Title: PSI 2016/2017 Quantum Information (Review) - Lecture 10 (Eduardo Martin-Martinez)

Date: Mar 07, 2017 09:00 AM

URL: http://pirsa.org/17030034

Abstract:

Atoms (or any complex system) will not survive a quantum bounce

Atoms (or any complex system) will not survive a quantum bounce

Imagine an ancient (pre-bounce) and very advanced civilization

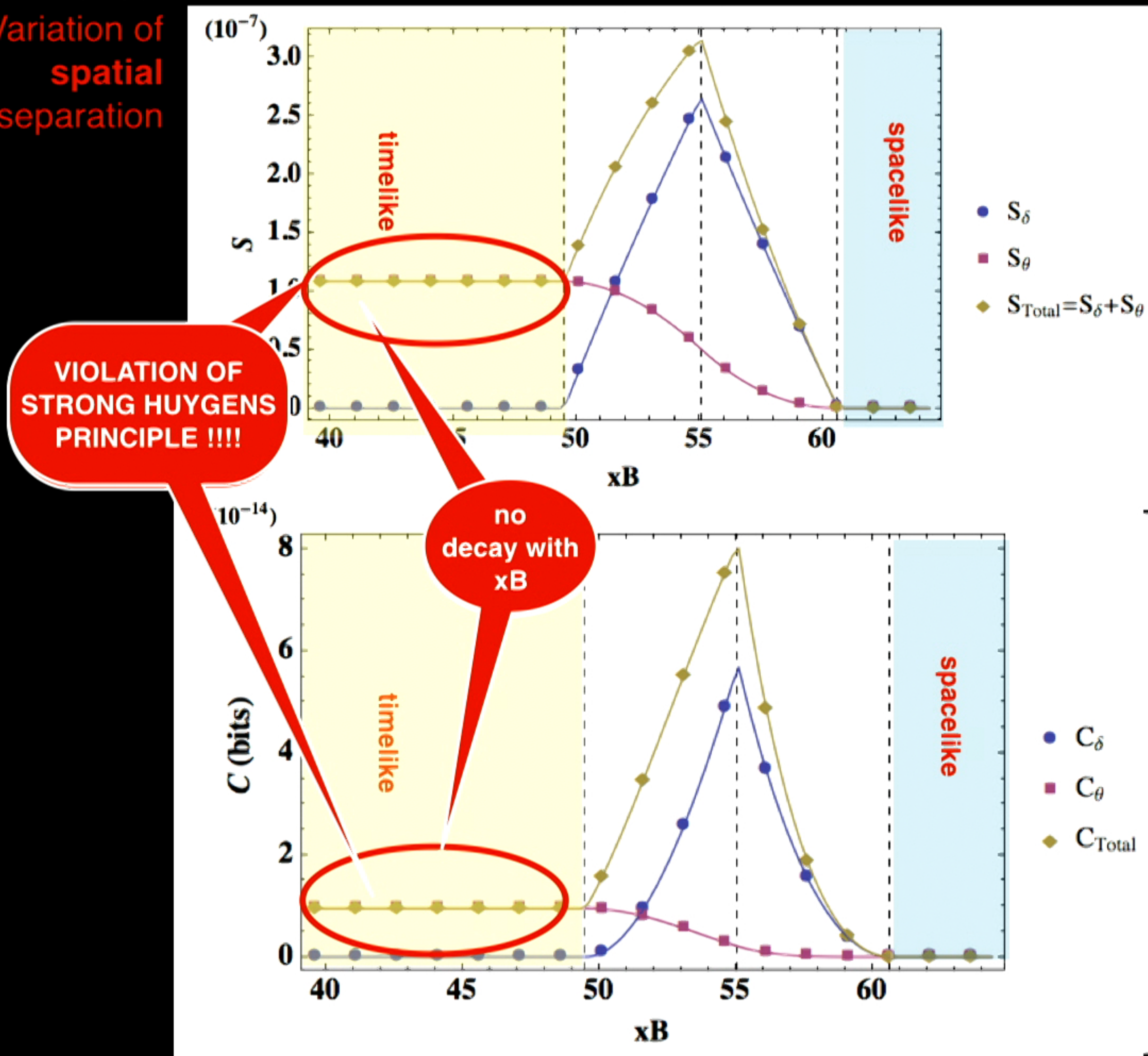
What would you do if you wanted your legacy to survive?

Encode the information in the quantum field: detectors and field get entangled.

Assuming optimality, how much information is recoverable nowadays?

Case:
Variation of **spatial** separation

MINIMAL COUPLING



SIGNALING ESTIMATOR, S

CHANNEL **CAPACITY**

Setting QUANTUM **BOUNCE**

Conclusions

The information can be, for example, Information about quantum gravity!

PHYSICAL REVIEW D **89,** 043510 (2014)

Echo of the quantum bounce

Luis J. Garay,[1,2] Mercedes Martín-Benito,[3] and Eduardo Martín-Martínez[3,4,5]

We identify a signature of quantum gravitational effects that survives from the early Universe to the current era: Fluctuations of quantum fields as seen by comoving observers are significantly influenced by the history of the early Universe. In particular, we show how the existence (or not) of a quantum bounce leaves a trace in the background quantum noise that is not damped and would be non-negligible even nowadays. Furthermore, we estimate an upper bound for the typical energy and length scales where quantum effects are relevant. We discuss how this signature might be observed and therefore used to build falsifiability tests of quantum gravity theories.

The response of a particle detector today carries the imprint of the specific dynamics of the spacetime in the early Universe

Decoherence mechanisms?

MOSTLY UNKNOWN

Quantum information impacted, not classical information!

IOP Publishing   Highlights of the Year 2012: Classical and Quantum Gravity

Class. Quantum Grav. **29** (2012) 224003 (30pp)   doi:10.1088/0264-9381/29/22/224003

Cosmological quantum entanglement

Eduardo Martín-Martínez[1] **and Nicolas C Menicucci**[2]


NAME THAT EXOPLANET
The rush to go down in astronomical history
NewScientist
Dig that reverb
Ancient echoes speak to us from the big bang
ScienceNews for Students
PHYSICS
How to pick up messages after they're gone
Scientists say there's a way to retrieve light-encoded messages even after the light has disappeared
BY ANDREW GRANT 7:00AM, MARCH 10, 2015
Today, astronomers read the history of the universe by looking at the light emitted by hot, distance objects. Sometimes, however, those light sources give out — and their light disappears. Good news: A new technique might be able to still read the information that had been carried in that long-gone light.

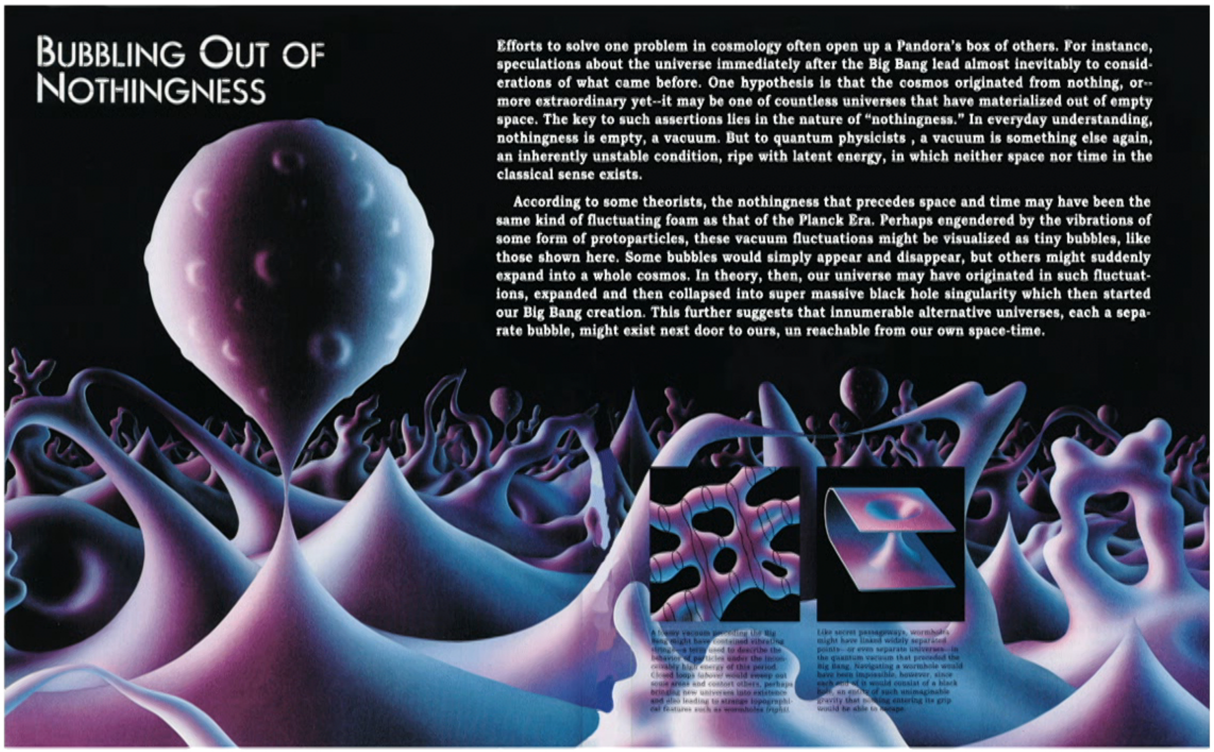
Conclusions

- ✓ All events that generate light signals also generate timelike signals (not mediated by massless quanta exchange), that decay slower.
- ✓ For a matter dominated universe we find that these signals do not decay with the spatial separation to the source. Temporal decay can be compensated by deploying a network of receivers inside the light-cone.
- ✓ We particularize the discussion to a concrete channel as a mere example to illustrate the non-decaying behaviour of the information capacity.
- ✓ Inflationary phenomena, early universe physics, primordial decouplings, etc, will also leave a timeline echo on top of the light signals that we receive from them.

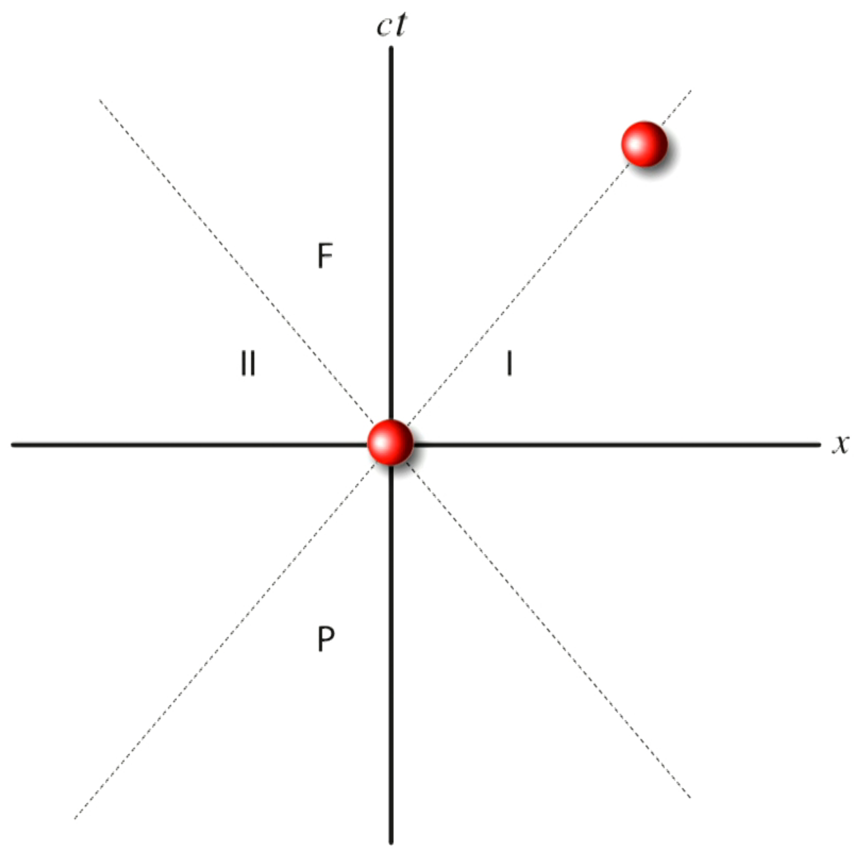
THIS MAY INSPIRE INVESTIGATING NOVEL WAYS TO LOOK AT THE EARLY UNIVERSE VIA THE TIMELIKE SIGNALS

