Title: PSI 2016/2017 String Theory (Review) - Lecture 9

Date: Mar 06, 2017 10:15 AM

URL: http://pirsa.org/17030025

Abstract:

$$\mathcal{A}_n = \sum_g g_s^{2g-2+n} \int_{\mathcal{M}_{g,n}} \cdots\cdots$$

$$\mathcal{A}_3 = g_s \mathcal{A}_{3,0} + g_s^3 \mathcal{A}_{3,1} \cdots\cdots$$

$$\mathcal{A}_n = \sum_g g_s^{2g-2+n} \int_{\mathcal{M}_{g,n}} \cdots$$

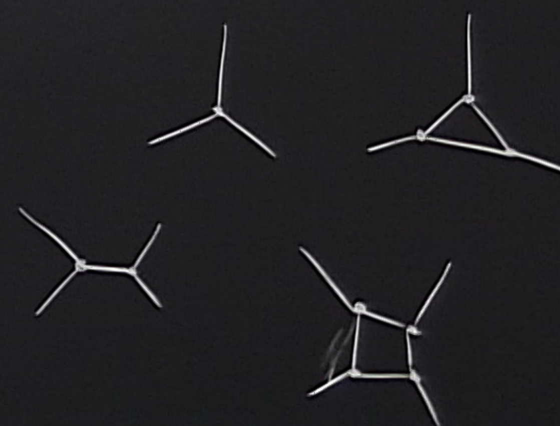
$$\mathcal{A}_3 = g_s \mathcal{A}_{3,0} + g_s^3 \mathcal{A}_{3,1} \cdots$$

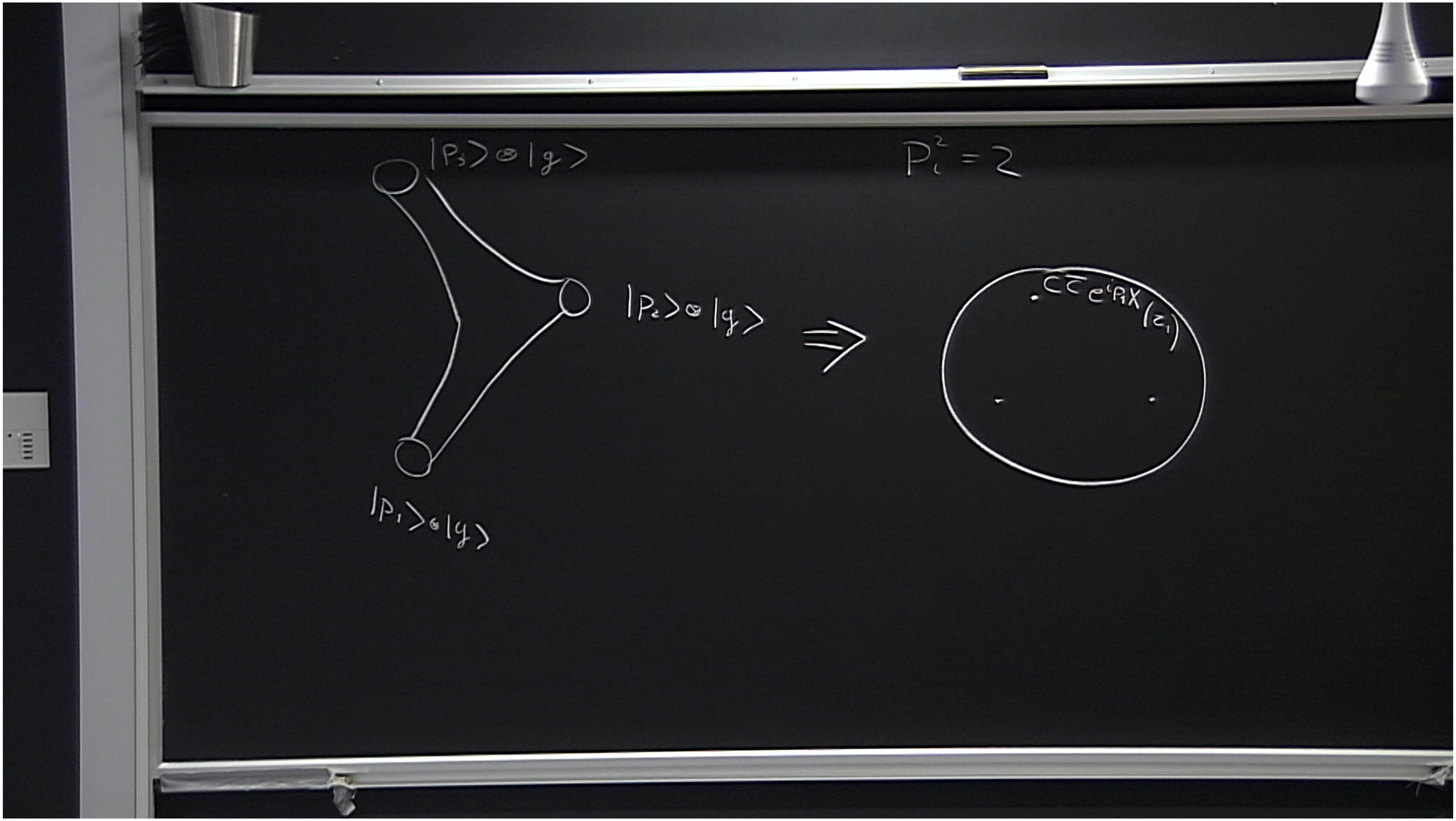
$$\mathcal{A}_4 = g_s^2 \mathcal{A}_{4,0} + \cdots$$

$$A_n = \sum_g g_s^{2g-2+n} \int_{\mathcal{M}_{g,n}} \cdots$$

$$A_3 = g_s A_{3,0} + g_s^3 A_{3,1} \cdots$$

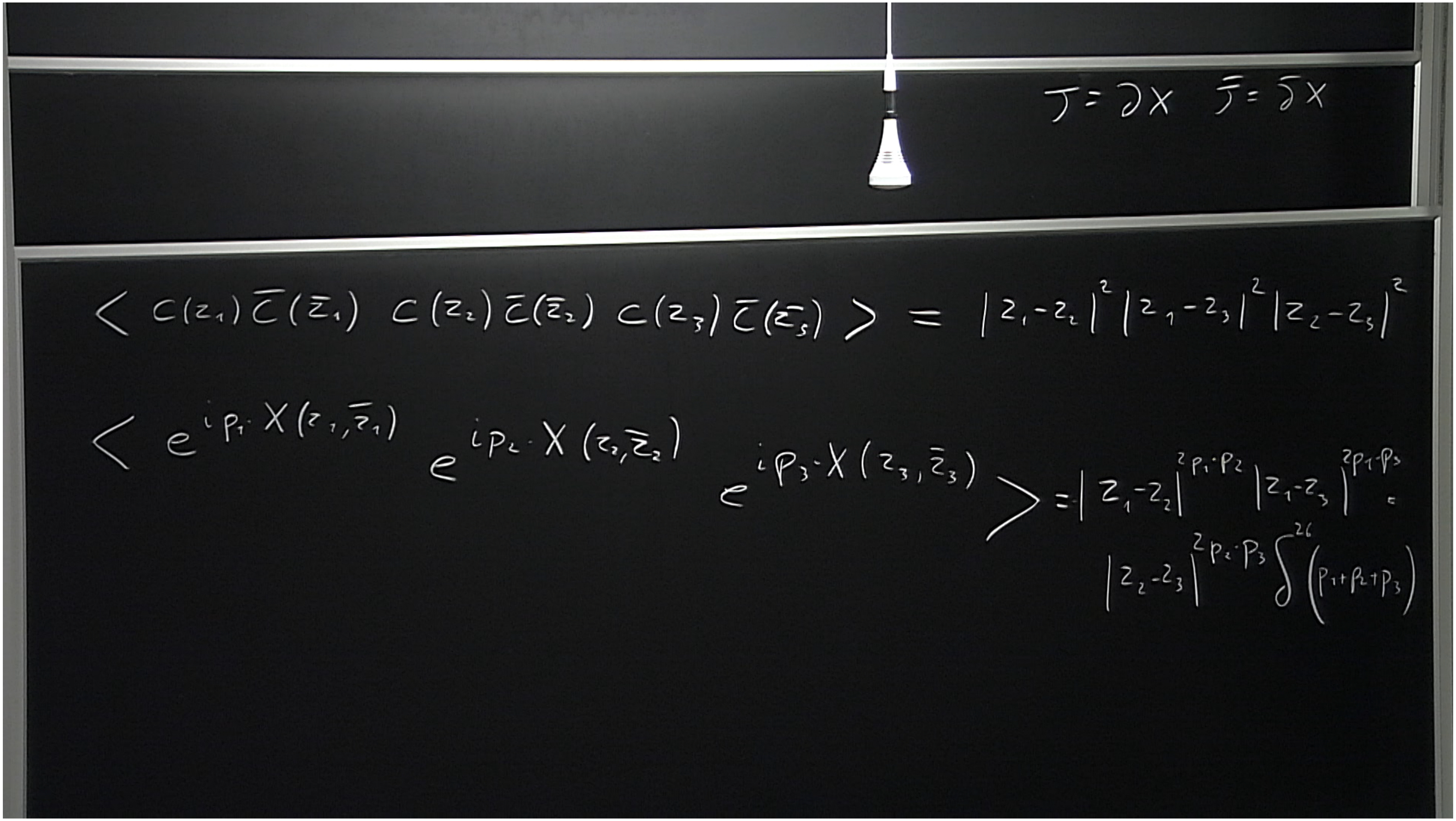
$$A_4 = g_s^2 A_{4,0} + g_s^4 A_{4,1} + \cdots$$

