Title: PSI 2016/2017 Introduction to Quantum Integrable Systems - Lecture 12

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URL: http://pirsa.org/17030043

Abstract:

Continuum 2d integrable theories

Thirring model:

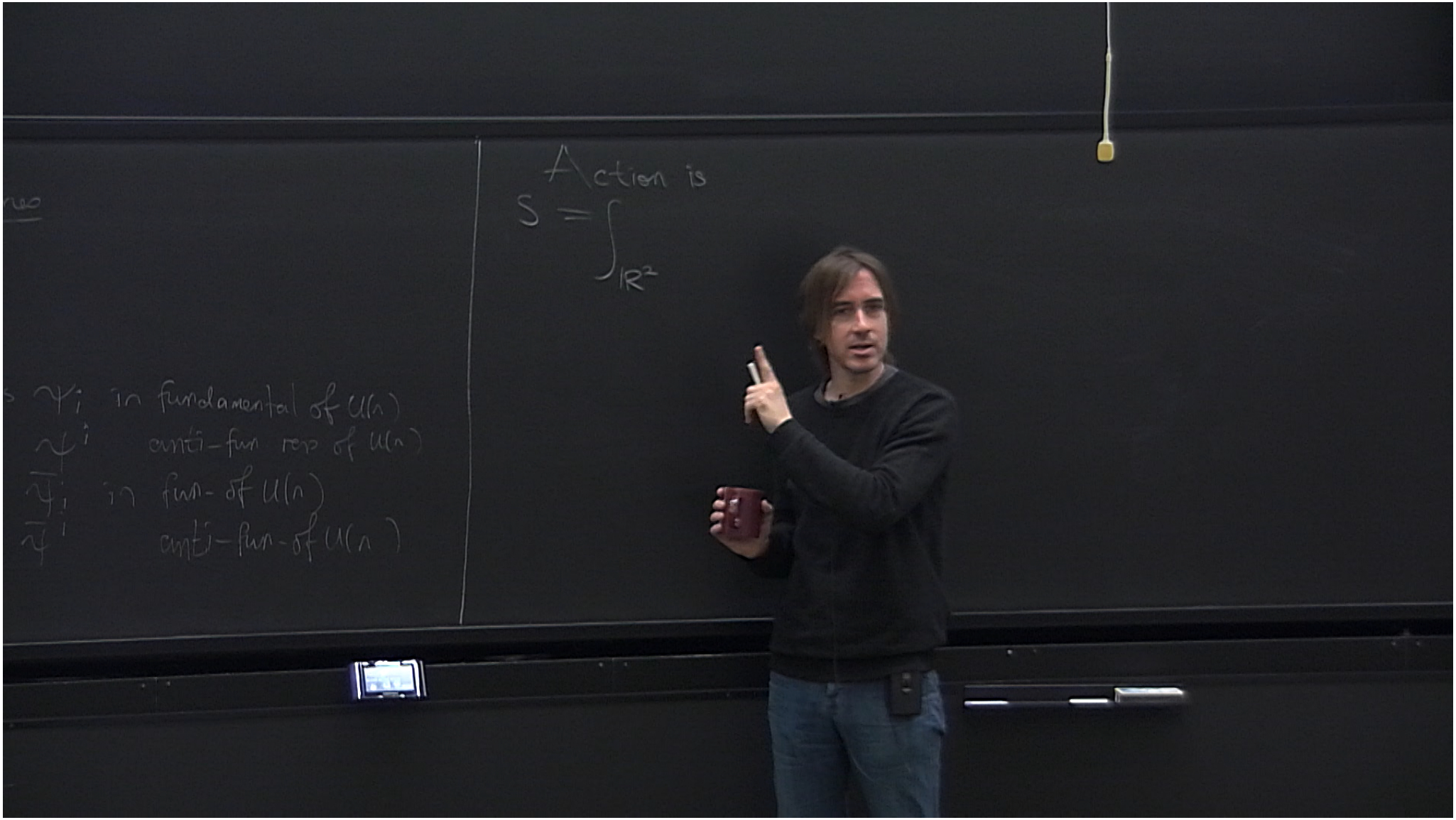
$U(n)$ symmetry

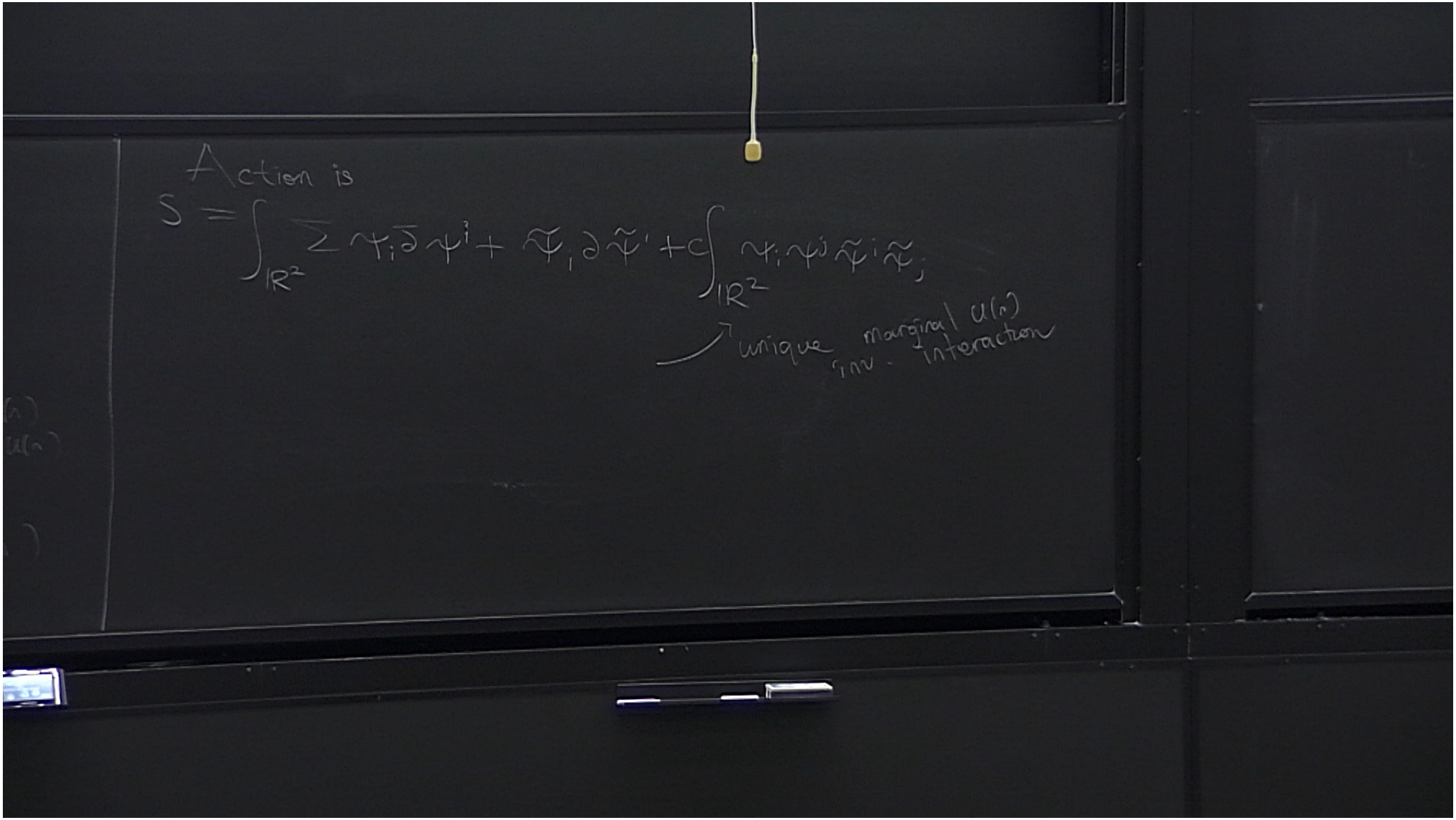
Fields are: n chiral fermions ψ_i in fundamental of $U(n)$

