

Title: PSI 2016/2017 String Theory (Review) - Lecture 9

Date: Mar 06, 2017 10:15 AM

URL: <http://pirsa.org/17030025>

Abstract:

$$A_n = \sum_g \binom{2g-2+n}{g_s} \mu_{g,n}$$

$$A_3 = g_s A_{3,0} + g_s^3 A_{3,1} \dots$$

$$A_n = \sum_g \binom{2g-2+n}{g_s} \mu_{g,n} \dots$$

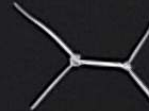
$$A_3 = g_s A_{3,0} + g_s^3 A_{3,1} \dots$$

$$A_4 = g_s^2 A_{4,0} + \dots$$

$$A_m = \sum_g \binom{2g-2+m}{g} \mathcal{M}_{g,m} \dots$$

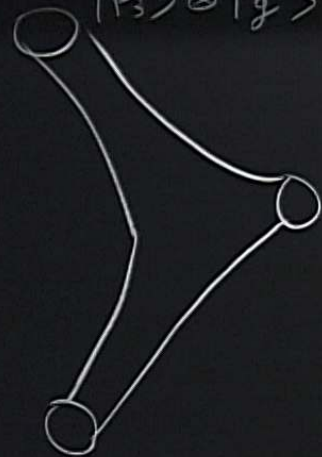
$$A_3 = g_s A_{3,0} + g_s^3 A_{3,1} \dots$$

$$A_4 = g_s^2 A_{4,0} + g_s^4 A_{4,1} \dots$$

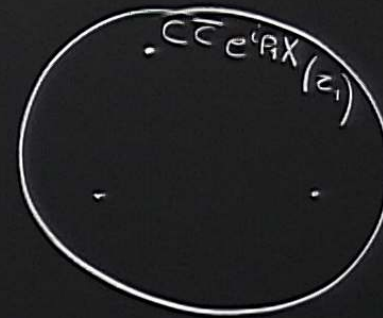


$|P_3\rangle \otimes |g\rangle$

$$P_i^2 = 2$$



$|P_2\rangle \otimes |g\rangle$



$|P_1\rangle \otimes |g\rangle$

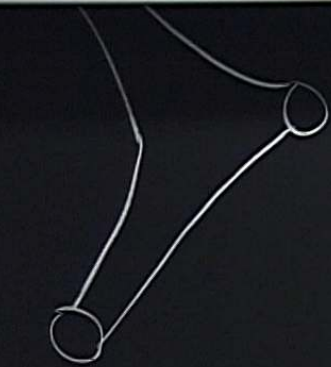
$$\langle c(z_1) \bar{c}(\bar{z}_1) c(z_2) \bar{c}(\bar{z}_2) c(z_3) \bar{c}(\bar{z}_3) \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$\langle e^{i p_1 \cdot X(z_1, \bar{z}_1)} e^{i p_2 \cdot X(z_2, \bar{z}_2)} e^{i p_3 \cdot X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 \cdot p_2} |z_1 - z_3|^{2 p_1 \cdot p_3} |z_2 - z_3|^{2 p_2 \cdot p_3} \int^{26} \binom{p_1 + p_2 + p_3}{}$$

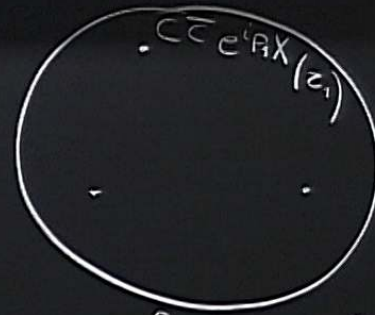
$$\mathcal{T} = \partial X \quad \bar{\mathcal{T}} = \bar{\partial} X$$

$$\langle c(z_1) \bar{c}(\bar{z}_1) c(z_2) \bar{c}(\bar{z}_2) c(z_3) \bar{c}(\bar{z}_3) \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$\langle e^{i p_1 \cdot X(z_1, \bar{z}_1)} e^{i p_2 \cdot X(z_2, \bar{z}_2)} e^{i p_3 \cdot X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 \cdot p_2} |z_1 - z_3|^{2 p_1 \cdot p_3} |z_2 - z_3|^{2 p_2 \cdot p_3} \int^{26} (p_1 + p_2 + p_3)$$



