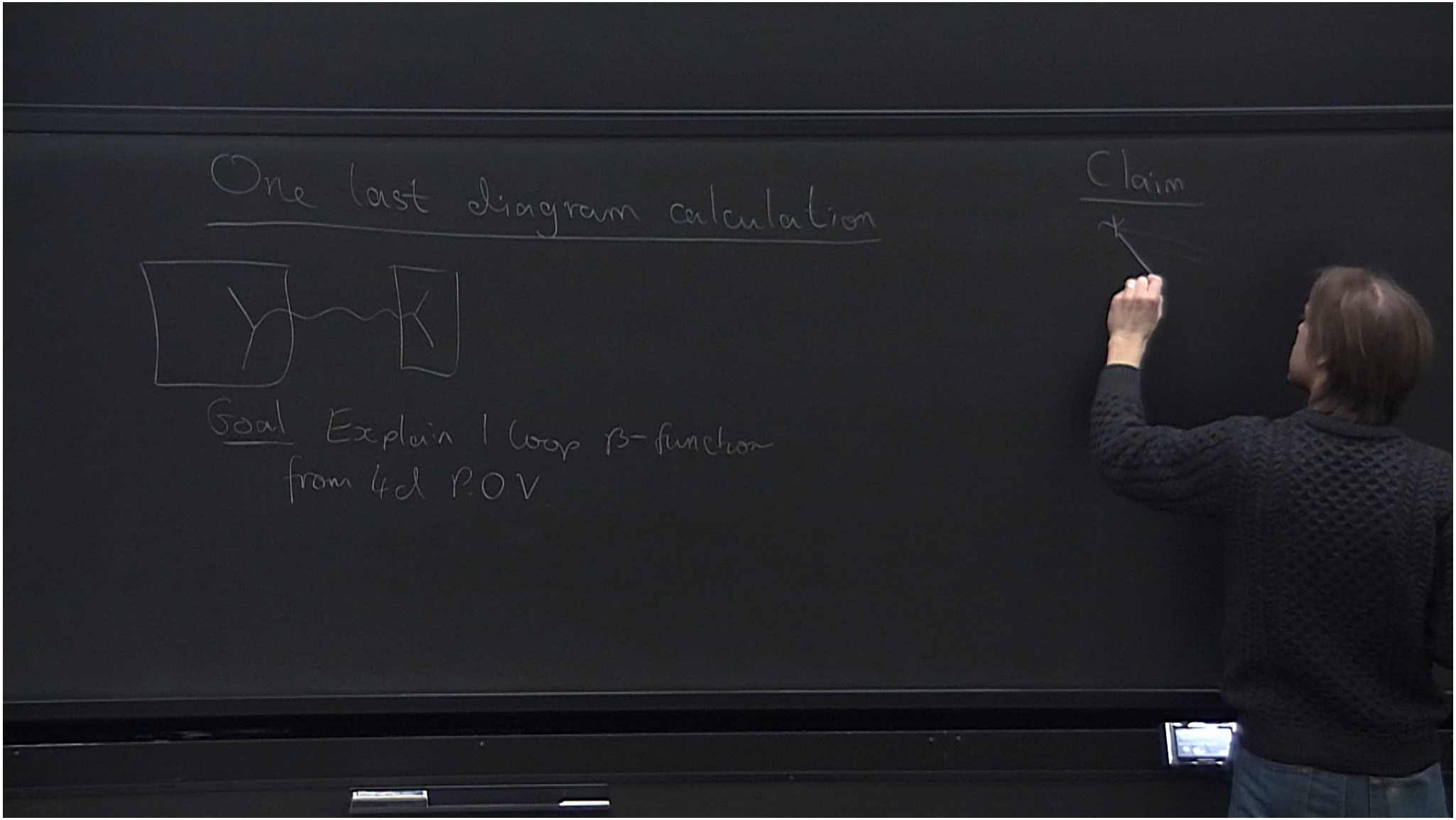
Title: PSI 2016/2017 Introduction to Quantum Integrable Systems - Lecture 14

Date: Mar 10, 2017 11:30 AM

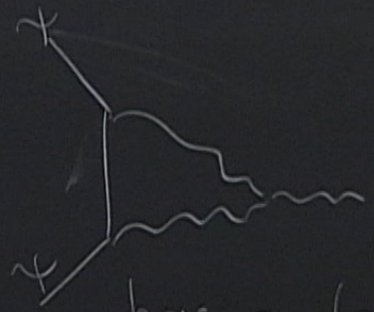
URL: http://pirsa.org/17030045

Abstract:


