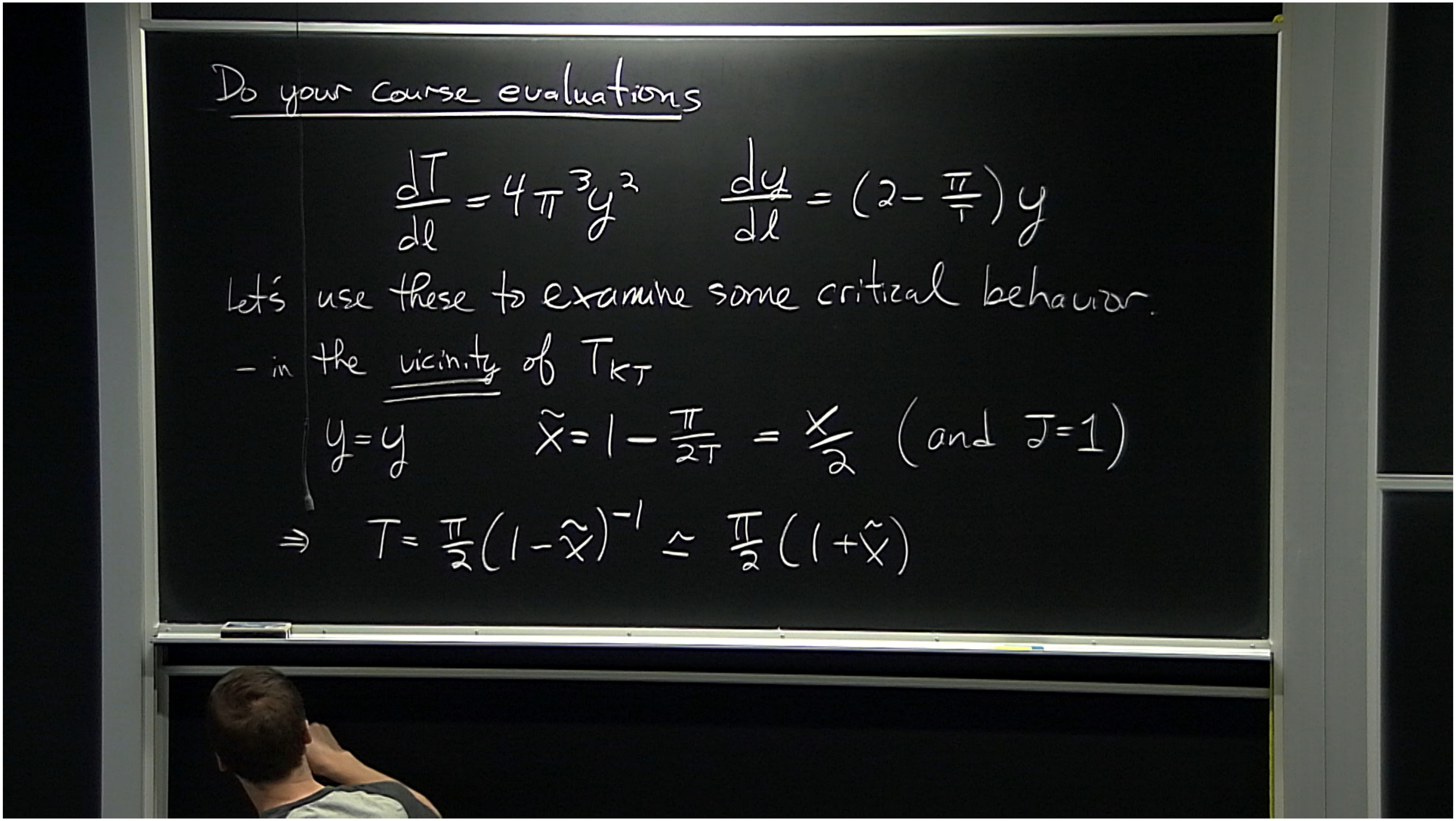
Title: 2016/2017 Statistical Mechanics 2 - Roger Melko - Lecture 26

Date: Mar 31, 2017 10:30 AM

URL: http://pirsa.org/17030010

Abstract:


