Title: PSI 2016/2017 Introduction to Quantum Integrable Systems - Lecture 9

Date: Mar 03, 2017 11:30 AM

URL: http://pirsa.org/17030040

Abstract:

Last time

Considered a theory on $\mathbb{R}^2 \times \mathbb{C}$ coords x, y, z

$A = A_x\,dx + A_y\,dy + A_{\bar{z}}\,d\bar{z}$ gauge field

Action functional

$$S(A) = \int dz\, \frac{A\,dA}{2} + \frac{1}{6}\,dz\, A \wedge [A, A]$$

$$= \int dz\, CS(A).$$

Wilson line

Wilson line in the x-y plane

In the x direction, put a Wilson line.

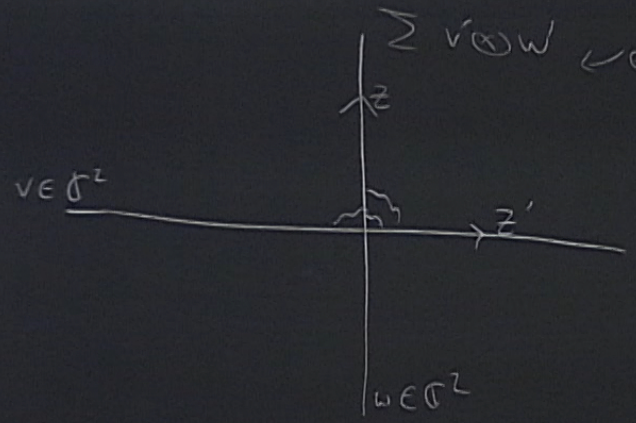
choose a rep. V of Lie alg g ($g = u(n)$, $V = \mathbb{C}^n$)

At $x = -\infty$ put an incoming state $v \in V$

$x = +\infty$ an outgoing state $v^* \in V^*$

Wilson line operator is $\langle v^* | \left(PE^{\int_x A_x dx} \right) v \rangle$

