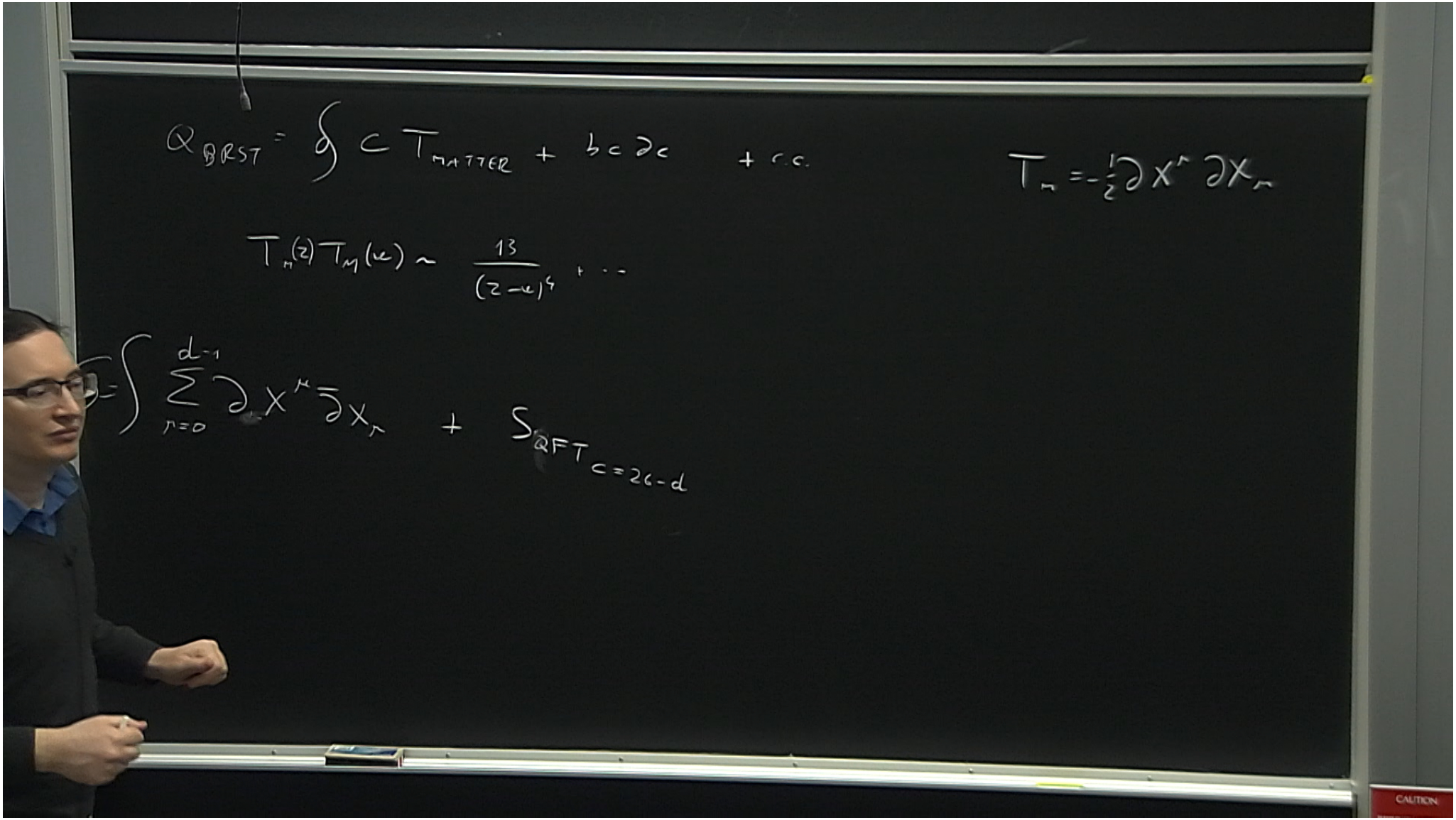
Title: PSI 2016/2017 String Theory (Review) - Lecture 10

Date: Mar 07, 2017 10:15 AM

URL: http://pirsa.org/17030026

Abstract:


$Q_{BRST} = \oint c\,T_{MATTER} + b\,c\,\partial c + c.c.$
$T_n = -\frac{1}{2}\partial X^\mu \partial X_\mu$
$T_n(z)\,T_m(w) \sim \frac{13}{(z-w)^4} + \cdots$
$\sum_{n=0}^{d-1} \partial X^\mu \bar{\partial} X_n + S_{QFT\,c=26-d}$
CAUTION

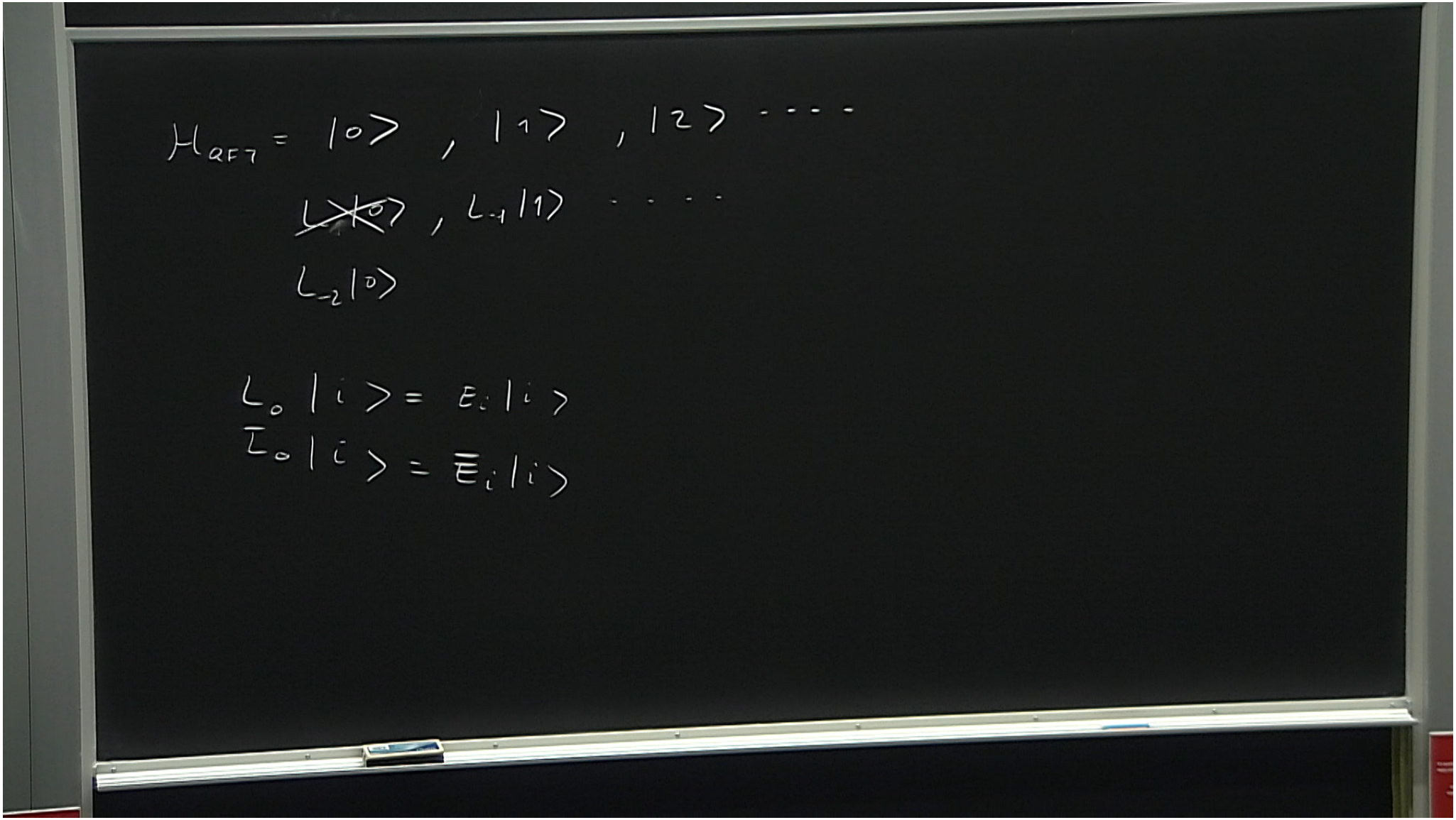
$$Q_{BRST} = \oint c\,T_{MATTER} + b\,c\,\partial c + \text{c.c.}$$

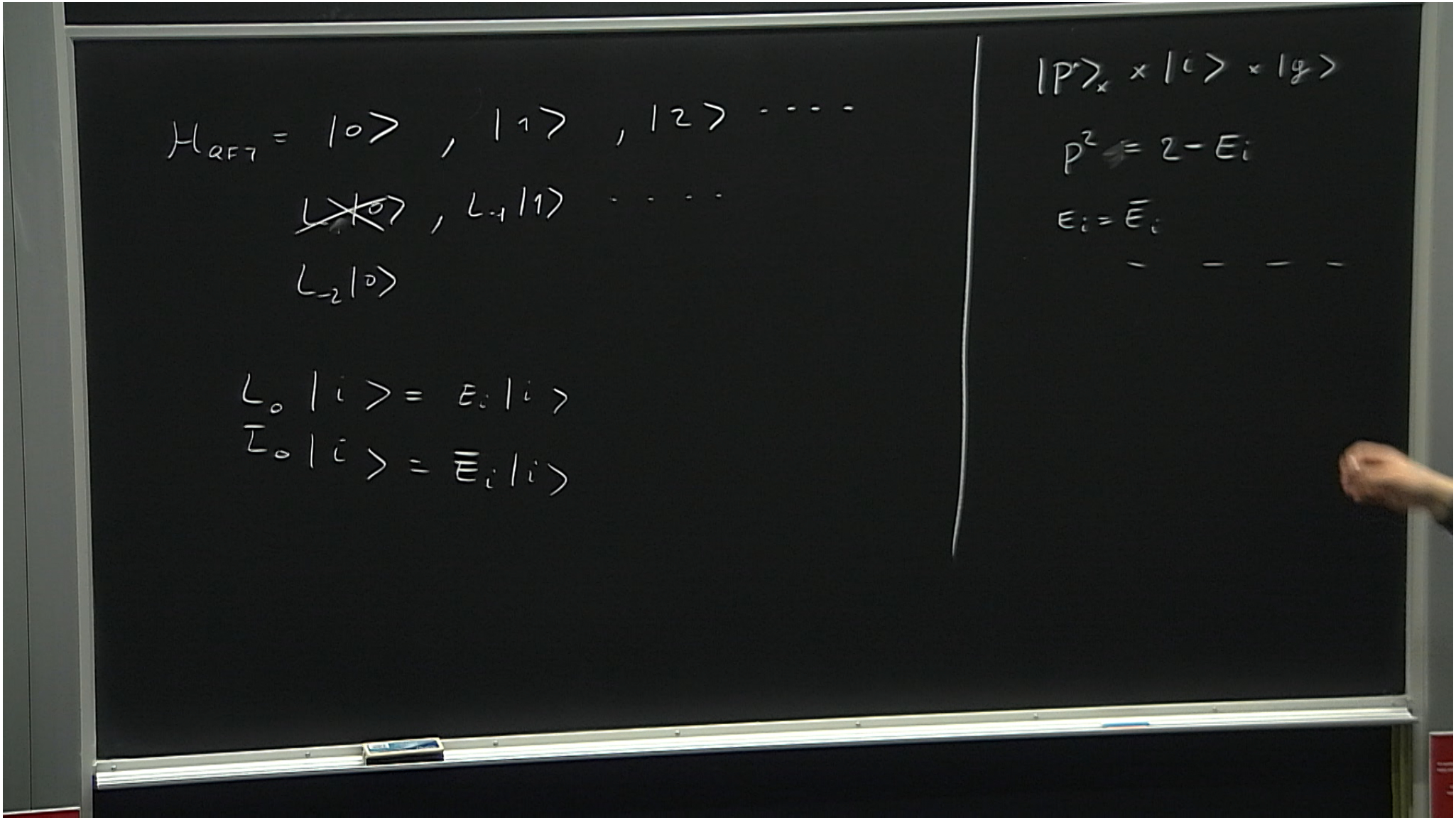
$$T_m = -\frac{1}{2}\partial X^\mu \partial X_\mu$$

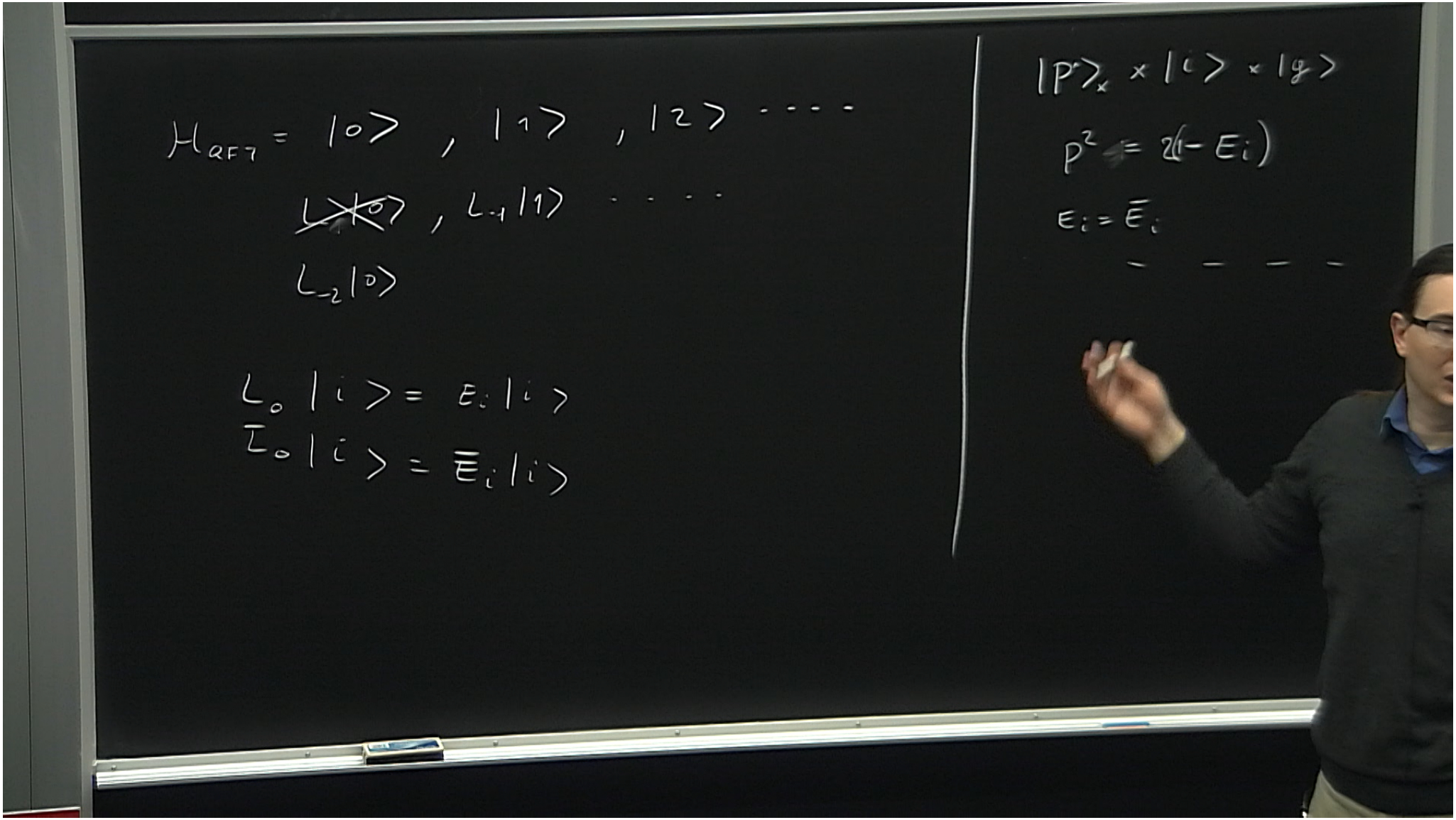
$$T_m(z)\,T_M(w) \sim \frac{13}{(z-w)^4} + \cdots$$

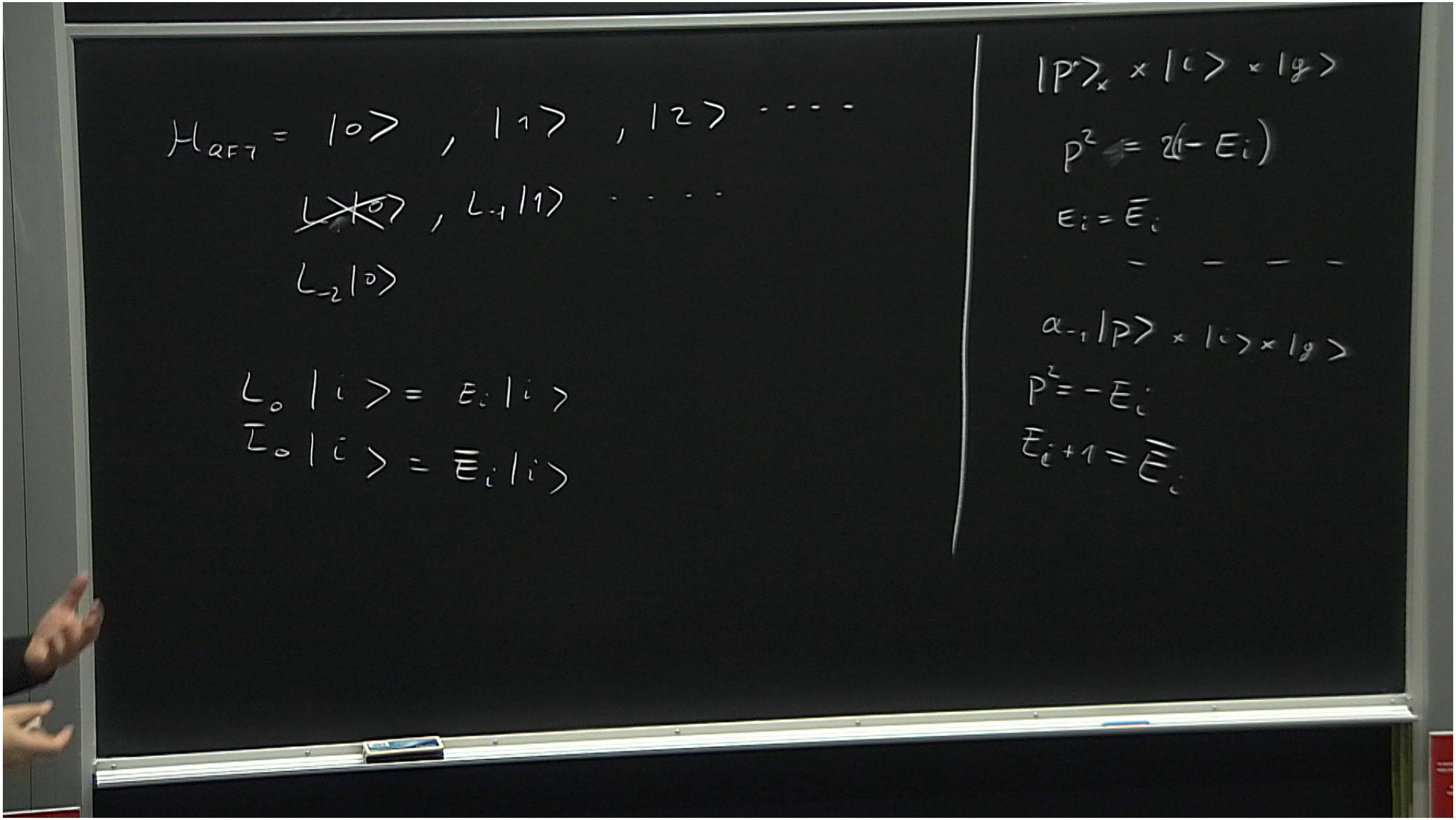
$$S = \int \sum_{\mu=0}^{d-1} \partial X^\mu \bar{\partial} X_\mu + S_{QFT\,c=26-d}$$

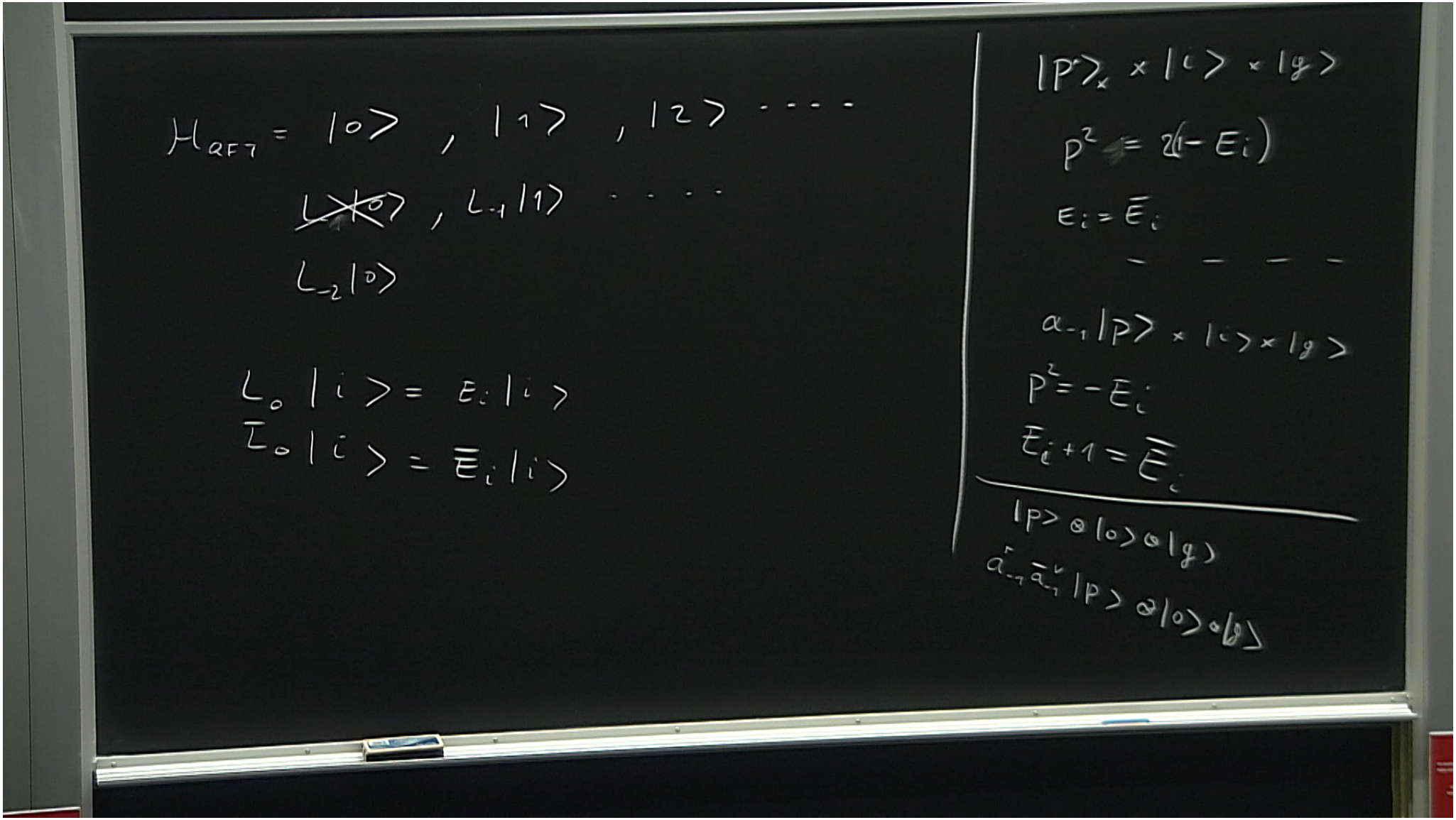
$$T_m = -\frac{1}{2}\partial X^\mu \partial X_\mu + T_{QFT}^{c=26-d}$$