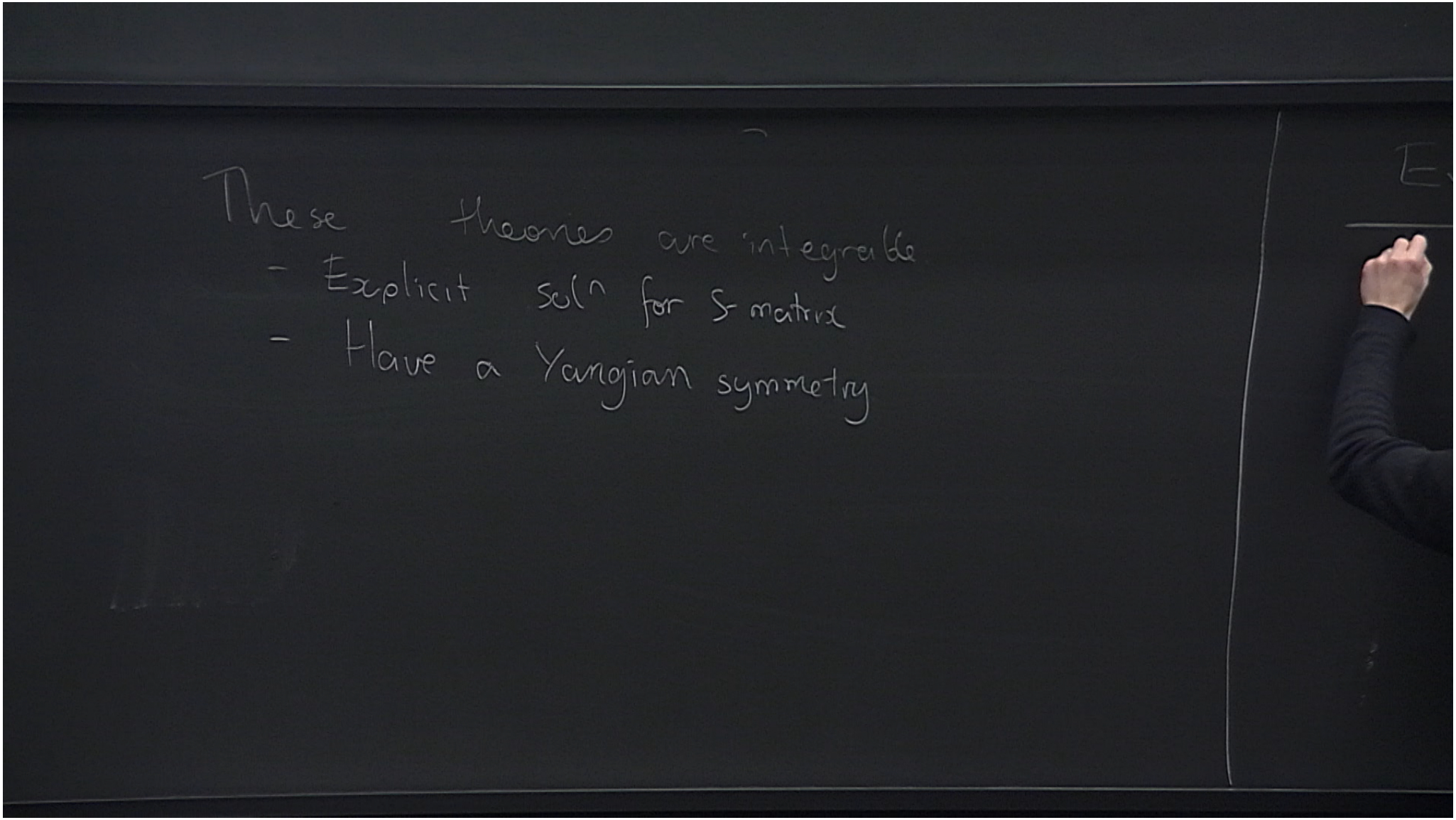
n " " χ^i anti-fun rep of $U(n)$

n anti-chiral $\bar{\psi}_i$ in fun- of $U(n)$

n anti-chiral $\bar{\chi}^i$ anti-fun- of $U(n)$


Action is
$S = \int_{\mathbb{R}^2}$
ψ_i in fundamental of $U(n)$
ψ^i anti-fun rep of $U(n)$
$\bar{\psi}_i$ in fun-of $U(n)$
$\bar{\psi}^i$ anti-fun-of $U(n)$


Action is
$S = \int_{\mathbb{R}^2} \sum \psi_i \bar{\partial} \psi^i + \tilde{\psi}_i \partial \tilde{\psi}^i + c \int_{\mathbb{R}^2} \psi_i \psi^j \tilde{\psi}^i \tilde{\psi}_j$
unique marginal $U(n)$ inv. interaction


These theories are integrable
- Explicit soln for S-matrix
- Have a Yangian symmetry

anti-fun-of $U(n)$

Engineering from 4d $(G = U(n))$

On $\mathbb{R}^2 \times \mathbb{C}$, we'll insert 2d surface defects at $\mathbb{R}^2 \times z_0$, $\mathbb{R}^2 \times z_1$