$\sum v \otimes w$ ← entangled state



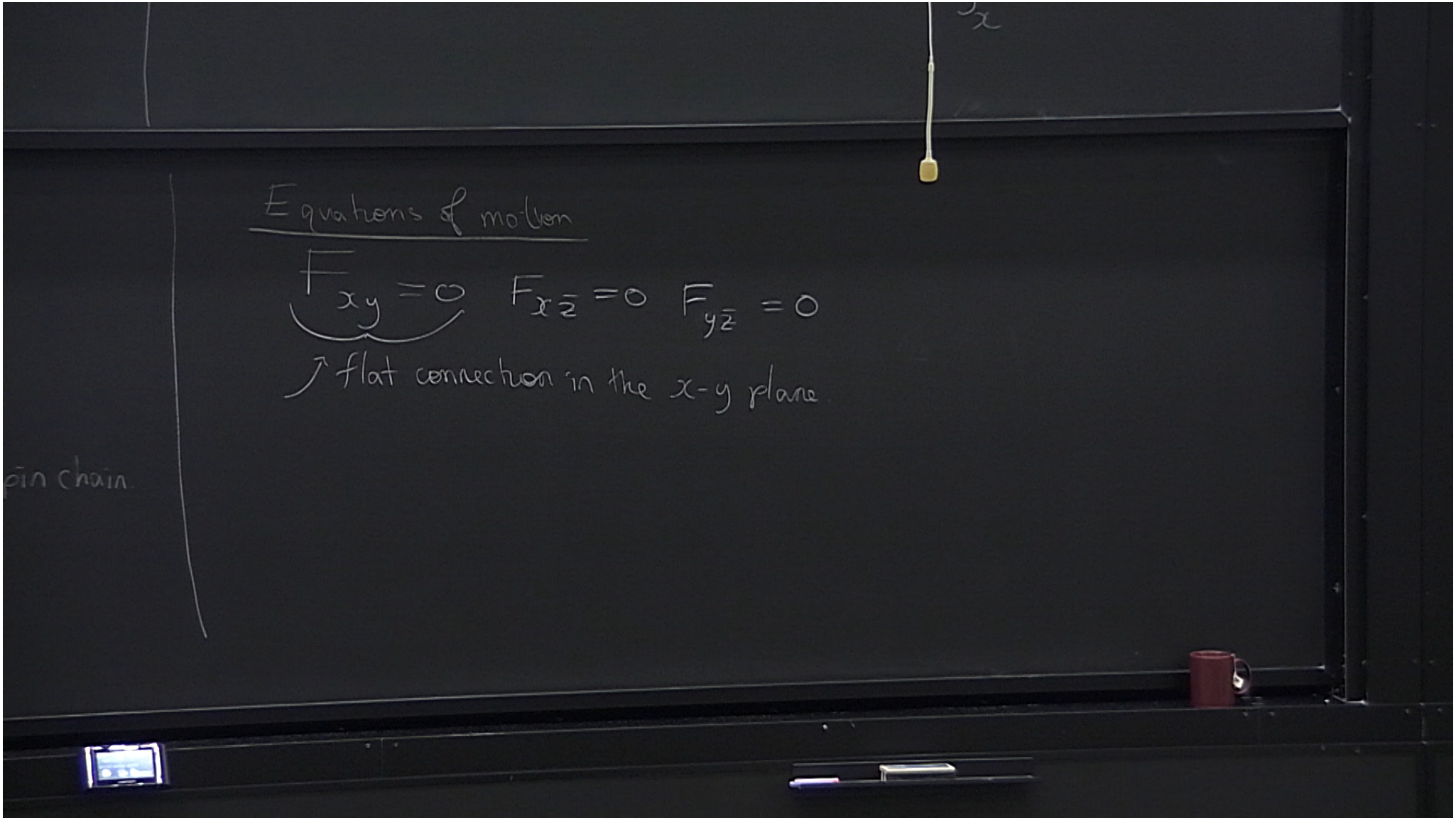
This operator is a linear map

$R(z-z'): \mathbb{C}^2 \otimes \mathbb{C}^2 \longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$

This is the R matrix of XXX spin chain

$g = su(2)$

$V = \mathbb{C}^2$

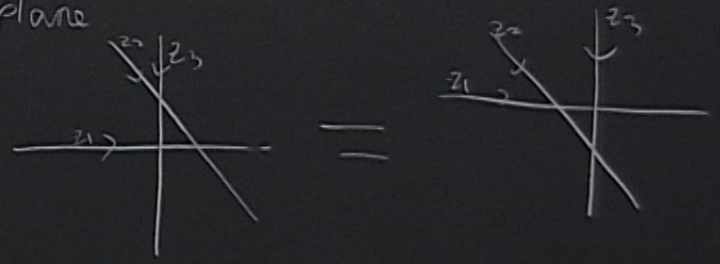

Equations of motion
$F_{xy} = 0$ $F_{x\bar{z}} = 0$ $F_{y\bar{z}} = 0$
flat connection in the x-y plane
pin chain