One last diagram calculation
Goal Explain 1 loop β-function from 4d P.O.V
Claim

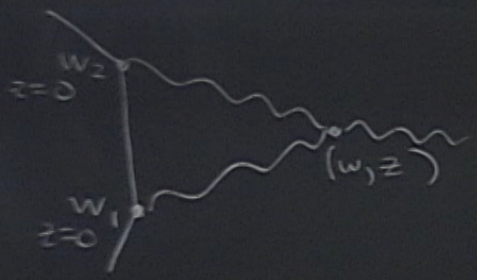
tion

Claim



has a log. divergence of the form

$$\log \varepsilon \int_{\mathbb{R}^2 \times z_0} \psi (\partial_z A) \psi$$



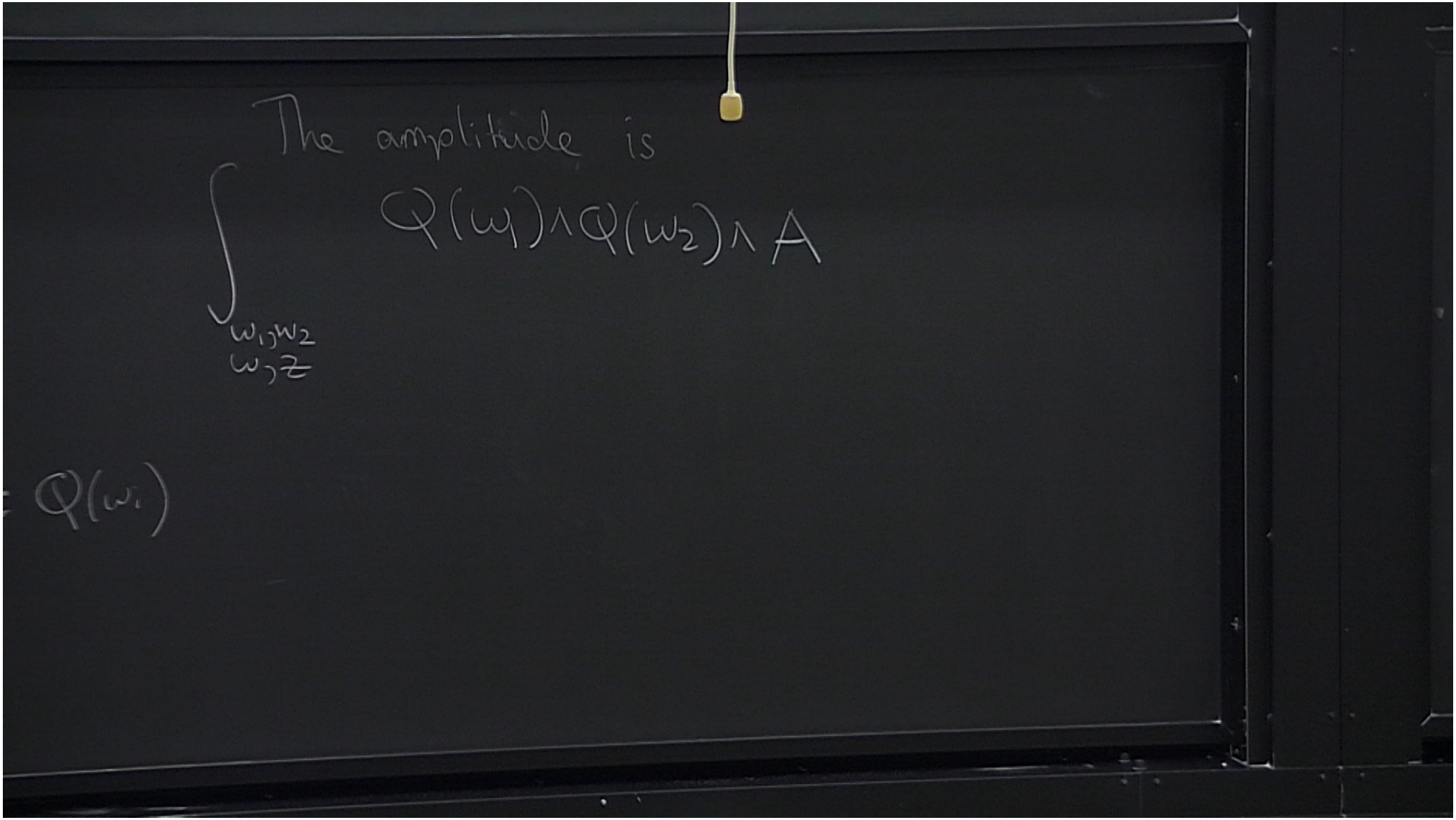
Coordinates w, z

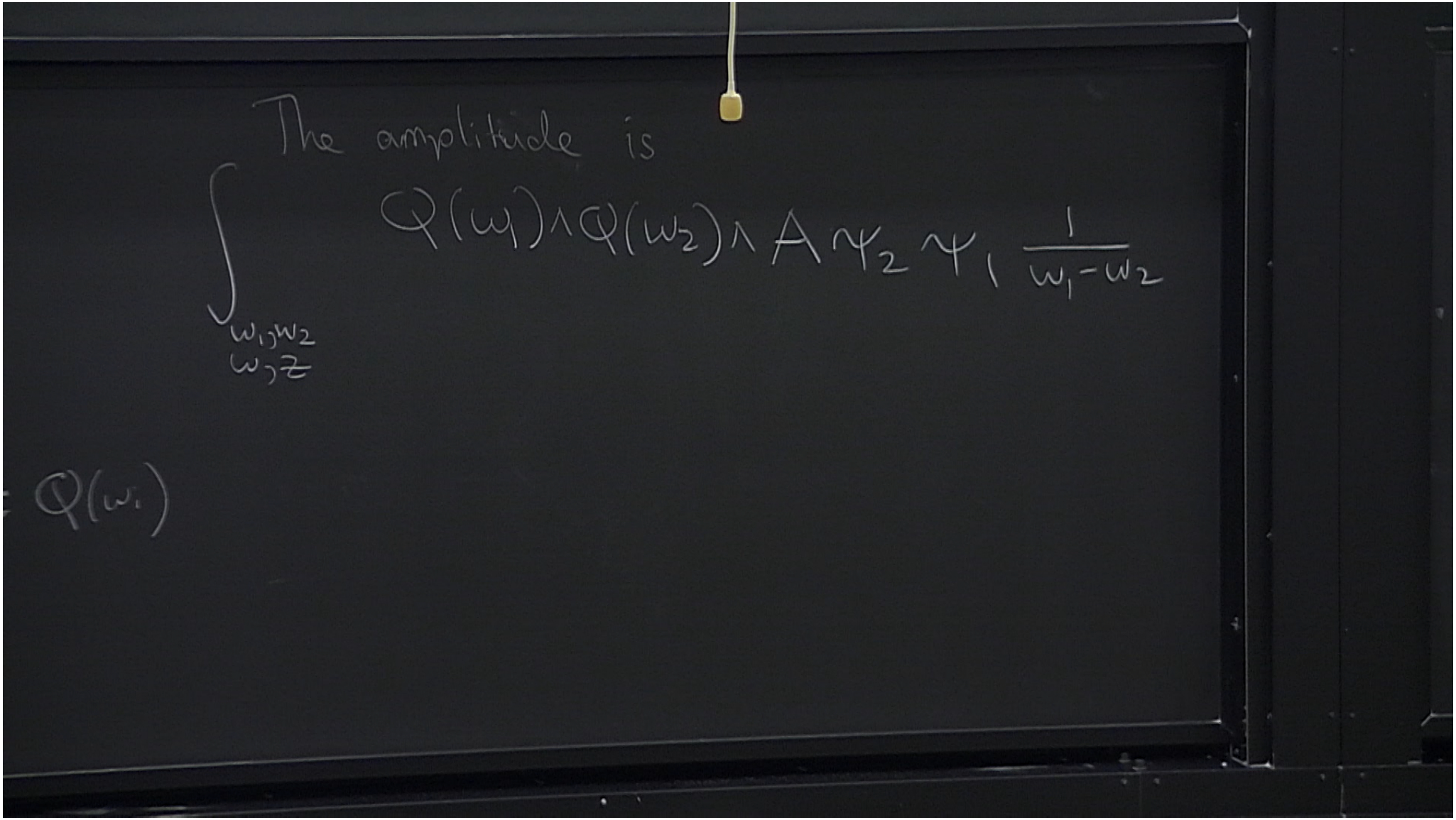
Propagators

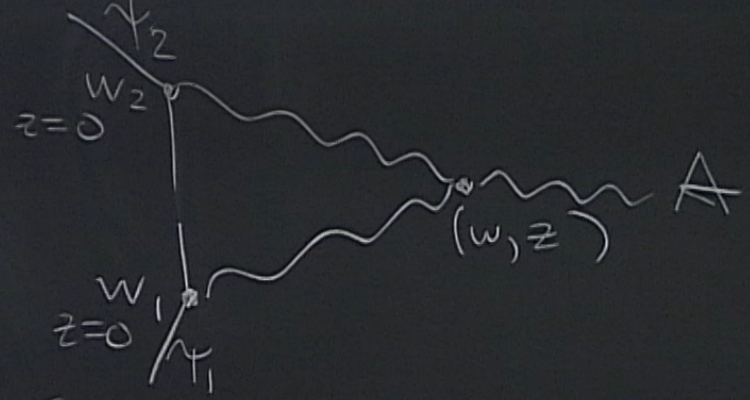
$|$ = free fermion prop. $\frac{1}{w_1 - w_2}$



$$d\bar{w}_i (dw\,\partial_z - d\bar{z}\,\partial_{\bar{w}}) \frac{1}{(|w-w_i|^2+|z|^2)} =: Q(w_i)$$


The amplitude is
$\int_{\substack{w_1,w_2 \\ w,z}} \Phi(w_1) \wedge \Phi(w_2) \wedge A$
$\Phi(w_i)$


The amplitude is
$\int_{\substack{w_1,w_2 \\ w,z}} \Phi(w_1) \wedge \Phi(w_2) \wedge A \, \psi_2 \, \psi_1 \, \frac{1}{w_1 - w_2}$
$\Phi(w_1)$



Coordinates w, z

Propagators

$|$ = free fermion prop. $\frac{1}{w_1 - w}$



$d\bar{w}_i (dw)_z$

The amplitude is

$$\int_{\substack{w_1, w_2 \\ w_3 z}} Q(w_1) \wedge Q(w_2) \wedge A \, \gamma_2 \, \gamma_1 \, \frac{1}{w_1 - w_2}$$

Assume γ_i = constant

$Q(w_i)$ Dimensional analysis: $A = z$

Integration by parts

$$\int_{w_i, z} \partial_{\bar{w}} \left(\frac{1}{(|w-w_1|^2+|z|^2)(|w-w_2|^2+|z|^2)} \right) \partial_z A \qquad \frac{1}{w_1 - w_2}$$

$A = z$

Trick: Integrate ↑ we recover

$$\frac{1}{\bar{w}_1 - \bar{w}_2}$$ ⟩ anti-chiral fermion propagator.

So we find the same log. divergence we did before.

$-\omega_z$

propagator.

