Title: PSI 2016/2017 String Theory (Review) - Lecture 9

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Abstract:

$$\mathcal{A}_n = \sum_g g_s^{2g-2+n} \int_{\mathcal{M}_{g,n}} \cdots$$

$$\mathcal{A}_3 = g_s \mathcal{A}_{3,0} + g_s^3 \mathcal{A}_{3,1} \cdots$$

$$\mathcal{A}_n = \sum_g g_s^{2g-2+n} \int_{\mathcal{M}_{g,n}} \cdots$$

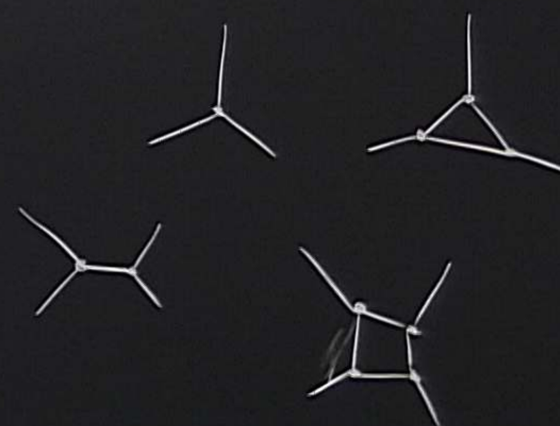
$$\mathcal{A}_3 = g_s \mathcal{A}_{3,0} + g_s^3 \mathcal{A}_{3,1} \cdots$$

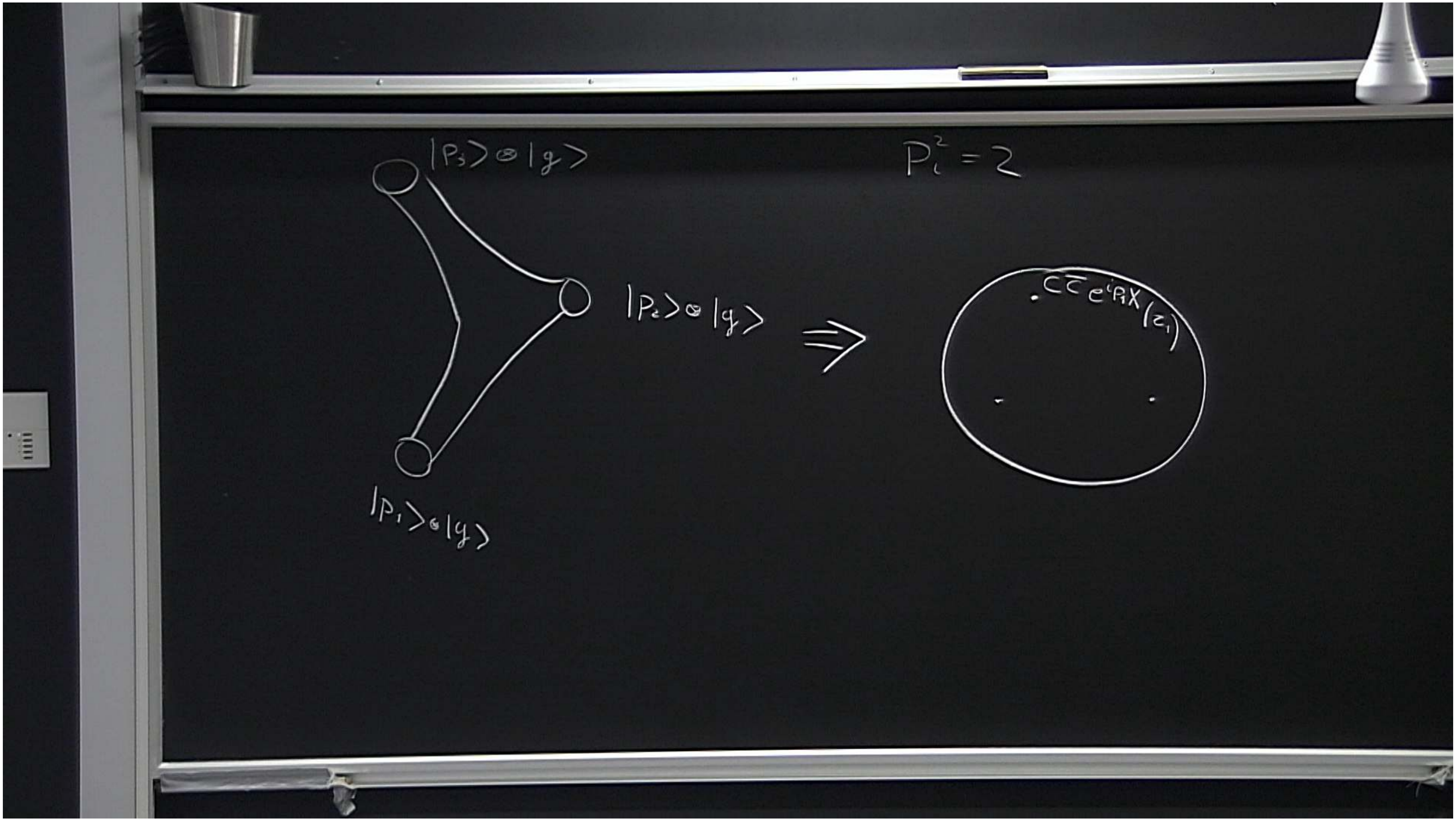
$$\mathcal{A}_4 = g_s^2 \mathcal{A}_{4,0} + \cdots$$