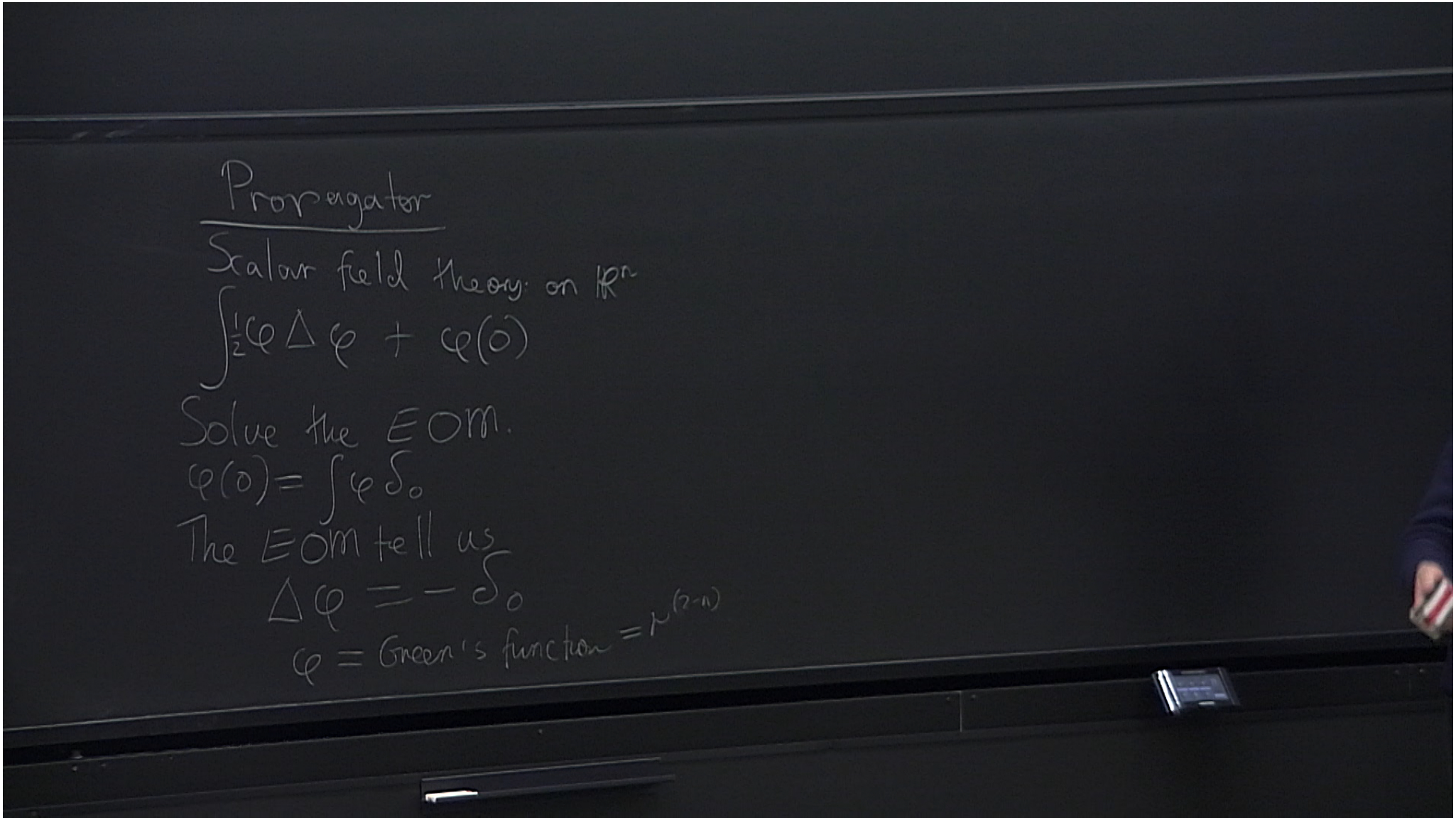
Equations of motion

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$F_{xy} = 0$: flat connection in the x-y plane