At z_0, we'll insert chiral free fermions in fun + anti-fun rep of $U(n)$

Extra fields $\psi_i(x,y)$ $\psi^i(x,y)$ with action

$$\Sigma \int \psi_i \frac{\partial}{\partial \bar{w}} \psi^i + \Sigma \int \psi_i A^{ij}_{\bar{w}} \psi^j$$

At z_1, we can introduce an anti-chiral free fermion $\tilde{\Psi}_1, \tilde{\Psi}^i$, $\int \tilde{\Psi}_1 \partial_w \tilde{\Psi}^i + \Sigma \int \tilde{\Psi}_1 A_w^{ij} \tilde{\Psi}^j$

Claim When we integrate out gauge field A, these models become coupled and give Thirring model.

Consider field sourced by surface op at z_0.

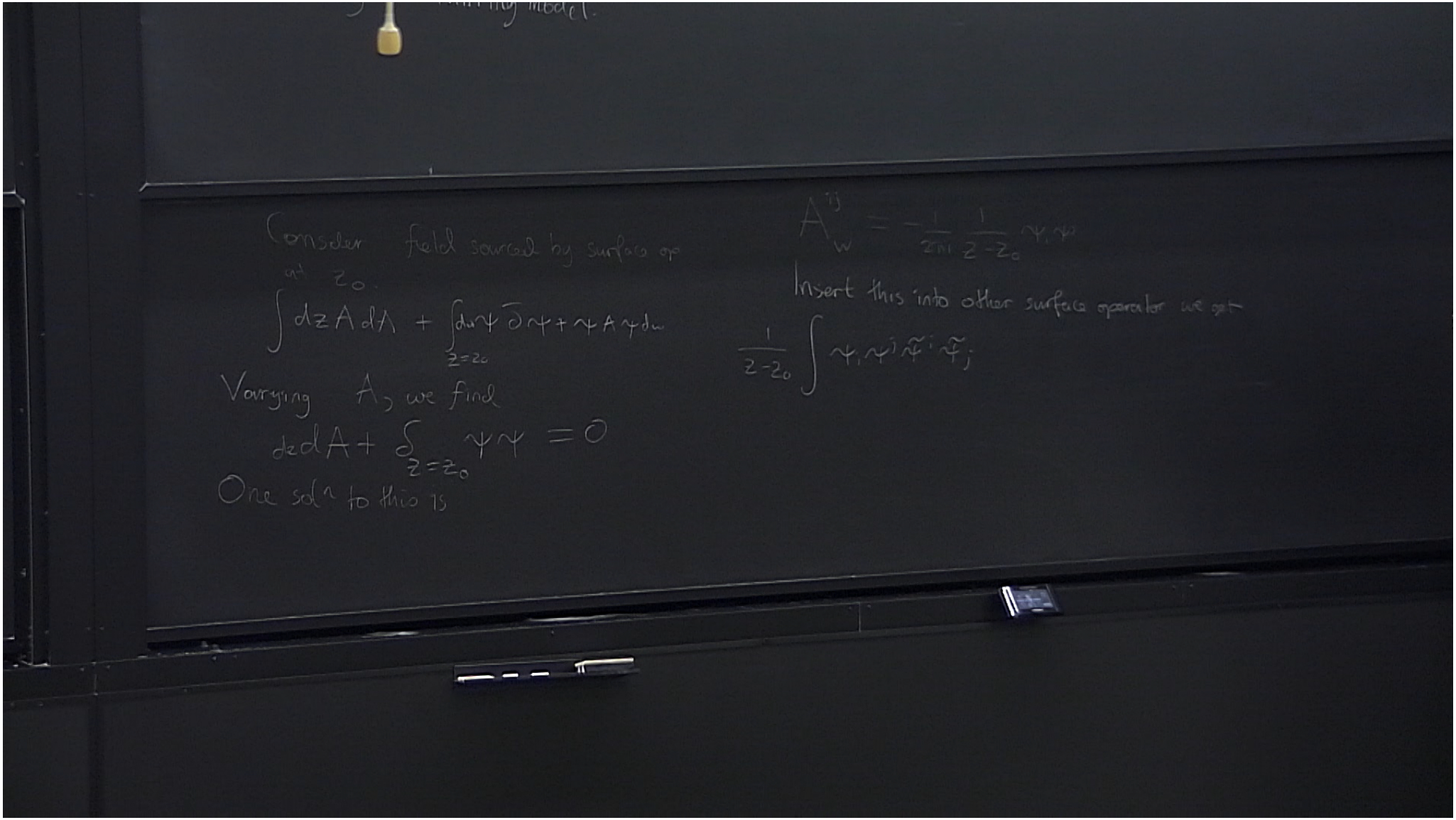
$$\int dz\, A\, d\Lambda + \int_{z=z_0} dw\, \psi\, \bar{\partial}\psi + \psi A \psi\, dw$$

