Title: PSI 2016/2017 String Theory (Review) - Lecture 8

Date: Mar 03, 2017 10:00 AM

URL: http://pirsa.org/17030024

Abstract:

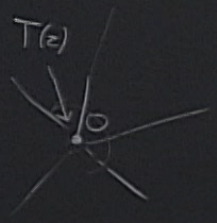
$z = e^{s} = e^{\tau - i\sigma}$

$L_{-1}|0\rangle = 0$
$\bar{L}_{-1}|0\rangle = 0$

$\frac{\partial}{\partial z} = e^{-s}\frac{\partial}{\partial s}$

TRANSLATIONS

z)



$T(z)$ DOES NOT DIVERGE AS $z \to 0$

s)

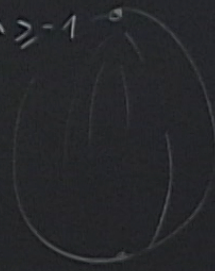
$L_n|0\rangle = 0 \quad n \geq -1$
$\bar{L}_n|0\rangle = 0 \quad n \geq -1$



$0 \to$ INFINITE PAST

$|0\rangle$

$1\!\!1 \longrightarrow |0\rangle$



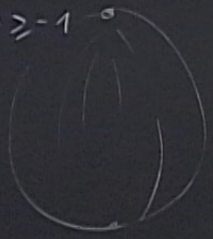
$|0\rangle = 0$

$\langle 0| = 0$

$L_n |0\rangle = 0 \quad n \geq -1$

$\bar{L}_n |0\rangle = 0 \quad n \geq -1$

$\Leftrightarrow$



$\frac{\partial}{\partial z} = e^{-s} \frac{\partial}{\partial s}$

$T(s) = -\frac{c}{24} + \sum L_n e^{-ns}$

$\mathcal{O}_N |0\rangle = ?$

$0 = L_{-1} \mathcal{O}_N |0\rangle = [L_{-1}, \mathcal{O}_N] |0\rangle$

$\propto \mathcal{O}_{N-1} |0\rangle$

$\mathcal{O}$ DIM Δ

$\mathcal{O}_{-0} |0\rangle \neq 0 \qquad \mathcal{O}_{n-0} |0\rangle = 0 \quad n > 0$

$|0\rangle$

SCALAR

$|g\rangle$

$b_{-1} \bar{b}_{-1} |g\rangle$

$= -\frac{c}{24} + \sum L_n e^{-nS}$

$|0\rangle = 0$

$|0\rangle = [L_{-1}, O_N]|0\rangle$

$\propto O_{N-1}|0\rangle$

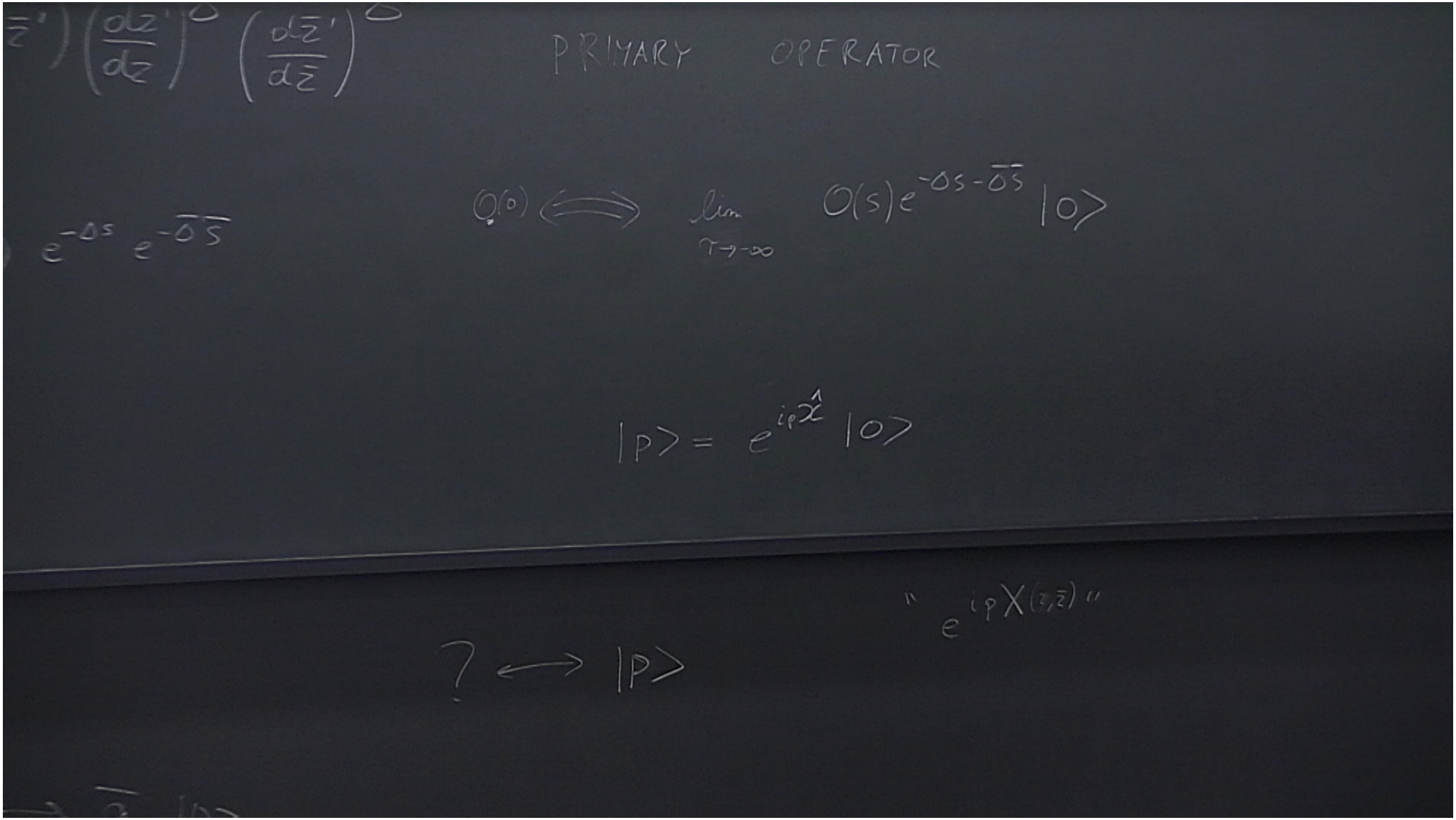
$O_{n-0}|0\rangle \neq 0 \quad O_{n-0}|0\rangle = 0 \quad n > 0$