Yang-Baxter Equation



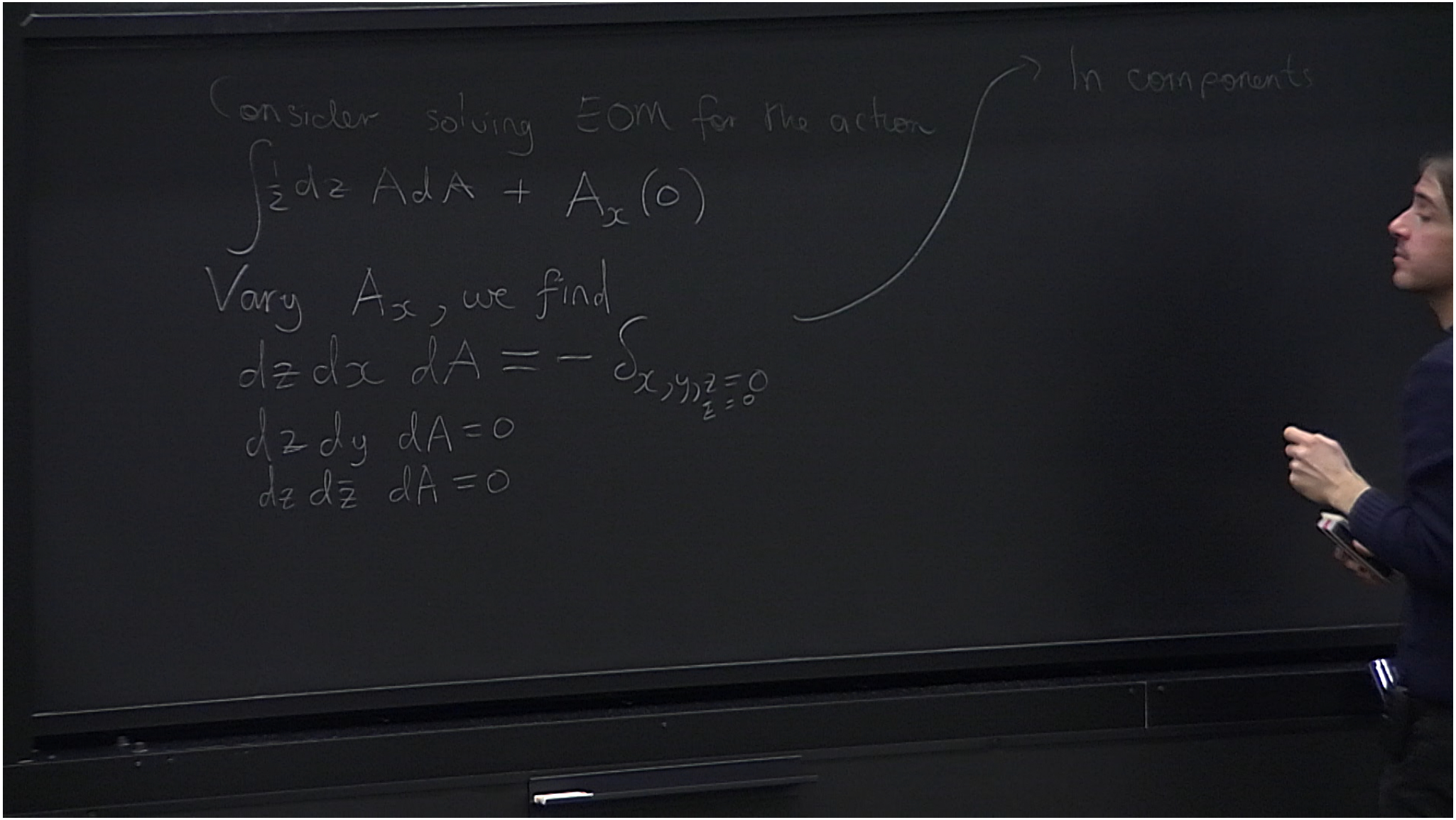

Propagator
Scalar field theory on $\mathbb{R}^n$
$\int \frac{1}{2}\varphi \Delta \varphi + \varphi(0)$
Solve the EOM.
$\varphi(0) = \int \varphi \delta_0$
The EOM tell us
$\Delta \varphi = -\delta_0$
φ = Green's function $= r^{(2-n)}$