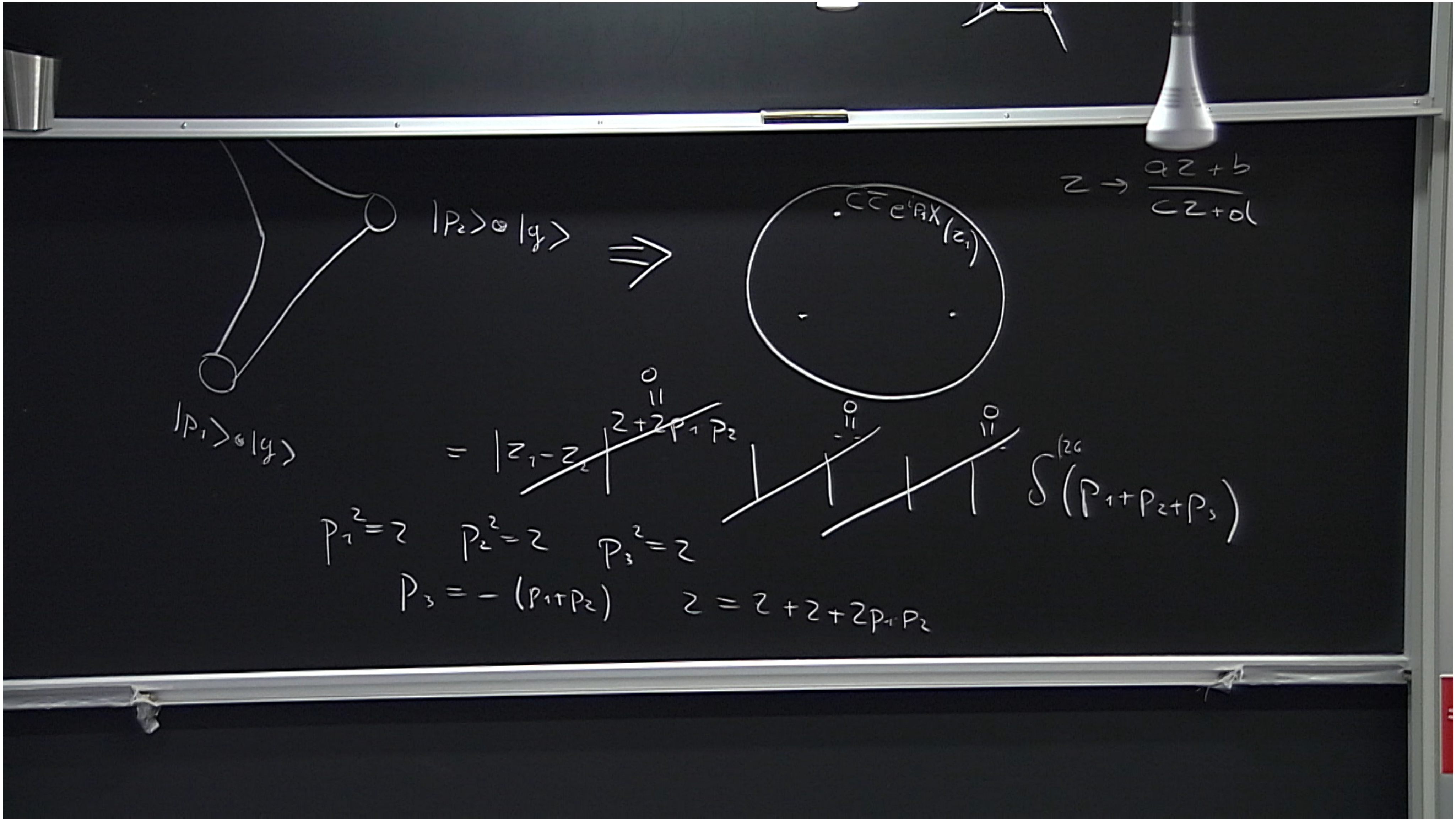



|P3> ⊗ |g>
Pi² = 2
|P2> ⊗ |g>
⇒
c c̄ e^{ip1X}(z1)
|P1> ⊗ |g>

$$\langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3)\rangle = |z_1-z_2|^2\,|z_1-z_3|^2\,|z_2-z_3|^2$$

$$\langle e^{ip_1\cdot X(z_1,\bar{z}_1)}\, e^{ip_2\cdot X(z_2,\bar{z}_2)}\, e^{ip_3\cdot X(z_3,\bar{z}_3)}\rangle = |z_1-z_2|^{2p_1\cdot p_2}\,|z_1-z_3|^{2p_1\cdot p_3}\,|z_2-z_3|^{2p_2\cdot p_3}\,\delta^{26}(p_1+p_2+p_3)$$

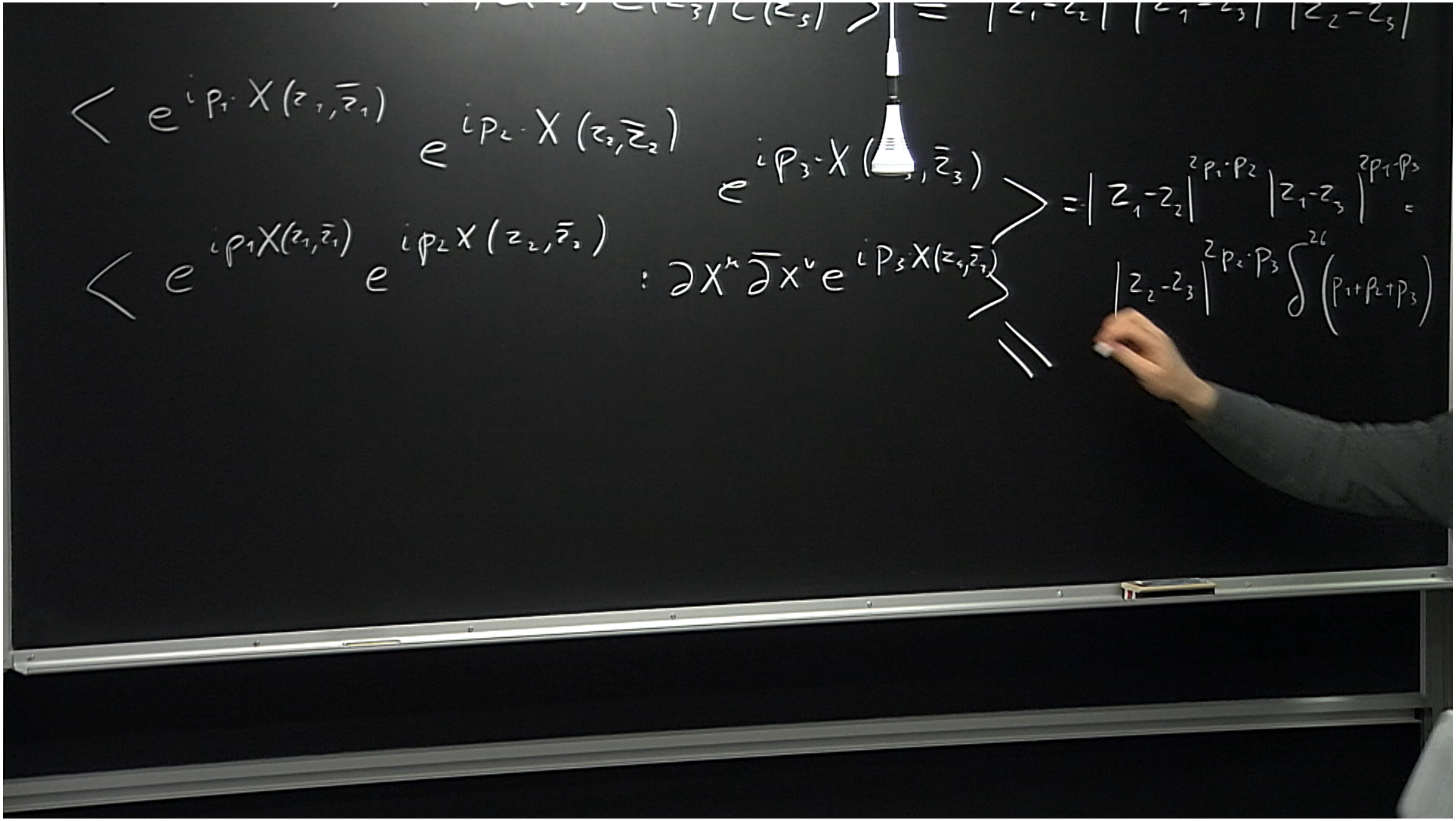

$J = \partial X \quad \bar{J} = \bar{\partial} X$
$\langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3) \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$
$\langle e^{ip_1\cdot X(z_1,\bar{z}_1)}\, e^{ip_2\cdot X(z_2,\bar{z}_2)}\, e^{ip_3\cdot X(z_3,\bar{z}_3)} \rangle = |z_1 - z_2|^{2p_1\cdot p_2} |z_1 - z_3|^{2p_1\cdot p_3} \cdot |z_2 - z_3|^{2p_2\cdot p_3} \delta^{26}(p_1+p_2+p_3)$


|p₂⟩ ⊗ |g⟩
|p₁⟩ ⊗ |g⟩
c c̄ e^{ip₃X}(z₁)
z → (az + b)/(cz + d)
0 = 2 + 2p₁·p₂
= |z₁ − z₂|^{2+2p₁·p₂} | | | | | δ^{26}(p₁+p₂+p₃)
p₁² = 2   p₂² = 2   p₃² = 2
p₃ = −(p₁+p₂)   2 = 2 + 2 + 2p₁·p₂

$$\langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3)\rangle = |z_1-z_2|^2\,|z_1-z_3|^2\,|z_2-z_3|^2$$

$$\langle e^{ip_1\cdot X(z_1,\bar{z}_1)}\, e^{ip_2\cdot X(z_2,\bar{z}_2)}\, e^{ip_3\cdot X(z_3,\bar{z}_3)}\rangle = |z_1-z_2|^{2p_1\cdot p_2}\,|z_1-z_3|^{2p_1\cdot p_3}\,|z_2-z_3|^{2p_2\cdot p_3}\,\delta^{26}(p_1+p_2+p_3)$$