$$A_n = \sum_g g_s^{2g-2+n} \int_{\mathcal{M}_{g,n}} \cdots\cdots$$

$$A_3 = g_s A_{3,0} + g_s^3 A_{3,1} \cdots\cdots$$

$$A_4 = g_s^2 A_{4,0} + g_s^4 A_{4,1} + \cdots$$




$|P_3\rangle \otimes |g\rangle$
$P_i^2 = 2$
$|P_2\rangle \otimes |g\rangle$
$\Rightarrow$
$c\bar{c}\, e^{ip_1 X}(z_1)$
$|P_1\rangle \otimes |g\rangle$

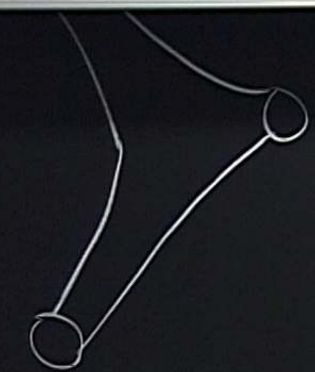
$$\langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3) \rangle = |z_1-z_2|^2 |z_1-z_3|^2 |z_2-z_3|^2$$

$$\langle e^{ip_1\cdot X(z_1,\bar{z}_1)}\, e^{ip_2\cdot X(z_2,\bar{z}_2)}\, e^{ip_3\cdot X(z_3,\bar{z}_3)} \rangle = |z_1-z_2|^{2p_1\cdot p_2} |z_1-z_3|^{2p_1\cdot p_3} |z_2-z_3|^{2p_2\cdot p_3}\, \delta^{26}(p_1+p_2+p_3)$$

$$J = \partial X \qquad \bar{J} = \bar{\partial} X$$

$$\langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3) \rangle = |z_1-z_2|^2 |z_1-z_3|^2 |z_2-z_3|^2$$

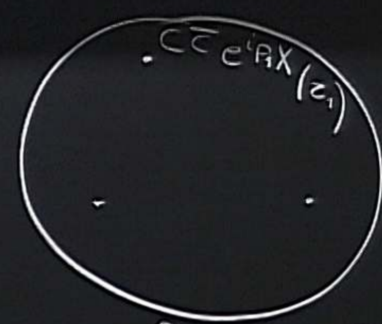
$$\langle e^{ip_1\cdot X(z_1,\bar{z}_1)}\, e^{ip_2\cdot X(z_2,\bar{z}_2)}\, e^{ip_3\cdot X(z_3,\bar{z}_3)} \rangle = |z_1-z_2|^{2p_1\cdot p_2} |z_1-z_3|^{2p_1\cdot p_3} \cdot |z_2-z_3|^{2p_2\cdot p_3}\, \delta^{26}(p_1+p_2+p_3)$$



$|p_2\rangle \otimes |g\rangle$

$|p_1\rangle \otimes |g\rangle$

$\Rightarrow$



$$z \to \frac{az+b}{cz+d}$$

$$= \cancel{|z_1 - z_2|^{\overset{0}{\overset{\shortparallel}{2+2p_1\cdot p_2}}}}\ \cancel{|\quad|^{\overset{0}{\shortparallel}}}\ \cancel{|\quad|^{\overset{0}{\shortparallel}}}\ \delta^{(26)}(p_1+p_2+p_3)$$

$$p_1^2 = 2 \qquad p_2^2 = 2 \qquad p_3^2 = 2$$

$$p_3 = -(p_1+p_2) \qquad 2 = 2+2+2p_1\cdot p_2$$

$$\langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3)\rangle = |z_1-z_2|^2 |z_1-z_3|^2 |z_2-z_3|^2$$

$$\langle e^{ip_1\cdot X(z_1,\bar{z}_1)}\, e^{ip_2\cdot X(z_2,\bar{z}_2)}\, e^{ip_3\cdot X(z_3,\bar{z}_3)}\rangle = |z_1-z_2|^{2p_1\cdot p_2} |z_1-z_3|^{2p_1\cdot p_3} \cdot |z_2-z_3|^{2p_2\cdot p_3}\, \delta^{26}(p_1+p_2+p_3)$$

$$\langle e^{ip_1X(z_1,\bar{z}_1)}\, e^{ip_2X(z_2,\bar{z}_2)}$$

$$\ldots c(z_3)\, c(z_3) \rangle = |z_1 - z_2|\,|z_1 - z_3|\,|z_2 - z_3|$$

$$\langle e^{ip_1\cdot X(z_1,\bar z_1)}\, e^{ip_2\cdot X(z_2,\bar z_2)}\, e^{ip_3\cdot X(z_3,\bar z_3)} \rangle = |z_1 - z_2|^{2p_1\cdot p_2}\,|z_1 - z_3|^{2p_1\cdot p_3}\,|z_2 - z_3|^{2p_2\cdot p_3}\,\delta^{26}(p_1+p_2+p_3)$$

