somewhere...

The (D+1)-dimensional Schwarzschild metric is…

$$ds^2 = \left(1 - \frac{R_D}{r^{D-2}}\right) dt^2 - \left(1 - \frac{R_D}{r^{D-2}}\right)^{-1} dr^2 - r^{D-1}\, d\Omega_{D-1}$$

where the horizon radius is….

$$R_D = \left(\frac{2\, G_D\, M}{D-2}\right)^{\frac{1}{D-2}} \quad \text{if} \quad D > 2$$

and Gravitational constant is defined as

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and Gravitational constant is defined as

$$G_D = \frac{\ell_D^{D-2}}{m_D}$$

Matches an exact solution!
Can derive BH thermodynamics!

[Mann, Shiekh, Tarasov, NP**B 341**, 134 (1990)]

(3+1)-D vs (1+1)-D Spacetime

(3+1)-D

$$g_{tt} = 1 - \frac{2G_N M}{r}$$

$$g_{rr} = -g_{tt}^{-1}$$

$$r_H \sim M$$

$$T \sim \frac{1}{M}$$

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(1+1)-D

$$g_{tt} = 1 - G_1 M|x|$$

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Sub-Planckian regime →

Dimensional reduction?

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Sub-Planckian regime →

Dimensional reduction?

The gravitational physics of the sub-Planckian regime is governed by an *effective* (1+1)-D

Dimensional Reduction, HEP, and QG

- Spectral (CDTs/EDTs, Hořava-Lifshitz, non-commutative)

  [Ambjorn, Jurkiewicz, Loll, PRD**72**:064014 (2005);
  Horava, PRL**102**, 161301 (2009);
  Modesto and Nicolini, PRD**81**:104040 (2010)]

- Evolving (Vanishing) Dimensions

  [Anchordoqui et al., PRD**83**,114046 (2011);
  JRM and Stojkovic, PRL **106**, 101101 (2011);
  Anchordoqui et al., MPLA**27**,1250021 (2012)]

- Brane-world

  [C. Callan, J. Maldacena, Nucl.Phys. B513, 198 (1998),
  N. Constable, R. Myers, O. Tafjord, PR**D61:**106009 (2000)]

- Multifractal gravity

  [Calcagni, PRL **104**, 251301(2010);
  Calcagni PLB**697**, 251 (2011);
  Arzano *et al.*, PRD**84**:125002 (2011)]

Black Holes are not Particles

Black Holes are not Particles

- Can ensure that the physical length scale of particles does not fall within their gravitational radius (*e.g.* neutrinos)

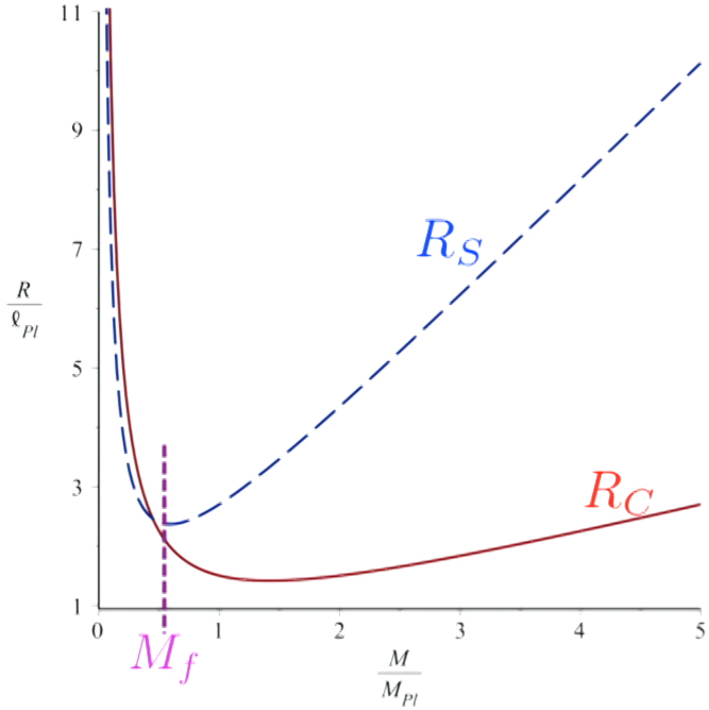
- Define two ways of approaching Planck scale [Carr 2013]:

  - Generalized Compton: $$R_C = \frac{1}{M}\left[1 + \alpha\left(\frac{M}{M_{\rm Pl}}\right)^2\right]$$

