Title: Towards a "hydrodynamic" approach to density matrices in quantum chaotic systems

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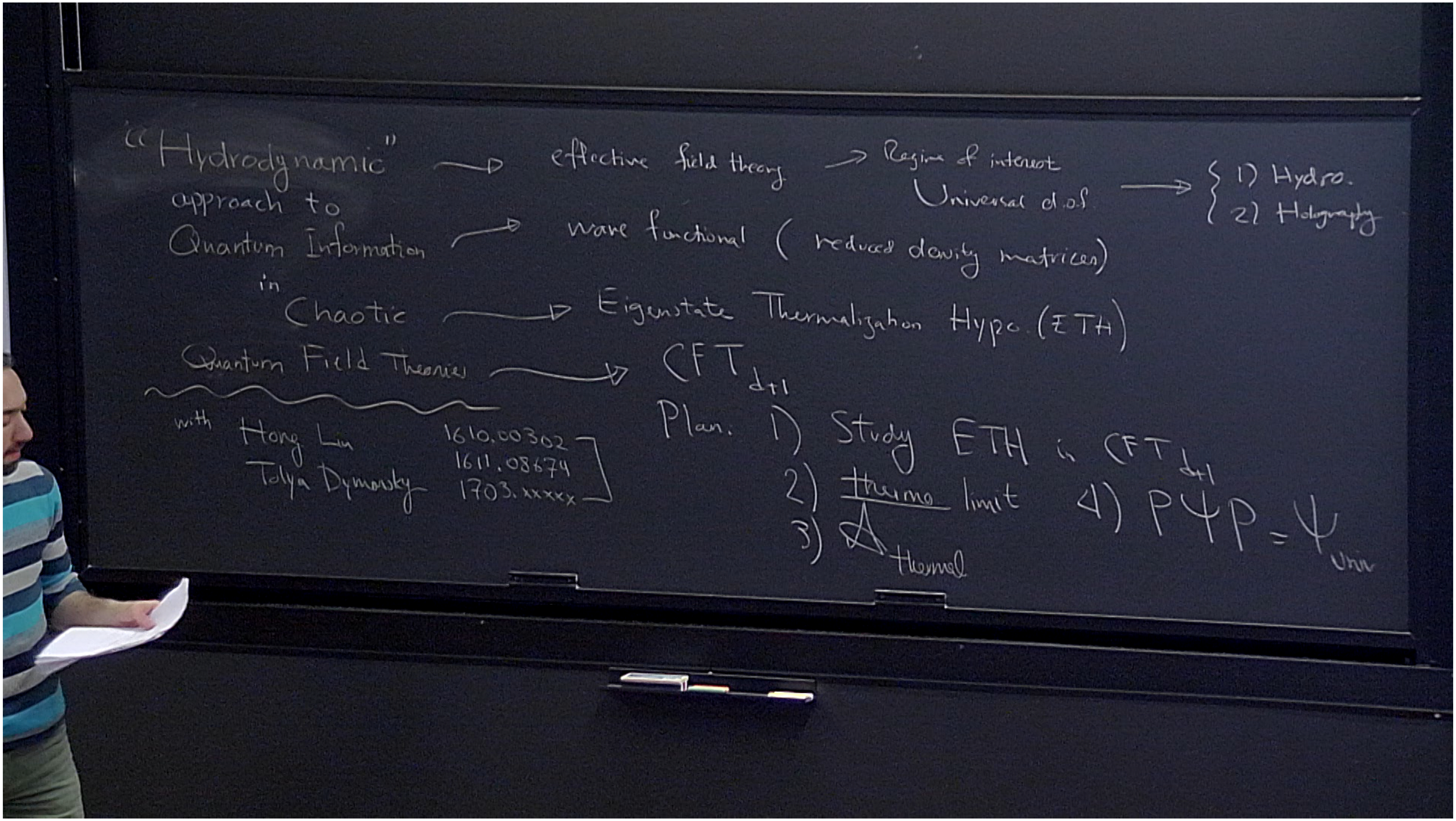
Abstract: <p>We study the eigenstate thermalization hypothesis in chaotic conformal ï¬•eld theories (CFTs) of arbitrary dimensions by computing the reduced density matrices of small size in energy eigenstates. We show that in the inï¬•nite volume limit this operator is well-approximated by a â€œuniversalâ€• density matrix which is its projection to the primary operators that have nonzero thermal one-point functions. These operators in all two-dimensional CFTs and holographic higher-dimensional CFTs are the polynomials of stress tensor. We compute the time-dependent two-point correlators and Renyi entropies of the universal density martix, and compare the results to the thermal Gibbs state and black holes in various dimensions. We demonstrate that the two-dimensional universal density matrix is close in trace distance to the reduced Gibbs state. We put forward the truncation of density matrix to the stress tensor sector as a â€œhydrodynamicâ€• method to study the out-of-equilibrium dynamics of strong-coupled ï¬•eld theories</p>