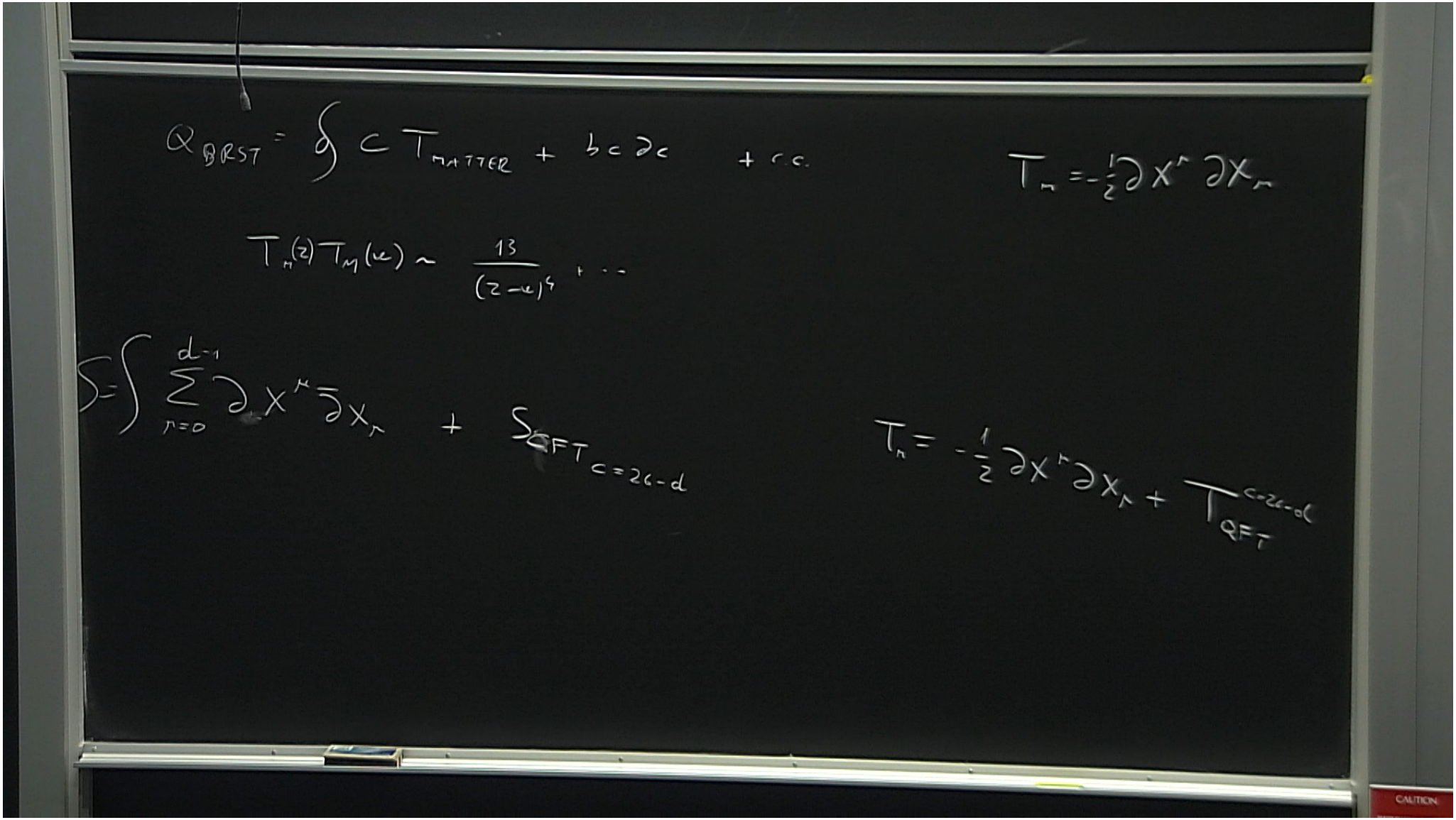

$\mathcal{H}_{QFT} = |0\rangle, |1\rangle, |2\rangle \cdots$
$\cancel{L_{-1}|0\rangle}, L_{-1}|1\rangle \cdots$
$L_{-2}|0\rangle$
$L_0|i\rangle = E_i|i\rangle$
$\bar{L}_0|i\rangle = \bar{E}_i|i\rangle$

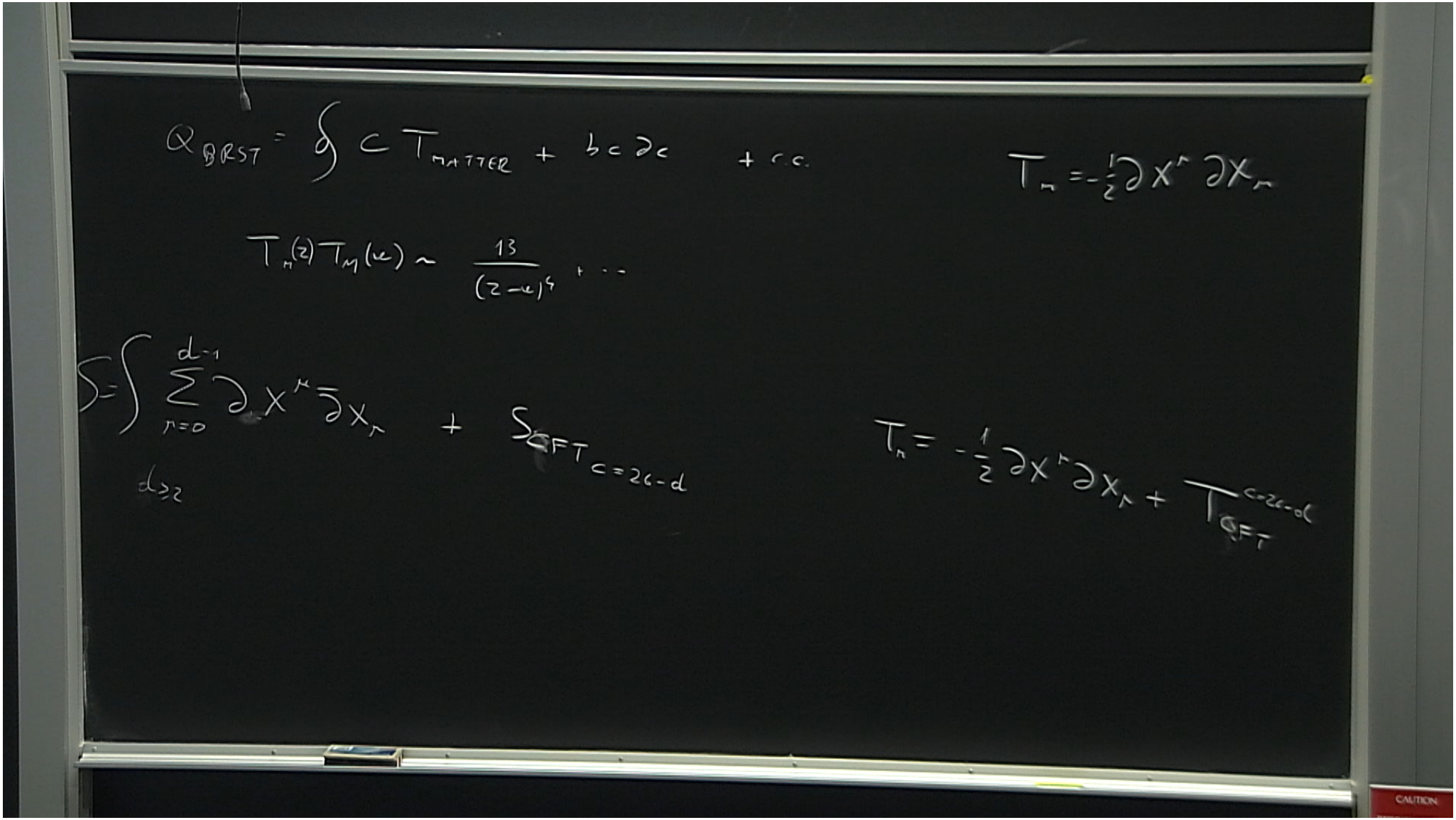

$\mathcal{H}_{QFT} = |0\rangle , |1\rangle , |2\rangle \cdots$
$L_{-1}|0\rangle , L_{-1}|1\rangle \cdots$
$L_{-2}|0\rangle$
$L_0|i\rangle = E_i|i\rangle$
$\bar{L}_0|i\rangle = \bar{E}_i|i\rangle$
$|p\rangle_x \times |i\rangle \times |g\rangle$
$p^2 = 2 - E_i$
$E_i = \bar{E}_i$

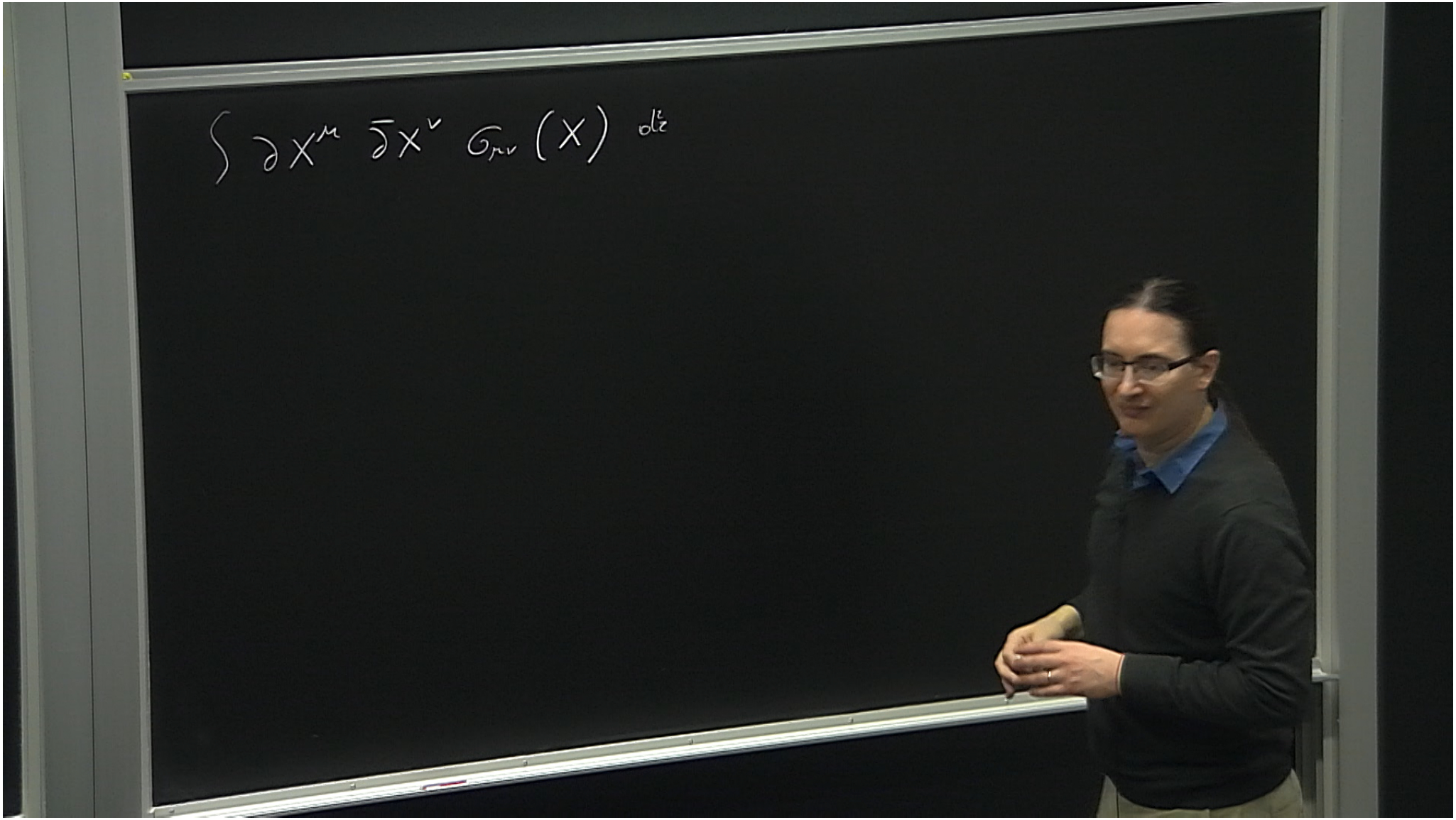

$H_{QFT} = |0\rangle , |1\rangle , |2\rangle \cdots$
$L_{-1}|1\rangle \cdots$
$L_{-2}|0\rangle$
$L_0|i\rangle = E_i|i\rangle$
$\bar{L}_0|i\rangle = \bar{E}_i|i\rangle$
$|p\rangle_x \times |i\rangle \times |g\rangle$
$p^2 = 2(1 - E_i)$
$E_i = \bar{E}_i$

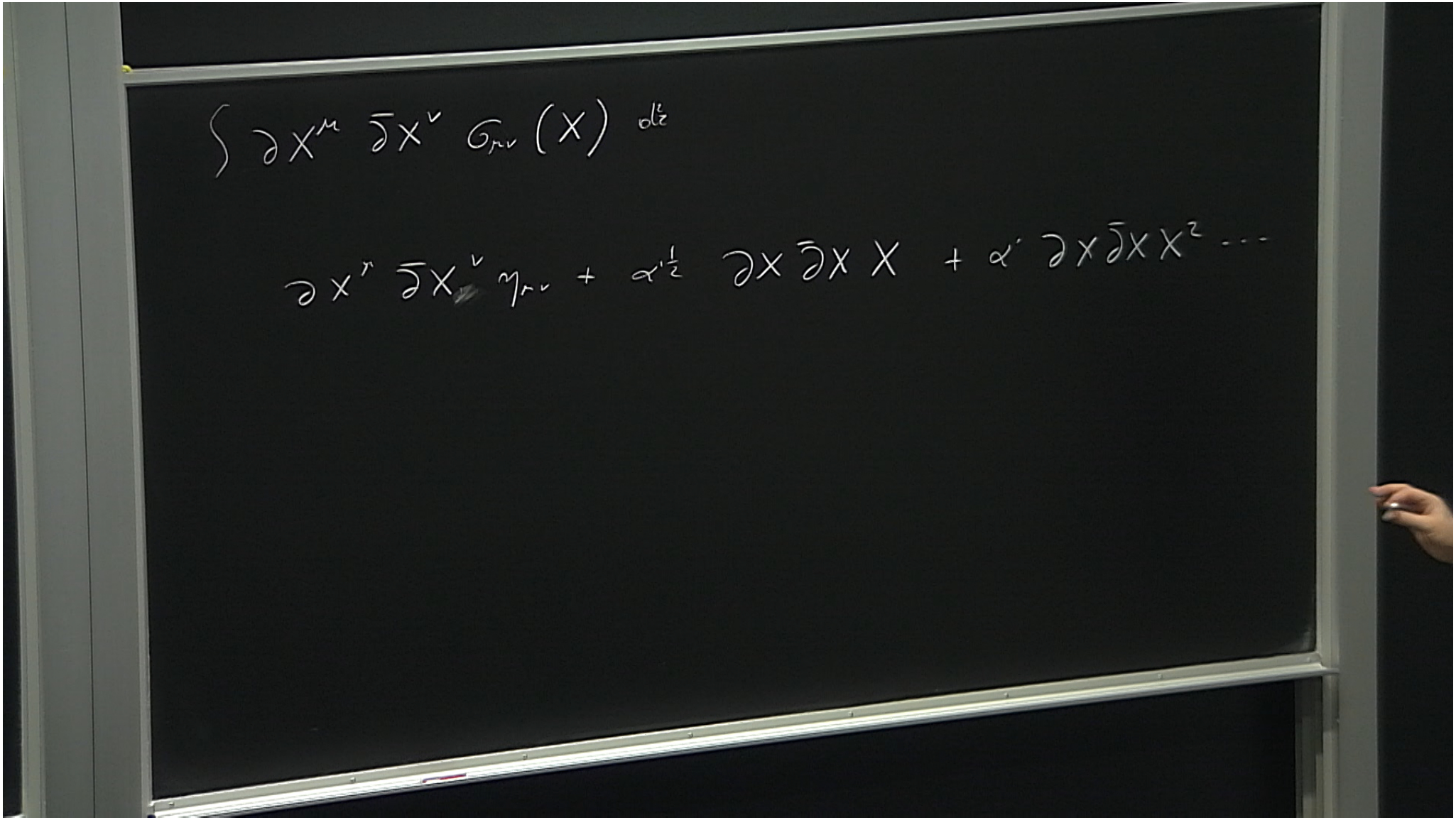

$H_{QFT} = |0\rangle , |1\rangle , |2\rangle \cdots$
$L_{-1}|0\rangle , L_{-1}|1\rangle \cdots$
$L_{-2}|0\rangle$
$L_0|i\rangle = E_i|i\rangle$
$\bar{L}_0|i\rangle = \bar{E}_i|i\rangle$
$|p\rangle_x \times |i\rangle \times |g\rangle$
$p^2 = 2(-E_i)$
$E_i = \bar{E}_i$
$a_{-1}|p\rangle \times |i\rangle \times |g\rangle$
$p^2 = -E_i$
$E_{i+1} = \bar{E}_i$

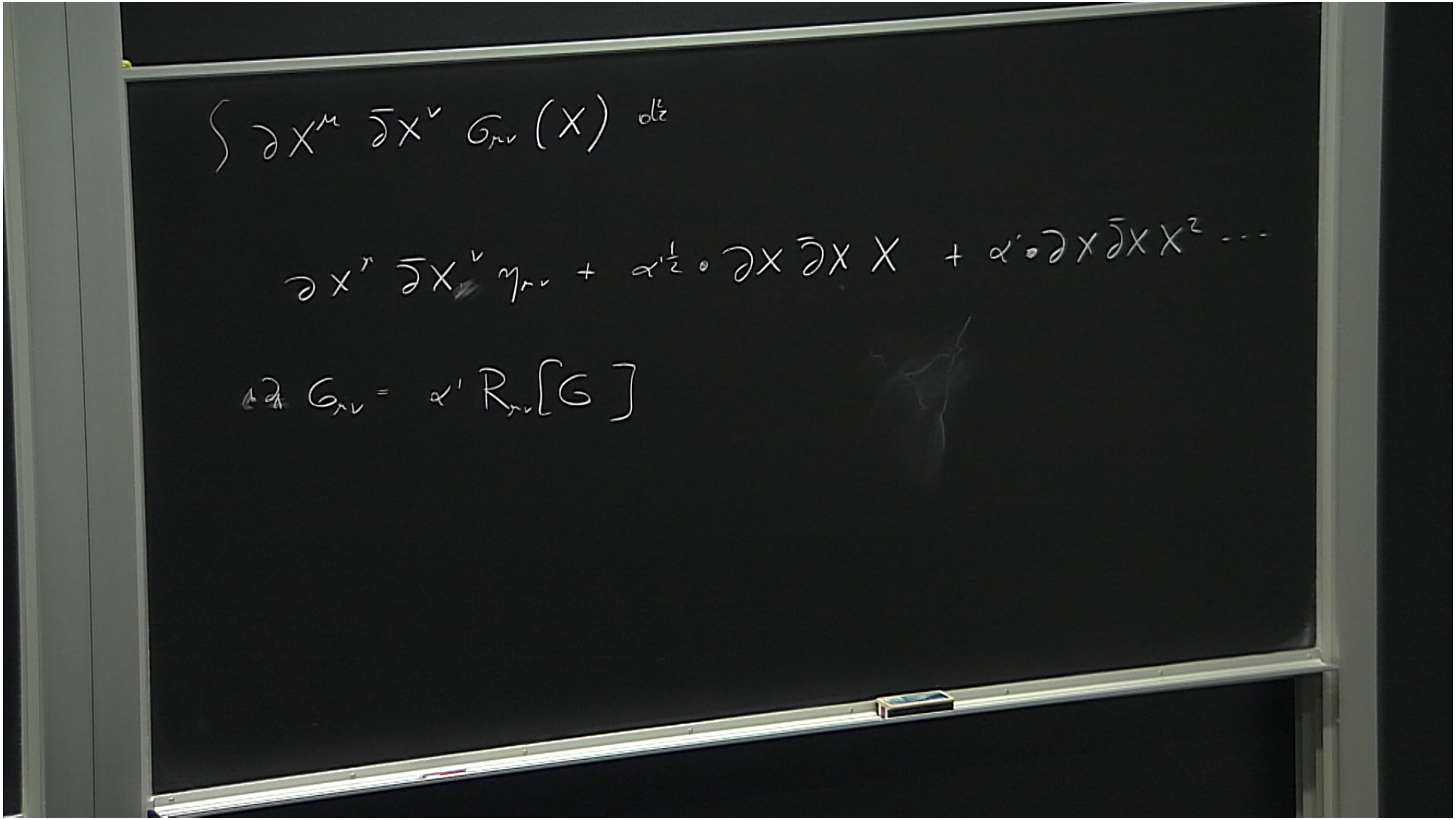

$\mathcal{H}_{QFT} = |0\rangle , |1\rangle , |2\rangle \cdots$
$L_{-1}|1\rangle \cdots$
$L_{-2}|0\rangle$
$L_0|i\rangle = E_i|i\rangle$
$\bar{L}_0|i\rangle = \bar{E}_i|i\rangle$
$|p\rangle_x \times |i\rangle \times |g\rangle$
$p^2 = 2(-E_i)$
$E_i = \bar{E}_i$
$a_{-1}|p\rangle \times |i\rangle \times |g\rangle$
$p^2 = -E_i$
$E_{i+1} = \bar{E}_i$
$|p\rangle \otimes |0\rangle \otimes |g\rangle$
$a_{-1}^{\mu} \bar{a}_{-1}^{\nu} |p\rangle \otimes |0\rangle \otimes |g\rangle$