$$\langle e^{ip_1 X(z_1,\bar z_1)}\, e^{ip_2 X(z_2,\bar z_2)} :\partial X^\mu \bar\partial X^\nu e^{ip_3 X(z_3,\bar z_3)} \rangle =$$

$$\langle c(z_1)\bar{c}(\bar{z}_1)\, c(z_2)\bar{c}(\bar{z}_2)\, c(z_3)\bar{c}(\bar{z}_3) \rangle = |z_1-z_2|^2 |z_1-z_3|^2 |z_2-z_3|^2$$

$$\langle e^{ip_1\cdot X(z_1,\bar{z}_1)}\, e^{ip_2\cdot X(z_2,\bar{z}_2)}\, e^{ip_3\cdot X(z_3,\bar{z}_3)} \rangle = |z_1-z_2|^{2p_1\cdot p_2} |z_1-z_3|^{2p_1\cdot p_3} |z_2-z_3|^{2p_2\cdot p_3} \delta^{26}(p_1+p_2+p_3)$$

$$\langle e^{ip_1X(z_1,\bar{z}_1)}\, e^{ip_2X(z_2,\bar{z}_2)}\, \epsilon_{\mu\nu} :\partial X^\mu \bar{\partial} X^\nu e^{ip_3X(z_3,\bar{z}_3)}: \rangle =$$

$$= \epsilon_{\mu\nu} \left(\frac{p_1^\mu}{z_1-z_3} + \frac{p_2^\mu}{z_2-z_3} \right) \left(\frac{p_1^\nu}{\bar{z}_1-\bar{z}_3} + \frac{p_2^\nu}{\bar{z}_2-\bar{z}_3} \right) |z_1-z_2|^{2p_1\cdot p_2} | \quad | \; | \quad | \, \delta(\quad)$$

$$|p_1\rangle \otimes |g\rangle$$

$$|p_2\rangle \otimes |g\rangle$$

$$\epsilon_{\mu\nu}\, a^{\mu}_{-1}\, \bar{a}^{\nu}_{-1}\, |p_3) \otimes |g\rangle$$

$$\sum p_i = 0$$

$$p_1^2 = 2$$

$$p_2^2 = 2$$

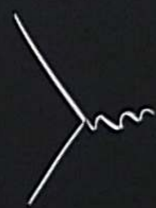
$$p_3^2 = 0$$

$$\epsilon_{\mu\nu} p_3^{\mu} = 0$$

$$A_{TTG} = \epsilon_{\mu\nu} \left(\frac{p_1^{\mu}}{z_1 - z_3} + \frac{p_2^{\mu}}{z_2 - z_3} \right) \left(\frac{p_1^{\nu}}{\bar{z}_1 - \bar{z}_3} + \frac{p_2^{\nu}}{\bar{z}_2 - \bar{z}_3} \right) |z_1 - z_2|^{-2} |z_1 - z_3|^{2} |z_2 - z_3|^{2}$$

$$= \epsilon_{\mu\nu} p_1^{\mu} p_2^{\nu} \cancel{\left| \frac{1}{z_1 - z_3} - \frac{1}{z_2 - z_3} \right| |\cdot| |\cdot| |\cdot|}$$

$$\delta(p_1 + p_2 + p_3)$$



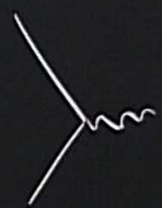
$$|p_1\rangle \otimes |g\rangle$$

$$|p_2\rangle \otimes |g\rangle$$

$$\epsilon_{\mu\nu}\, a^{\mu}_{-1}\, \bar{a}^{\nu}_{-1}\, |p_3) \otimes |g\rangle$$