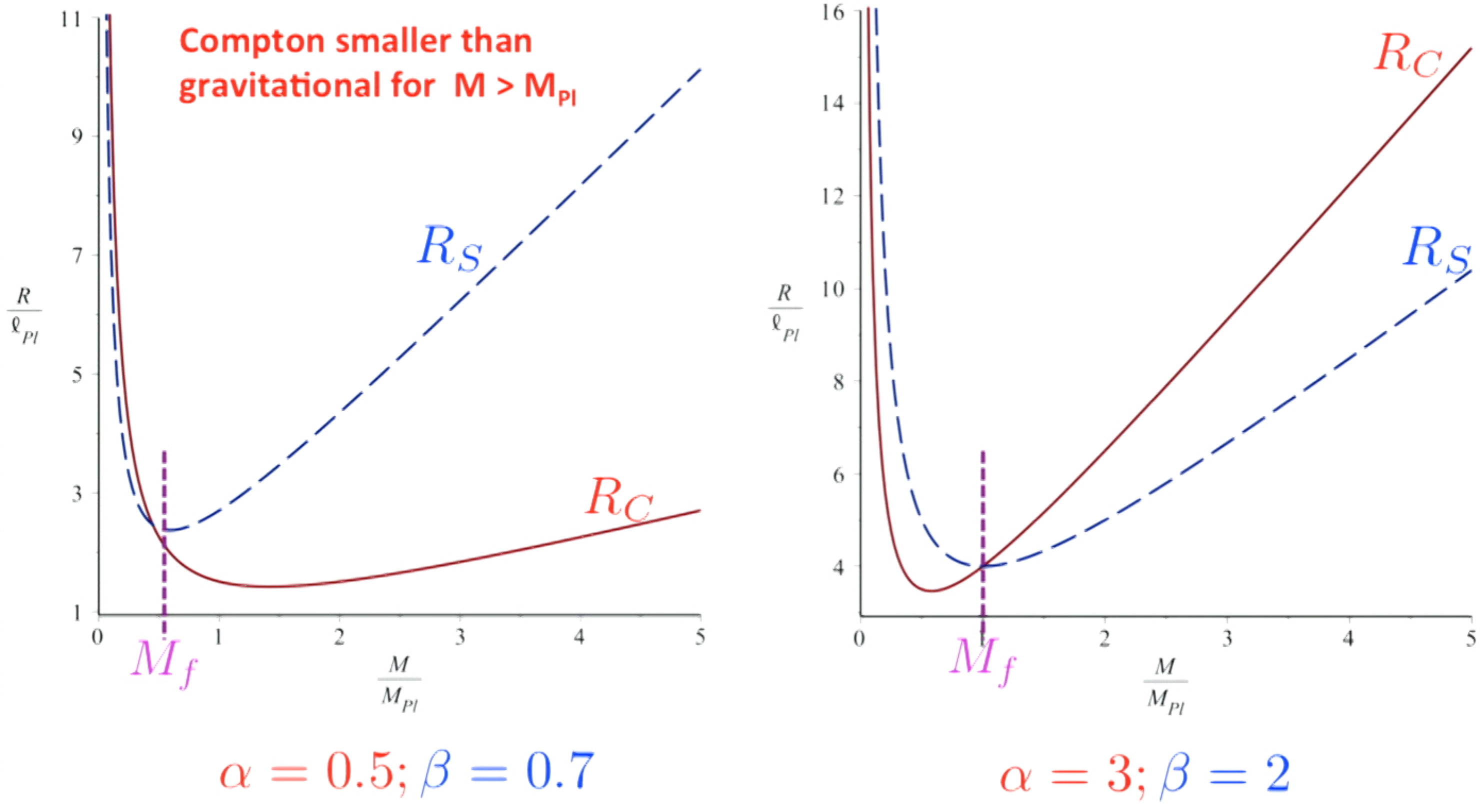
  - Generalized event horizon: $$R_S = 2M\left[1 + \frac{\beta}{2}\left(\frac{M_{\rm Pl}}{M}\right)^2\right]$$

- Coincide at the mass scale

$$M_{\rm f} = \sqrt{\frac{1-\beta}{2-\alpha}}\, M_{\rm Pl}\,,$$



$\alpha = 0.5; \beta = 0.7$



Compton smaller than gravitational for M > M_Pl
R_S
R_C
M_f
$\frac{R}{\ell_{Pl}}$
$\frac{M}{M_{Pl}}$
$\alpha = 0.5; \beta = 0.7$
$\alpha = 3; \beta = 2$

Production of Sub-Planckian BHs

- How can we understand production of (sub-Planckian) black holes in collisions?

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- **Horizon Wavefunction Formalism**

Given a particle created in e.g. a collision, what is the probability that…

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- **Horizon Wavefunction Formalism**

Given a particle created in e.g. a collision, what is the probability that…

1. … the distribution is within a spherical region $R \leq r_H$
2. … the particle's gravitational (horizon) radius is r_H
3. … the particle lies completely within r_H

Production of Sub-Planckian BHs

- Given an eigenstate

$$\hat{H}|\psi_E\rangle = E|\psi_E\rangle \qquad E = \frac{M_{\rm Pl} R_H}{2\ell_{\rm Pl}}$$

← ***Hoop conjecture***

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← ***Hoop conjecture***

- Define the horizon wavefunction:

$$|\psi_{\rm S}\rangle = \sum_E C(E)|\psi_E\rangle \qquad \psi_H(r_H) = C(M_{\rm Pl}\, r_H / 2\, \ell_{\rm Pl})$$

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- Define the spatial distribution of the particle:

$$\psi_S(r) = \text{Some (gaussian) distribution} \quad \text{width } \ell \geq \lambda_{\rm C}$$

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← ***Hoop conjecture***

- Define the horizon wavefunction:

$$|\psi_{\rm S}\rangle = \sum_E C(E)|\psi_E\rangle \qquad \psi_H(r_H) = C(M_{\rm Pl}\, r_H/2\,\ell_{\rm Pl})$$

- Define the spatial distribution of the particle:

$$\psi_S(r) = \text{Some (gaussian) distribution} \quad \text{width } \ell \geq \lambda_{\rm C}$$

- Probability of being a black hole
  - The particle lies completely within r_H, and r_H is its horizon radius!