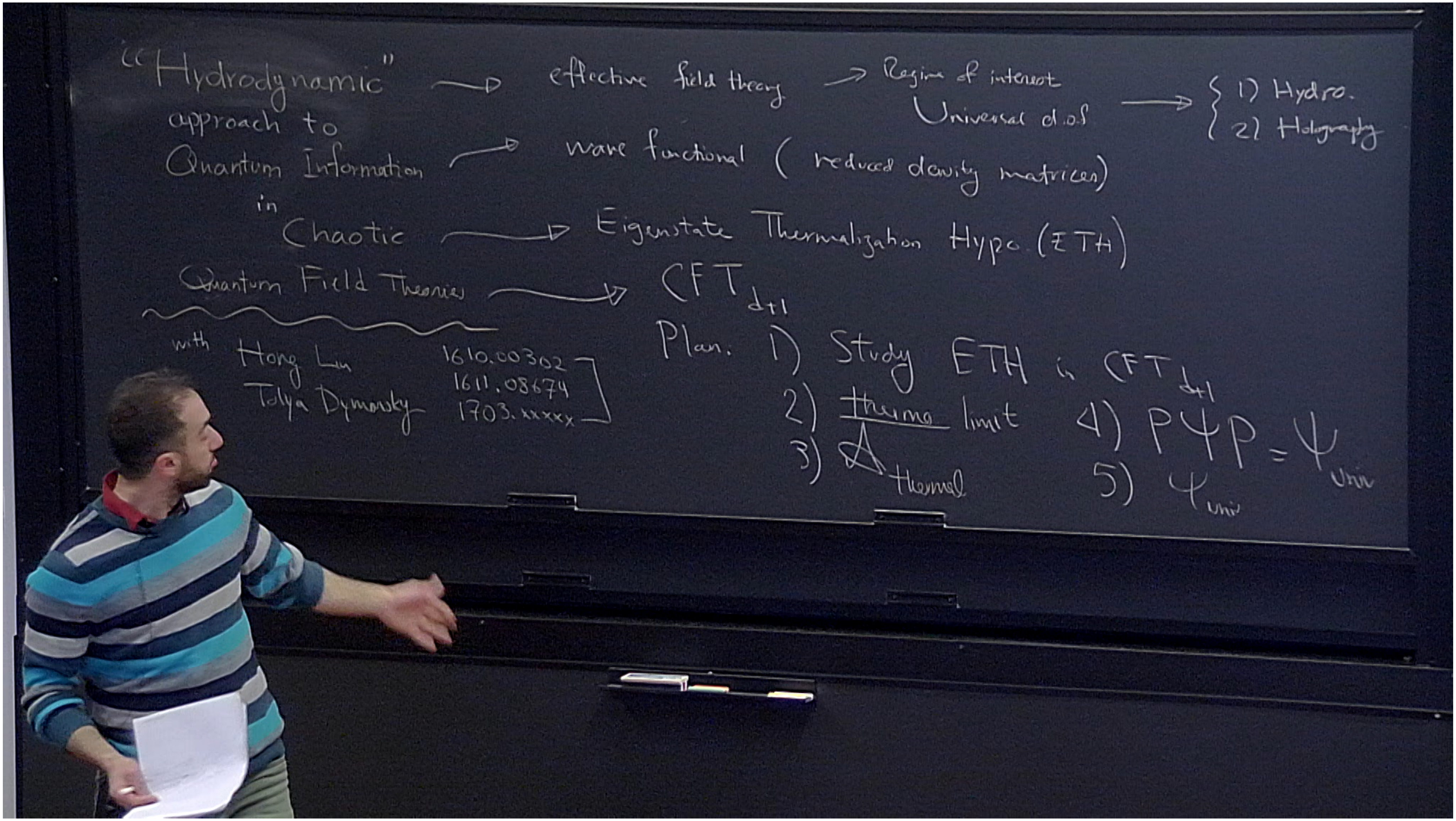
"Hydrodynamic" approach to Quantum Information in Chaotic Quantum Field Theories $\longrightarrow$ effective field theory $\rightarrow$ Regime of interest

Universal d.o.f $\longrightarrow$ $\left\{ \begin{array}{l} \text{1) Hydro.} \\ \text{2) Holography} \end{array} \right.$

with Hong Liu, Tolya Dymarsky

$\left.\begin{array}{l} \text{1610.00302} \\ \text{1611.08674} \\ \text{1703.xxxxx} \end{array}\right]$


"Hydrodynamic" approach to Quantum Information in Chaotic Quantum Field Theories
effective field theory → Regime of interest
Universal d.o.f → 1) Hydro. 2) Holography
wave functional (reduced density matrices)
Eigenstate Thermalization Hypo. (ETH)
CFT d+1
with Hong Liu 1610.00302
Tolya Dymarsky 1611.08674
1703.xxxxx
Plan. 1) Study ETH in CFT d+1
2) thermo limit
3) A thermal
4) PΨP = Ψ univ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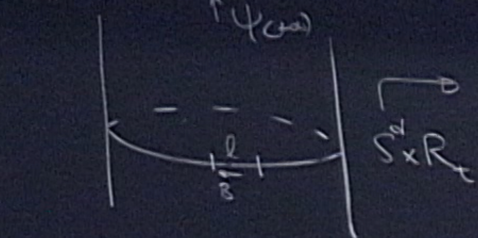
5) Ψ univ

3) $\mathcal{A}_{\text{thermal}}$

5) ψ_{univ} $\quad = \psi_{\text{univ}}$

Thermo. limit



$$\begin{cases} \dfrac{\ell}{L} \longrightarrow 0 \\ h_a \sim L^{d+1} \end{cases}$$

$$\varepsilon = \langle \psi_a | T_{00} | \psi_a \rangle \sim h_a L^{-(d+1)} \longrightarrow \lambda_T = \left(\frac{\varepsilon}{C} \right)^{-\frac{1}{d+1}}$$

Results:

$$|\psi_a\rangle \leftrightarrow \psi_a \qquad \lim_{L\to\infty} \left\| \psi_a\left(\frac{\ell}{\lambda_T}\right) - \psi_{\text{univ}}\left(\frac{\ell}{\lambda_T}\right) \right\| = 0$$

$$\operatorname{tr}(\psi_{\text{univ}} \cdots) = \sum_{p \in \mathcal{A}_{\text{thermal}}} f_p \left(\frac{\ell}{\lambda_T}\right)^{h_p} \operatorname{tr}(\hat{O}_p \cdots)$$