$$\langle e^{ip_1 X(z_1,\bar{z}_1)}\, e^{ip_2 X(z_2,\bar{z}_2)}$$


$\langle e^{ip_1\cdot X(z_1,\bar z_1)} e^{ip_2\cdot X(z_2,\bar z_2)} e^{ip_3\cdot X(z_3,\bar z_3)} \rangle = |z_1-z_2|^{2p_1\cdot p_2} |z_1-z_3|^{2p_1\cdot p_3} |z_2-z_3|^{2p_2\cdot p_3} \delta^{26}(p_1+p_2+p_3)$
$\langle e^{ip_1 X(z_1,\bar z_1)} e^{ip_2 X(z_2,\bar z_2)} :\partial X^\mu \bar\partial X^\nu e^{ip_3\cdot X(z_3,\bar z_3)} \rangle =$

$$\langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3)\rangle = |z_1-z_2|^2 |z_1-z_3|^2 |z_2-z_3|^2$$

$$\langle e^{ip_1\cdot X(z_1,\bar{z}_1)}\, e^{ip_2\cdot X(z_2,\bar{z}_2)}\, e^{ip_3\cdot X(z_3,\bar{z}_3)}\rangle = |z_1-z_2|^{2p_1\cdot p_2} |z_1-z_3|^{2p_1\cdot p_3} |z_2-z_3|^{2p_2\cdot p_3}\, \delta^{26}(p_1+p_2+p_3)$$

$$\langle e^{ip_1 X(z_1,\bar{z}_1)}\, e^{ip_2 X(z_2,\bar{z}_2)}\, \epsilon_{\mu\nu} :\partial X^\mu \bar{\partial} X^\nu e^{ip_3 X(z_3,\bar{z}_3)}:\rangle =$$

$$= \epsilon_{\mu\nu}\left(\frac{p_1^\mu}{z_1-z_3} + \frac{p_2^\mu}{z_2-z_3}\right)\left(\frac{p_1^\nu}{\bar{z}_1-\bar{z}_3} + \frac{p_2^\nu}{\bar{z}_2-\bar{z}_3}\right)|z_1-z_2|^{2p_1\cdot p_2}\, |\quad|\quad|\quad|\, \delta(\quad)$$

$|p_1\rangle \otimes |g\rangle$

$|p_2\rangle \otimes |g\rangle$

$\epsilon_{\mu\nu}\, a^{\mu}_{-1}\, \bar{a}^{\nu}_{-1}\, |p_3) \otimes |g\rangle$