Production of Sub-Planckian BHs

- Given an eigenstate

$$\hat{H}|\psi_E\rangle = E|\psi_E\rangle \qquad E = \frac{M_{\rm Pl} R_H}{2\ell_{\rm Pl}}$$

← ***Hoop conjecture***

- Define the horizon wavefunction:

$$|\psi_{\rm S}\rangle = \sum_E C(E)|\psi_E\rangle \qquad \psi_H(r_H) = C(M_{\rm Pl}\, r_H / 2\,\ell_{\rm Pl})$$

- Define the spatial distribution of the particle:

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- Probability of being a black hole
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Production of Sub-Planckian BHs

[Casadio, Cavalcanti, Giugno, JM, PLB **760**:36 (2016)]

- Probability that particle is inside a D-ball of radius :

$$P_{\rm S}(r < r_H) = \Omega_{D-1} \int_0^{r_H} |\psi_{\rm S}(r)|^2\, r^{D-1}\, \text{r}$$

- Probablity that radius of the horizon is r_H:

$$\mathcal{P}_{\rm H}(r_H) = \Omega_{D-1}\, r^{D-1}\, |\psi_{\rm H}(r_H)|^2$$

- Probability that particle lies within its horizon radius:

$$\mathcal{P}_<(r < r_H) \;=\; P_{\rm S}(r < r + H)\, \mathcal{P}_{\rm H}(r_H)$$

- **Probability that particle is a black hole:**

$$P_{\rm BH} \;=\; \int_0^{\infty} \mathcal{P}_<(r < r_H)\, dr_H$$

The (D+1)-dimensional Schwarzschild metric is...

$$ds^2 = \left(1 - \frac{R_D}{r^{D-2}}\right) dt^2 - \left(1 - \frac{R_D}{r^{D-2}}\right)^{-1} dr^2 - r^{D-1}\, d\Omega_{D-1}$$

where the horizon radius is....

$$R_D = \left(\frac{2\, G_D\, M}{D-2}\right)^{\frac{1}{D-2}} \quad \text{if} \quad D > 2 \quad = \frac{1}{2G_1 M} \text{ if } D = 1$$

and Gravitational constant is defined as

$$G_D = \frac{\ell_D^{D-2}}{m_D}$$

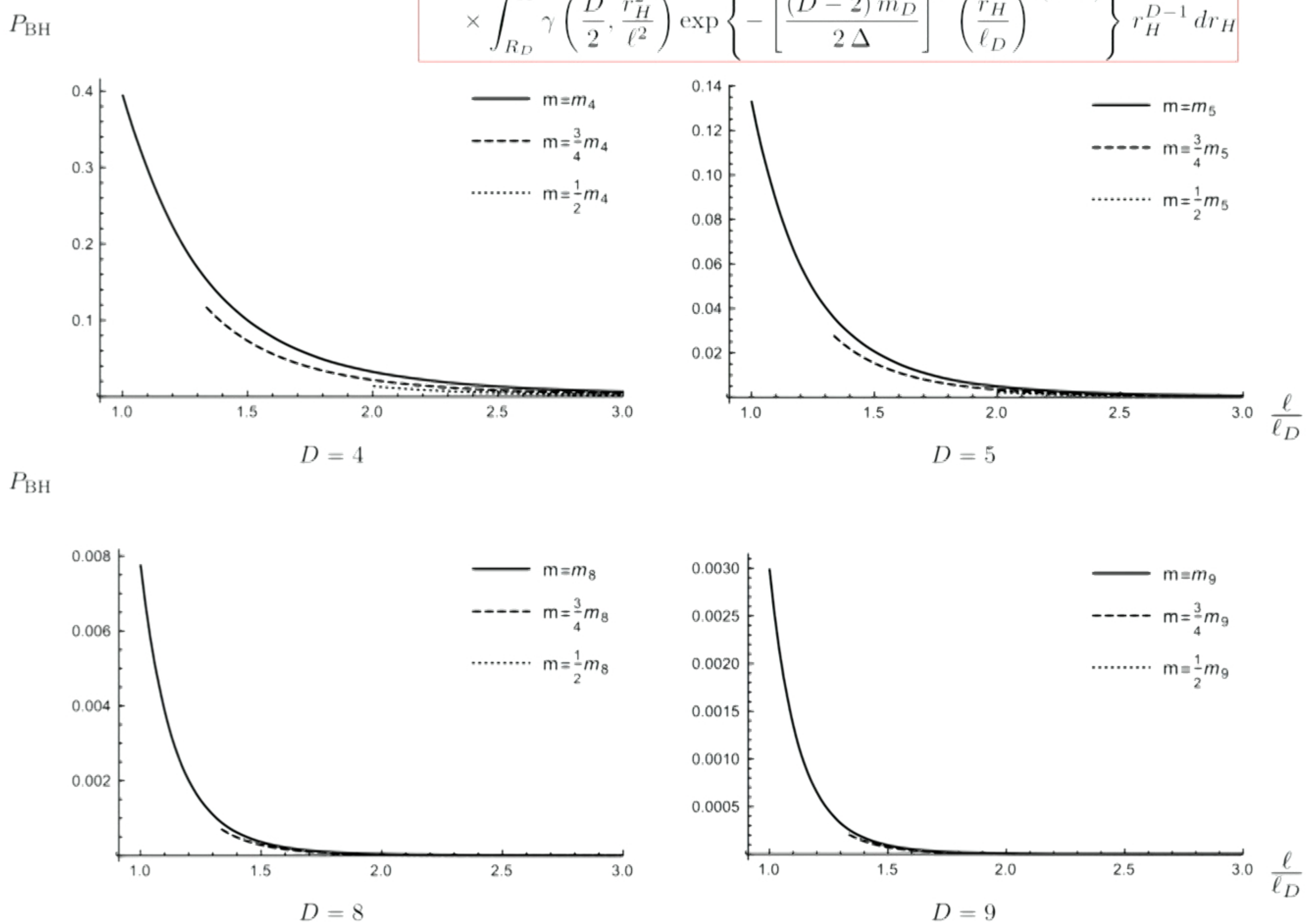
Matches an exact solution!