$5)\ \Psi_{\text{univ}}$

$\Rightarrow \lambda_T = \left(\frac{\varepsilon}{c}\right)^{-\frac{1}{d+1}}$

$= 0$

$\left(\frac{\ell}{\lambda_T}\right)^{h_p} \mathrm{tr}(\hat{O}_p \cdots)$

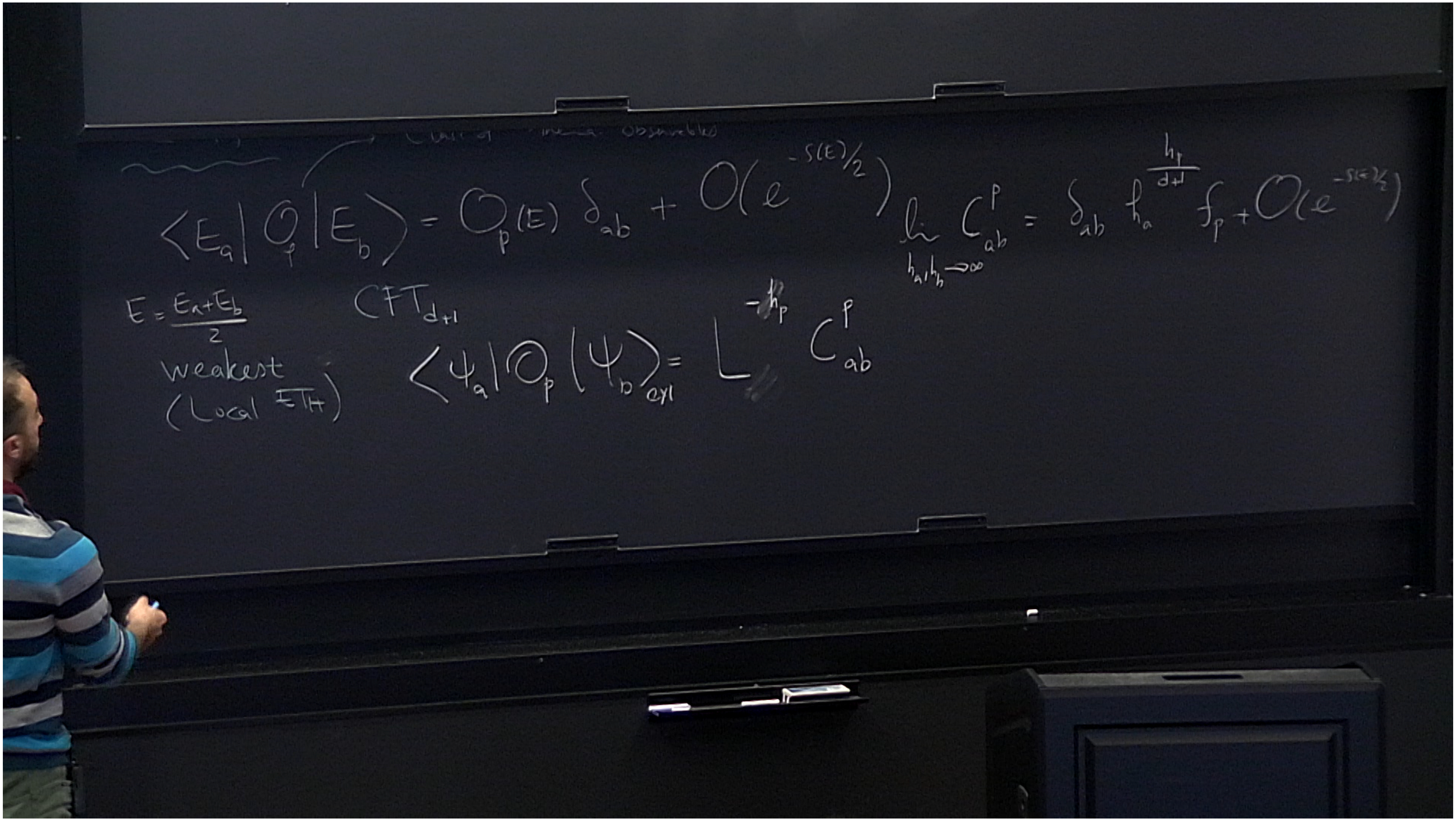
$$A_{\text{thermal}} = \left\{ O_p \ \text{s.t.}\ \frac{\mathrm{tr}(O_p e^{-\beta H})}{\mathrm{tr}(e^{-\beta H})} \sim \beta^{-h_p} f_p \right.$$