$$\sum p_i = 0 \qquad 2p_1 p_2 = -4 \qquad 2p_2 p_3 = 0$$

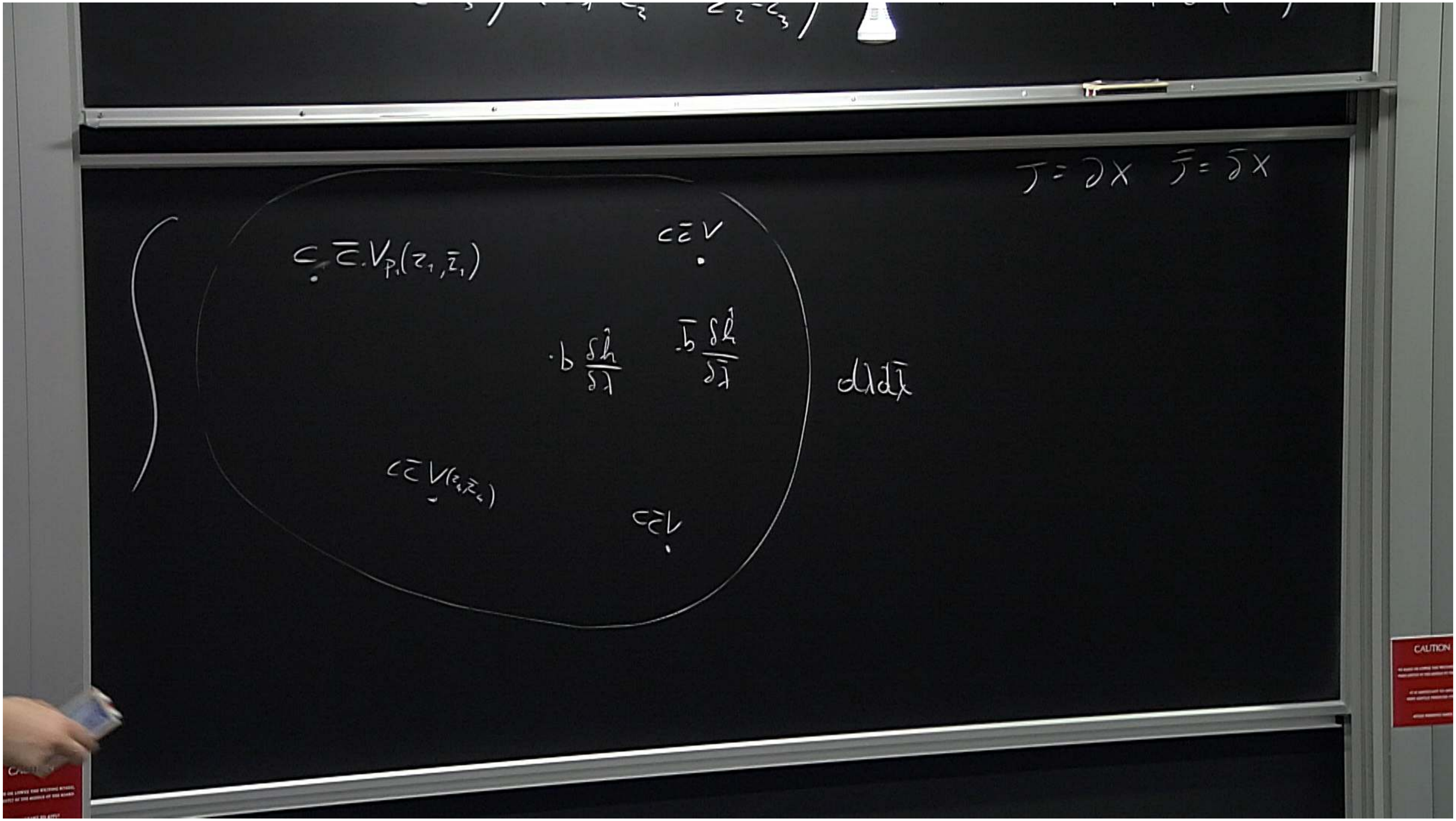
$$p_1^2 = 2 \qquad p_2^2 = 2 \qquad p_3^2 = 0 \qquad \epsilon_{\mu\nu} p_3^{\mu} = 0$$