$\sum p_i = 0$

$p_1^2 = 2$

$p_2^2 = 2$

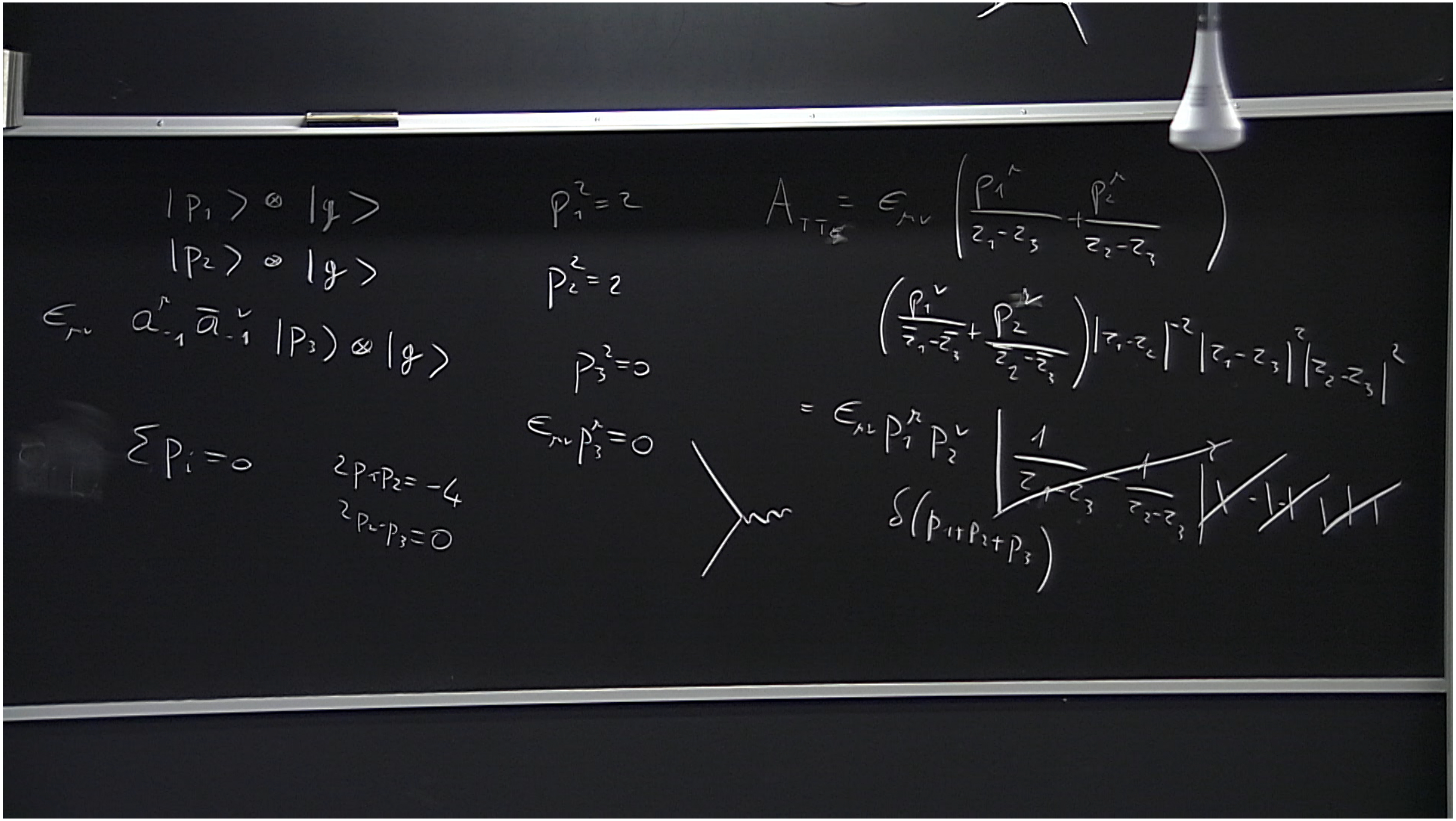
$p_3^2 = 0$

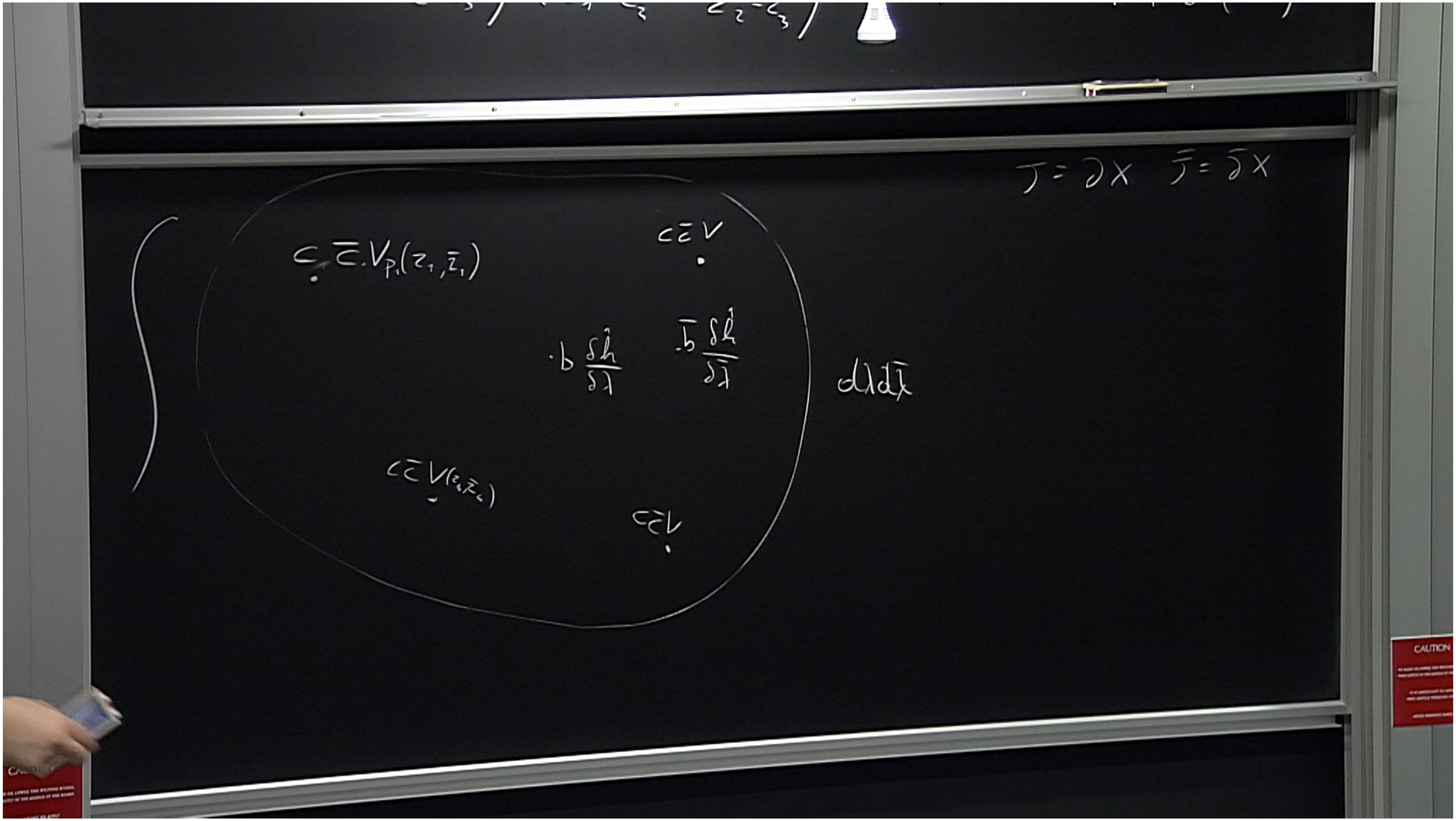
$\epsilon_{\mu\nu}\, p_3^{\mu} = 0$

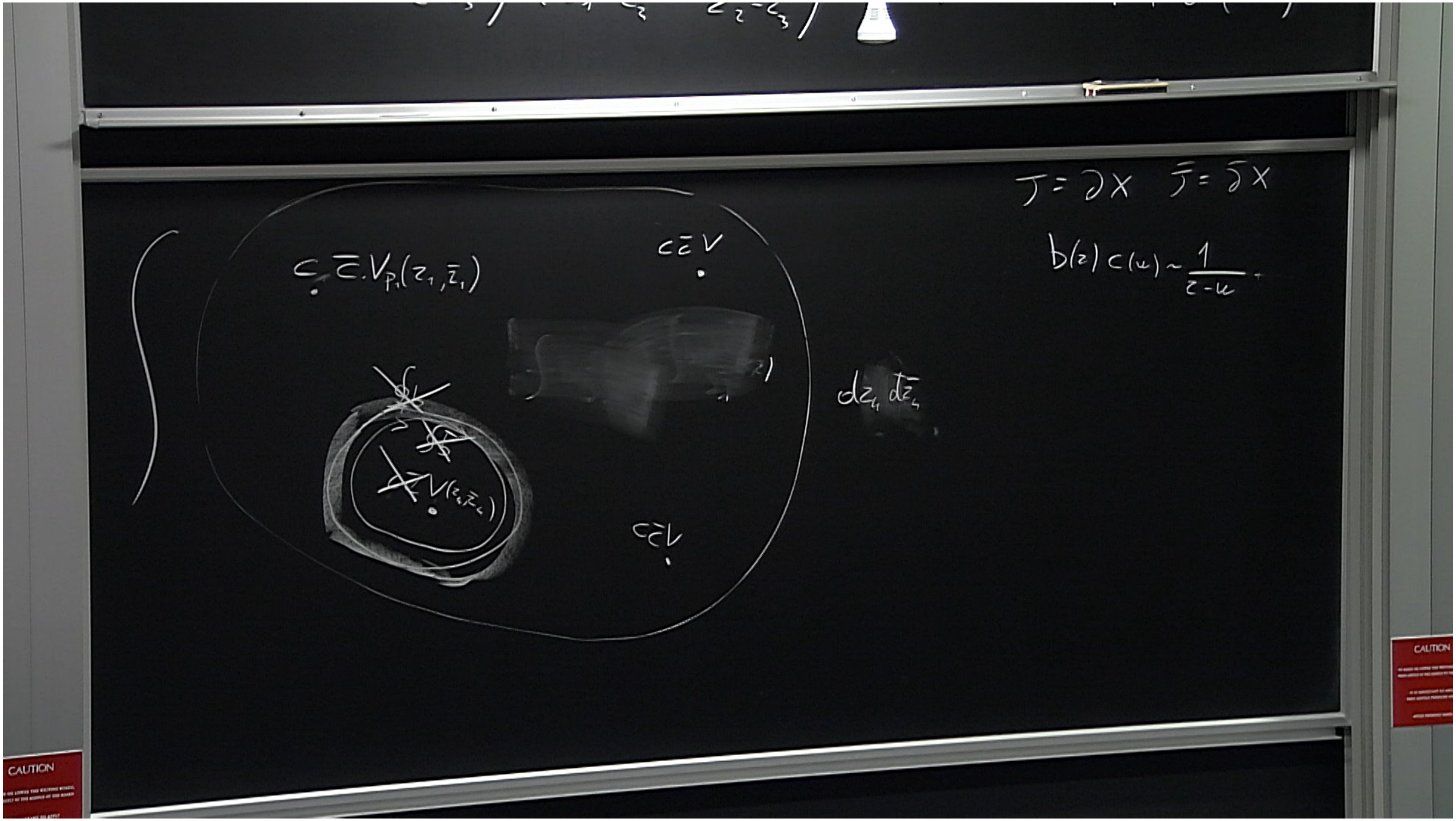
$$A_{TTG} = \epsilon_{\mu\nu}\left(\frac{p_1^{\mu}}{z_1-z_3}+\frac{p_2^{\mu}}{z_2-z_3}\right)\left(\frac{p_1^{\nu}}{\bar{z}_1-\bar{z}_3}+\frac{p_2^{\nu}}{\bar{z}_2-\bar{z}_3}\right)|z_1-z_2|^{-2}|z_1-z_3|^{2}|z_2-z_3|^{2}$$

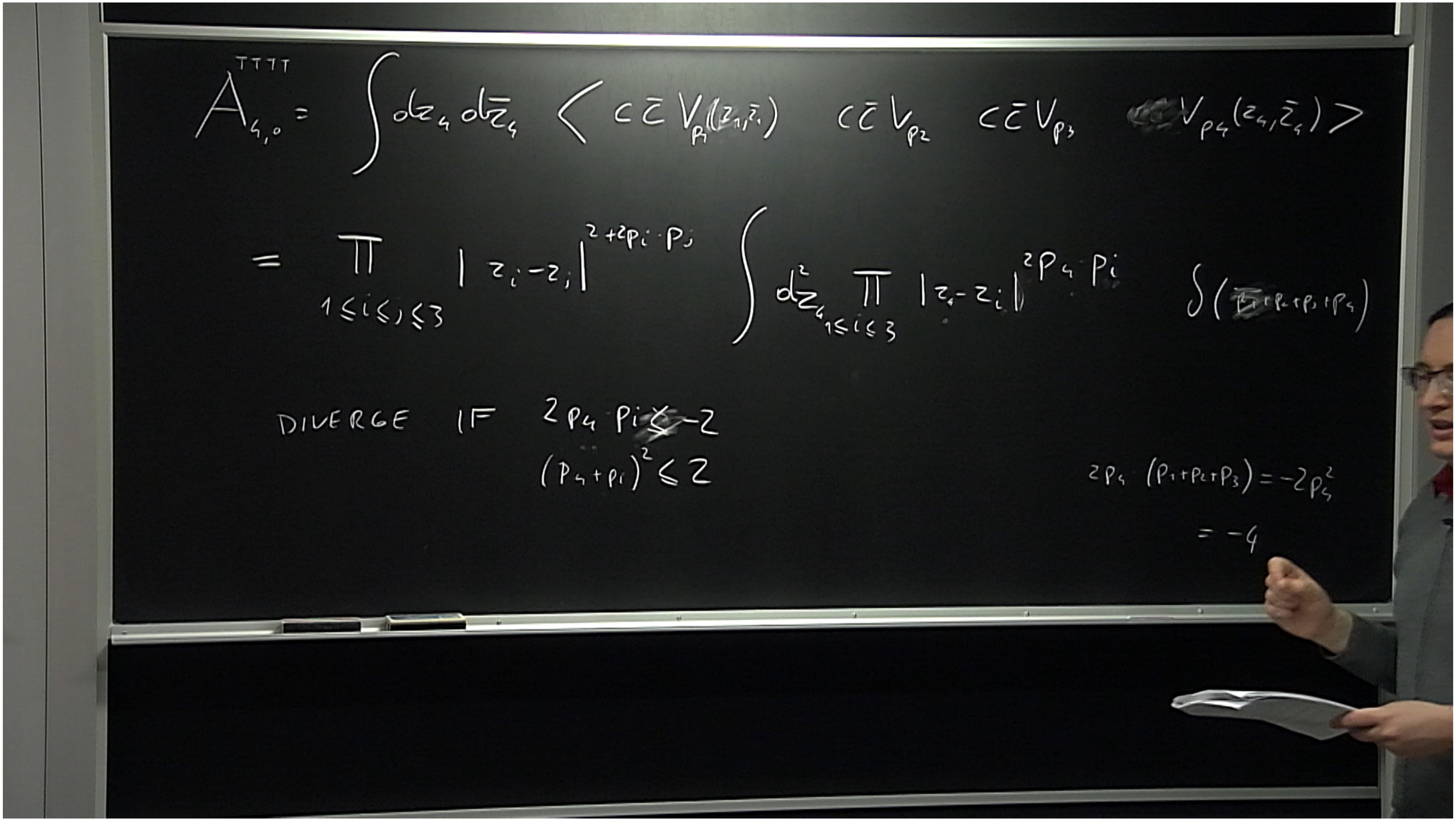
$$= \epsilon_{\mu\nu}\, p_1^{\mu}\, p_2^{\nu} \left|\frac{1}{z_1-z_3}-\frac{1}{z_2-z_3}\right|^2$$

$\delta(p_1+p_2+p_3)$


|p1 > ⊗ |g >
|p2 > ⊗ |g >
ε_μν a^μ_{-1} ā^ν_{-1} |p3) ⊗ |g >
Σ pi = 0
2p1+p2 = −4
2p2−p3 = 0
p1^2 = 2
p2^2 = 2
p3^2 = 0
ε_μν p3^μ = 0
A_TTG = ε_μν (p1^μ/(z1−z3) + p2^μ/(z2−z3))
(p1^ν/(z̄1−z̄3) + p2^ν/(z̄2−z̄3)) |z1−z2|^−2 |z1−z3|^2 |z2−z3|^2
= ε_μν p1^μ p2^ν
δ(p1+p2+p3)


$J = \partial X \quad \bar{J} = \bar{\partial} X$
$c\,\bar{c}\,V_{p_1}(z_1, \bar{z}_1)$
$c\bar{c}V$
$b\,\frac{\delta h}{\delta \lambda}$
$\bar{b}\,\frac{\delta \bar{h}}{\delta \bar{\lambda}}$
$d\lambda\, d\bar{\lambda}$
$c\bar{c}V(z_4, \bar{z}_4)$
$c\bar{c}V$