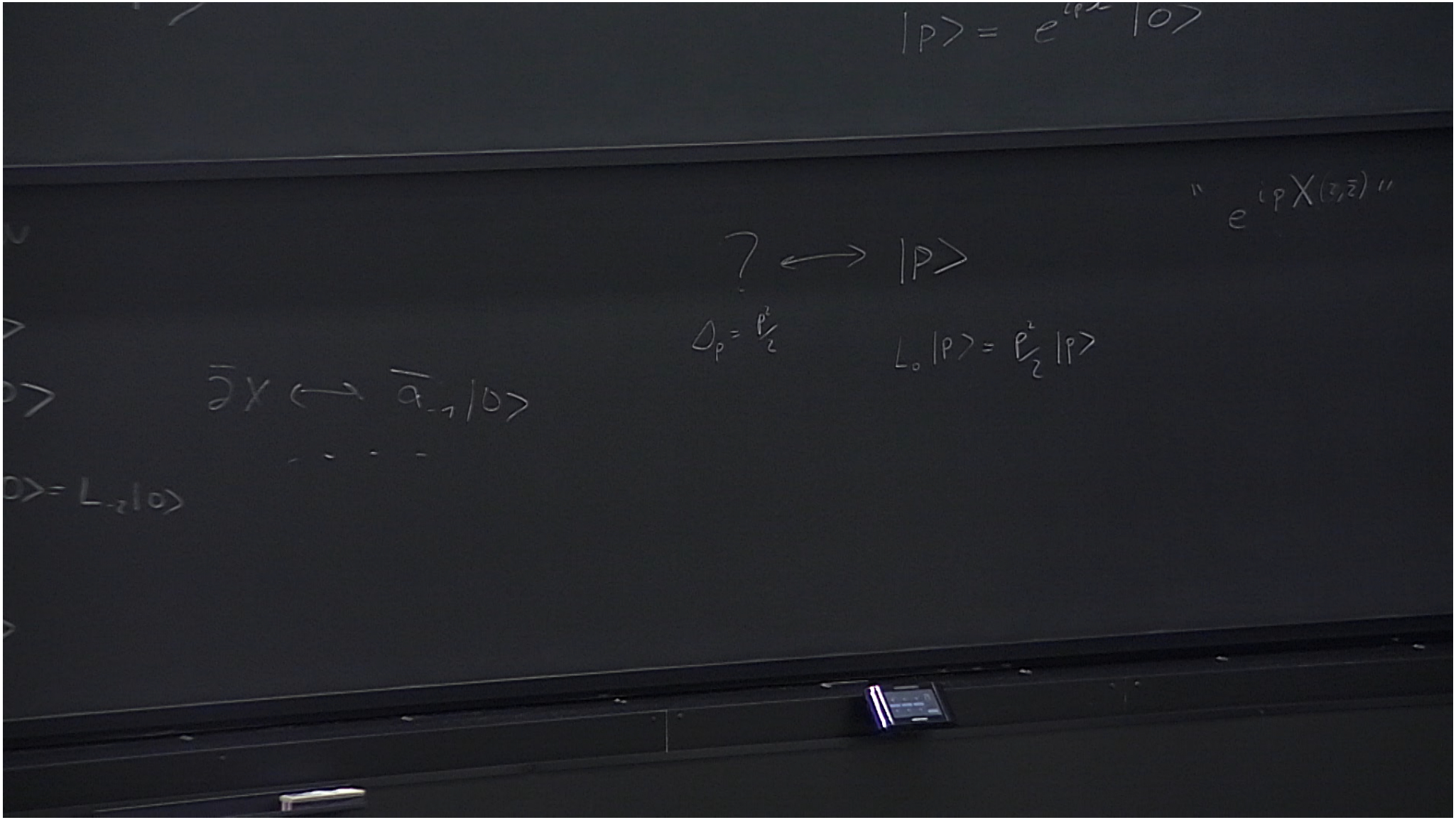
$|0\rangle$ $\qquad L_{-1}|p\rangle = p\, a_{-1}|p\rangle$