[Mann, Shiekh, Tarasov, NP**B 341**, 134 (1990)]

In (D+1)-dimensions…

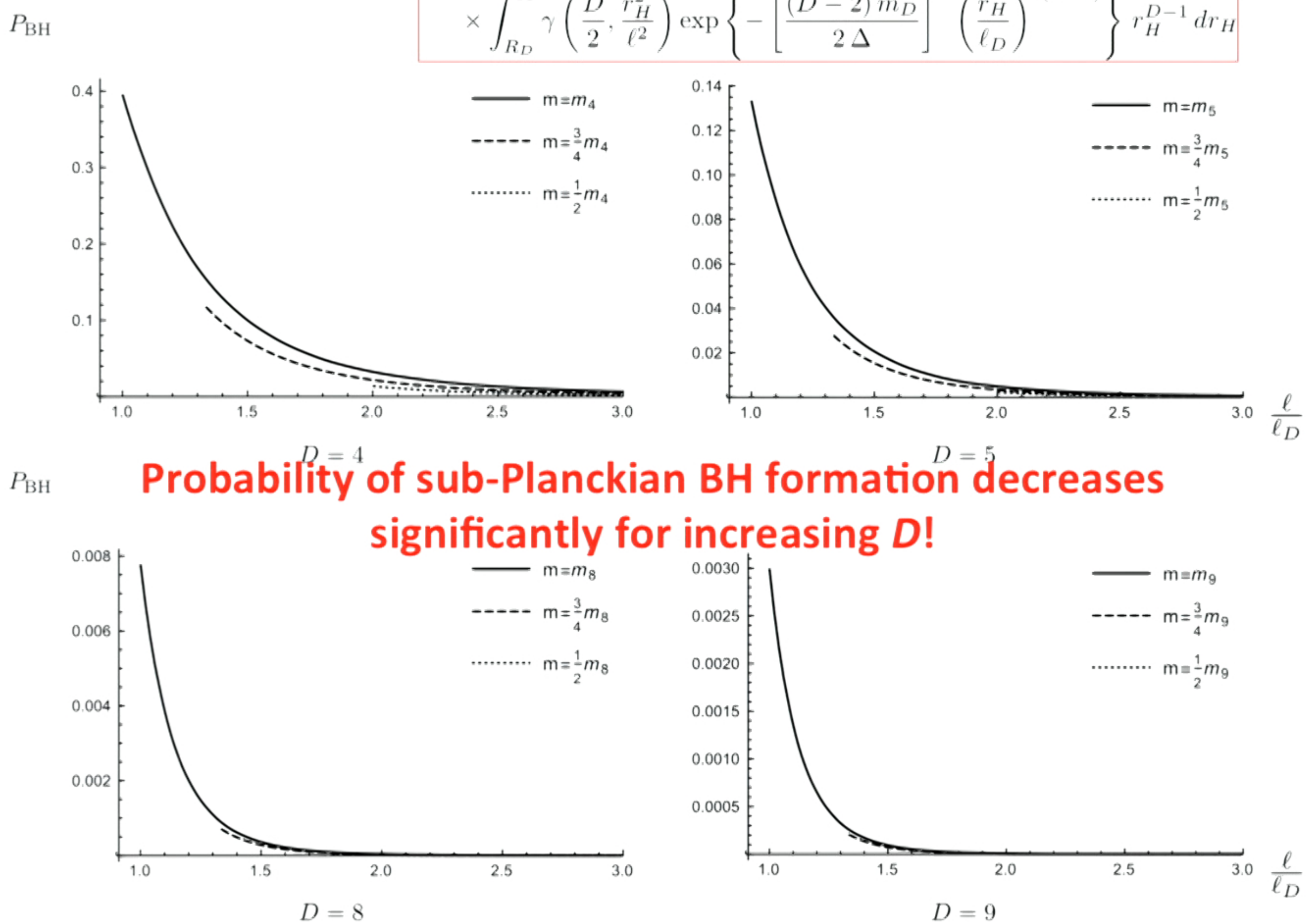
$$P_{\rm BH} = \frac{2}{\ell_D^D} \left(\frac{(D-2)\, m_D}{2\,\Delta}\right)^{\frac{D}{D-2}} \frac{D-2}{\Gamma\left(\frac{D}{2D-4}, \frac{m^2}{\Delta^2}\right)\Gamma\left(\frac{D}{2}\right)}$$

$$\times \int_{R_D}^{\infty} \gamma\left(\frac{D}{2}, \frac{r_H^2}{\ell^2}\right) \exp\left\{-\left[\frac{(D-2)\, m_D}{2\,\Delta}\right]^2 \left(\frac{r_H}{\ell_D}\right)^{2(D-2)}\right\} r_H^{D-1}\, dr_H$$



