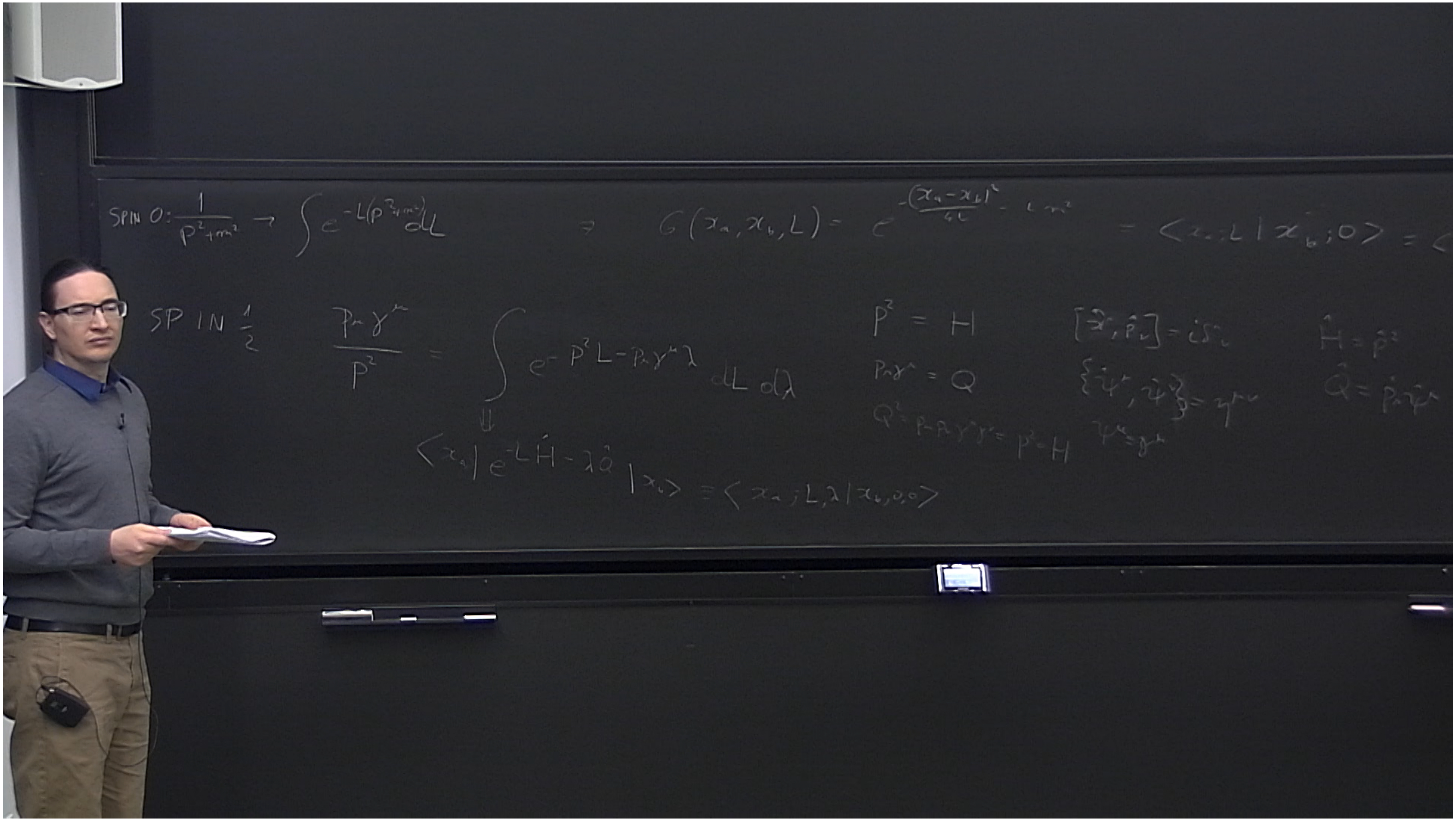
Title: PSI 2016/2017 String Theory (Review) - Lecture 12

Date: Mar 09, 2017 10:00 AM

URL: http://pirsa.org/17030028

Abstract:


SPIN 0: $\frac{1}{p^2+m^2} \to \int e^{-L(p^2+m^2)} dL \Rightarrow G(x_a, x_b, L) = e^{-\frac{(x_a-x_b)^2}{4L} - Lm^2} = \langle x_a; L | x_b; 0 \rangle =$
SPIN $\frac{1}{2}$ $\frac{p_\mu \gamma^\mu}{p^2} = \int e^{-p^2 L - p_\mu \gamma^\mu \lambda} dL \, d\lambda$
$\Downarrow$
$\langle x_a | e^{-L\hat{H} - \lambda \hat{Q}} | x_b \rangle = \langle x_a ; L, \lambda | x_b, 0, 0 \rangle$
$p^2 = H$
$p_\mu \gamma^\mu = Q$
$Q^2 = p_\mu p_\nu \gamma^\mu \gamma^\nu = p^2 = H$
$[\hat{x}^\mu, \hat{p}_\nu] = i\delta^\mu_\nu$
$\{\hat{\psi}^\mu, \hat{\psi}^\nu\} = \eta^{\mu\nu}$
$\psi^\mu = \gamma^\mu$
$\hat{H} = \hat{p}^2$
$\hat{Q} = \hat{p}_\mu \hat{\psi}^\mu$

$$(x_a, x_b; L) = e^{-\frac{(x_a - x_b)^2}{4L} - L m^2} = \langle x_a; L | x_b; 0 \rangle = \langle x_a | e^{-L(p^2 + m^2)} | x_b \rangle$$

$$\hat{H} = p^2 + m^2$$

$$p^2 = H \qquad [\hat{x}^\mu, \hat{p}_\nu] = i\delta^\mu_\nu \qquad \hat{H} = \hat{p}^2$$

$$p_\mu \gamma^\mu = Q \qquad \{\hat{\psi}^\mu, \hat{\psi}^\nu\} = \eta^{\mu\nu} \qquad \hat{Q} = \hat{p}_\mu \hat{\psi}^\mu$$

$$dL\, d\lambda$$

$$Q^2 = p_\mu p_\nu \gamma^\mu \gamma^\nu = p^2 = H \qquad \psi^\mu = \gamma^\mu$$

$$= \langle x_a; L, \lambda | x_b, 0, 0 \rangle$$

$$\hat{Q}\theta$$

$$x^\mu(\tau,\theta) = e^{\hat{H}\tau + \hat{Q}\theta}\,\hat{x}^\mu\,e^{-\hat{H}\tau - \hat{Q}\theta}$$

$$\psi^\mu(\tau,\theta) = \cdots\cdots\ \psi^\mu$$

$$p^\mu(\tau,\theta) = \cdots\cdots\ \hat{p}\ \cdots = \partial_\tau x(\tau,\theta)$$

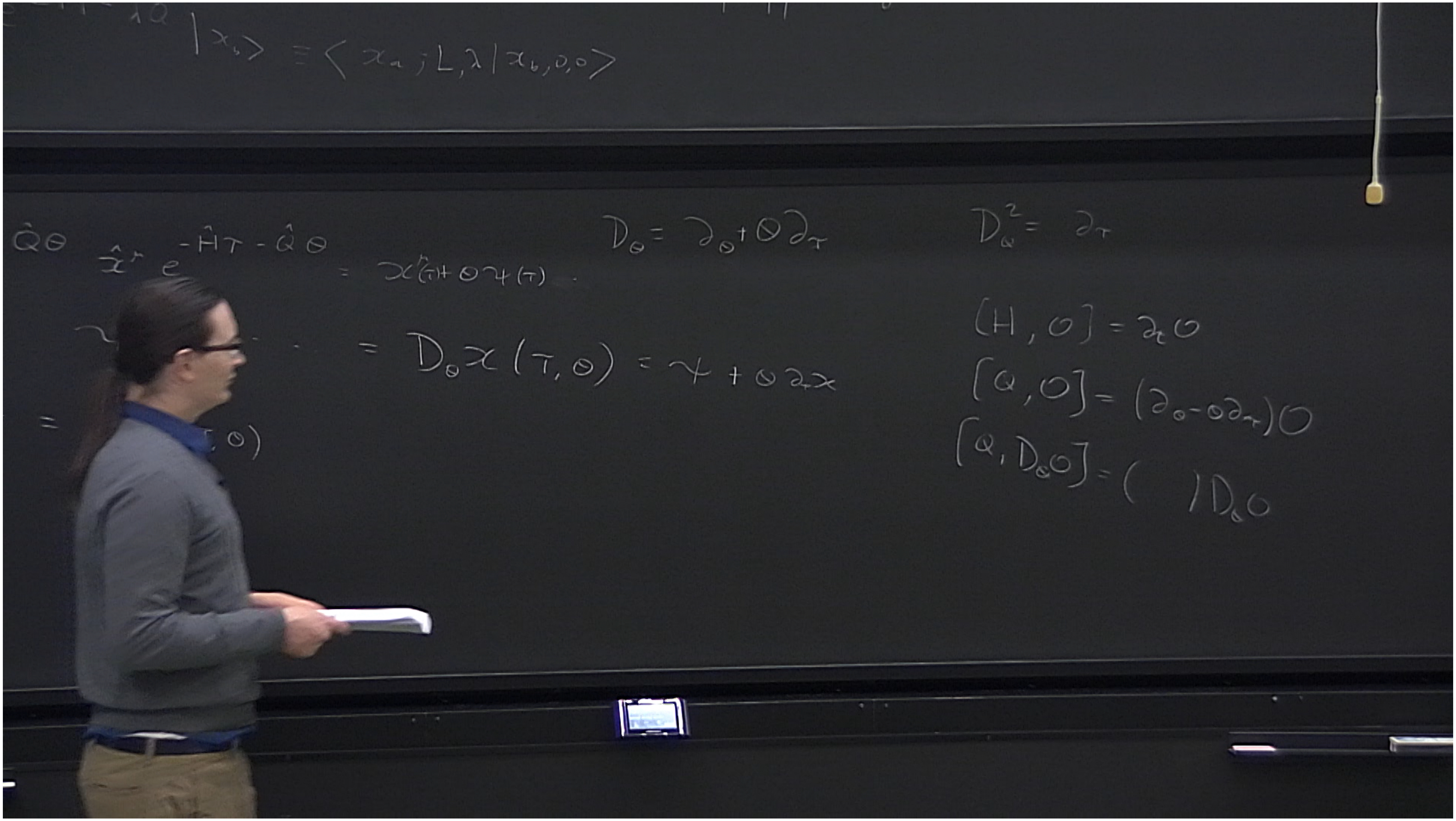
$p_\mu p_\nu \gamma^\mu \gamma^\nu = p^2 = H \qquad \psi^\mu = \gamma^\mu$

$$\langle x_a | e^{-L\hat{H} - \lambda\hat{Q}} | x_b \rangle \equiv \langle x_a ; L, \lambda | x_b, 0, 0 \rangle$$

$$x^\mu(\tau,\theta) = e^{\hat{H}\tau + \hat{Q}\theta}\, \hat{x}^\mu\, e^{-\hat{H}\tau - \hat{Q}\theta} = x(\tau) + \theta\psi(\tau) \qquad D_\theta = \partial_\theta - \theta\partial_\tau \qquad D_\theta^2 = \partial_\tau$$