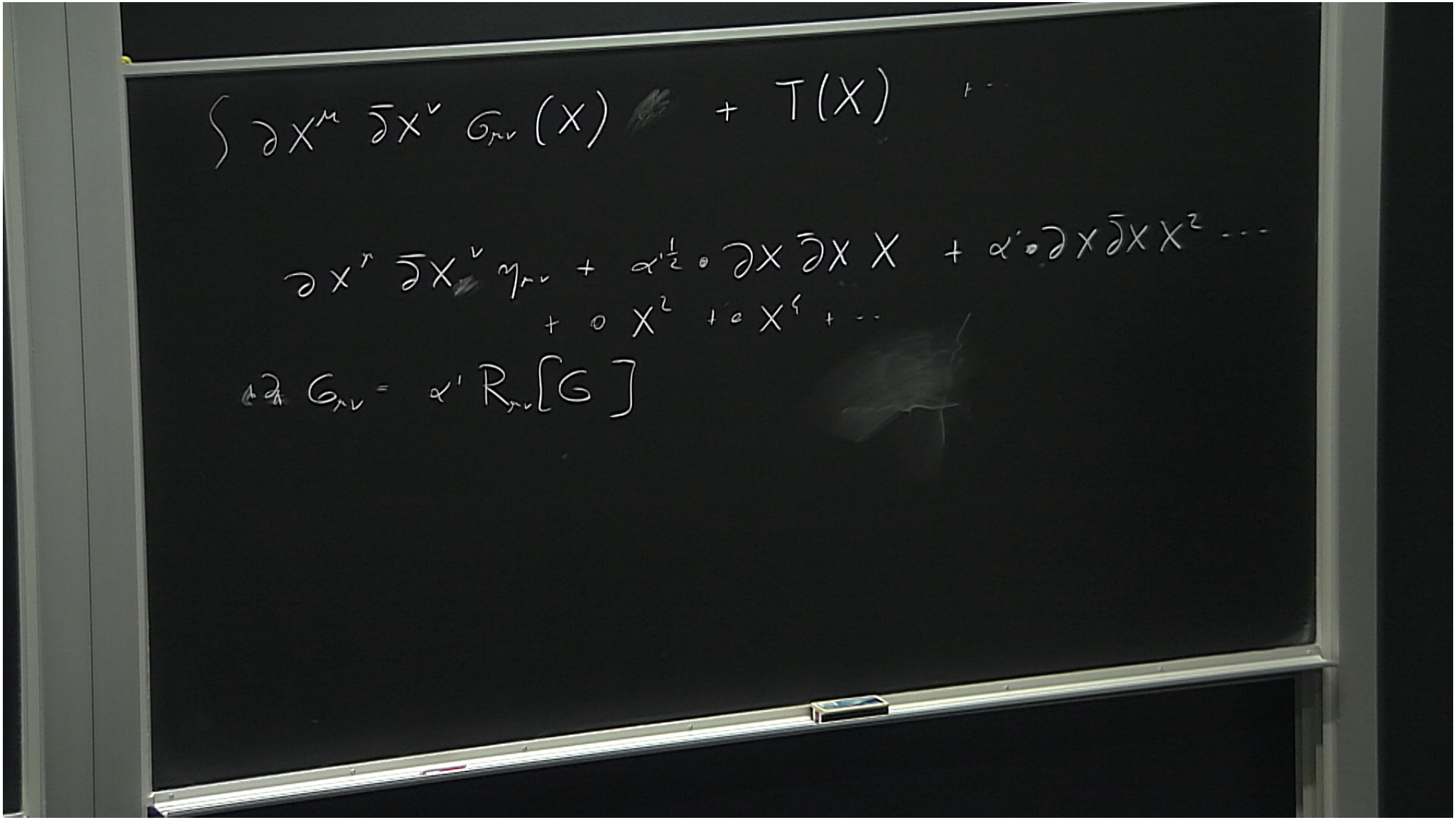

$Q_{BRST} = \oint c\,T_{MATTER} + b\,c\,\partial c + c.c.$
$T_n = -\frac{1}{2}\partial X^\mu \partial X_\mu$
$T_n(z)\,T_m(w) \sim \frac{13}{(z-w)^4} + \cdots$
$S = \int \sum_{\mu=0}^{d-1} \partial X^\mu \bar{\partial} X_\mu + S_{QFT\,c=26-d}$
$T_n = -\frac{1}{2}\partial X^\mu \partial X_\mu + T_{QFT}^{c=26-d}$

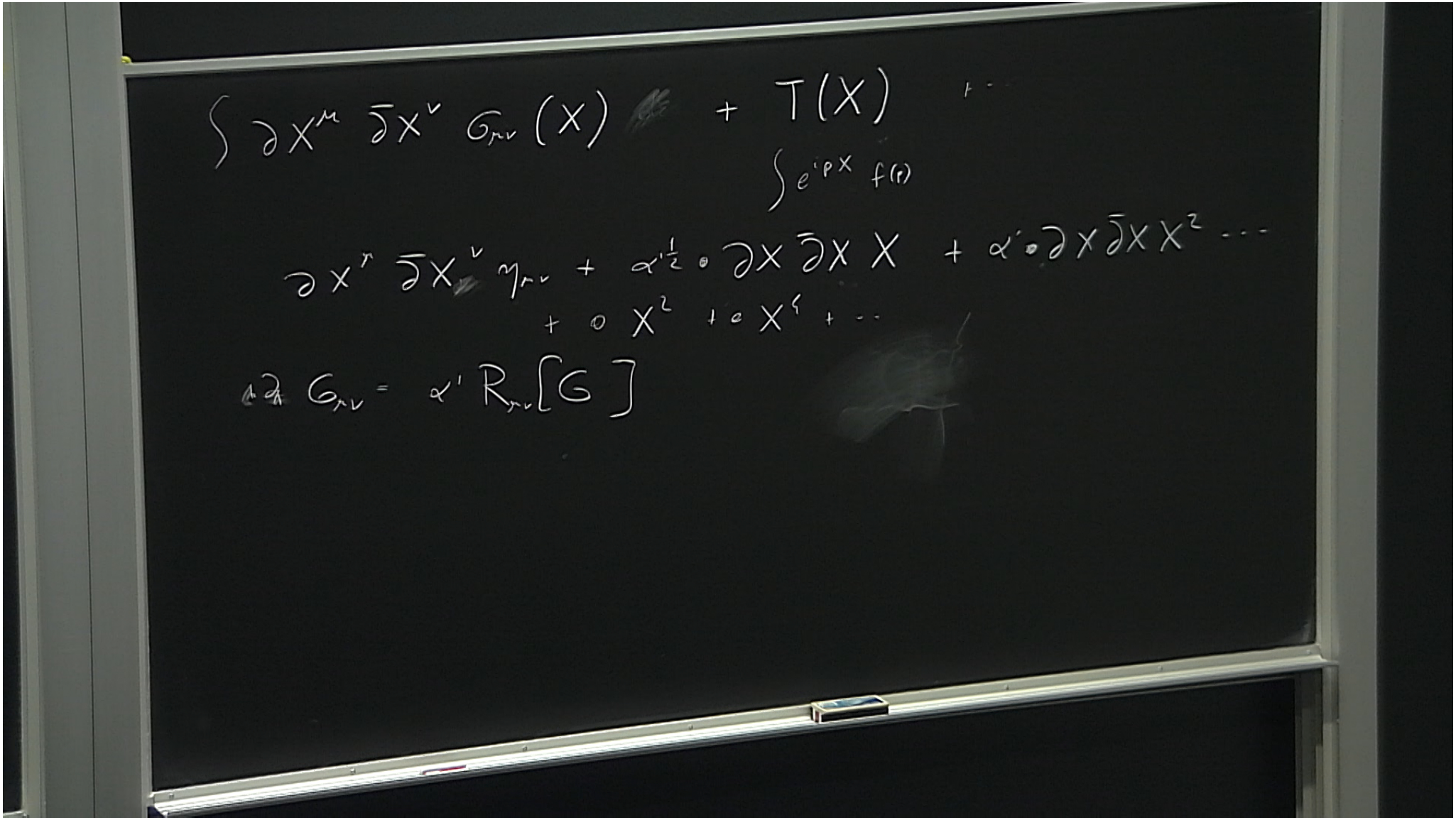

$Q_{BRST} = \oint c\,T_{MATTER} + b\,c\,\partial c + c.c.$
$T_n = -\frac{1}{2}\partial X^\mu \partial X_\mu$
$T_n(z)\,T_m(w) \sim \frac{13}{(z-w)^4} + \cdots$
$S = \int \sum_{\mu=0}^{d-1} \partial X^\mu \bar{\partial} X_\mu + S_{QFT\ c=26-d}$
$d \geq 2$
$T_n = -\frac{1}{2}\partial X^\mu \partial X_\mu + T_{QFT}^{c=26-d}$

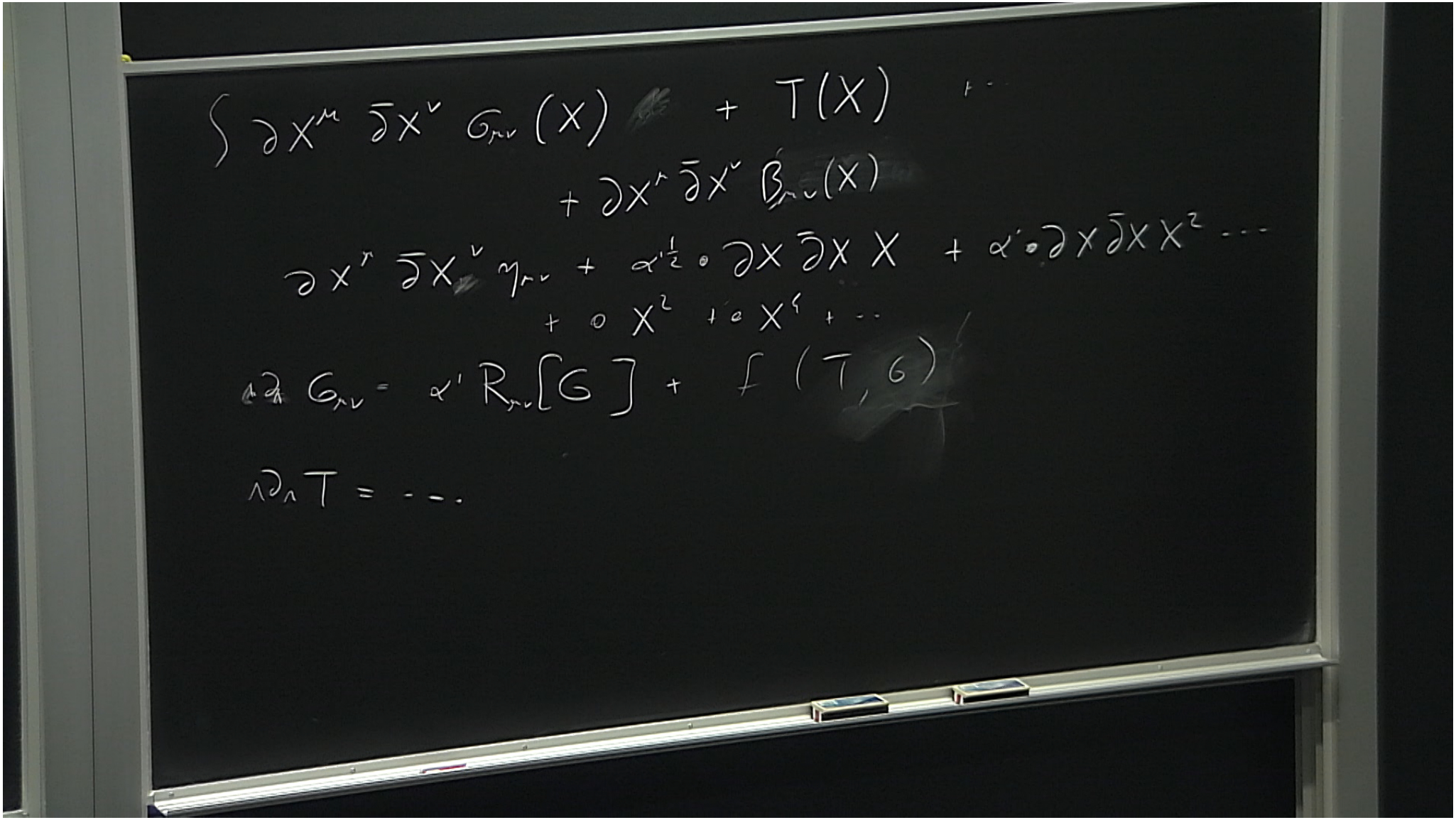

$\int \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) \, d^2z$

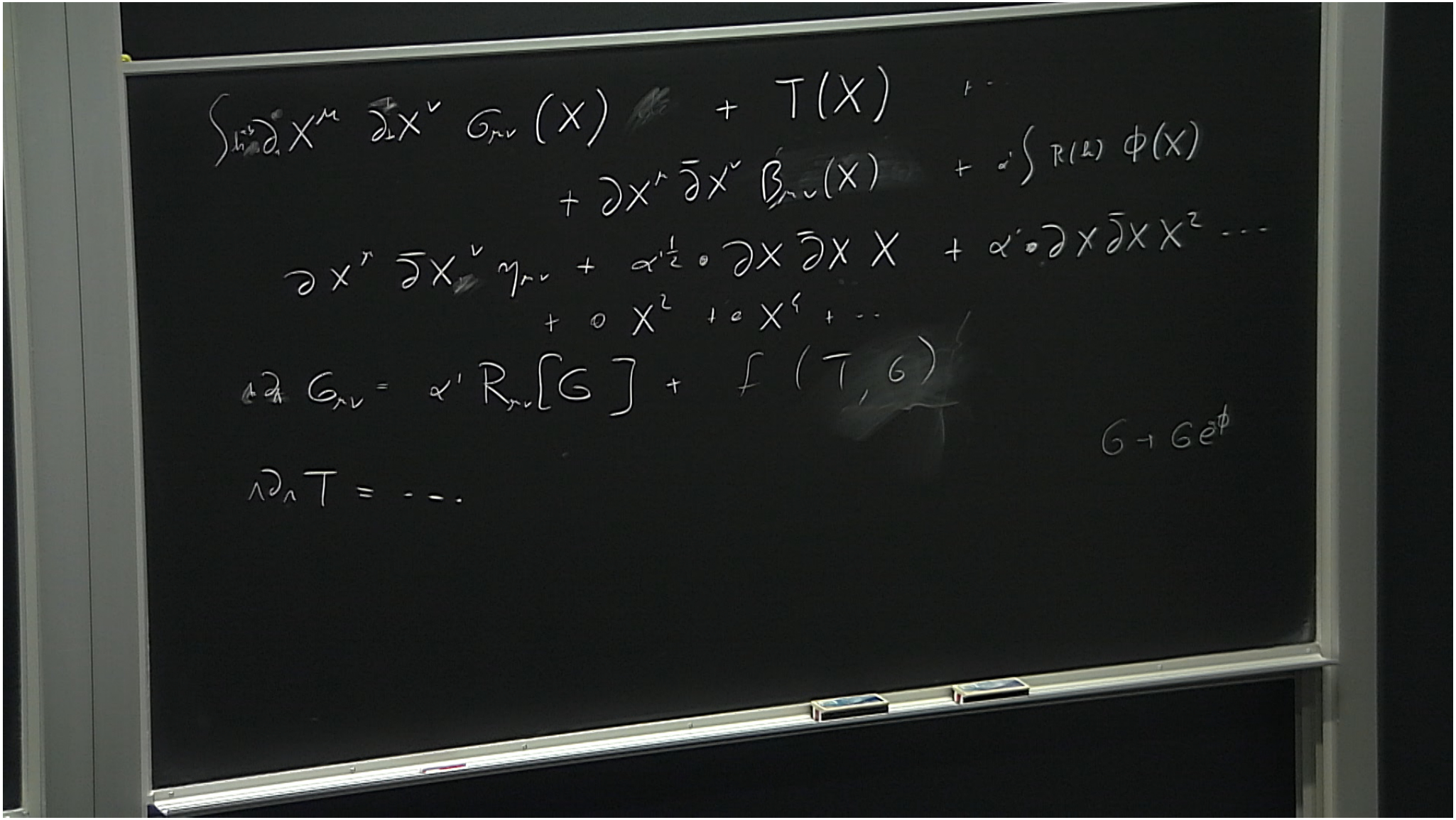

$\int \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) \, d^2z$
$\partial x^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha'^{\frac{1}{2}} \partial x \bar{\partial} x X + \alpha' \partial x \bar{\partial} x X^2 \cdots$

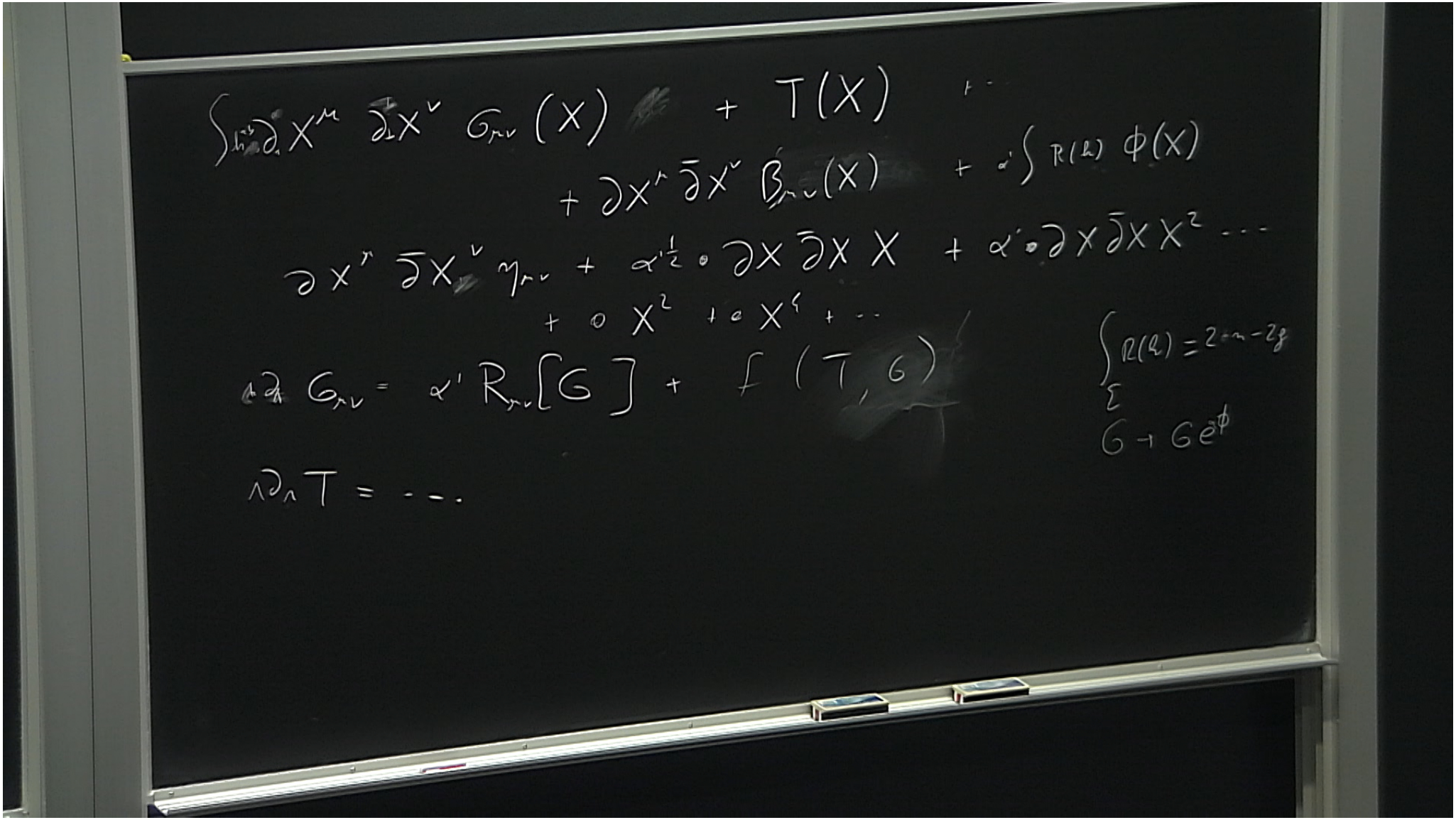

$\int \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X)$
$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha' \partial X \bar{\partial} X X + \alpha' \partial X \bar{\partial} X X^2 \cdots$
$\beta G_{\mu\nu} = \alpha' R_{\mu\nu}[G]$