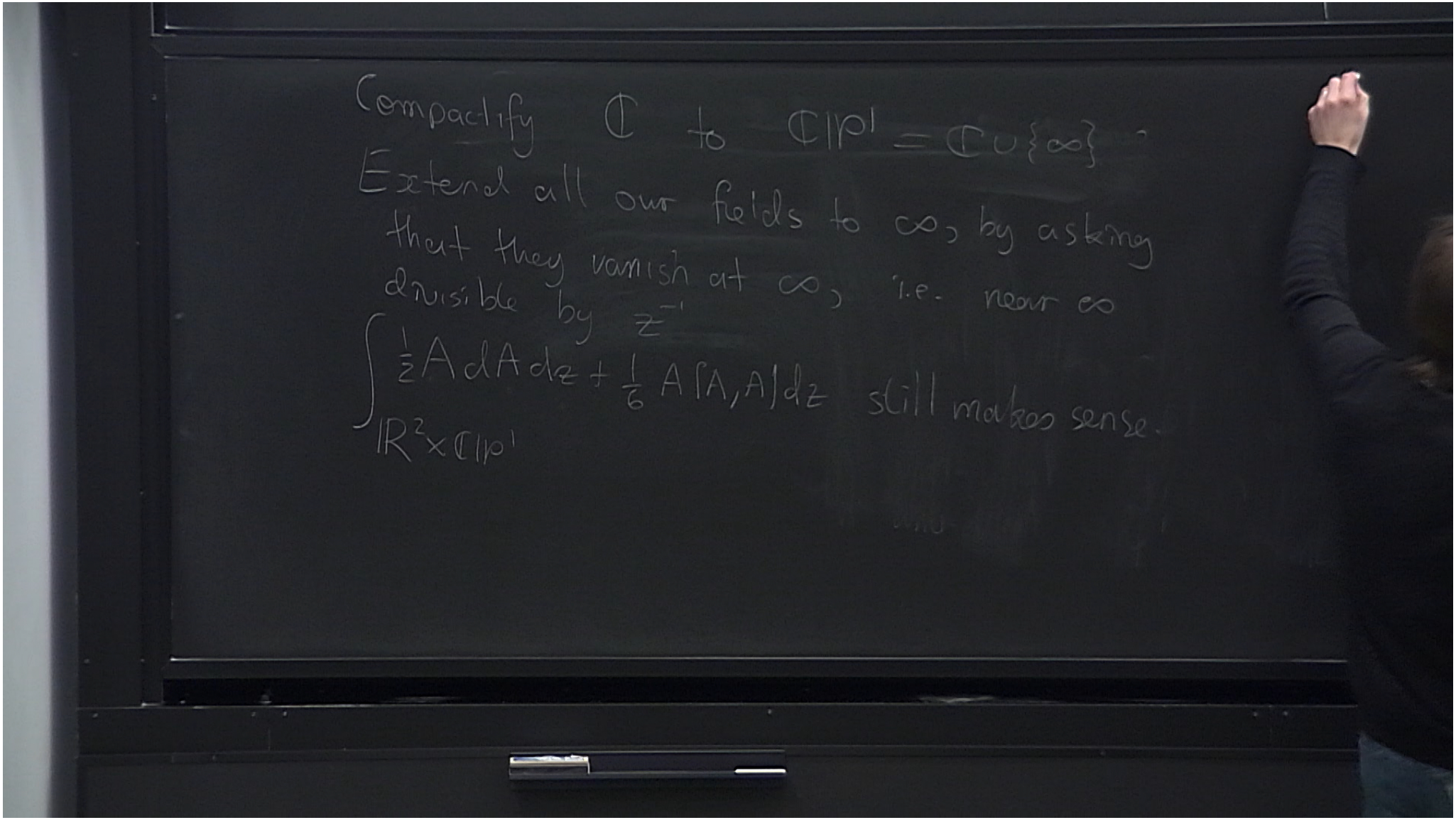
Varying A, we find

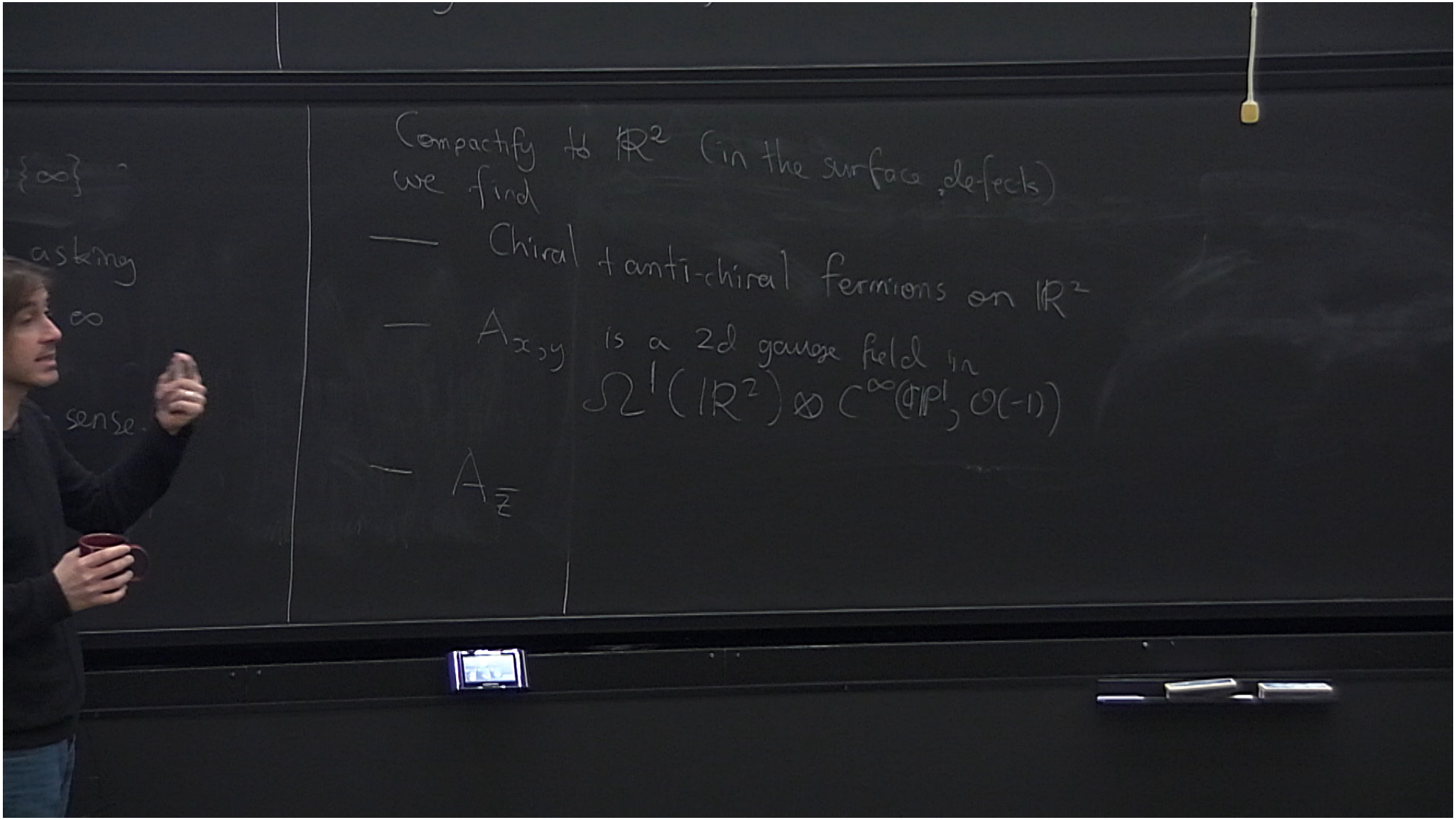
$$dz\, d\Lambda + \oint_{z=z_0} \psi\psi = 0$$

One sol^n to this is

$A^{ij}_{\bar{z}}$


Consider field sourced by surface op at z_0.
$\int dz A d\Lambda + \int_{z=z_0} du\, \psi \bar{\partial} \gamma + \gamma A \psi du$
Varying A_z we find
$dz dA + \delta_{z=z_0} \psi\gamma = 0$
One sol^n to this is
$A^{ij}_w = -\frac{1}{2\pi i} \frac{1}{z - z_0} \psi_i \psi_j$
Insert this into other surface operator we get
$\frac{1}{z - z_0} \int \psi_i \psi^j \tilde{\psi}^i \tilde{\psi}_j$


Compactify $\mathbb{C}$ to $\mathbb{C}\mathbb{P}^1 = \mathbb{C} \cup \{\infty\}$.
Extend all our fields to ∞, by asking that they vanish at ∞, i.e. near ∞ divisible by z^{-1}
$\int_{\mathbb{R}^2 \times \mathbb{C}\mathbb{P}^1} \frac{1}{2} A dA dz + \frac{1}{6} A[A,A] dz$ still makes sense.


