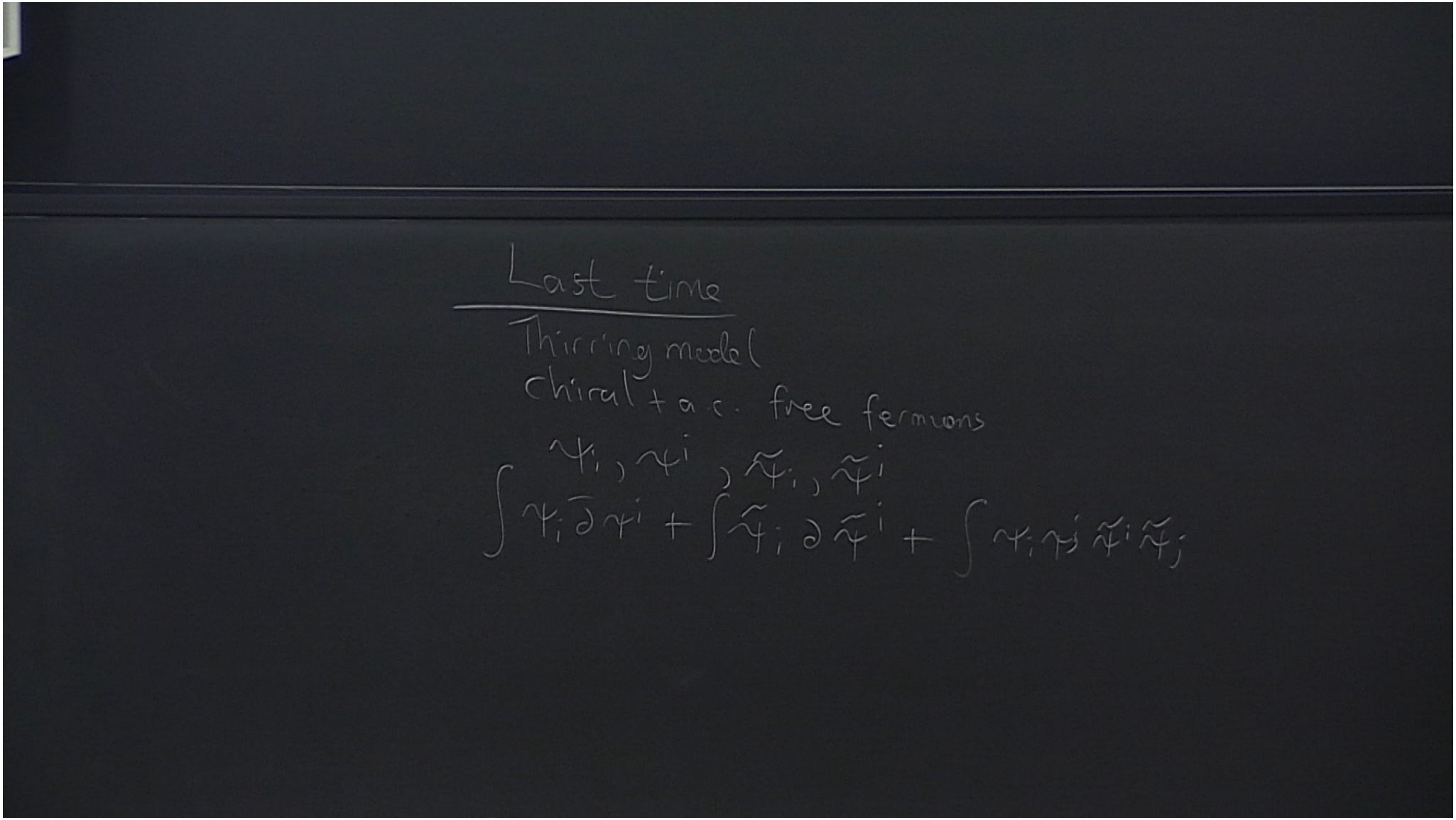
Title: PSI 2016/2017 Introduction to Quantum Integrable Systems - Lecture 13

Date: Mar 09, 2017 11:30 AM

URL: http://pirsa.org/17030044

Abstract:


Last time
Thirring model
chiral + a.c. free fermions
$\psi_i, \psi^i, \tilde{\psi}_i, \tilde{\psi}^i$
$\int \psi_i \partial \psi^i + \int \tilde{\psi}_i \partial \tilde{\psi}^i + \int \psi_i \psi_j \tilde{\psi}^i \tilde{\psi}_j$