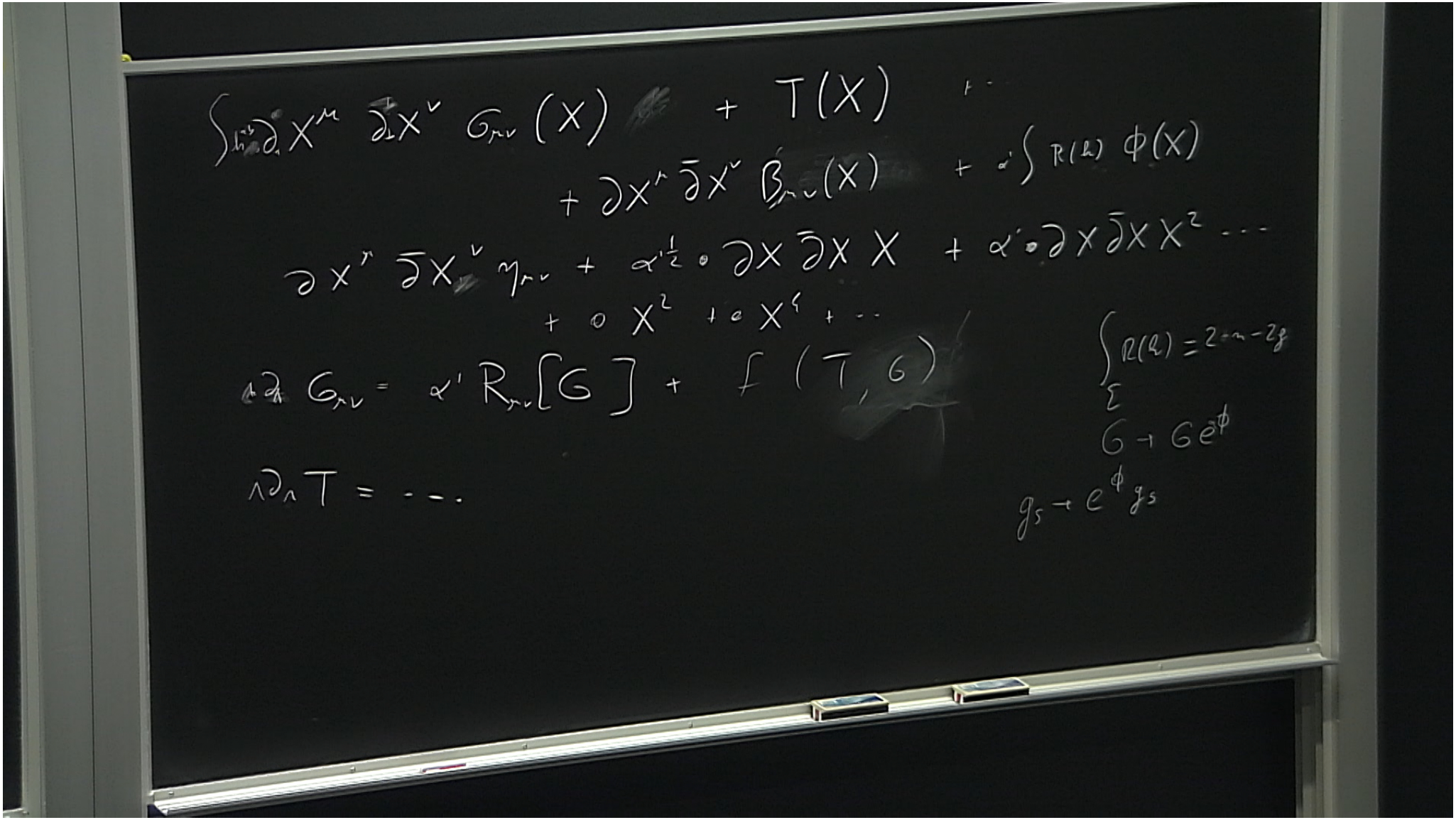

$\int \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) + T(X) + \cdots$
$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha' \partial X \bar{\partial} X X + \alpha' \partial X \bar{\partial} X X^2 \cdots$
$+ \partial X^2 + \partial X^4 + \cdots$
$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu}[G]$

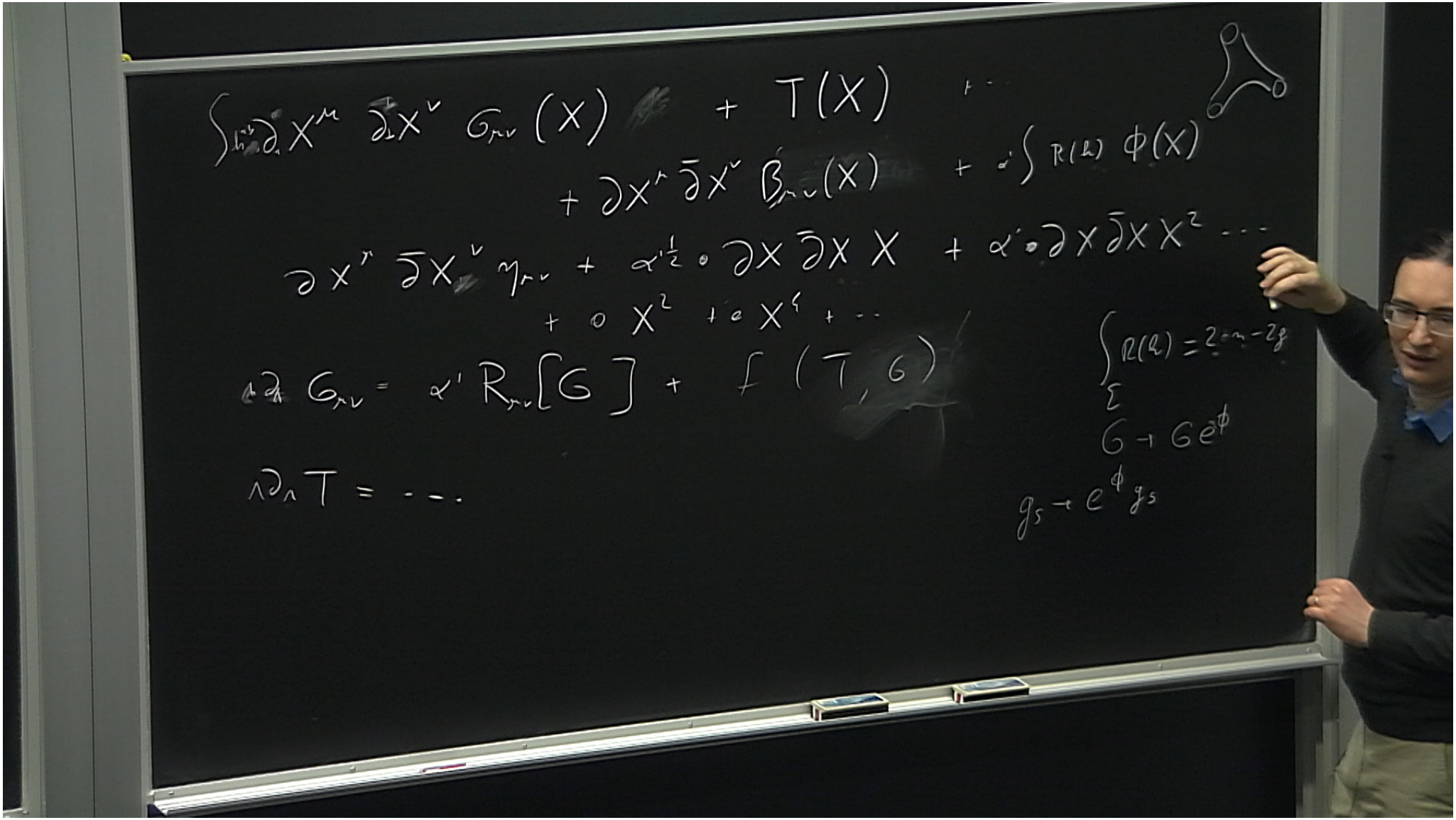

$\int \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) + T(X) \cdots$
$\int e^{ipX} f(r)$
$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha' \partial X \bar{\partial} X X + \alpha' \partial X \bar{\partial} X X^2 \cdots$
$+ \partial X^2 + \partial X^3 + \cdots$
$\Delta G_{\mu\nu} = \alpha' R_{\mu\nu}[G]$

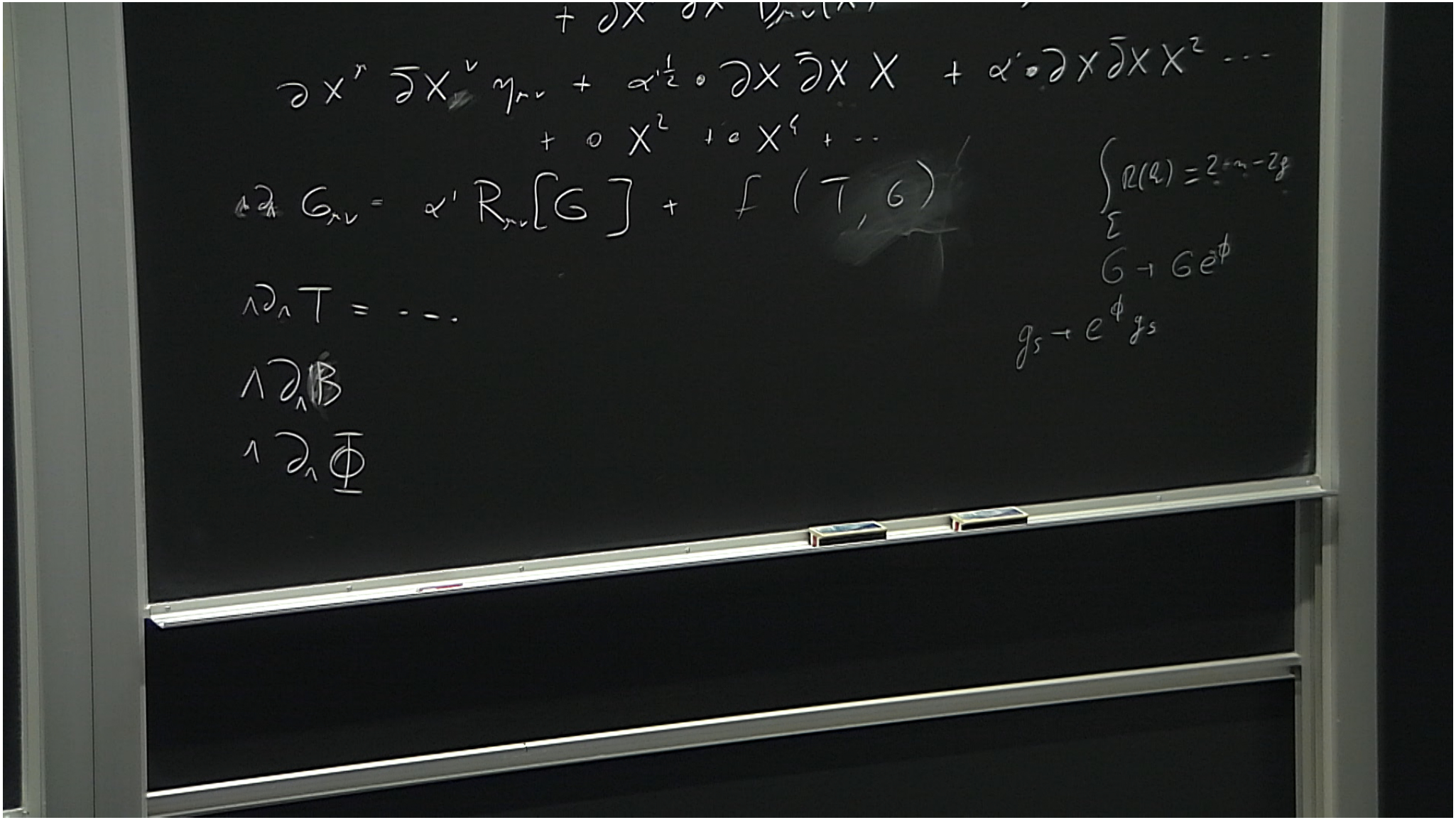

$\int \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) + T(X) + \ldots$
$+ \partial X^\mu \bar{\partial} X^\nu B_{\mu\nu}(X)$
$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha'^{\frac{1}{2}} \circ \partial X \bar{\partial} X X + \alpha' \circ \partial X \bar{\partial} X X^2 \ldots$
$+ \circ X^2 + \circ X^4 + \ldots$
$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu}[G] + f(T, G)$
$\nabla_\mu T = \ldots$

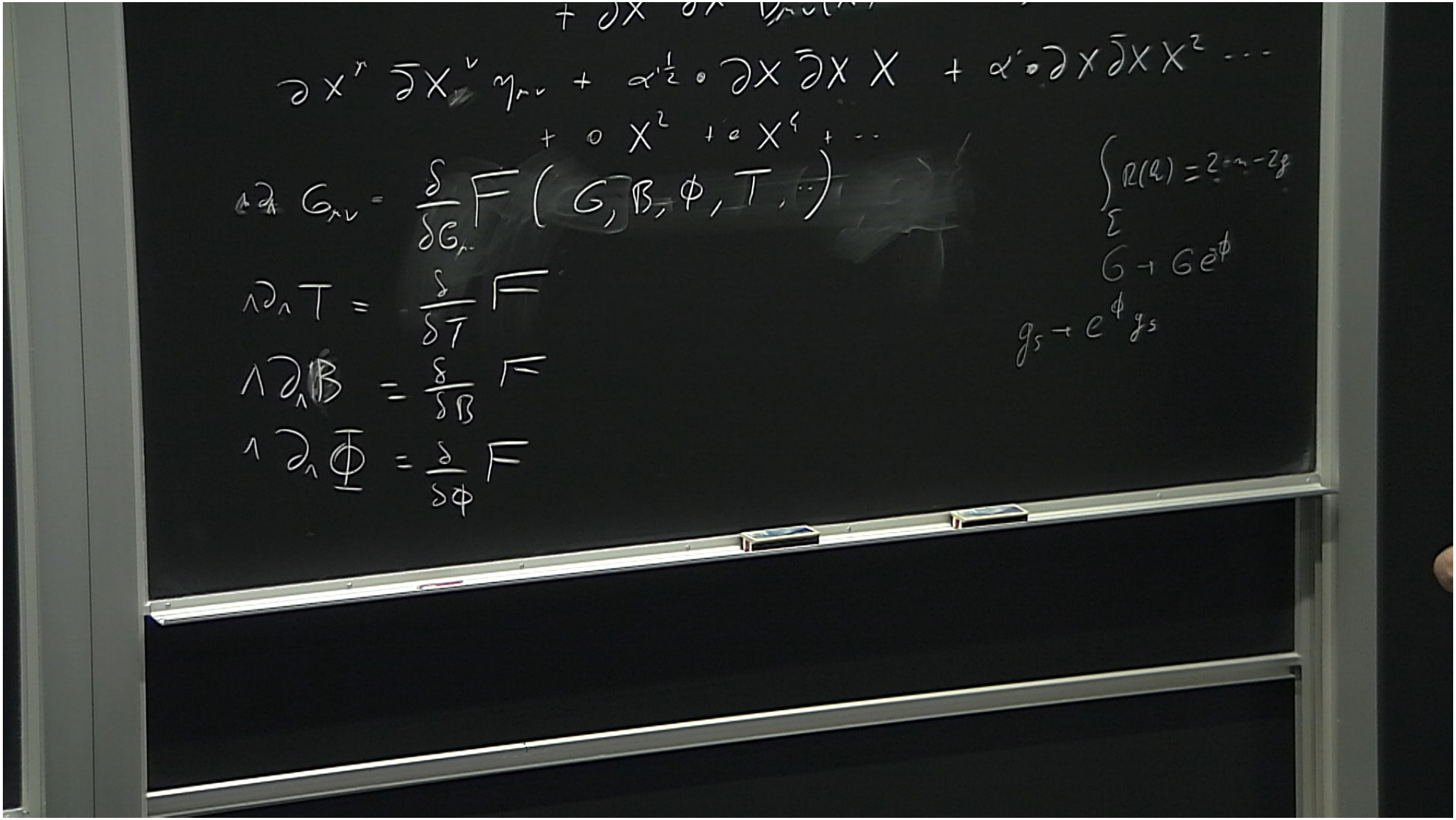

$\int h^{ab} \partial_a X^\mu \bar{\partial}_b X^\nu G_{\mu\nu}(X) + T(X) + \ldots$
$+ \partial X^\mu \bar{\partial} X^\nu B_{\mu\nu}(X) + \alpha' \int R(h) \Phi(X)$
$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha' \partial X \bar{\partial} X X + \alpha' \partial X \bar{\partial} X X^2 \ldots$
$+ \partial X^2 + \partial X^4 + \ldots$
$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu}[G] + f(T, G)$
$G \to G e^{\Phi}$
$\nabla_\mu T = \ldots$

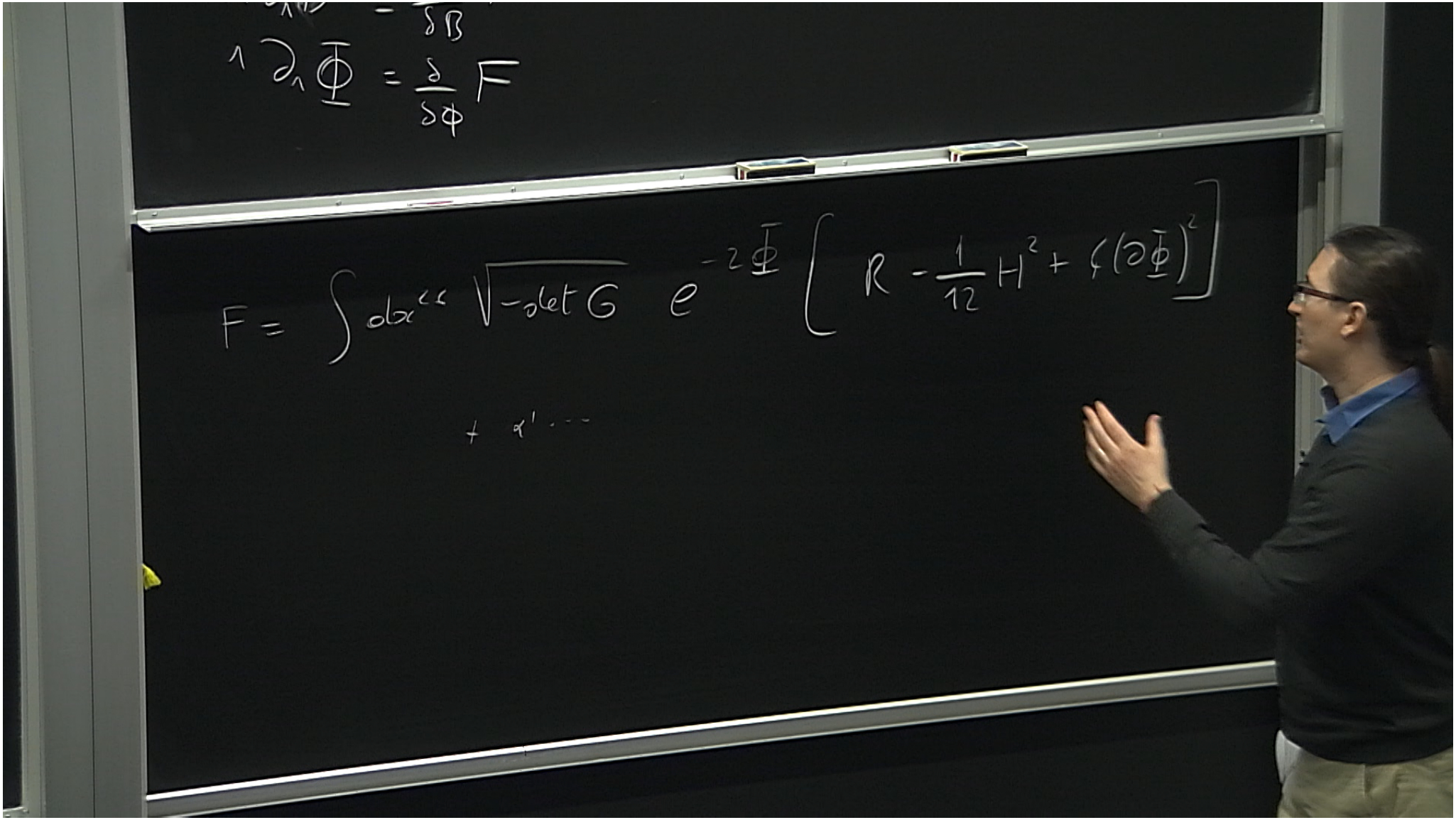

$\int_\Sigma \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) + T(X) + \ldots$
$+ \partial X^\mu \bar{\partial} X^\nu B_{\mu\nu}(X) + \alpha' \int R(h)\, \Phi(X)$
$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha' \partial X \bar{\partial} X X + \alpha' \partial X \bar{\partial} X X^2 \ldots$
$+ \partial X^2 + \partial X^4 + \ldots$
$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu}[G] + f(T, G)$
$\int_\Sigma R(h) = 2 - n - 2g$
$G \to G e^{\Phi}$
$\partial_\mu T = \ldots$

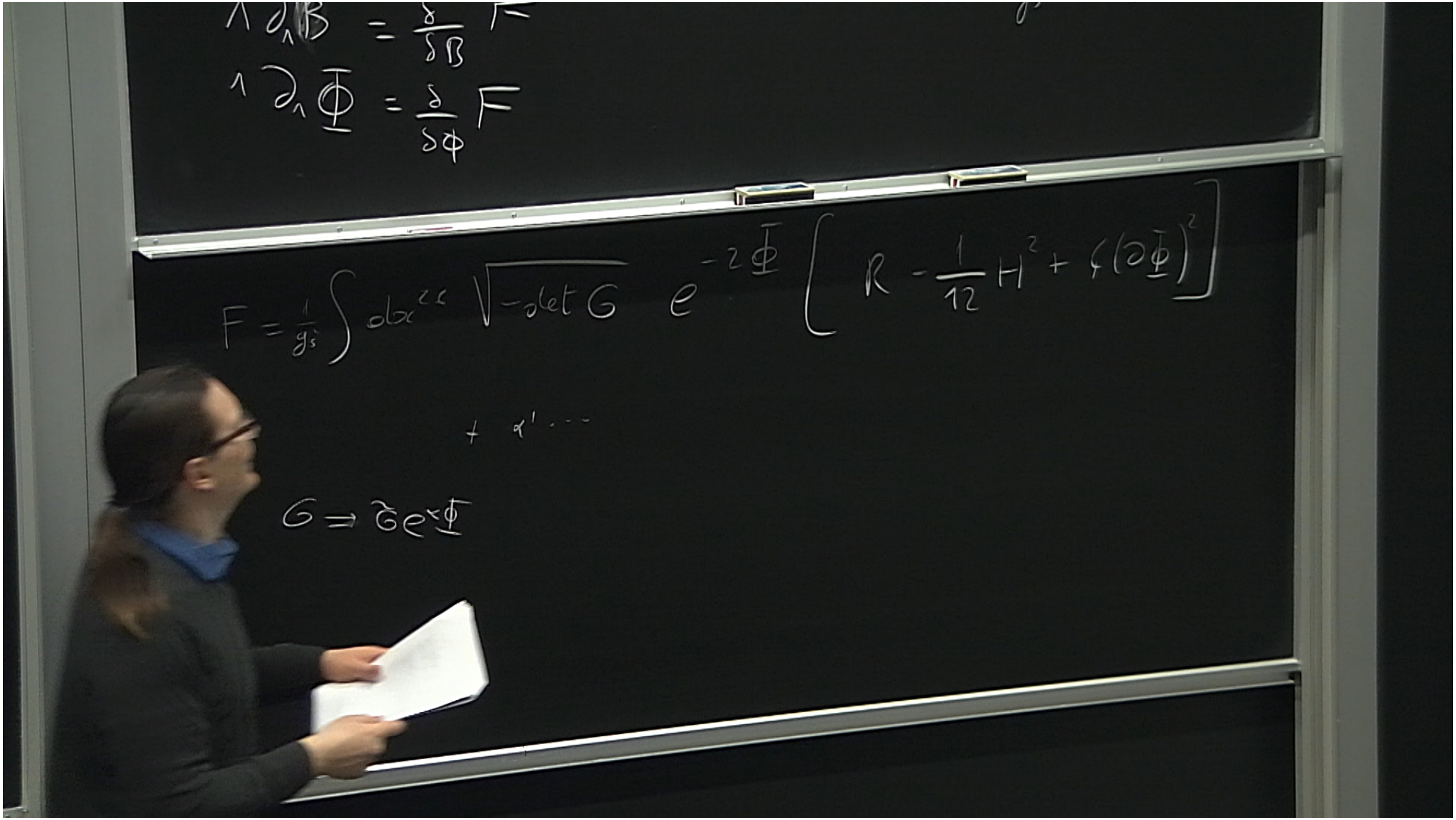

$\int_\Sigma \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) + T(X) + \ldots$
$+ \partial X^\mu \bar{\partial} X^\nu B_{\mu\nu}(X) + \alpha' \int R^{(2)} \Phi(X)$
$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha' \partial X \bar{\partial} X X + \alpha' \partial X \bar{\partial} X X^2 \ldots$
$+ \partial X^2 + \partial X^4 + \ldots$
$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu}[G] + f(T, G)$
$\partial^\mu T_{\mu\nu} = \ldots$
$\int_\Sigma R^{(2)} = 2 - n - 2g$
$G \to G e^{\Phi}$
$g_s \to e^{\Phi} g_s$