In (D+1)-dimensions…

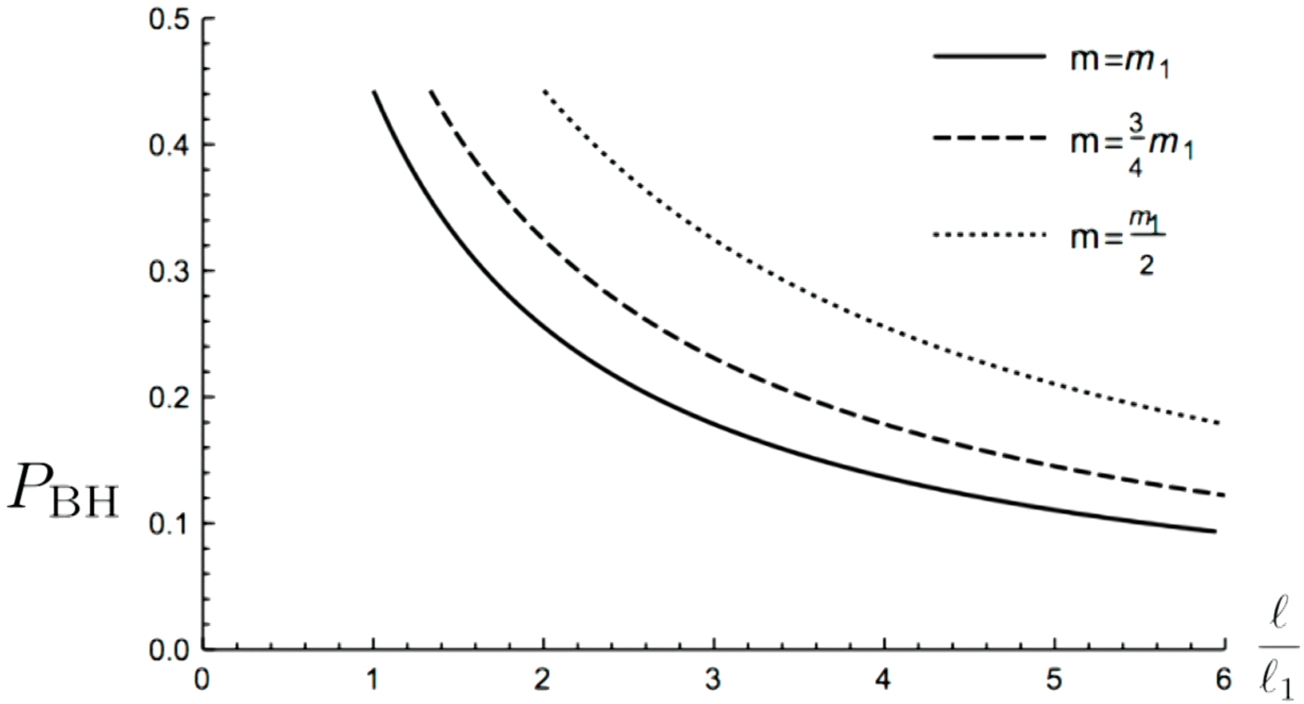
$$P_{\rm BH} = \frac{2}{\ell_D^D}\left(\frac{(D-2)\,m_D}{2\,\Delta}\right)^{\frac{D}{D-2}} \frac{D-2}{\Gamma\left(\frac{D}{2D-4},\frac{m^2}{\Delta^2}\right)\Gamma\left(\frac{D}{2}\right)} \times \int_{R_D}^{\infty} \gamma\left(\frac{D}{2},\frac{r_H^2}{\ell^2}\right) \exp\left\{-\left[\frac{(D-2)\,m_D}{2\,\Delta}\right]^2 \left(\frac{r_H}{\ell_D}\right)^{2(D-2)}\right\} r_H^{D-1}\, dr_H$$



Probability of sub-Planckian BH formation decreases significantly for increasing *D*!

In (1+1)-dimensions…

$$P_{\rm BH} = \frac{4/\ell}{\Gamma\left(-\frac{1}{2}, \frac{m^2}{\Delta^2}\right)} \int_0^{R_1} \mathrm{erf}\left(\frac{r_H}{\ell}\right) \exp\left\{-\frac{\ell^2}{4\,R_H^2}\right\} dr_H$$



HWF and GUP

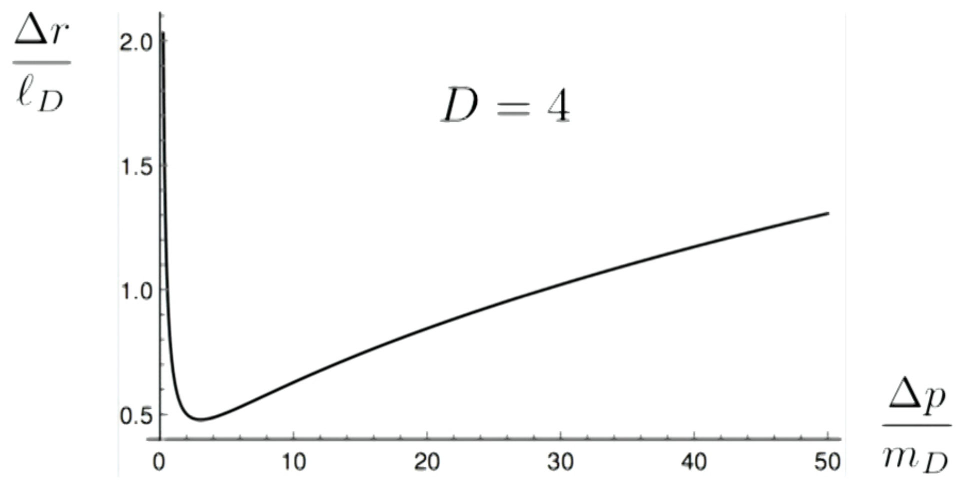
[Casadio, Cavalcanti, Giugno, JM, PLB **760**:36 (2016)]