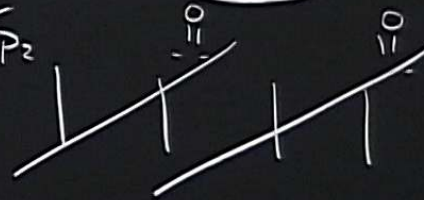
$$|P_2\rangle \circ |q\rangle \Rightarrow$$



$$z \rightarrow \frac{az+b}{cz+d}$$

$$|P_1\rangle \circ |q\rangle$$

$$= |z_1 - z_2| \frac{z + z_0}{z + z_1}$$



$$\int_{\gamma} (P_1 + P_2 + P_3)$$

$$P_1^2 = z \quad P_2^2 = z \quad P_3^2 = z$$

$$P_3 = -(P_1 + P_2) \quad z = z + z + z P_1 P_2$$

$$\langle c(z_1) \bar{c}(\bar{z}_1) c(z_2) \bar{c}(\bar{z}_2) c(z_3) \bar{c}(\bar{z}_3) \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$\langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} e^{i p_3 X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 p_2} |z_1 - z_3|^{2 p_1 p_3} |z_2 - z_3|^{2 p_2 p_3} \int^{26} (p_1 + p_2 + p_3)$$

$$\begin{aligned}
 & \langle e^{i p_1 \cdot X(z_1, \bar{z}_1)} e^{i p_2 \cdot X(z_2, \bar{z}_2)} e^{i p_3 \cdot X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 \cdot p_2} |z_1 - z_3|^{2 p_1 \cdot p_3} \\
 & \langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} : \partial X^\mu \bar{\partial} X^\nu e^{i p_3 X(z_3, \bar{z}_3)} \rangle = |z_2 - z_3|^{2 p_2 \cdot p_3} \int^{26} (p_1 + p_2 + p_3)
 \end{aligned}$$

$$\langle c(z_1) \bar{c}(\bar{z}_1) c(z_2) \bar{c}(\bar{z}_2) c(z_3) \bar{c}(\bar{z}_3) \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$\langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} e^{i p_3 X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 \cdot p_2} |z_1 - z_3|^{2 p_1 \cdot p_3} |z_2 - z_3|^{2 p_2 \cdot p_3} \int^{26} (p_1 + p_2 + p_3)$$

$$\langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} \epsilon_{\mu\nu} : \partial X^\mu \bar{\partial} X^\nu e^{i p_3 X(z_3, \bar{z}_3)} \rangle =$$

$$= \epsilon_{\mu\nu} \left( \frac{p_1^\mu}{z_1 - z_3} + \frac{p_2^\mu}{z_2 - z_3} \right) \left( \frac{p_1^\nu}{\bar{z}_1 - \bar{z}_2} + \frac{p_2^\nu}{\bar{z}_2 - \bar{z}_3} \right) |z_1 - z_2|^{2 p_1 \cdot p_2} \dots \delta(\dots)$$

$$|p_1\rangle \otimes |g\rangle$$

$$|p_2\rangle \otimes |g\rangle$$

$$\epsilon_{\mu\nu} a_{-1}^{\mu} \bar{a}_{-1}^{\nu} |p_3\rangle \otimes |g\rangle$$

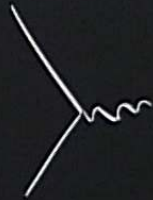
$$\sum p_i = 0$$

$$p_1^2 = 2$$

$$p_2^2 = 2$$

$$p_3^2 = 0$$

$$\epsilon_{\mu\nu} p_3^{\mu} = 0$$



$$A_{TTT} = \epsilon_{\mu\nu} \left( \frac{p_1^{\mu}}{z_1 - z_3} + \frac{p_2^{\mu}}{z_2 - z_3} \right)$$

$$\left( \frac{p_1^{\nu}}{z_1 - z_3} + \frac{p_2^{\nu}}{z_2 - z_3} \right) |z_1 - z_2|^{-2} |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$= \epsilon_{\mu\nu} p_1^{\mu} p_2^{\nu} \left| \frac{1}{z_1 - z_3} + \frac{1}{z_2 - z_3} \right| \delta(p_1 + p_2 + p_3)$$