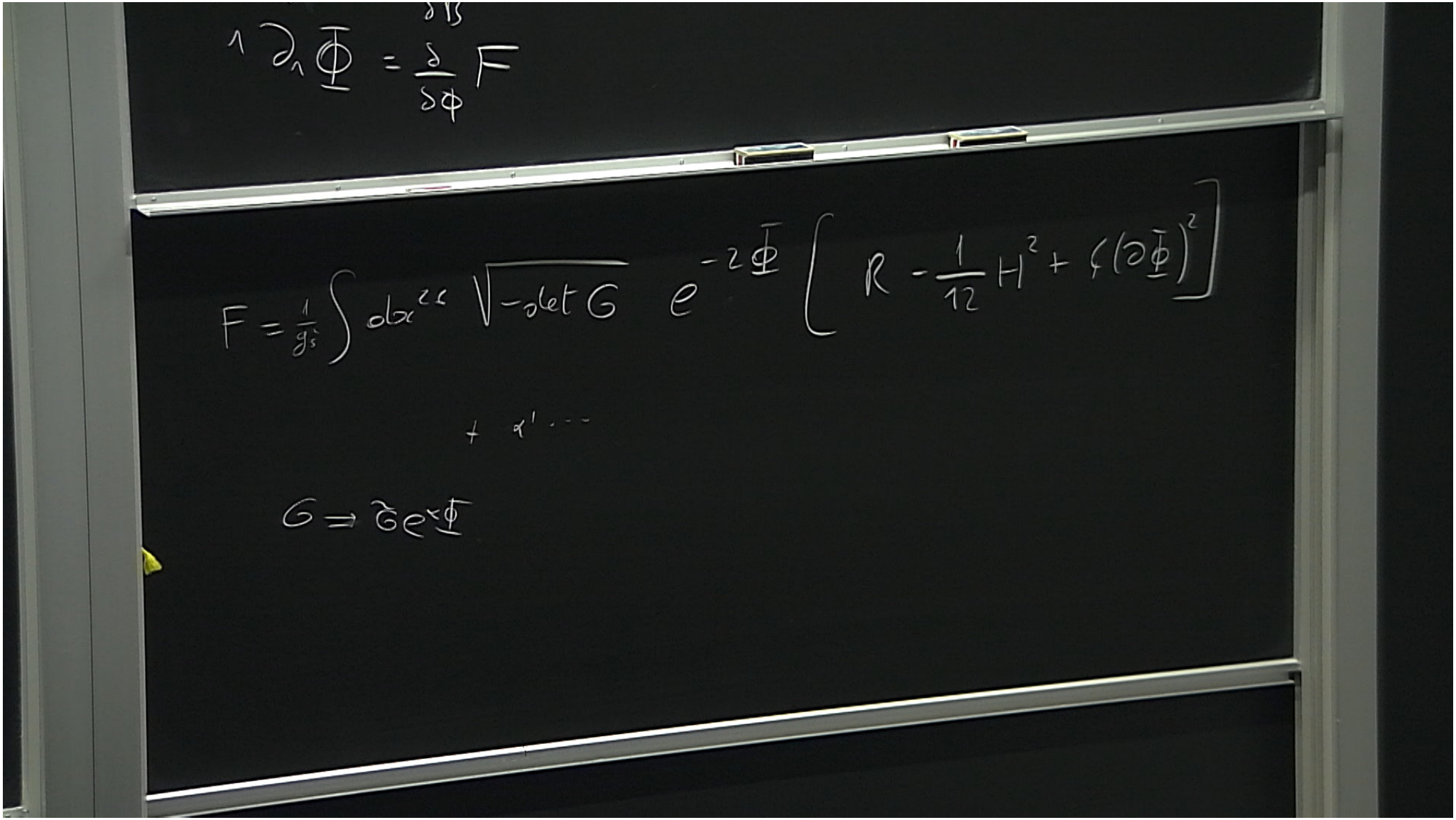

$\int_\Sigma \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) + T(X) + \ldots$
$+ \partial X^\mu \bar{\partial} X^\nu B_{\mu\nu}(X) + \alpha' \int R(h)\, \Phi(X)$
$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha' \partial X \bar{\partial} X X + \alpha' \partial X \bar{\partial} X X^2 \ldots$
$+ \partial X^2 + \partial X^4 + \ldots$
$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu}[G] + f(T, G)$
$\nabla_\mu T = \ldots$
$\int_\Sigma R(h) = 2 - n - 2g$
$G \to G e^{\Phi}$
$g_s \to e^{\Phi} g_s$

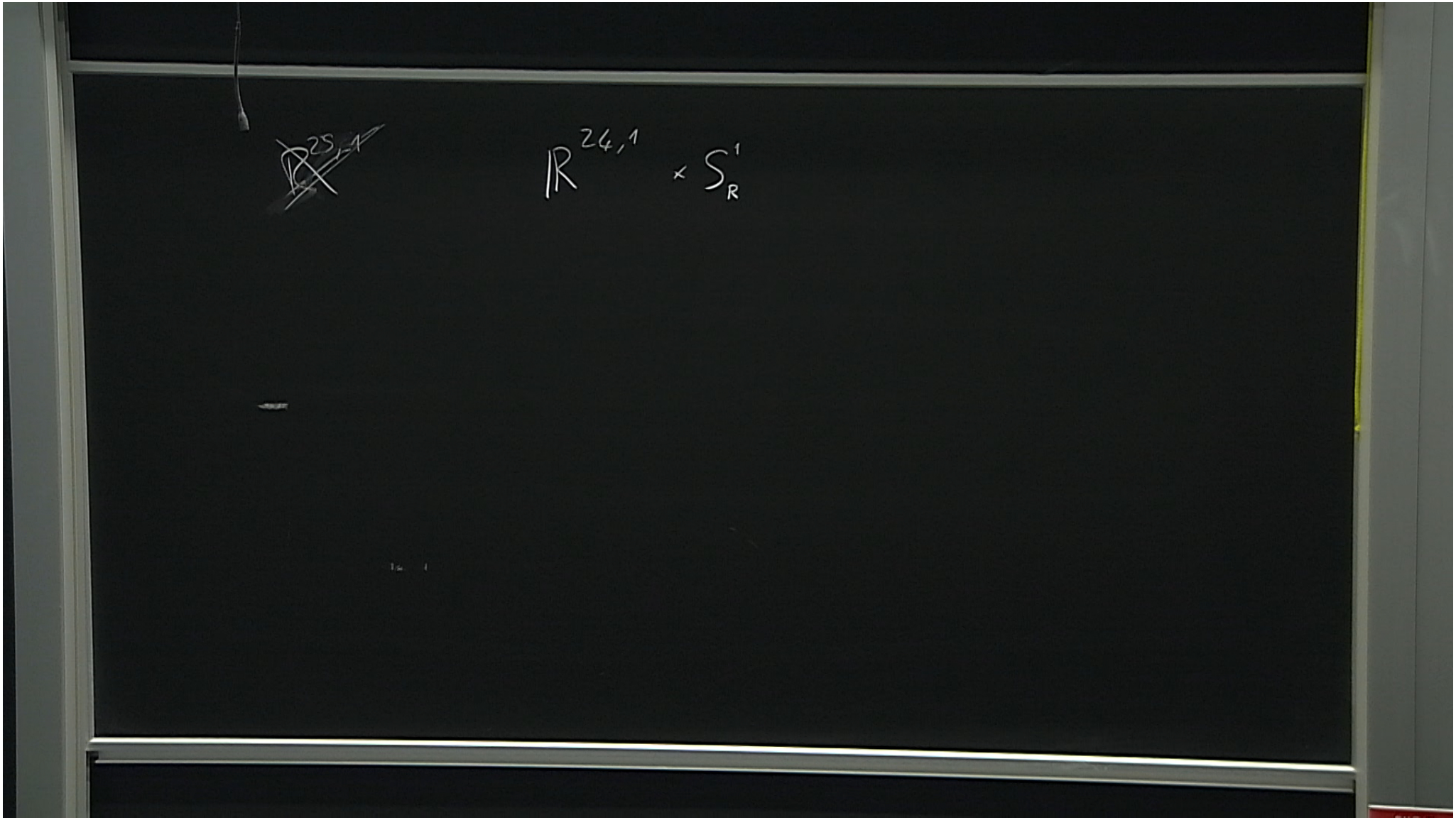

$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha' \partial X \bar{\partial} X X + \alpha' \partial X \bar{\partial} X X^2 \ldots$
$+ \partial X^2 + \partial X^4 + \ldots$
$\beta^G_{\mu\nu} = \alpha' R_{\mu\nu}[G] + f(T, G)$
$\int_\Sigma R(4) = 2 - n - 2g$
$G \to G e^\phi$
$g_s \to e^\phi g_s$
$\partial_\mu T = \ldots$
$\partial_\mu B$
$\partial_\mu \Phi$

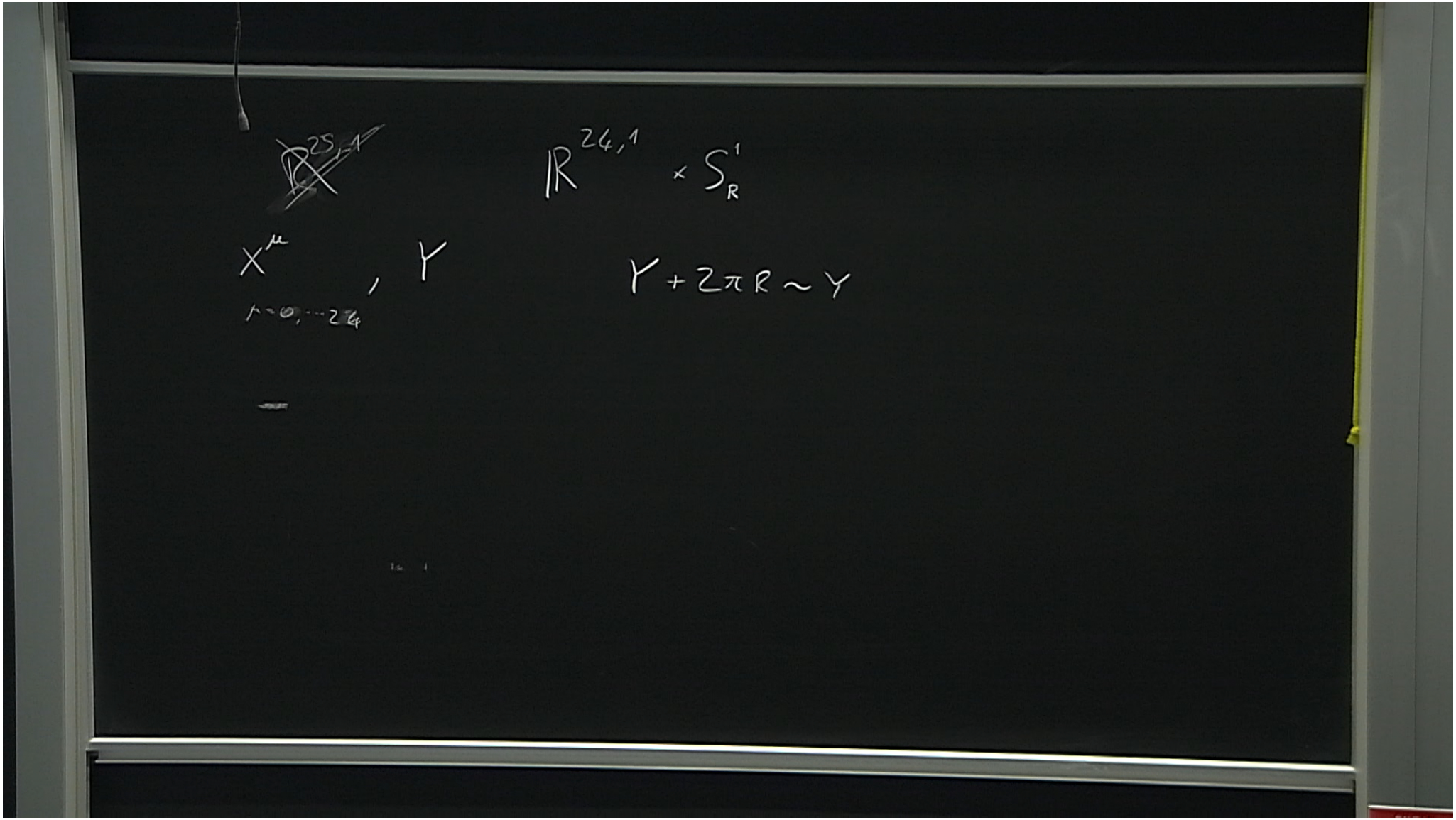

$\partial X^\mu \bar{\partial} X^\nu \eta_{\mu\nu} + \alpha' \partial X \bar{\partial} X X + \alpha' \partial X \bar{\partial} X X^2 \cdots$
$+ \partial X^2 + \partial X^4 + \cdots$
$\beta G_{\mu\nu} = \frac{\delta}{\delta G_{\mu\nu}} F(G, B, \Phi, T, \cdots)$
$\beta_T = \frac{\delta}{\delta T} F$
$\beta_B = \frac{\delta}{\delta B} F$
$\beta_\Phi = \frac{\delta}{\delta \Phi} F$
$\int_\Sigma R(g) = 2 - n - 2g$
$G \to G e^{\phi}$
$g_s \to e^{\phi} g_s$

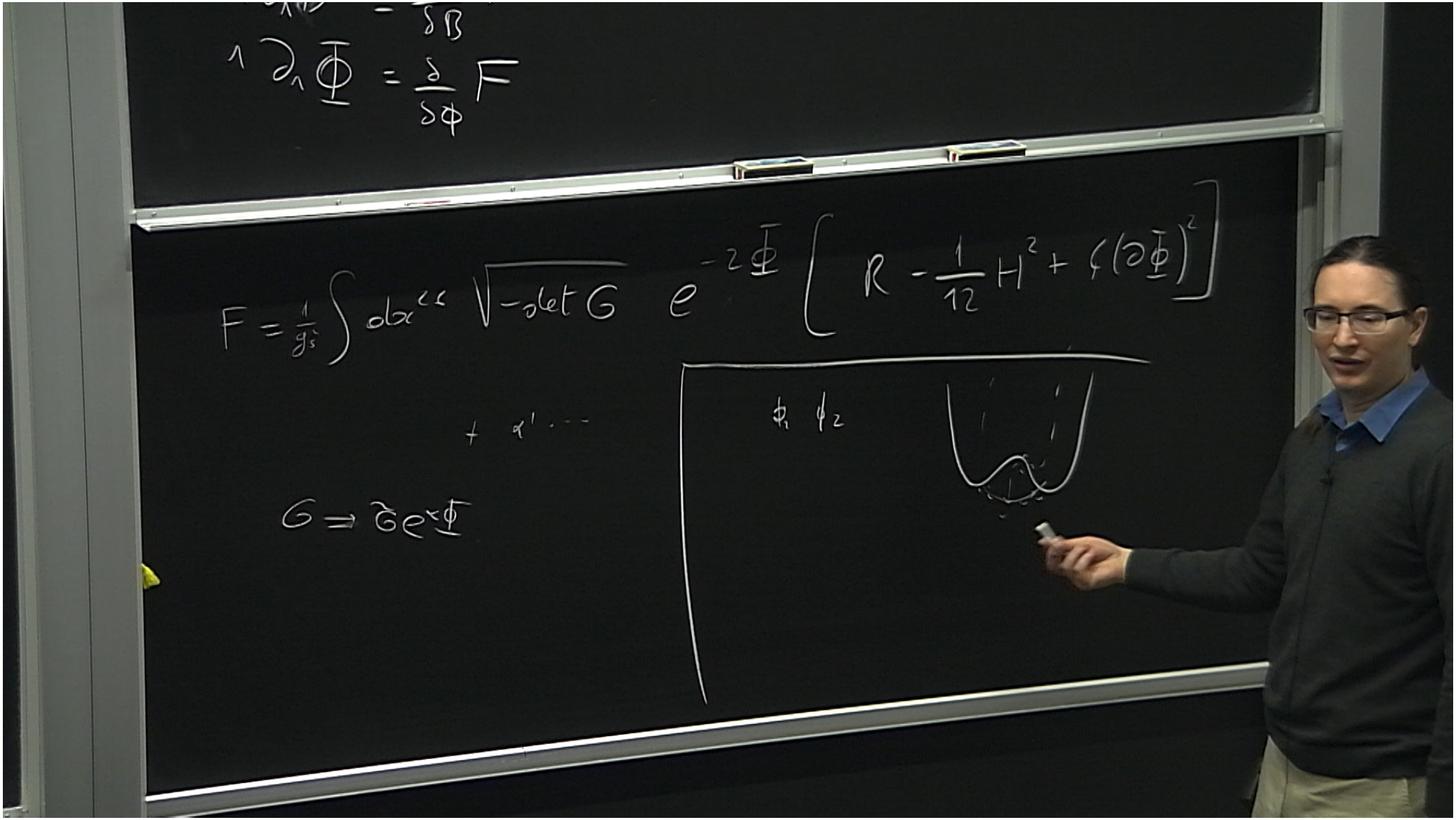

$\frac{\delta}{\delta\beta}$
$\frac{1}{2}\partial_1\bar{\Phi} = \frac{\delta}{\delta\Phi}F$
$F = \int d^{26}x \sqrt{-\det G}\; e^{-2\bar{\Phi}}\left[R - \frac{1}{12}H^2 + 4(\partial\bar{\Phi})^2\right]$
$+ \alpha' \cdots$

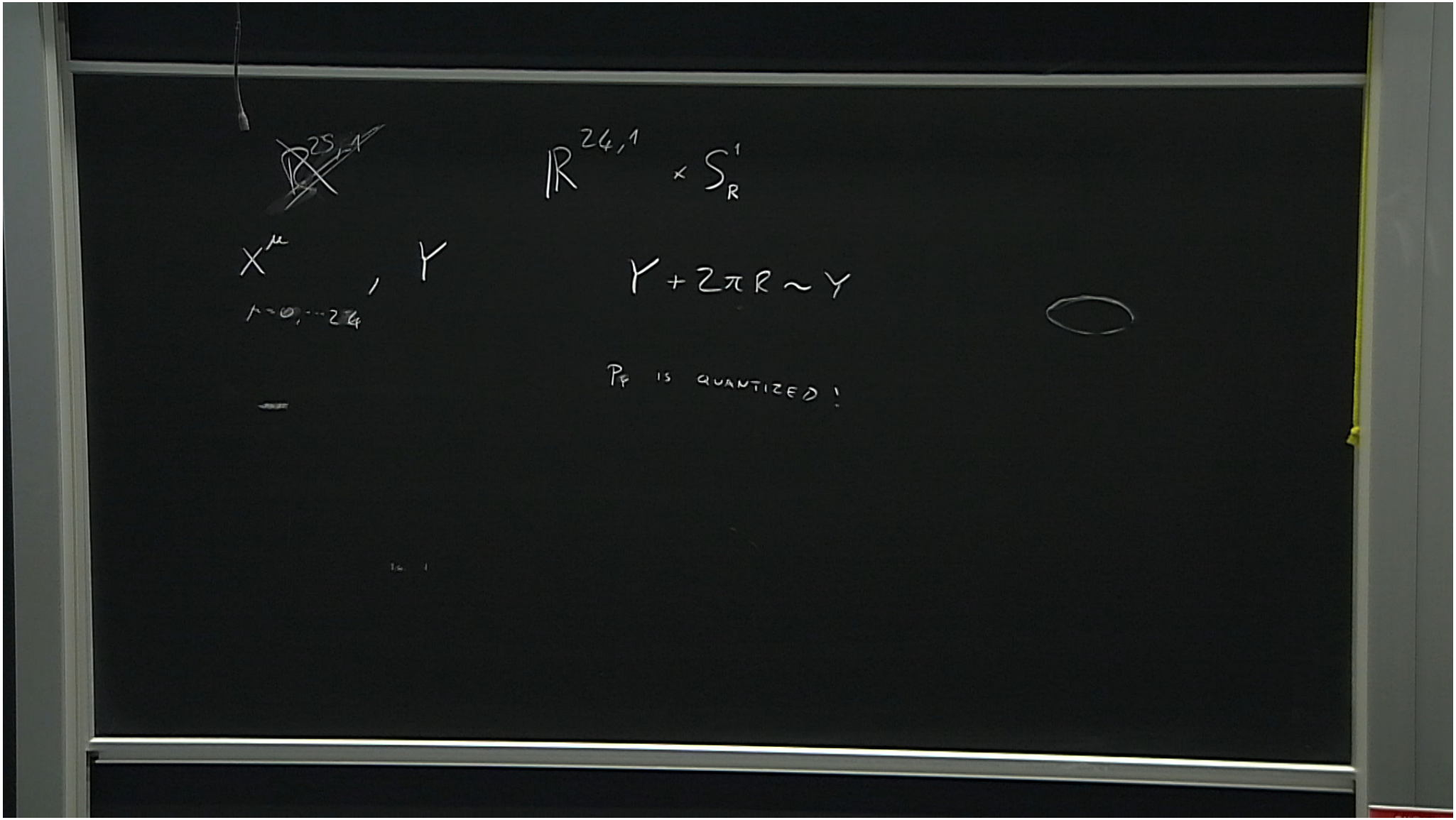

$\frac{\delta}{\delta B}$
$\frac{\delta}{\delta \Phi}$
$F = \frac{1}{g_s^2} \int d^{26}x \sqrt{-\det G}\, e^{-2\Phi} \left[R - \frac{1}{12} H^2 + 4 (\partial \Phi)^2 \right]$
$+ \alpha' \ldots$
$G \Rightarrow \tilde{G} e^{\frac{4}{D-2}\Phi}$