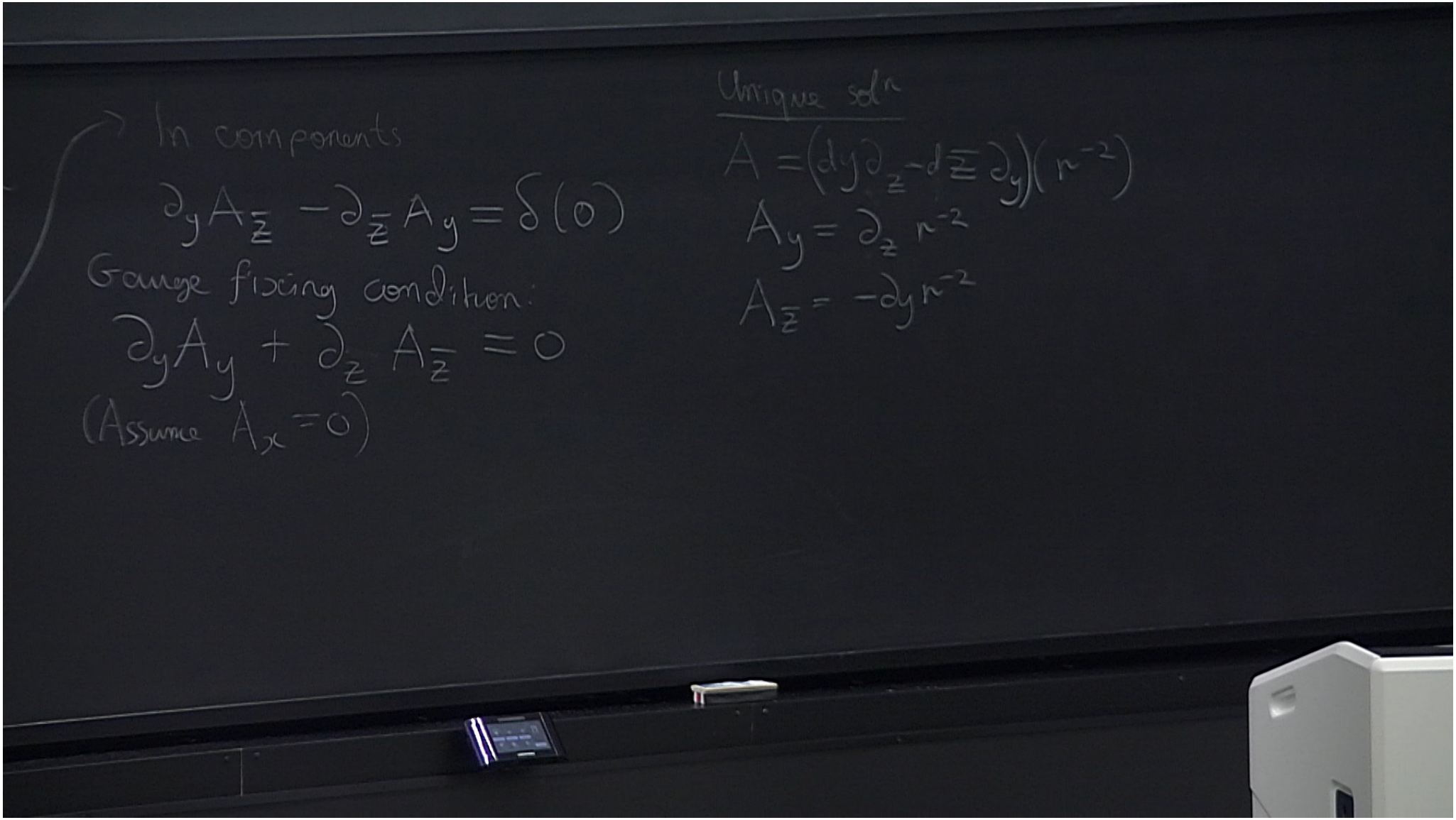
In a gauge theory, there are two differences

1) There is more than one component.
   3 components, so propagator can have 9 components
   $P_{x,y'}$ $P_{x,z'}$, ...