The Quantum Vacuum

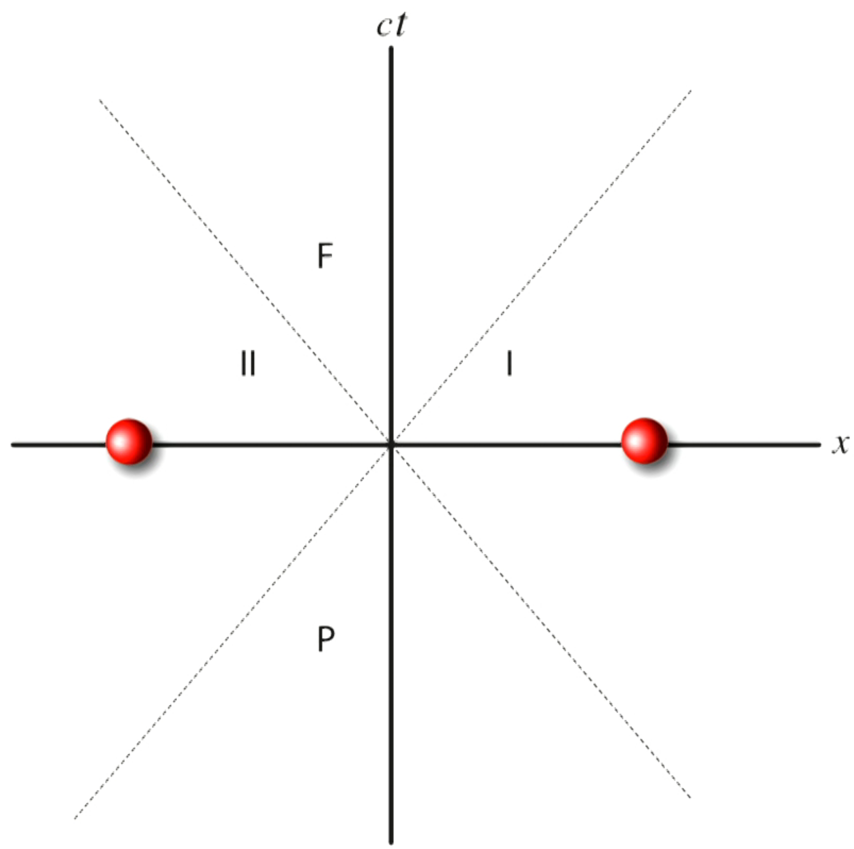
The Vacuum is not empty



Entanglement Harvesting

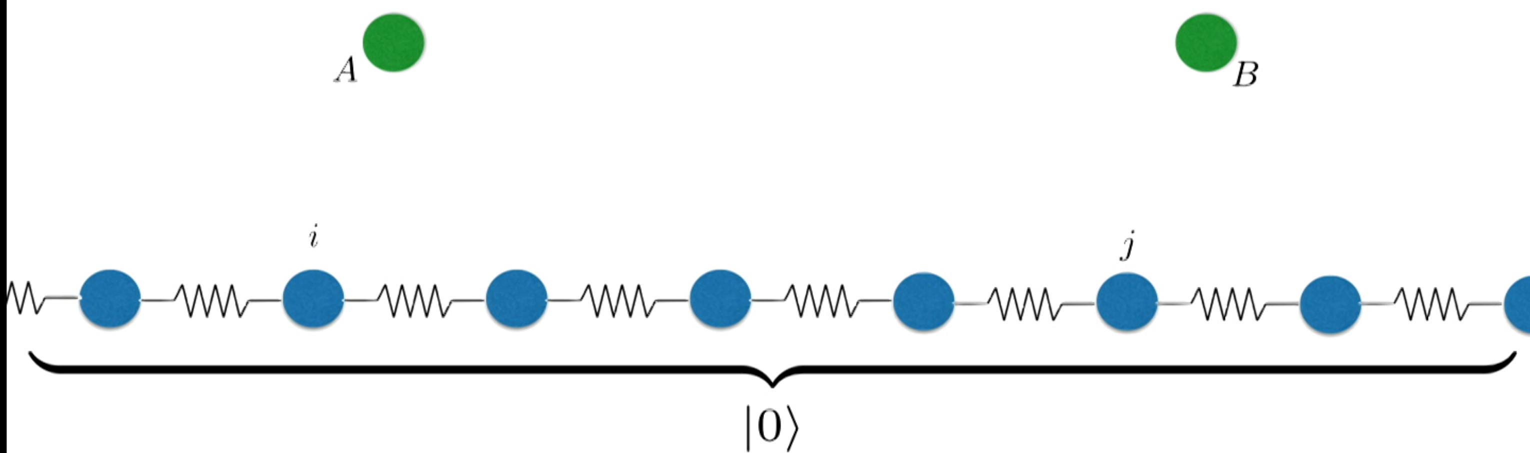


(Spacelike) Entanglement Harvesting



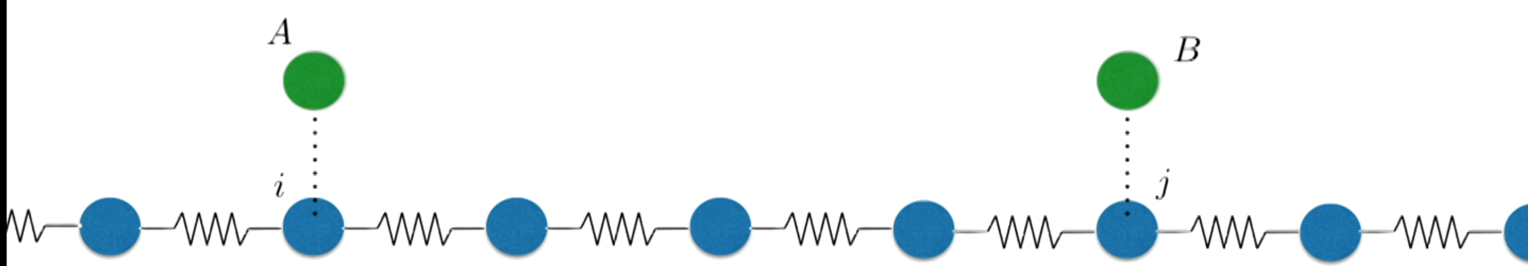
1-D Harmonic lattice in the Ground state

How do we get two systems entangled by means of local interactions with a lattice in the ground state?



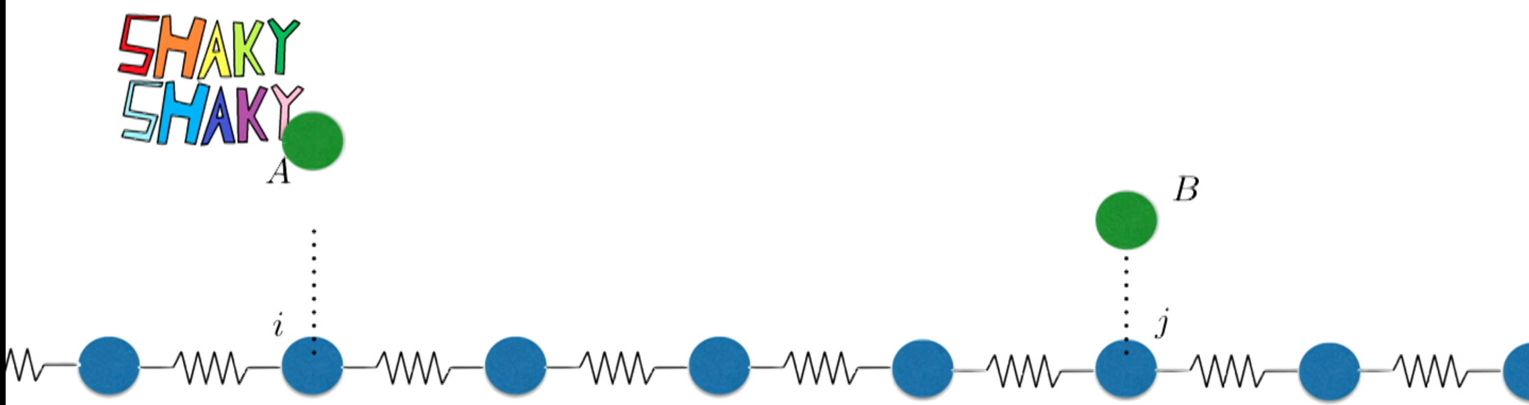
Two possible mechanisms.

1-D Harmonic lattice in the Ground state



1) Communication via phonons

1-D Harmonic lattice in the Ground state



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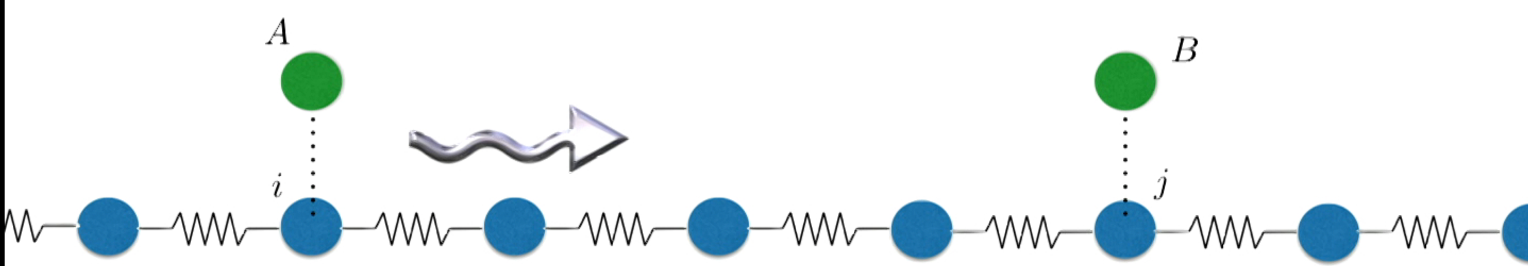

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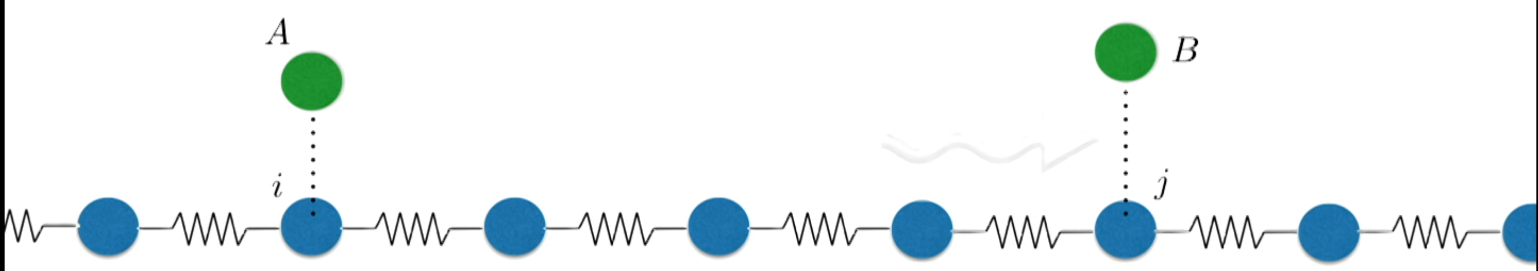

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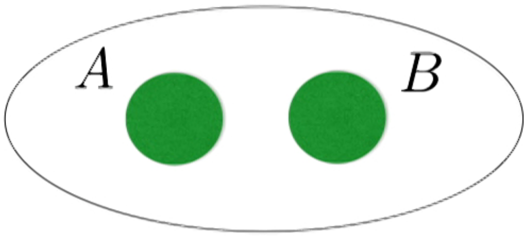


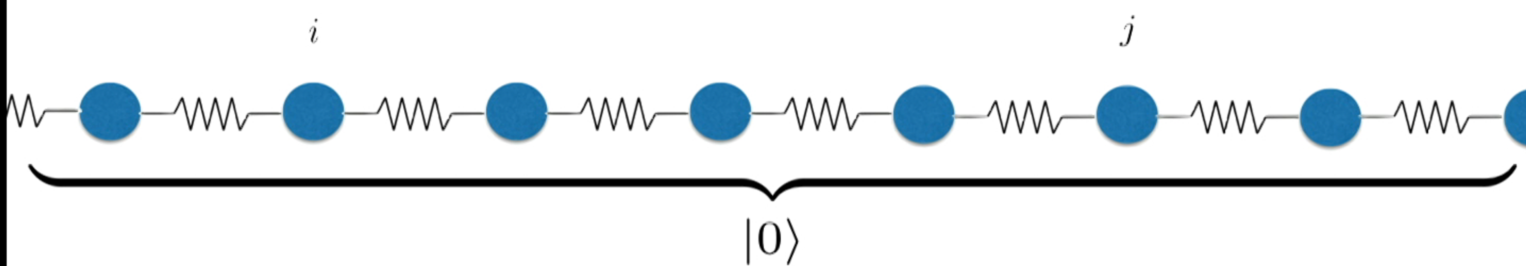
$$\rho_{AB} \neq \sum_i p_i \rho_A^i \otimes \rho_B^i$$

Limited by the speed of 'sound'

1-D Harmonic lattice in the Ground state

There's another possibility:

Take advantage of pre-existent entanglement

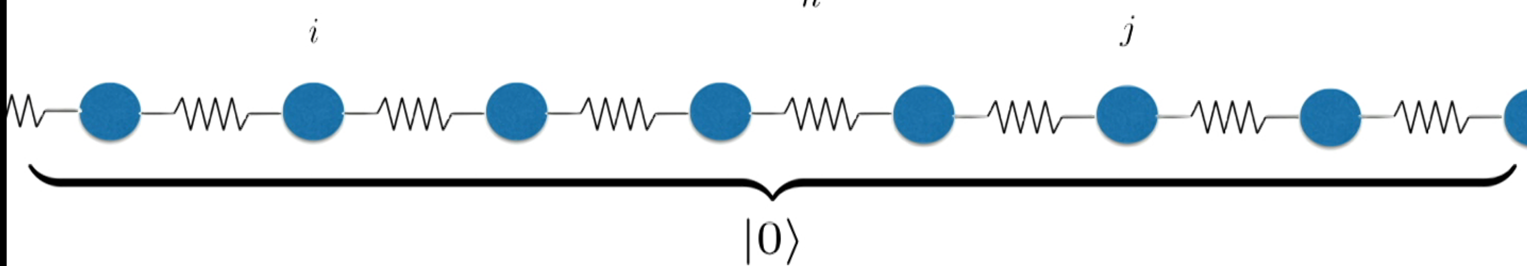


1-D Harmonic lattice in the Ground state

'Non-local' basis: Normal modes $|0\rangle\,,|1\rangle\,,|2\rangle\,,\ldots$

'Local' basis: individual number states $\{|n_1,\ldots,n_i,\ldots,n_j,\ldots\rangle\}$