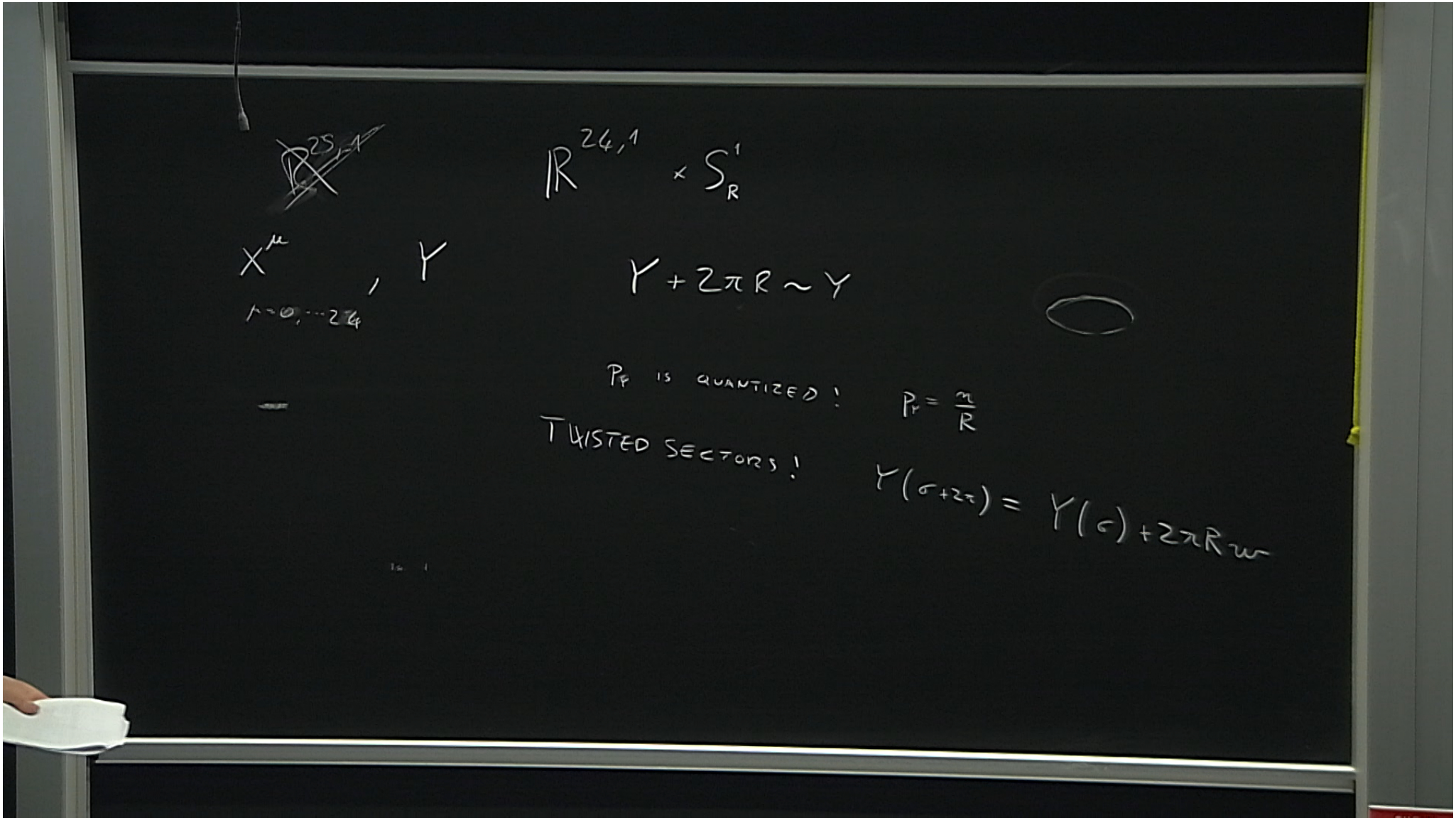

$\frac{\partial}{\partial\phi}F$
$F = \frac{1}{g_s^2}\int d^{26}x\sqrt{-\det G}\; e^{-2\bar{\Phi}}\left[R - \frac{1}{12}H^2 + c(\partial\bar{\Phi})^2\right]$
$+ \alpha' \cdots$
$G \Rightarrow \tilde{G}e^{\kappa\bar{\Phi}}$


$\mathbb{R}^{25,1}$
$\mathbb{R}^{24,1} \times S^1_R$


$\mathbb{R}^{25,1}$
$\mathbb{R}^{24,1} \times S^1_R$
X^μ, Y
$\mu = 0, \cdots 24$
$Y + 2\pi R \sim Y$


$\frac{\delta}{\delta\beta}$
$\partial_1 \Phi = \frac{\delta}{\delta\phi} F$
$F = \frac{1}{g_s^2} \int d^{26}x \sqrt{-\det G}\, e^{-2\Phi} \left[R - \frac{1}{12} H^2 + c (\partial\Phi)^2 \right]$
$+ \alpha' \cdots$
$G \Rightarrow \tilde{G} e^{\kappa\phi}$
ϕ_1 ϕ_2


$\mathbb{R}^{25,1}$
$\mathbb{R}^{24,1} \times S^1_R$
X^μ , Y
$\mu = 0, \cdots 24$
$Y + 2\pi R \sim Y$
P_F IS QUANTIZED !


$\mathbb{R}^{25,1}$
$\mathbb{R}^{24,1} \times S^1_R$
X^μ , Y
$\mu = 0, \cdots 24$
$Y + 2\pi R \sim Y$
P_Y IS QUANTIZED! $P_Y = \frac{n}{R}$
TWISTED SECTORS! $Y(\sigma + 2\pi) = Y(\sigma) + 2\pi R w$

$$Y = x - 2i\frac{n}{R}\tau + w R\sigma + \sum_{n\neq 0} \frac{i}{n} a_n e^{in(\tau+\sigma)} + \bar{a}\cdots$$

$$\partial Y \qquad a_0 = \frac{n}{R} - \frac{Rw}{2} \qquad L_0 = \frac{a_0^2}{2}$$

$$\bar{\partial} Y \qquad \bar{a}_0 = \frac{n}{R} + \frac{Rw}{2} \qquad \bar{L}_0 = \frac{\bar{a}_0^2}{2}$$

$$L_0 + \bar{L}_0 = \frac{n^2}{R^2} + \frac{R^2 w^2}{4} \qquad L_0 - \bar{L}_0$$

$$Y = x - 2i\frac{n}{R}\tau + w R\sigma + \sum_{n\neq 0} \frac{i}{n} a_n e^{in(\sigma - \tau)} + \bar{a}\ldots$$

$$\partial Y \quad a_0 = \frac{n}{R} - \frac{Rw}{2} \qquad L_0 = \frac{a_0^2}{2}$$

$$\bar{\partial} Y \quad \bar{a}_0 = \frac{n}{R} + \frac{Rw}{2} \qquad \bar{L}_0 = \frac{\bar{a}_0^2}{2}$$

$$L_0 + \bar{L}_0 = \frac{n^2}{R^2} + \frac{R^2 w^2}{4} \qquad L_0 - \bar{L}_0 = -nw$$

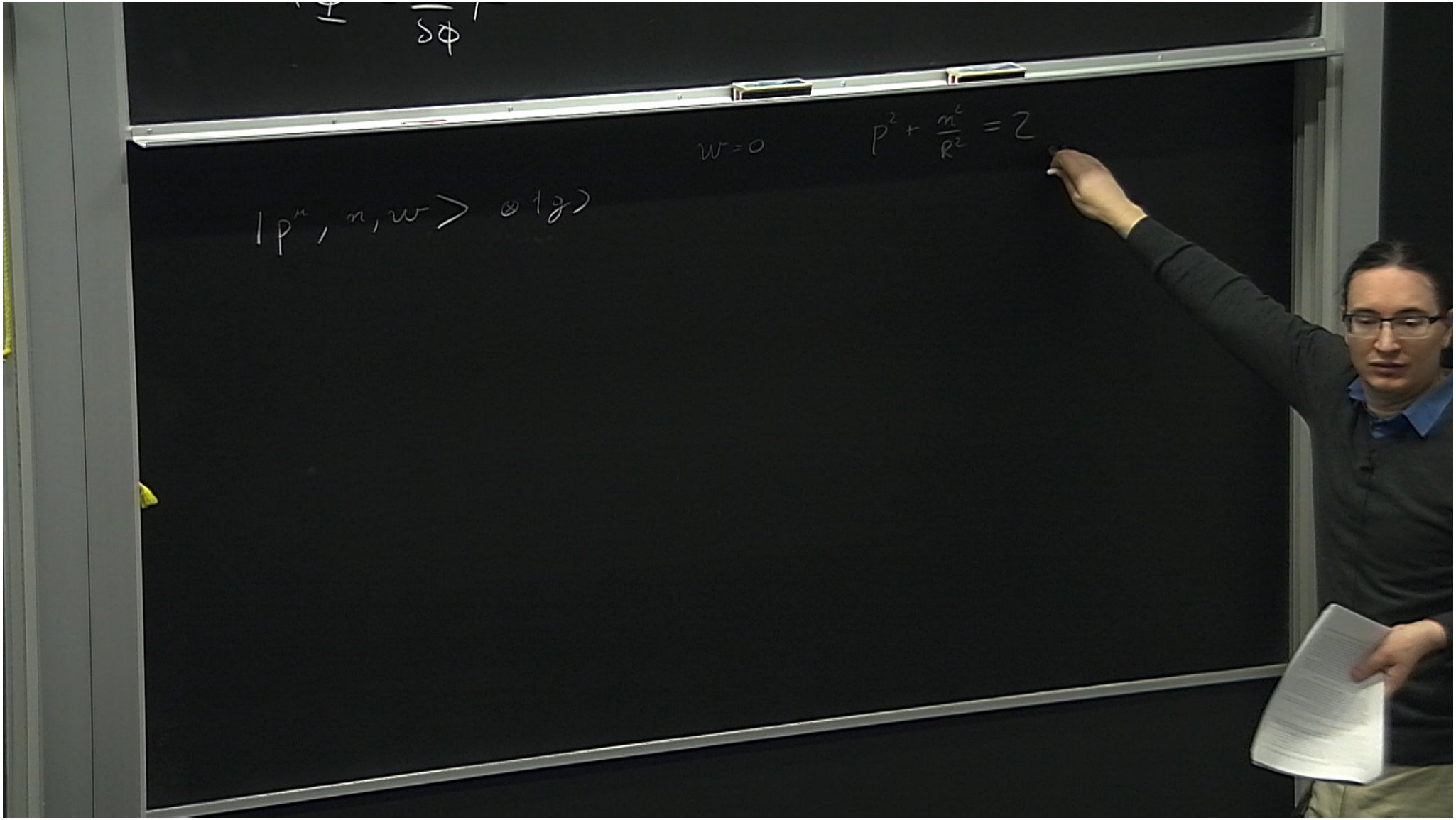
$$Y = x - 2i\frac{n}{R}\tau + wR\sigma + \sum_{n\neq 0} \frac{i}{n} a_n e^{in(\tau-\sigma)} + \bar{a}\ldots$$

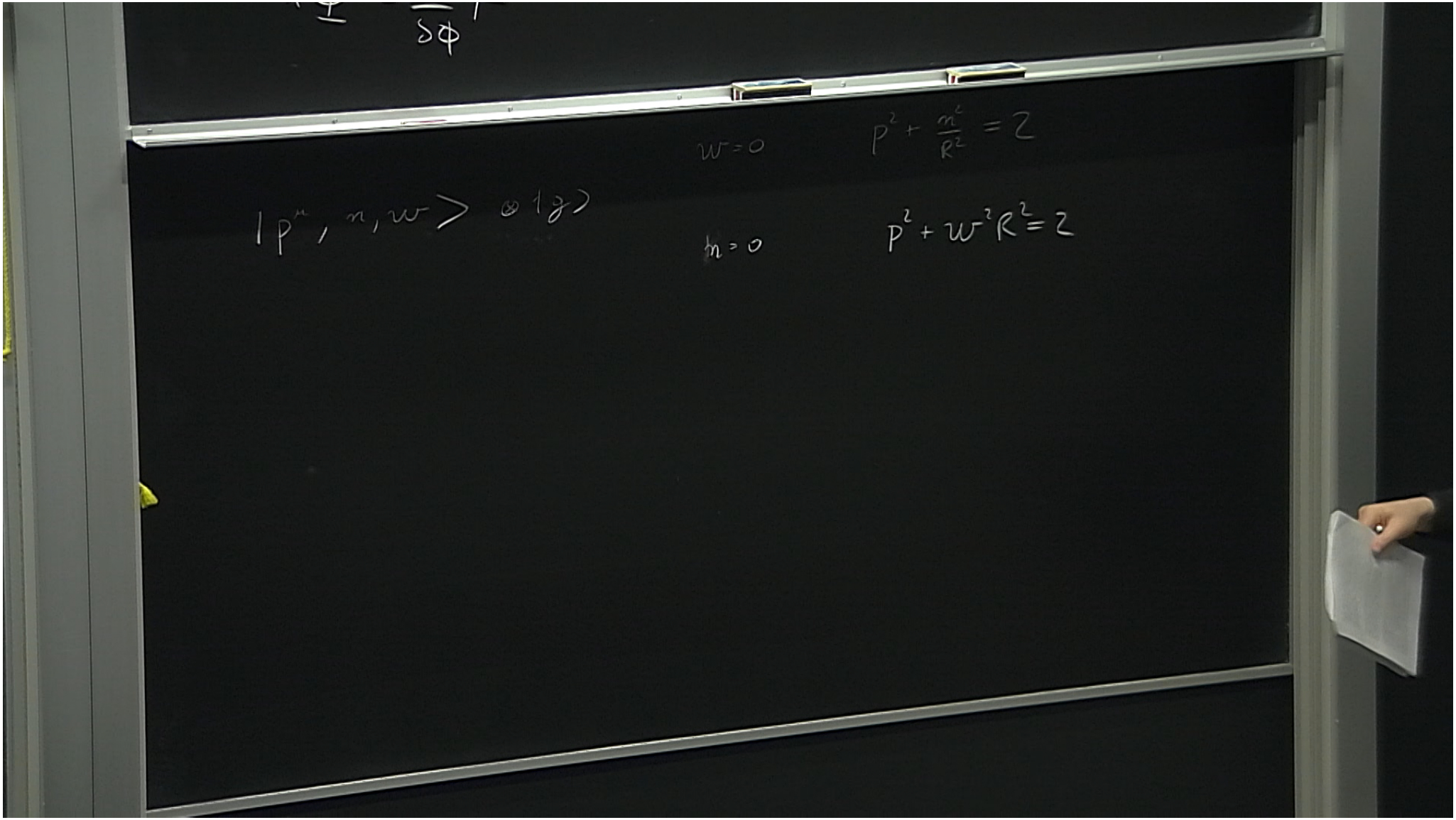
$$\partial Y \quad a_0 = \frac{n}{R} - \frac{Rw}{2} \qquad L_0 = \frac{a_0^2}{2}$$

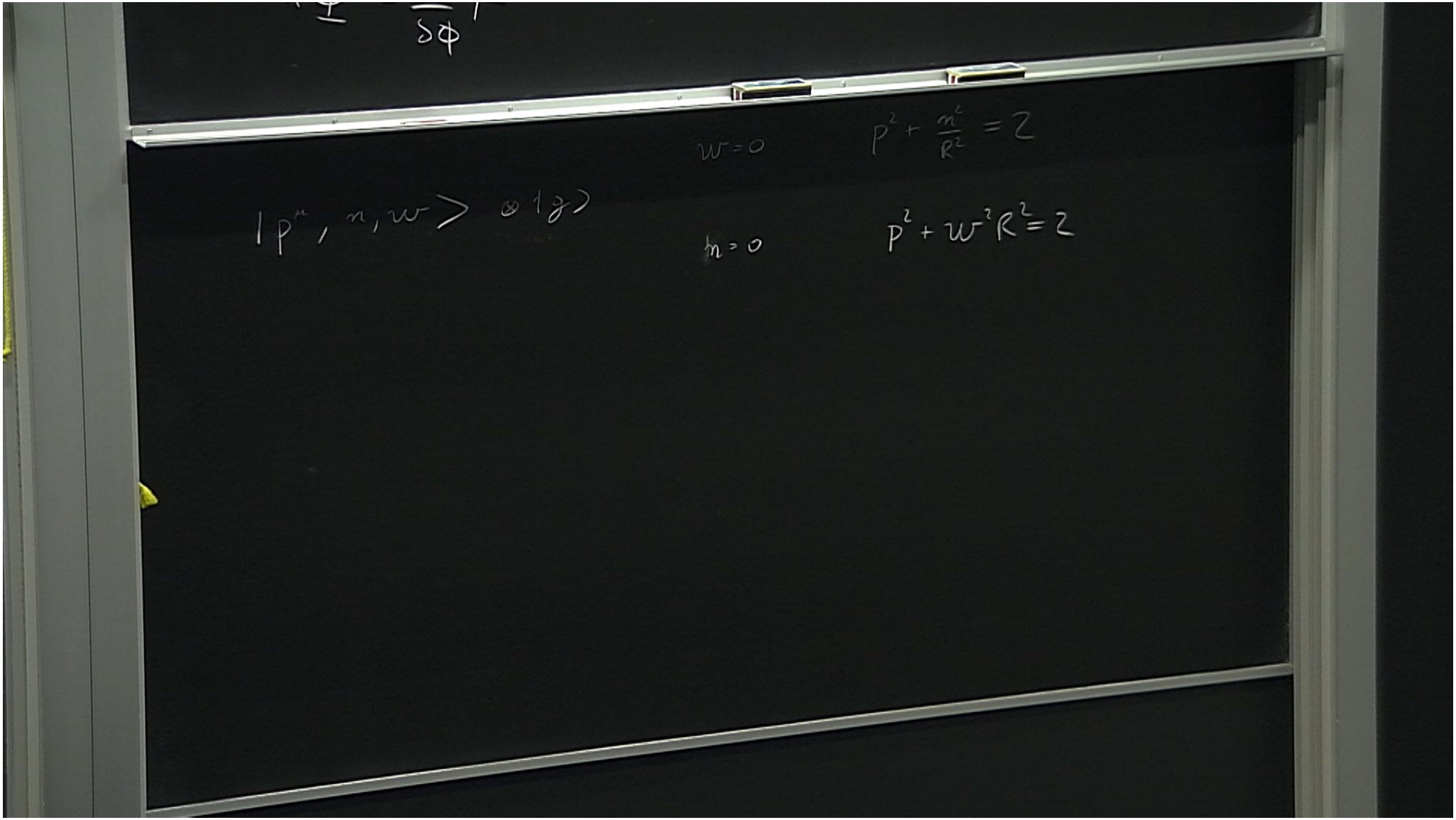
$$\bar{\partial} Y \quad \bar{a}_0 = \frac{n}{R} + \frac{Rw}{2} \qquad \bar{L}_0 = \frac{\bar{a}_0^2}{2}$$

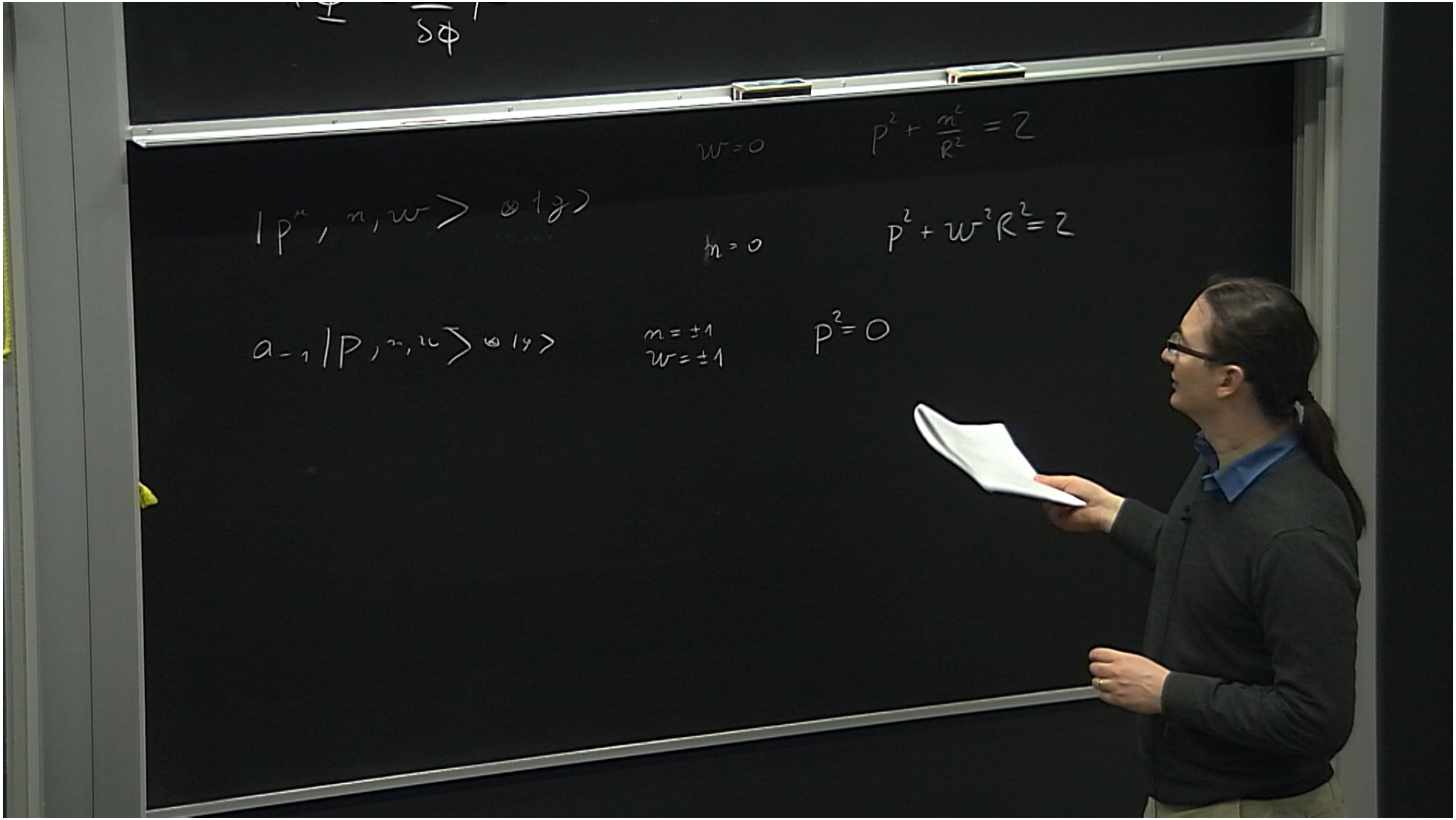
$$L_0 + \bar{L}_0 = \frac{n^2}{R^2} + \frac{R^2 w^2}{4} \qquad L_0 - \bar{L}_0 = -nw$$

