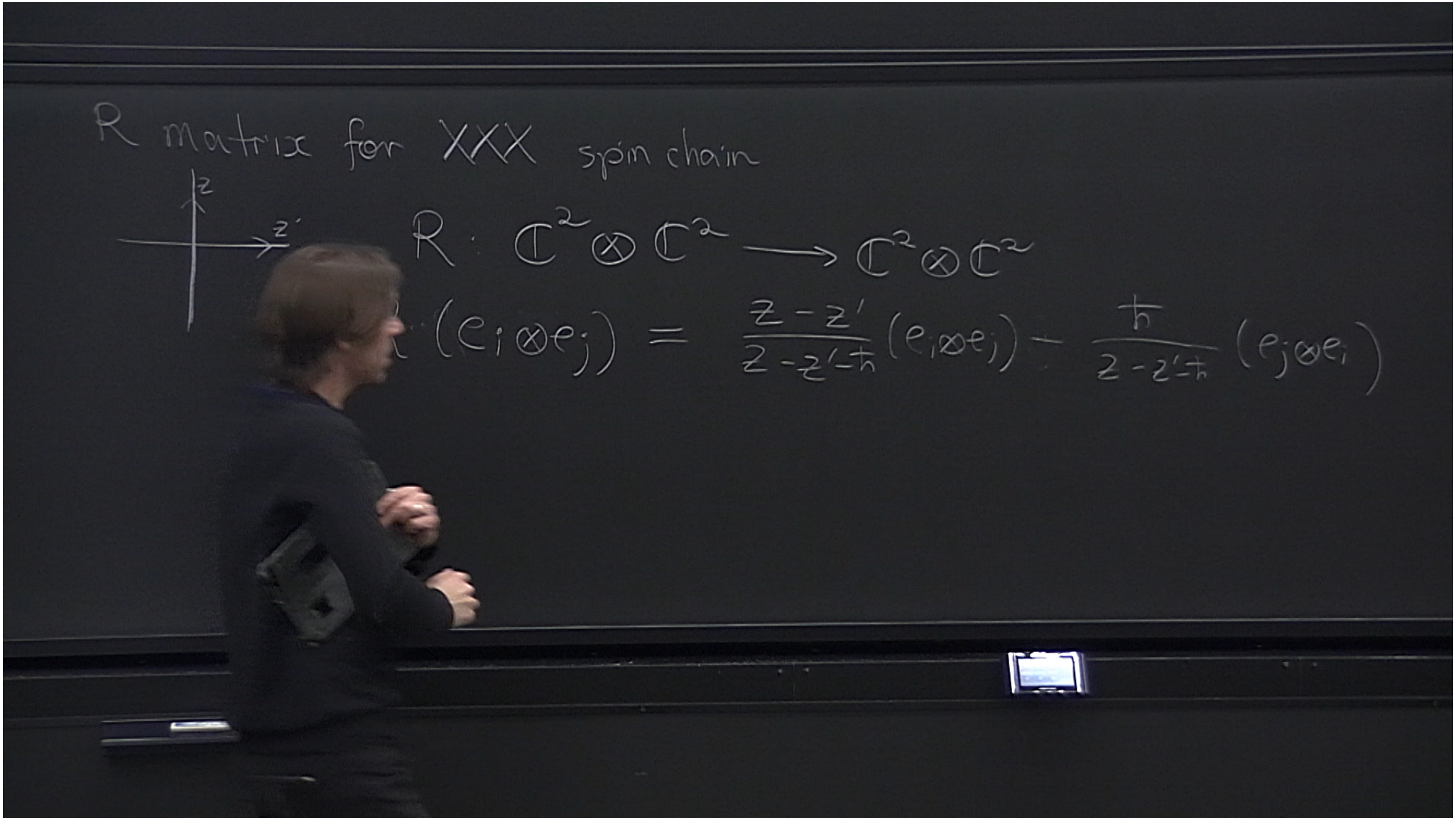
Title: PSI 2016/2017 Introduction to Quantum Integrable Systems - Lecture 8

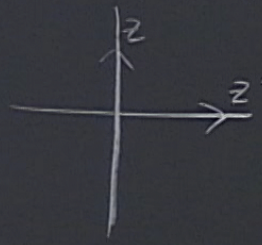
Date: Mar 02, 2017 11:30 AM

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Abstract:

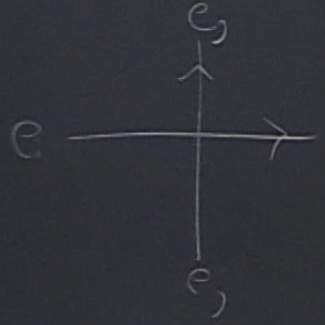

R matrix for XXX spin chain
z
z'
$R : \mathbb{C}^2 \otimes \mathbb{C}^2 \longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$
$(e_i \otimes e_j) = \frac{z - z'}{z - z' - \hbar}(e_i \otimes e_j) - \frac{\hbar}{z - z' - \hbar}(e_j \otimes e_i)$

R matrix for XXX spin chain

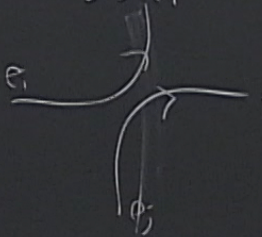


$$R : \mathbb{C}^2 \otimes \mathbb{C}^2 \longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$$

$$R(e_i \otimes e_j) = \frac{z - z'}{z - z' - \hbar}(e_i \otimes e_j) - \frac{\hbar}{z - z' - \hbar}(e_j \otimes e_i)$$

