

Title: PSI 2016/2017 String Theory (Review) - Lecture 9

Date: Mar 06, 2017 10:15 AM

URL: <http://pirsa.org/17030025>

Abstract:

$$A_n = \sum_g \binom{2g-2+n}{g_s} \mu_{g,n} \dots$$

$$A_3 = g_s A_{3,0} + g_s^3 A_{3,1} \dots$$

$$A_n = \sum_g \binom{2g-2+n}{g_s} \mu_{g,n} \dots$$

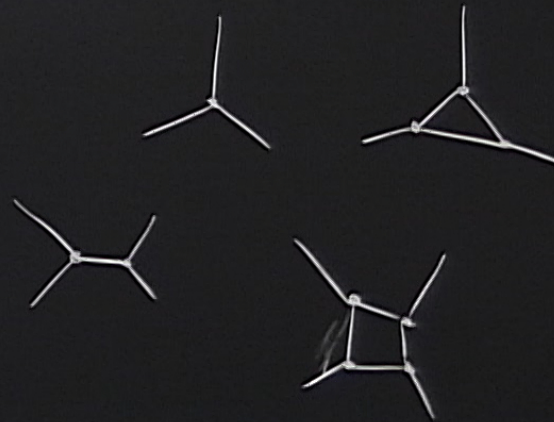
$$A_3 = g_s A_{3,0} + g_s^3 A_{3,1} \dots$$

$$A_4 = g_s^2 A_{4,0} + \dots$$

$$A_m = \sum_g \binom{2g-2+m}{g} \mathcal{M}_{g,m} \dots$$

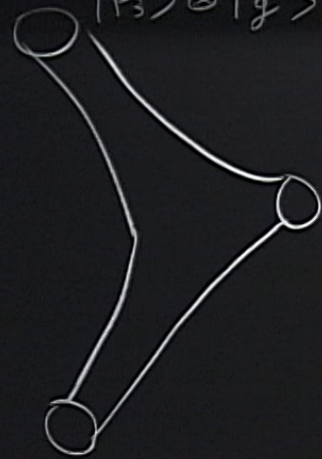
$$A_3 = g_s A_{3,0} + g_s^3 A_{3,1} \dots$$

$$A_4 = g_s^2 A_{4,0} + g_s^4 A_{4,1} \dots$$

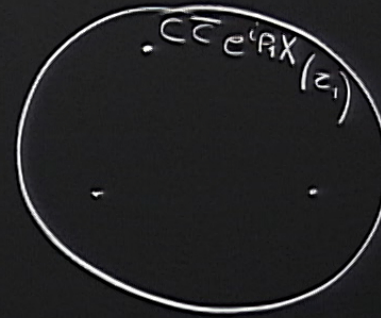
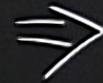


$|P_3\rangle \otimes |g\rangle$

$$P_i^2 = 2$$



$|P_2\rangle \otimes |g\rangle$



$|P_1\rangle \otimes |g\rangle$

$$\langle c(z_1) \bar{c}(\bar{z}_1) c(z_2) \bar{c}(\bar{z}_2) c(z_3) \bar{c}(\bar{z}_3) \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$\langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} e^{i p_3 X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 p_2} |z_1 - z_3|^{2 p_1 p_3} |z_2 - z_3|^{2 p_2 p_3} \int^{26} \binom{p_1 + p_2 + p_3}{}$$

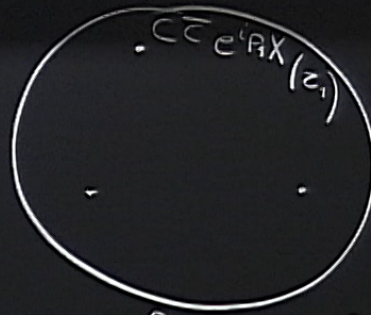
$$\mathcal{J} = \partial X \quad \bar{\mathcal{J}} = \bar{\partial} X$$

$$\langle c(z_1) \bar{c}(\bar{z}_1) c(z_2) \bar{c}(\bar{z}_2) c(z_3) \bar{c}(\bar{z}_3) \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$\langle e^{i p_1 \cdot X(z_1, \bar{z}_1)} e^{i p_2 \cdot X(z_2, \bar{z}_2)} e^{i p_3 \cdot X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 \cdot p_2} |z_1 - z_3|^{2 p_1 \cdot p_3} |z_2 - z_3|^{2 p_2 \cdot p_3} \int^{26} (p_1 + p_2 + p_3)$$