Compactify to $\mathbb{R}^2$ (in the surface, defects) we find
— Chiral + anti-chiral fermions on $\mathbb{R}^2$
— $A_{x,y}$ is a 2d gauge field in $\Omega^1(\mathbb{R}^2) \otimes C^\infty(\mathbb{CP}^1, \mathcal{O}(-1))$
— $A_{\bar{z}}$
asking
sense.

Compactify to $\mathbb{R}^2$ (in the surface defects) we find

— Chiral + anti-chiral fermions on $\mathbb{R}^2$

— $A_{x,y}$ is a 2d gauge field in $\Omega^1(\mathbb{R}^2) \otimes C^\infty(\mathbb{CP}^1, \mathcal{O}(-1))$

— $A_{\bar{z}}$ is a 2d scalar $B \in \Omega^0(\mathbb{R}^2) \otimes \Omega^{0,1}(\mathbb{CP}^1, \mathcal{O}(-1))$

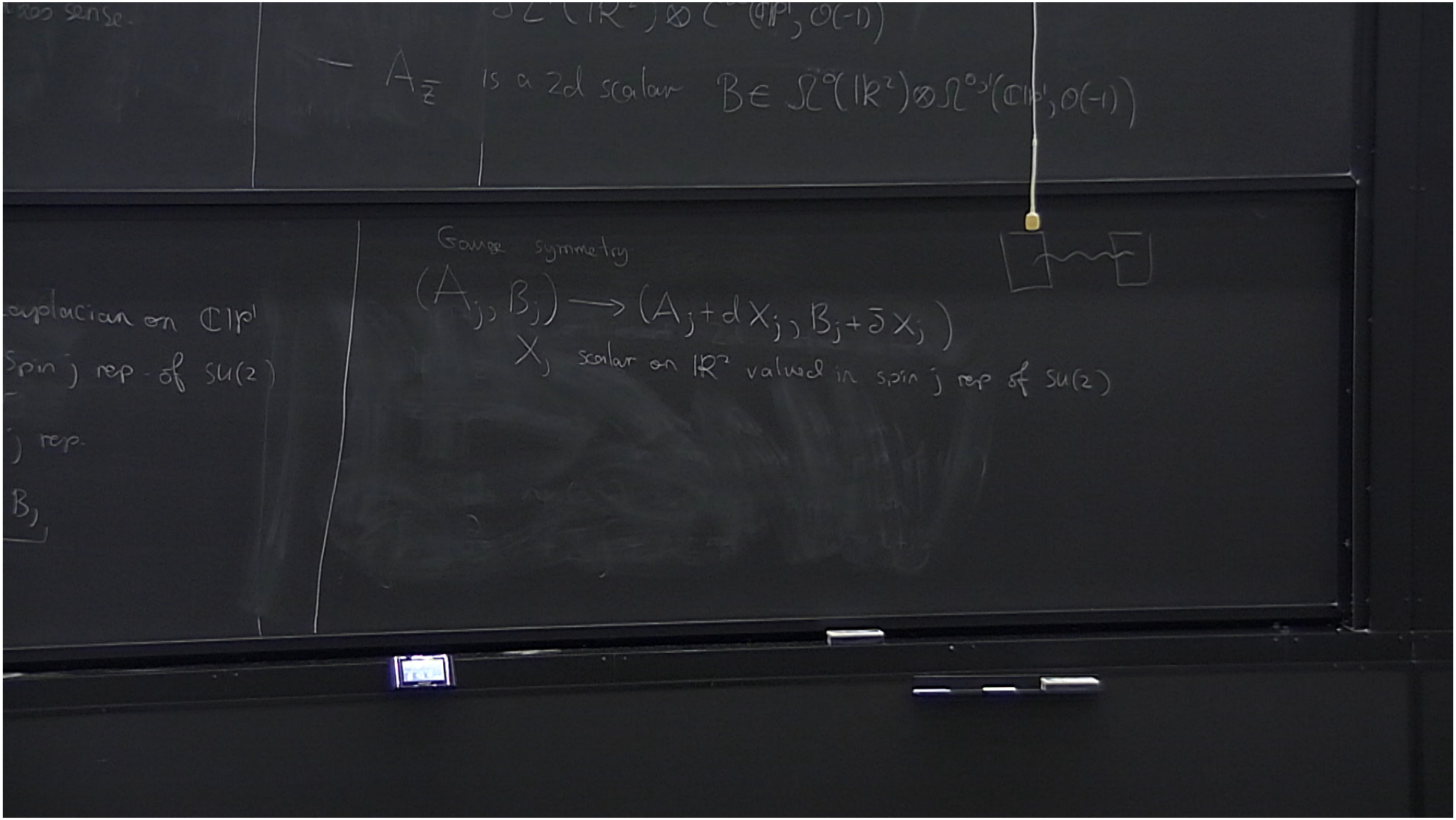
Decompose into eigenvalues of Laplacian on $\mathbb{CP}^1$

A_j . . . 1-form valued in Spin j rep of $SU(2)$

j $\frac{1}{2}$-integer

B_j scalar in Spin j rep.