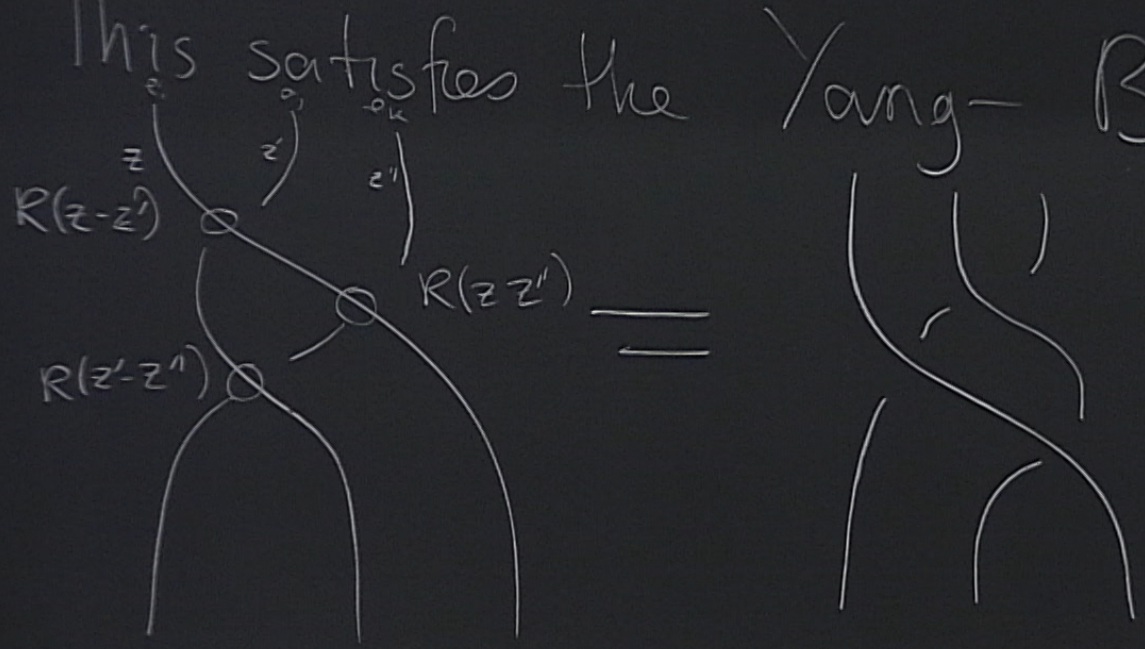


$$\frac{z - z'}{z - z' - \hbar} -$$

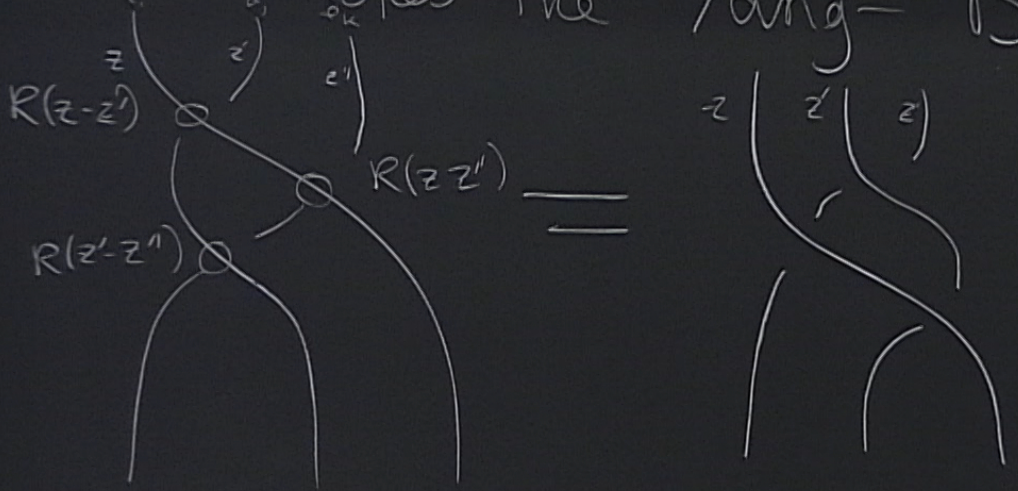


$$\frac{\hbar}{z - z' - \hbar}$$

This satisfies the Yang-Baxter equation



This satisfies the Yang-Baxter equation

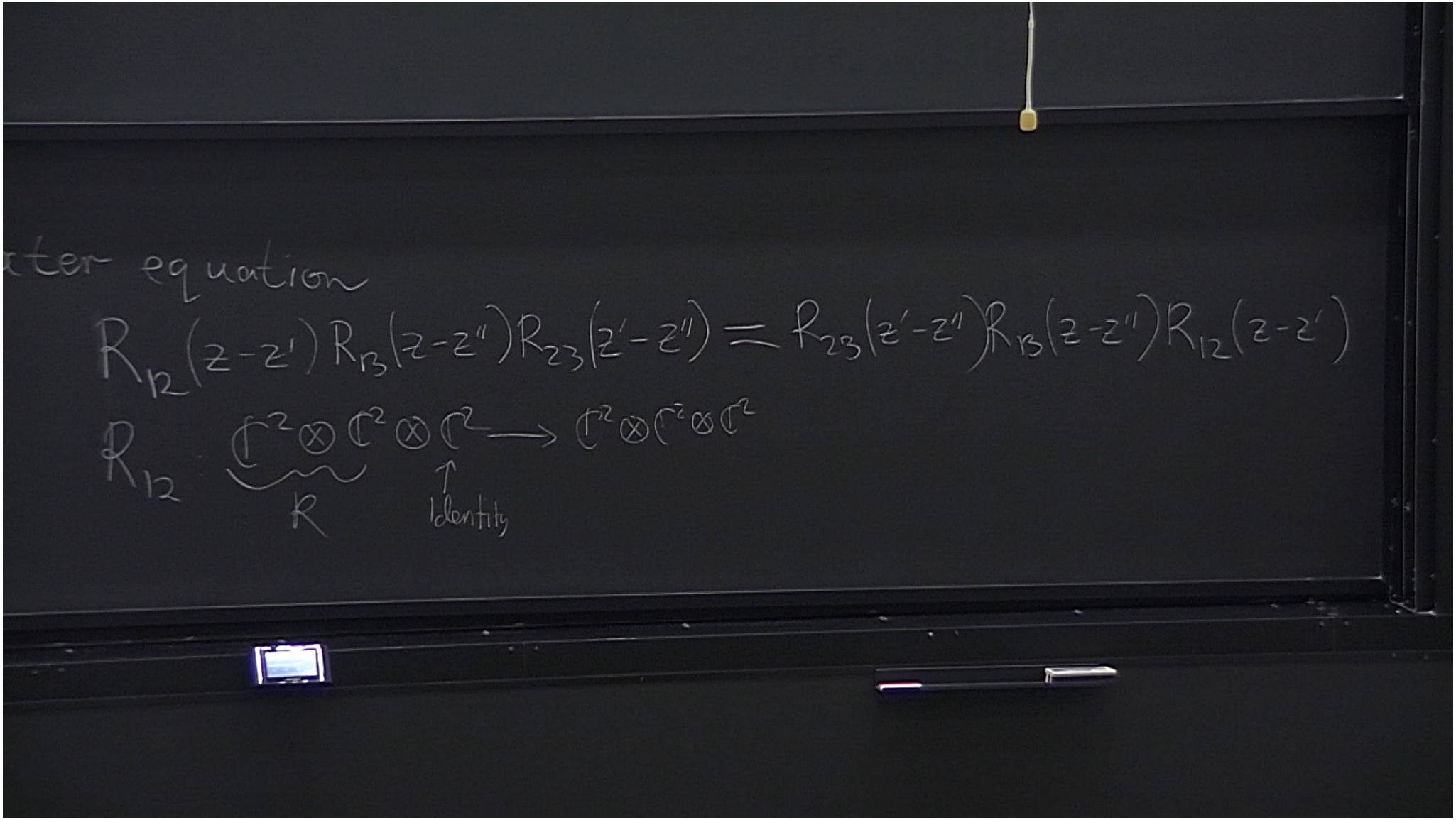


=

$$R_{12}(z-z')\,R_{13}(z-z'')\,R_{2}$$

ter equation

$$R_{12}(z-z')\,R_{13}(z-z'')\,R_{23}(z'-z'') = R_{23}(z'-z'')\,R_{13}(z-z'')\,R_{12}(z-z')$$


ter equation
$R_{12}(z-z')R_{13}(z-z'')R_{23}(z'-z'') = R_{23}(z'-z'')R_{13}(z-z'')R_{12}(z-z')$
$R_{12}: \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2 \longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$
R
Identity

Gauge Theories $\Longrightarrow$ Sol's of YBE

3d Chern-Simons theory. This has Wilson line operators

Strand $\Rightarrow$ Wilson Line

Configurations of Wilson Lines $\Longrightarrow$ Sol's of YBE

4d version: $\Rightarrow$ R matrix w. spectral param.

4d gauge theory

$\mathbb{R}^2 \times \mathbb{C}$ coords x, y, z

Fundamental field is a connection

$$A = A_x dx + A_y dy + A_z dz + A_{\bar{z}} d\bar{z}$$

Action is

$$S(A) = \frac{1}{2}\int dz \wedge A \wedge dA + \frac{1}{6}\int dz \wedge A \wedge [A, A]$$

Wilson Lines

Given any gauge theory w. gauge field A, on $\mathbb{R}^n$, we can define a line operator called a Wilson line by $\mathrm{Tr}\, P \mathrm{Exp} \int_{S^1} A$

$S^1 \subseteq \mathbb{R}^n$

A is a gauge field for gl_n

$$P\,Exp\int_{S^1} A = 1 + \int_{t\in S^1} A + \int_{t_1<t_2} A_{t_1} \cdot A_{t_2} + \cdots$$

Alternate Picture

Introduce extra fields along the line, which are coupled to the gauge field.