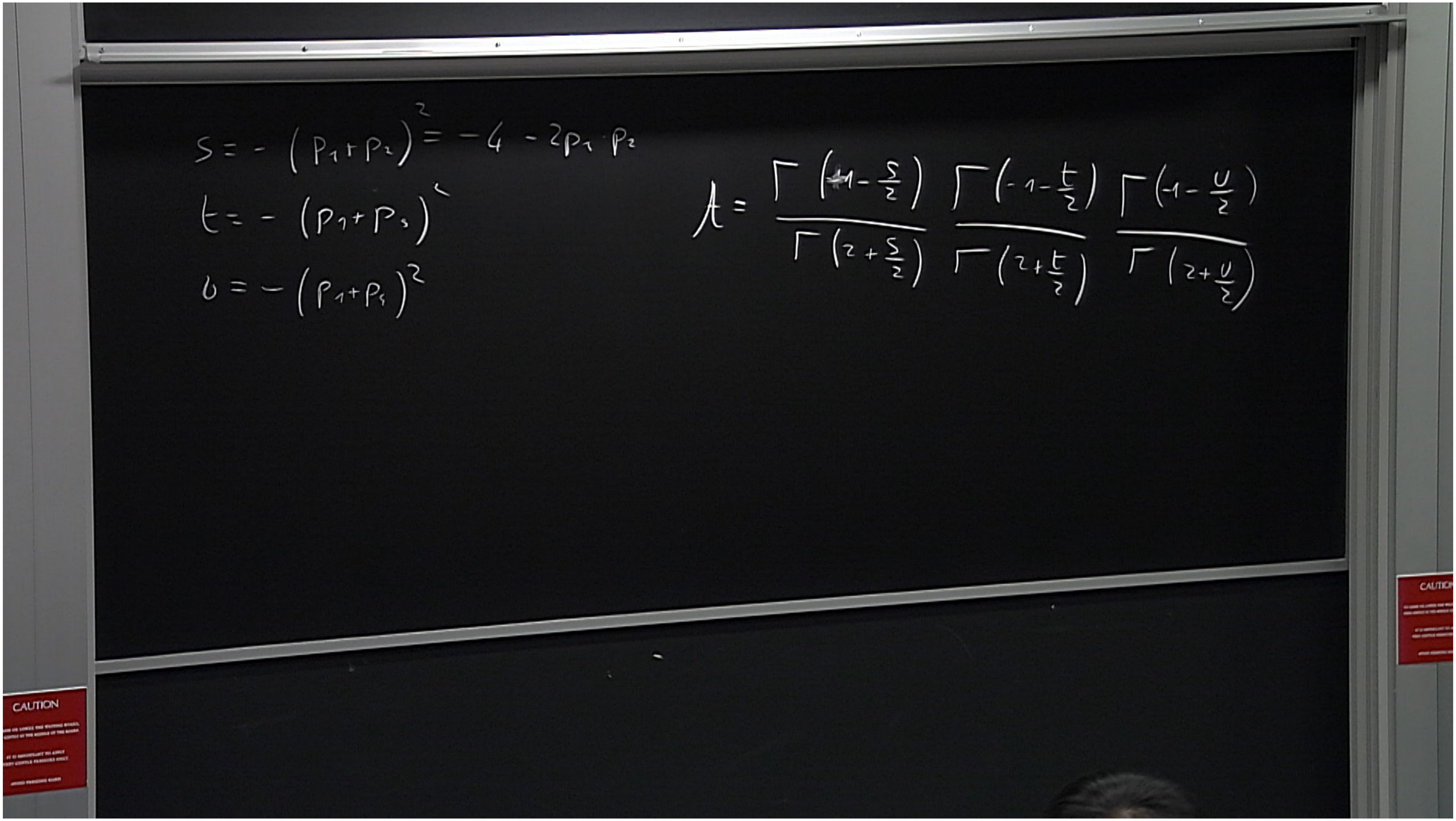

J = ∂X   J̄ = ∂̄X
b(z) c(w) ~ 1/(z-w)
c c̄ V_{p_1}(z_1, z̄_1)
c c̄ V
c c̄ V
V(z_4, z̄_4)
dz_4 dz̄_4
CAUTION

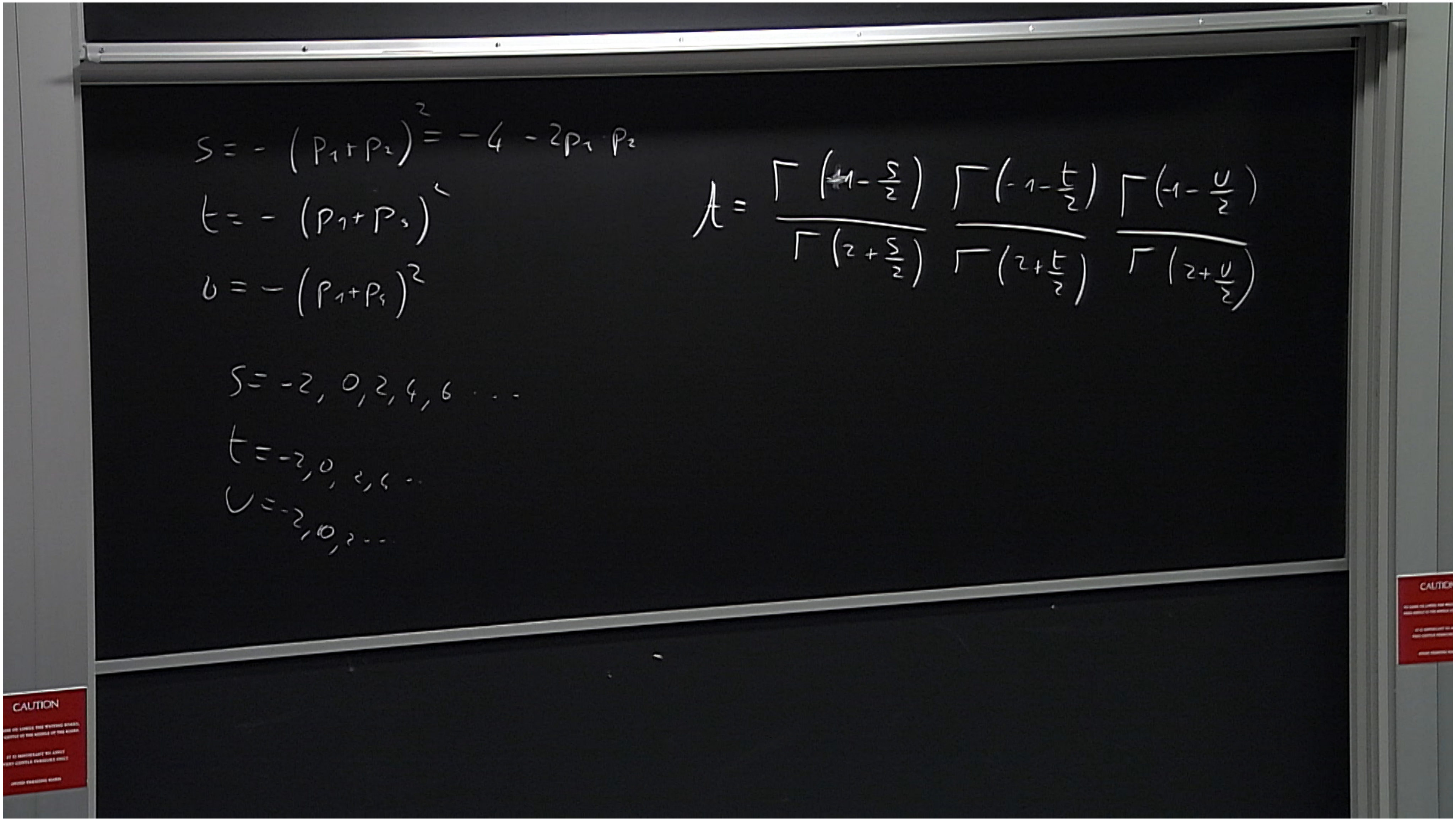

DIVERGE IF

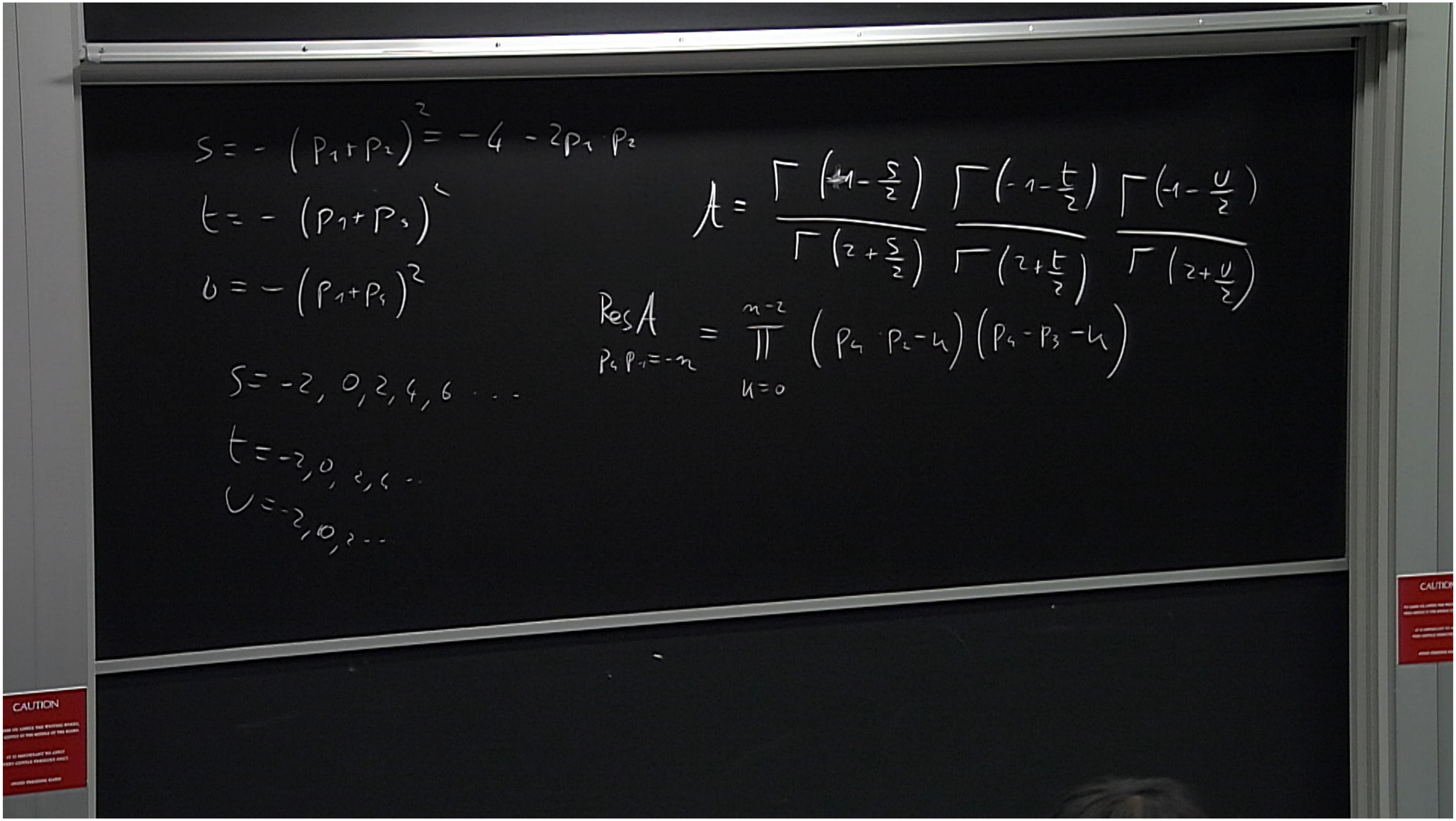
$$t = \frac{z_4 - z_1}{z_4 - z_3} \cdot \frac{z_2 - z_3}{z_2 - z_1}$$