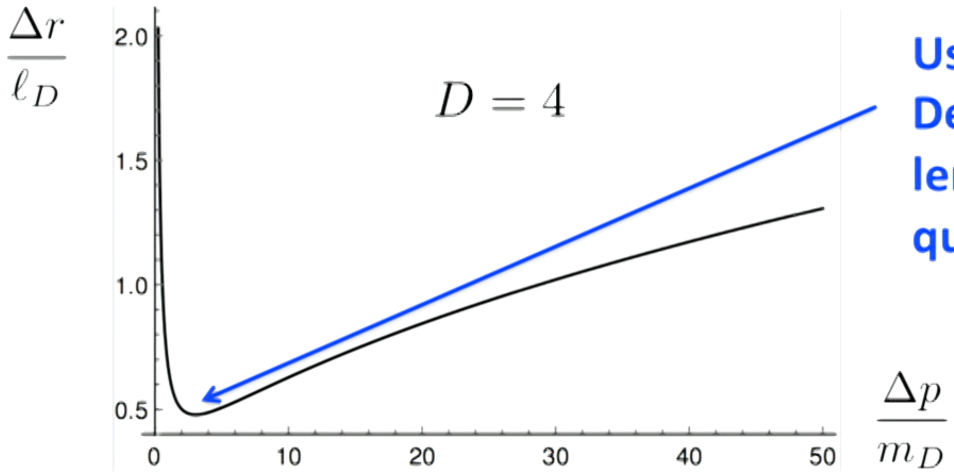
- *The Horizon Wavefunction formalism naturally implies the GUP*
- **Horizon and position expectations and uncertainties:**

$$\langle r_H^n \rangle = \Omega_{D-1} \int_0^\infty \tilde{\psi_H}^*(r)\, r_H^n\, \tilde{\psi_H}(r)\, r^{D-1}\, dr$$

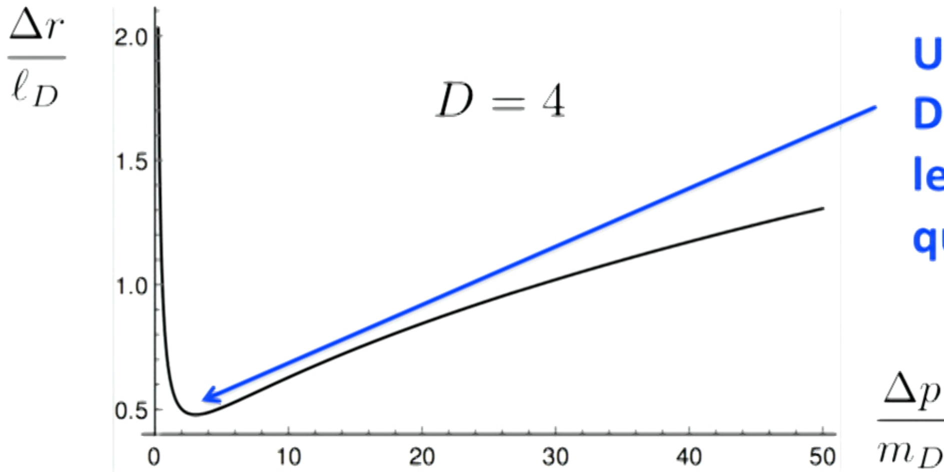
$$\langle r^n \rangle = \Omega_{D-1} \int_0^\infty \tilde{\psi_S}^*(r)\, r^n\, \tilde{\psi_S}(r)\, r^{D-1}\, dr$$

- **Total spatial uncertainty:** $\Delta r_{\text{tot}} = \Delta r + \alpha \Delta r_H$
- **Define momentum uncertainty similarly using** $\psi_S(p)$
- **Find** Δr_{tot} **in terms of** Δp

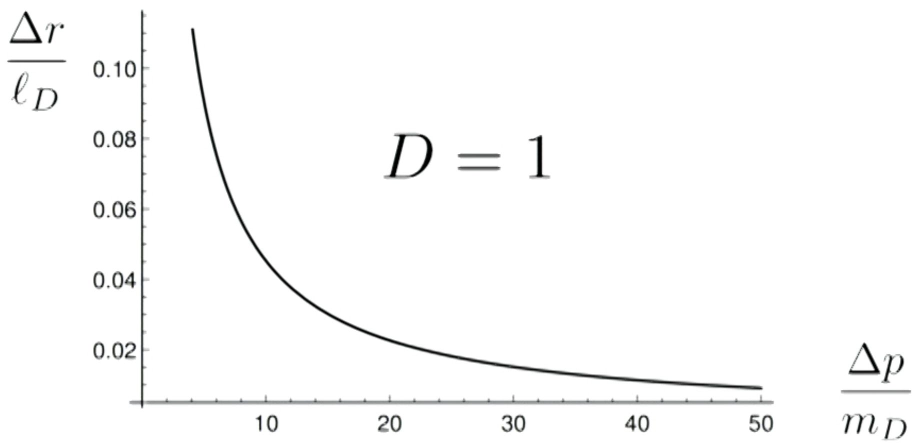

$\frac{\Delta r}{\ell_D}$
$D = 4$
$\frac{\Delta p}{m_D}$
2.0
1.5
1.0
0.5
0
10
20
30
40
50