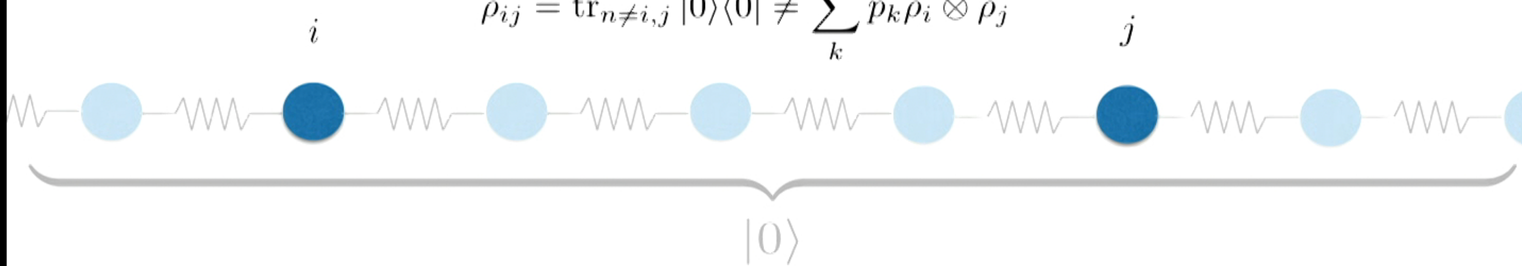
$$|0\rangle \neq \bigotimes_n |0_n\rangle$$



1-D Harmonic lattice

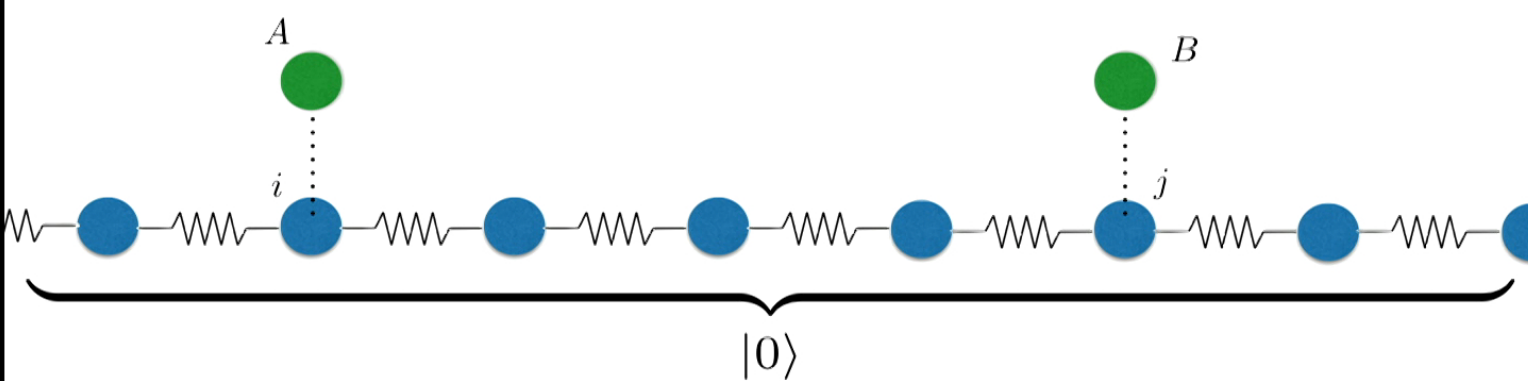
$$|0\rangle \neq \bigotimes_n |0_n\rangle$$

$$\rho_{ij} = \mathrm{tr}_{n\neq i,j}\, |0\rangle\langle 0| \neq \sum_k p_k \rho_i \otimes \rho_j$$



1-D Harmonic lattice in the Ground state

2) Swapping ground state entanglement



1-D Harmonic lattice in the Ground state

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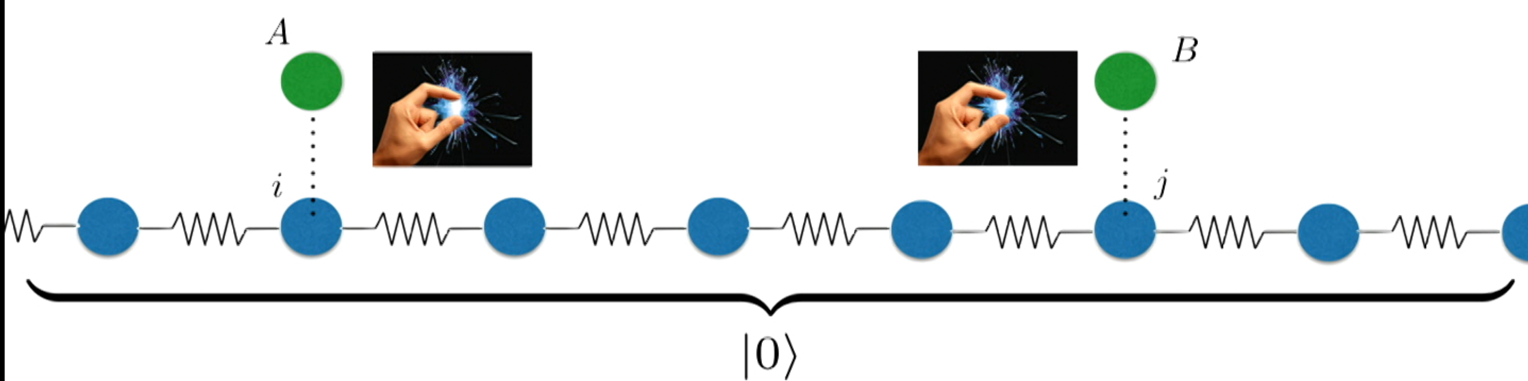
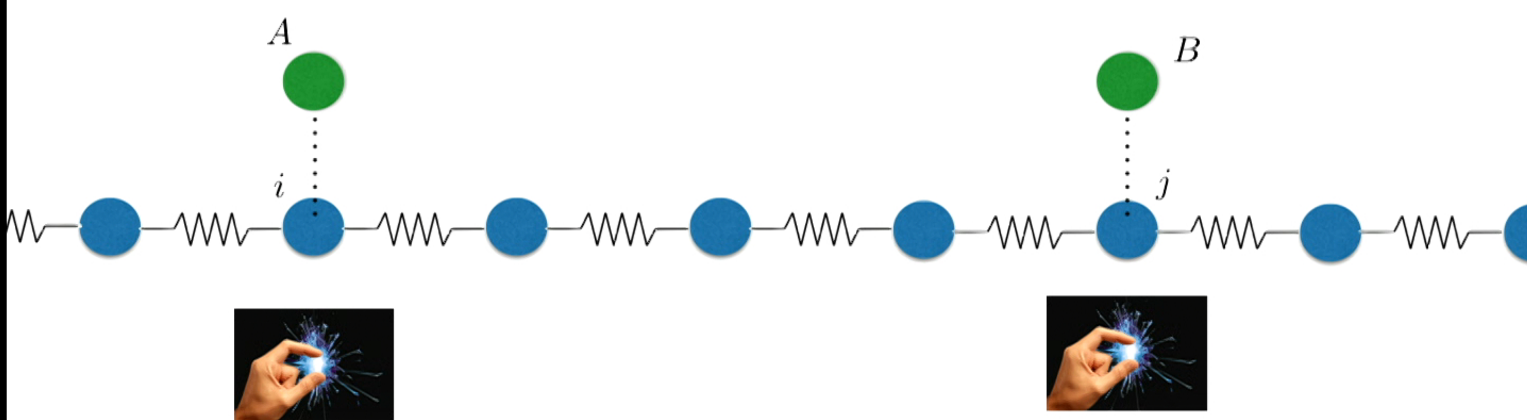


1-D Harmonic lattice in the Ground state

Local coupling to the vacuum: Observed fluctuations are correlated



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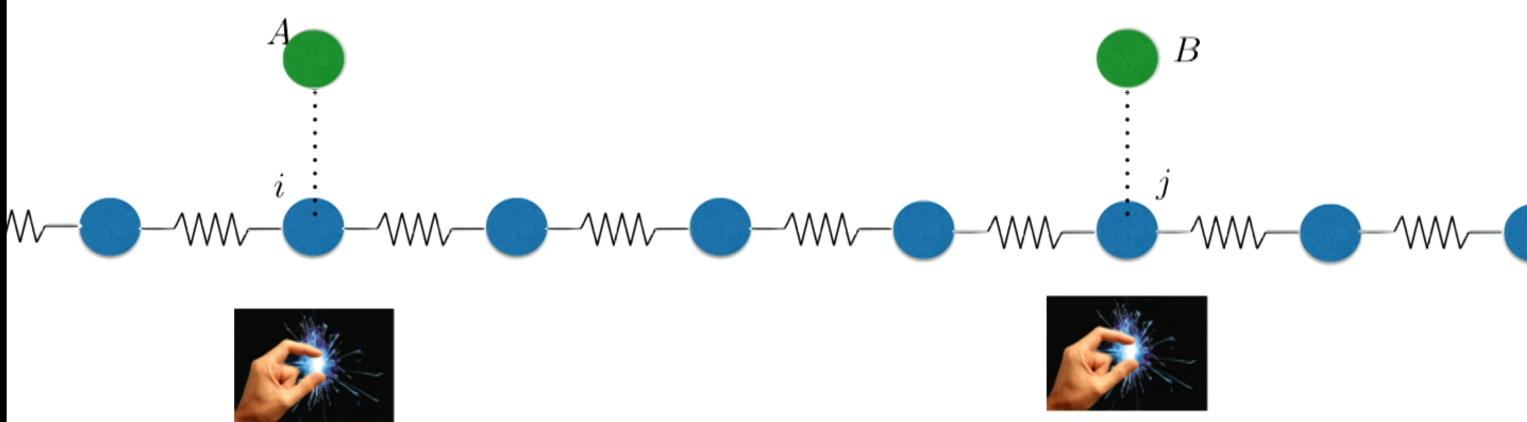

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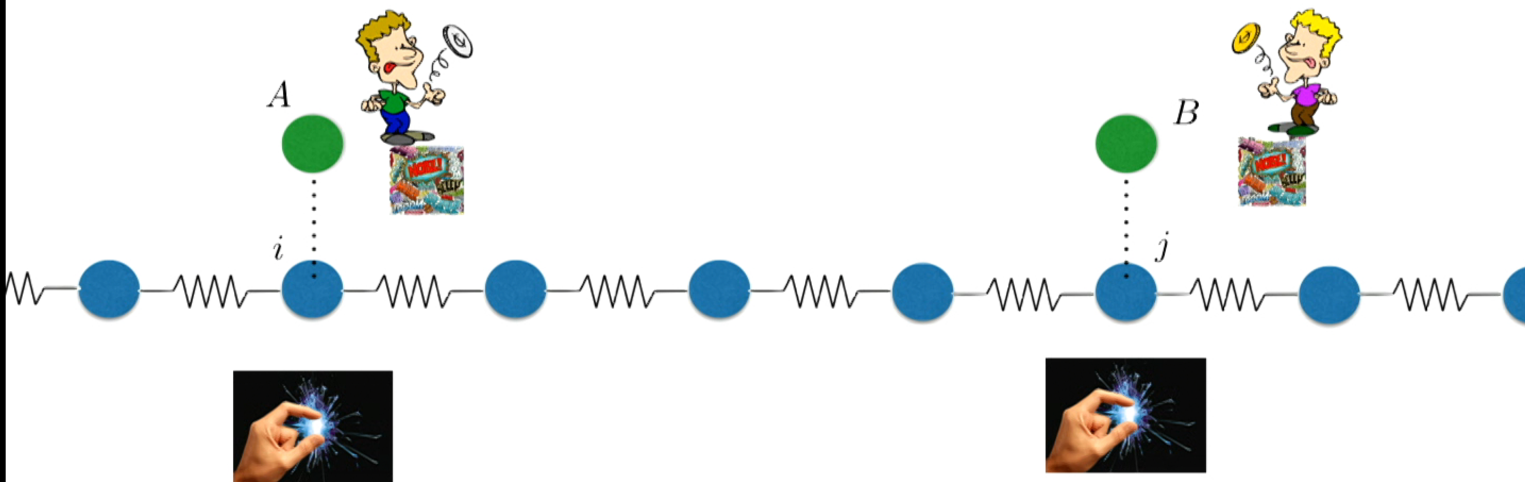
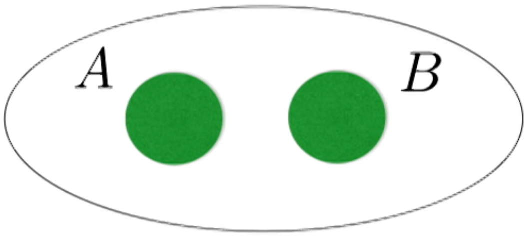


1-D Harmonic lattice in the Ground state

2) Swapping ground state entanglement

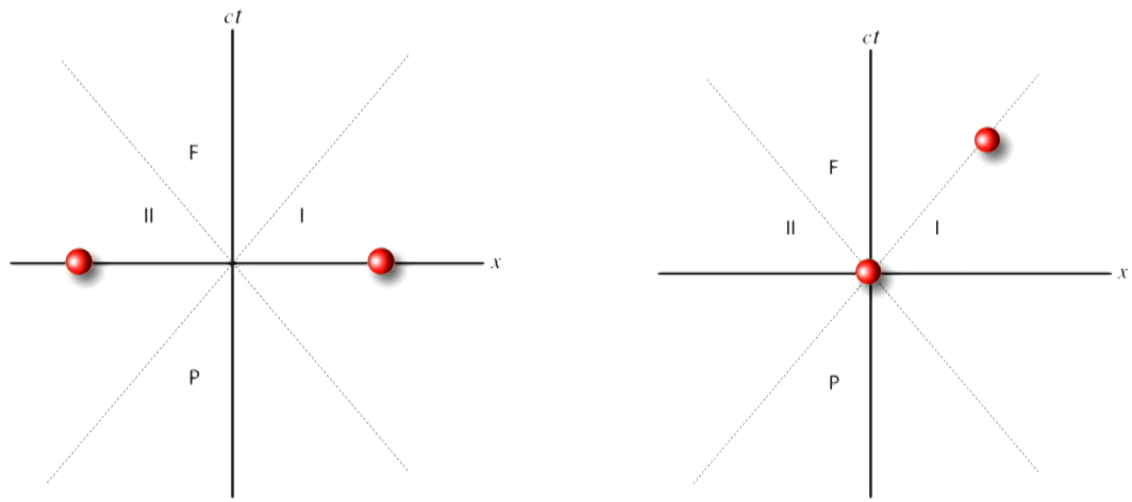


$$\rho_{AB} \neq \sum_i p_i \rho_A^i \otimes \rho_B^i$$

NOT Limited by the speed of 'sound'

Quantum Fields

A 1D quantum field can be thought as the 'continuum limit' of such a lattice



Two mechanisms to get 'atoms' entangled via interaction with quantum fields:

1) Via exchange of real field quanta

2) Swapping vacuum entanglement

Can we extract vacuum entanglement?