So, we find if we rescale the w plane we change the surface defect by a one loop coupling

$$\hbar \int_{\mathbb{R}^2 \times z_0} \psi \partial_z A \psi$$

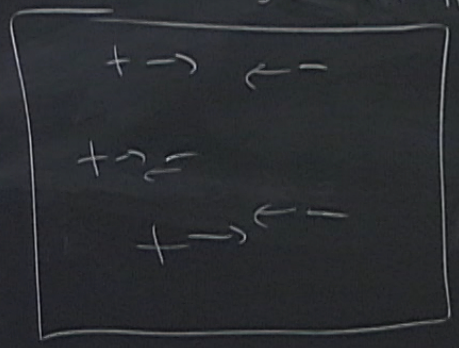
$$= \hbar \partial_z \int_{\mathbb{R}^2 \times z_0} \psi \bar{\partial}_A \psi = \hbar \partial_{z_0} \int \psi \bar{\partial}_A \psi$$

One-loop β-function moves z_0

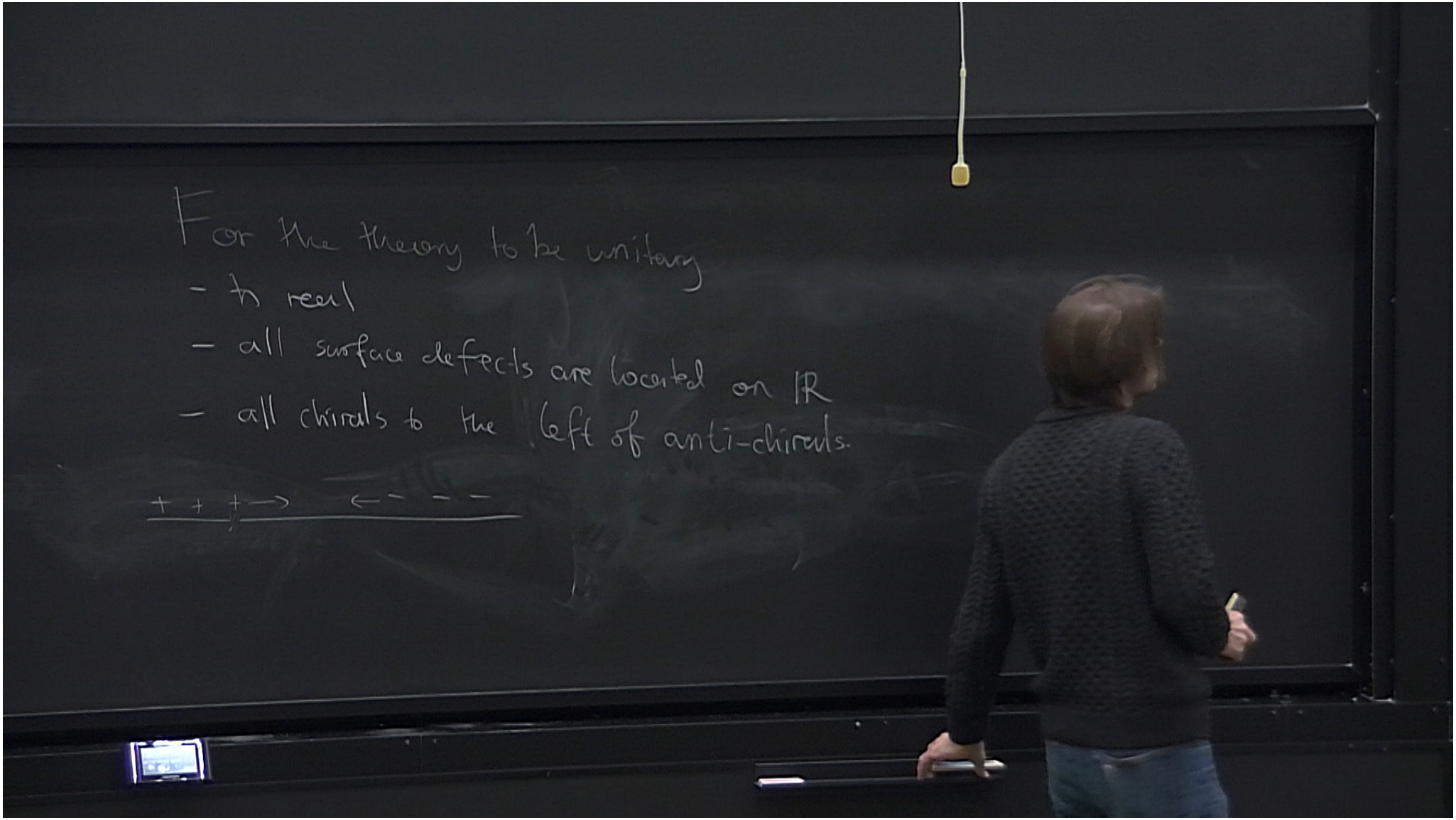
If we used an anti-chiral surface defect at z_1,

we would change z_1 to $z_1 - \hbar$

If we have a number of chiral/anti-chiral defects in the z planes, then β- fn looks like



z plane
+ chiral
− anti-chiral


For the theory to be unitary
- h real
- all surface defects are located on IR
- all chirals to the left of anti-chirals.
+ + + → ← - - -