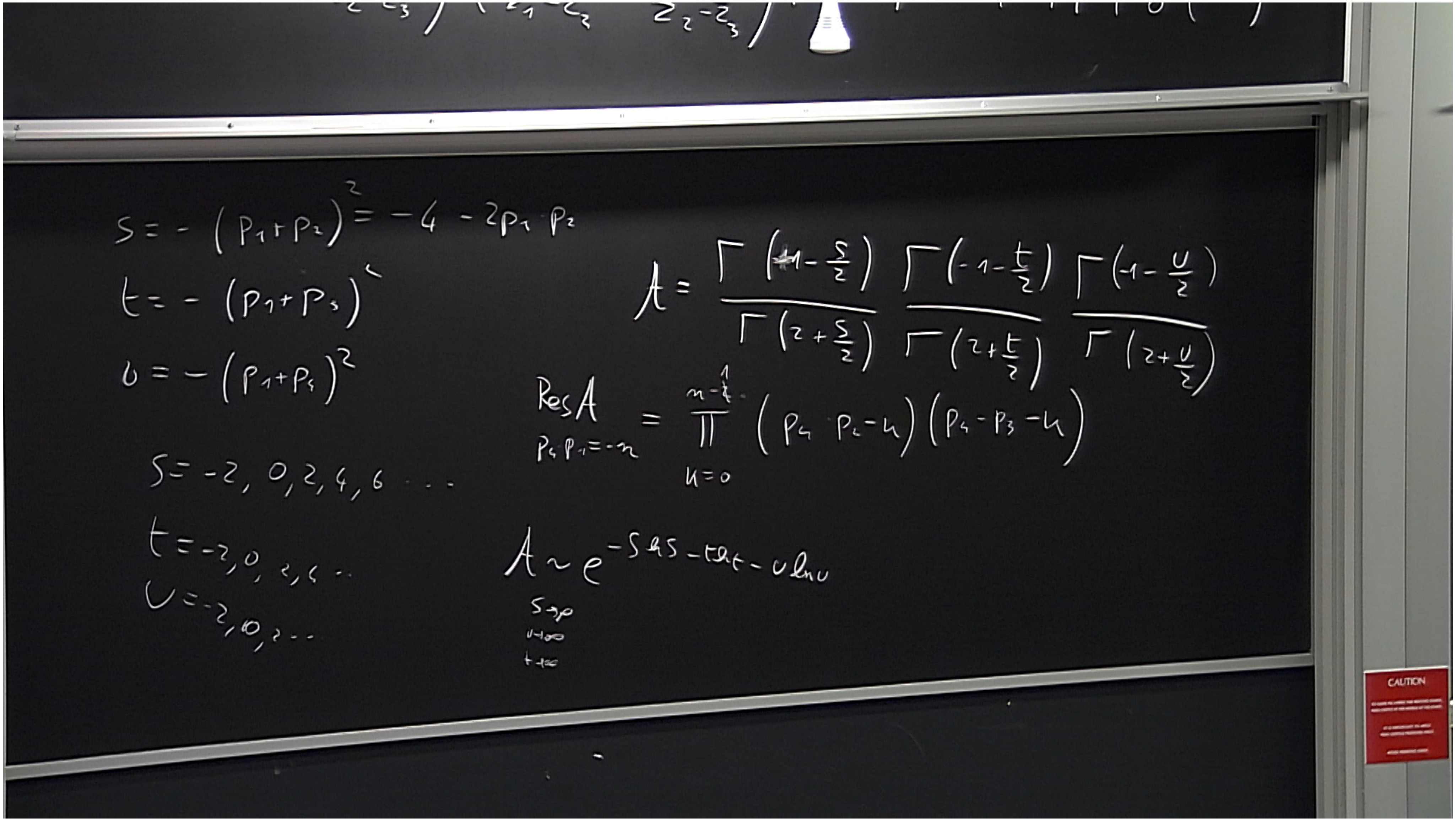
$$A_{4,0} = \int d^2t \, |t|^{2p_4 \cdot p_1} \, |1-t|^{2p_4 \cdot p_2} \, \delta^{(26)}(\quad)$$

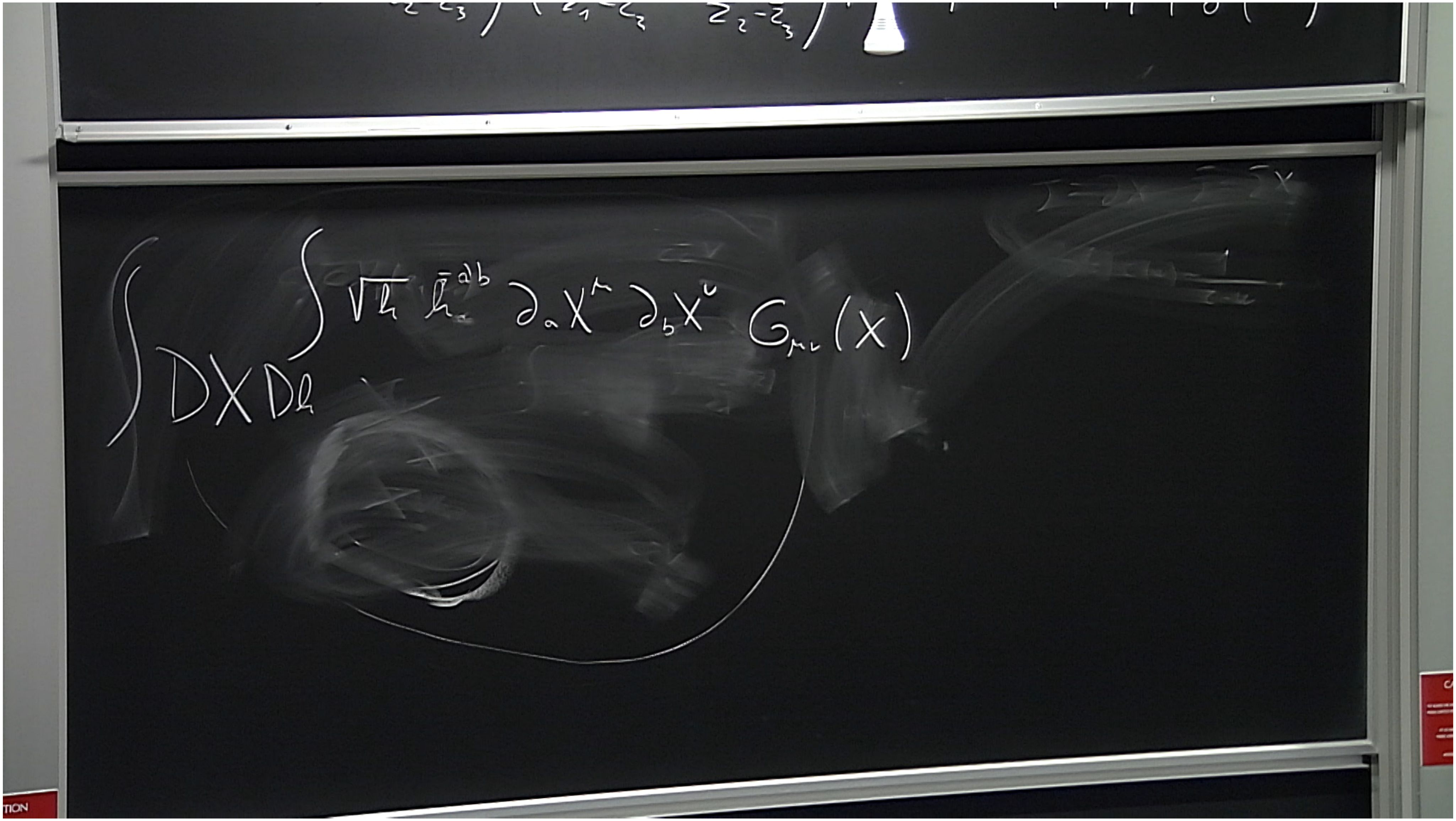
$$= \frac{\Gamma(1 + p_4 \cdot p_1)\,\Gamma(1 + p_4 \cdot p_2)\,\Gamma(1 + p_4 \cdot p_3)}{\Gamma(-p_4 \cdot p_1)\,\Gamma(-p_4 \cdot p_2)\,\Gamma(-p_4 \cdot p_3)}$$