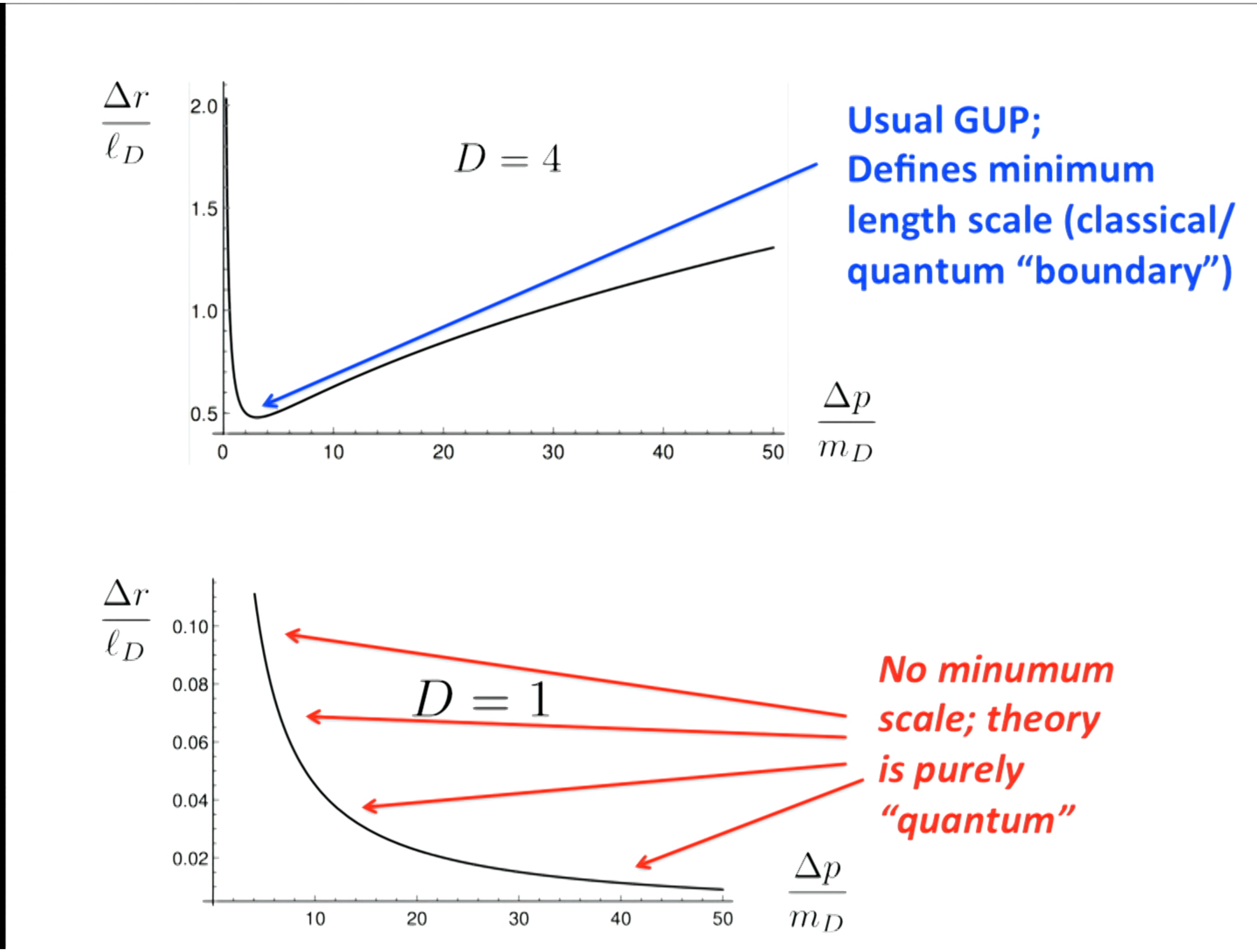


Usual GUP;
Defines minimum length scale (classical/ quantum “boundary”)



Usual GUP;
Defines minimum length scale (classical/ quantum "boundary")