Scalar fields and Unruh-DeWitt detectors:

- A. Valentini, Phys. Lett. A,153, 321 (1991)
- B. Reznik, Found. Phys. 33, 167 (2003)
- A. Pozas-Kerstjens and E. Martín-Martínez, Phys. Rev. D 92, 064042 (2015)

Electromagnetic fields and atoms:

- A. Pozas-Kerstjens and E. Martín-Martínez, Phys. Rev. D 94, 064074 (2016).

Sensitivity to spacetime geometry:

- G. V. Steeg and N. C. Menicucci, Phys. Rev. D 79, 044027 (2009)

Sensitivity to spacetime topology:

- E. Martín-Martínez, A. R. H. Smith and D. R. Terno, Phys. Rev. D, 93, 044001 (2016)

Experimental proposals:

- S. J. Olson and T. C. Ralph, Phys. Rev. Lett. 106, 110404 (2011).
- C. Sabín, B. Peropadre, M. del Rey & E. Martín-Martínez, Phys. Rev. Lett. 109, 033602 (2012)

What are the limits?

Can we repeat the process cyclically?

Is the vacuum entanglement in a cavity replenishable?

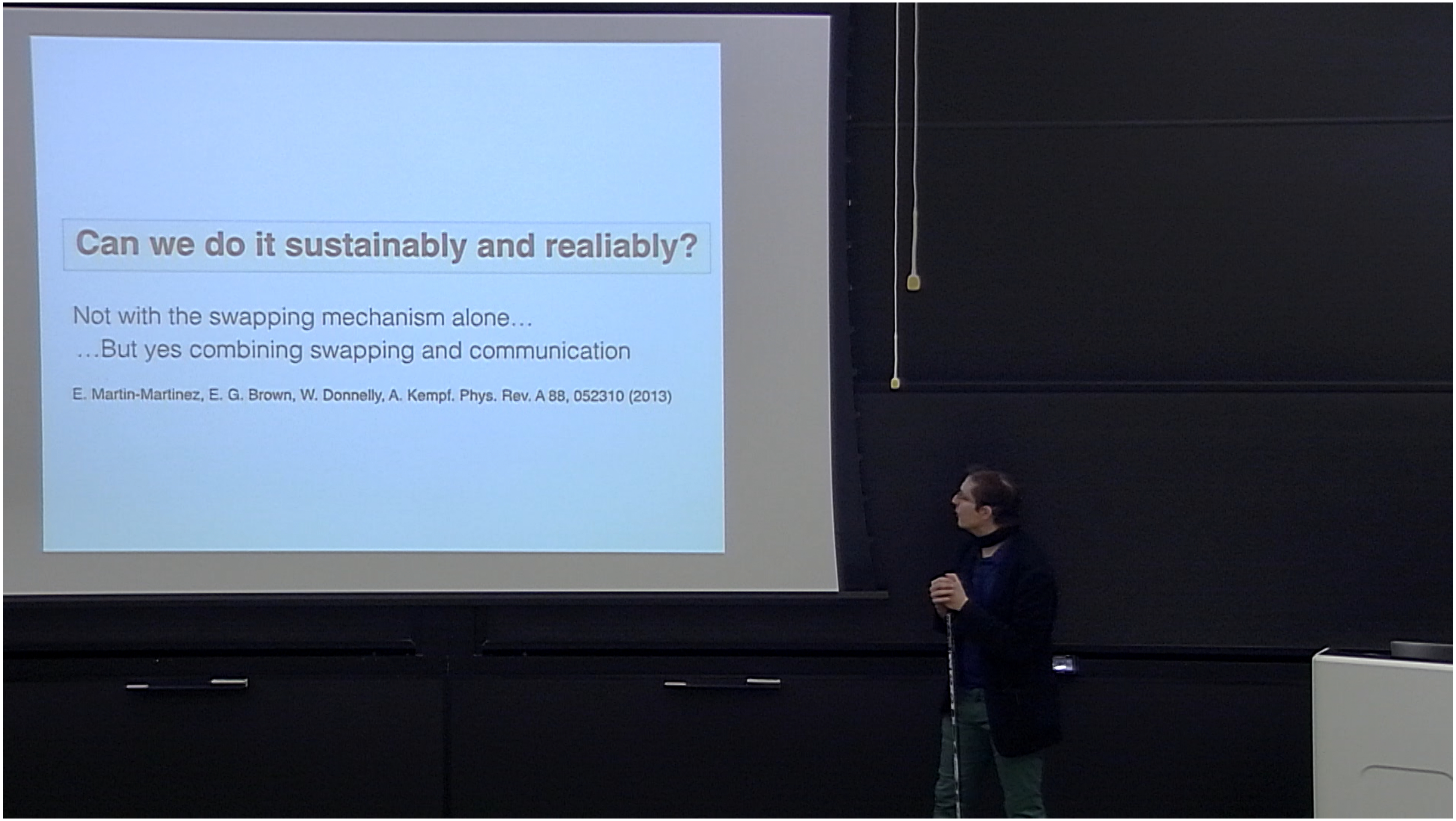
Is there a 'Carnot-like' optimal extraction cycle?

Can we do it sustainably and realiably?

Not with the swapping mechanism alone…

Entanglement resources get exhausted: Entropy increase: Heating, mixedness,…

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)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…But yes combining swapping and communication
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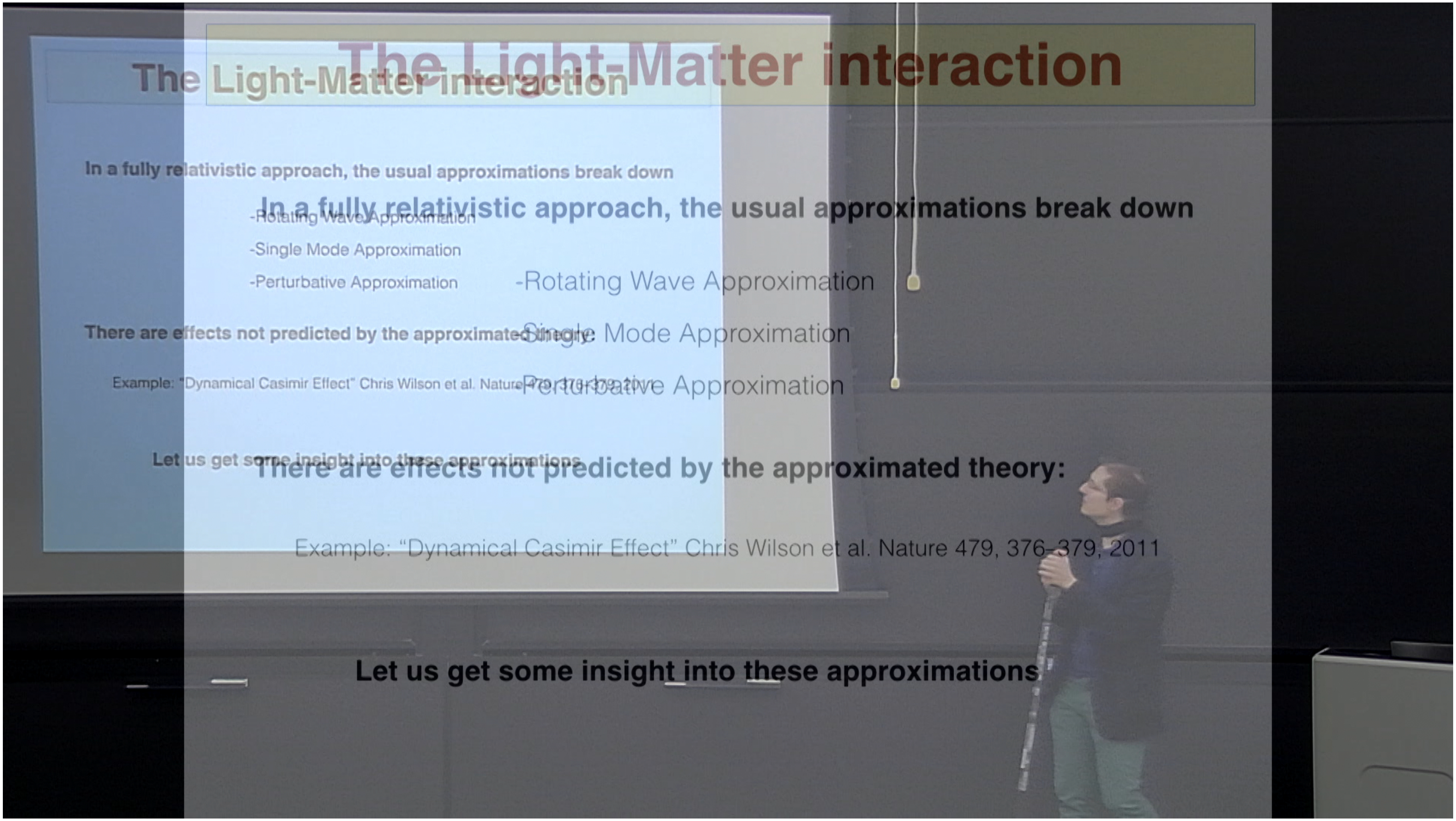
How do we do it?

Requirement: Go beyond the usual approximations in Quantum Optics.


The Light-Matter interaction
In a fully relativistic approach, the usual approximations break down
-Rotating Wave Approximation
-Single Mode Approximation
-Perturbative Approximation
There are effects not predicted by the approximated theory:
Example: "Dynamical Casimir Effect" Chris Wilson et al. Nature 479, 376-379, 2011
Let us get some insight into these approximations

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-Rotating Wave Approximation

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Example: "Dynamical Casimir Effect" Chris Wilson et al. Nature 479, 376–379, 2011

Let us get some insight into these approximations

The Light-Matter interaction

$$H_I = \lambda\left(\sigma^+ e^{i\Omega\tau} + \sigma^- e^{-i\Omega\tau}\right)\sum_{j=1}^{\infty}\left[a_j^\dagger e^{i\omega_j t(\tau)} + a_j e^{-i\omega_j t(\tau)}\right]\sin k_j x(\tau),$$

Models the interaction of a two-level system with a scalar field

$\sigma^+ a_j,\ \ \sigma^- a_j^\dagger$	Rotating-wave terms	$e^{i[\Omega\tau - \omega_j t(\tau)]}$
$\sigma^- a_j,\ \ \sigma^+ a_j^\dagger$	Counter-rotating wave terms	$e^{i[\Omega\tau + \omega_j t(\tau)]}$

Atom at rest: $x(\tau) = x_0, \quad t(\tau) = \tau$

Two kinds of terms

$$H_0 = \frac{\Omega}{2}\sigma_z + \sum_j \omega_j a_j^\dagger a_j$$

$$H_I = \lambda\left(\sigma^+ + \sigma^-\right)\sum_{j=1}^{\infty}\left[a_j^\dagger + a_j\right]\sin k_j x,$$

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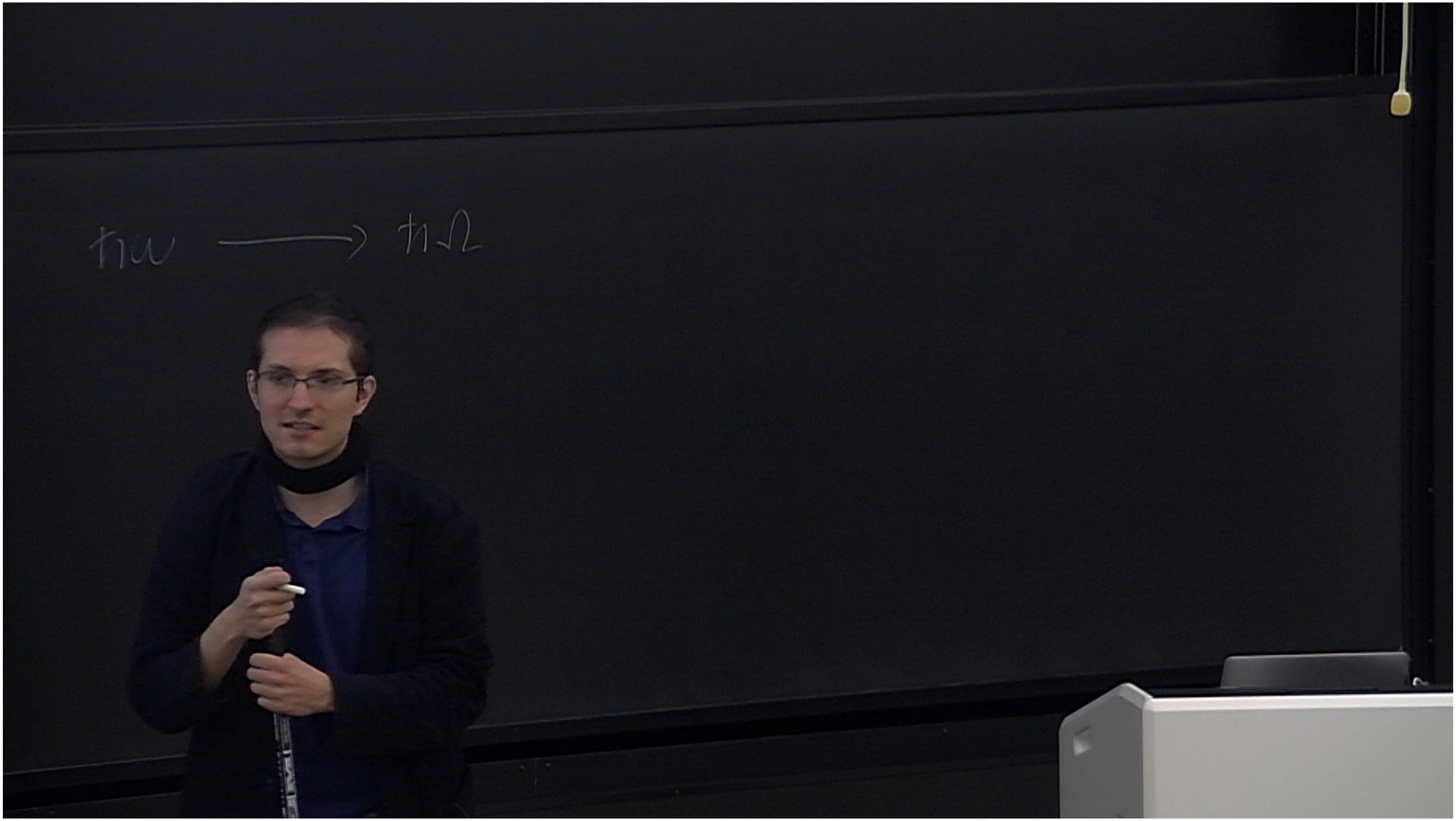
$$\sigma^+ a_\omega \left|1_\omega\right\rangle \left|g\right\rangle \longrightarrow \left|0\right\rangle \left|e\right\rangle$$