Elliptic Case

Replace $\mathbb{C}$ by $\mathbb{C}/\mathbb{Z}\oplus\tau\mathbb{Z}$



Take our gauge

Take our gauge field to live on $\mathbb{R}^2 \times E$

$A_x, A_y, A_{\bar{z}}$ as before

Take our gauge field to live on $\mathbb{R}^2 \times E$

A_x, A_y, A_z as before

Problem If we want to integrate out A_z need to know there are no zero modes

Take our gauge field to live on $\mathbb{R}^2 \times E$

$A_x, A_y, A_{\bar{z}}$ as before

Problem If we want to integrate out $A_{\bar{z}}$ need to know there are no zero modes

Zero modes arise if $\exists f$ on E s.t. $f d\bar{z}$ is not $\bar{\partial}$ of something

Or, if $\exists f$ on E $\bar{\partial} f = 0$

Sol'n $g = sl_2$ $\sigma_1, \sigma_2, \sigma_3$ Pauli matrices

We ask that σ_i components of A have twisted periodicity

$$A^{\sigma_1}(x,y,z) = A^{\sigma_1}(x,y,z+1)$$

$$A^{\sigma_1}(x,y,z) = -A^{\sigma_1}(x,y,z+\tau)$$

A^{σ_2} periodic under $z \to z+\tau$
anti-periodic $z \to z+1$

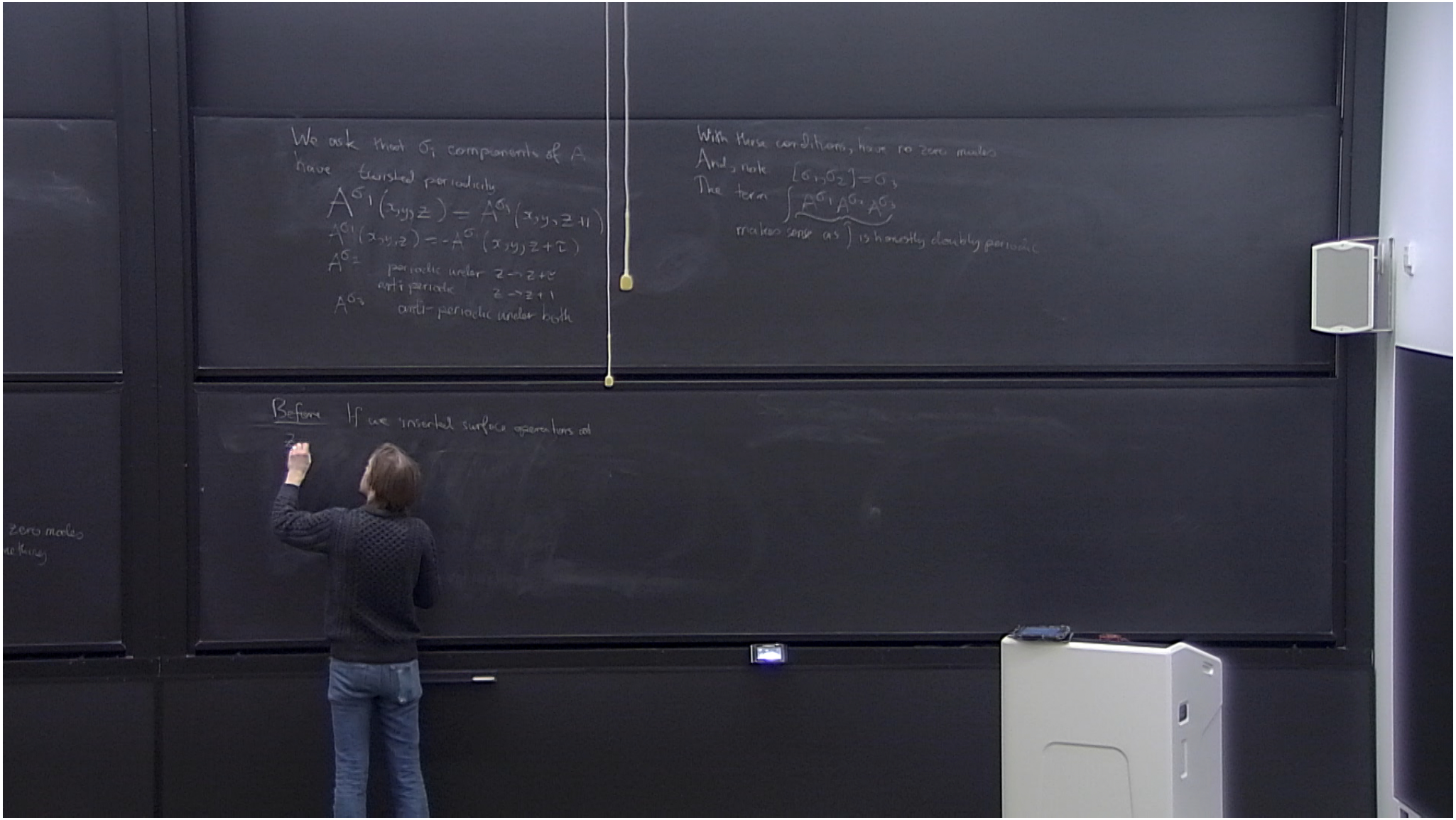
A^{σ_3} anti-periodic under both.