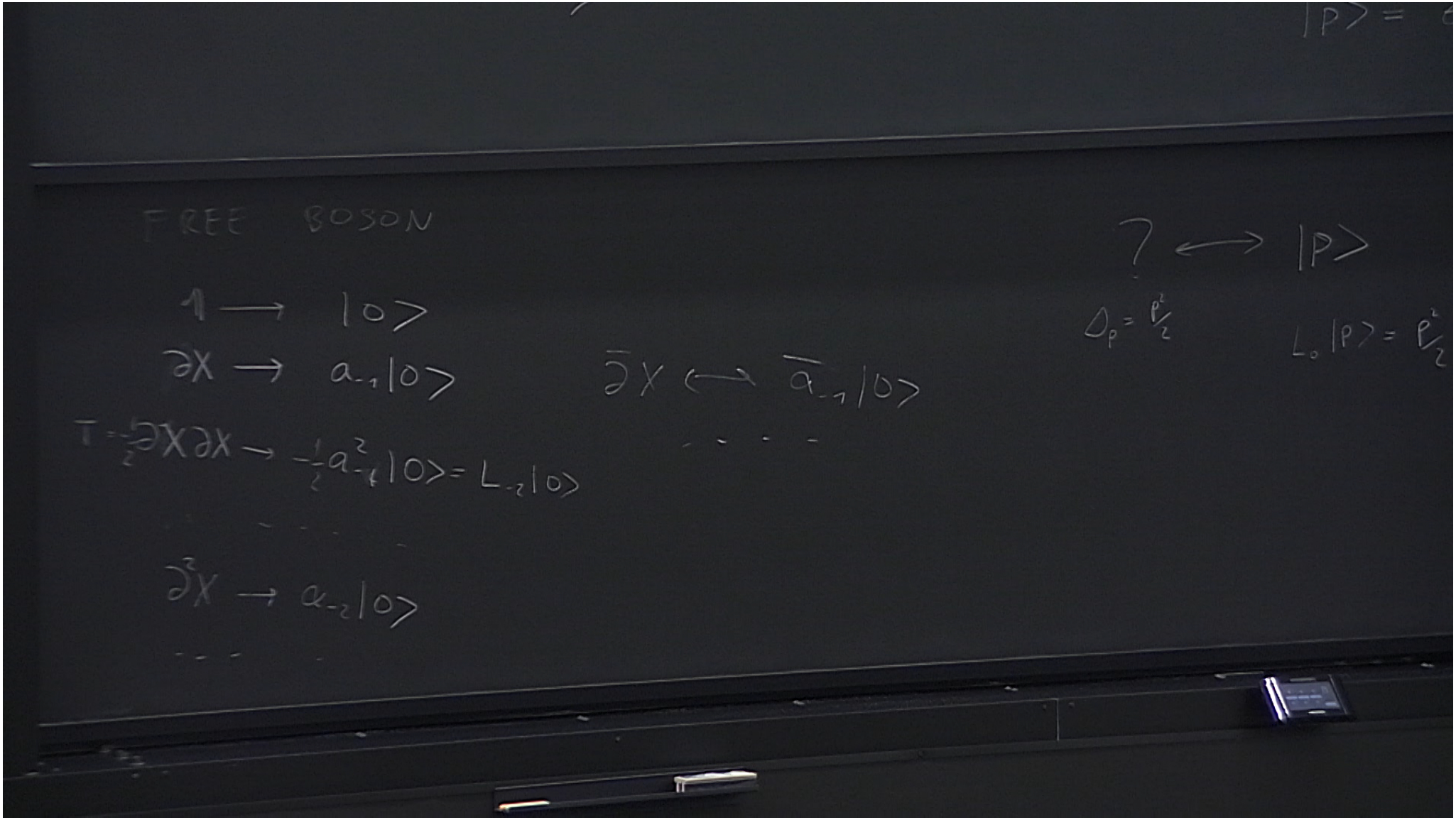
SCALAR $\qquad p = 0$

$|g\rangle$ NOT VACUUM !

$b_{-1}\bar{b}_{-1}|g\rangle = |0\rangle$ VACUUM !


PRIMARY OPERATOR
$\mathcal{O}(0) \Longleftrightarrow \lim_{\tau \to -\infty} \mathcal{O}(s) e^{-\Delta s - \bar{\Delta} s} |0\rangle$
$e^{-\Delta s}\, e^{-\bar{\Delta} s}$
$|p\rangle = e^{ip\hat{x}} |0\rangle$
" $e^{ipX(z,\bar{z})}$ "
$? \longleftrightarrow |p\rangle$

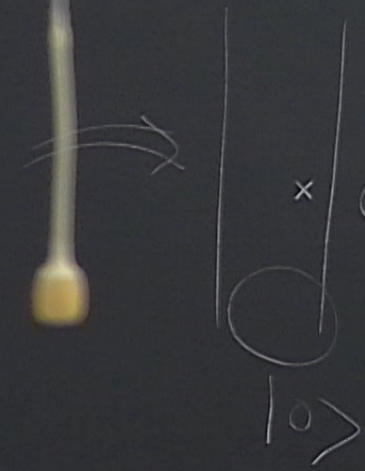

$|p\rangle = e^{ip\cdot x}|0\rangle$
" $e^{ipX(z,\bar{z})}$ "
$? \longleftrightarrow |p\rangle$
$\Delta_p = p^2/2$
$L_0|p\rangle = p^2/2\,|p\rangle$
$\partial X \longleftrightarrow \alpha_{-1}|0\rangle$
$|\rangle = L_{-2}|0\rangle$


FREE BOSON
$1 \longrightarrow |0\rangle$
$\partial X \to a_{-1}|0\rangle$
$\bar{\partial} X \hookrightarrow \bar{a}_{-1}|0\rangle$
$T = -\frac{1}{2}\partial X \partial X \to -\frac{1}{2}a_{-1}^2|0\rangle = L_{-2}|0\rangle$
$\partial^2 X \to a_{-2}|0\rangle$
$? \longleftrightarrow |p\rangle$
$\Delta_p = p^2/2$
$L_0|p\rangle = p^2/2$

$$\mathcal{O}_{\Delta,\bar{\Delta}}(z,\bar{z}) = \mathcal{O}_{\Delta,\bar{\Delta}}(z',\bar{z}')\left(\frac{dz'}{dz}\right)^{\Delta}\left(\frac{d\bar{z}'}{d\bar{z}}\right)^{\bar{\Delta}}$$

PRIMARY

$\mathcal{O}(z)$



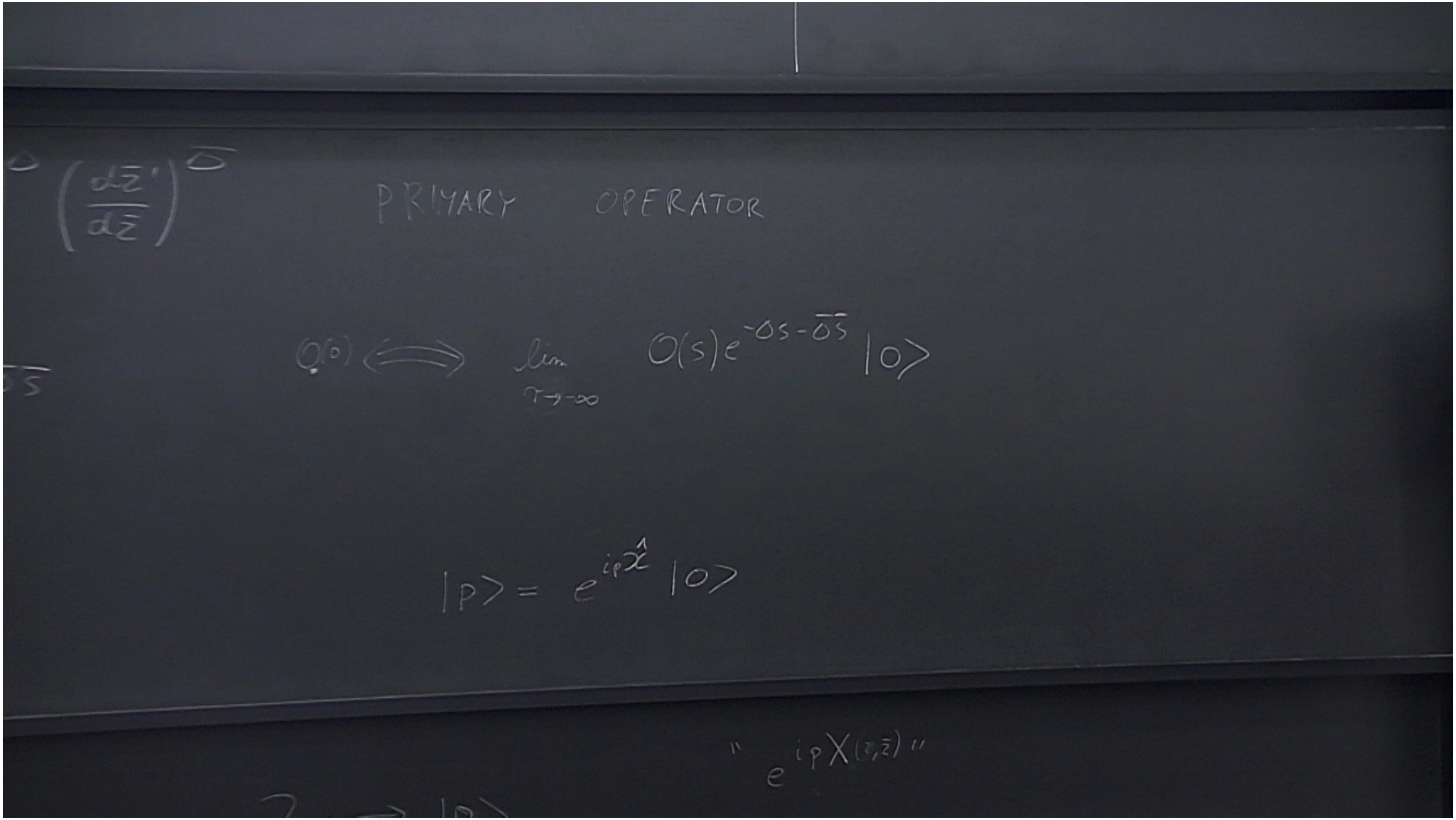
$\mathcal{O}(s)\ e^{-\Delta s}\ e^{-\bar{\Delta}\bar{s}}$