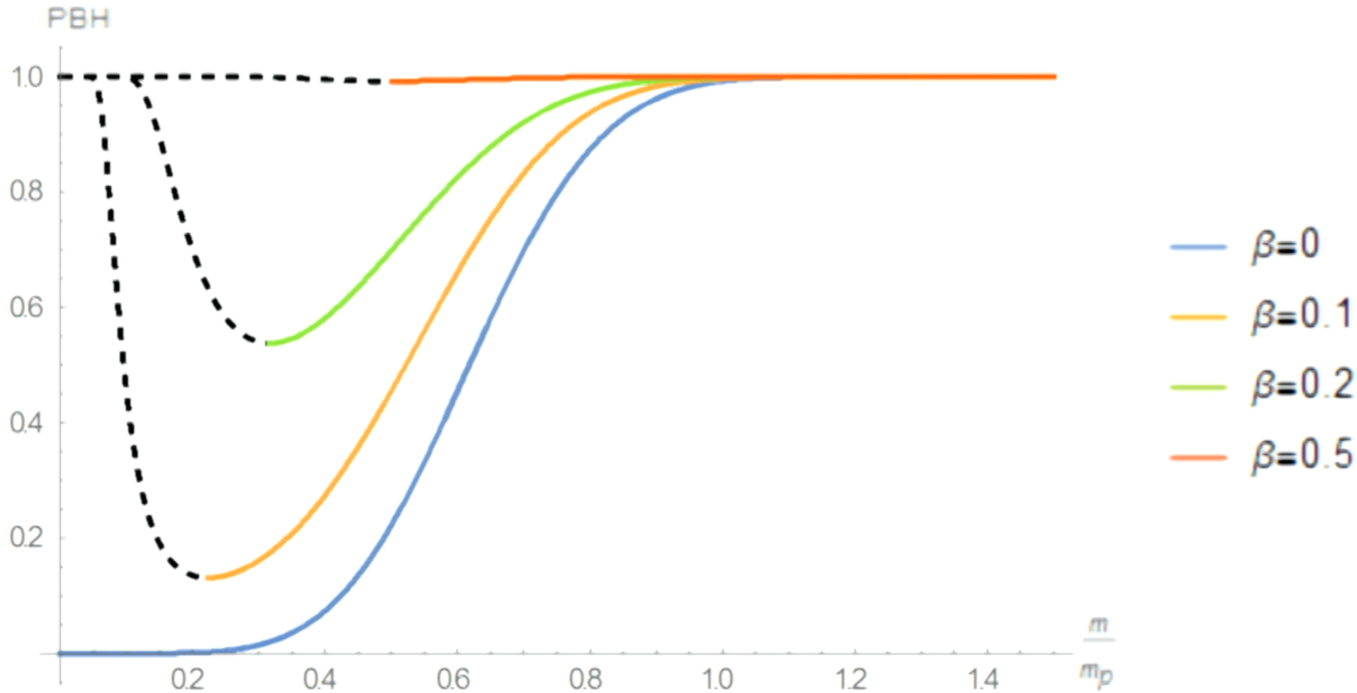

Δr / ℓ_D
D = 4
Usual GUP;
Defines minimum
length scale (classical/
quantum "boundary")
Δp / m_D
D = 1
No minumum
scale; theory
is purely
"quantum"

GUP Black Hole Production and HWF

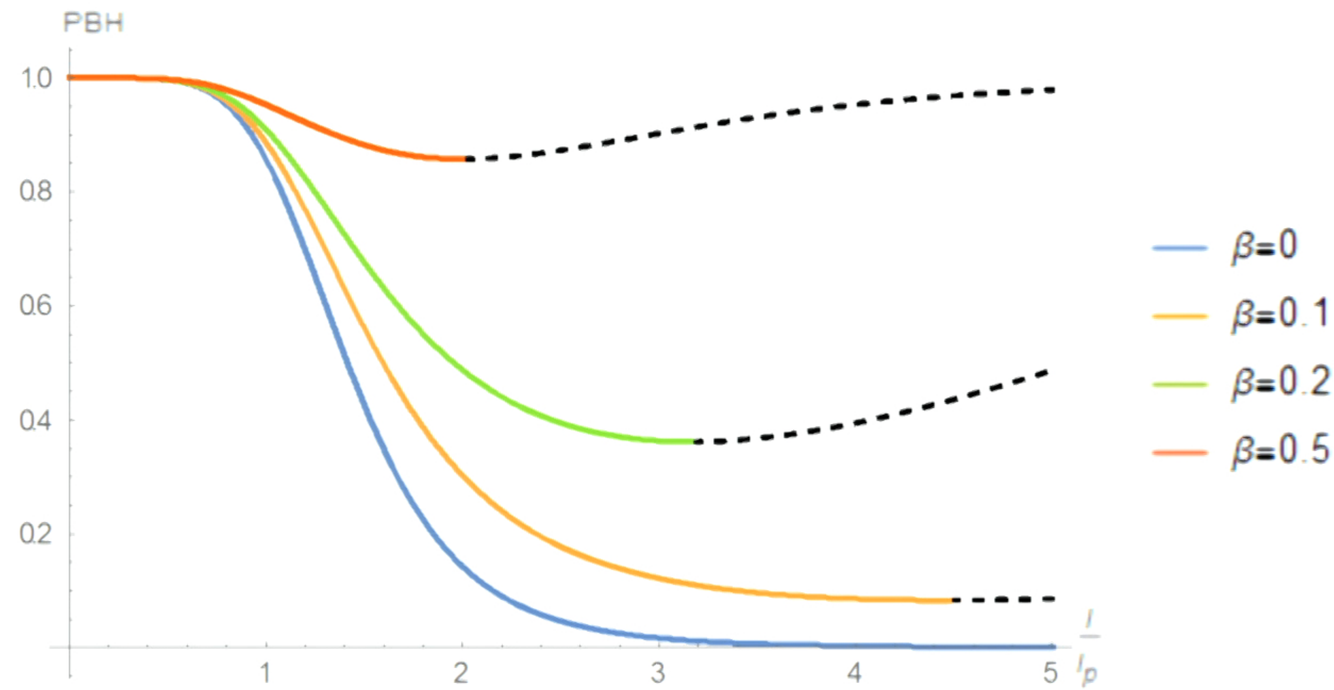
Manfredi and JM, AHEP **2016**:1543741 (2016)

- Given the GUP metric described earlier, the HWF is

$$\psi_{\rm H}(r_H) = \frac{1}{4\ell_{\rm Pl}^3}\sqrt{\frac{\ell^3}{\pi\Gamma\left(\frac{3}{2},1\right)}}\,\Theta(r_H - R_{\rm H})\,e^{-\frac{\ell^2 r_H^2}{8\ell^4}}$$



Manfredi and JM, AHEP **2016**:1543741 (2016)



Conclusions and Future Musings

- "Encoding" the GUP duality in the mass gives a metric that exhibits dimensional reduction in the sub-Planckian regime
- Cures thermodynamic instability of evaporating Schwarzschild black holes
- Production of quantum black holes is "easier" in lower dimensions than higher (particularly sub-Planckian)
- GUP emerges again as a natural characteristic of gravitation
- Instead of a two regimes governed by different theories (GR and QM), we have a consistent theory (gravity) in two different spacetime dimensions

KSM 2017

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Frankfurt Institute for Advanced Studies
24-28 July 2017

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Thank you!

jmureika@lmu.edu

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