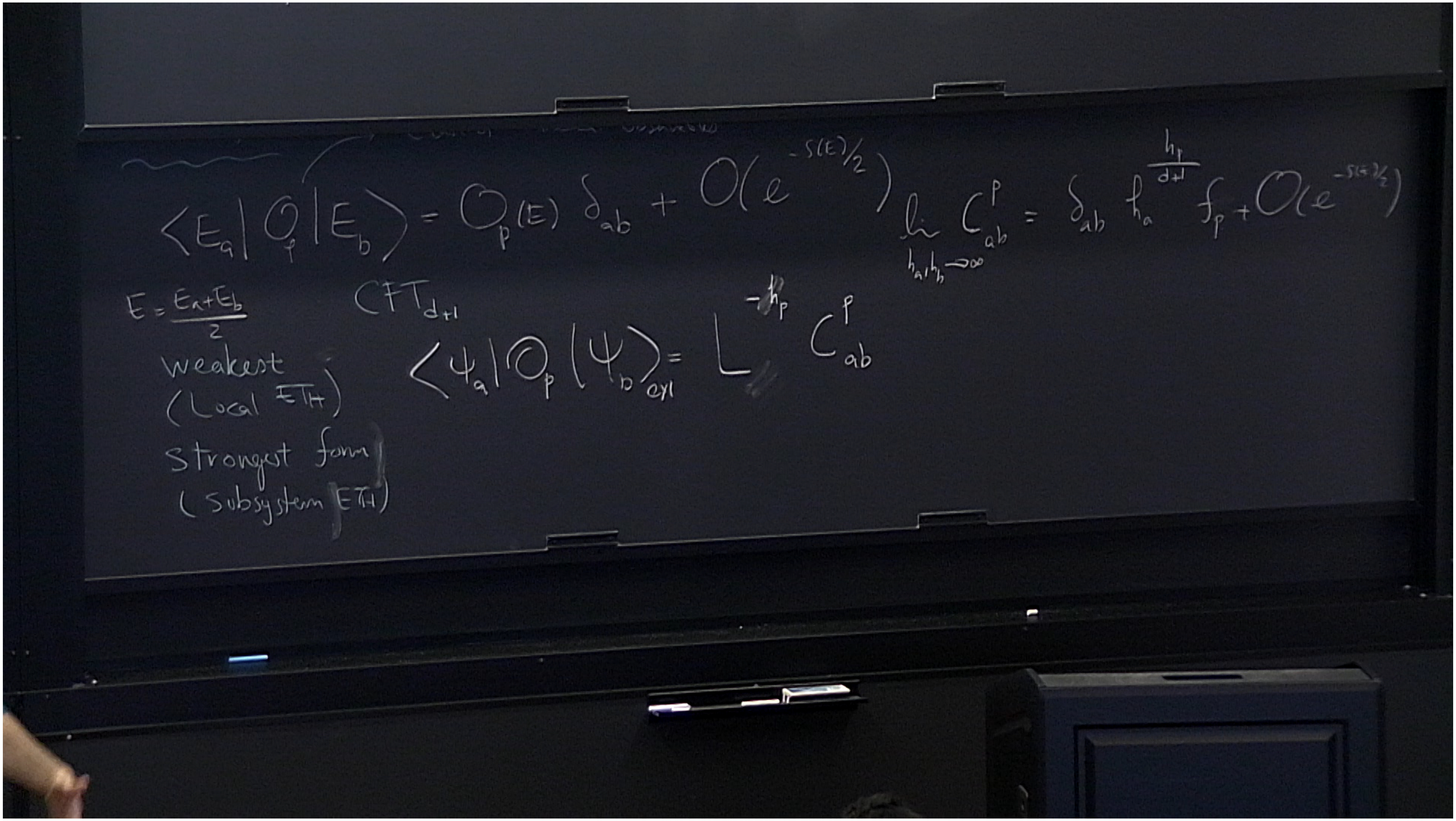
CFT_{d+1}

$\mathrm{CFT}_2 \quad \lim_{L\to\infty} \| \Psi_{\text{uni}} - \mathrm{tr}_{B^c} \rho_T \| = 0$

$$\rho_T \longrightarrow (\mathrm{tr}_{B^c} \rho_T)$$

$$S(\Psi_{\text{uni}}) \to S(BH)$$


$\langle E_a | O_p | E_b \rangle = O_p(E)\,\delta_{ab} + O(e^{-S(E)/2})$
$\lim_{h_a,h_b\to\infty} C^p_{ab} = \delta_{ab}\, h_a^{\frac{h_p}{d+1}} f_p + O(e^{-S(E)/2})$
$E = \frac{E_a+E_b}{2}$
CFT_{d+1}
weakest
(Local ETH)
$\langle \psi_a | O_p | \psi_b \rangle_{cyl} = L^{-h_p} C^p_{ab}$


$\langle E_a | O_\varphi | E_b \rangle = O_p(E)\,\delta_{ab} + O(e^{-S(E)/2})$
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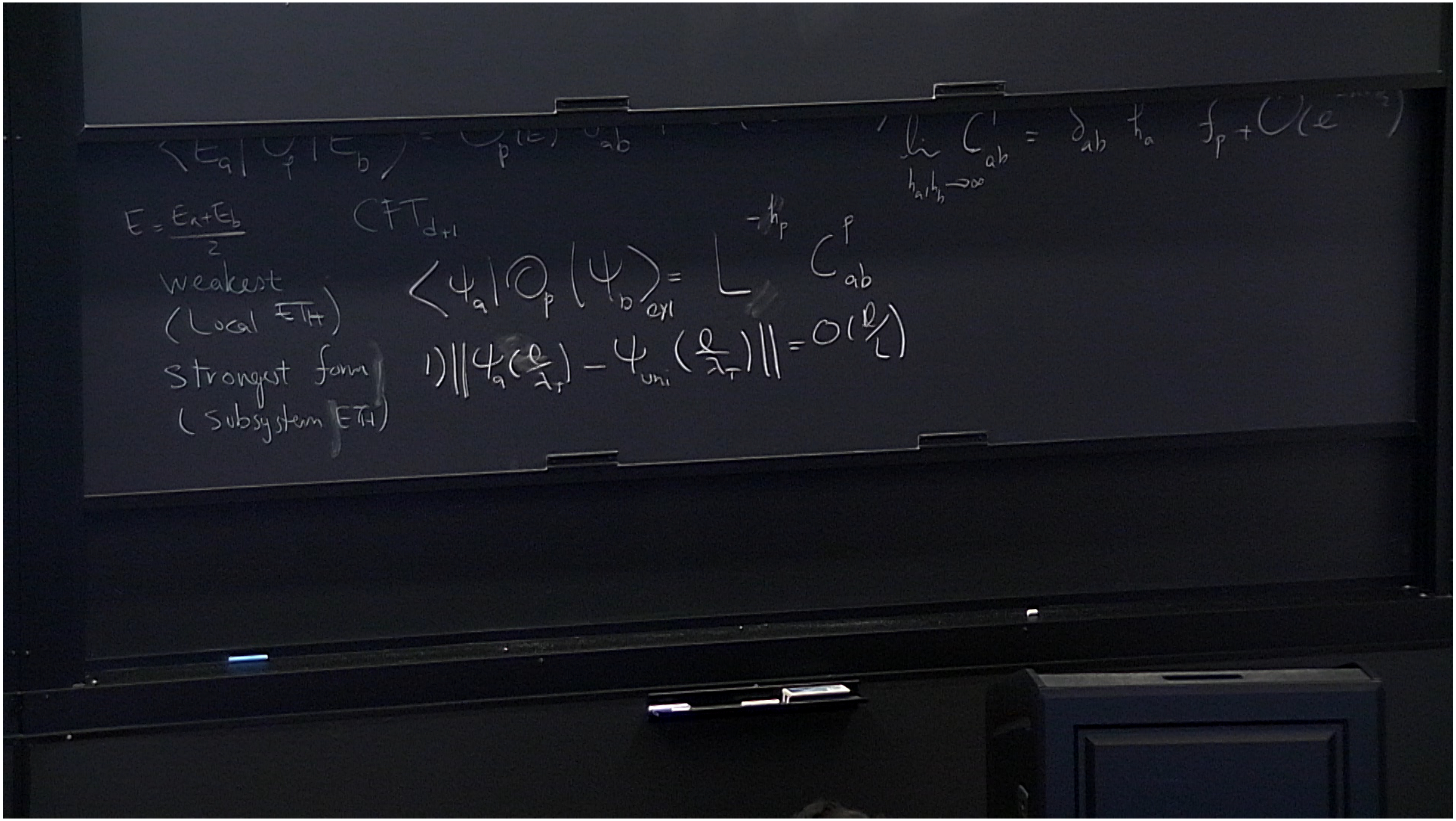
CFT_{d+1}
$\langle \psi_a | O_p | \psi_b \rangle_{cyl} = L^{-h_p} C^p_{ab}$
weakest
(Local ETH)
strongest form
(subsystem ETH)