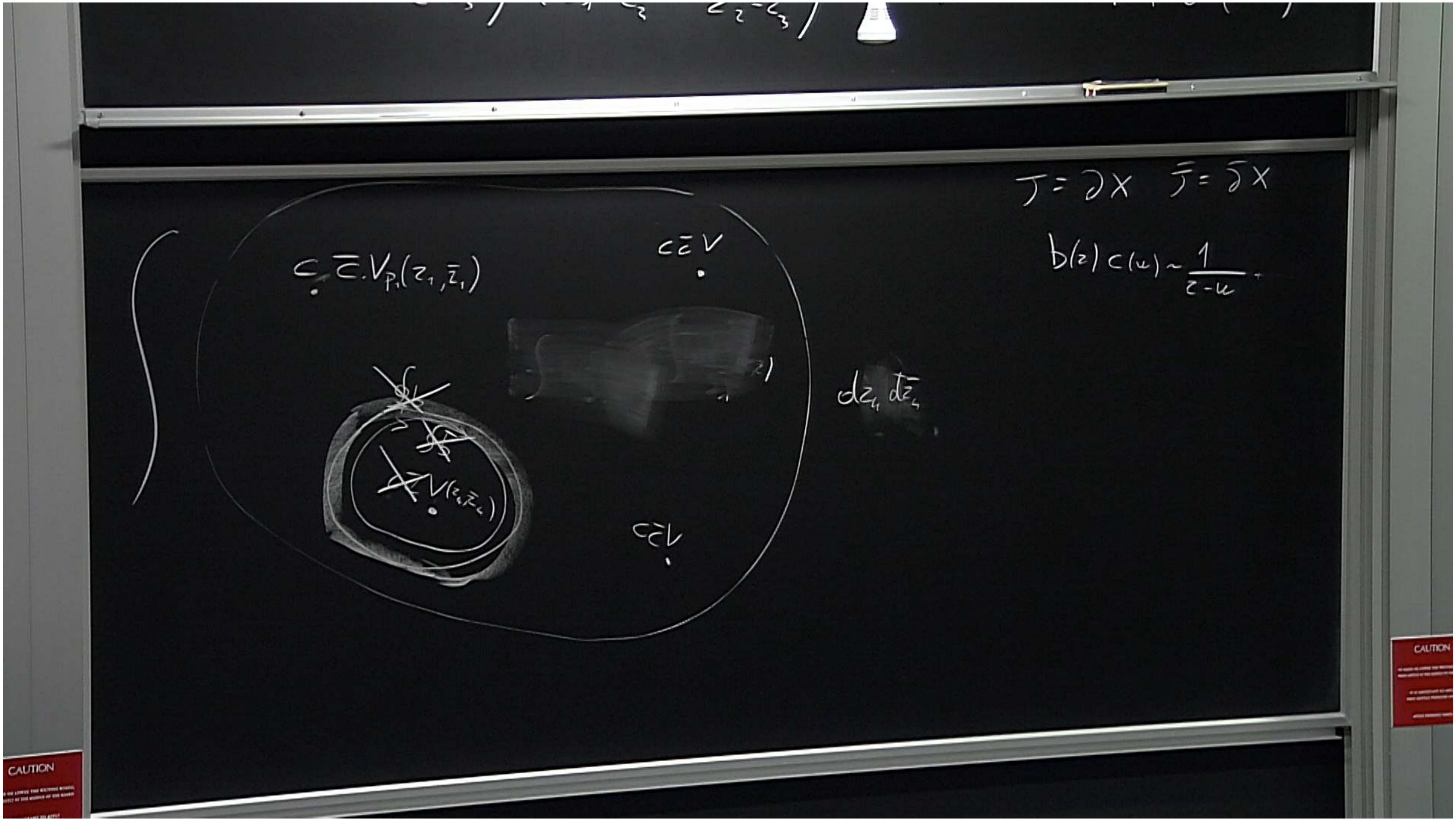


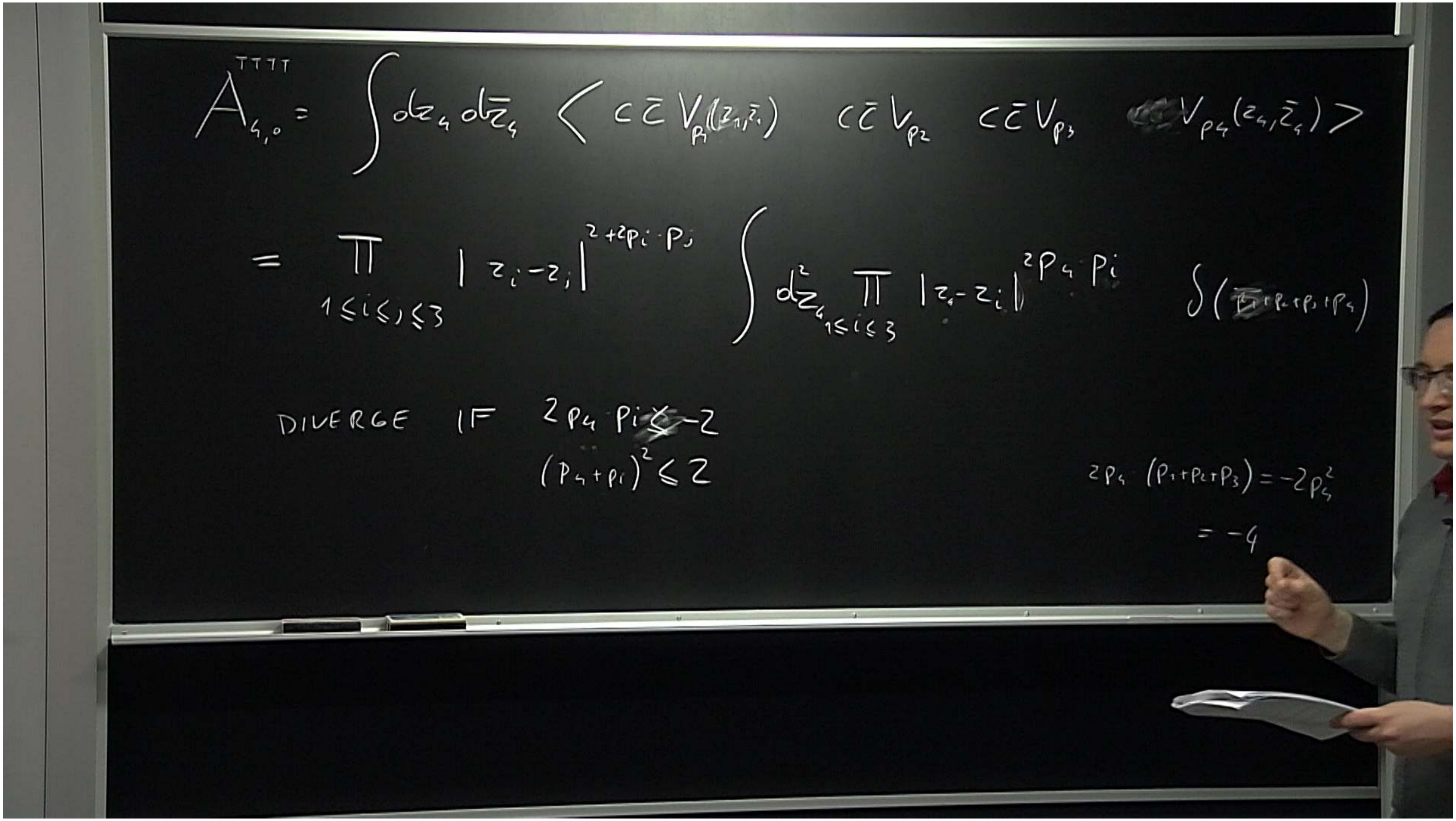
$$A_{TTG} = \epsilon_{\mu\nu}\left(\frac{p_1^{\mu}}{z_1 - z_3} + \frac{p_2^{\mu}}{z_2 - z_3}\right)\left(\frac{p_1^{\nu}}{\bar{z}_1 - \bar{z}_3} + \frac{p_2^{\nu}}{\bar{z}_2 - \bar{z}_3}\right)|z_1 - z_2|^{-2}|z_1 - z_3|^{2}|z_2 - z_3|^{2}$$