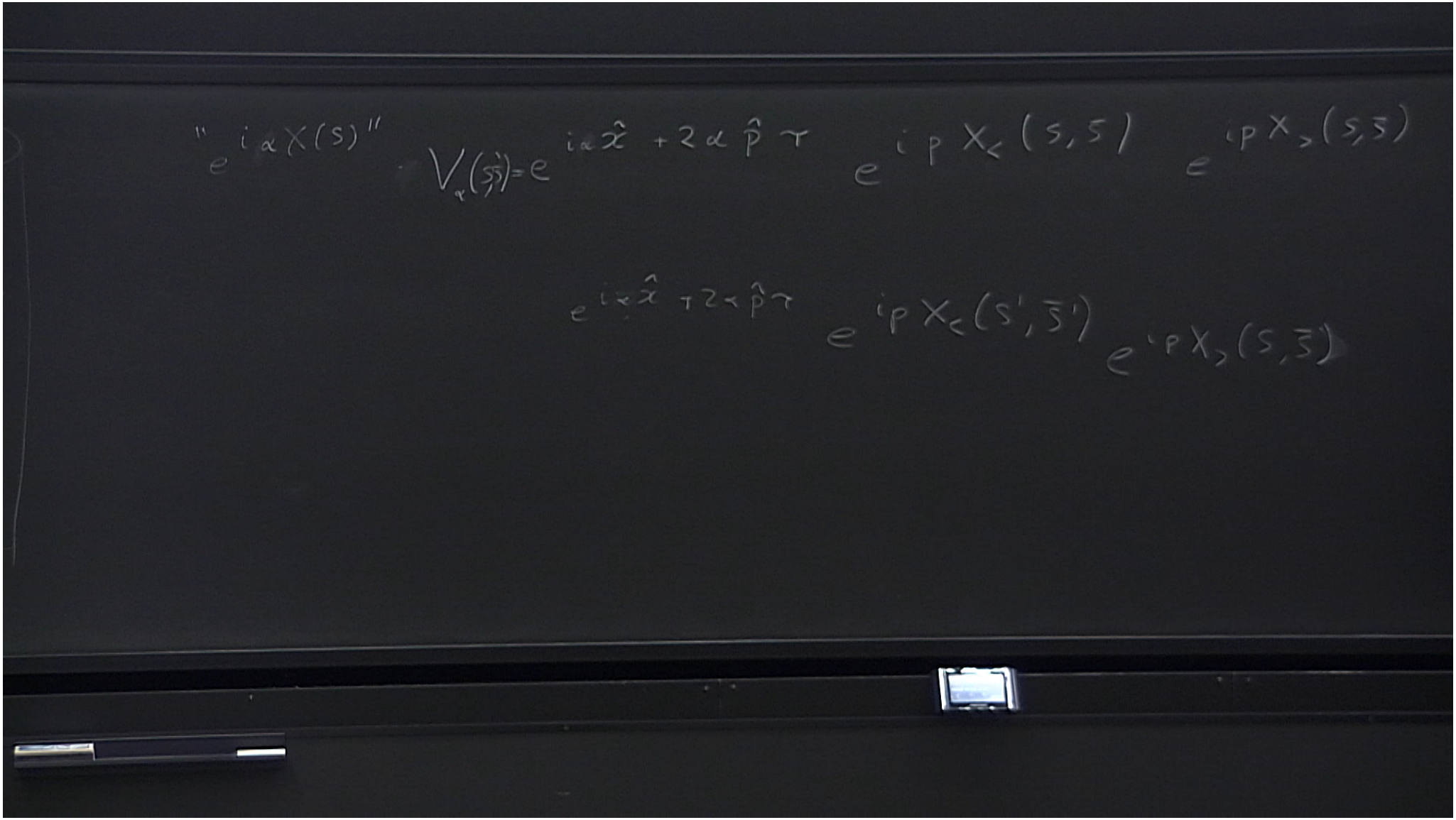
$\mathcal{O}(0) \Longleftrightarrow \lim_{\tau \to -\infty}$