$\sigma^+ a_j, \quad \sigma^- a_j^\dagger$ Rotating-wave terms

$\sigma^- a_j, \quad \sigma^+ a_j^\dagger$ Counter-rotating wave terms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$\sigma^+ a_j,\ \ \sigma^- a_j^\dagger$ Rotating-wave terms

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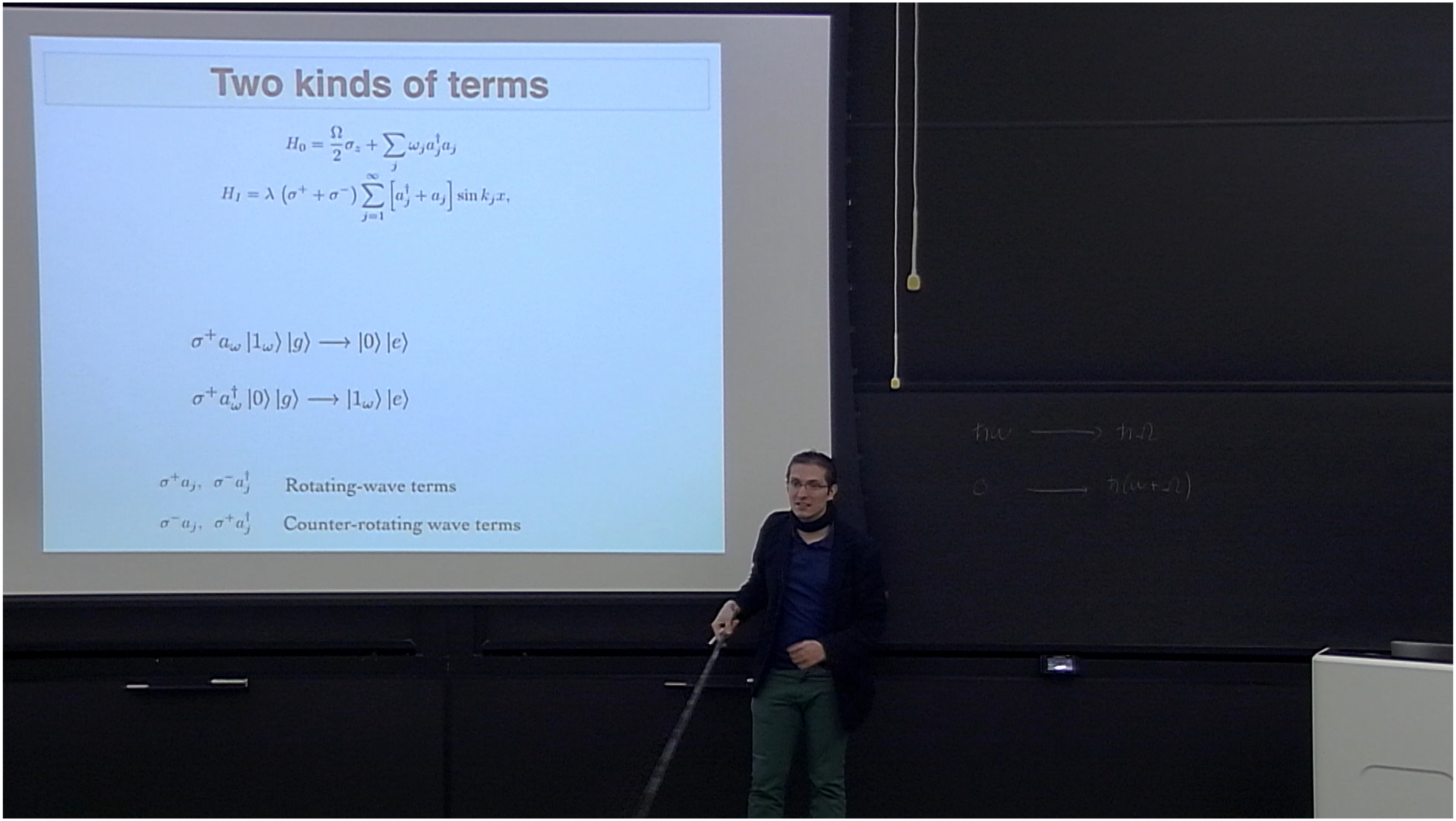
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$\sigma^+ a_\omega^\dagger |0\rangle |g\rangle \longrightarrow |1_\omega\rangle |e\rangle$
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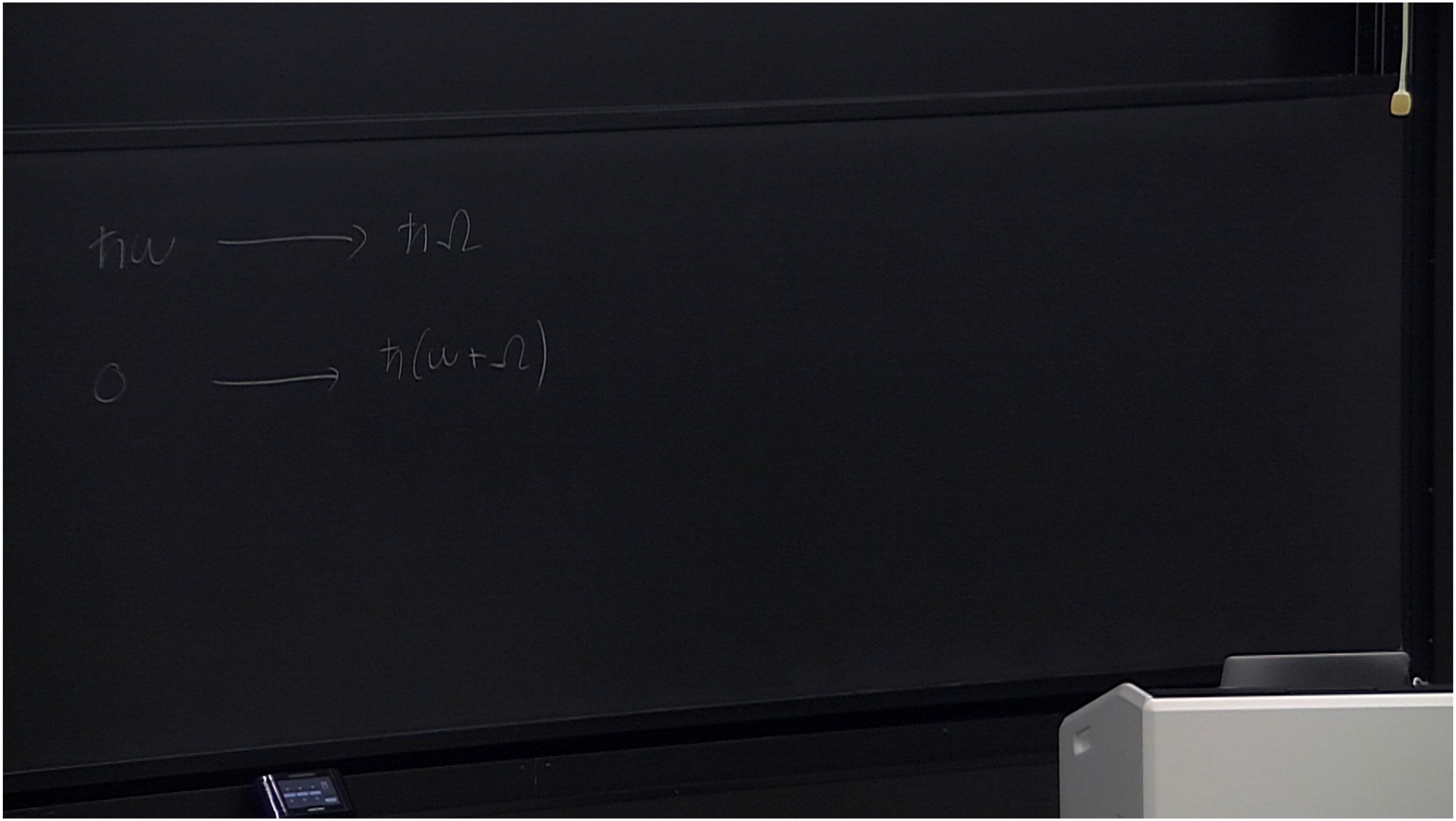
$\sigma^- a_j,\ \sigma^+ a_j^\dagger$ Counter-rotating wave terms
$\hbar\omega \longrightarrow \hbar\Omega$
$0 \longrightarrow \hbar(\omega+\Omega)$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Two kinds of terms

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$\sigma^+ a_j,\ \sigma^- a_j^\dagger$ Rotating-wave terms

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usual approximations in QO

Typical approximations made in quantum optics when $\omega_0 = \omega_j$

$\Delta T \gg \omega_0^{-1}$ Single mode approximation

$$\lim_{\substack{t\to\infty \\ t_0\to-\infty}} \int_{t_0}^{t} \mathrm{d}t_1\, e^{\mathrm{i}(\omega_0-\omega_j)t_1} \sim \Delta T$$

$$\lim_{\substack{t\to\infty \\ t_0\to-\infty}} \int_{t_0}^{t} \mathrm{d}t_1\, e^{\mathrm{i}(\omega_0-\omega_n)t_1} \sim \Delta T \left\langle e^{-\mathrm{i}(\omega_0-\omega_n)t_1} \right\rangle \sim \frac{1}{(\omega_0-\omega_n)}$$

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$\Delta T \gg (2\omega_0)^{-1}$ Rotating-wave approximation

$$\lim_{\substack{t\to\infty \\ t_0\to-\infty}} \int_{t_0}^{t} \mathrm{d}t_1 \, e^{\mathrm{i}(\omega_0+\omega_n)t_1} \sim \Delta T \left\langle e^{\mathrm{i}(\omega_0+\omega_n)t_1} \right\rangle \sim \frac{1}{(\omega_0+\omega_n)}$$

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Rotating-wave approximation $\Rightarrow$ No 'vacuum' fluctuations

The usual 'non-relativistic' Q.O. fails

- -Rotating wave approximation ❌
- -Single (or few) mode approximation ❌

The usual 'non-relativistic' Q.O. fails

- -Rotating wave approximation ❌
- -Single (or few) mode approximation ❌

-Perturbation Theory

The usual ‘non-relativistic’ Q.O. fails

- -Rotating wave approximation ✗
- -Single (or few) mode approximation ✗
- -Perturbation Theory ✗

The usual 'non-relativistic' Q.O. fails

-Rotating wave approximation ✗

-Single (or few) mode approximation ✗

-Perturbation Theory ✗

How do we do non-perturbative calculations in Q.O.?

- Harmonic model.
- Gaussian methods

Non-perturbative / Non-RWA / Non-SM Relativistic light-matter interaction

PHYSICAL REVIEW D **87,** 084062 (2013)

Detectors for probing relativistic quantum physics beyond perturbation theory

Eric G. Brown,[1] Eduardo Martín-Martínez,[1,2,3] Nicolas C. Menicucci,[4] and Robert B. Mann[1,3]

We develop a general formalism for a nonperturbative treatment of harmonic-oscillator particle detectors in relativistic quantum field theory using continuous-variable techniques. By means of this we forgo perturbation theory altogether and reduce the complete dynamics to a readily solvable set of first-order, linear differential equations. The formalism applies unchanged to a wide variety of physical setups, including arbitrary detector trajectories, any number of detectors, arbitrary time-dependent quadratic couplings, arbitrary Gaussian initial states, and a variety of background spacetimes. As a first set of concrete results, we prove nonperturbatively—and without invoking Bogoliubov transformations—that an accelerated detector in a cavity evolves to a state that is very nearly thermal with a temperature proportional to its acceleration, allowing us to discuss the universality of the Unruh effect. Additionally we quantitatively analyze the problems of considering single-mode approximations in cavity field theory and show the emergence of causal behavior when we include a sufficiently large number of field modes in the analysis. Finally, we analyze how the harmonic particle detector can harvest entanglement from the vacuum. We also study the effect of noise in time-dependent problems introduced by suddenly switching on the interaction versus ramping it up slowly (adiabatic activation).