From 4d point of view

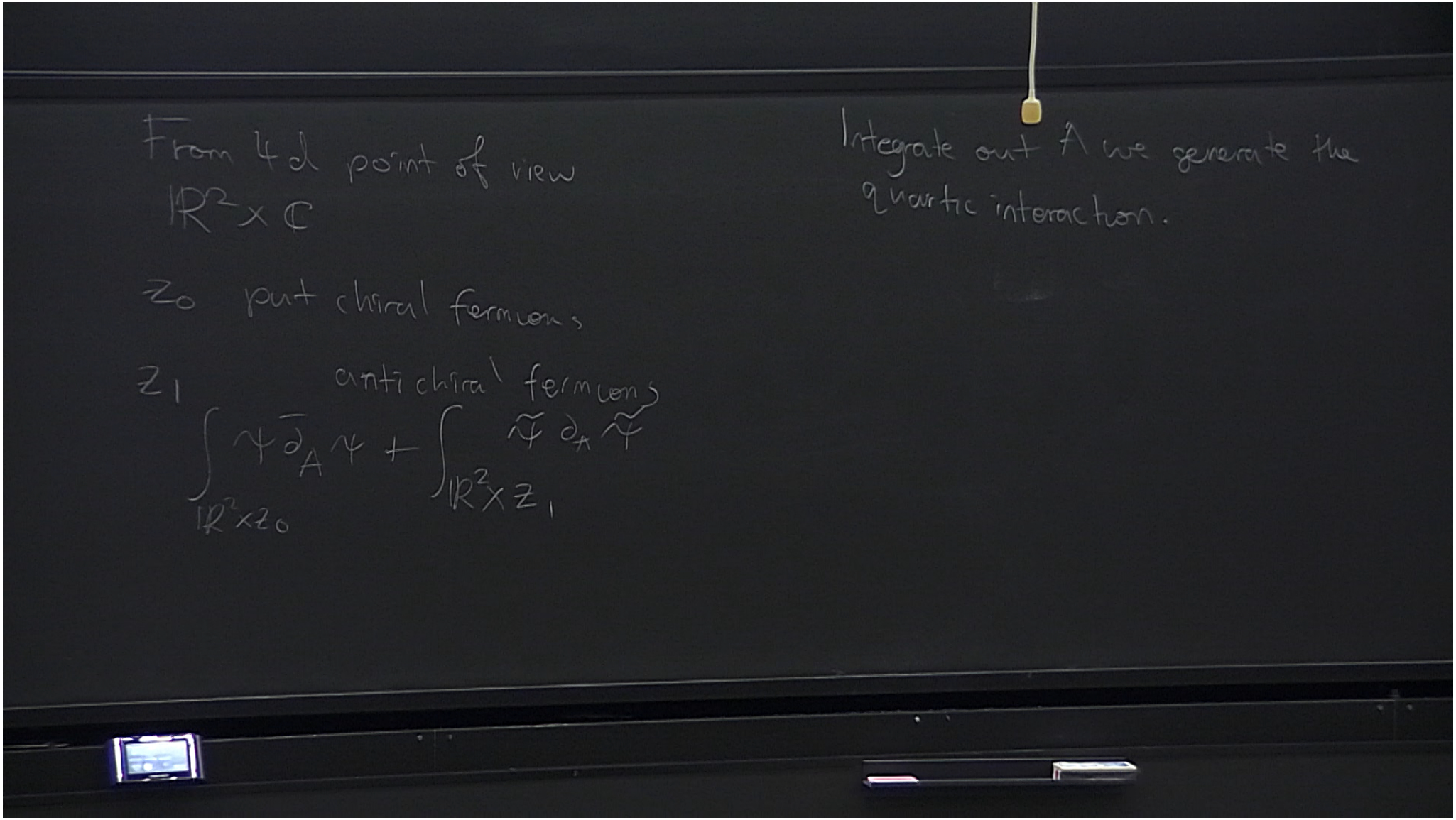
$\mathbb{R}^2 \times \mathbb{C}$

z_0 put chiral fermions

z_1 antichiral fermions

$$\int_{\mathbb{R}^2 \times z_0} \psi \bar{\partial}_A \psi + \int_{\mathbb{R}^2 \times z_1} \tilde{\psi} \partial_A \tilde{\psi}$$

Integrate out A we generate the quartic interaction.