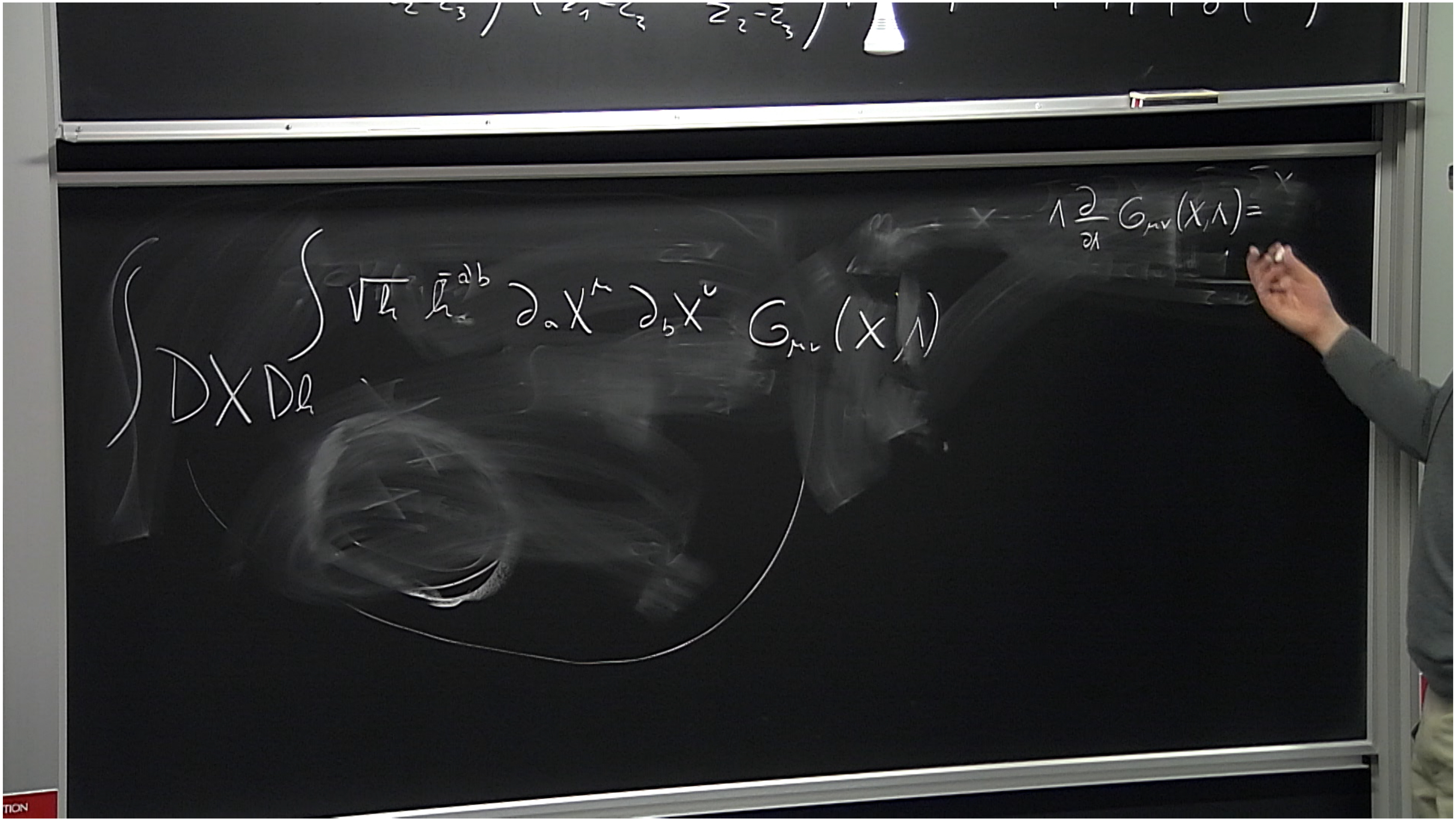

$s = -(p_1+p_2)^2 = -4 - 2p_1 \cdot p_2$
$t = -(p_1+p_3)^2$
$u = -(p_1+p_4)^2$
$A = \frac{\Gamma(-1-\frac{s}{2})}{\Gamma(2+\frac{s}{2})} \frac{\Gamma(-1-\frac{t}{2})}{\Gamma(2+\frac{t}{2})} \frac{\Gamma(-1-\frac{u}{2})}{\Gamma(2+\frac{u}{2})}$


$s = -(p_1+p_2)^2 = -4 - 2p_1 \cdot p_2$
$t = -(p_1+p_3)^2$
$u = -(p_1+p_4)^2$
$A = \frac{\Gamma(-1-\frac{s}{2})}{\Gamma(2+\frac{s}{2})} \frac{\Gamma(-1-\frac{t}{2})}{\Gamma(2+\frac{t}{2})} \frac{\Gamma(-1-\frac{u}{2})}{\Gamma(2+\frac{u}{2})}$
$s = -2, 0, 2, 4, 6 \ldots$
$t = -2, 0, 2, 4 \ldots$
$u = -2, 0, 2 \ldots$


$s = -(p_1+p_2)^2 = -4 - 2p_1 \cdot p_2$
$t = -(p_1+p_3)^2$
$u = -(p_1+p_4)^2$
$A = \frac{\Gamma(-1-\frac{s}{2})}{\Gamma(2+\frac{s}{2})} \frac{\Gamma(-1-\frac{t}{2})}{\Gamma(2+\frac{t}{2})} \frac{\Gamma(-1-\frac{u}{2})}{\Gamma(2+\frac{u}{2})}$
$\mathrm{Res}_{p_4 \cdot p_1 = -n} A = \prod_{k=0}^{n-2} (p_4 \cdot p_2 - k)(p_4 \cdot p_3 - k)$
$s = -2, 0, 2, 4, 6 \ldots$
$t = -2, 0, 2, 4 \ldots$
$u = -2, 0, 2 \ldots$
CAUTION
CAUTION

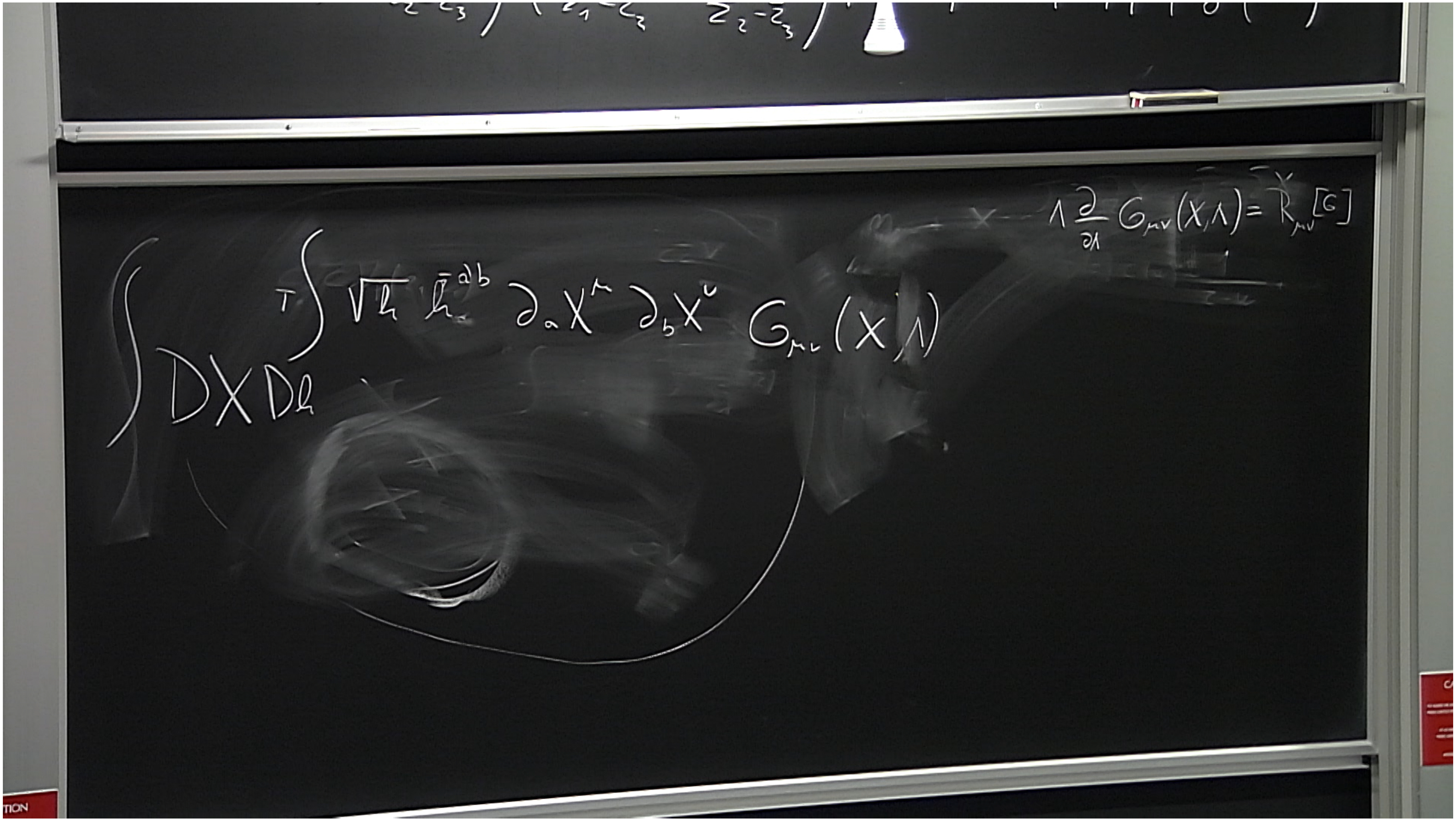

$s = -(p_1+p_2)^2 = -4 - 2p_1 \cdot p_2$
$t = -(p_1+p_3)^2$
$u = -(p_1+p_4)^2$
$s = -2, 0, 2, 4, 6 \ldots$
$t = -2, 0, 2, 4 \ldots$
$u = -2, 0, 2 \ldots$
$A = \frac{\Gamma(-1-\frac{s}{2})}{\Gamma(2+\frac{s}{2})} \frac{\Gamma(-1-\frac{t}{2})}{\Gamma(2+\frac{t}{2})} \frac{\Gamma(-1-\frac{u}{2})}{\Gamma(2+\frac{u}{2})}$
$\underset{p_4 \cdot p_1 = -n}{\mathrm{Res}\, A} = \prod_{k=0}^{n-1} (p_4 \cdot p_2 - k)(p_4 \cdot p_3 - k)$
$A \sim e^{-s \ln s - t \ln t - u \ln u}$
$s \to \infty$
$u \to -\infty$
$t \to \infty$