$\langle E_a | O_p | E_b \rangle = O_p(E)\, \delta_{ab}$
$\lim_{h_a, h_b \to \infty} C^p_{ab} = \delta_{ab}\, h_a\, f_p + O(e^{\cdots})$
$E = \frac{E_a + E_b}{2}$
CFT_{d+1}
weakest (Local ETH)
$\langle \psi_a | O_p | \psi_b \rangle_{cyl} = L^{-h_p}\, C^p_{ab}$
strongest form (subsystem ETH)
$1)\ \| \psi_a(\frac{\ell}{\lambda_T}) - \psi_{uni}(\frac{\ell}{\lambda_T}) \| = O(\frac{\ell}{L})$

$$\langle E_a | O_p | E_b \rangle = O_p(E)\,\delta_{ab} + O\left(e^{-S(E)/2}\right)$$

$$\lim_{h_a, h_b \to \infty} C^p_{ab} = \delta_{ab}\, h_a^{\frac{h_p}{d+1}} f_p + O\left(e^{-S(E)/2}\right)$$

$E = \frac{E_a + E_b}{2}$ $\qquad CFT_{d+1}$

$$\sigma_{ab} = \mathrm{tr}_{B^c} |E_a\rangle\langle E_b|$$

weakest (Local ETH) $\qquad \langle \psi_a | O_p | \psi_b \rangle_{cyl} = L^{-h_p}\, C^p_{ab}$

Strongest form (subsystem ETH)

1) $\left\| \psi_a\left(\frac{\ell}{\lambda_T}\right) - \psi_{uni}\left(\frac{\ell}{\lambda_T}\right) \right\| = O\left(\ell/L\right)$

2) $\left\| \sigma_{ab}\, e^{i\alpha} + \sigma_{ba}\, e^{-i\alpha} \right\| = O\left(1/L\right)$

$$\langle E_a | \mathcal{O}_p | E_b \rangle = \mathcal{O}_p(E)\, \delta_{ab} + O\left(e^{-S(E)/2}\right)$$

$$\lim_{h_a, h_b \to \infty} C^p_{ab} = \delta_{ab}\, h_a^{\frac{h_p}{d+1}} f_p + O\left(e^{-S(E)/2}\right)$$

$E = \frac{E_a + E_b}{2}$

CFT_{d+1}

weakest (Local ETH)

$$\langle \psi_a | \mathcal{O}_p | \psi_b \rangle_{cyl} = L^{-h_p}\, C^p_{ab}$$

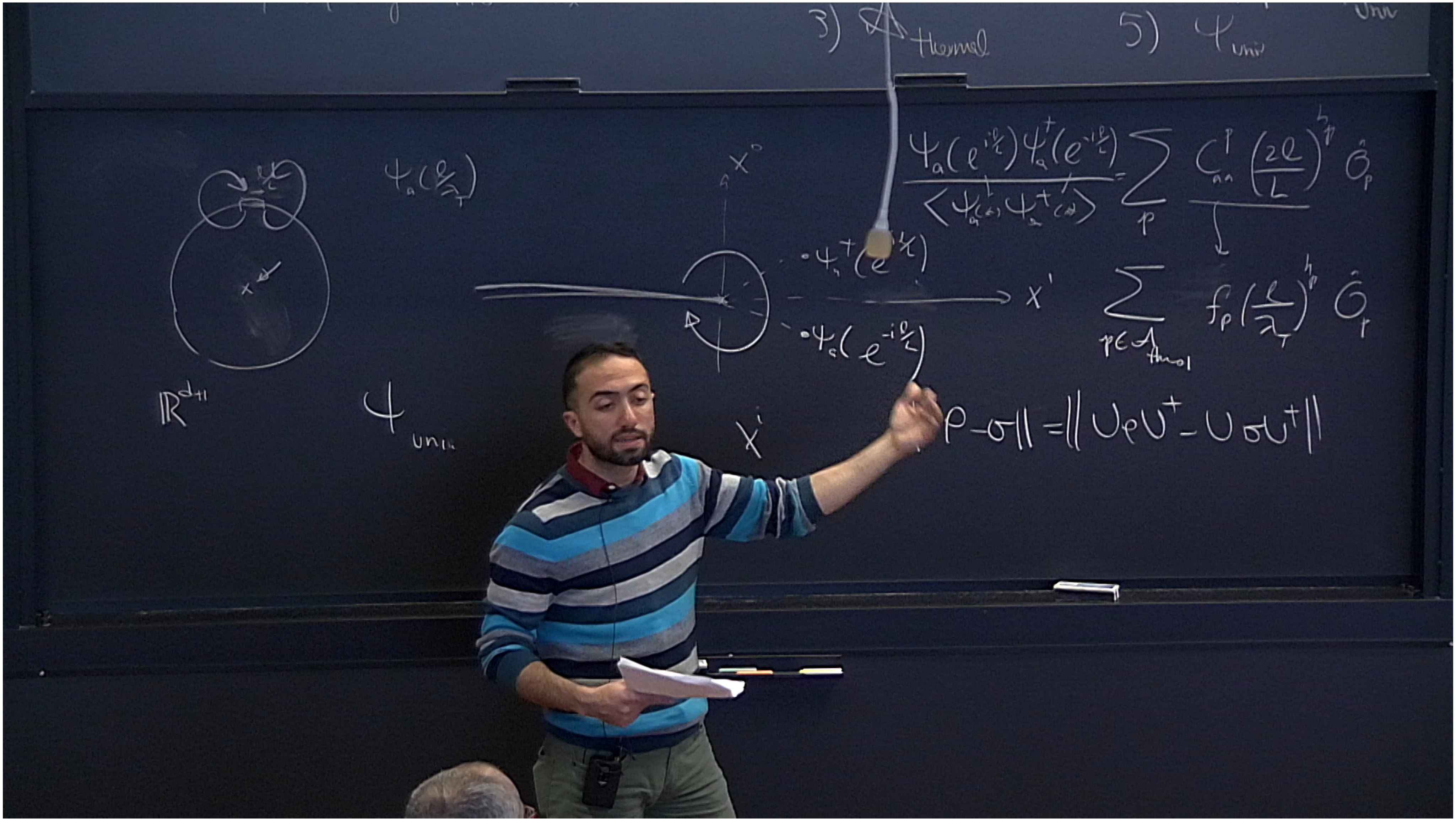
$$\sigma_{ab} = \mathrm{tr}_{B^c} |E_a\rangle\langle E_b|$$