From 4d point of view
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Integrate out A we generate the quartic interaction.

Yangian Symmetry

Chiral fermion has a current given by how it couples to a gauge field

If A is a 2d gauge field

$$\int \psi \bar{\partial}_A \psi = \int \psi_i \bar{\partial} \psi^i + \int \psi_i (A_x^{ij} - i A_y^{ij}) \psi^j$$

$$J_x^{ch} = \psi_i \psi^j$$

$$J_y^{ch} = -i\, \psi_i \psi^j$$

$$J_x = J_x^{ch} + J_x^{ac}$$

$$J_y = J_y^{ch} + J_y^{ac}$$

Anti-chiral theory

$$J_x^{a\cdot c} = \tilde{\psi}_i \tilde{\psi}^j$$

$$J_y^{a\cdot c} = i\, \tilde{\psi}_i \tilde{\psi}^j$$

$= -iA_y^{ij})\psi^j$

Define an operator (charge)

$$t^i_j(0) = \int_x J^i_{j,x}\, dx$$

Level 1 generator is defined by

$$t^i_j(1) = \int_x J^i_{j;y}\, dx + \int_{x<x'} J^i_{k;x}\, J^k_{j;x'}\, dx\, dx'$$

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Claim

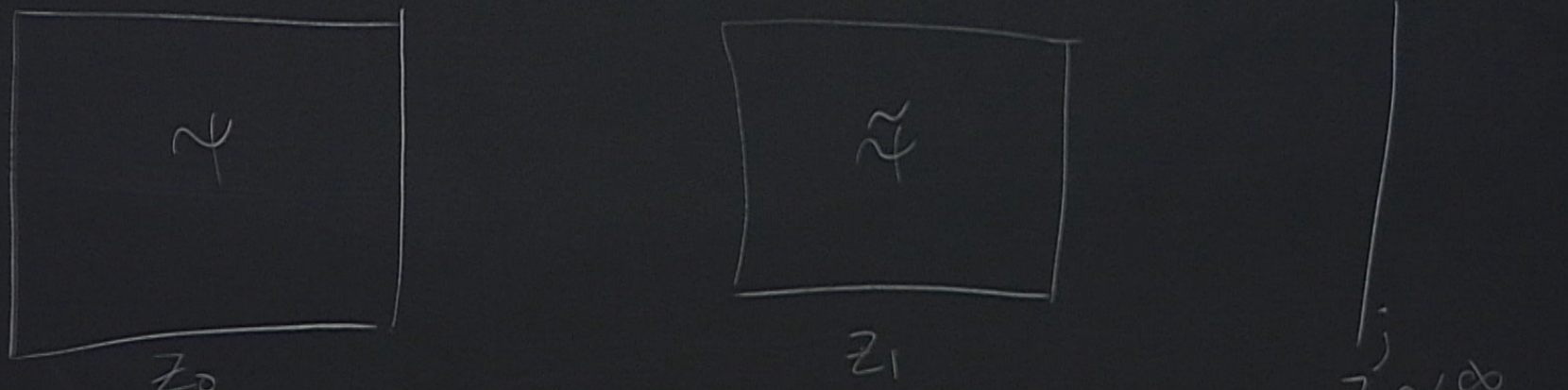
1) These are conserved charges:
we can move the contour of integration freely

2) These operators satisfy the relations of the Yangian

$$[t^i_j(0), t^k_l(0)] = \delta^k_j t^i_l(0) - \delta^i_l t^k_j(0)$$

$[t^i_j(1), t^k_l(1)]$ includes non-trivial correction given by the RTT relation.

From the 4d field theory point of view:



If we $\int$ out the gauge field A,

then

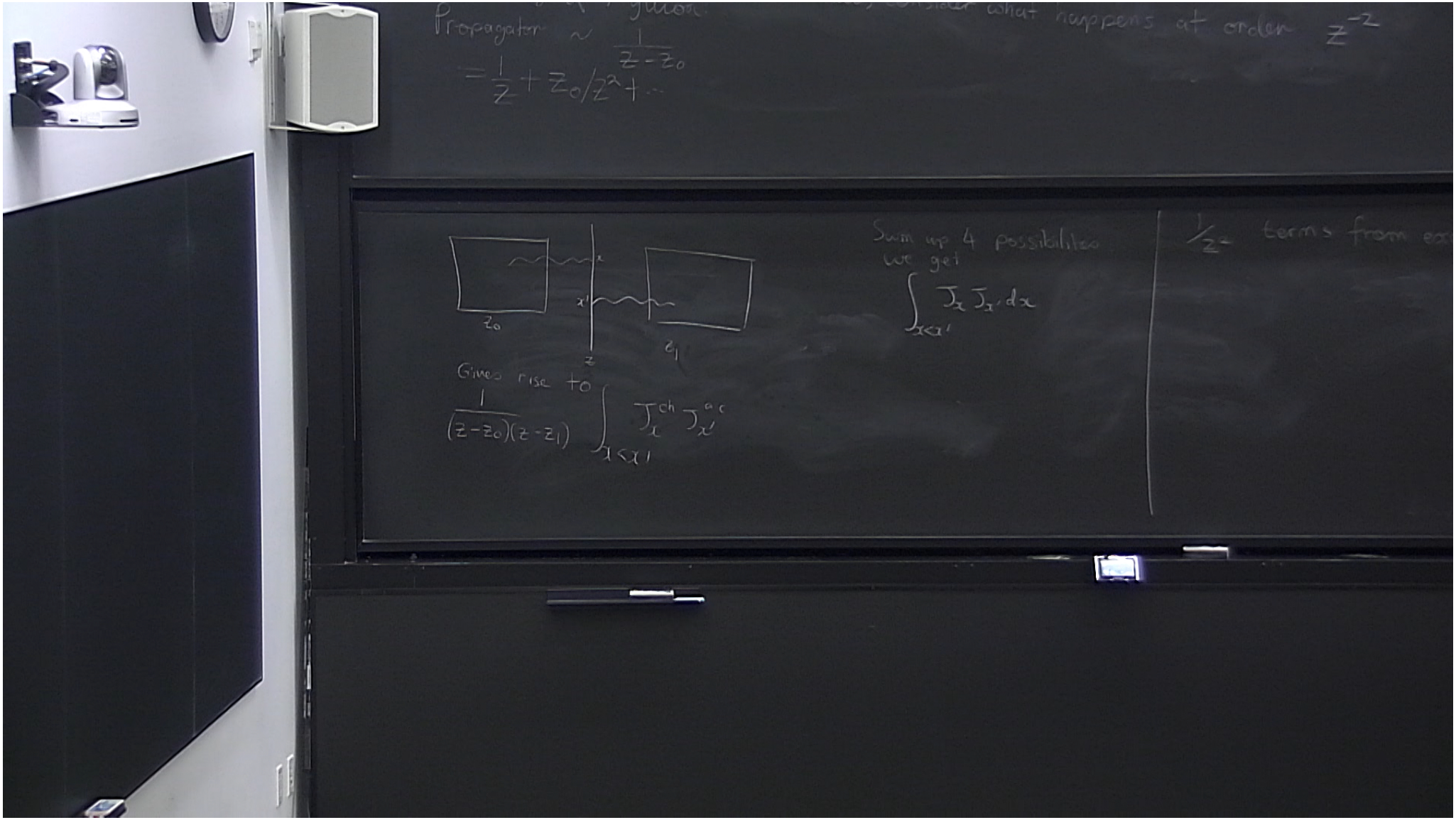
- The fermionic systems are coupled
- The Wilson line gives a line defect

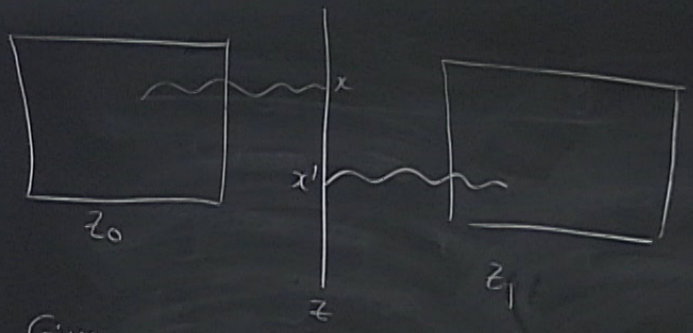
If we $\int$ out the gauge field A,

then

- The fermionic systems are coupled
- The Wilson line gives a line defect
- This line defect is conserved
- Operators $T^i_j(z) = \delta^i_j + \hbar \sum t^i_j(n) z^{-n-1}$

  define an action of the Yangian on the Hilbert space of 2d theory


Propagator ~ $\frac{1}{z-z_0}$
$= \frac{1}{z} + z_0/z^2 + \cdots$
what happens at order z^{-2}
z_0
x
x'
z
z_1
Sum up 4 possibilities we get
$\int_{x<x'} J_x J_{x'} dx$
$\frac{1}{z^2}$ terms from
Gives rise to
$\frac{1}{(z-z_0)(z-z_1)} \int_{x<x'} J_x^{ch} J_{x'}^{a c}$