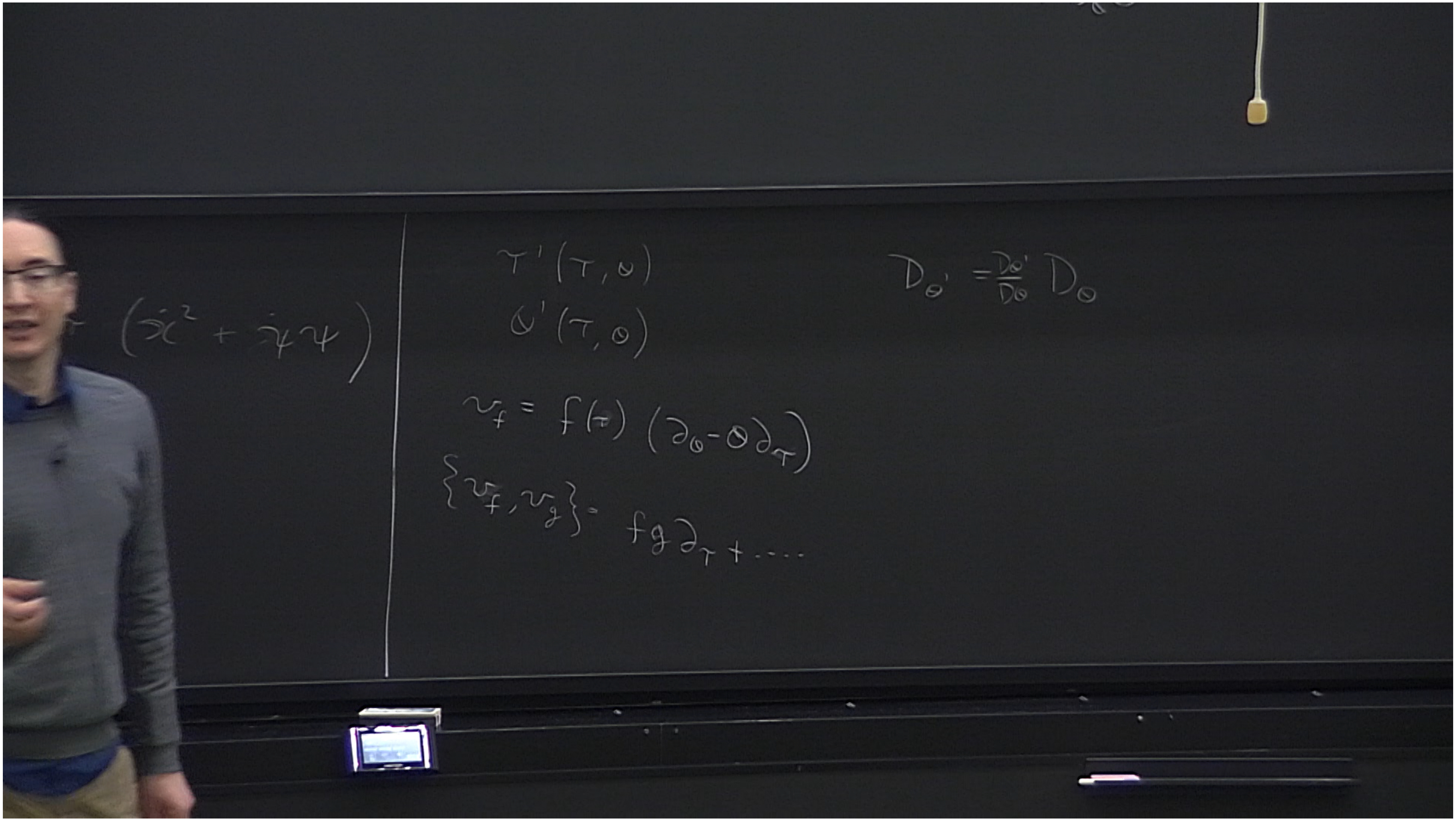
$$\psi^\mu(\tau,\theta) = \cdots\cdots \psi^\mu \cdots\cdots = D_\theta x(\tau,\theta) = \psi - \theta\partial_\tau x$$

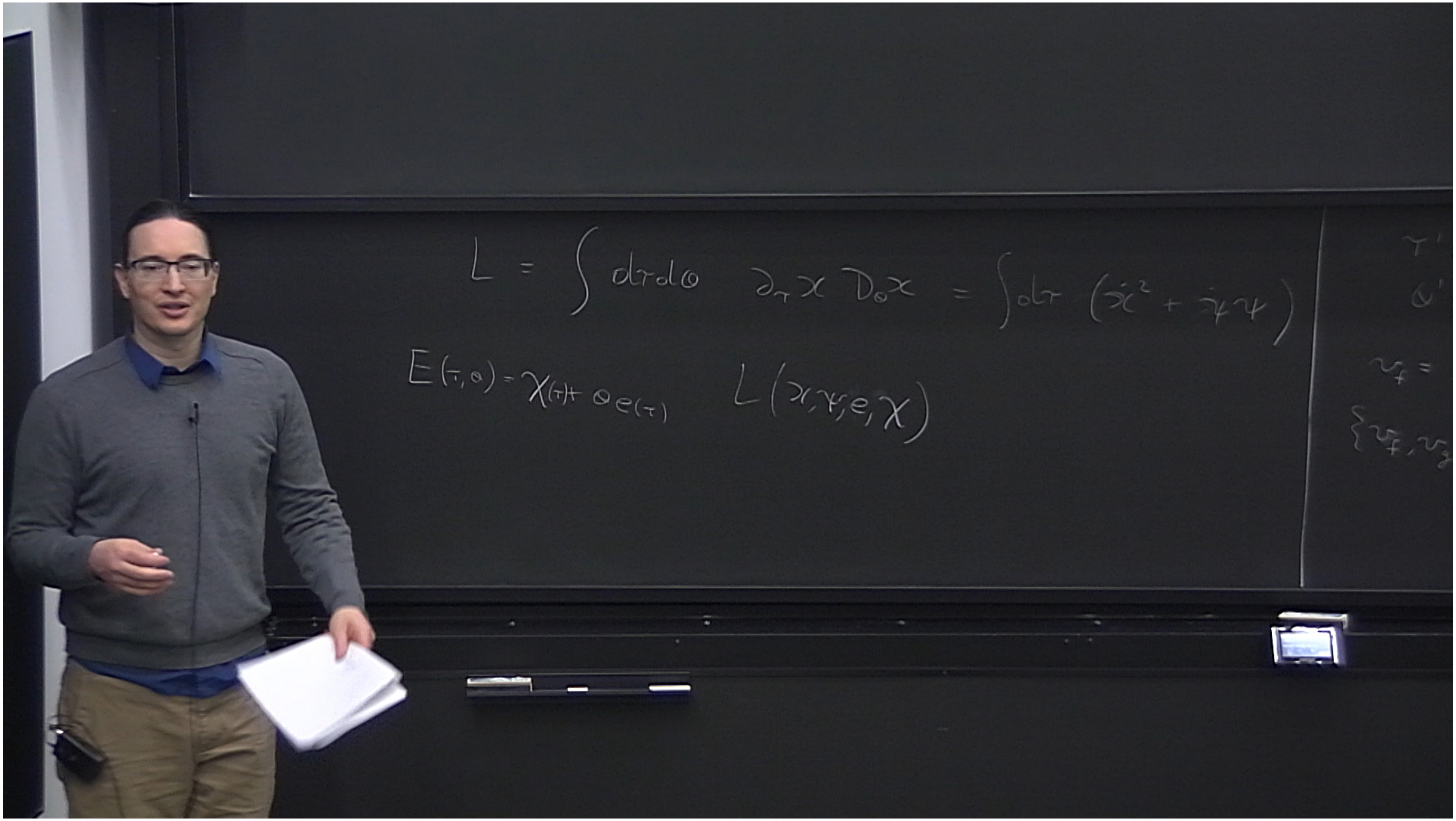
$$p^\mu(\tau,\theta) = \cdots\cdots \hat{p} \cdots = \partial_\tau x(\tau,\theta)$$

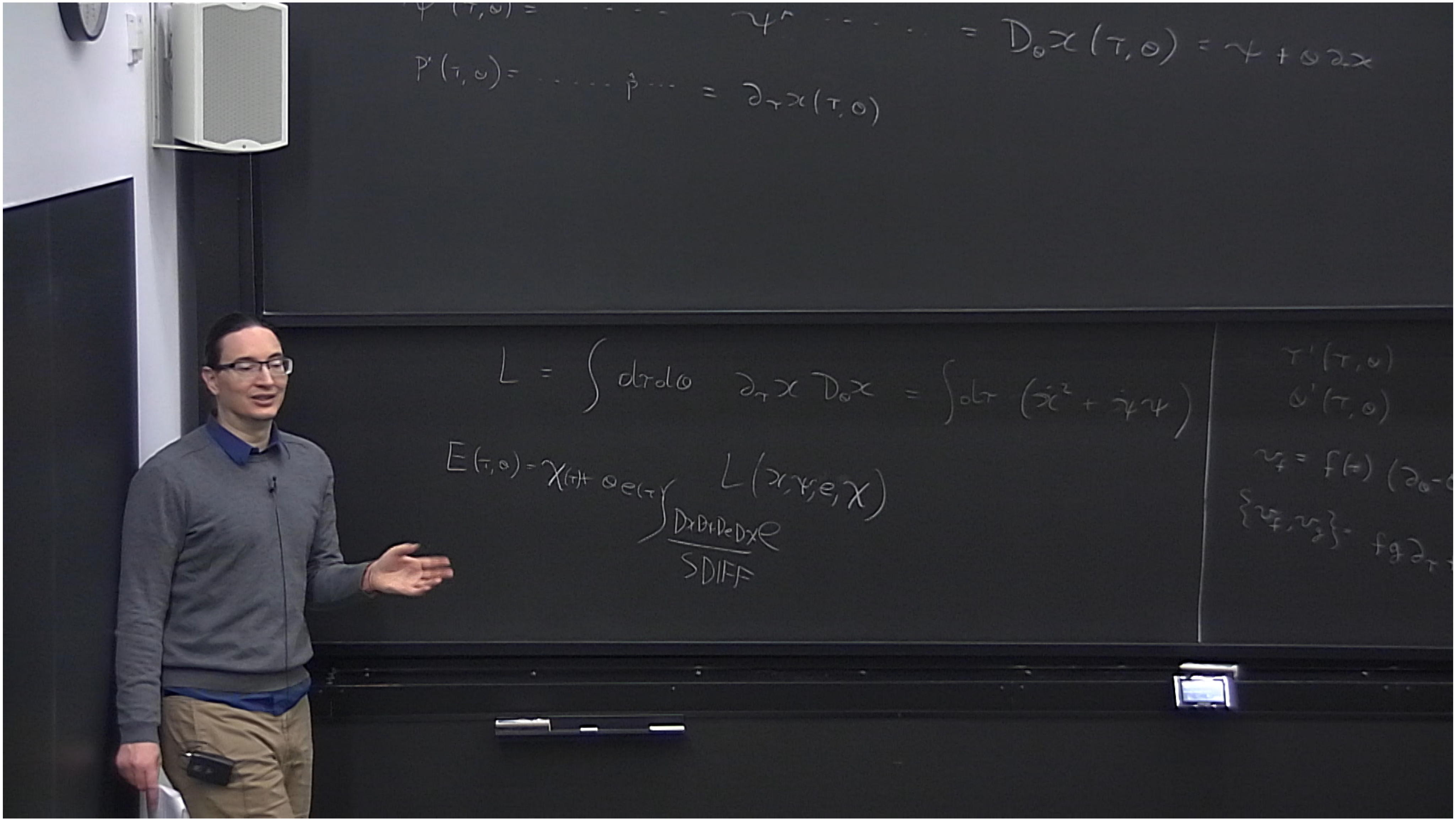


$|x_b\rangle \equiv \langle x_a ; L, \lambda | x_b, 0, 0\rangle$
$D_\theta = \partial_\theta + \theta \partial_\tau$
$D_\theta^2 = \partial_\tau$
$e^{-\hat{H}\tau - \hat{Q}\theta} = x(\tau) + \theta \psi(\tau)$
$= D_\theta x(\tau, \theta) = \psi + \theta \partial_\tau x$
$[H, O] = \partial_\tau O$
$[Q, O] = (\partial_\theta - \theta \partial_\tau) O$
$[Q, D_\theta O] = (\quad) D_\theta O$

$$L = \int d\tau\, d\theta \;\; \partial_\tau x \, D_\theta x \;=\; \int d\tau \left(\dot{x}^2 + \dot{\psi}\psi \right)$$