Do your course evaluations
$\frac{dT}{dl} = 4\pi^3 y^2$ $\frac{dy}{dl} = (2 - \frac{\pi}{T}) y$
Let's use these to examine some critical behavior.
- in the vicinity of T_{KT}
$y = y$ $\tilde{x} = 1 - \frac{\pi}{2T} = \frac{x}{2}$ (and $J=1$)
$\Rightarrow T = \frac{\pi}{2}(1-\tilde{x})^{-1} \simeq \frac{\pi}{2}(1+\tilde{x})$

then $\frac{dT}{dl} = \frac{\pi}{2}\frac{d\tilde{x}}{dl} \Rightarrow$

$$\boxed{\begin{aligned} \frac{d\tilde{x}}{dl} &= 8\pi^2 y^2 \\ \frac{dy}{dl} &= -2\tilde{x}y \end{aligned}}$$

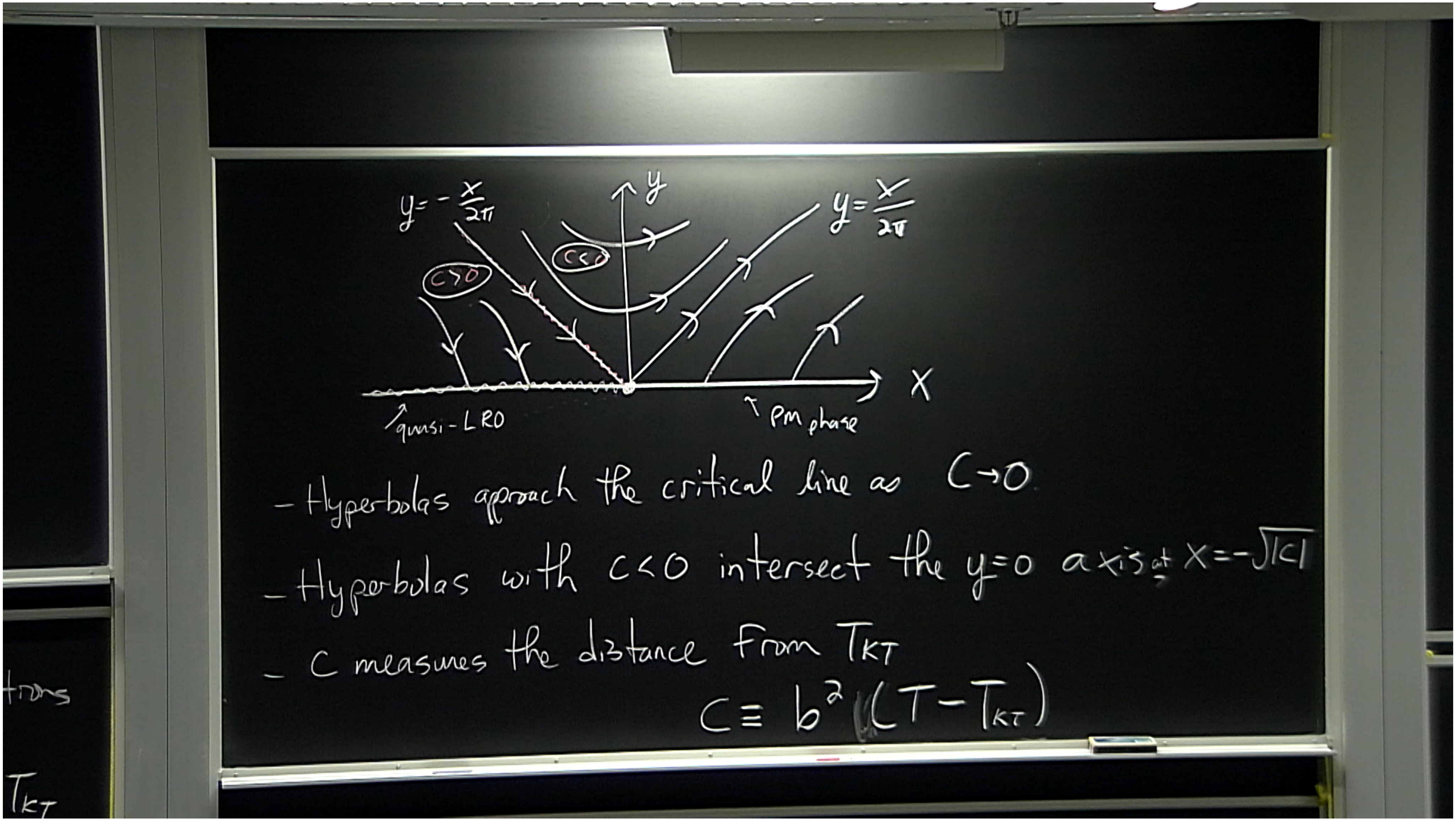
Kosterlitz RG equations

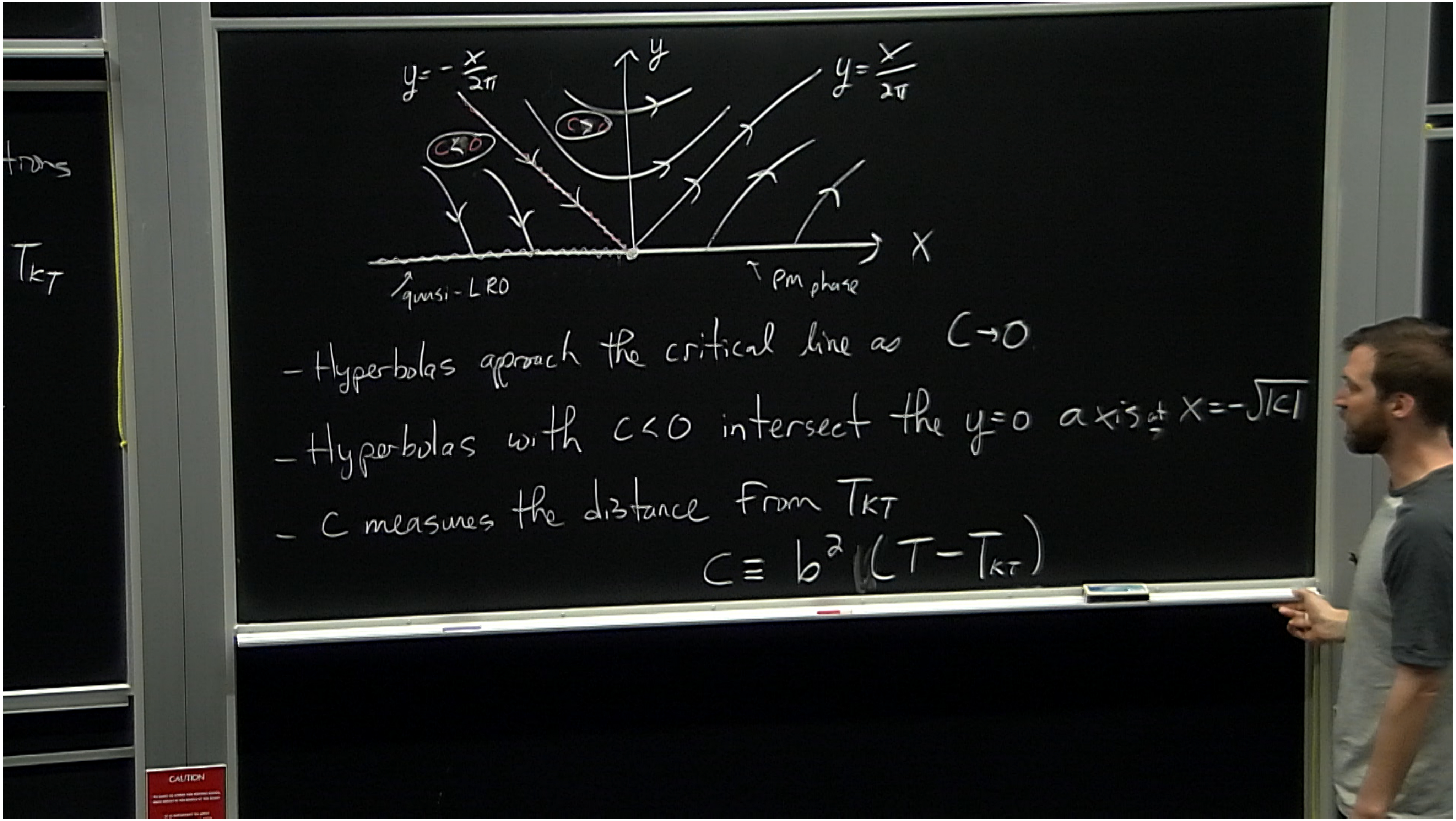
near $T = T_{KT}$

- Drop ~ on $\tilde{x}$, and eliminate l
- use the chain rule $\frac{d(x^2)}{dl} = 2x\frac{dx}{dl}$ etc to get