$$= \epsilon_{\mu\nu} p_1^{\mu} p_2^{\nu} \left|\frac{1}{z_1 - z_3} - \frac{1}{z_2 - z_3}\right|^2 \cdots$$

$$\delta(p_1 + p_2 + p_3)$$


J = ∂X   J̄ = ∂̄X
c c̄ V_{p_1}(z_1, z̄_1)
c c̄ V
b δh/δλ
b̄ δh/δλ̄
dλ dλ̄
c c̄ V(z_4, z̄_4)
c c̄ V
CAUTION


$J = \partial X \quad \bar{J} = \bar{\partial} X$
$b(z)\, c(w) \sim \frac{1}{z-w} +$
$c\, \bar{c}\, V_{p_1}(z_1, \bar{z}_1)$
$c \bar{c} V$
$dz_4\, d\bar{z}_4$
$V(z_4, \bar{z}_4)$
$c \bar{c} V$


$A^{TTTT}_{4,0} = \int dz_4\, d\bar{z}_4 \langle c\bar{c}V_{p_1}(z_1,\bar{z}_1)\; c\bar{c}V_{p_2}\; c\bar{c}V_{p_3}\; V_{p_4}(z_4,\bar{z}_4)\rangle$
$= \prod_{1\le i<j\le 3} |z_i - z_j|^{2+2p_i\cdot p_j} \int d^2 z_4 \prod_{1\le i\le 3} |z_4 - z_i|^{2p_4\cdot p_i}$
$\delta(p_1+p_2+p_3+p_4)$
DIVERGE IF $2p_4\cdot p_i \le -2$
$(p_4+p_i)^2 \le 2$
$2p_4\cdot(p_1+p_2+p_3) = -2p_4^2$
$= -4$

$$t = \frac{z_4 - z_1}{z_4 - z_3} \cdot \frac{z_2 - z_3}{z_2 - z_1}$$

$$A_{4,0} = \int d^2t \, |t|^{2p_4 \cdot p_1} \, |1-t|^{2p_4 \cdot p_2} \, \delta^{(26)}(\quad)$$

$$= \frac{\Gamma(1+p_4 \cdot p_1)\,\Gamma(1+p_4 \cdot p_2)\,\Gamma(1+p_4 \cdot p_3)}{\Gamma(-p_4 \cdot p_1)\,\Gamma(-p_4 \cdot p_2)\,\Gamma(-p_4 \cdot p_3)}$$

$$s = -(p_1+p_2)^2 = -4 - 2p_1 \cdot p_2$$

$$t = -(p_1+p_3)^2$$

$$u = -(p_1+p_4)^2$$

$$A = \frac{\Gamma\left(-1-\frac{s}{2}\right)}{\Gamma\left(2+\frac{s}{2}\right)} \frac{\Gamma\left(-1-\frac{t}{2}\right)}{\Gamma\left(2+\frac{t}{2}\right)} \frac{\Gamma\left(-1-\frac{u}{2}\right)}{\Gamma\left(2+\frac{u}{2}\right)}$$

$$s = -(p_1+p_2)^2 = -4 - 2p_1 \cdot p_2$$

$$t = -(p_1+p_3)^2$$

$$u = -(p_1+p_4)^2$$

$$A = \frac{\Gamma\left(-1-\frac{s}{2}\right)}{\Gamma\left(2+\frac{s}{2}\right)} \frac{\Gamma\left(-1-\frac{t}{2}\right)}{\Gamma\left(2+\frac{t}{2}\right)} \frac{\Gamma\left(-1-\frac{u}{2}\right)}{\Gamma\left(2+\frac{u}{2}\right)}$$

$s = -2, 0, 2, 4, 6 \ldots$

$t = -2, 0, 2, 4 \ldots$

$u = -2, 0, 2 \ldots$

$$s = -(p_1+p_2)^2 = -4 - 2p_1\cdot p_2$$

$$t = -(p_1+p_3)^2$$

$$u = -(p_1+p_4)^2$$

$$s = -2, 0, 2, 4, 6 \ldots$$

$$t = -2, 0, 2, 4 \ldots$$

$$u = -2, 0, 2 \ldots$$

$$A = \frac{\Gamma\left(-1-\frac{s}{2}\right)}{\Gamma\left(2+\frac{s}{2}\right)} \frac{\Gamma\left(-1-\frac{t}{2}\right)}{\Gamma\left(2+\frac{t}{2}\right)} \frac{\Gamma\left(-1-\frac{u}{2}\right)}{\Gamma\left(2+\frac{u}{2}\right)}$$

$$\underset{p_4\cdot p_1 = -n}{\mathrm{Res}\, A} = \prod_{k=0}^{n-2} \left(p_4\cdot p_2 - k\right)\left(p_4\cdot p_3 - k\right)$$