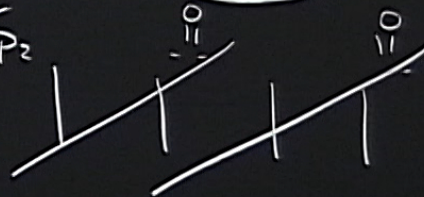
$$|p_2\rangle \otimes |q\rangle \Rightarrow$$



$$z \rightarrow \frac{az+b}{cz+d}$$

$$|p_1\rangle \otimes |q\rangle$$

$$= |z_1 - z_2| \frac{z + z_0}{z + z_1} \frac{z + z_2}{z + z_3}$$



$$\int_{z_0}^{z_1} (P_1 + P_2 + P_3)$$

$$P_1^2 = z \quad P_2^2 = z \quad P_3^2 = z$$

$$P_3 = -(P_1 + P_2) \quad z = z + z + z P_1 P_2$$



$$\langle c(z_1) \bar{c}(\bar{z}_1) c(z_2) \bar{c}(\bar{z}_2) c(z_3) \bar{c}(\bar{z}_3) \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$\langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} e^{i p_3 X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 p_2} |z_1 - z_3|^{2 p_1 p_3} |z_2 - z_3|^{2 p_2 p_3} \int^{2\ell} (p_1 + p_2 + p_3)$$

$$\langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} \rangle$$

$$\begin{aligned}
 & \langle e^{i p_1 \cdot X(z_1, \bar{z}_1)} e^{i p_2 \cdot X(z_2, \bar{z}_2)} e^{i p_3 \cdot X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 \cdot p_2} |z_1 - z_3|^{2 p_1 \cdot p_3} \\
 & \langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} : \partial X^\mu \bar{\partial} X^\nu e^{i p_3 X(z_3, \bar{z}_3)} \rangle = |z_2 - z_3|^{2 p_2 \cdot p_3} \int^{26} (p_1 + p_2 + p_3)
 \end{aligned}$$

$$\langle c(z_1) \bar{c}(\bar{z}_1) c(z_2) \bar{c}(\bar{z}_2) c(z_3) \bar{c}(\bar{z}_3) \rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$\langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} e^{i p_3 X(z_3, \bar{z}_3)} \rangle = |z_1 - z_2|^{2 p_1 \cdot p_2} |z_1 - z_3|^{2 p_1 \cdot p_3} |z_2 - z_3|^{2 p_2 \cdot p_3} \int^{26} (p_1 + p_2 + p_3)$$

$$\langle e^{i p_1 X(z_1, \bar{z}_1)} e^{i p_2 X(z_2, \bar{z}_2)} \epsilon_{\mu\nu} : \partial X^\mu \bar{\partial} X^\nu e^{i p_3 X(z_3, \bar{z}_3)} \rangle =$$

$$= \epsilon_{\mu\nu} \left( \frac{p_1^\mu}{z_1 - z_3} + \frac{p_2^\mu}{z_2 - z_3} \right) \left( \frac{p_1^\nu}{\bar{z}_1 - \bar{z}_2} + \frac{p_2^\nu}{\bar{z}_2 - \bar{z}_3} \right) |z_1 - z_2|^{2 p_1 \cdot p_2} \dots \delta(\dots)$$

$$|p_1\rangle \otimes |g\rangle$$

$$|p_2\rangle \otimes |g\rangle$$

$$\epsilon_{\mu\nu} a_{-1}^{\mu} \bar{a}_{-1}^{\nu} |p_3\rangle \otimes |g\rangle$$

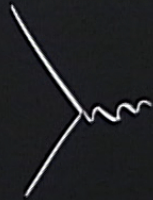
$$\sum p_i = 0$$

$$p_1^2 = 2$$

$$p_2^2 = 2$$

$$p_3^2 = 0$$

$$\epsilon_{\mu\nu} p_3^{\mu} = 0$$



$$A_{TTT} = \epsilon_{\mu\nu} \left( \frac{p_1^{\mu}}{z_1 - z_3} + \frac{p_2^{\mu}}{z_2 - z_3} \right)$$

$$\left( \frac{p_1^{\nu}}{z_1 - z_3} + \frac{p_2^{\nu}}{z_2 - z_3} \right) |z_1 - z_2|^{-2} |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$= \epsilon_{\mu\nu} p_1^{\mu} p_2^{\nu} \left| \frac{1}{z_1 - z_3} + \frac{1}{z_2 - z_3} \right| \left| \frac{1}{z_1 - z_2} \right|$$