$(\dot{x}^2 + \dot{\psi}\psi)$
$\tau'(\tau, \theta)$
$\theta'(\tau, \theta)$
$D_{\theta'} = \frac{D\theta'}{D\theta} D_\theta$
$v_f = f(\tau)(\partial_\theta - \theta\partial_\tau)$
$\{v_f, v_g\} = fg\partial_\tau + \dots$


$L = \int d\tau\, d\theta \;\; \partial_\tau x \, D_\theta x = \int d\tau \left(\dot{x}^2 + \dot{\psi}\psi \right)$
$E(\tau, \theta) = \chi(\tau) + \theta e(\tau)$
$L(x, \psi, e, \chi)$


p'(τ, θ) = ... p ... = ∂τx(τ, θ)
= Dθx(τ, θ) = ψ + θ∂τx
L = ∫dτdθ ∂τx Dθx = ∫dτ (ẋ² + ψ̇ψ)
L(x, ψ, e, χ)
DτDψDeDχe
SDIFF

$$\int \frac{DX^{\mu} D\psi^{\mu} Dh\, DX}{\text{SDIFF} \times \text{SWEYL}}\, e^{S[\cdots]}$$

$$\int \frac{DX^\mu D\psi^\mu Dh\, DX}{SDIFF \times SWEYL}\, e^{S(\cdots]}$$

$U^1, U^2, \theta^1, \theta^2$ $\quad Q^1, Q^2$

$$D_{\theta^\alpha} = \partial_{\theta^\alpha} + \sigma^a_{\alpha\beta}\frac{\partial}{\partial U^a} \qquad \{Q^\alpha, Q^\beta\} = \sigma^{\alpha\beta}_a P^a$$

z, θ

$\bar{z}, \bar{\theta}$

$f(z)\left(\partial_\theta + \theta\partial_z\right) \quad , \quad g(z)\,\partial_z + \cdots$

X^μ

z, θ

$\bar{z}, \bar{\theta}$

$f(z)\left(\partial_\theta + \theta \partial_z\right) \quad , \quad g(z)\,\partial_z + \cdots$

γ	$-\frac{1}{2}$	c	-1
β	$\frac{3}{2}$	b	2

$\{Q_{BRST}, \beta\} = G \qquad \{Q_{BRST}, b\} = T$

$$\mathbb{X}^\mu(z,\bar{z},\theta,\bar{\theta}) = X^\mu + \theta\psi^\mu + \bar{\theta}\bar{\psi}^\mu + \theta\bar{\theta}F$$

$$\partial X\bar{\partial}X + \psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi} + \cancel{F^2}$$

z, θ

$\bar{z}, \bar{\theta}$

$f(z)\left(\partial_\theta + \theta \partial_z\right) \quad , \quad g(z)\partial_z + \cdots$

$\gamma \quad -\frac{1}{2}$

$\beta \quad \frac{3}{2}$

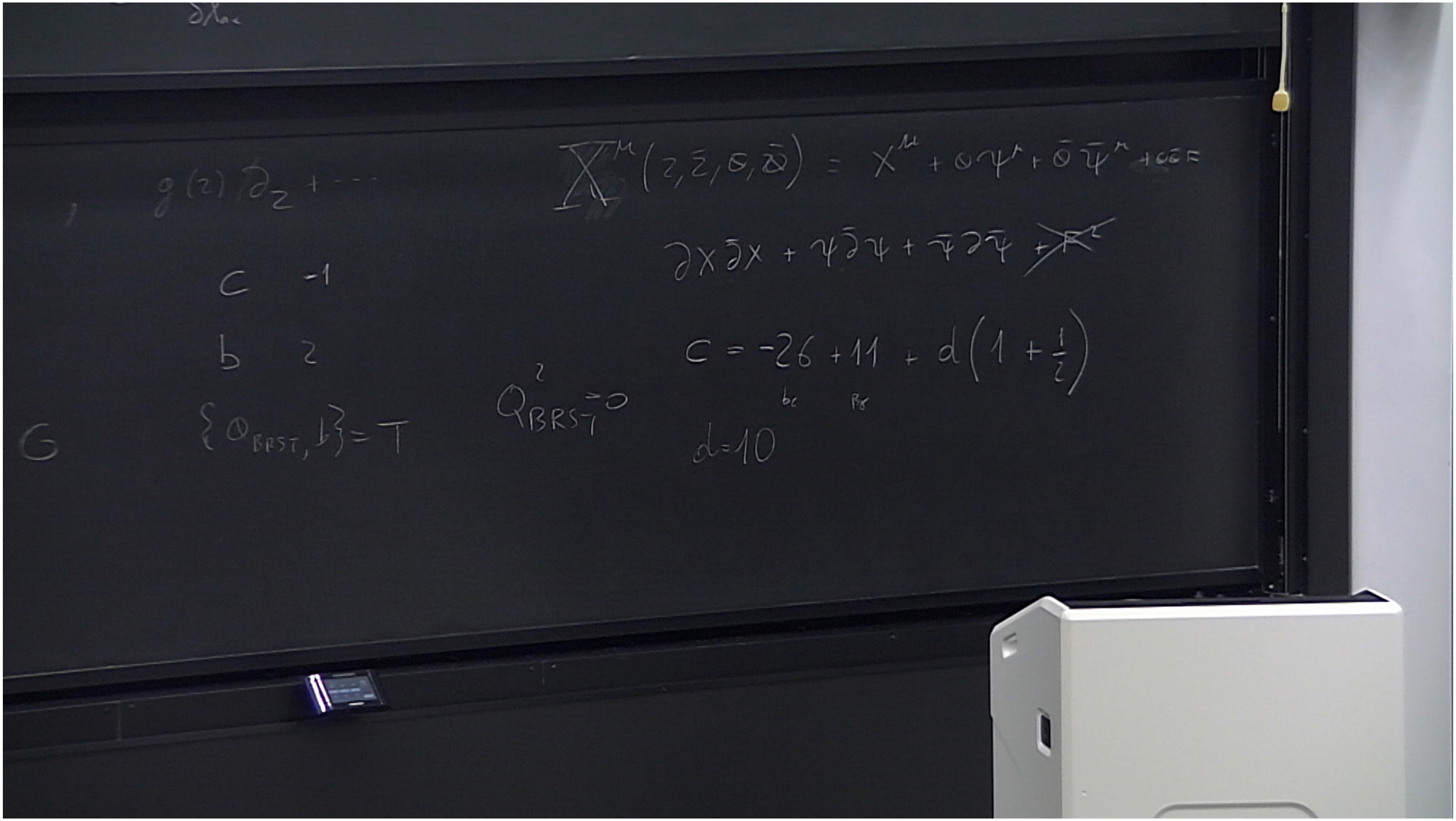
$c \quad -1$

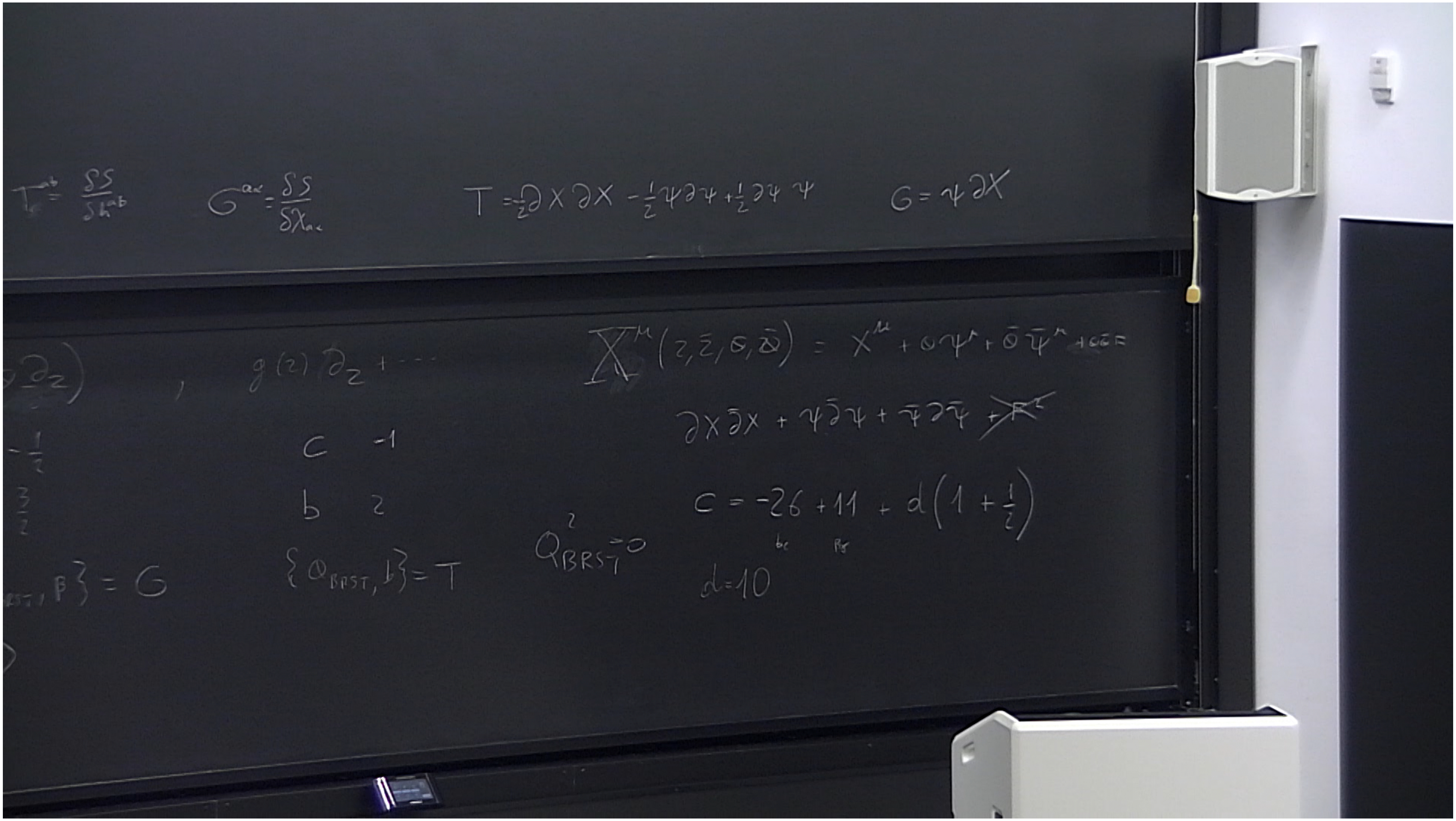
$b \quad 2$

$\{Q_{BRST}, \beta\} = G$

$\{Q_{BRST}, b\} = T$

$$\sum_g g_s^{\cdots} \int_{\hat{\mathcal{M}}_{g,n}} \langle (b\, dt)(\beta\, d\zeta)\, V_1 \cdots V_n \rangle$$


$g(z)\partial_z + \cdots$
$\mathbb{X}^\mu(z,\bar z,\theta,\bar\theta) = X^\mu + \theta\psi^\mu + \bar\theta\bar\psi^\mu + \theta\bar\theta F$
$\partial X\bar\partial X + \psi\bar\partial\psi + \bar\psi\partial\bar\psi + \cancel{F^2}$
c -1
b 2
$c = \underset{bc}{-26} + \underset{\beta\gamma}{11} + d\left(1 + \frac{1}{2}\right)$
$Q_{BRST}^2 = 0$
G $\{Q_{BRST}, b\} = T$
$d = 10$