$$s = -(p_1+p_2)^2 = -4 - 2p_1\cdot p_2$$

$$t = -(p_1+p_3)^2$$

$$u = -(p_1+p_4)^2$$

$$A = \frac{\Gamma\left(-1-\frac{s}{2}\right)}{\Gamma\left(2+\frac{s}{2}\right)}\frac{\Gamma\left(-1-\frac{t}{2}\right)}{\Gamma\left(2+\frac{t}{2}\right)}\frac{\Gamma\left(-1-\frac{u}{2}\right)}{\Gamma\left(2+\frac{u}{2}\right)}$$

$$\operatorname*{Res}_{p_4\cdot p_1=-n} A = \prod_{k=0}^{n-1}\left(p_4\cdot p_2-k\right)\left(p_4\cdot p_3-k\right)$$

$$s = -2, 0, 2, 4, 6 \ldots$$

$$t = -2, 0, 2, 4 \ldots$$

$$u = -2, 0, 2 \ldots$$

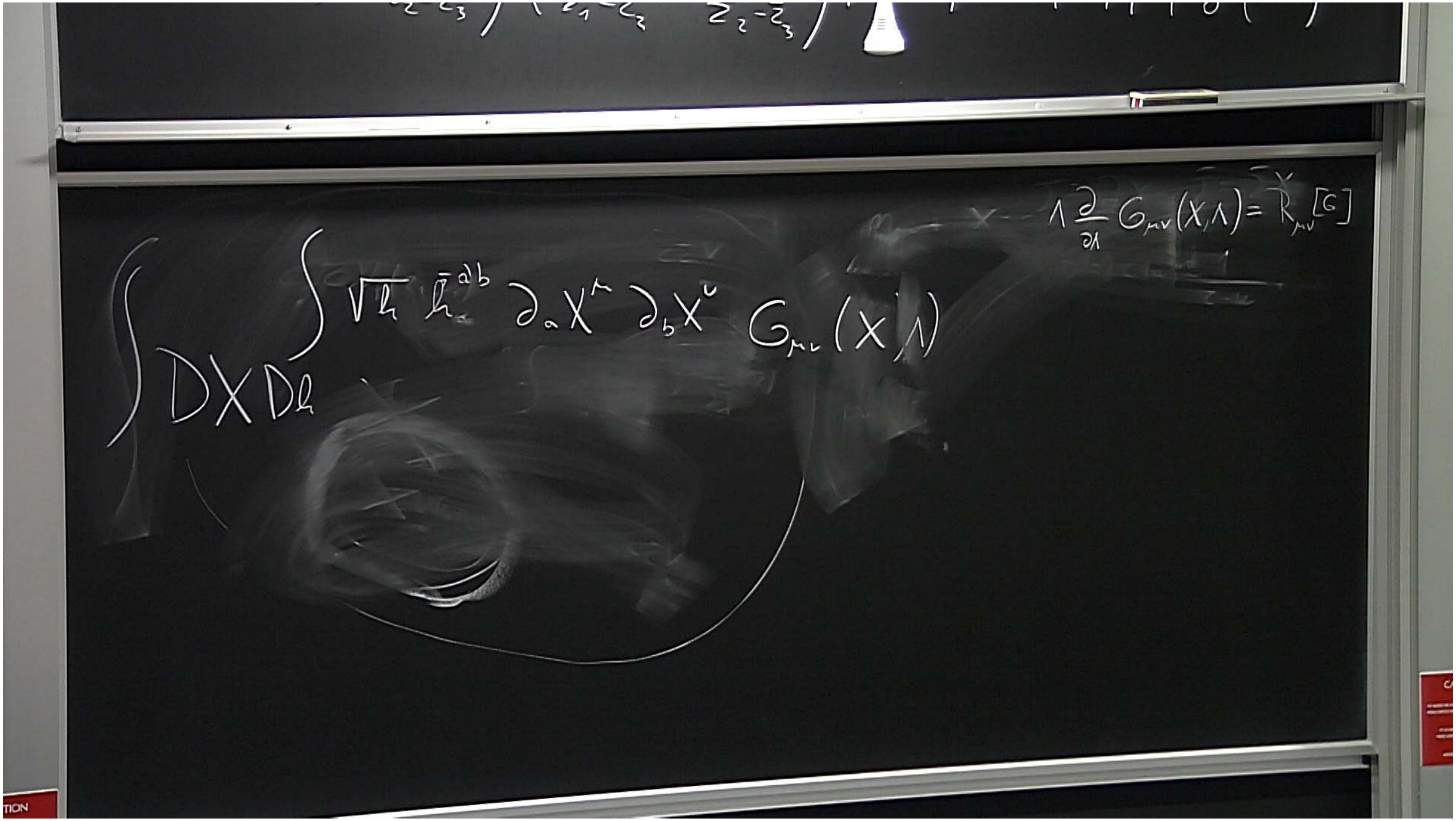
$$A \sim e^{-s\ln s - t\ln t - u\ln u}$$