$$|p_1\rangle \otimes |g\rangle$$

$$|p_2\rangle \otimes |g\rangle$$

$$\epsilon_{\mu\nu} a_{-1}^{\mu} \bar{a}_{-1}^{\nu} |p_3\rangle \otimes |g\rangle$$

$$\sum p_i = 0$$

$$2p_1 p_2 = -4$$

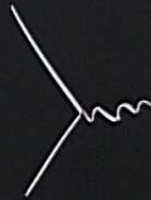
$$2p_2 p_3 = 0$$

$$p_1^2 = 2$$

$$p_2^2 = 2$$

$$p_3^2 = 0$$

$$\epsilon_{\mu\nu} p_3^{\mu} = 0$$

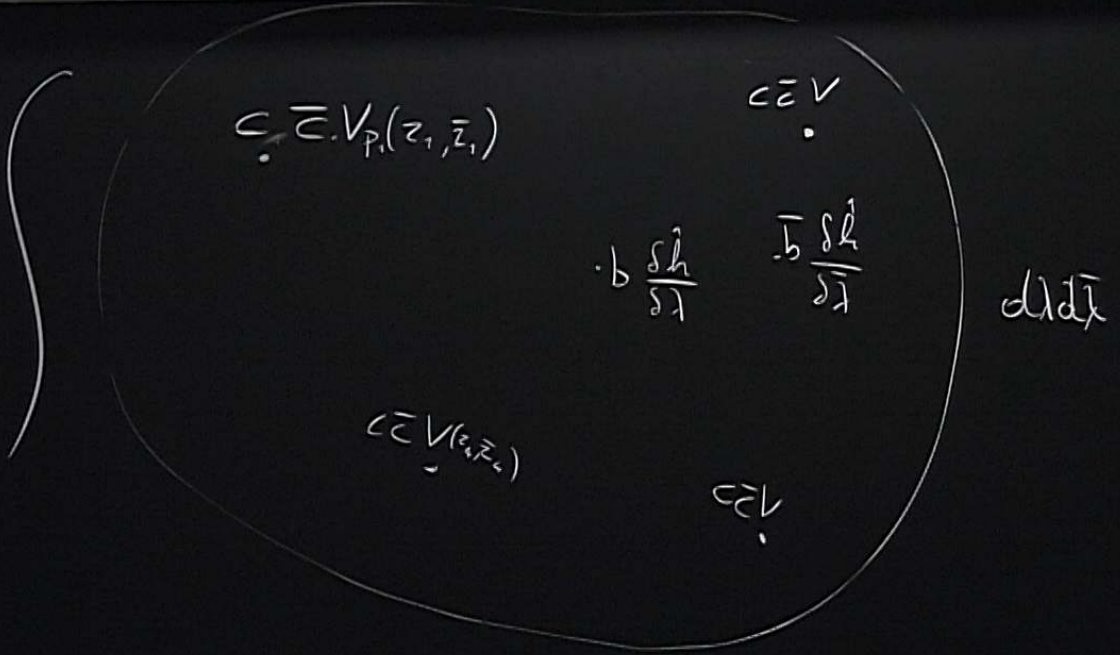


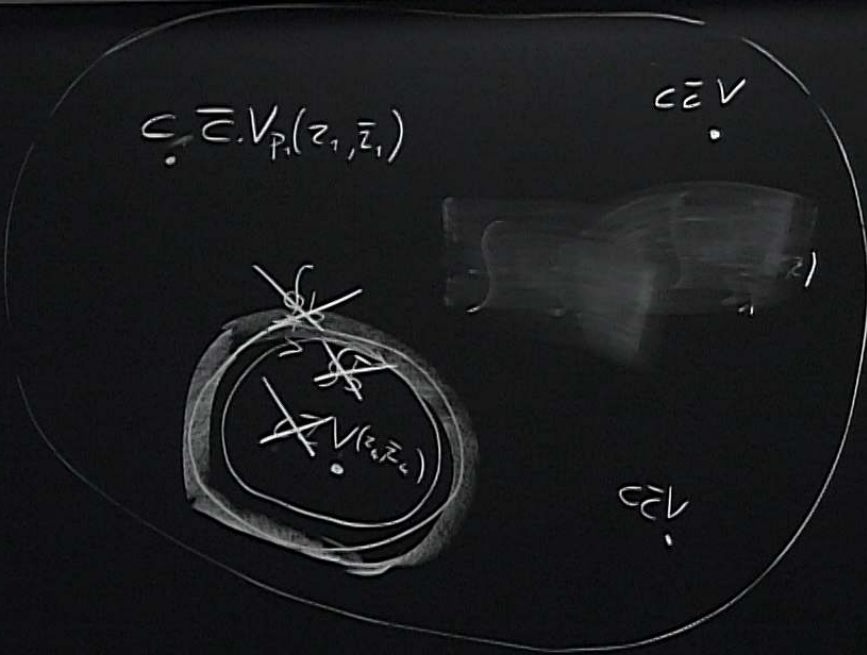
$$A_{TT} = \epsilon_{\mu\nu} \left( \frac{p_1^{\mu}}{z_1 - z_3} + \frac{p_2^{\mu}}{z_2 - z_3} \right)$$

$$\left( \frac{p_1^{\nu}}{z_1 - z_3} + \frac{p_2^{\nu}}{z_2 - z_3} \right) |z_1 - z_2|^{-2} |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$= \epsilon_{\mu\nu} p_1^{\mu} p_2^{\nu} \sqrt{\frac{1}{z_1 - z_3} + \frac{1}{z_2 - z_3}} \delta(p_1 + p_2 + p_3)$$

$$J = \partial X \quad \bar{J} = \bar{\partial} X$$





$$\mathcal{J} = \partial X \quad \bar{\mathcal{J}} = \bar{\partial} X$$

$$b(z) c(w) \sim \frac{1}{z-w} +$$

$$dz_1, d\bar{z}_1$$

CAUTION  
 Do not touch the chalkboard  
 when it is being used.