2) We must impose gauge fixing condition when we solve the equation


Consider solving EOM for the action
$\int_z^1 dz\, A dA + A_x(0)$
In components
Vary A_x, we find
$dz\, dx\, dA = -\delta_{x,y,z=0}$
$dz\, dy\, dA = 0$
$dz\, d\bar{z}\, dA = 0$


In components
$\partial_y A_{\bar{z}} - \partial_{\bar{z}} A_y = \delta(0)$
Gauge fixing condition:
$\partial_y A_y + \partial_z A_{\bar{z}} = 0$
(Assume $A_x = 0$)
Unique soln
$A = (dy\,\partial_z - d\bar{z}\,\partial_y)(r^{-2})$
$A_y = \partial_z r^{-2}$
$A_{\bar{z}} = -\partial_y r^{-2}$

Summary description of the propagator

P is a 2-form on $(\mathbb{R}^2 \times \mathbb{C})^2$ coords are

x, y, z

x', y', z'

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$$r^2 = (x-x')^2 + (y-y')^2 + |z-z'|^2$$

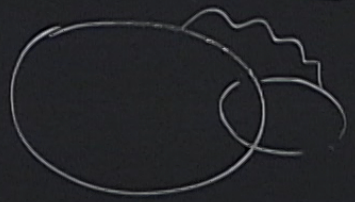
$$P = \frac{(x-x')d(y-y')d(\bar{z}-\bar{z}') - (y-y')d(x-x')d(\bar{z}-\bar{z}') + \frac{1}{2}(\bar{z}-\bar{z}')d(x-x')d(y-y')}{r^4}$$

Exercise

Consider U(1) Chern-Simons on $\mathbb{R}^3$

Derive a similar expression for the propagator.

Given 2 linked Wilson lines



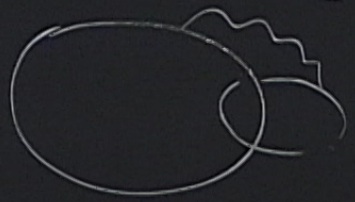
$\int$ where 1 leg of propagator is on one line the other on the other line

Exercise

Consider $U(1)$ Chern-Simons on $\mathbb{R}^3$

Derive a similar expression for the propagator.

Given 2 linked Wilson lines



$\int$ where 1 leg of propagator is on one line the other on the other line

Show this gives Gauss's formula for linking number

$P \in \Omega^2((\mathbb{R}^2 \times \mathbb{C})^2)$

$d(z - z')\, dP = \delta_{\text{Diagonal}}$

CS.

$P_{CS} \in \Omega^2((\mathbb{R}^3)^2)$

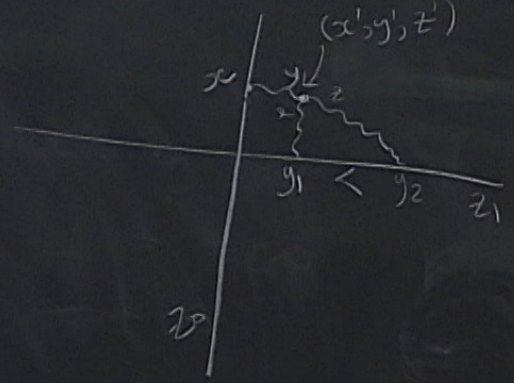
$dP_{CS} = \delta_{\text{Diagonal}}$

$\frac{\partial}{\partial x} P_{CS}$

$= (d(z)\partial/\partial x + \overline{(z)}\partial/\partial \bar{x}) P_{CS}$

$= d(z)\partial/\partial x P_{CS}$

How do we do Feynman diagrammatics?



The result of this will be

$$\int_x \int_{y_1<y_2} \int_{x',y',z'} \frac{\frac{1}{2}\,dx\,dy\,(\bar{z}'-\bar{z}_0)}{\left((x-x')^2+y'^2+|z'-z_0|^2\right)^2}$$