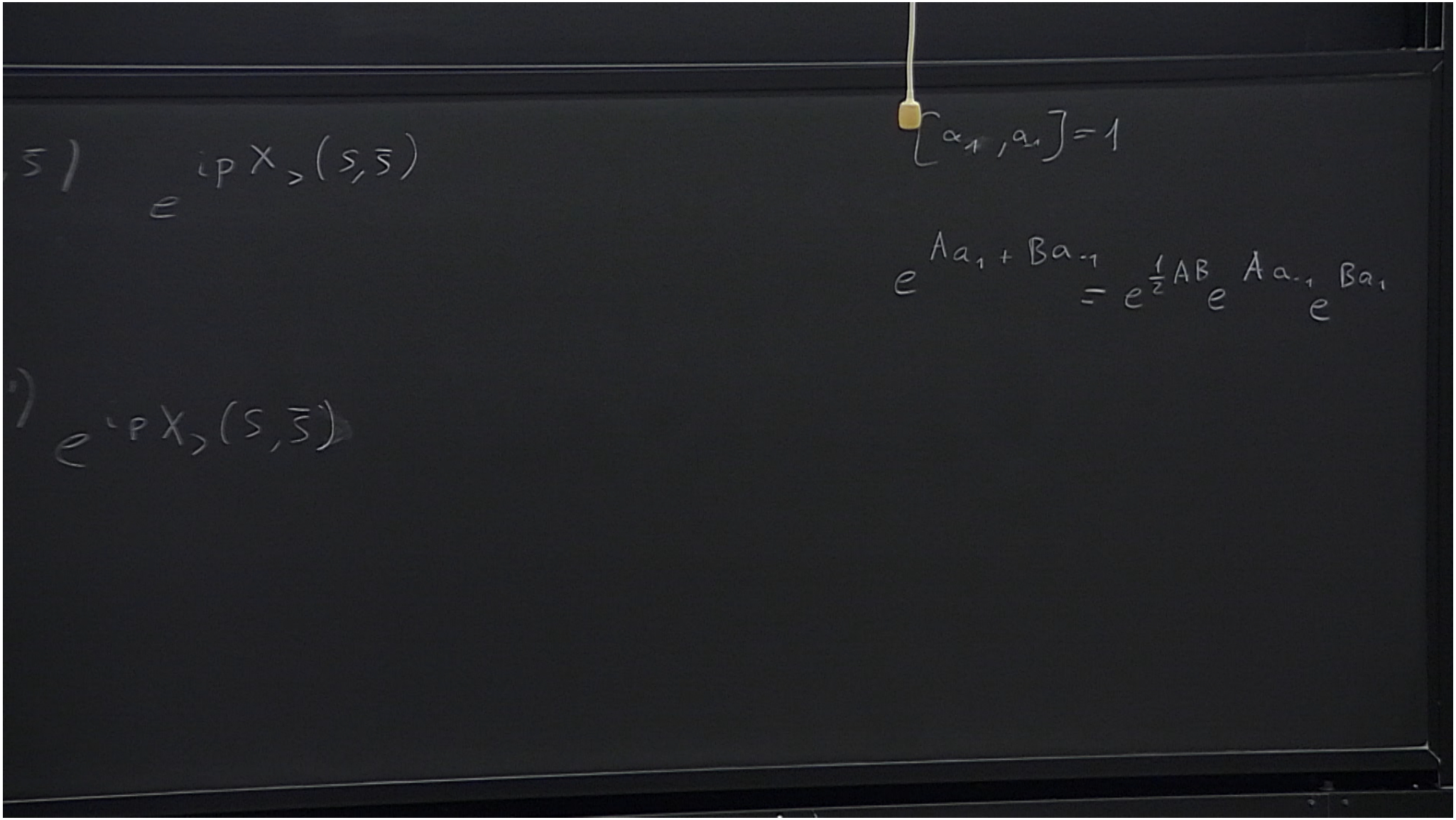
$|0\rangle$

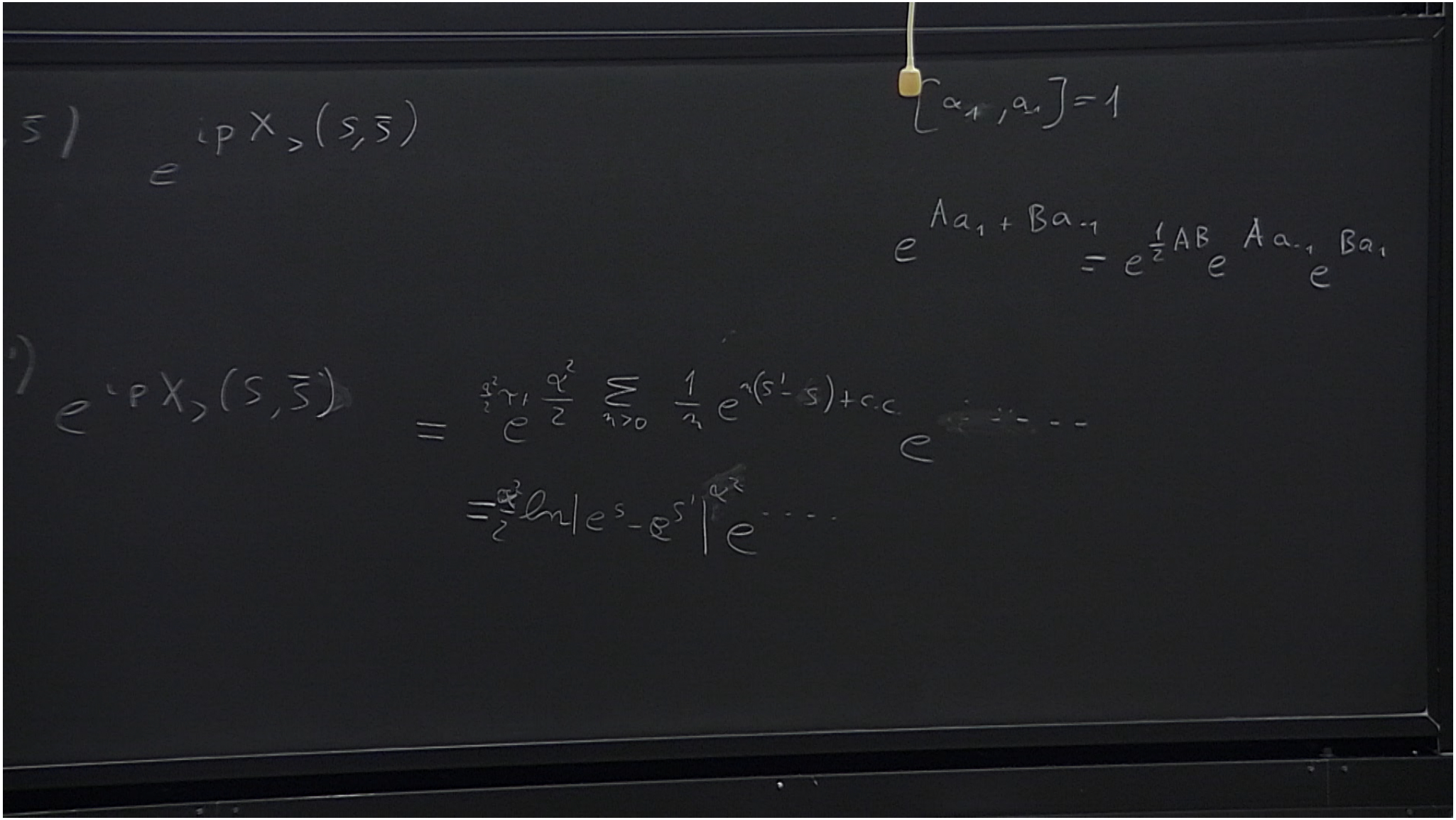
$|p\rangle =$

FREE BOSON


PRIMARY OPERATOR
$\lim_{\tau \to -\infty}$
$|p\rangle = e^{ip\hat{x}}|0\rangle$
" $e^{ipX(z,\bar{z})}$ "