Sum up 4 possibilities we get

$$\int_{x<x'} J_x J_{x'} dx$$

Gives rise to

$$\frac{1}{(z-z_0)(z-z_1)} \int_{x<x'} J_x^{ch} J_{x'}^{a.c}$$

Sum up 4 possibilities we get

$$\int_{x<a'} J_x J_{x'} dx$$

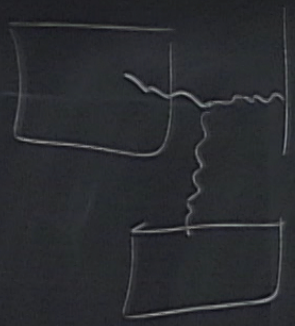
$1/z^2$ terms from exchange of a single gluon:

$$z_0/z^2 \int J_x^{ch} + z_1/z^2 \int J_x^{a.c.}$$

Assume $z_1 = -z_0$

$$\frac{z_0}{z^2} \int J_x^{ch} - J_x^{a.c.} = \frac{z_0}{z^2} \int J_y$$

One more term

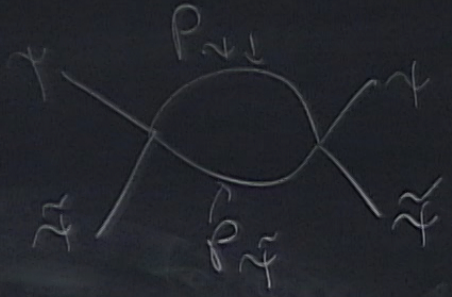


Beta Function

The theory

$$\int \tau \bar{\partial} \tau + \int \tau \partial \tau + c \int \tau \tau \tilde{\tau} \tilde{\tau}$$

has a 1-loop β-function which changes the parameter c.