Action $\int_{\mathbb{R}^2} \underbrace{A_j \, \bar{\partial}_{\mathbb{CP}^1} A_j}_{\text{mass term}} + \int_{\mathbb{R}^2} \underbrace{dA_j \, B_j}_{BF}$


$\Omega^0(\mathbb{R}^2) \otimes \Omega^{0,1}(\mathbb{CP}^1, \mathcal{O}(-1))$
$A_{\bar{z}}$ is a 2d scalar $B \in \Omega^0(\mathbb{R}^2) \otimes \Omega^{0,1}(\mathbb{CP}^1, \mathcal{O}(-1))$
Gauge symmetry
$(A_j, B_j) \longrightarrow (A_j + d X_j, B_j + \bar{\partial} X_j)$
X_j scalar on $\mathbb{R}^2$ valued in spin j rep of $SU(2)$
Laplacian on $\mathbb{CP}^1$
spin j rep of $SU(2)$
j rep.
B_j

Gauge symmetry:

$$(A_j, B_j) \longrightarrow (A_j + d X_j, B_j + \bar{\partial} X_j)$$

X_j scalar on $\mathbb{R}^2$ valued in spin j rep of $SU(2)$

Dolbeault coho. of $\Omega^0(\mathbb{CP}^1, \mathcal{O}(-1)) \xrightarrow{\bar{\partial}} \Omega^{0,1}(\mathbb{CP}^1, \mathcal{O}(-1))$ is zero

$A_{\bar{z}}$ is a 2d scalar $B \in \Omega^0(\mathbb{R}^2) \otimes \Omega^{0,1}(\mathbb{CP}^1, \mathcal{O}(-1))$

Gauge symmetry

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Choose a gauge where $B_j = 0$ Completely fixes the gauge.

Introduce $\psi, \tilde{\psi}$ again

$$\int_{\mathbb{R}^2} \psi \bar{\partial}_A \psi + \int_{\mathbb{R}^2} \tilde{\psi} \partial_A \tilde{\psi} + \int A \bar{\partial}_z A$$

Integrating out A gives a <u>local</u> interaction.

Because $\partial_z^{-1} = \frac{1}{z}$

we find $\frac{1}{z_0 - z_1} \int \psi \psi \tilde{\psi} \tilde{\psi}$.

free fermion

field

nd

In 4d on
$\mathbb{R}^2 \times \mathbb{CP}^1$

metric on $\mathbb{CP}^1$ does not play a role in the action.
It does play a role in gauge fix, ask

$$d^* A_{z,y} + \bar{\partial}^b A_{\bar{z}} = 0$$

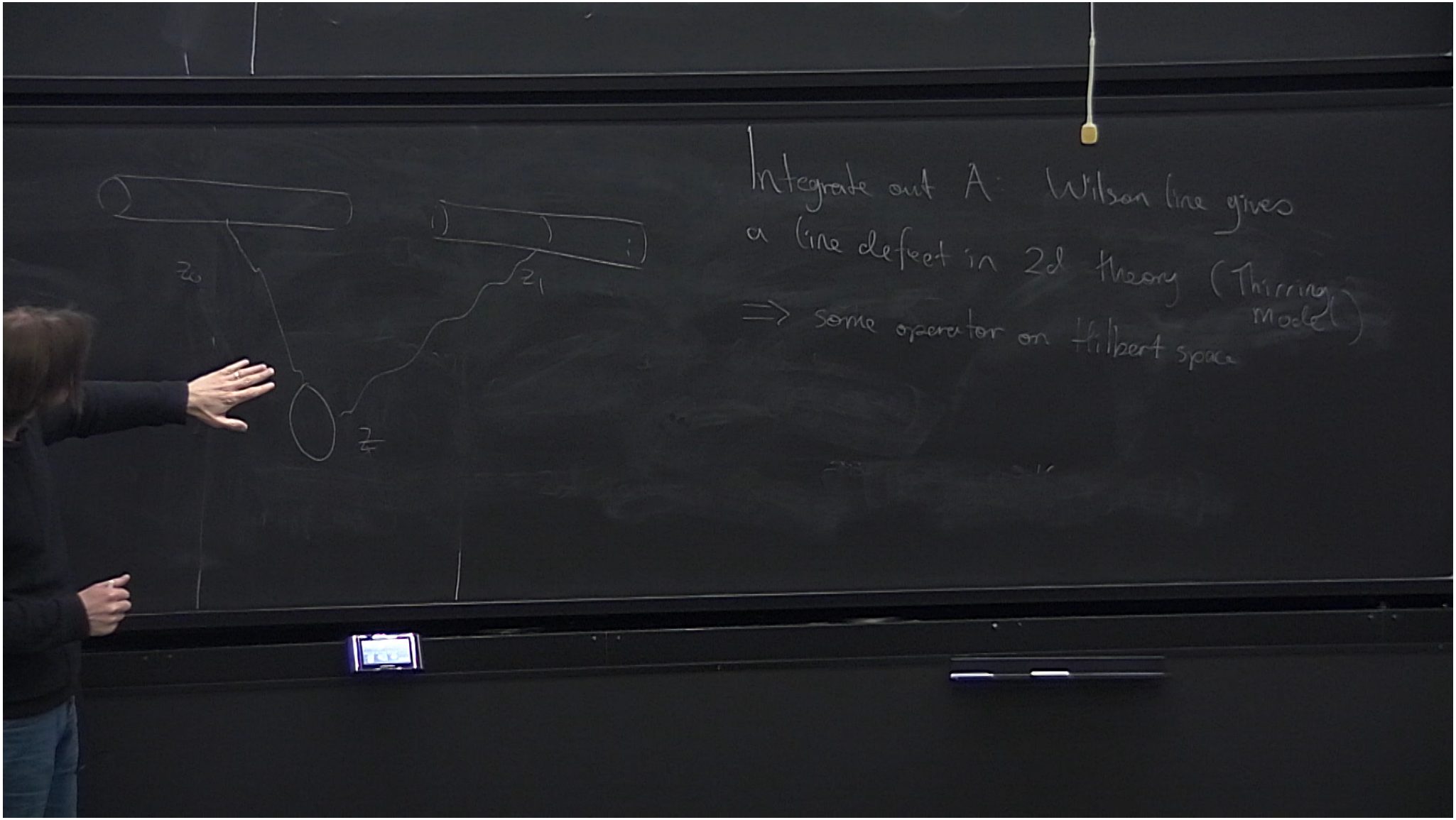
The gauge that leads to local interaction is when $Vol(\mathbb{CP}^1) \longrightarrow 0$