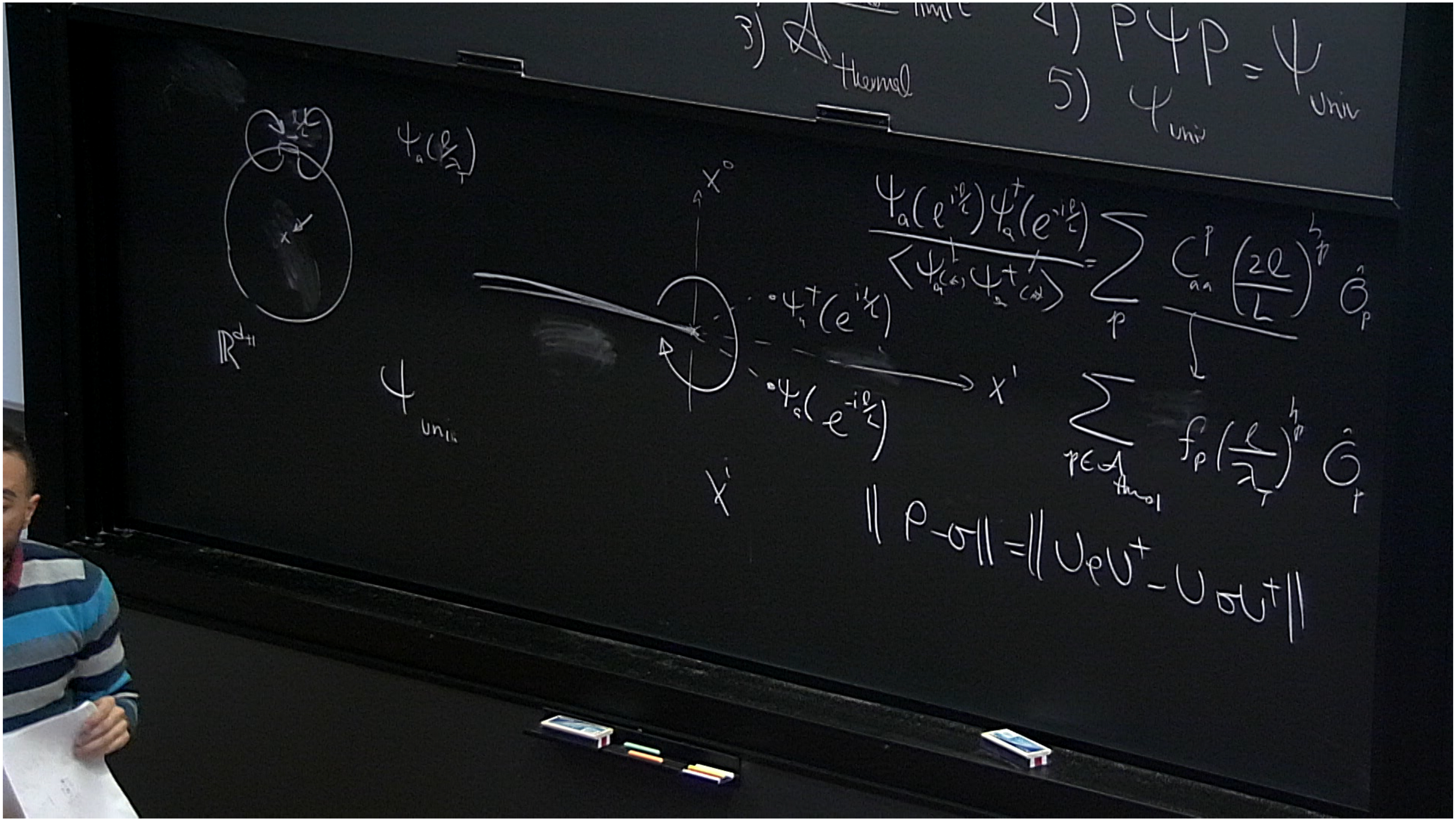
CFT_{d+1} : local ETH $\longrightarrow$ Subsystem ETH

Strongest form (subsystem ETH)