For gl_n, introduce ψ, ψ' fermions on the line $\int \psi \partial_t \psi' + \psi A_t \psi'$

ψ fun. rep

ψ' anti-fun. rep

$\int_{t\in S^1} A + \int_{t_1<t_2} A_{t_1} \cdot A_{t_2} + \cdots$

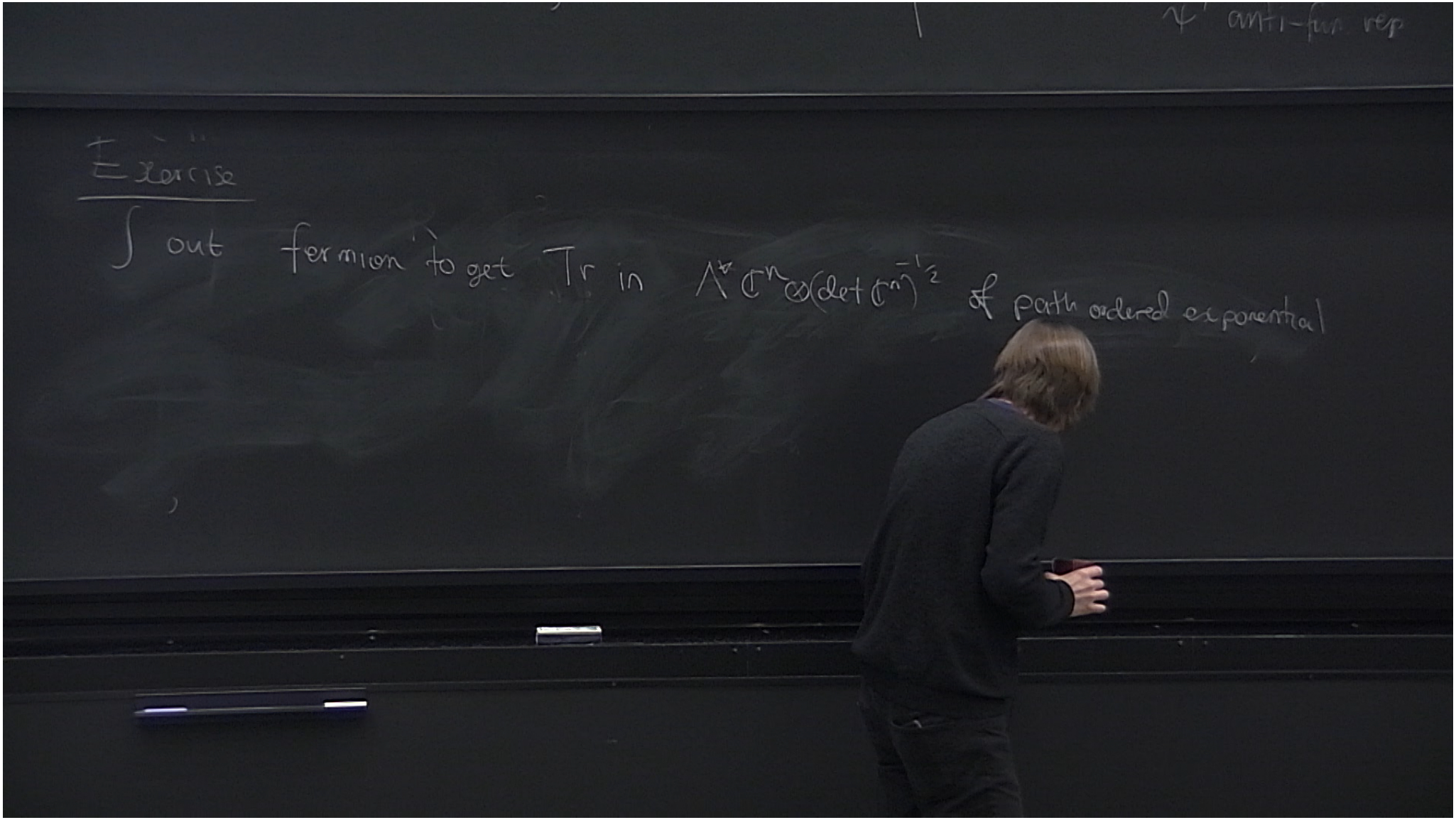
cture

extra fields along the line, which are coupled to the gauge

duce ψ, ψ' fermions on the line $\int \psi^i \partial_t \psi_i' + \cdots \psi^i A_i^j \psi_j'$

ψ fun. rep

ψ' anti-fun. rep


anti-fun rep
Exercise
∫ out fermion to get Tr in $\Lambda^* \mathbb{C}^n \otimes (\det \mathbb{C}^n)^{-\frac{1}{2}}$ of path ordered exponential

Λ is a gauge field for gl_n

ψ fun. rep

ψ' anti-fun. rep

Exercise

$\int$ out fermion to get Tr in $\Lambda^* \mathbb{C}^n \otimes (\det \mathbb{C}^n)^{-\frac{1}{2}}$ of path ordered exponential

In 4d theory with action $\int dz\, CS(A)$
we can put Wilson lines
only in the plane $\mathbb{R}^2$, at a fixed value of z.
Gauge symmetry $A \to A + f dz$ prohibits Wilson lines moving in z plane.