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$$A_{4,0}^{\text{T T T T}} = \int dz_4 d\bar{z}_4 \langle c \bar{c} V_{p_1}(z_1, \bar{z}_1) c \bar{c} V_{p_2} c \bar{c} V_{p_3} V_{p_4}(z_4, \bar{z}_4) \rangle$$

$$= \prod_{1 \leq i < j \leq 3} |z_i - z_j|^{2 + 2p_i \cdot p_j} \int dz_4 \prod_{1 \leq i \leq 3} |z_4 - z_i|^{2p_4 \cdot p_i} \int (\text{divergent})$$

DIVERGE IF  $2p_4 \cdot p_i \leq -2$   
 $(p_4 + p_i)^2 \leq 2$

$$2p_4 \cdot (p_1 + p_2 + p_3) = -2p_4^2 = -4$$

$$t = \frac{z_4 - z_1}{z_4 - z_3} \cdot \frac{z_2 - z_3}{z_2 - z_1}$$

$$A_{4,0} = \int dt |t|^{2p_4 \cdot p_1} |1-t|^{2p_4 \cdot p_2} \delta^{(26)}(\quad)$$

$$= \frac{\Gamma(1+p_4 \cdot p_1) \Gamma(1+p_4 \cdot p_2) \Gamma(1+p_4 \cdot p_3)}{\Gamma(-p_4 \cdot p_1) \Gamma(-p_4 \cdot p_2) \Gamma(-p_4 \cdot p_3)}$$

CAUTION

DO NOT TOUCH THE BOARD WHEN THE BOARD IS IN USE

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$$S = - (P_1 + P_2)^2 = -4 - 2P_1 \cdot P_2$$

$$t = - (P_1 + P_3)^2$$

$$u = - (P_1 + P_3)^2$$

$$A = \frac{\Gamma\left(-1 - \frac{S}{2}\right) \Gamma\left(-1 - \frac{t}{2}\right) \Gamma\left(-1 - \frac{u}{2}\right)}{\Gamma\left(2 + \frac{S}{2}\right) \Gamma\left(2 + \frac{t}{2}\right) \Gamma\left(2 + \frac{u}{2}\right)}$$