$$\delta(p_1 + p_2 + p_3)$$

$$|p_1\rangle \otimes |g\rangle$$

$$|p_2\rangle \otimes |g\rangle$$

$$\epsilon_{\mu\nu} a_{-1}^{\mu} \bar{a}_{-1}^{\nu} |p_3\rangle \otimes |g\rangle$$

$$\sum p_i = 0$$

$$2p_1 p_2 = -4$$

$$2p_2 p_3 = 0$$

$$p_1^2 = 2$$

$$p_2^2 = 2$$

$$p_3^2 = 0$$

$$\epsilon_{\mu\nu} p_3^{\mu} = 0$$

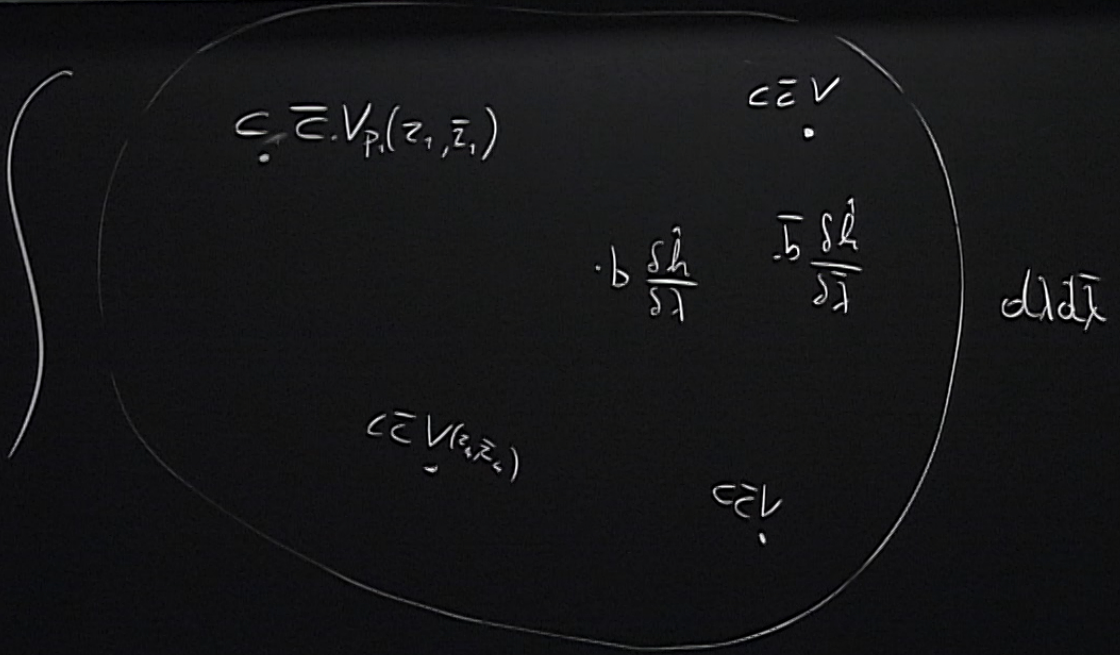


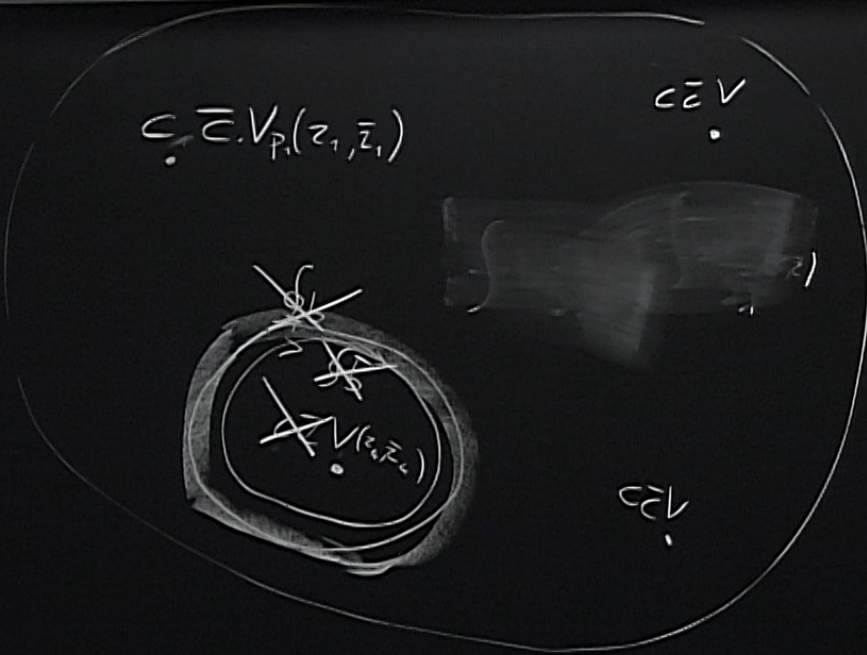
$$A_{TTT} = \epsilon_{\mu\nu} \left( \frac{p_1^{\mu}}{z_1 - z_3} + \frac{p_2^{\mu}}{z_2 - z_3} \right)$$