A is a gauge field for gl_N

ψ fun. rep

ψ' anti-fun. rep

Claim

Consider $g = sl_2$

Consider Wilson lines in the fundamental representation

One at z, vertical

z' horizontal



Claim

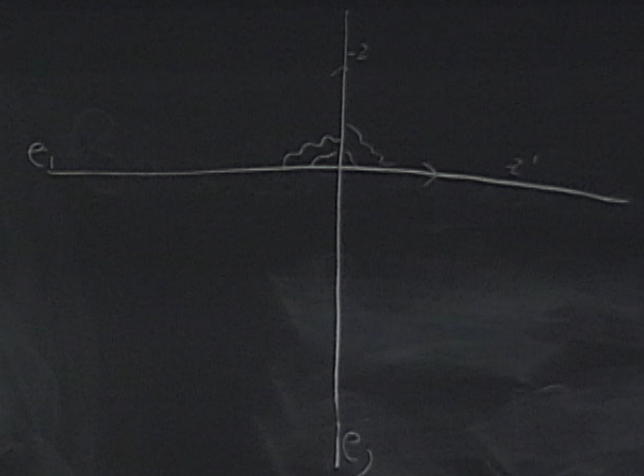
Consider $g = sl_2$

Consider Wilson lines in the fundamental representation

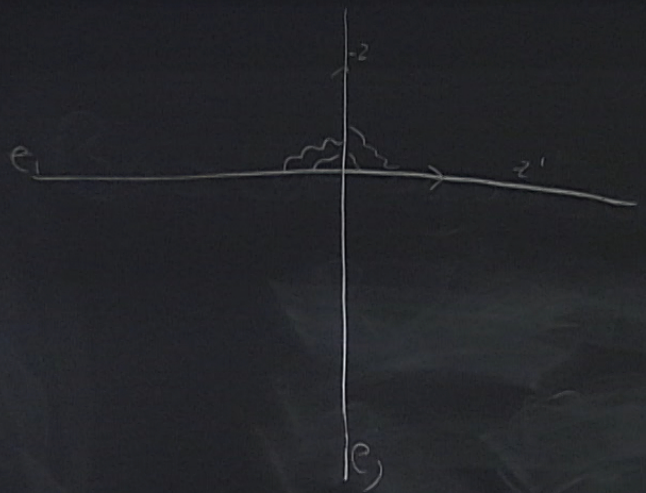
One at z, vertical

z' horizontal

States at the end of Wilson line are $\mathbb{C}^2$



ψ' anti-fun. rep



The operator depicted here is

$R(z-z') : \mathbb{C}^2 \otimes \mathbb{C}^2 \longrightarrow \mathbb{C}^2 \otimes \mathbb{C}^2$

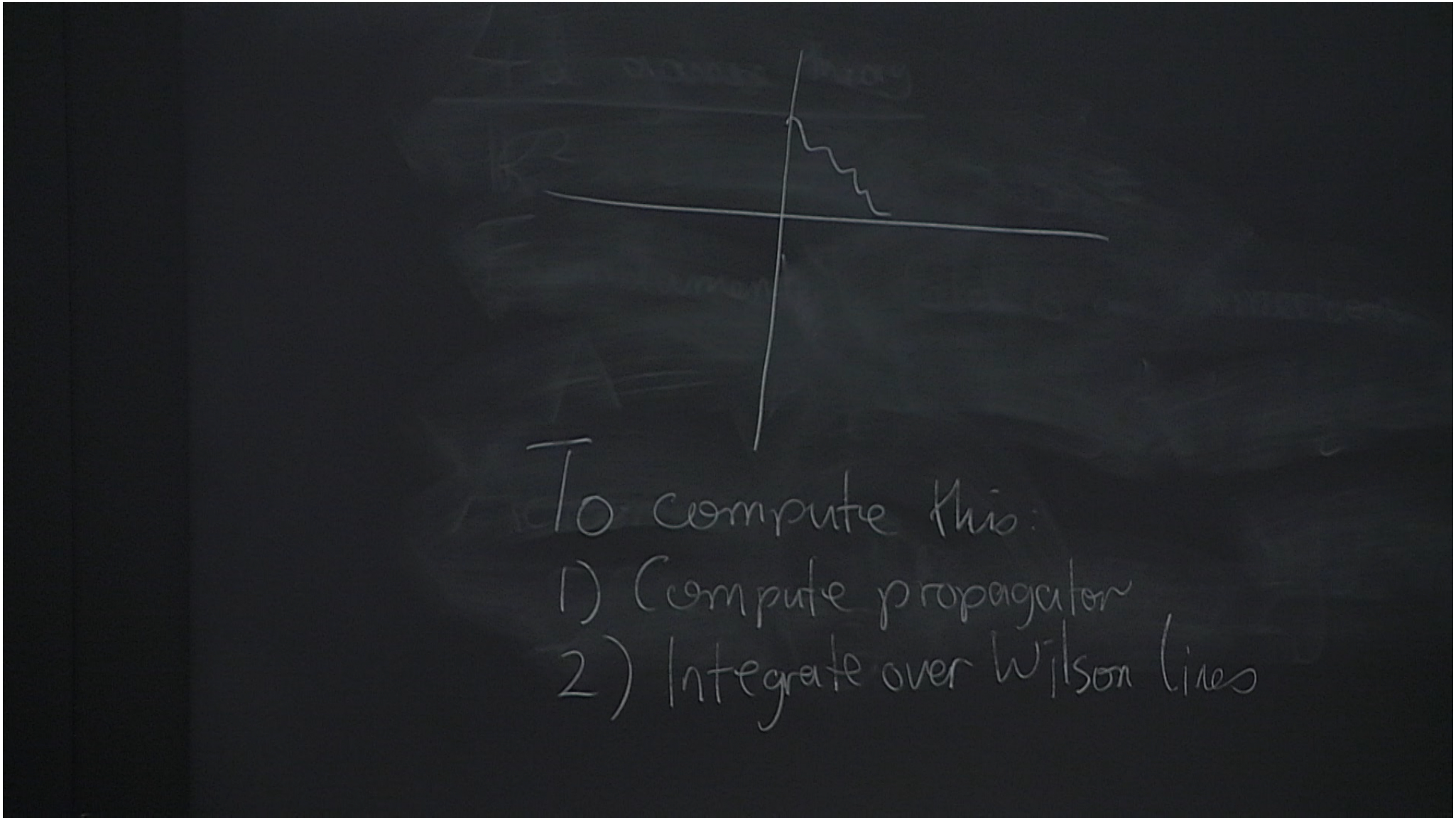
The YBE follows from the fact that the theory is top' on $\mathbb{R}^2$