$$\frac{d(x^2)}{dl} = 16\pi^2 x y^2 \quad \text{and} \quad \frac{d(y^2)}{dl} = 4xy^2$$

giving $\frac{dx^2}{dy^2} = 4\pi^2$ or $y^2 = \frac{1}{4\pi^2}(x^2 + C)$


y = - x/2π
y
y = x/2π
c<0
c>0
x
quasi-LRO
PM phase
- Hyperbolas approach the critical line as c→0
- Hyperbolas with c<0 intersect the y=0 axis at x=-√|c|
- c measures the distance from T_KT
c ≡ b² (T - T_KT)



tions
T_KT
y = -x/2π
y
y = x/2π
c<0
c>0
X
quasi-LRO
pm phase
- Hyperbolas approach the critical line as C→0
- Hyperbolas with c<0 intersect the y=0 axis at x=-√|c|
- C measures the distance from T_KT
C ≡ b²(T−T_KT)

Let's use this to see how the correlation length diverges at T_{KT}

from $T < T_{KT}$, $C < 0$, $|C| = x^2 - 4\pi^2 y^2 = b^2(T - T_{KT})$

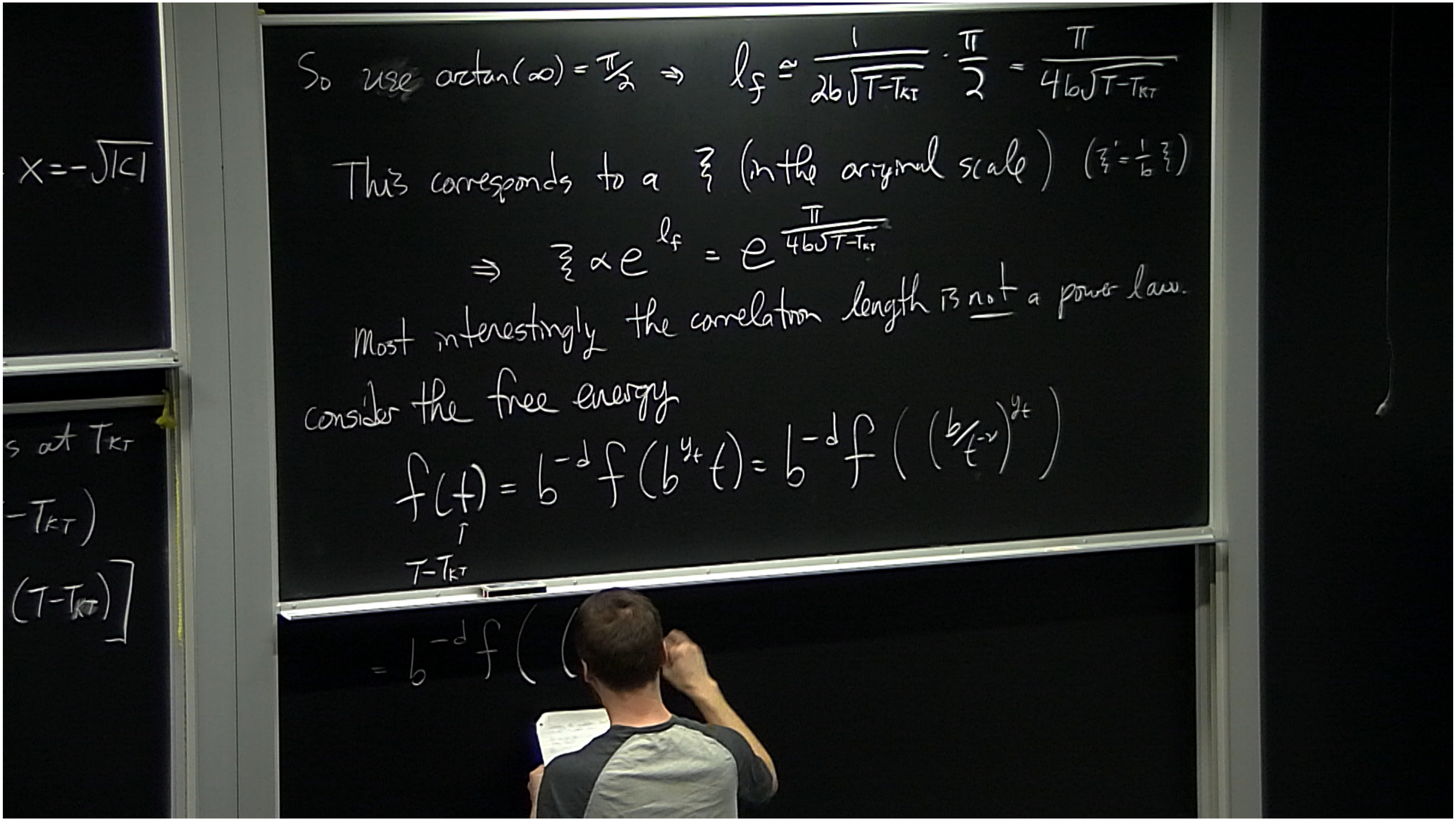
and $$\frac{dx}{dl} = 8\pi^2 y^2 = 2(x^2 + C) = 2\left[x^2 + b^2(T - T_{KT})\right]$$