"$e^{i\alpha X(s)}$"
$V_\alpha(s,\bar{s}) = e^{i\alpha\hat{x} + 2\alpha\hat{p}\tau} e^{ipX_L(s,\bar{s})} e^{ipX_R(s,\bar{s})}$
$e^{i\alpha\hat{x} + 2\alpha\hat{p}\tau} e^{ipX_L(s',\bar{s}')} e^{ipX_R(s,\bar{s})}$


$e^{ipX_>(s,\bar{s})}$
$[a_1, a_{-1}] = 1$
$e^{Aa_1 + Ba_{-1}} = e^{\frac{1}{2}AB} e^{Aa_{-1}} e^{Ba_1}$
$e^{ipX_>(s,\bar{s})}$


$[a_1, a_{-1}] = 1$
$e^{Aa_1 + Ba_{-1}} = e^{\frac{1}{2}AB} e^{Aa_{-1}} e^{Ba_1}$
$e^{ipX_>(s,\bar{s})}$
$= e^{\frac{q^2}{2}\gamma_1} e^{\frac{q^2}{2}} \sum_{\gamma>0} \frac{1}{\gamma} e^{\gamma(s'-s)+c.c.}$
$= e^{\frac{q^2}{2}\ln|e^s - e^{s'}|^2}$

$$[a_1, a_{-1}] = 1$$

$$e^{Aa_1 + Ba_{-1}} = e^{\frac{1}{2}AB} e^{Aa_{-1}} e^{Ba_1}$$

$$e^{ipX_>(s,\bar{s})}$$

$$e^{-ipX_>(s,\bar{s})} = e^{\frac{q^2}{2}\gamma_1} e^{\frac{q^2}{2}} \sum_{n>0} \frac{1}{n} e^{n(s'-s)+c.c.} e^{\cdots}$$

$$= e^{\frac{q^2}{2} \ln|e^s - e^{s'}|^2} = e^{-\ln|e^s - e^{s'}|^{q^2}} e^{\cdots}$$

$$\lim_{\epsilon \to 0} \int DX\, e^{S(X) + \sum_i i p_i \int d^2z\, f_\epsilon(z,\bar{z}, z_i, \bar{z}_i)\, X(z,\bar{z}) - \sum_q \frac{p_q^2}{2} \int dz_1 d\bar{z}_1 \, f_\epsilon(z_1, \bar{z}_1, z_q, \bar{z}_q)}$$