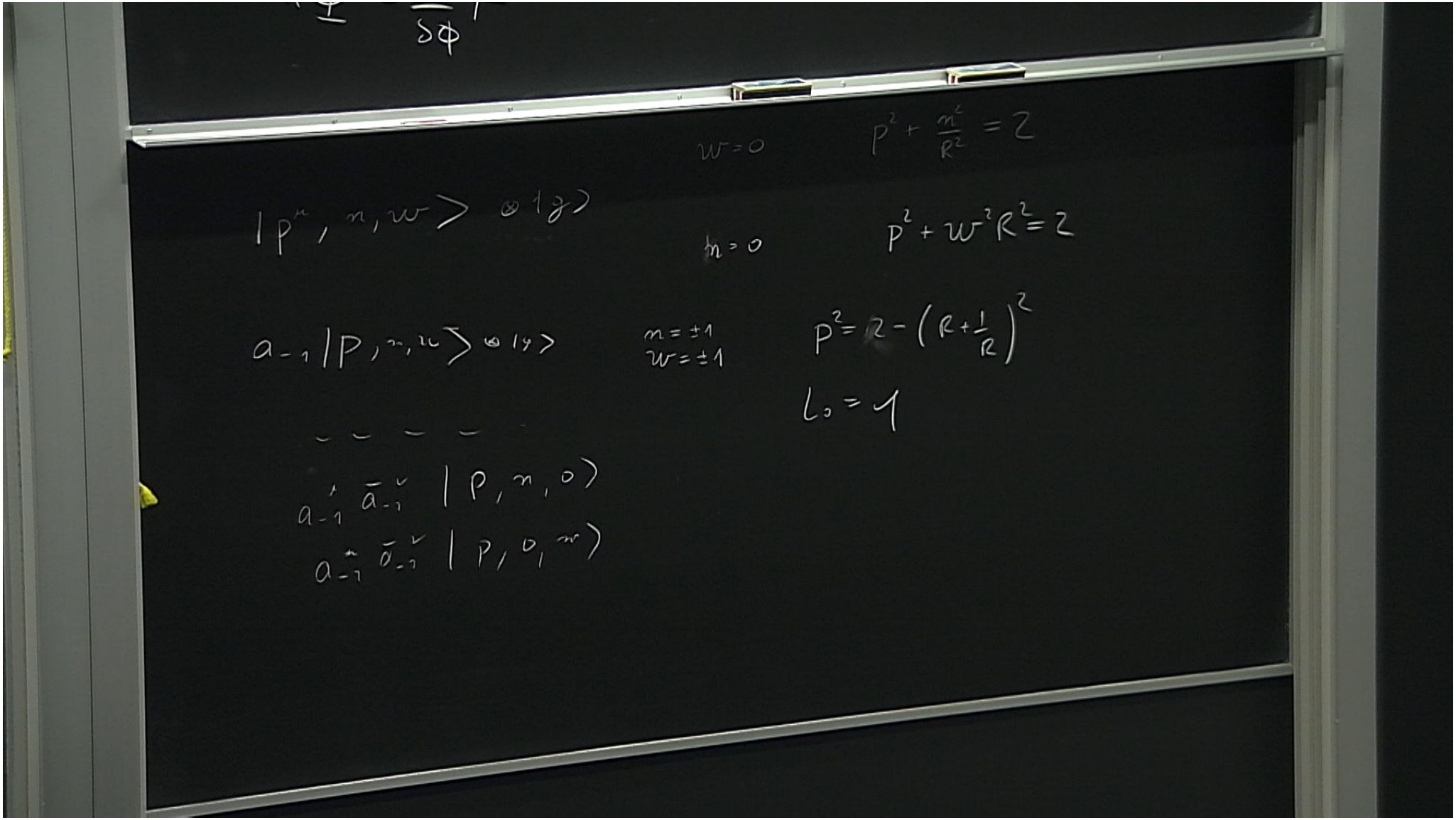
$$|n, w\rangle \quad a_{-1}^\dagger |\,\rangle \ldots$$


$\frac{\delta}{\delta\phi}$
$w=0$
$p^2 + \frac{n^2}{R^2} = 2$
$|p^\mu, n, w\rangle \otimes |g\rangle$


$\frac{\partial}{\partial \phi}$
$w = 0$
$p^2 + \frac{n^2}{R^2} = 2$
$|p^\mu, n, w\rangle \otimes |g\rangle$
$n = 0$
$p^2 + w^2 R^2 = 2$


$\frac{\delta}{\delta\phi}$
$w=0$
$p^2+\frac{n^2}{R^2}=2$
$|p^\mu, n, w\rangle \otimes |g\rangle$
$n=0$
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$\frac{\delta}{\delta\phi}$
$w=0$
$p^2+\frac{n^2}{R^2}=2$
$|p^\mu, n, w\rangle \otimes |g\rangle$
$n=0$
$p^2+w^2R^2=2$
$a_{-1}|p,\ldots\rangle \otimes |\psi\rangle$
$n=\pm 1$
$w=\pm 1$
$p^2=0$


$\frac{\delta}{\delta\phi}$
$w=0$ $\quad p^2+\frac{n^2}{R^2}=2$
$|p^\mu, n, w\rangle \otimes |g\rangle$
$n=0$ $\quad p^2+w^2R^2=2$
$a_{-1}|p,n,w\rangle \otimes |g\rangle$ $\quad n=\pm1$ $\quad w=\pm1$ $\quad p^2=2-\left(R+\frac{1}{R}\right)^2$
$L_0=1$
$a_{-1}^\dagger \bar{a}_{-1}^\nu |p,n,0\rangle$
$a_{-1}^\dagger \bar{a}_{-1}^\nu |p,0,w\rangle$

$$Y = x - 2i\frac{n}{R}\tau + w R\sigma + \sum_{n\neq 0}$$

$$\partial Y \qquad a_0 = \frac{n}{R} - \frac{Rw}{2} \qquad L_0 = \frac{a_0^2}{2}$$

$$\bar{\partial} Y \qquad \bar{a}_0 = \frac{n}{R} + \frac{Rw}{2} \qquad \bar{L}_0 = \frac{\bar{a}_0^2}{2}$$

$$L_0 + \bar{L}_0 = \frac{n^2}{R^2} + \frac{R^2 w^2}{4} \qquad L_0 - \bar{L}_0 = -nw$$

$$\tilde{Y}(\sigma,\tau) = \tilde{x} - 2\frac{n}{R}\sigma + iwR\tau \qquad \sum \cdots a_n \quad -\bar{a}_n$$

$$\tilde{Y} \to \tilde{Y} + \frac{2\pi}{R}$$

$$Y = x - 2i\frac{n}{R}\tau + w R \sigma + \sum_{n\neq 0}$$

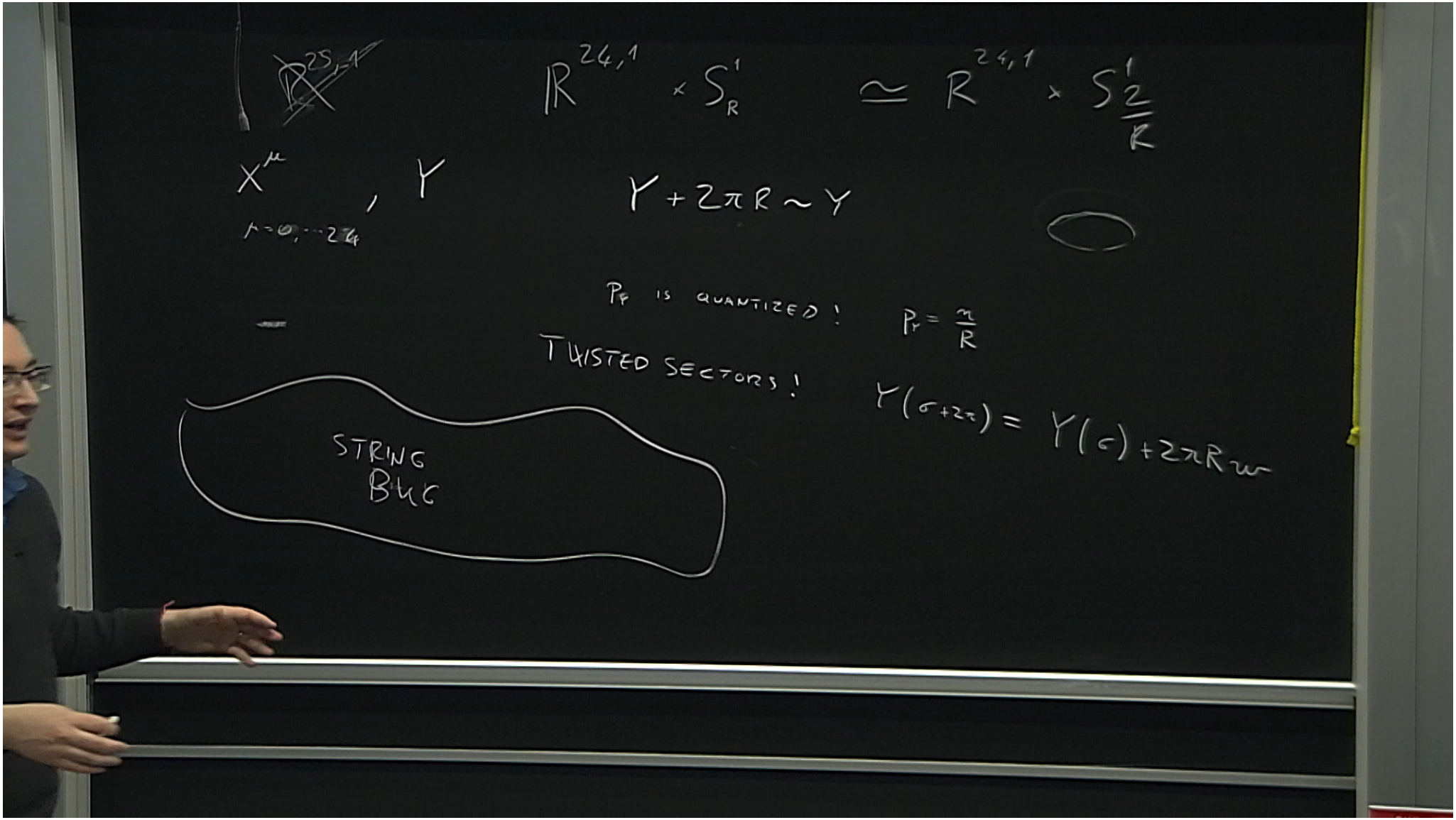
$$\partial Y \quad a_0 = \frac{n}{R} - \frac{Rw}{2} \qquad L_0 = \frac{a_0^2}{2}$$

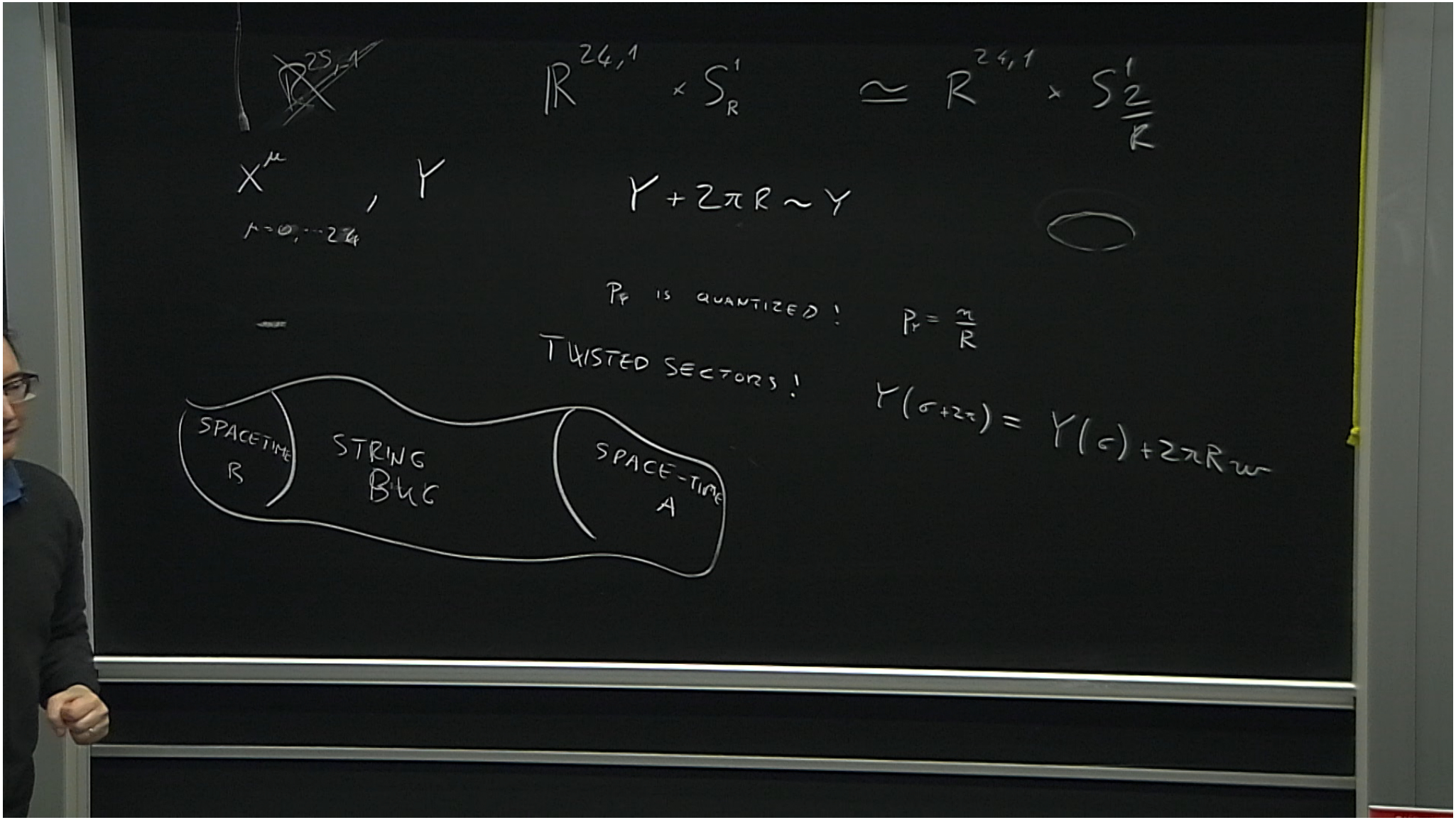
$$\bar{\partial} Y \quad \bar{a}_0 = \frac{n}{R} + \frac{Rw}{2} \qquad \bar{L}_0 = \frac{\bar{a}_0^2}{2}$$

$$L_0 + \bar{L}_0 = \frac{n^2}{R^2} + \frac{R^2 w^2}{4} \qquad L_0 - \bar{L}_0 = -nw$$

$$\tilde{Y}(\sigma,\tau) = \tilde{x} - 2\frac{n}{R}\sigma + iwR\tau \qquad \sum \cdots a_n \quad -\bar{a}_n$$

$$\tilde{Y} \to \tilde{Y} + \frac{4\pi}{R}$$


$\mathbb{R}^{25,1}$
$\mathbb{R}^{24,1} \times S^1_R \simeq \mathbb{R}^{24,1} \times S^1_{\frac{2}{R}}$
X^μ , Y
$\mu = 0, \cdots 24$
$Y + 2\pi R \sim Y$
P_Y IS QUANTIZED! $P_Y = \frac{n}{R}$
TWISTED SECTORS!
$Y(\sigma + 2\pi) = Y(\sigma) + 2\pi R w$
STRING BLOB


$\mathbb{R}^{25,1}$
$\mathbb{R}^{24,1} \times S^1_R \simeq \mathbb{R}^{24,1} \times S^1_{\frac{2}{R}}$
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TWISTED SECTORS!
$Y(\sigma + 2\pi) = Y(\sigma) + 2\pi R w$
SPACETIME B
STRING BLG
SPACE-TIME A

$\mathbb{R}^{25,1}$

$\mathbb{R}^{24,1} \times S^1_{R l_s} \simeq \mathbb{R}^{24,1} \times S^1_{\frac{2 l_s}{R}}$

X^μ , Y

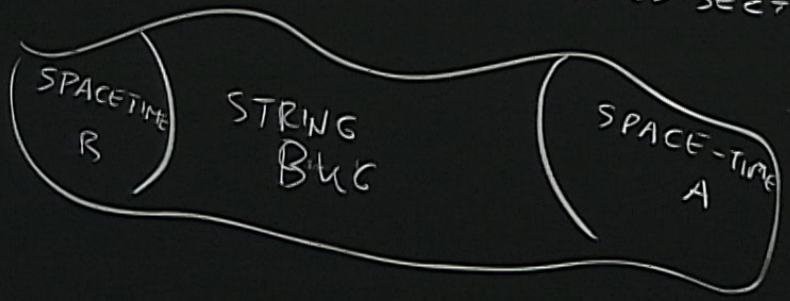
$\mu = 0, \dots 24$

$Y + 2\pi R \sim Y$

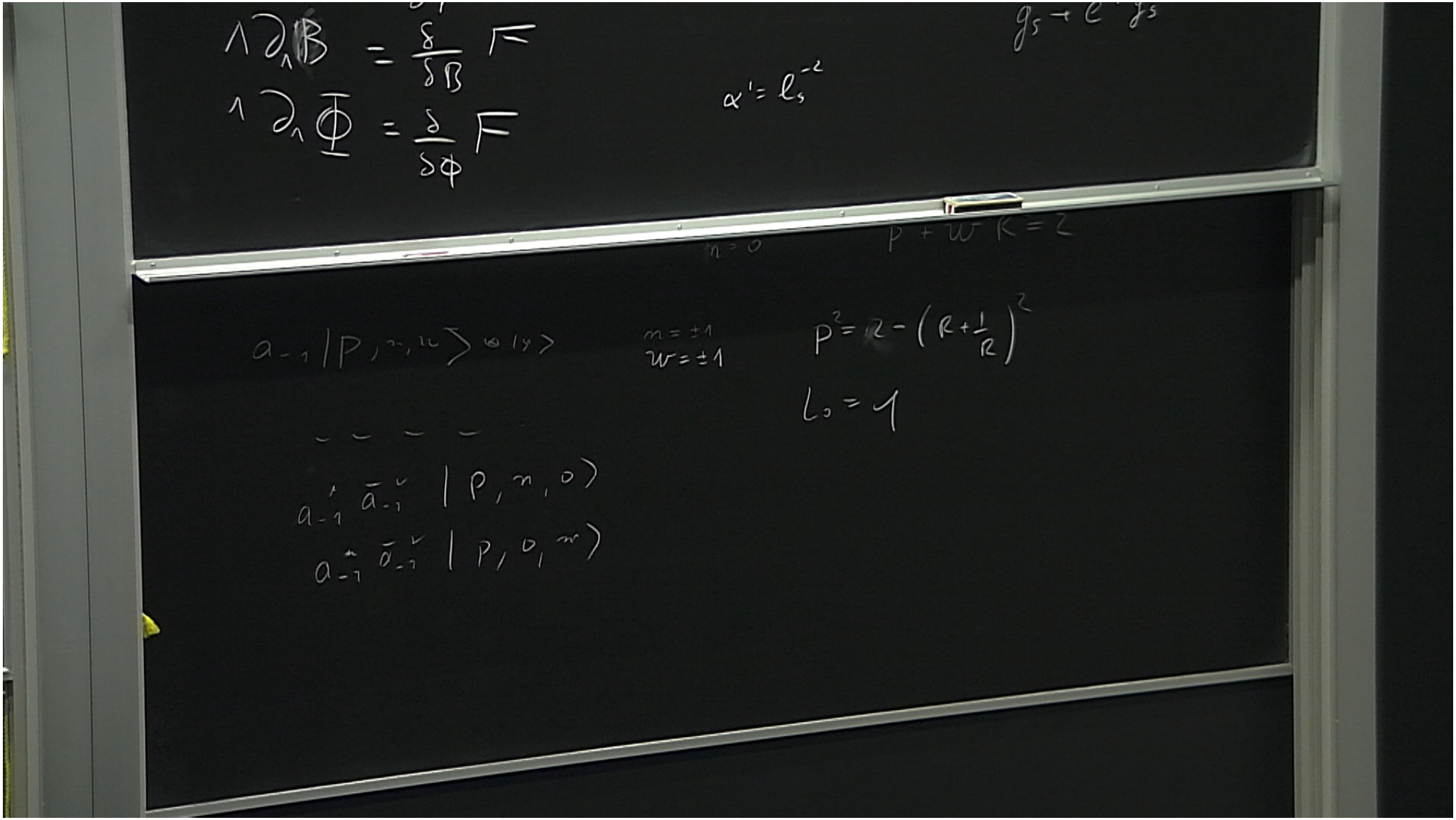
P_Y IS QUANTIZED! $P_Y = \frac{n}{R}$

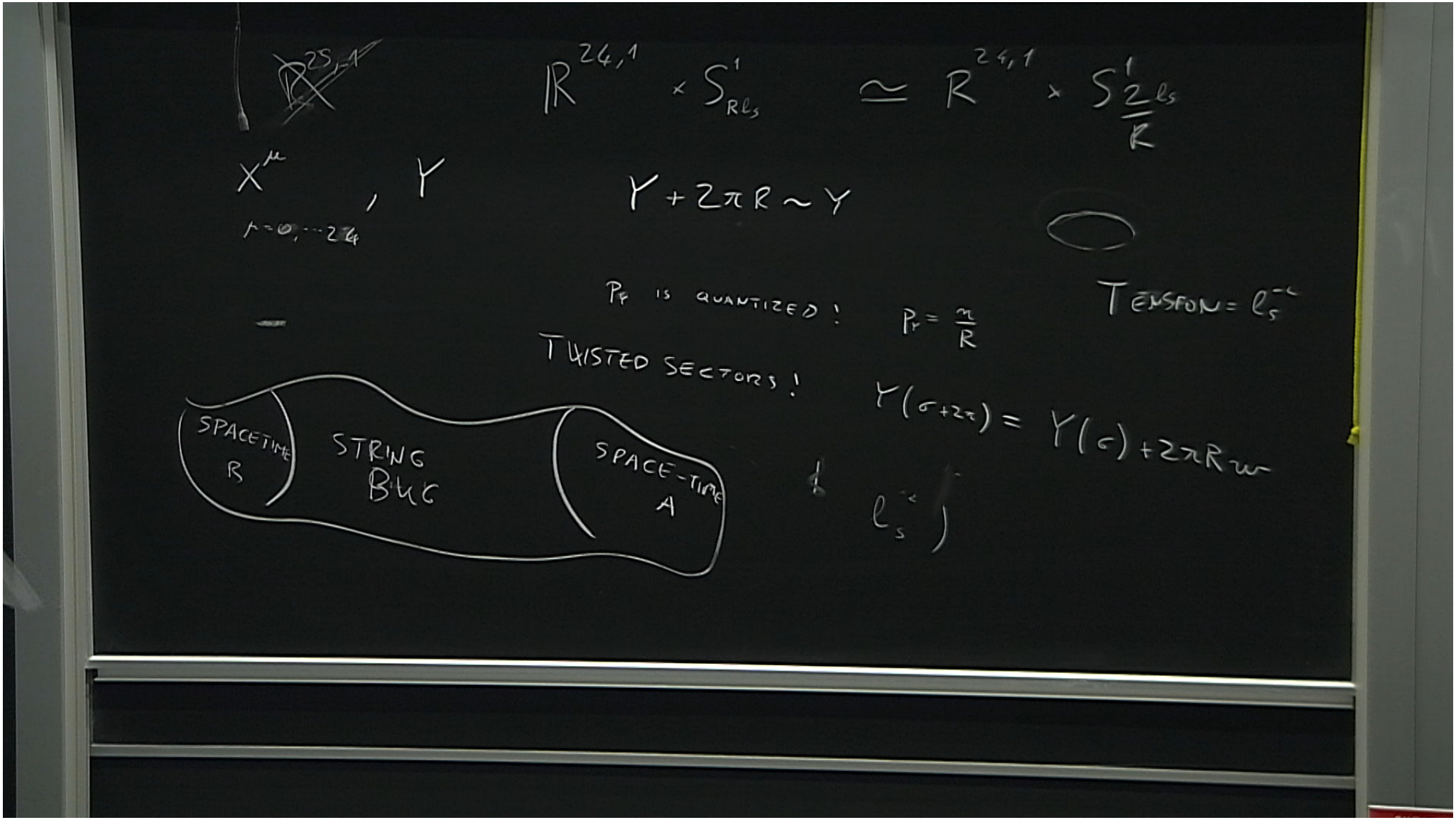
TWISTED SECTORS!