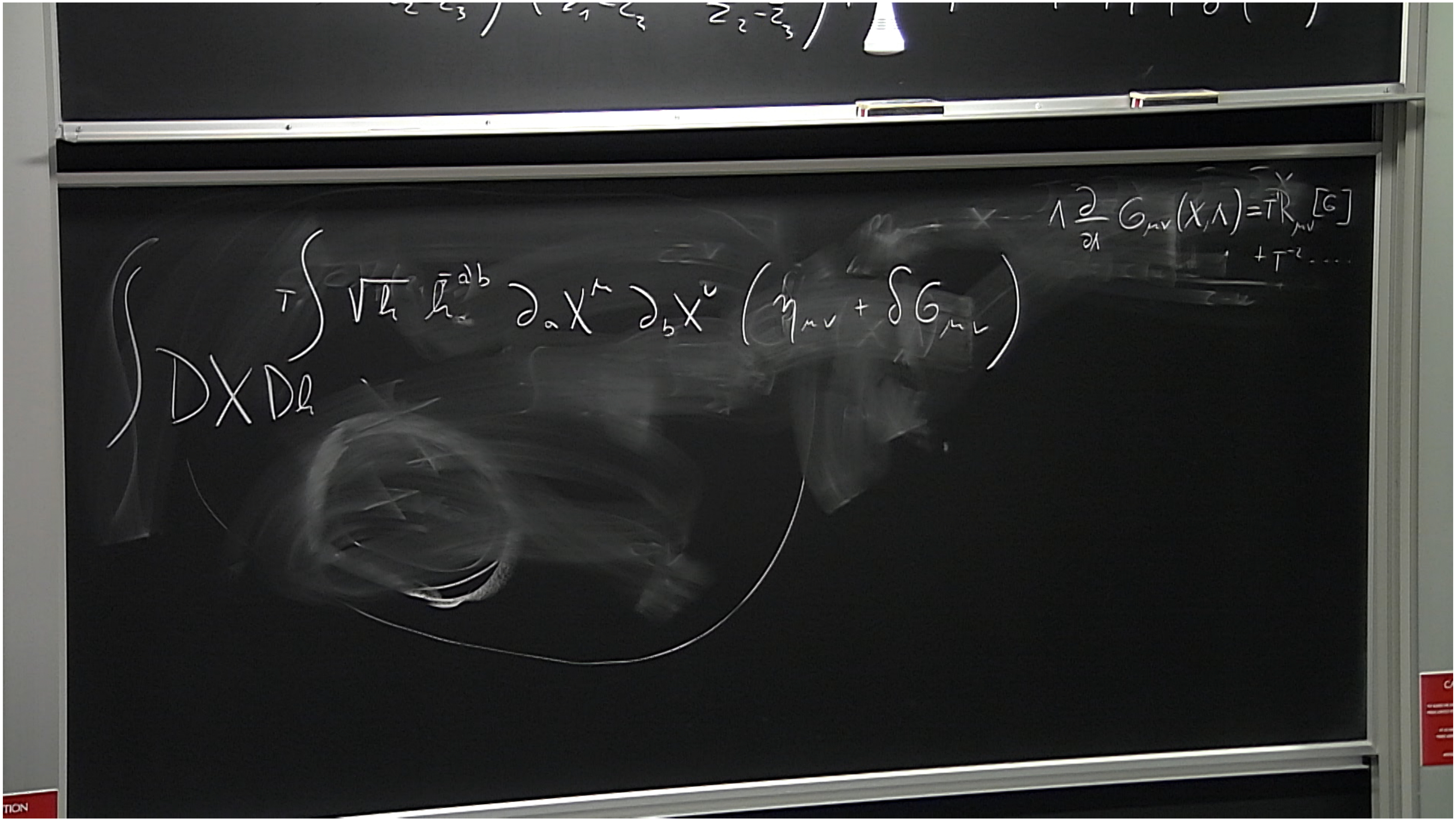

$\int DX\, Dh \int \sqrt{h}\, h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu\, G_{\mu\nu}(X)$


$\int DX Dh \int \sqrt{h}\, h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X,\Lambda)$
$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X,\Lambda) =$

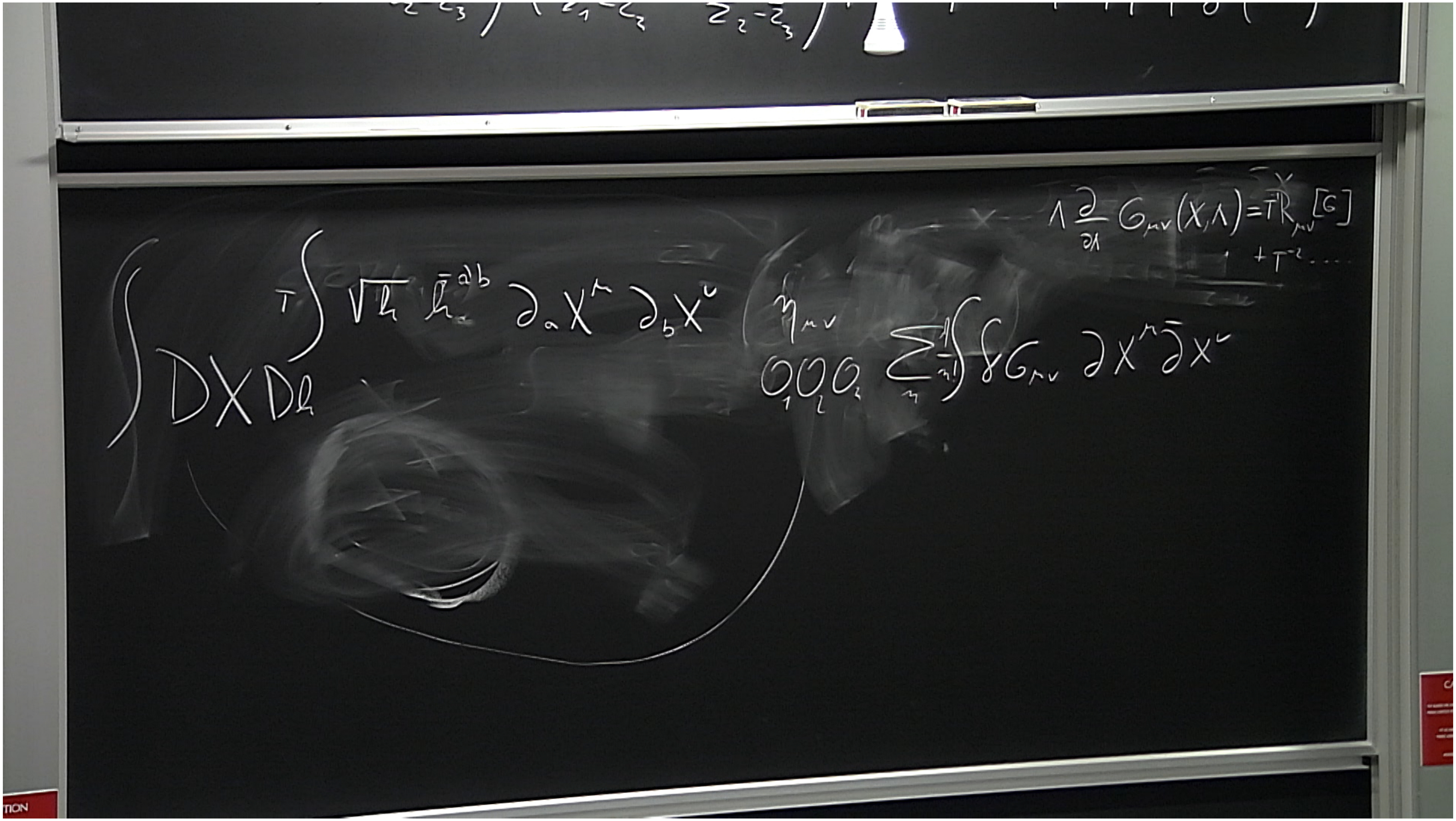
$$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X,\Lambda) = R_{\mu\nu}[G]$$

$$\int DX\, Dh \int \sqrt{h}\, h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu\, G_{\mu\nu}(X,\Lambda)$$


$\int DX\,Dh \; T\int \sqrt{h}\, h^{ab}\, \partial_a X^\mu \partial_b X^\nu \, G_{\mu\nu}(X,\Lambda)$
$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X,\Lambda) = R_{\mu\nu}[G]$



$\int DX\,Dh\; T\int \sqrt{h}\, h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu \left(\eta_{\mu\nu} + \delta G_{\mu\nu}\right)$
$\Lambda \frac{\partial}{\partial\Lambda} G_{\mu\nu}(X,\Lambda) = T R_{\mu\nu}[G]$
$+ T^{-2} \ldots$


Λ ∂/∂Λ G_μν(X,Λ) = T R_μν[G]
+ T^-2 ...
∫ DX Dh T∫ √h h^ab ∂_a X^μ ∂_b X^ν
η_μν
O_1 O_2 O_3 Σ_n 1/n! ∫ δG_μν ∂X^μ ∂̄X^ν