Integrate <u>directly</u> to obtain

$$2l = \frac{1}{b\sqrt{T - T_{KT}}} \arctan\left(\frac{x}{b\sqrt{T - T_c}}\right)$$

Limits of integration: $x \sim 0$ and $x \sim O(1)$

(where perturbation theory breaks down)


So use $\arctan(\infty) = \pi/2 \Rightarrow l_f \simeq \frac{1}{2b\sqrt{T-T_{KT}}} \cdot \frac{\pi}{2} = \frac{\pi}{4b\sqrt{T-T_{KT}}}$
This corresponds to a ξ (in the original scale) $(\xi' = \frac{1}{b}\xi)$
$\Rightarrow \xi \propto e^{l_f} = e^{\frac{\pi}{4b\sqrt{T-T_{KT}}}}$
Most interestingly the correlation length is not a power law.
Consider the free energy
$f(t) = b^{-d} f(b^{y_t} t) = b^{-d} f\left(\left(\frac{b}{t^{-\nu}}\right)^{y_t}\right)$
$T-T_{KT}$
$= b^{-d} f(($
$x = -\sqrt{|c|}$
s at T_{KT}
$-T_{KT})$
$(T-T_{KT})]$

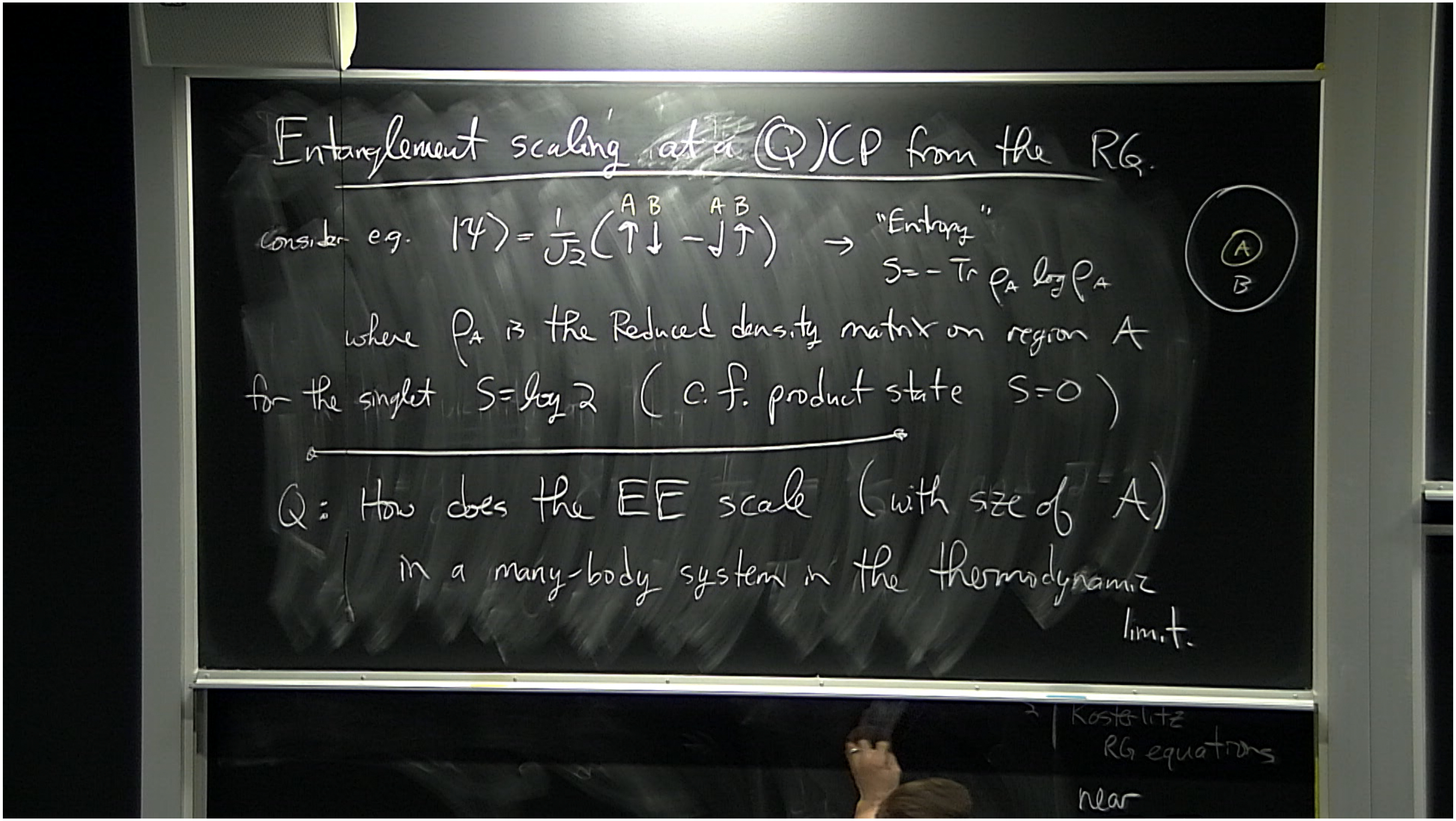