With these conditions, have no zero modes

And, note. $[\sigma_1, \sigma_2] = \sigma_3$

The term $\int A^{\sigma_1} A^{\sigma_2} A^{\sigma_3}$

makes sense as $\int$ is honestly doubly periodic

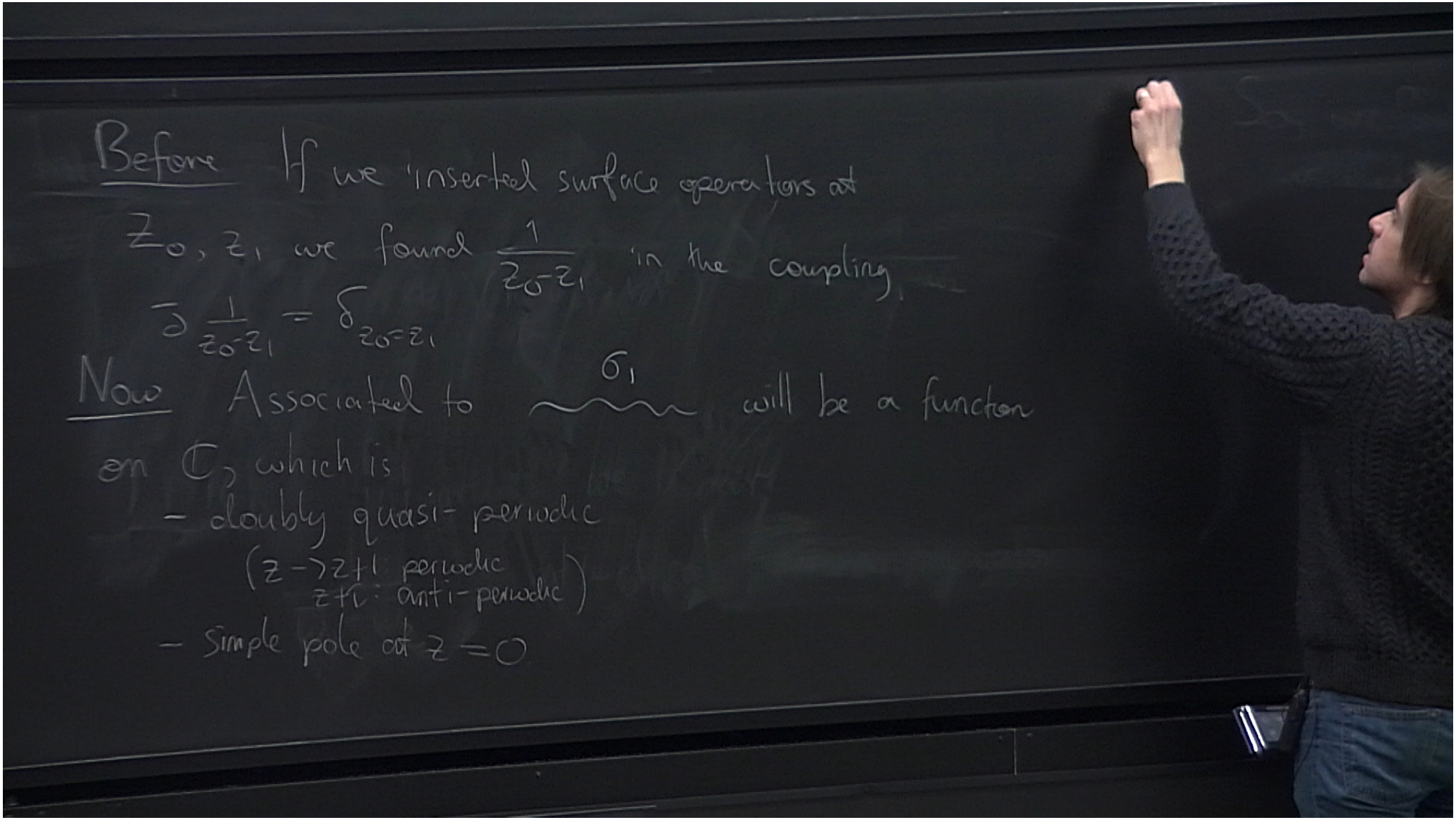

We ask that σ_i components of A have twisted periodicity
$A^{\sigma_1}(x,y,z) = A^{\sigma_1}(x,y,z+1)$
$A^{\sigma_1}(x,y,z) = -A^{\sigma_1}(x,y,z+\tau)$
A^{σ_2} periodic under $z \to z+\tau$
anti-periodic $z \to z+1$
A^{σ_3} anti-periodic under both
With Hirsa conditions, have no zero modes
And, note $[\sigma_1,\sigma_2] = \sigma_3$
The term $\int A^{\sigma_1} A^{\sigma_2} A^{\sigma_3}$
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Before If we inserted surface operations at
zero modes