$$= \left\langle \prod_q V_{p_q}(z_q, \bar{z}_q) \right\rangle$$

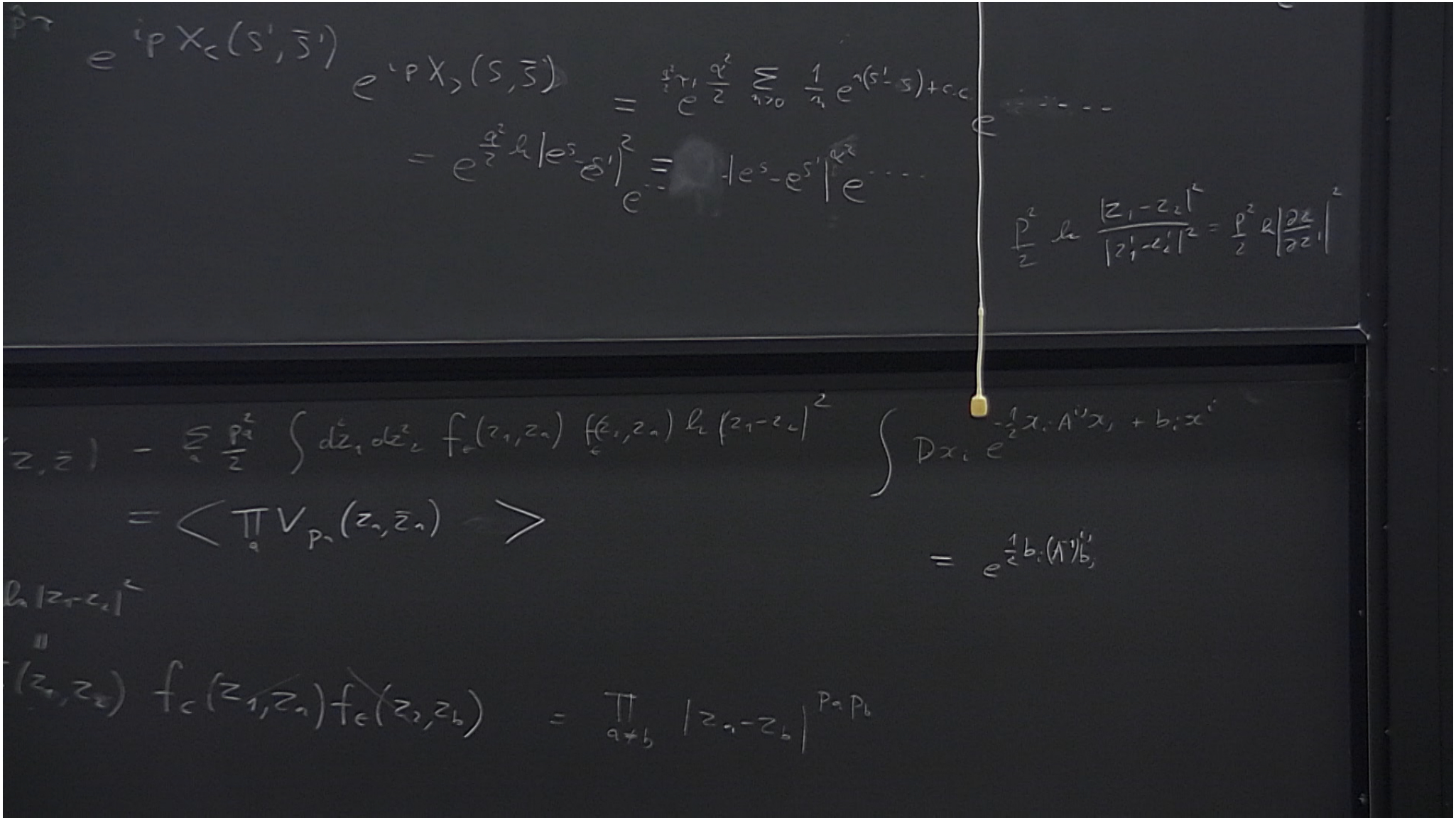
$$\int f_\epsilon \, d^2z = 1$$

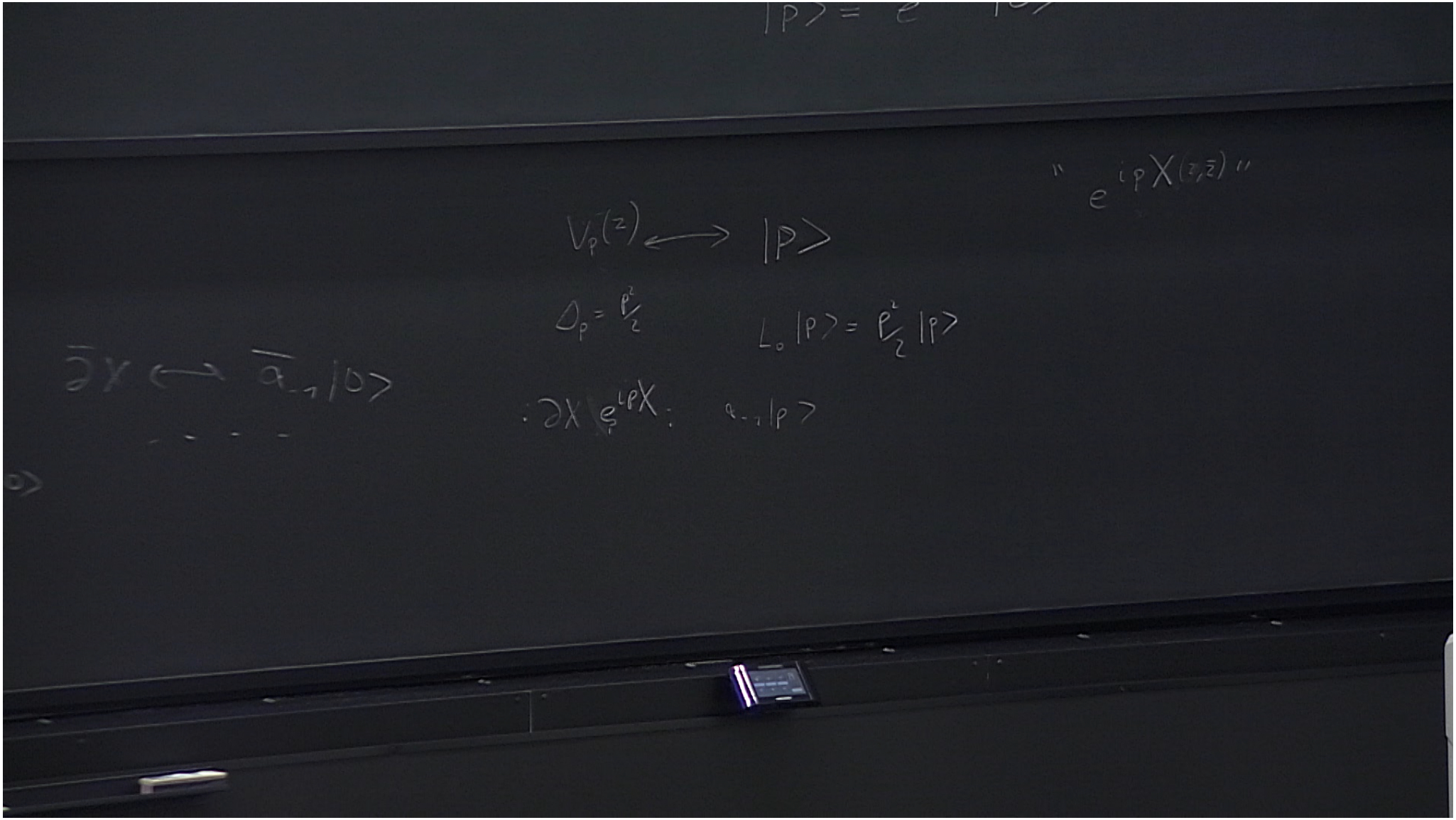
$$f_\epsilon = 0 \quad |z - z_i| > \epsilon$$