CAUTION

DO NOT TOUCH THE ELECTRIC BOARD  
UNLESS YOU ARE A QUALIFIED ELECTRICIAN  
OR YOU ARE ADVISED BY A QUALIFIED ELECTRICIAN  
UNLESS OTHERWISE ADVISED

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$$S = -2, 0, 2, 4, 6, \dots$$

$$t = -2, 0, 2, 4, \dots$$

$$u = -2, 0, 2, \dots$$

CAUTION

DO NOT TOUCH THE ELECTRIC BOARD  
WHILE IT IS POWERED ON  
IF IT IS NECESSARY TO ADJUST  
THEY MUST BE POWERED OFF  
FIRST

CAUTION

$$S = - (P_1 + P_2)^2 = -4 - 2P_1 P_2$$

$$t = - (P_1 + P_3)^2$$

$$u = - (P_1 + P_4)^2$$

$$S = -2, 0, 2, 4, 6, \dots$$

$$t = -2, 0, 2, 4, \dots$$

$$u = -2, 0, 2, \dots$$

$$A = \frac{\Gamma\left(-1 - \frac{S}{2}\right) \Gamma\left(-1 - \frac{t}{2}\right) \Gamma\left(-1 - \frac{u}{2}\right)}{\Gamma\left(2 + \frac{S}{2}\right) \Gamma\left(2 + \frac{t}{2}\right) \Gamma\left(2 + \frac{u}{2}\right)}$$

$$\text{Res } A = \prod_{\substack{n=2 \\ P_4 P_1 = -n}}^{\infty} (P_4 - P_2 - n) (P_4 - P_3 - n)$$

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DO NOT TOUCH THE ELECTRIC BOARD  
WHILE IT IS ON OR THE SURFACE OF THE BOARD  
IS HOT. ALWAYS BE CAREFUL  
WHEN USING THE BOARD.

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$$s = -(p_1 + p_2)^2 = -4 - 2p_1 p_2$$

$$t = -(p_1 + p_3)^2$$

$$u = -(p_1 + p_4)^2$$

$$s = -2, 0, 2, 4, 6, \dots$$

$$t = -2, 0, 2, 4, \dots$$

$$u = -2, 0, 2, \dots$$

$$A = \frac{\Gamma(-1 - \frac{s}{2}) \Gamma(-1 - \frac{t}{2}) \Gamma(-1 - \frac{u}{2})}{\Gamma(z + \frac{s}{2}) \Gamma(z + \frac{t}{2}) \Gamma(z + \frac{u}{2})}$$

$$\text{Res}_A = \frac{n-1}{\prod_{k=0}^{n-1} (p_4 - p_k - u) (p_4 - p_3 - u)}$$

$$A \sim e^{-s \ln s - t \ln t - u \ln u}$$

$s \rightarrow \infty$   
 $u \rightarrow \infty$   
 $t \rightarrow \infty$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$$

$z_3 / (z_1 - z_2) \quad z_2 - z_3 / (z_1 - z_2)$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X, \Lambda)$$

$$\frac{1}{\kappa} \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X, \Lambda) =$$

$z_3 / (z_1 - z_2) \quad z_2 - z_3 / (z_1 - z_2)$

$$\frac{1}{2} \frac{\partial}{\partial \lambda} G_{\mu\nu}(X, \lambda) = \dot{R}_{\mu\nu}^{\lambda}[G]$$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X, \lambda)$$

$z_3 / (z_1 - z_2) \quad z_2 - z_3 / (z_1 - z_2)$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X, \Lambda)$$

$$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X, \Lambda) = \tilde{R}_{\mu\nu}^{\lambda\sigma} [G]$$

$z_1 - z_2 \quad z_2 - z_3 \quad \dots$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu (\eta_{\mu\nu} + \delta G_{\mu\nu})$$

$$\frac{\delta}{\delta \lambda} G_{\mu\nu}(X, \lambda) = T^{\mu\nu}[G]$$

$z_3 / (z_1 - z_2) \quad z_2 - z_3 / (z_1 - z_2)$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu$$

$$\eta_{\mu\nu} \sum_{n!} \delta G_{\mu\nu} \partial X^\mu \partial X^\nu$$

$$\lambda \frac{\partial}{\partial \lambda} G_{\mu\nu}(X, \lambda) = \bar{T}^{\mu\nu}[G]$$