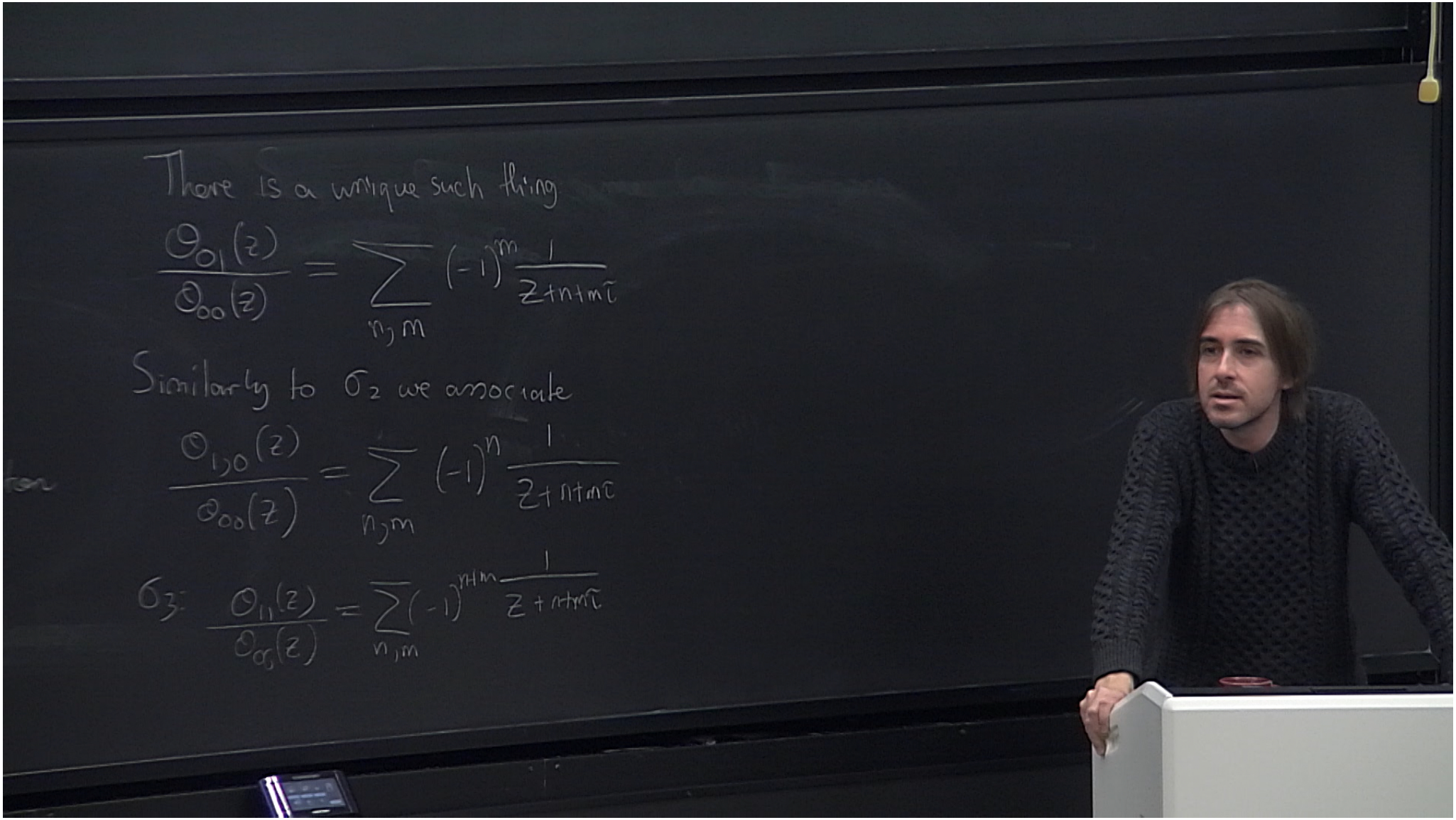
Before If we inserted surface operators at z_0, z_1 we found $\frac{1}{z_0 - z_1}$ in the coupling.

$$\bar{\partial} \frac{1}{z_0 - z_1} = \delta_{z_0 = z_1}$$

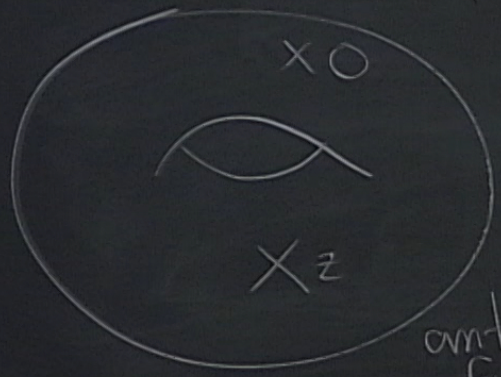
Now Associated to σ_1 (wavy line) will be a function on $\mathbb{C}$, which is

- doubly quasi-periodic
  ($z \to z+1$ periodic
  $z \to z+\tau$: anti-periodic)
- simple pole at $z = 0$




There is a unique such thing
$\frac{\Theta_{01}(z)}{\Theta_{00}(z)} = \sum_{n,m} (-1)^m \frac{1}{z+n+m\tau}$
Similarly to σ_2 we associate
$\frac{\Theta_{10}(z)}{\Theta_{00}(z)} = \sum_{n,m} (-1)^n \frac{1}{z+n+m\tau}$
σ_3: $\frac{\Theta_{11}(z)}{\Theta_{00}(z)} = \sum_{n,m} (-1)^{n+m} \frac{1}{z+n+m\tau}$

3 chiral fermions in 3d rep of su(2)



anti-chiral fermions 3d rep of su(2)

If a

If we integrate out gauge field, using this prescription, we generate a quartic interaction

$$\psi_2 \tilde{\psi}_2 \psi_3 \tilde{\psi}_3 \frac{\Theta_{01}(z)}{\Theta_{00}(z)} + \psi_1 \tilde{\psi}_1 \psi_3 \tilde{\psi}_3 \frac{\Theta_{10}(z)}{\Theta_{00}(z)} + \psi_1 \tilde{\psi}_1 \psi_2 \tilde{\psi}_2 \frac{\Theta_{11}(z)}{\Theta_{00}(z)}$$