Gaussian methods in QM

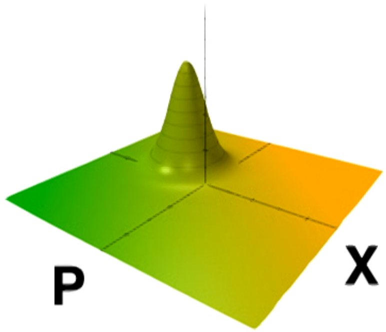
Quantum Mechanics is computationally difficult

- Set of N harmonic oscillators
- Density operators are infinite dimensional

Not the whole Hilbert space is needed here.

Gaussian States

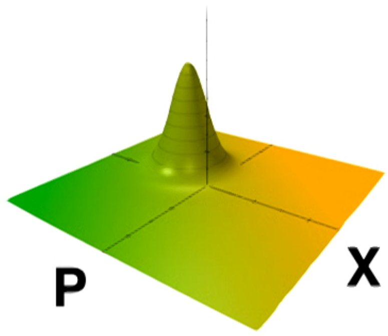
G.S. are states whose Wigner function is Gaussian



- Thermal states
- Coherent states
- Squeezed states
- Squeezed thermal states

Gaussian States

G.S. are states whose Wigner function is Gaussian



- Thermal states
- Coherent states
- Squeezed states
- Squeezed thermal states
- The vacuum state

Gaussian States

A zero mean Gaussian state can be characterized by its first and second moments

$$\sigma_{ij} \equiv \langle \hat{x}_i \hat{x}_j + \hat{x}_j \hat{x}_i \rangle - 2\langle \hat{x}_i \rangle \langle \hat{x}_j \rangle$$

Set of M+N harmonic oscillators

$$\hat{\mathbf{x}} := \left(\hat{q}_{d_1}, \ldots, \hat{q}_{d_M}, \hat{q}_1, \ldots, \hat{q}_N, \hat{p}_{d_1}, \ldots, \hat{p}_{d_M}, \hat{p}_1, \ldots, \hat{p}_N\right)^{\mathrm{T}},$$

$$\hat{q}_i = \frac{1}{\sqrt{2}}\left(\hat{a}_i + \hat{a}_i^\dagger\right), \quad \hat{p}_i = \frac{\mathrm{i}}{\sqrt{2}}\left(\hat{a}_i^\dagger - \hat{a}_i\right)$$

Gaussian Evolution

Evolution under quadratic Hamiltonians

$$\hat{H} = \hat{\mathbf{x}}^{\mathrm{T}}\mathbf{F}(t)\hat{\mathbf{x}} = (\hat{\mathbf{a}}^{\dagger})^{\mathrm{T}}\mathbf{w}(t)\hat{\mathbf{a}} + (\hat{\mathbf{a}}^{\dagger})^{\mathrm{T}}\mathbf{g}(t)\hat{\mathbf{a}}^{\dagger} + \hat{\mathbf{a}}^{\mathrm{T}}\mathbf{g}(t)^{\mathrm{H}}\hat{\mathbf{a}}$$

Preserves Gaussianity

$$\hat{\mathbf{a}} := (\hat{a}_{d_1}, \ldots, \hat{a}_{d_M}, \hat{a}_1, \ldots \hat{a}_N)^{\mathrm{T}},$$
$$\hat{\mathbf{a}}^{\dagger} := (\hat{a}^{\dagger}_{d_1}, \ldots, \hat{a}^{\dagger}_{d_M}, \hat{a}^{\dagger}_1, \ldots \hat{a}^{\dagger}_N)^{\mathrm{T}}.$$

$$\hat{\mathbf{x}} := (\hat{q}_{d_1}, \ldots, \hat{q}_{d_M}, \hat{q}_1, \ldots, \hat{q}_N, \hat{p}_{d_1}, \ldots, \hat{p}_{d_M}, \hat{p}_1, \ldots, \hat{p}_N)^{\mathrm{T}},$$

$$\hat{q}_i = \frac{1}{\sqrt{2}}\left(\hat{a}_i + \hat{a}_i^{\dagger}\right), \quad \hat{p}_i = \frac{\mathrm{i}}{\sqrt{2}}\left(\hat{a}_i^{\dagger} - \hat{a}_i\right)$$

E. G. Brown, E. Martín-Martínez, N. C. Menicucci, R. B. Mann. Phys. Rev. D 87, 084062 (2013)

Gaussian Evolution (no displacements)

Unitary transformations in Hilbert space

$$\hat{\mathbf{x}}(t) = \hat{U}^\dagger(t)\,\hat{\mathbf{x}}_0\,\hat{U}(t) = \mathbf{S}(t)\,\hat{\mathbf{x}}_0$$

Symplectic transformations in phase space

$$\boldsymbol{\sigma}(t) = \mathbf{S}(t)\boldsymbol{\sigma}_0\mathbf{S}(t)^{\mathrm{T}},$$

$$\frac{d}{dt}\hat{\mathbf{x}}(t) = i\big[\hat{H}(t),\hat{\mathbf{x}}(t)\big] \Rightarrow \quad \frac{d}{dt}\mathbf{S}(t) = \boldsymbol{\Omega}\mathbf{F}^{\mathrm{sym}}(t)\mathbf{S}(t).$$

$$[\hat{\mathbf{x}},\hat{\mathbf{x}}^{\mathrm{T}}] = i\begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{pmatrix} =: i\boldsymbol{\Omega} \qquad \mathbf{F}^{\mathrm{sym}} = (\mathbf{F} + \mathbf{F}^{\mathrm{T}})$$

Applied to the light-matter interaction?

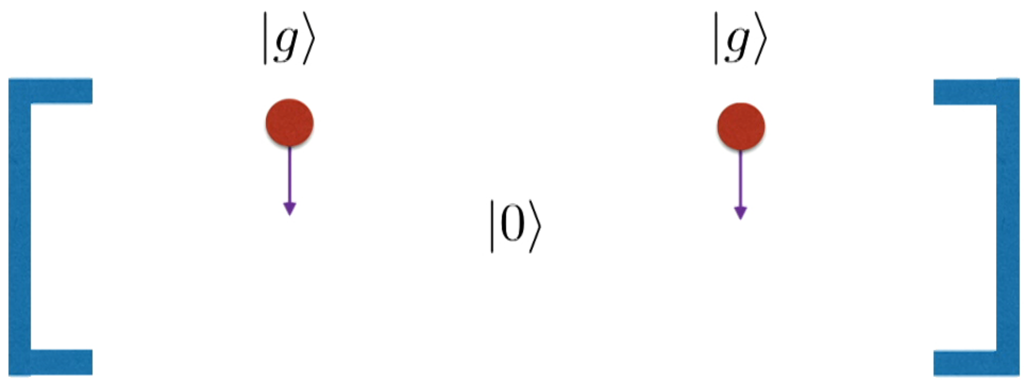
The first M harmonic oscillators are atoms

The next $N \to \infty$ harmonic oscillators are modes of a quantum field in an optical cavity

- Non-perturbative
- Generically time-dependent problems
- Ideal for relativistic approaches
- Ideal for cases where you need to relax approximations

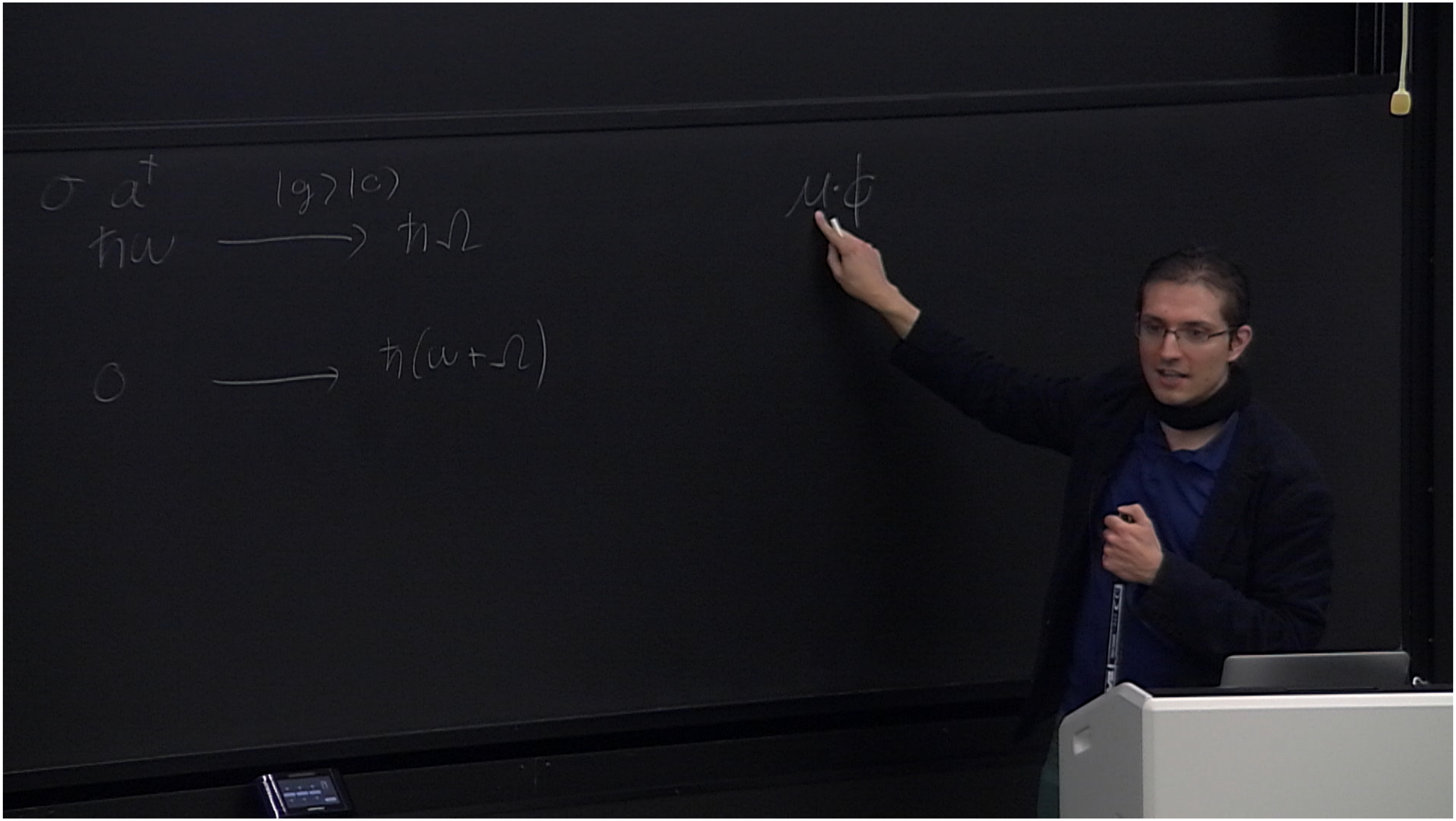
Towards Entanglement Farming

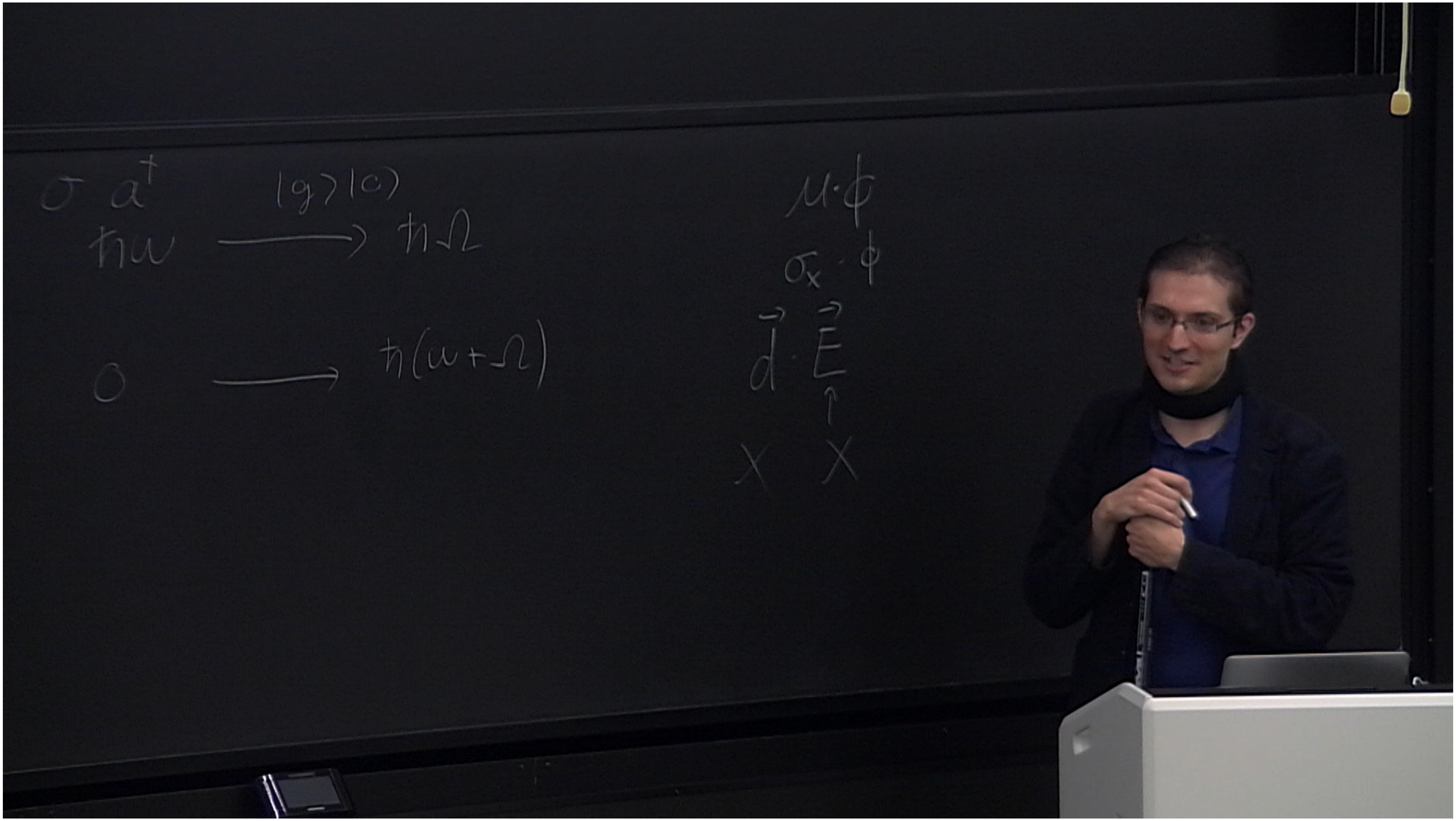
Let us consider the following setting:

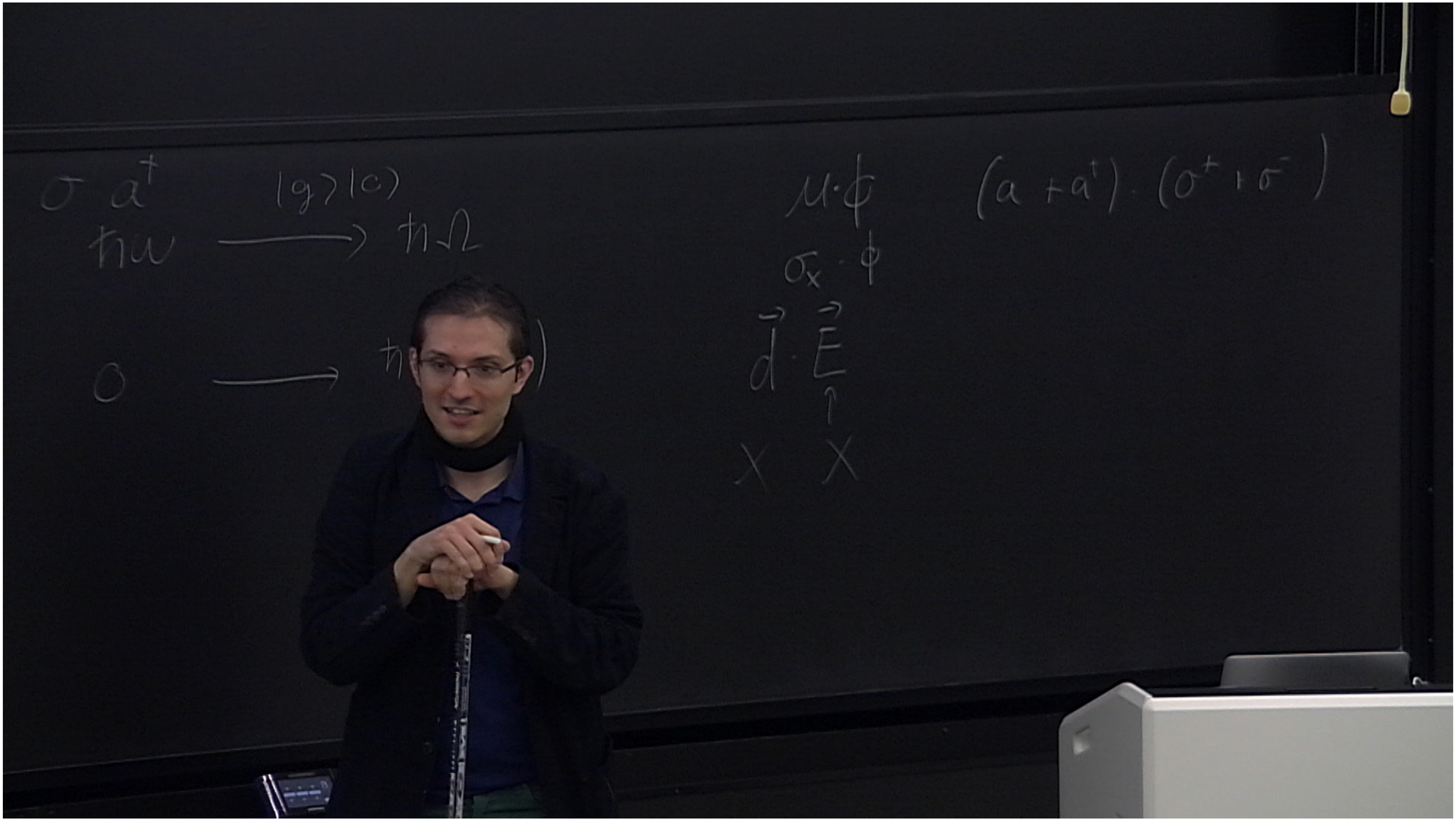


Two atoms going through an optical cavity prepared in the vacuum

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)


σ a†
|g⟩|e⟩
ħω ⟶ ħΩ
μ·φ
0 ⟶ ħ(ω+Ω)


σ a†
|g⟩|0⟩
ħω ⟶ ħΩ
0 ⟶ ħ(ω+Ω)
μ·φ
σx · φ
d · E
X X


σ a†
|g⟩|0⟩
ħω → ħΩ
0 →
μ·φ
(a + a†)·(σ⁺ + σ⁻)
σx · φ
d · E

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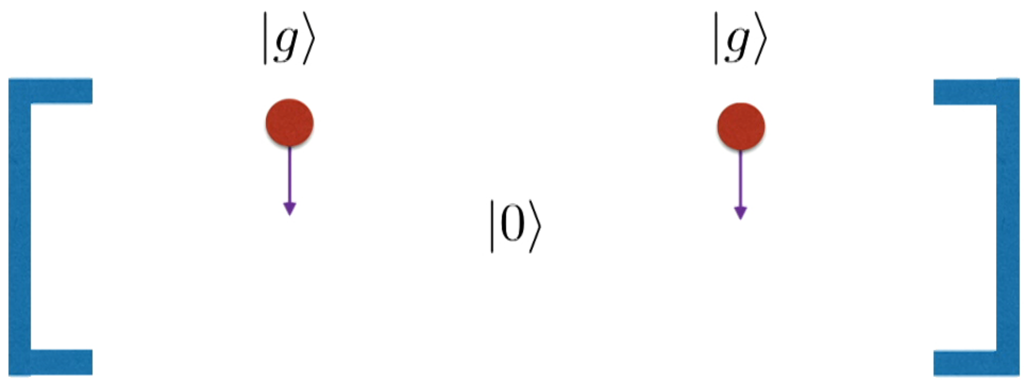

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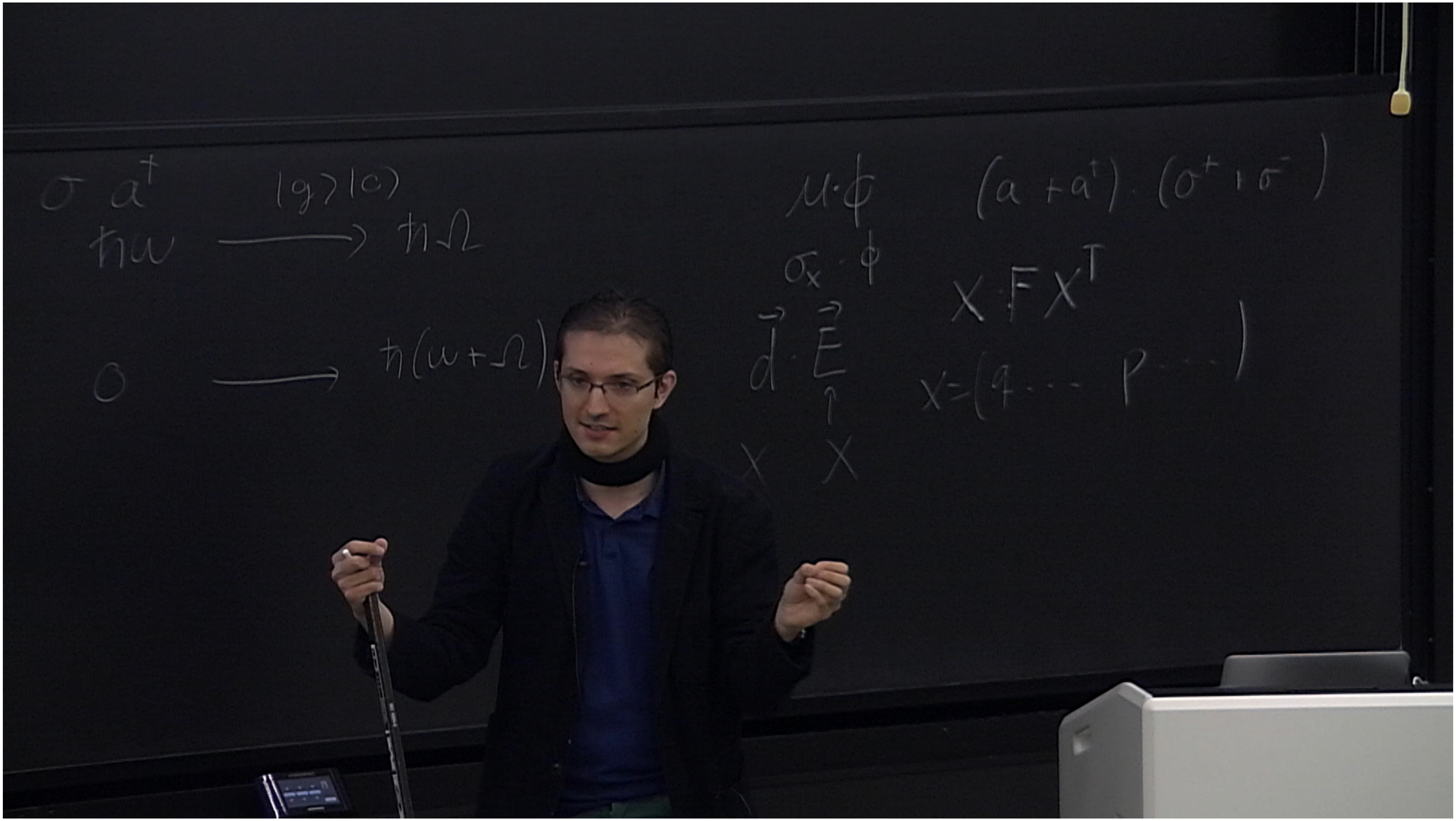
σ a†
|g⟩|0⟩
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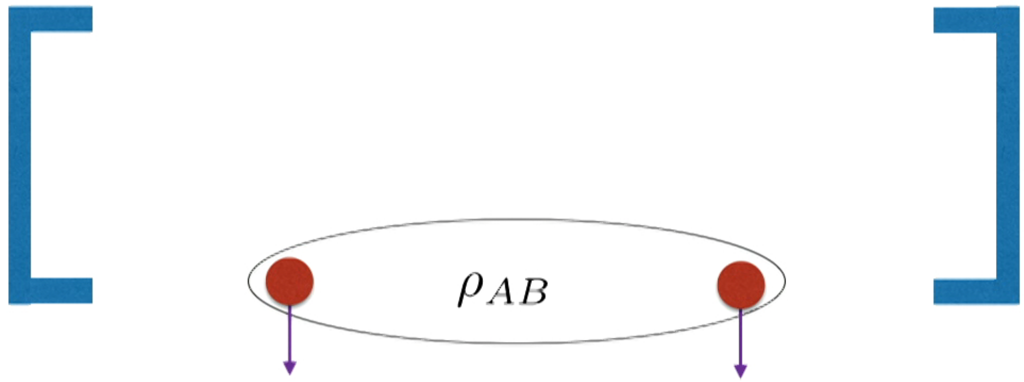
μ·φ
σx · φ
d · E
X X
(a + a†) · (σ+ + σ−)
X · F X^T
X = (q . . . . p . . . .)