$$\left( \frac{p_1^{\nu}}{z_1 - z_3} + \frac{p_2^{\nu}}{z_2 - z_3} \right) |z_1 - z_2|^{-2} |z_1 - z_3|^2 |z_2 - z_3|^2$$

$$= \epsilon_{\mu\nu} p_1^{\mu} p_2^{\nu} \sqrt{\frac{1}{z_1 - z_3} + \frac{1}{z_2 - z_3}} \delta(p_1 + p_2 + p_3)$$

$$J = \partial X \quad \bar{J} = \bar{\partial} X$$





$$\mathcal{J} = \partial X \quad \bar{\mathcal{J}} = \bar{\partial} X$$

$$b(z) c(w) \sim \frac{1}{z-w} +$$

$$dz_1, d\bar{z}_1$$

CAUTION  
 Do not touch the chalkboard  
 when it is being used.

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$$A_{4,0}^{\text{T T T T}} = \int dz_4 d\bar{z}_4 \langle c \bar{c} V_{p_1}(z_1, \bar{z}_1) c \bar{c} V_{p_2} c \bar{c} V_{p_3} V_{p_4}(z_4, \bar{z}_4) \rangle$$

$$= \prod_{1 \leq i < j \leq 3} |z_i - z_j|^{2 + 2p_i \cdot p_j} \int dz_4 \prod_{1 \leq i \leq 3} |z_4 - z_i|^{2p_4 \cdot p_i} \int (\text{divergent})$$

DIVERGE IF  $2p_4 \cdot p_i \leq -2$   
 $(p_4 + p_i)^2 \leq 2$

$$2p_4 \cdot (p_1 + p_2 + p_3) = -2p_4^2 = -4$$



$$t = \frac{z_4 - z_1}{z_4 - z_3} \cdot \frac{z_2 - z_3}{z_2 - z_1}$$

$$A_{3,0} = \int dt |t|^{2p_4 \cdot p_1} |1-t|^{2p_4 \cdot p_2} \delta^{(26)}(\quad)$$

$$= \frac{\Gamma(1+p_4 \cdot p_1) \Gamma(1+p_4 \cdot p_2) \Gamma(1+p_4 \cdot p_3)}{\Gamma(-p_4 \cdot p_1) \Gamma(-p_4 \cdot p_2) \Gamma(-p_4 \cdot p_3)}$$

CAUTION

THE BOARD OR CHALK MAY BE DAMAGED BY  
HEAVY USE OR IN THE AREA OF THE BOARD

IF AN ACCIDENT OCCURS  
PLEASE REPORT TO THE  
APPROPRIATE OFFICE

AVOID EXCESSIVE USE

$$S = - (P_1 + P_2)^2 = -4 - 2P_1 P_2$$

$$t = - (P_1 + P_3)^2$$

$$u = - (P_1 + P_3)^2$$

$$A = \frac{\Gamma\left(-1 - \frac{S}{2}\right) \Gamma\left(-1 - \frac{t}{2}\right) \Gamma\left(-1 - \frac{u}{2}\right)}{\Gamma\left(2 + \frac{S}{2}\right) \Gamma\left(2 + \frac{t}{2}\right) \Gamma\left(2 + \frac{u}{2}\right)}$$

CAUTION

DO NOT TOUCH THE ELECTRIC WIRES  
BEHIND THE BOARD OF THE BOARD  
IT IS DANGEROUS TO TOUCH  
THEY ARE NOT INSULATED  
UNLESS OTHERWISE NOTED