$T_{ab} = \frac{\delta S}{\delta h^{ab}}$
$G^{a\alpha} = \frac{\delta S}{\delta \chi_{a\alpha}}$
$T = -\frac{1}{2}\partial X \partial X - \frac{1}{2}\psi\partial\psi + \frac{1}{2}\partial\psi\,\psi$
$G = \psi\partial X$
$g(z)\,\partial_z + \cdots$
$\mathbb{X}^\mu(z,\bar z,\theta,\bar\theta) = X^\mu + \theta\psi^\mu + \bar\theta\bar\psi^\mu + \theta\bar\theta F$
$\partial X\bar\partial X + \psi\bar\partial\psi + \bar\psi\partial\bar\psi + \cancel{F^2}$
$c\quad -1$
$b\quad 2$
$c = -26 + 11 + d\left(1 + \frac{1}{2}\right)$
bc
βγ
$Q_{BRST}^2 = 0$
$\{Q_{BRST}, b\} = T$
$\{Q_{BRST}, \beta\} = G$
$d = 10$

$$L = \int d\tau\, d\theta \;\; \partial_\tau x \, D_\theta x \; = \int d\tau \left(\dot{x}^2 + \dot{\psi}\psi \right)$$

$$E(\tau,\theta) = \chi(\tau) + \theta e(\tau) \qquad L(x, \psi, e, \chi)$$

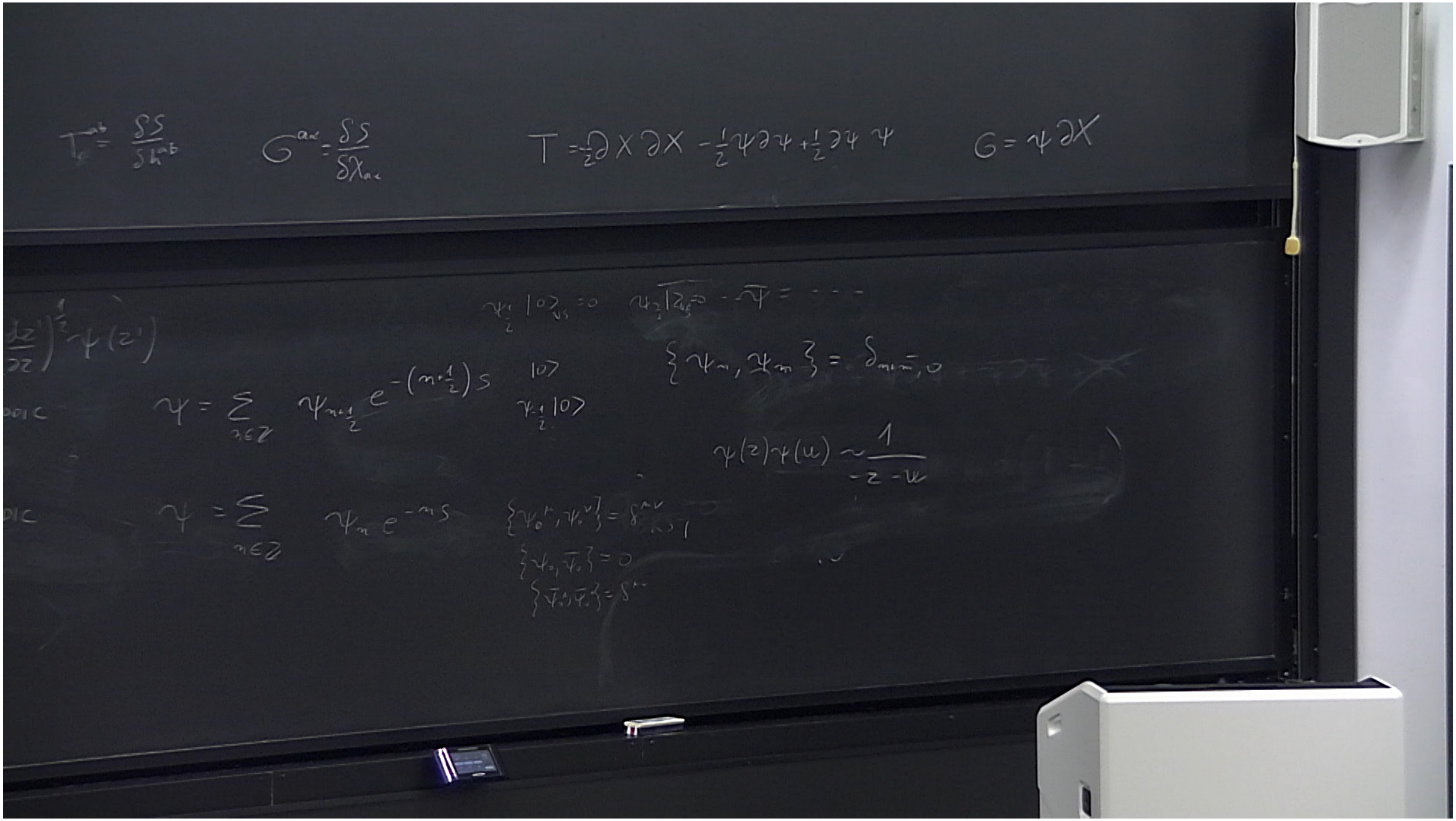
$$\int \frac{Dx\, D\psi\, De\, D\chi\, e}{SDIFF}$$

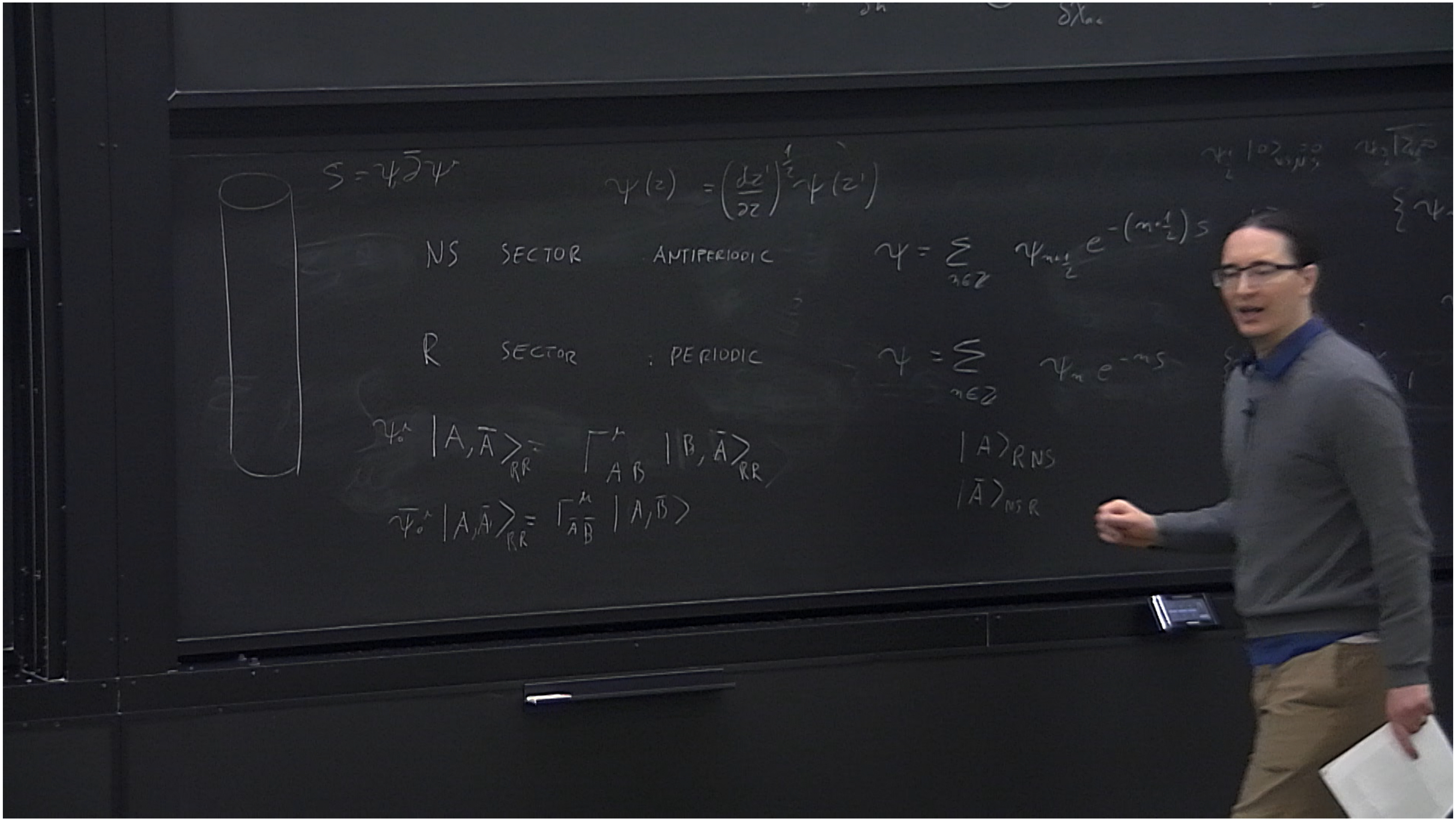
$S = \psi \bar{\partial} \psi$

$$\psi(z) = \left(\frac{dz'}{dz}\right)^{\frac{1}{2}} \psi(z')$$

NS SECTOR ANTIPERIODIC $\psi = \sum_{n \in \mathbb{Z}} \psi_{n+\frac{1}{2}} e^{-(n+\frac{1}{2})s}$

R SECTOR : PERIODIC $\psi = \sum_{n \in \mathbb{Z}} \psi_n e^{-ns}$


$T^{ab} = \frac{\delta S}{\delta h^{ab}}$
$G^{a\alpha} = \frac{\delta S}{\delta \chi_{a\alpha}}$
$T = -\frac{1}{2}\partial X \partial X - \frac{1}{2}\psi\partial\psi + \frac{1}{2}\partial\psi\,\psi$
$G = \psi\,\partial X$
$\{\psi_n, \psi_m\} = \delta_{n+m,0}$
$\psi = \sum_{n\in\mathbb{Z}} \psi_{n+\frac{1}{2}}\, e^{-(n+\frac{1}{2})s}$
$|0\rangle$
$\psi_{-\frac{1}{2}}|0\rangle$
$\psi(z)\psi(w) \sim \frac{1}{z-w}$
$\psi = \sum_{n\in\mathbb{Z}} \psi_n\, e^{-ns}$


NS SECTOR ANTIPERIODIC
R SECTOR : PERIODIC

$$T_{ab} = \frac{\delta S}{\delta h^{ab}} \qquad G^{a\alpha} = \frac{\delta S}{\delta \chi_{a\alpha}} \qquad T = -\frac{1}{2}\partial X \partial X - \frac{1}{2}\psi\partial\psi + \frac{1}{2}\partial\psi\,\psi \qquad G = \psi\,\partial$$

$$\psi(z) = \left(\frac{dz'}{dz}\right)^{\frac{1}{2}} \psi(z')$$

$\psi_{\frac{1}{2}} |0\rangle_{NS,NS} = 0$ $\quad \psi_{\frac{1}{2}} \overline{|0\rangle_{NS}} = \ldots \overline{\psi} = \ldots$