$s\to\infty$
$u\to\infty$
$t\to\infty$

$$\int DX\,Dh \int \sqrt{h}\, h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu\, G_{\mu\nu}(X)$$

$$\int DX\, Dh \int \sqrt{h}\, h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu\, G_{\mu\nu}(X,\Lambda)$$

$$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X,\Lambda) =$$

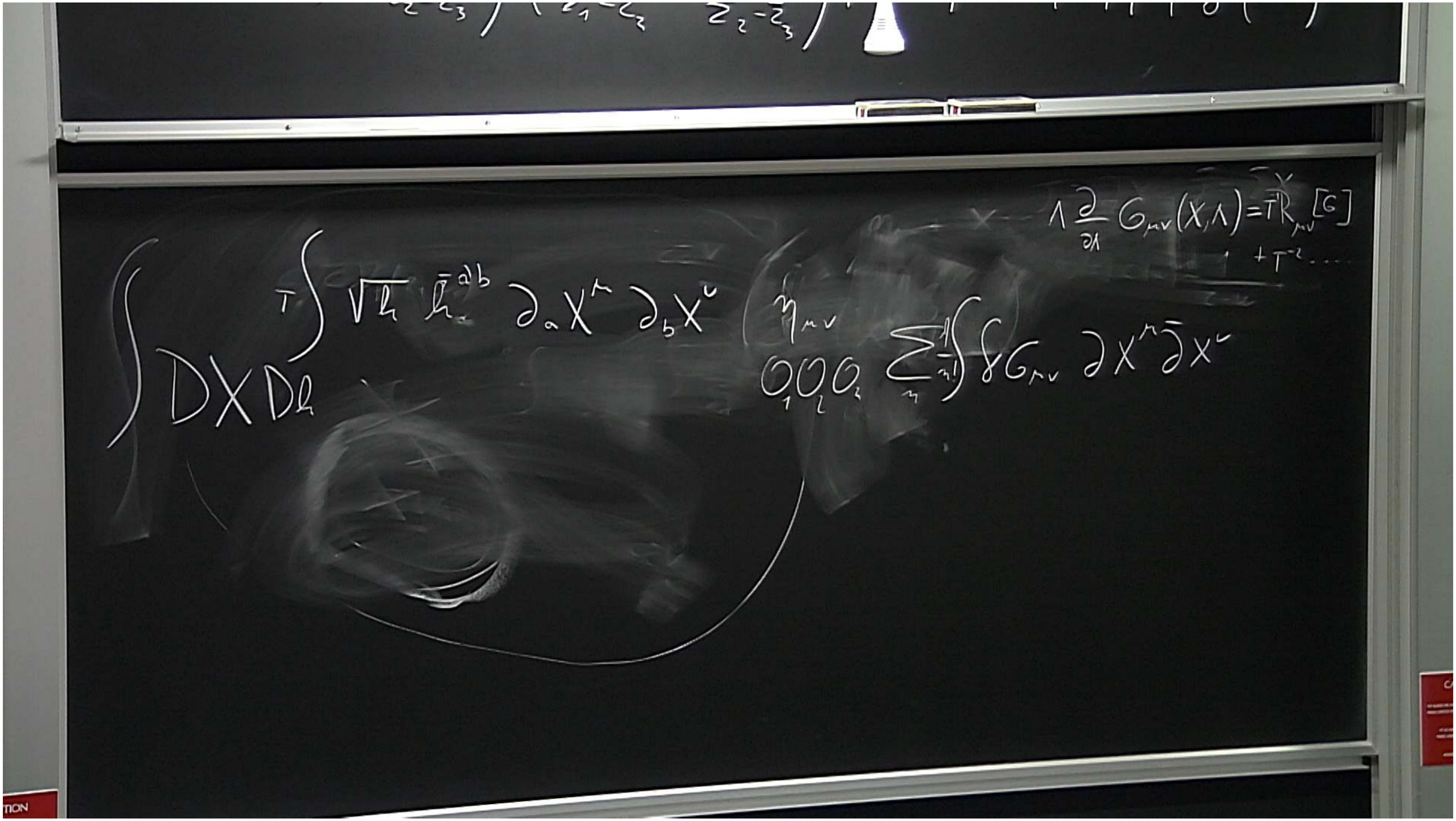

$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X,\Lambda) = R_{\mu\nu}[G]$
$\int DX\,Dh \int \sqrt{h}\, h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu\, G_{\mu\nu}(X,\Lambda)$

$$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X,\Lambda) = R_{\mu\nu}[G]$$

$$\int DX\,Dh \quad T\int \sqrt{h}\, h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu\, G_{\mu\nu}(X,\Lambda)$$

$$\int DX\,Dh\; e^{\,T\int \sqrt{h}\, h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu \left(\eta_{\mu\nu} + \delta G_{\mu\nu}\right)}$$

$$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X,\Lambda) = T^{-1} R_{\mu\nu}[G] + T^{-2} \ldots$$


$\Lambda \frac{\partial}{\partial \Lambda} G_{\mu\nu}(X,\Lambda) = T^{-1} R_{\mu\nu}[G]$
$+ T^{-2} \ldots$
$T \int \sqrt{h}\, h^{ab}\, \partial_a X^\mu \partial_b X^\nu \, \eta_{\mu\nu}$
$\int DX\, Dh$
$O_1 O_2 O_3 \sum_n \frac{1}{n!} \int \delta G_{\mu\nu}\, \partial X^\mu \bar{\partial} X^\nu$