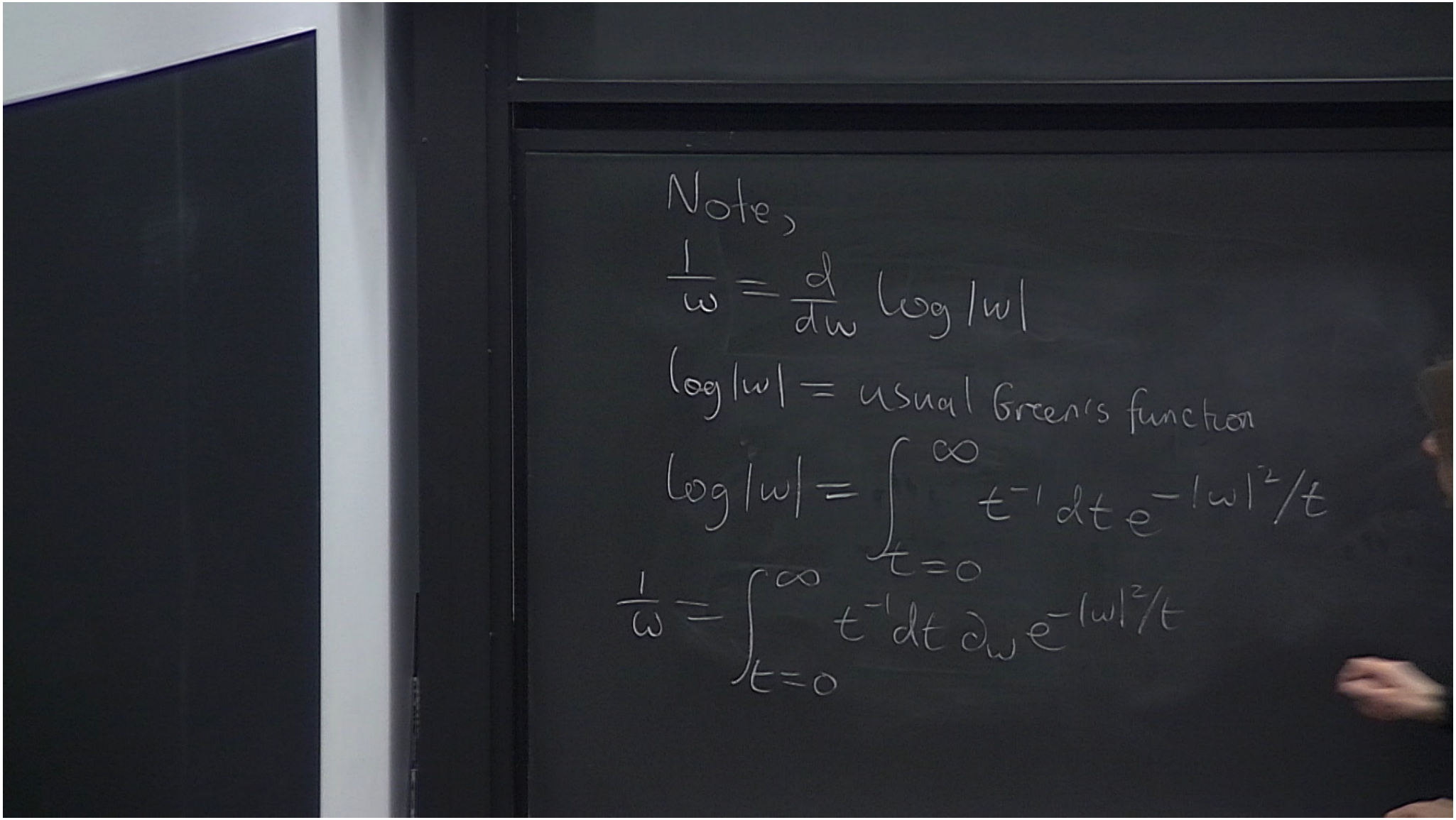


This diagram has a logarithmic divergence.

$\int \gamma \frac{d}{d\bar{w}} \gamma$ leads to a propagator

$$\frac{1}{d\bar{w}} = \frac{1}{2\pi i}\frac{1}{w}$$

$$\frac{d}{d\bar{w}}\frac{1}{w} = 2\pi i \delta_0$$


Note,
$\frac{1}{w} = \frac{d}{dw} \log|w|$
$\log|w|$ = usual Green's function
$\log|w| = \int_{t=0}^{\infty} t^{-1} dt\, e^{-|w|^2/t}$
$\frac{1}{w} = \int_{t=0}^{\infty} t^{-1} dt\, \partial_w e^{-|w|^2/t}$

$= -\int t^{-2}\, \bar{w}\, e^{-|w|^2/t}\, dt$



Similarly, anti-chiral propagator is

$-\int_{s=0}^{\infty} s^{-2}\, w\, e^{-|w|^2/s}\, ds$

UV cut-off integrate s,t from ε to ∞

$\rightsquigarrow \int_w \int_{s,t>\varepsilon} s^{-2} t^{-2} |w|^2 e^{-|w|^2(t^{-1}+s^{-1})}\, ds\, dt\, dw\, d\bar{w}$

$= -\int t^{-2}\, \bar{w}\, e^{-|w|^2/t}\, dt$



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UV cut-off integrate s,t from ε to ∞

$\rightsquigarrow \int_w \int_{s,t\geq\varepsilon} s^{-2} t^{-2} |w|^2 e^{-|w|^2(t^{-1}+s^{-1})}\, ds\, dt\, dw\, d\bar{w}$

$\tilde{w} = w\sqrt{t^{-1}+s^{-1}}$

$\int_{\tilde{w}} e^{-|\tilde{w}|^2} d\tilde{w}\, d\bar{\tilde{w}} \int_{t,s} \frac{s^{-2}t^{-2}}{(t^{-1}+s^{-1})^2}\, ds\, dt$