$\{\psi_m, \psi_n\} = \delta_{m+n,0}$

R ANTIPERIODIC $\quad \psi = \sum_{n\in\mathbb{Z}} \psi_{n+\frac{1}{2}}\, e^{-(n+\frac{1}{2})s}$ $\quad |0\rangle$ $\quad \psi_{-\frac{1}{2}}|0\rangle$

$$\psi(z)\psi(w) \sim \frac{1}{z-w}$$

R : PERIODIC $\quad \psi = \sum_{n\in\mathbb{Z}} \psi_n\, e^{-ns}$

$\{\psi_0^\mu, \psi_0^\nu\} = \delta^{\mu\nu}$

$\{\psi_0, \overline{\psi}_0\} = 0$

$\{\overline{\psi}_0^\mu, \overline{\psi}_0^\nu\} = \delta^{\mu\nu}$

$\Gamma^\mu_{AB}\, |B, \bar{A}\rangle_{RR}$ $\qquad |A\rangle_{R\,NS}$

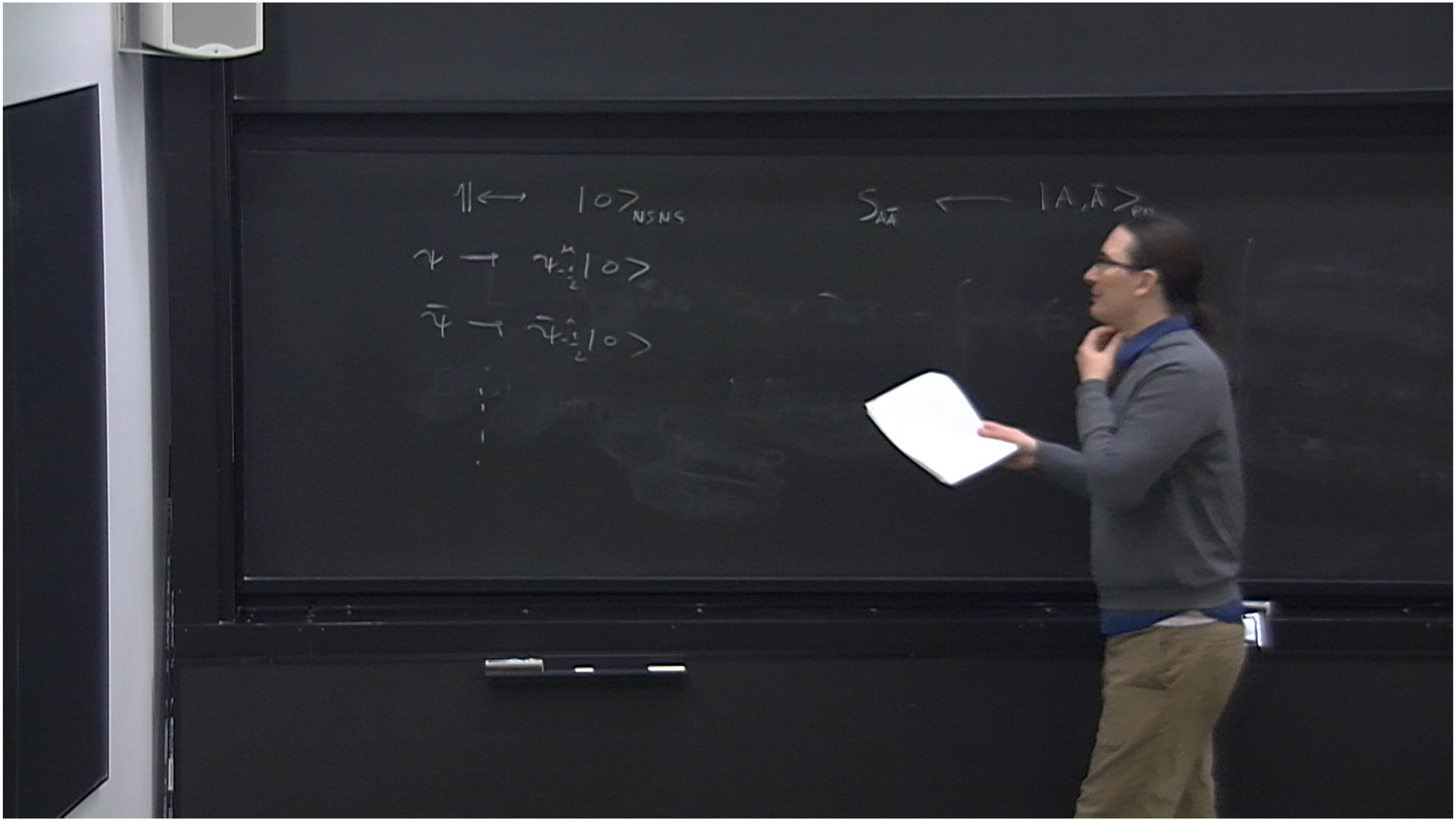
$\Gamma^\mu_{\bar{A}\bar{B}}\, |A, \bar{B}\rangle$ $\qquad |\bar{A}\rangle_{NS\,R}$

$$\mathbb{1} \longleftrightarrow |0\rangle_{NSNS} \qquad S_{A\bar{A}} \longleftarrow |A,\bar{A}\rangle_{RR}$$

$$\psi \mapsto \psi_{-\frac{1}{2}}|0\rangle$$

$$\bar{\psi} \mapsto \bar{\psi}_{-\frac{1}{2}}|0\rangle$$

$$\vdots$$


𝟙 ⟷ |0>NSNS
SAĀ ⟵ |A,Ā>
ψ → ψ-1/2 |0>
ψ̄ → ψ̄-1/2 |0>

$\mathbb{1} \longleftrightarrow |0\rangle_{NSNS}$

$S_{A\bar{A}} \longleftarrow |A,\bar{A}\rangle_{RR}$

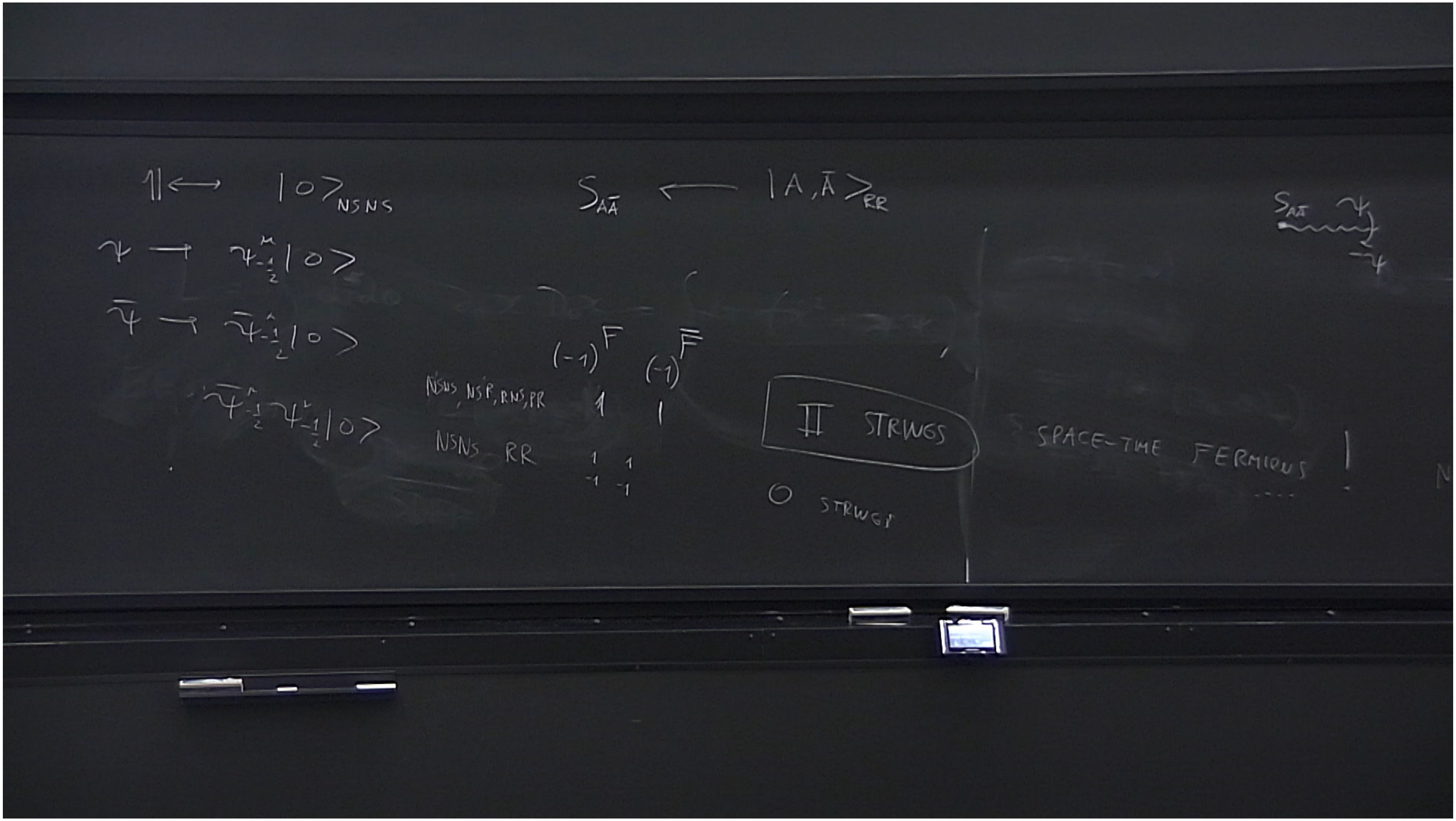
$\psi \rightarrow \psi^{\mu}_{-\frac{1}{2}}|0\rangle$

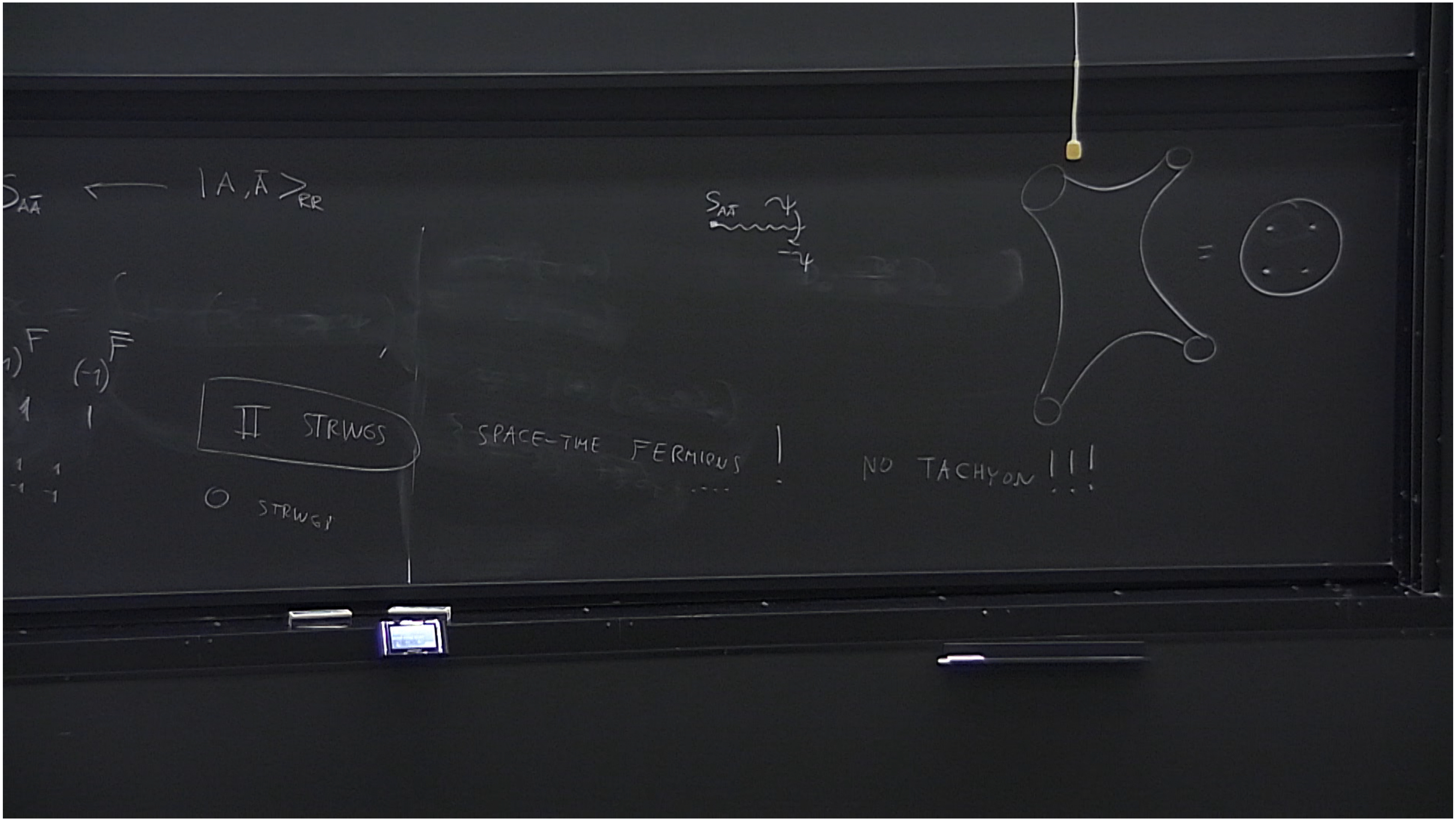
$\bar{\psi} \rightarrow \bar{\psi}^{\mu}_{-\frac{1}{2}}|0\rangle$