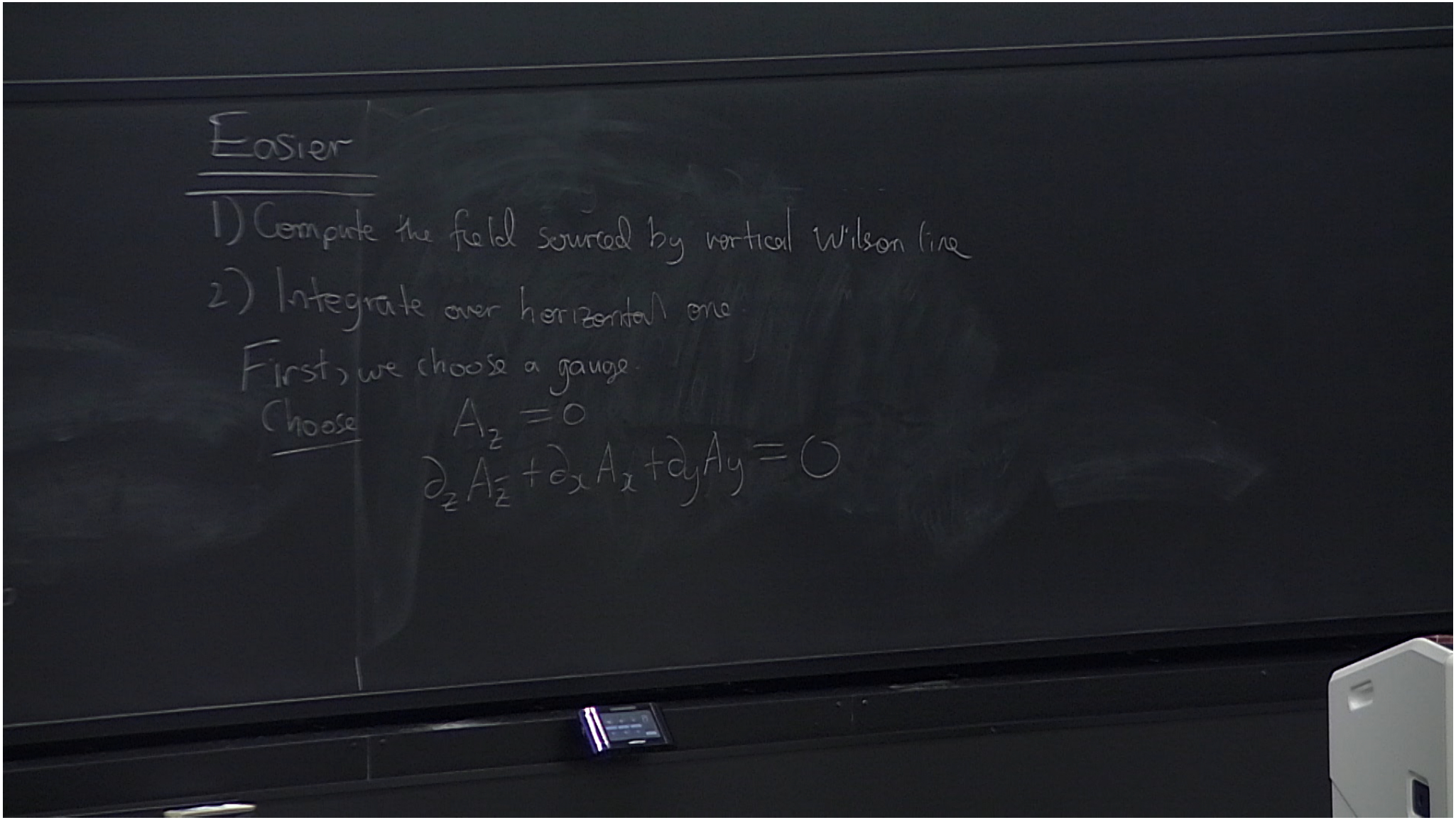


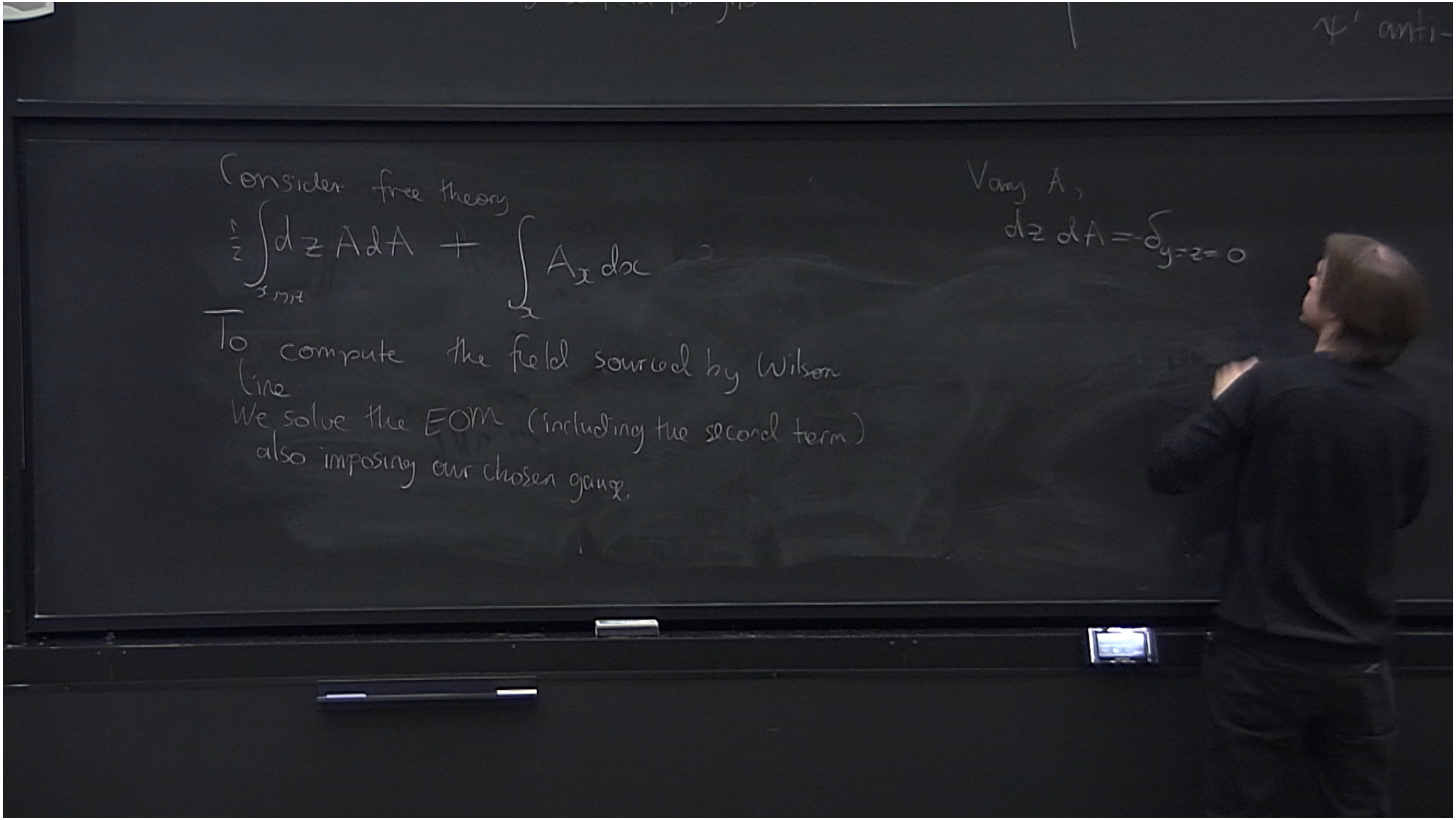
$$R(z-z') = Id + \frac{\hbar}{z-z'}\,(\Sigma e_{ij} \otimes e_{ji}) + O(\hbar^2)$$

$\hbar$ = loop expansion param. of gauge theory

Goal: Compute from gauge theory this 1 loop R matrix


To compute this:
1) Compute propagator
2) Integrate over Wilson lines


Easier
1) Compute the field sourced by vertical Wilson line
2) Integrate over horizontal one
First, we choose a gauge.
Choose $A_z = 0$
$\partial_z A_z + \partial_x A_x + \partial_y A_y = 0$


ψ' anti-
Consider free theory
$\frac{1}{2}\int_{x,y,z} dz\, A\, dA + \int_x A_x\, dx$
Vary A,
$dz\, dA = -\delta_{y=z=0}$
To compute the field sourced by Wilson line
We solve the EOM (including the second term)
also imposing our chosen gauge.

ψ' anti-fun rep

Vary A,

$$dz\, d\bar{A} = -\delta_{y=z=0}$$

The solution to this equation is

$$A = (dy\,\partial_{\bar{z}} - d\bar{z}\,\partial_y)\frac{1}{(y^2+z\bar{z})^{1/2}}$$

(Wilson

ord term)