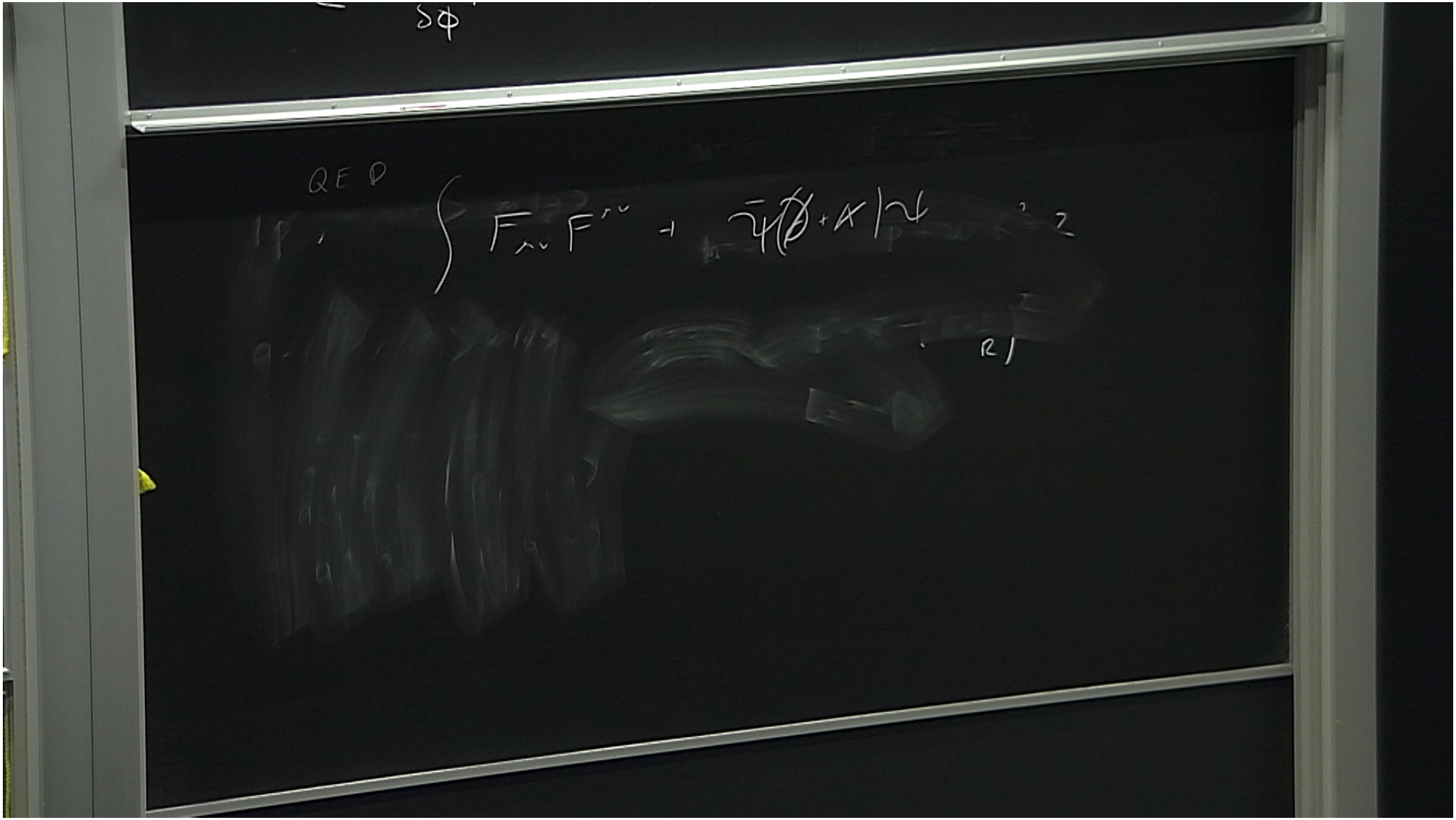
$Y(\sigma + 2\pi) = Y(\sigma) + 2\pi R w$

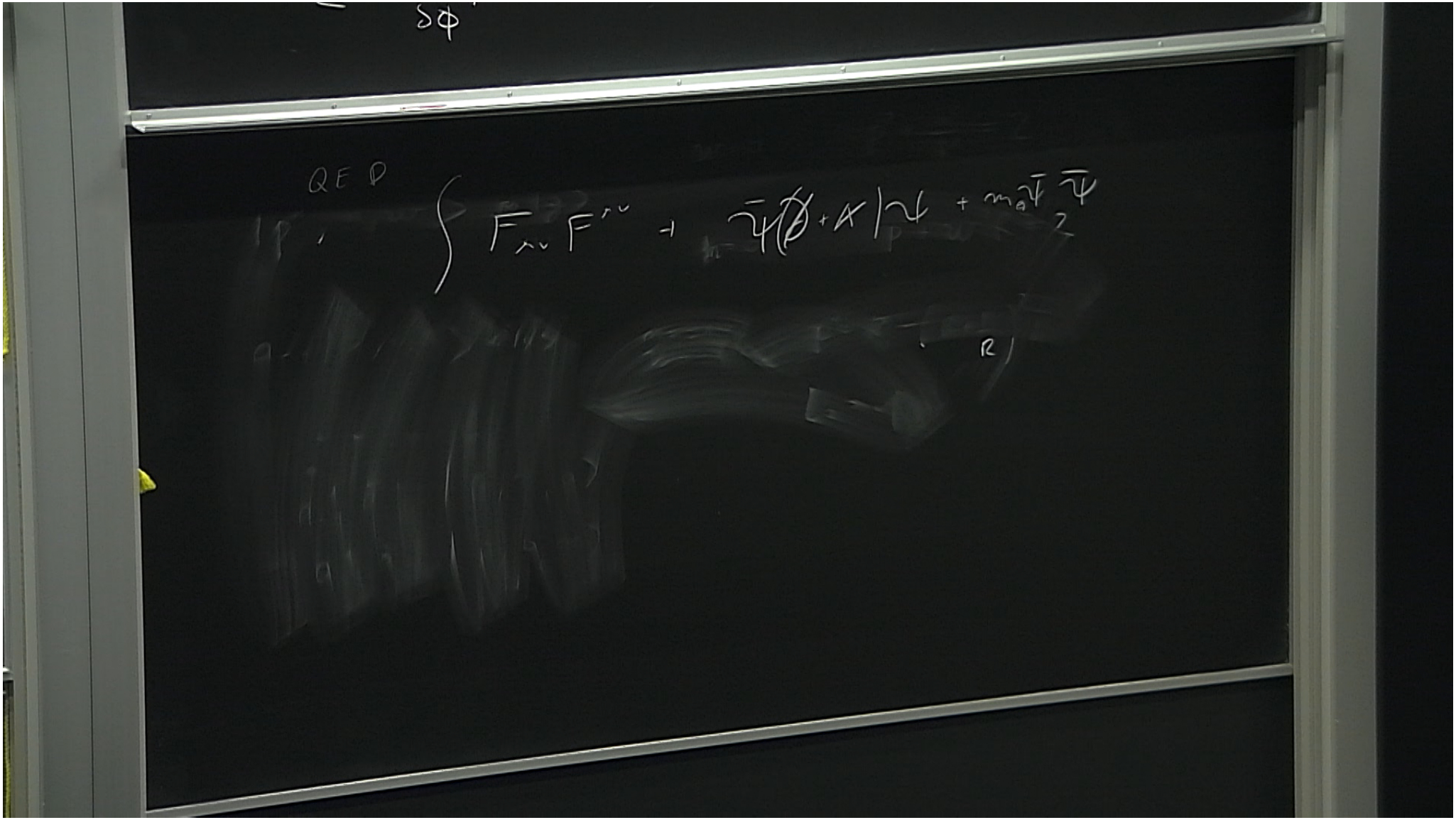


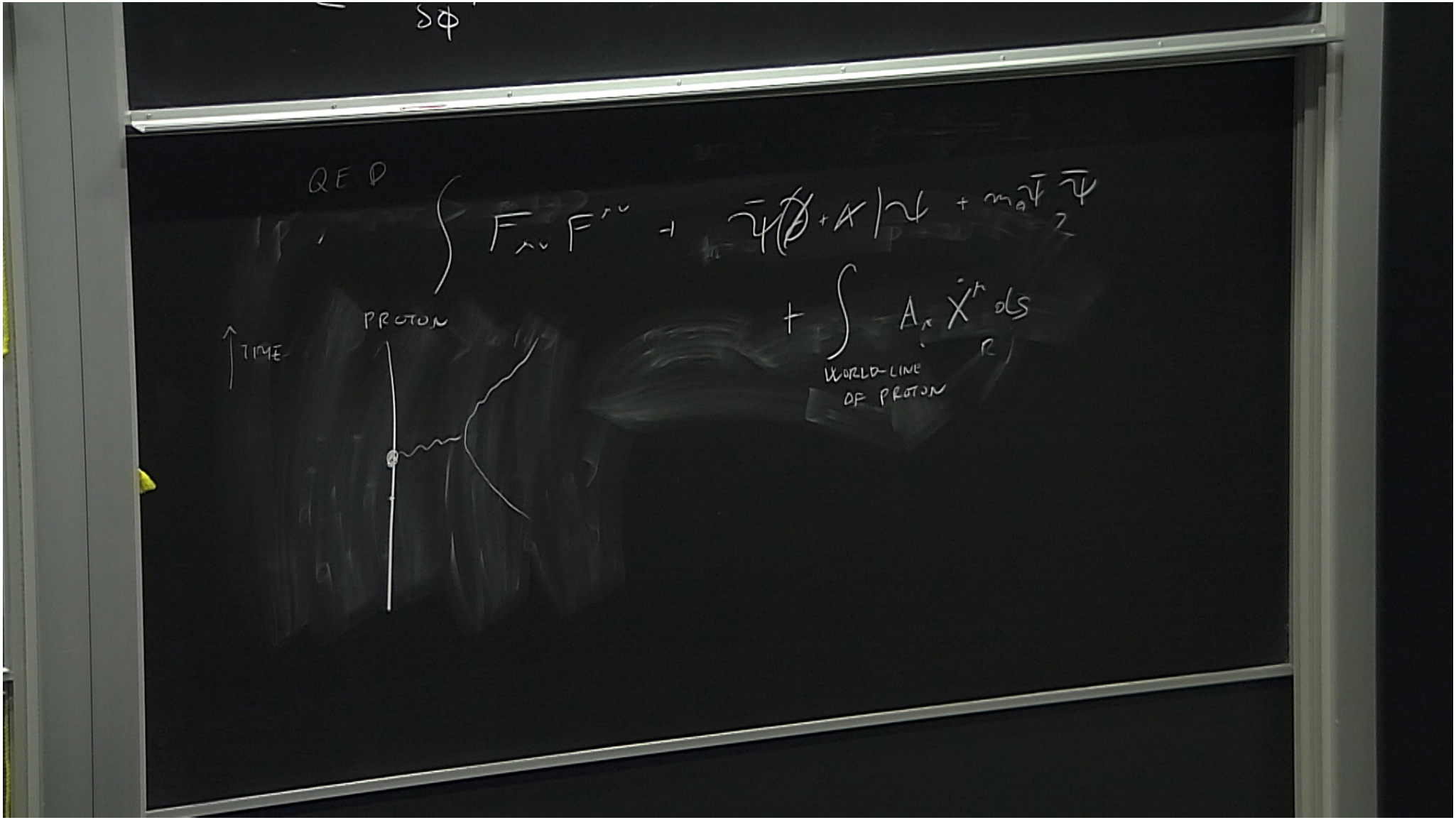
ℓ_s !

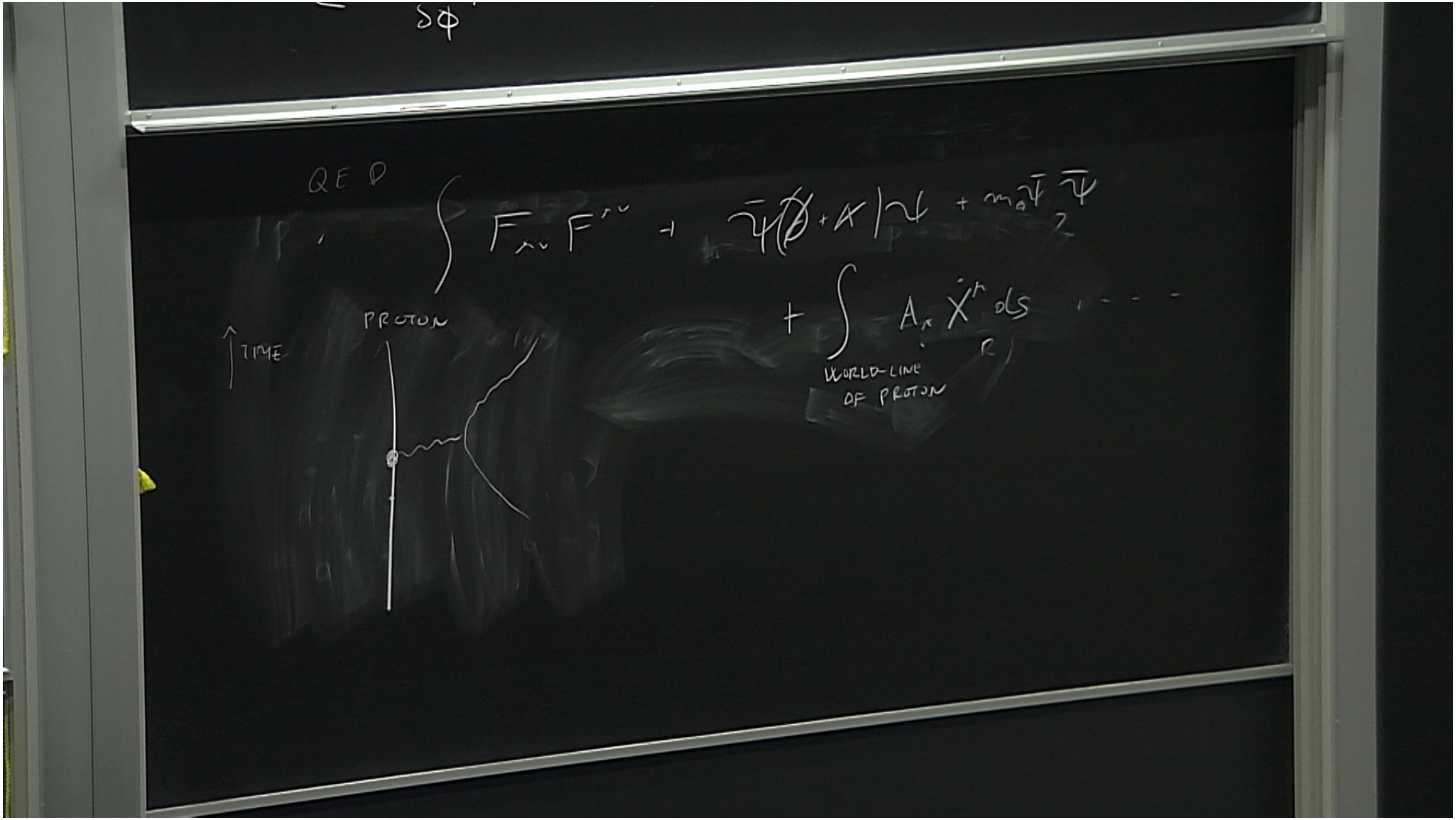

$\partial_1 B = \frac{\delta}{\delta B} F$
$\partial_1 \Phi = \frac{\delta}{\delta \Phi} F$
$\alpha' = \ell_s^{-2}$
$g_s \to e^{\phi} g_s$
$n = 0$
$p + w \; K = 2$
$a_{-1} |p, n, 1\rangle \otimes |\psi\rangle$
$n = \pm 1$
$w = \pm 1$
$p^2 = 2 - \left(R + \frac{1}{R}\right)^2$
$L_0 = 1$
$a_{-1}^{\dagger} \bar{a}_{-1}^{\nu} |p, n, 0\rangle$
$a_{-1}^{\dagger} \bar{a}_{-1}^{\nu} |p, 0, m\rangle$

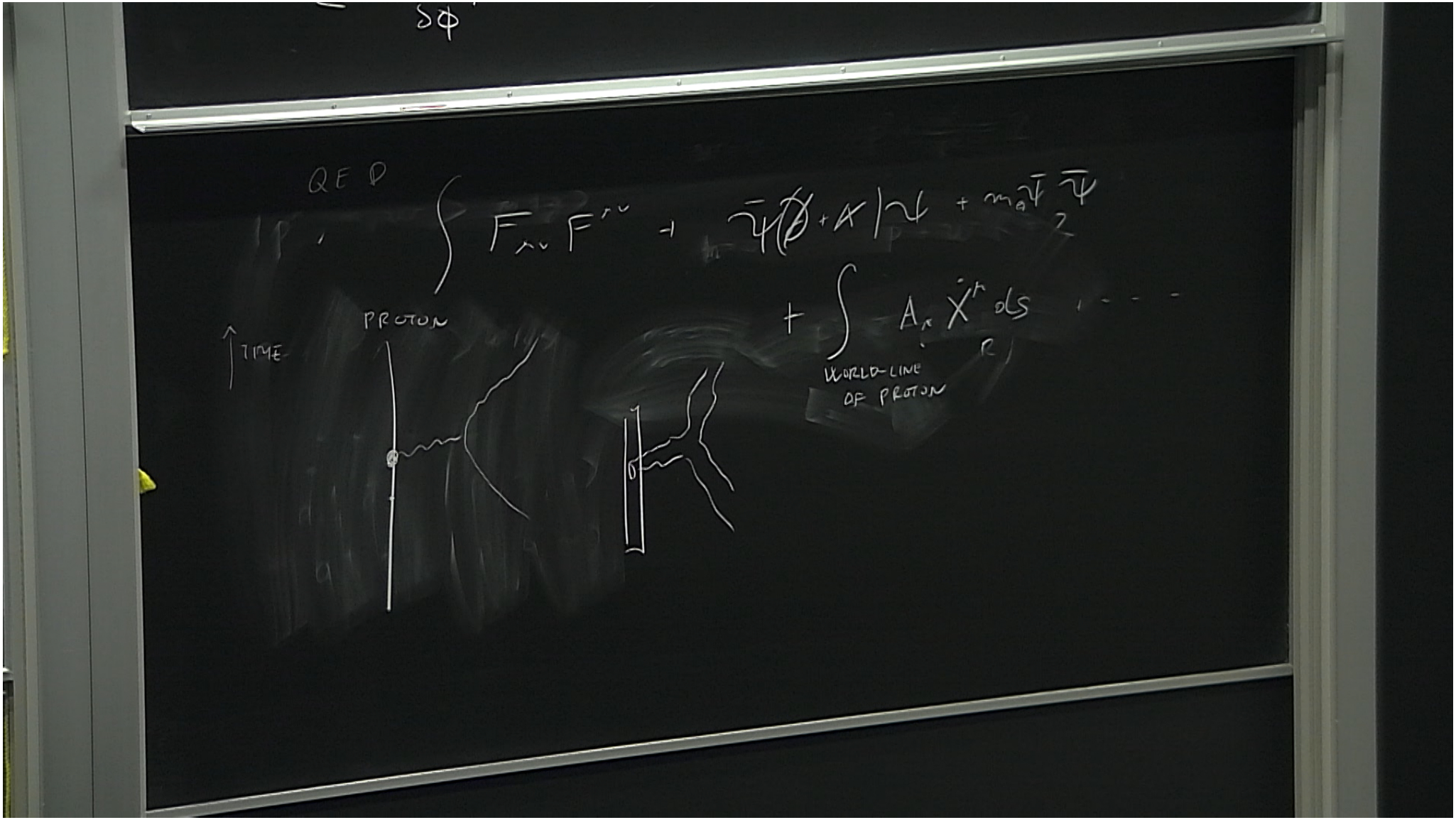

$\mathbb{R}^{25,1}$
$\mathbb{R}^{24,1} \times S^1_{R\ell_s} \simeq \mathbb{R}^{24,1} \times S^1_{\frac{2\ell_s}{R}}$
X^μ , Y
$\mu = 0, \dots 24$
$Y + 2\pi R \sim Y$
P_Y IS QUANTIZED! $P_Y = \frac{n}{R}$
TENSION $= \ell_s^{-2}$
TWISTED SECTORS!
$Y(\sigma + 2\pi) = Y(\sigma) + 2\pi R \cdot w$
SPACETIME R
STRING BLG
SPACE-TIME A
ℓ_s


QED


QED
$\int F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(\partial + A)\psi + m_0 \bar{\psi}\psi$


QED
PROTON
TIME
WORLD-LINE
OF PROTON


QED
PROTON
TIME
WORLD-LINE
OF PROTON


QED
PROTON
TIME
WORLD-LINE
OF PROTON