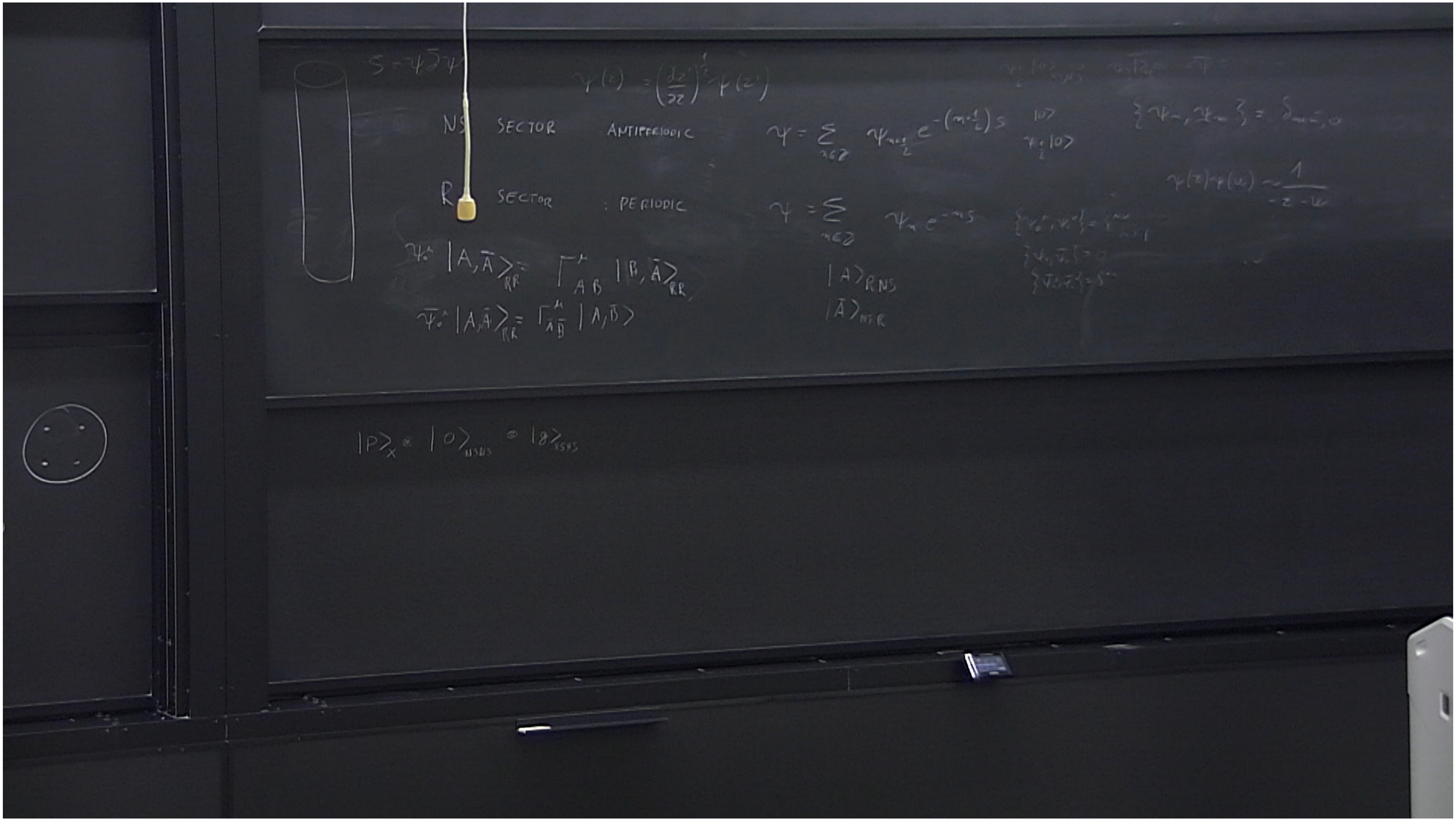
$\bar{\psi}^{\mu}_{-\frac{1}{2}}\psi^{\nu}_{-\frac{1}{2}}|0\rangle$

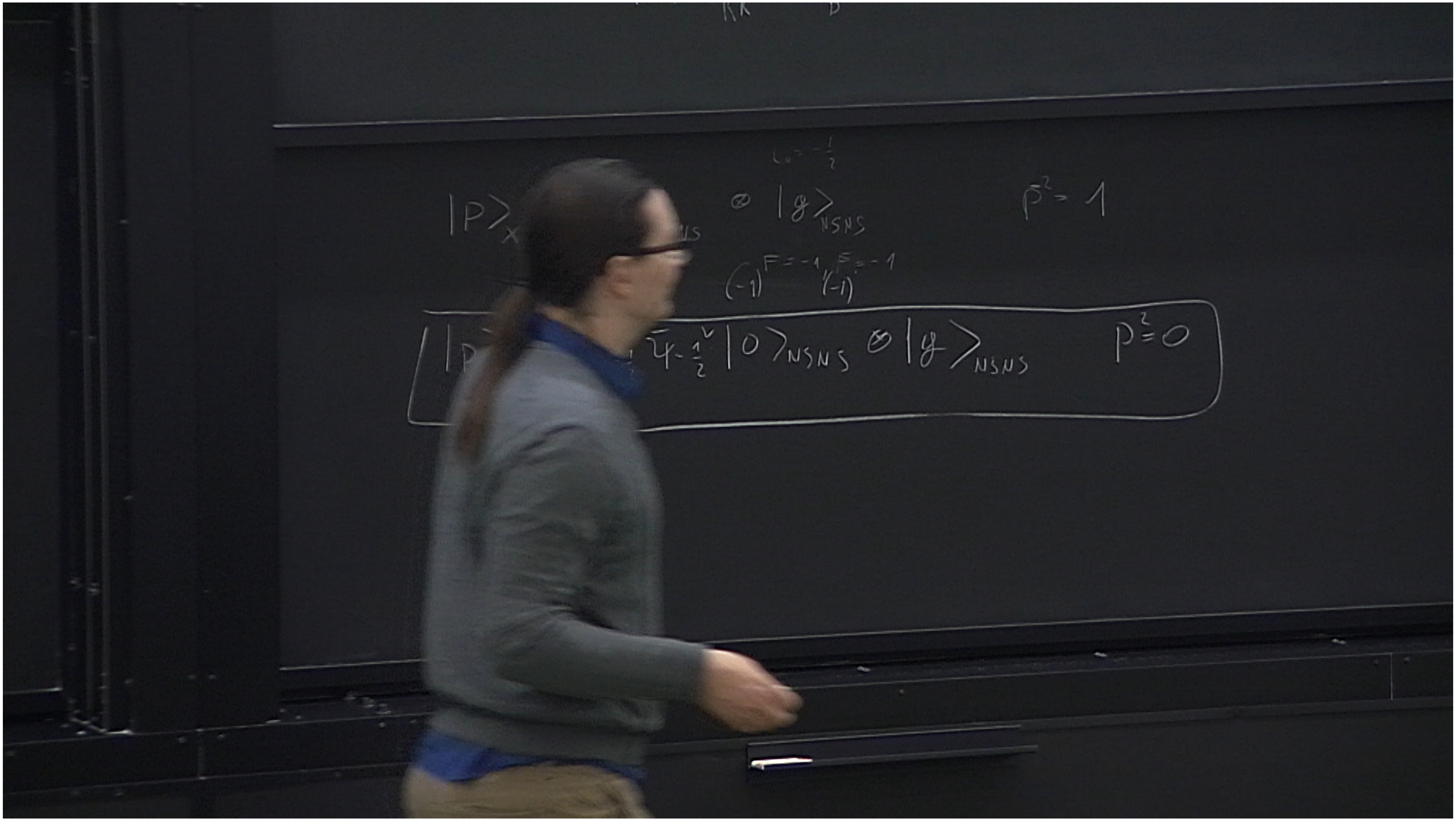
	$(-1)^F$	$(-1)^{\bar{F}}$
NSNS, NSR, RNS, RR	1	1

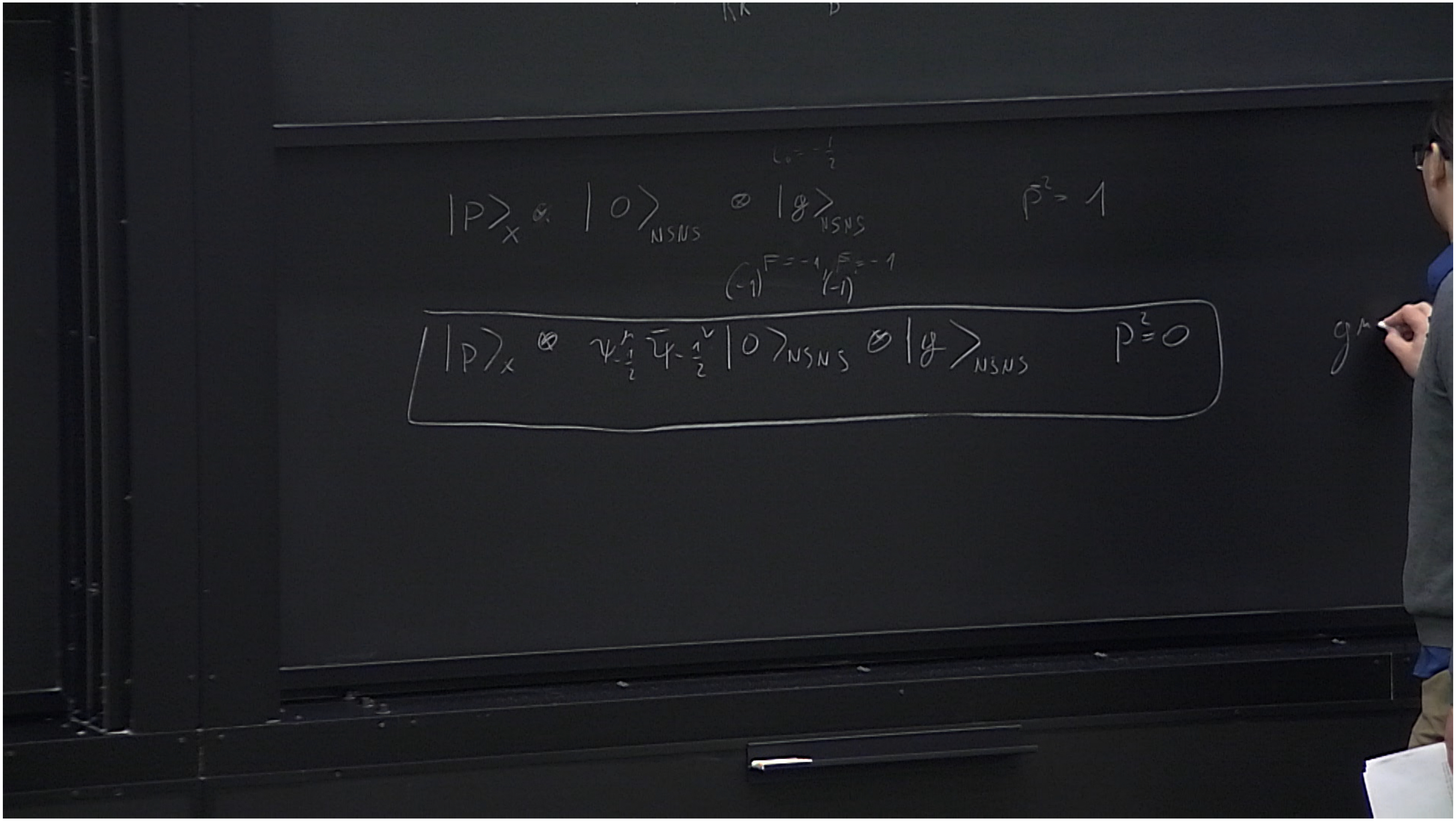
II STRINGS


1 ⟷ |0>NSNS
ψ → ψ-1/2 |0>
ψ̄ → ψ̄-1/2 |0>
ψ̄-1/2 ψ-1/2 |0>
SAĀ ⟵ |A, Ā>RR
(-1)F (-1)F̄
NSNS, NSR, RNS, RR 1 1
NSNS RR 1 1 -1 -1
II STRINGS
0 STRINGS
SAĀ ψ ψ̄
SPACE-TIME FERMIONS !