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$$S = - (P_1 + P_2)^2 = -4 - 2P_1 P_2$$

$$t = - (P_1 + P_3)^2$$

$$u = - (P_1 + P_4)^2$$

$$A = \frac{\Gamma\left(-1 - \frac{S}{2}\right) \Gamma\left(-1 - \frac{t}{2}\right) \Gamma\left(-1 - \frac{u}{2}\right)}{\Gamma\left(2 + \frac{S}{2}\right) \Gamma\left(2 + \frac{t}{2}\right) \Gamma\left(2 + \frac{u}{2}\right)}$$

$$S = -2, 0, 2, 4, 6, \dots$$

$$t = -2, 0, 2, 4, \dots$$

$$u = -2, 0, 2, \dots$$

CAUTION

DO NOT TOUCH THE ELECTRIC PARTS  
OF THE BOARD OF THE BOARD  
IF IT IS NECESSARY TO DO SO  
PLEASE CONTACT THE  
TECHNICAL SUPPORT

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$$S = -2, 0, 2, 4, 6, \dots$$

$$t = -2, 0, 2, 4, \dots$$

$$u = -2, 0, 2, \dots$$

$$A = \frac{\Gamma\left(-1 - \frac{S}{2}\right) \Gamma\left(-1 - \frac{t}{2}\right) \Gamma\left(-1 - \frac{u}{2}\right)}{\Gamma\left(2 + \frac{S}{2}\right) \Gamma\left(2 + \frac{t}{2}\right) \Gamma\left(2 + \frac{u}{2}\right)}$$

$$\text{Res } A = \prod_{\substack{n=2 \\ P_4 P_1 = -n}}^{\infty} (P_4 - P_2 - n) (P_4 - P_3 - n)$$

CAUTION  
DO NOT TOUCH THE ELECTRIC PARTS  
OF THE BOARD OF THE BOARD  
IF IT IS NECESSARY TO ASSESS  
THE BOARD THROUGH THE  
APPROPRIATE ROUTE

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$$s = -(p_1 + p_2)^2 = -4 - 2p_1 p_2$$

$$t = -(p_1 + p_3)^2$$

$$u = -(p_1 + p_4)^2$$

$$s = -2, 0, 2, 4, 6, \dots$$

$$t = -2, 0, 2, 4, \dots$$

$$u = -2, 0, 2, \dots$$

$$A = \frac{\Gamma(-1 - \frac{s}{2}) \Gamma(-1 - \frac{t}{2}) \Gamma(-1 - \frac{u}{2})}{\Gamma(z + \frac{s}{2}) \Gamma(z + \frac{t}{2}) \Gamma(z + \frac{u}{2})}$$

$$\text{Res}_A = \prod_{k=0}^{n-1} (p_4 - p_2 - k) (p_4 - p_3 - k)$$

$k=0$

$$A \sim e^{-s \ln s - t \ln t - u \ln u}$$

$s \rightarrow \infty$   
 $t \rightarrow \infty$   
 $u \rightarrow \infty$

$z_1 - z_2$   $z_2 - z_3$   $z_3 - z_1$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$$

$z_1 - z_2$   $z_2 - z_3$   $z_3 - z_1$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X, \Lambda)$$

$$\frac{1}{\kappa} \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X, \Lambda) =$$

$z_3 / (z_1 - z_2) \quad z_2 - z_3 / (z_1 - z_2)$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X, \Lambda)$$

$$\Lambda \frac{\delta}{\delta \Lambda} G_{\mu\nu}(X, \Lambda) = \tilde{R}_{\mu\nu}[G]$$



$z_3 / (z_1 - z_2 \quad z_2 - z_3)$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X, \Lambda)$$

$$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X, \Lambda) = \tilde{R}_{\mu\nu}^{\Lambda}[G]$$

$z_3 / (z_1 - z_2) \quad z_2 - z_3 / (z_1 - z_2)$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu (\eta_{\mu\nu} + \delta G_{\mu\nu})$$

$$\lambda \frac{\partial}{\partial \lambda} G_{\mu\nu}(X, \lambda) = \bar{T}^{\mu\nu}[G]$$

$z_3 / (z_1 - z_2) \quad z_2 - z_3 / (z_1 - z_2)$

$$\int DX Dg \int \sqrt{|h|} h^{ab} \partial_a X^\mu \partial_b X^\nu$$

$$\eta_{\mu\nu} \sum_{n!} \delta G_{\mu\nu} \partial X^\mu \partial X^\nu$$

$$\lambda \frac{\partial}{\partial \lambda} G_{\mu\nu}(X, \lambda) = \bar{T}^{\mu\nu}[G]$$