C

$\int_{t,s\geq\varepsilon} \frac{1}{(t+s)^2}\, ds\, dt \sim \log \varepsilon$

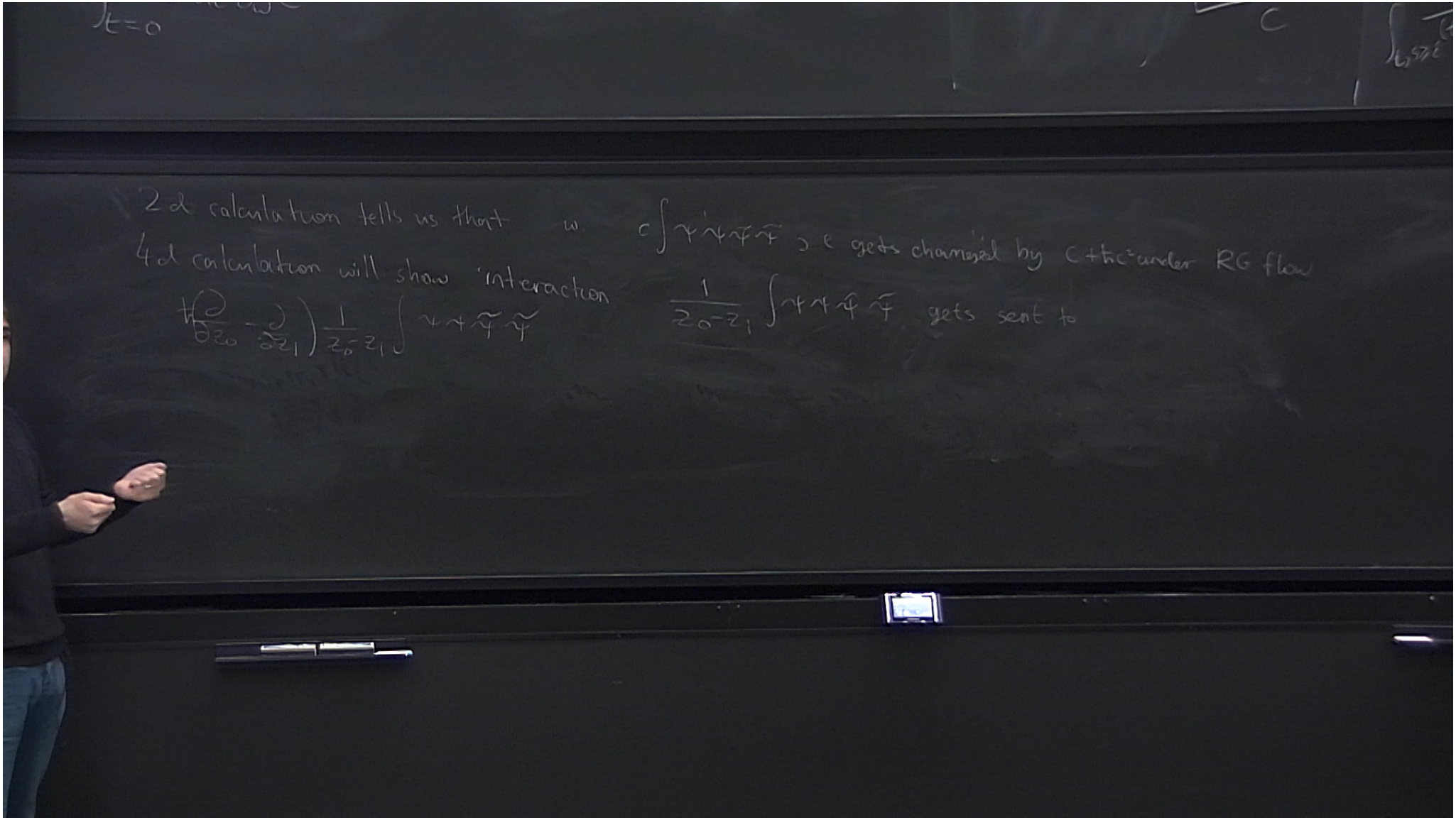
4d Version

There's a one-loop log divergence supported on each surface operator which gives a β-function in x-y plane

$\therefore$ The RG flow under scaling x-y plane moves the surface operators in z-plane,

Chiral surface operator at z_0 is moved to $z_0 + t$

A.c. at z_1 is moved to $z_1 - t$


2d calculation tells us that
4d calculation will show interaction
gets changed by
under RG flow
gets sent to

2d calculation tells us that $c\int \psi\psi\bar{\psi}\bar{\psi}$ gets changed by $C+\hbar c^2$ under RG flow

4d calculation will show interaction $\frac{1}{z_0 - z_1}\int \psi\psi\bar{\psi}\bar{\psi}$ gets sent to

$$\left(\frac{\partial}{\partial z_0} - \frac{\partial}{\partial z_1}\right)\frac{1}{z_0 - z_1}\int \psi\psi\bar{\psi}\bar{\psi}$$