II STRINGS
O STRINGS
SPACE-TIME FERMIONS !
NO TACHYON !!!


$S = \psi \bar{\partial} \psi$
$\psi(z) = \left(\frac{dz'}{dz}\right)^{\frac{1}{2}} \psi(z')$
NS SECTOR ANTIPERIODIC
$\psi = \sum_{n \in \mathbb{Z}} \psi_{n+\frac{1}{2}} e^{-(n+\frac{1}{2})s}$
$|0\rangle$
$\psi_{\frac{1}{2}} |0\rangle$
R SECTOR : PERIODIC
$\psi = \sum_{n \in \mathbb{Z}} \psi_n e^{-ns}$
$\psi(z)\psi(w) \sim \frac{1}{z-w}$
$\psi_0^\mu |A, \bar{A}\rangle_{RR} = \Gamma^\mu_{AB} |B, \bar{A}\rangle_{RR}$
$\bar{\psi}_0^\mu |A, \bar{A}\rangle_{RR} = \Gamma^\mu_{\bar{A}\bar{B}} |A, \bar{B}\rangle$
$|A\rangle_{RNS}$
$|\bar{A}\rangle_{NSR}$
$|P\rangle_X \otimes |0\rangle_{NSNS} \otimes |g\rangle_{NSNS}$




$L_0 = -\frac{1}{2}$
$|p\rangle_X \otimes |0\rangle_{NSNS} \otimes |g\rangle_{NSNS}$
$p^2 = 1$
$(-1)^F = -1, \; (-1)^{\bar{F}} = -1$
$|p\rangle_X \otimes \psi^\mu_{-\frac{1}{2}} \bar{\psi}^\nu_{-\frac{1}{2}} |0\rangle_{NSNS} \otimes |g\rangle_{NSNS}$
$p^2 = 0$
g^μ

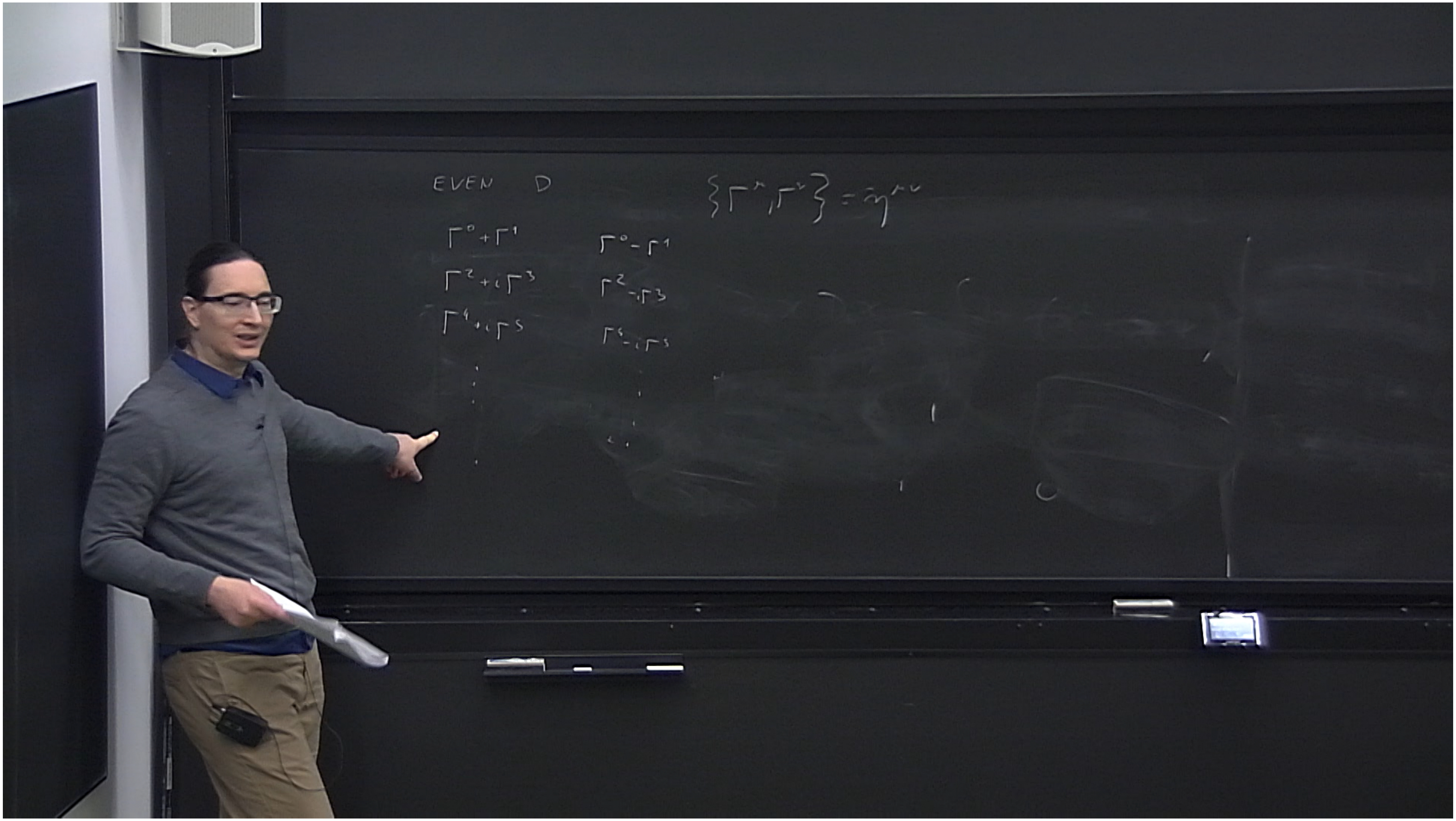
$L_m |PHYS\rangle = 0 \quad m > 0$

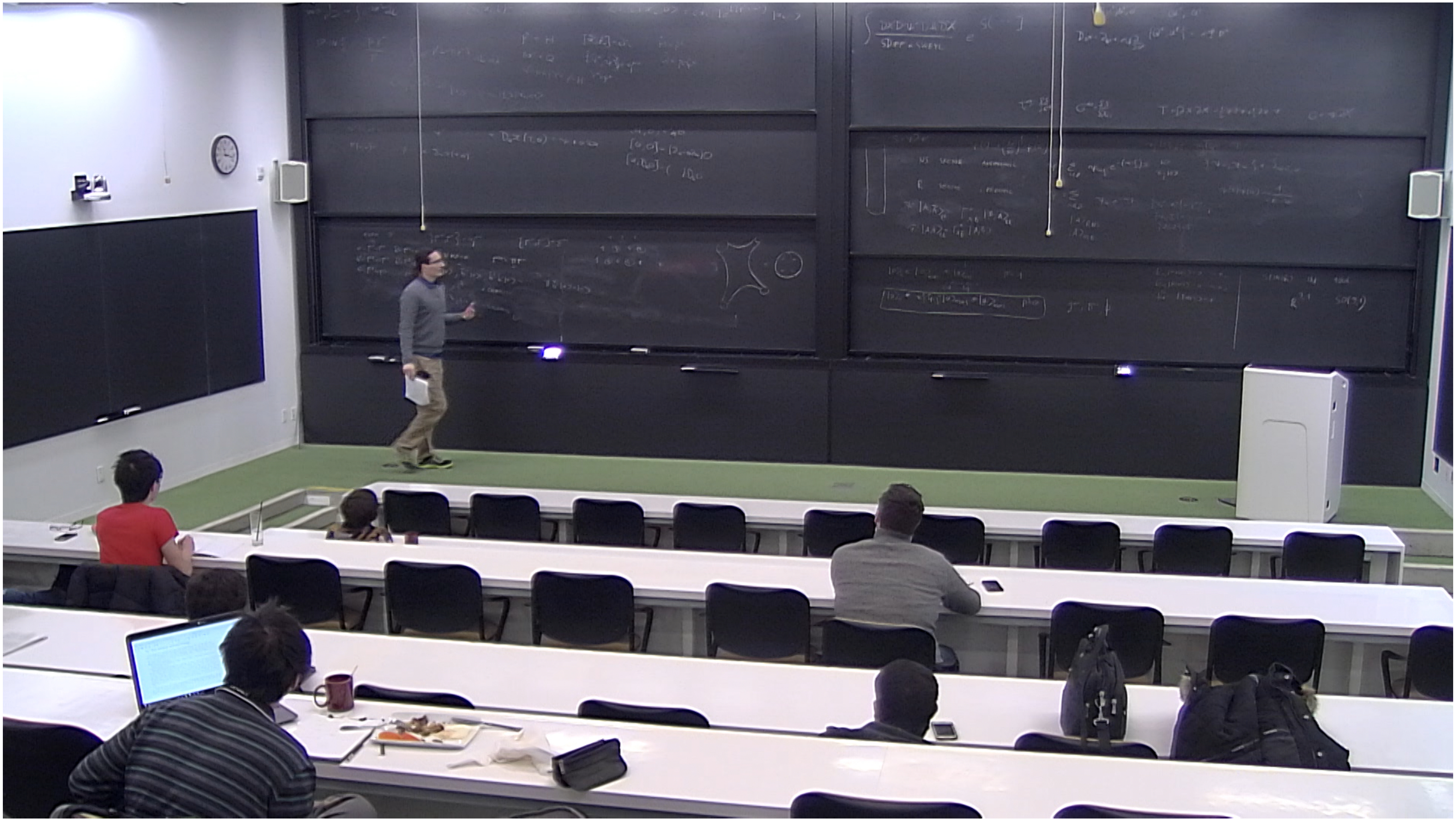
$G_m |PHYS\rangle = 0 \quad m > 0$

$L_0^{tot} |PHYS\rangle = 0$

SPINORS IN 10d

$\mathbb{R}^{9,1} \quad SO(9,1)$


EVEN D
$\{\Gamma^\mu, \Gamma^\nu\} = \eta^{\mu\nu}$
$\Gamma^0 + \Gamma^1$
$\Gamma^0 - \Gamma^1$
$\Gamma^2 + i\Gamma^3$
$\Gamma^2 - i\Gamma^3$
$\Gamma^4 + i\Gamma^5$
$\Gamma^4 - i\Gamma^5$