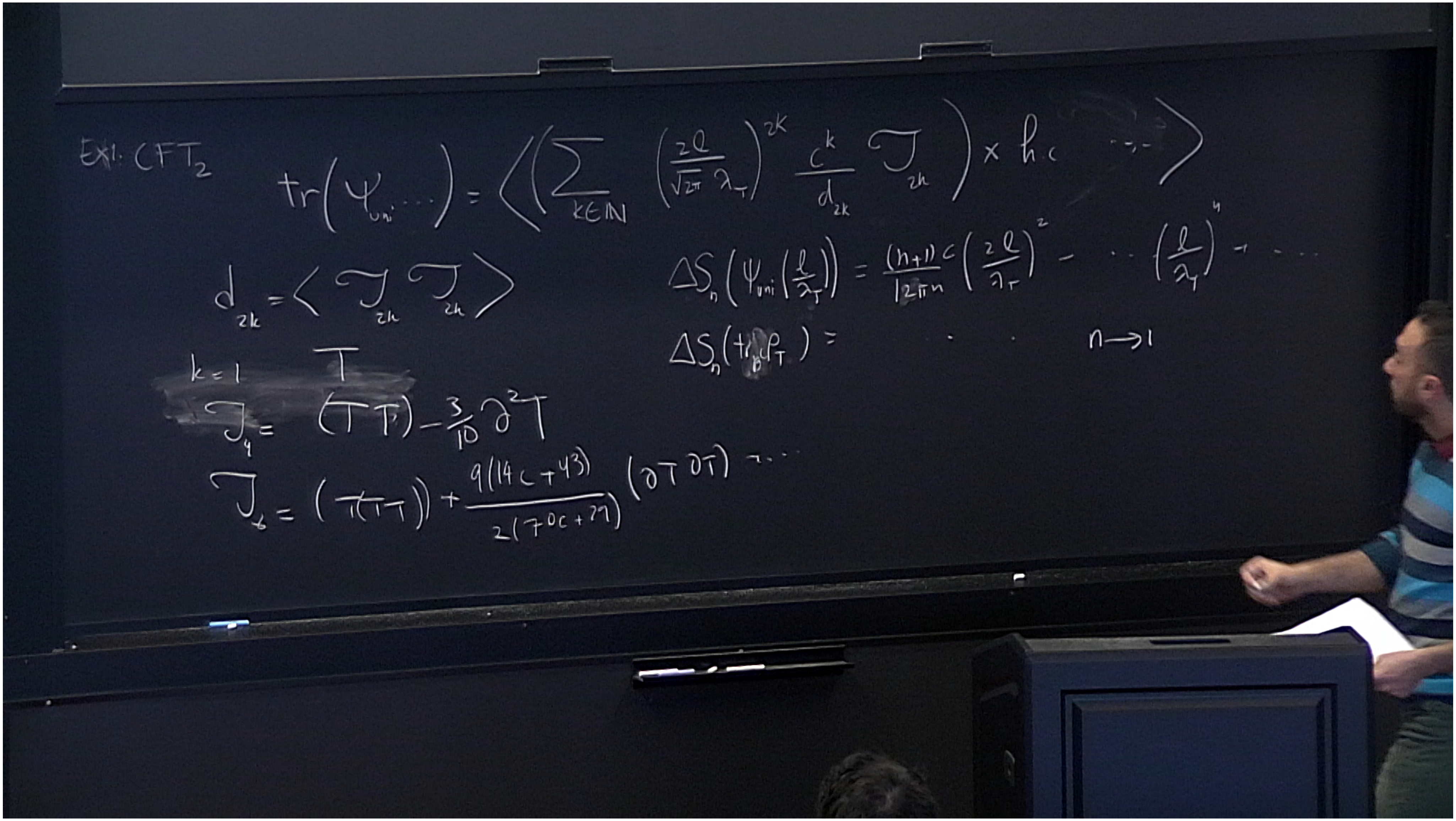
1) $\left\| \psi_a\left(\frac{\ell}{\lambda_T}\right) - \psi_{uni}\left(\frac{\ell}{\lambda_T}\right) \right\| = O(\ell/L)$

2) $\left\| \sigma_{ab} e^{i\alpha} + \sigma_{ba} e^{-i\alpha} \right\| = O(1/L)$





3) A thermal
4) ρ Ψ ρ = Ψ univ
5) Ψ univ
Ψa(ℓ/λT)
ℝd+1
Ψ univ
X0
X1
Σ p
Σ p∈A thermal
||ρ−σ|| = ||UρU+ − UσU+||


Ex1: CFT_2
$\mathrm{tr}(\psi_{uni} \ldots) = \left\langle \left(\sum_{k \in \mathbb{N}} \left(\frac{2\ell}{\sqrt{2\pi}\,\lambda_T}\right)^{2k} \frac{c^k}{d_{2k}} \mathcal{T}_{2k} \right) \times h.c. \ldots \right\rangle$
$d_{2k} = \langle \mathcal{T}_{2k} \mathcal{T}_{2k} \rangle$
$k=1 \quad T$
$\mathcal{T}_4 = (TT) - \frac{3}{10}\partial^2 T$
$\mathcal{T}_6 = (T(TT)) + \frac{9(14c+43)}{2(70c+29)}(\partial T \partial T) + \ldots$
$\Delta S_n\left(\psi_{uni}\left(\frac{\ell}{\lambda_T}\right)\right) = \frac{(n+1)c}{12\pi n}\left(\frac{2\ell}{\lambda_T}\right)^2 - \ldots \left(\frac{\ell}{\lambda_T}\right)^4 + \ldots$
$\Delta S_n(\mathrm{tr}_{B^c}\rho_T) = \quad . \quad . \quad n \to 1$

Ex1: CFT_2

$$\mathrm{tr}(\psi_{uni} \ldots) = \left\langle \left(\sum_{k \in \mathbb{N}} \left(\frac{2\ell}{\sqrt{2\pi}\,\lambda_T} \right)^{2k} \frac{c^k}{d_{2k}} \mathcal{T}_{2k} \right) \times h_c \ldots \right\rangle$$

$$d_{2k} = \langle \mathcal{T}_{2k} \mathcal{T}_{2k} \rangle$$

$k=1$ $\quad T$

$$\mathcal{T}_4 = (TT) - \frac{3}{10} \partial^2 T$$

$$\mathcal{T}_6 = (T(TT)) + \frac{9(14c+43)}{2(70c+29)} (\partial T \partial T) + \ldots$$

$$\Delta S_n\left(\psi_{uni}\left(\frac{\ell}{\lambda_T}\right)\right) = \frac{(n+1)c}{12\pi n} \left(\frac{2\ell}{\lambda_T}\right)^2 - \ldots \left(\frac{\ell}{\lambda_T}\right)^4 + \ldots$$

$$\Delta S_n(\mathrm{tr}_{B^c} \rho_T) = \ldots$$

$$S(\psi_{uni} \,\|\, \mathrm{tr}_{B^c} \rho_T) \overset{n \to 1}{=} \cancel{\Delta} H_{\rho_T} - \cancel{\Delta} S = O(1/\ell)$$