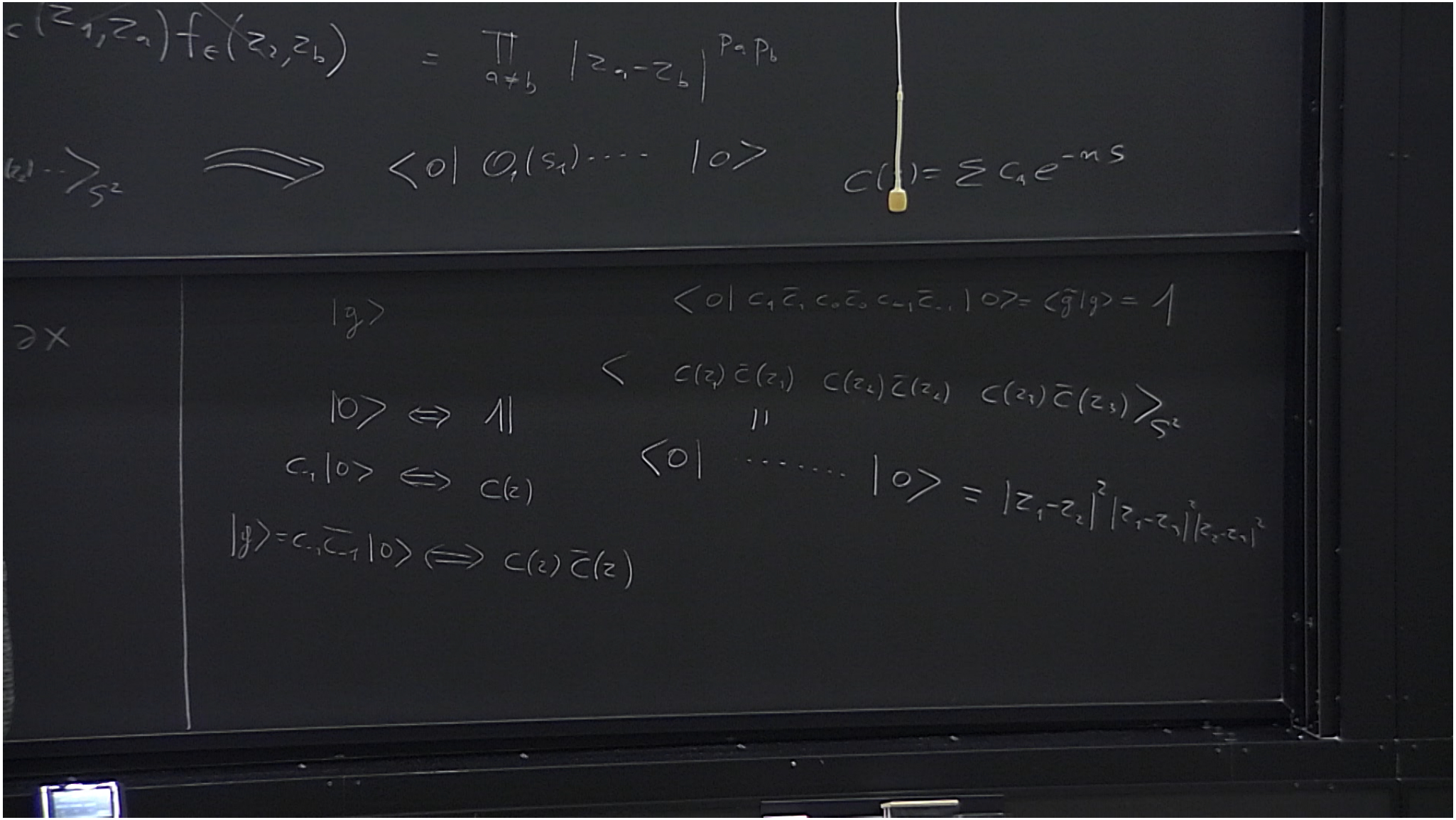
$$\| \qquad\qquad -h_0 |z_1 - z_2|^2$$

$$e^{-\sum_{a<b} p_a p_b \int dz_1 dz_2 \, G(z_1, z_2) f_\epsilon(z_1, z_a) f_\epsilon(z_2, z_b)}$$

$$\|$$

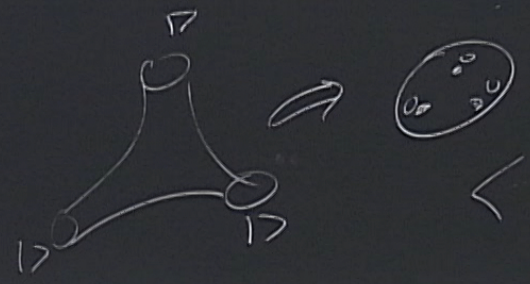

$e^{ipX_c(s',\bar{s}')}\, e^{ipX_3(s,\bar{s})} = e^{\frac{p^2}{2}\frac{q^2}{2}} \sum_{n>0} \frac{1}{n} e^{n(s'-s)} + c.c$
$= e^{\frac{q^2}{2}\ln|e^s - e^{s'}|^2} = |e^s - e^{s'}|^{q^2}$
$\frac{p^2}{2}\ln\frac{|z_1-z_2|^2}{|z_1'-z_2'|^2} = \frac{p^2}{2}\ln\left|\frac{\partial z}{\partial z'}\right|^2$
$(z,\bar{z}) - \sum_c \frac{p_c^2}{2}\int dz_1 d\bar{z}_1\, f_c(z_1,z_a)\, f_c(z_1,z_a)\ln|z_1-z_c|^2$
$= \langle \prod_a V_{p_a}(z_a,\bar{z}_a) \rangle$
$\int Dx_i\, e^{-\frac{1}{2}x_i A^{ij} x_j + b_i x^i}$
$= e^{\frac{1}{2}b_i (A^{-1})^{ij} b_j}$
$\ln|z_a-z_c|^2$
$(z_a,z_b)\, f_c(z_1,z_a) f_c(z_2,z_b) = \prod_{a\neq b} |z_a-z_b|^{p_a p_b}$


$|p\rangle = e$
" $e^{ipX(z,\bar{z})}$ "
$V_p(z) \longleftrightarrow |p\rangle$
$\Delta_p = p^2/2$
$L_0|p\rangle = p^2/2\,|p\rangle$
$\partial X \hookrightarrow \bar{a}_{-1}|0\rangle$
$:\partial X\, e^{ipX}:$
$a_{-1}|p\rangle$


$c(z_1,z_a) f_\epsilon(z_2,z_b) = \prod_{a\neq b} |z_a - z_b|^{P_a P_b}$
$\langle z_1 \ldots \rangle_{S^2} \Longrightarrow \langle 0| \mathcal{O}_1(s_1) \cdots |0\rangle$
$c(s) = \sum c_n e^{-ns}$
∂X
$|g\rangle$
$|0\rangle \Leftrightarrow 1$
$c_1|0\rangle \Leftrightarrow c(z)$
$|g\rangle = c_1 \bar{c}_1 |0\rangle \Leftrightarrow c(z)\bar{c}(z)$
$\langle 0| c_1 \bar{c}_1 c_0 \bar{c}_0 c_{-1} \bar{c}_{-1} |0\rangle = \langle \tilde{g}|g\rangle = 1$
$< c(z_1)\bar{c}(z_1)\ c(z_2)\bar{c}(z_2)\ c(z_3)\bar{c}(z_3) >_{S^2}$
$\langle 0| \cdots\cdots |0\rangle = |z_1 - z_2|^2 |z_1 - z_3|^2 |z_2 - z_3|^2$

$$e^{-\sum_{a\neq b} P_a P_b \int dz_1 dz_2 \, G(z_1,z_2)} \quad \| \quad f_\epsilon(z_1,z_a) f_\epsilon(z_2,z_b) = \prod_{a\neq b} |z_a - z_b|^{P_a P_b}$$

$|z - z_a| > \epsilon$



$$\langle O_1(z_1)\, O_2(z_2) \cdots \rangle_{S^2} \Longrightarrow \langle 0 | \, O_1(z_1) \cdots | 0 \rangle$$

$$\partial_{\bar z} \frac{1}{z-w} \sim \frac{P}{z-w} V_P(w)$$

$$\partial_{\bar w} \frac{1}{\bar z - \bar w} \sim \frac{P}{\bar z - \bar w} V_P(w,\bar w)$$

$$T = -\frac{1}{2} \partial X \, \partial X \qquad p^2 = 2$$

$$|P\rangle \otimes |g\rangle \Downarrow c\bar c e^{ipX}$$

$$\langle c\bar c e^{ip_1 X}(z_1) \; c\bar c e^{ip_2 X}(z_2) \; c\bar c e^{ip_3 X}(z_3) \rangle$$

$|g\rangle$ $\qquad \langle 0 | c_1 \bar c$

$|0\rangle \Leftrightarrow 1\!\!1 \qquad \langle c(z)\bar c(z_1)$

$c_1|0\rangle \Leftrightarrow c(z) \qquad \| \quad \langle 0 | \cdots$

$|g\rangle = c_1 \bar c_1 |0\rangle \Leftrightarrow c(z)\bar c(z)$