ψ' anti-fun rep

Vary A,

$$dz\, d\bar{A} = -\delta_{y=z=0}$$

The solution to this equation is

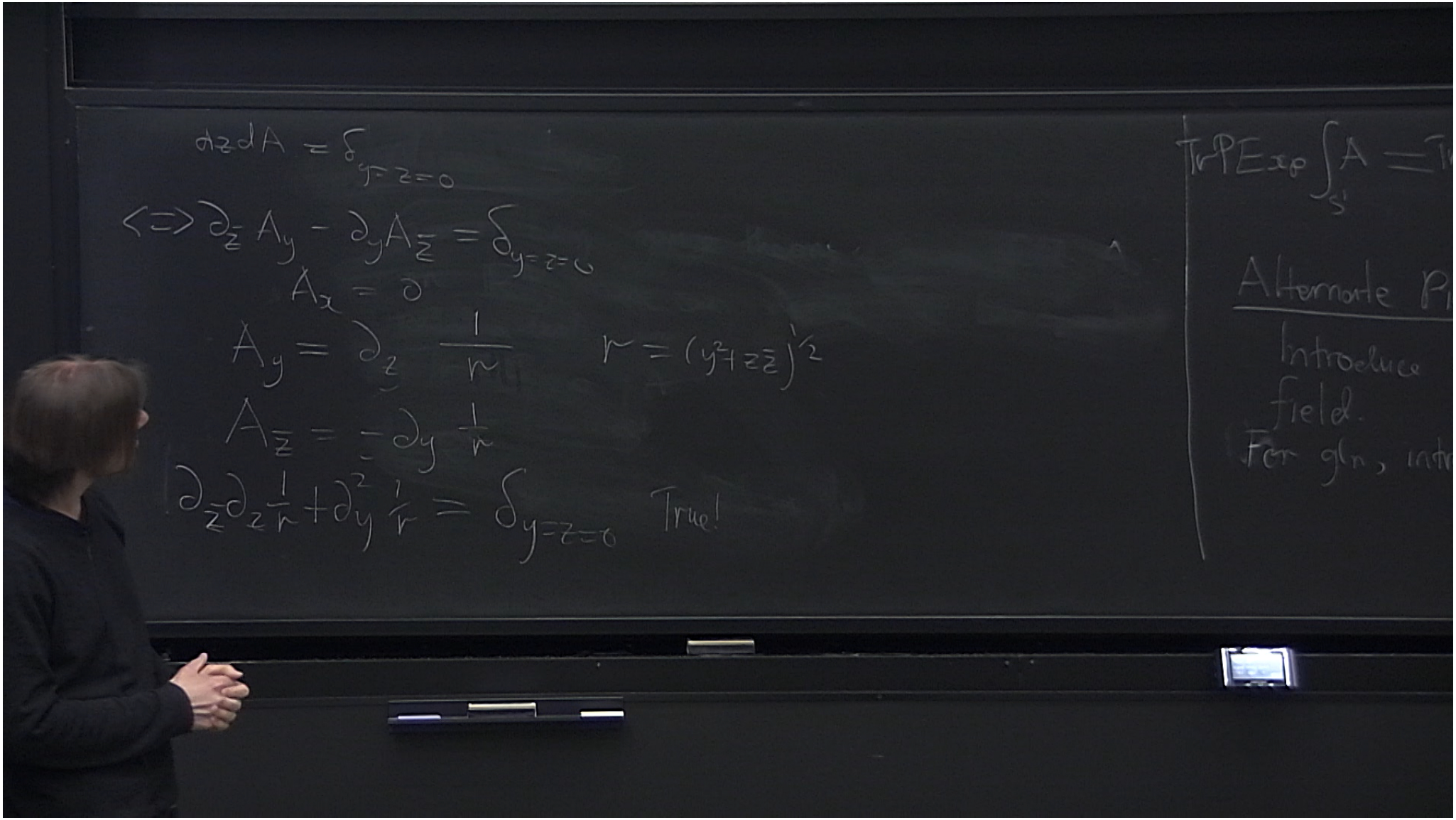
$$A = (dy\,\partial_z - d\bar{z}\,\partial_y)\frac{1}{(y^2+z\bar{z})^{1/2}}$$

$$\partial_y A_y + \partial_z A_{\bar{z}} = 0$$

dx

by Wilson

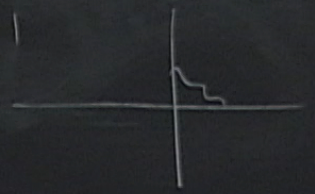
the second term)


$dz\,dA = \delta_{y=z=0}$
$\Leftrightarrow \partial_z A_y - \partial_y A_z = \delta_{y=z=0}$
$A_z = 0$
$A_y = \partial_z \frac{1}{r}$ $r = (y^2+z^2)^{1/2}$
$A_z = -\partial_y \frac{1}{r}$
$\partial_z \partial_z \frac{1}{r} + \partial_y^2 \frac{1}{r} = \delta_{y=z=0}$ True!
Alternate
Introduce
field.

A is field sourced by Wilson line at $z0$ in x direction

Want:

$$\int_{y,z=z_0} A_y \sim \frac{1}{z_0}$$

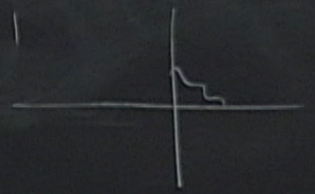


= leading term in the expansion of R matrix

A is field sourced by Wilson line at $z=0$ in x direction

Want:

$$\int_{y,z \text{ fixed}} A_y \sim \frac{1}{z}$$



= leading term in the expansion of R matrix

$$= \partial_z \int \frac{1}{(y^2+z\bar{z})^{1/2}} dy \sim \partial_z \log z\bar{z} = \frac{1}{z}$$