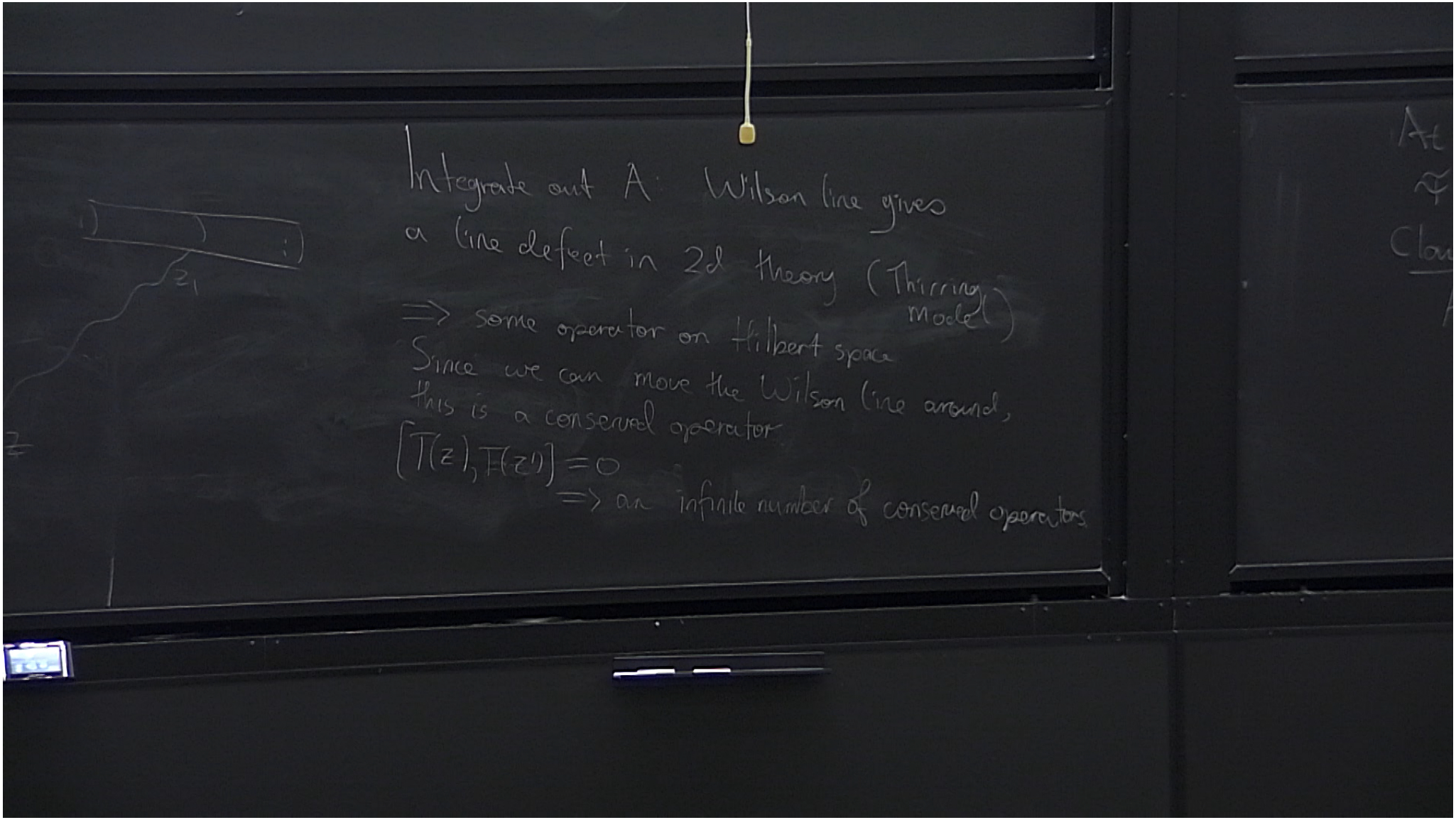
Yangian action / Integrability

Consider 4d theory on $\mathbb{R} \times S^1 \times \mathbb{C}$

Surface operators at z_0, z_1

Put a Wilson line wraps S^1 at $z \sim \infty$


Integrate out A: Wilson line gives
a line defect in 2d theory (Thirring model)
⇒ some operator on Hilbert space
z_0
z_1
z_f


Integrate out A: Wilson line gives
a line defect in 2d theory (Thirring model)
⇒ some operator on Hilbert space
Since we can move the Wilson line around,
this is a conserved operator
$[T(z), T(z')] = 0$
⇒ an infinite number of conserved operators