EVEN D

$b^0 = \Gamma^0 + \Gamma^1$ $\quad \bar{b}^0 = \Gamma^0 - \Gamma^1$

$b^1 = \Gamma^2 + i\Gamma^3$ $\quad \bar{b}^1 = \Gamma^2 - i\Gamma^3$

$b^2 = \Gamma^4 + i\Gamma^5$ $\quad \ldots \Gamma^4 - i\Gamma^5$

$\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$

$[\Gamma^\mu, \Gamma^\nu] = \Sigma^{\mu\nu}$

$b^i |+\rangle = 0$

$\Gamma^{D+1} = \prod_\mu \Gamma^\mu$

$|+\rangle, \ \bar{b}^i |+\rangle, \ \bar{b}^i \bar{b}^j |+\rangle \ldots$

$\prod \bar{b}^i |+\rangle = |-\rangle$

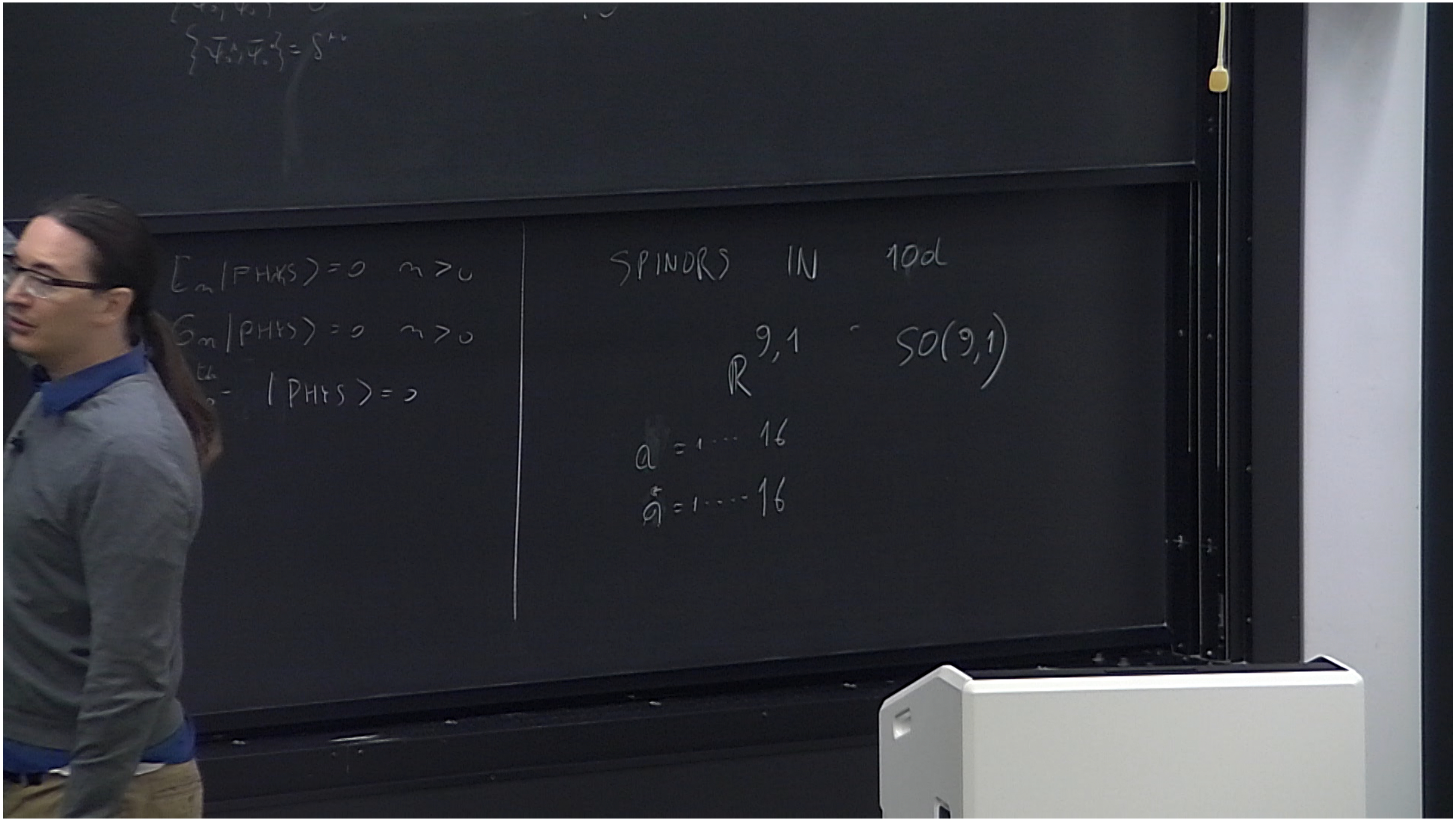
$2^{\frac{D}{2}} \nearrow 2^{\frac{D}{2}-1}$ + CHIRALITY

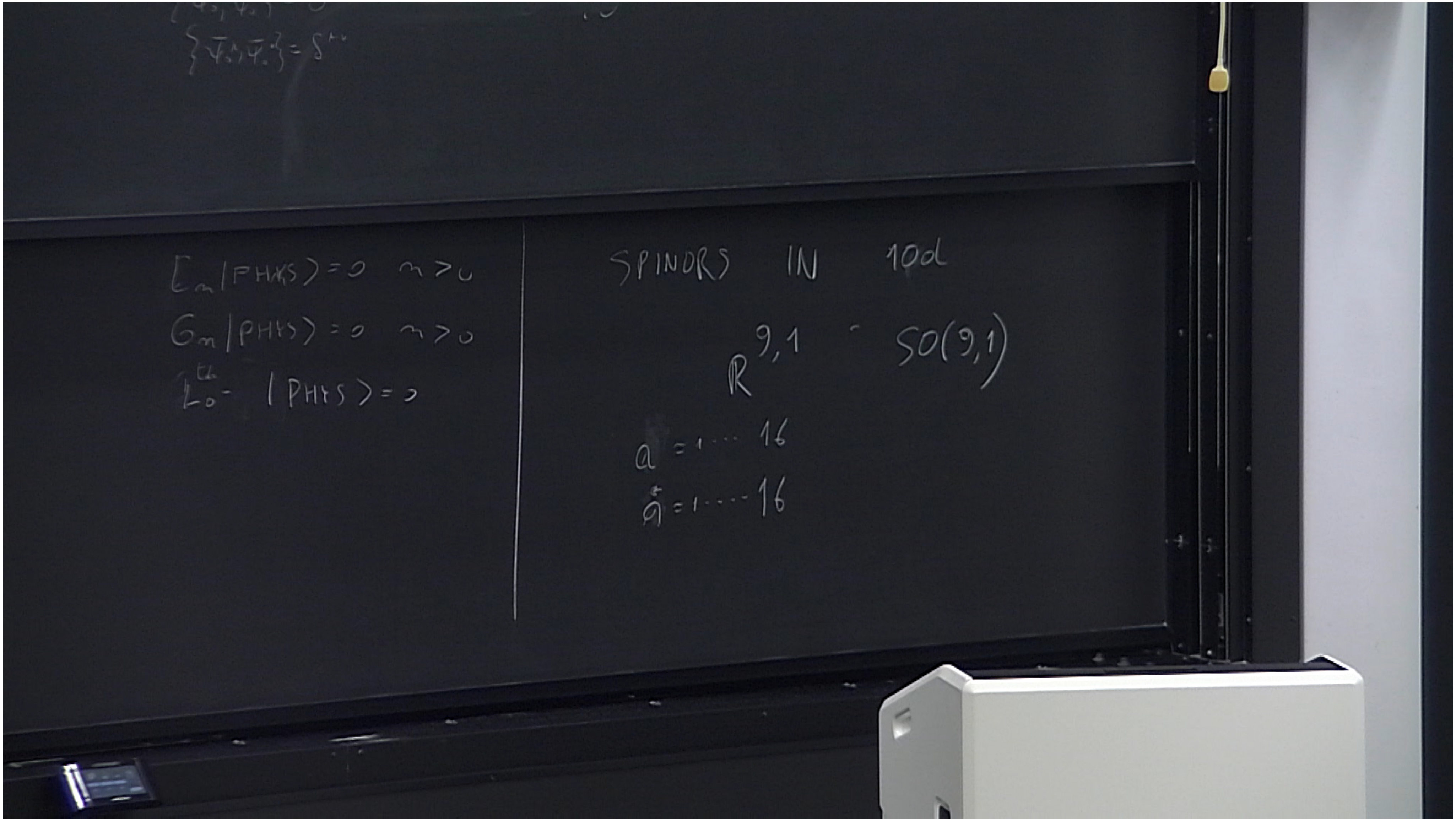
$\searrow 2^{\frac{D}{2}-1}$ − CHIRALITY

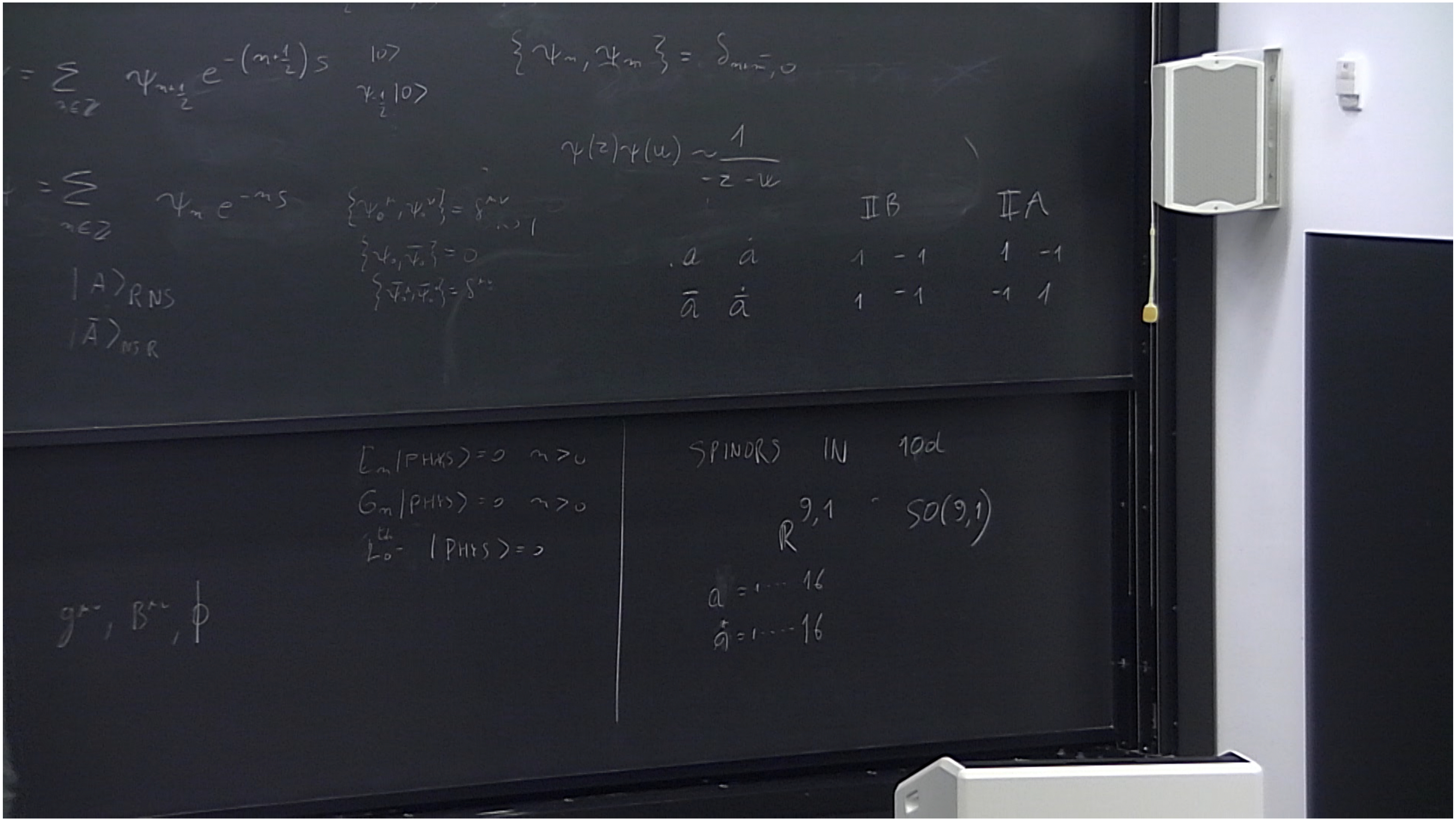
1 2 1

1 3 3 1

1 4 6 4 1


L_n |PHYS> = 0 n > 0
G_n |PHYS> = 0 n > 0
SPINORS IN 10d
R^{9,1} SO(9,1)
a = 1 … 16
ȧ = 1 … 16


L_m |PHYS> = 0   m > 0
G_m |PHYS> = 0   m > 0
L_0 - |PHYS> = 0
SPINORS IN 10d
R^9,1   SO(9,1)
a = 1 ... 16
ȧ = 1 16


SPINORS IN 10d
$\mathbb{R}^{9,1}$ $SO(9,1)$
$a = 1 \cdots 16$
$\dot{a} = 1 \cdots 16$
IIB IIA
$L_n|PHYS\rangle = 0$ $n>0$
$G_n|PHYS\rangle = 0$ $n>0$
$g^{\mu\nu}, B^{\mu\nu}, \phi$
$|A\rangle_{RNS}$
$|\bar{A}\rangle_{NSR}$