Towards Farming

Let us consider the following setting:

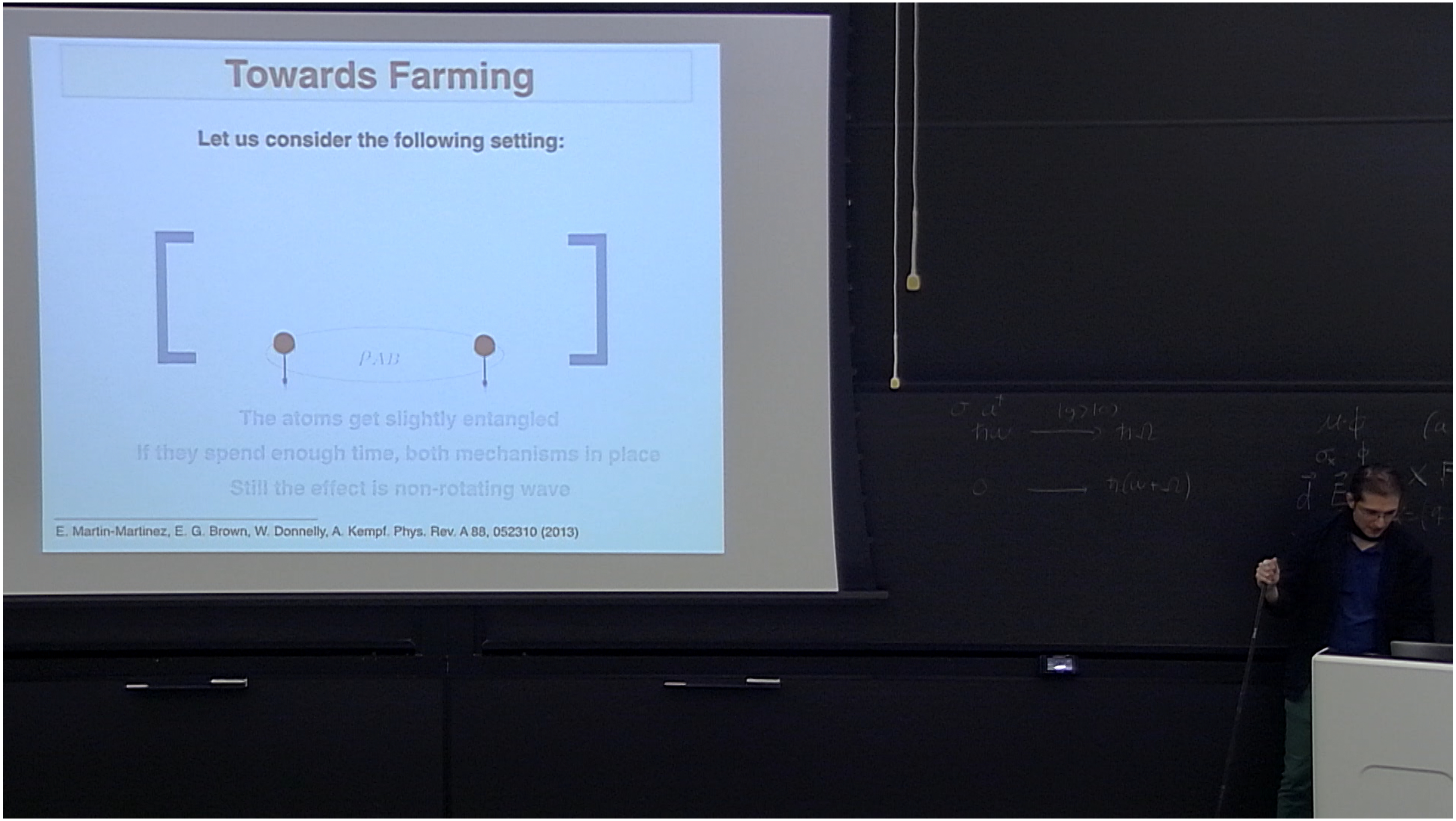


The atoms get slightly entangled

If they spend enough time, both mechanisms in place

Still the effect is non-rotating wave

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)


Towards Farming
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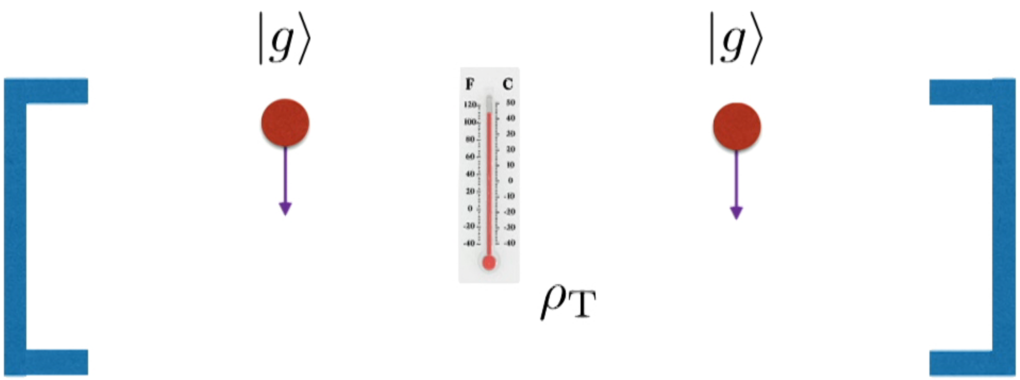

Two atoms going through an optical cavity prepared in a thermal state

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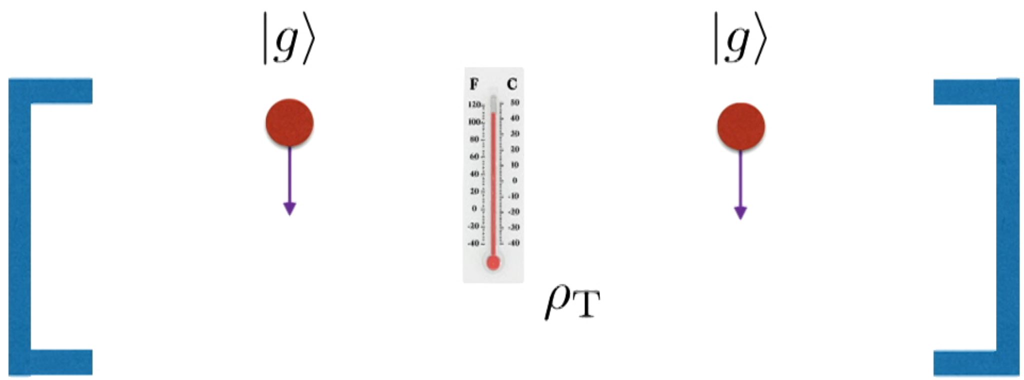

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Towards Farming

Let us consider the following setting:

$$\rho_{AB} = \sum_i p_i \, \rho_A \otimes \rho_B$$

The atoms will NOT get entangled

Too much 'local' noise

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Towards Farming

It would seem this setting is not so great to get atoms entangled

Not robust under finite temperatures:

The amount of entanglement extracted **vanishes** quickly as the temperature increases

What if we repeat the process iteratively?

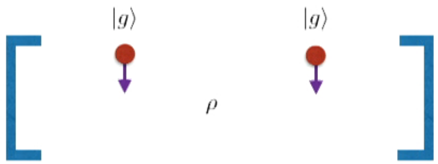
We send many pairs of atoms initialized in the ground state

Let us analyze the dynamics of this process

E. Martin-Martinez, E. G. Brown, W. Donnelly, A. Kempf. Phys. Rev. A 88, 052310 (2013)

Towards Farming

Step 1a: field in arbitrary state. Atoms in the ground state



$$\sigma_0 = \sigma_{A,g} \oplus \sigma_{B,g} \oplus \sigma_{\mathrm{f}}$$

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$$\downarrow$$

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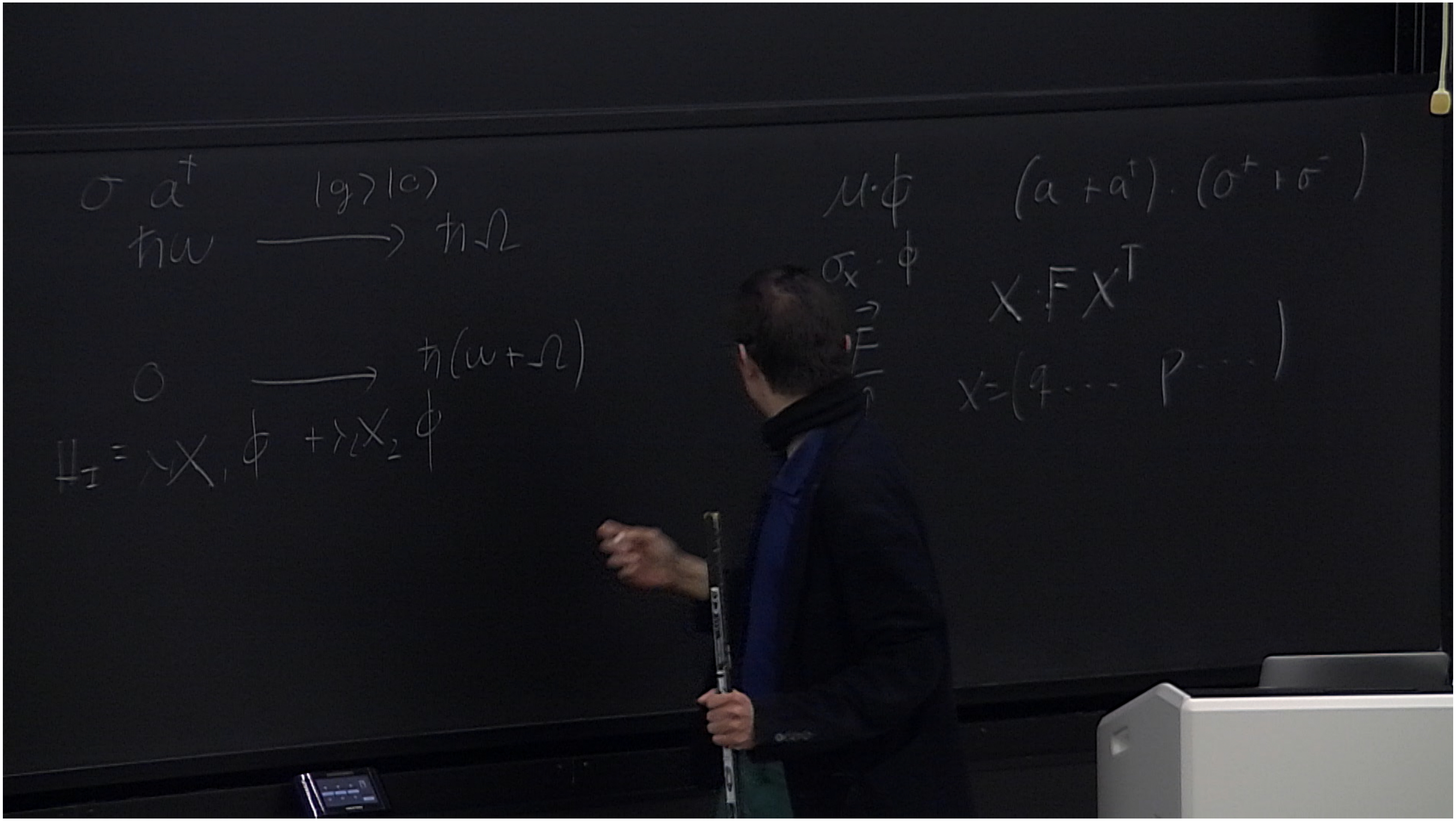
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Step 1b: Remove the atoms and prepare a fresh pair

$$\sigma_{1\mathrm{b}} = \begin{pmatrix} \sigma_{A,g} & 0 & 0 \\ 0 & \sigma_{B,g} & 0 \\ 0 & 0 & \sigma_{f,1} \end{pmatrix}$$

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Step 2: Repeat the process

$$\downarrow$$

$$\sigma_2 = \mathbf{S}\sigma_{1\mathrm{b}}\mathbf{S}^{\mathrm{T}}$$

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Step 3: Iterate to obtain $\sigma_3, \sigma_4, \sigma_5, \ldots$

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Fixed point

Is there a fixed point in this iterative process?

Consider the symplectic matrix $$\mathbf{S} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$$

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The partial state of the field at the step k+1 becomes

$$\boldsymbol{\sigma}_{f,(k+1)} = \mathbf{D}\boldsymbol{\sigma}_{f,k}\mathbf{D}^T + \mathbf{C}\mathbf{C}^T.$$

Recast the problem in a more familiar form

$$\mathbf{v}^{(k+1)} = (\mathbf{D} \otimes \mathbf{D})\mathbf{v}^{(k)} + \mathbf{c}$$

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If the eigenvalues of D are within the unit circle, there is a fixed point

$$\mathbf{v}_{\text{fixed}} = (\mathbf{I} - \mathbf{D} \otimes \mathbf{D})^{-1}\mathbf{c}.$$

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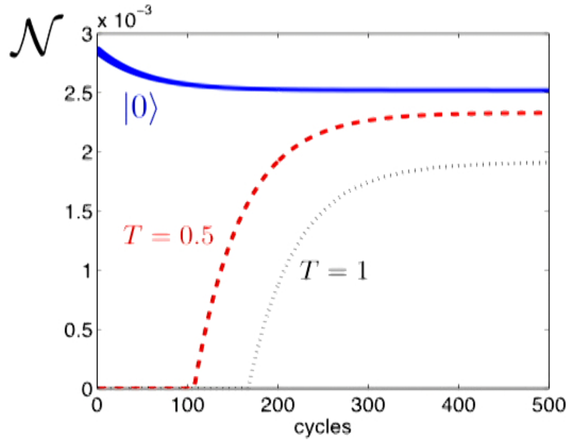
Fixed point

- **How common is that fixed point?**

  If the interaction time is long enough there always exists a fixed point

- **Can we extract entanglement from that fixed point?**
- **How fast do we go towards the fixed point depending on the initial state?**

Entanglement can be extracted!



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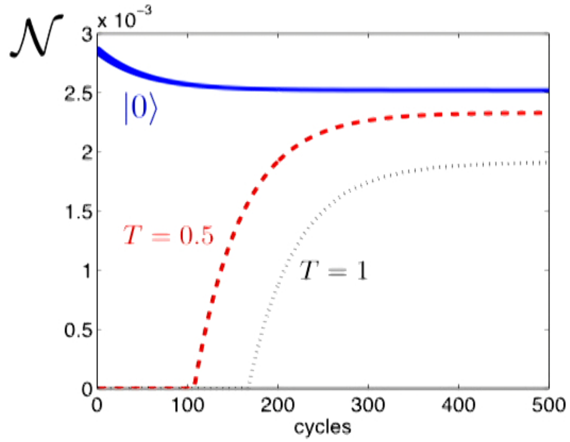

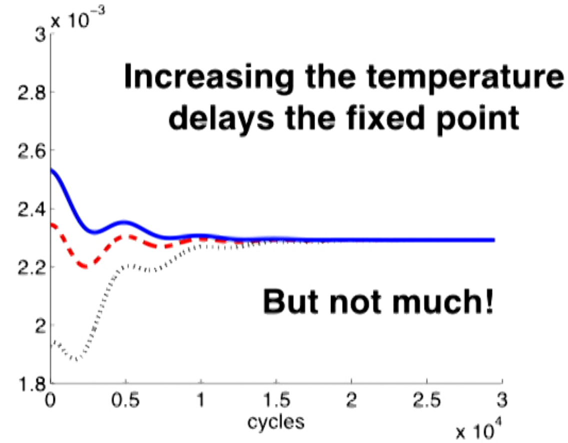

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