Ex1: CFT_2

$$\mathrm{tr}(\psi_{uni} \cdots) = \left\langle \left(\sum_{k \in \mathbb{N}} \left(\frac{2\ell}{\sqrt{2\pi}\,\lambda_T} \right)^{2k} \frac{c^k}{d_{2k}} \mathcal{T}_{2k} \right) \times h.c. \cdots \right\rangle$$

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$$\Delta S_n(\mathrm{tr}_{B^c} \rho_T) =$$

$$S(\psi_{uni} \,\|\, \mathrm{tr}_{B^c} \rho_T) \overset{n \to 1}{=} \cancel{\Delta} H_{\rho_T} - \cancel{\Delta} S + O(\ell)$$

β

1611.08879
Tolya Dymarsky 1703.xxxxx

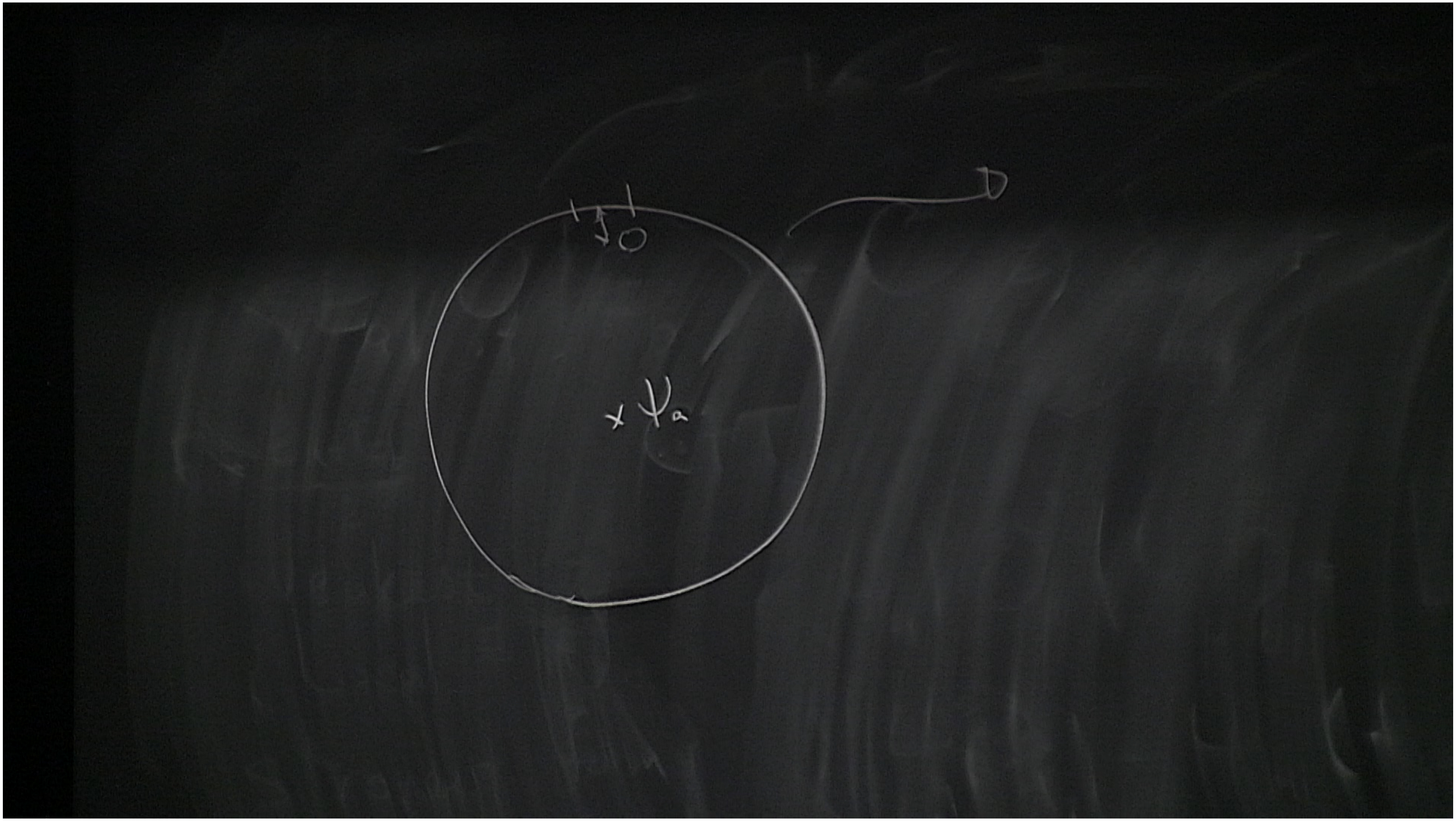
2) Thermo limit
3) Δ_{thermal}
4) $P \Psi P = \Psi_{\text{univ}}$
5) Ψ_{univ}

$$\mathrm{tr}(\Psi_{\text{uni}} \ \ ) = \left\langle \left(1 + \frac{d+1}{d} \left(\frac{2\ell}{\lambda_T}\right)^{d+1} n^\mu n^\nu T_{\mu\nu} + \cdots \right) \cdots \right\rangle$$

$\forall \ \mathcal{O}_\varphi$ rel $\quad f_p = 0$

$$\Delta S_1\left(\ell/\lambda_T\right) = \frac{4\pi^{d/2+1}}{d(d+2)\Gamma(d/2)} \left(\frac{\ell}{\lambda_T}\right)^{d+1} - \frac{2(d+1)^2\,\Gamma(d+3)\,\Gamma(d)\, d_T}{2\Gamma(5+2d)} \left(\frac{2\ell}{\lambda_T}\right)^{2(d+1)} + \cdots$$

$$\Delta S_1 = \quad \checkmark \qquad \checkmark$$


$\times \psi_a$