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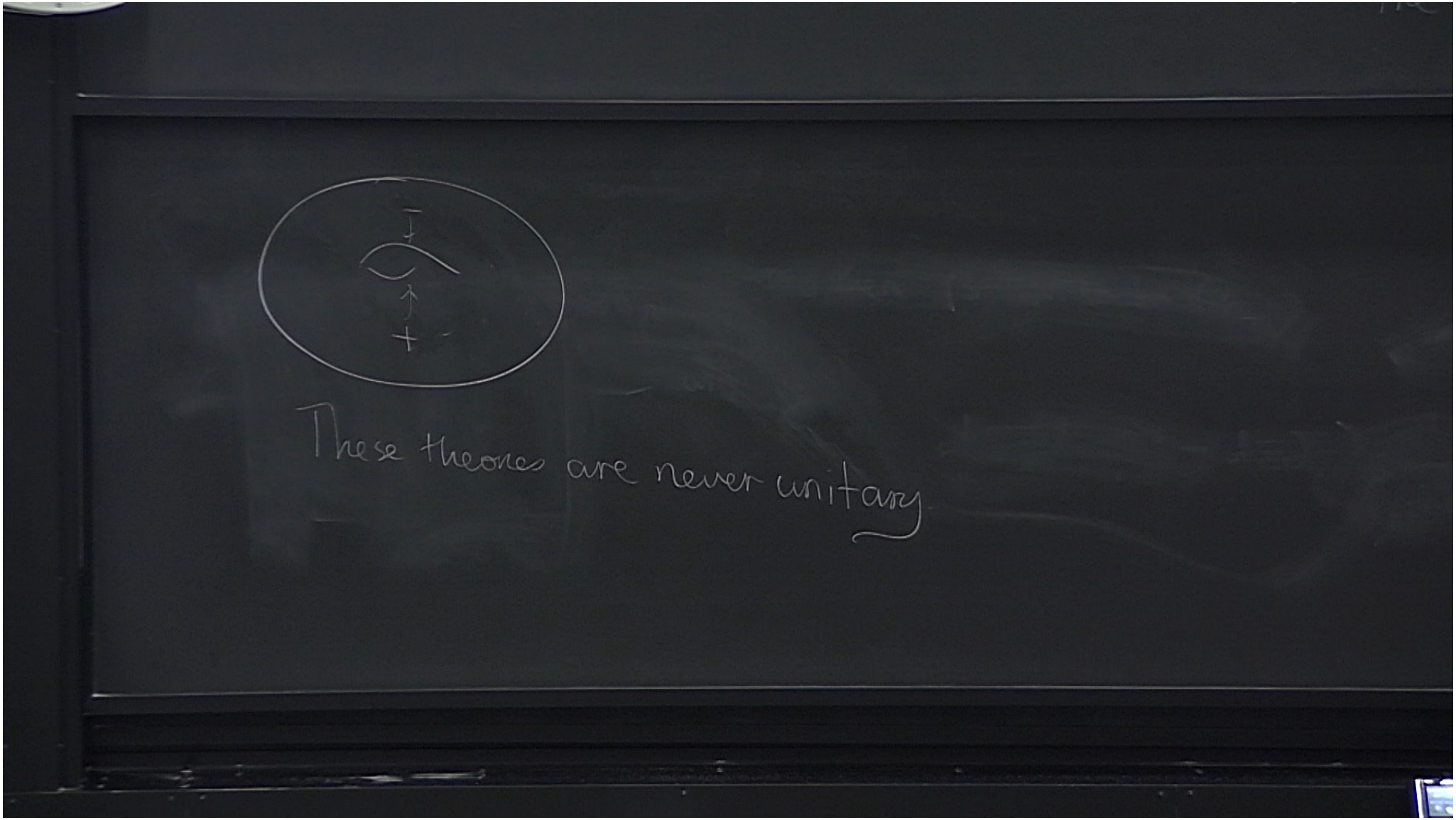
$$\psi_2\tilde{\psi}_2\psi_3\tilde{\psi}_3\,\frac{\Theta_{01}(z)}{\Theta_{00}(z)} + \psi_1\tilde{\psi}_1\psi_3\tilde{\psi}_3\,\frac{\Theta_{10}(z)}{\Theta_{00}(z)} + \psi_1\tilde{\psi}_1\psi_2\tilde{\psi}_2\,\frac{\Theta_{11}(z)}{\Theta_{00}(z)}$$

Claim
- This theory is always integrable
- Has symmetry related to the elliptic quantum group

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$$\psi_2\tilde{\psi}_2\psi_3\tilde{\psi}_3\,\frac{\Theta_{01}(z)}{\Theta_{00}(z)} + \psi_1\tilde{\psi}_1\psi_3\tilde{\psi}_3\,\frac{\Theta_{10}(z)}{\Theta_{00}(z)} + \psi_1\tilde{\psi}_1\psi_2\tilde{\psi}_2\,\frac{\Theta_{11}(z)}{\Theta_{00}(z)}$$

Claim
- This theory is always integrable
- Has symmetry related to the elliptic quantum group
- The β-function is realized by moving points around one


These theories are never unitary.