$$= b^{-d} f\left(\left(\frac{b}{\xi}\right)^{y_t}\right) \equiv b^{-d} g\left(\frac{b}{\xi}\right)$$

set $O(1)$, $b \sim \xi$

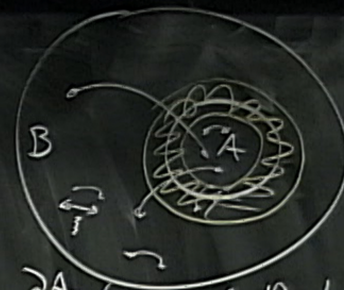
$$f(t) = \xi^{-d} g(O(1)) \simeq e^{-\frac{\text{const}}{\sqrt{T-T_{KT}}}} \quad (d=2)$$

- This is the "essential" singularity (∞-order transition)
- all derivatives are finite
- the specific heat doesn't have a singularity

→ e.g. through T_{KT} (unobservable experimentally in typical bulk measurements)


Entanglement scaling at a (Q)CP from the RG.
Consider e.g. |Ψ⟩ = 1/√2 (↑↓ − ↓↑) → "Entropy"
S = − Tr ρA log ρA
where ρA is the Reduced density matrix on region A
for the singlet S = log 2 (c.f. product state S = 0)
Q: How does the EE scale (with size of A)
in a many-body system in the thermodynamic limit.
A
B
Kosterlitz
RG equations
near

Case 1: not critical, ξ = finite



$\Rightarrow$ notion of the "area law"

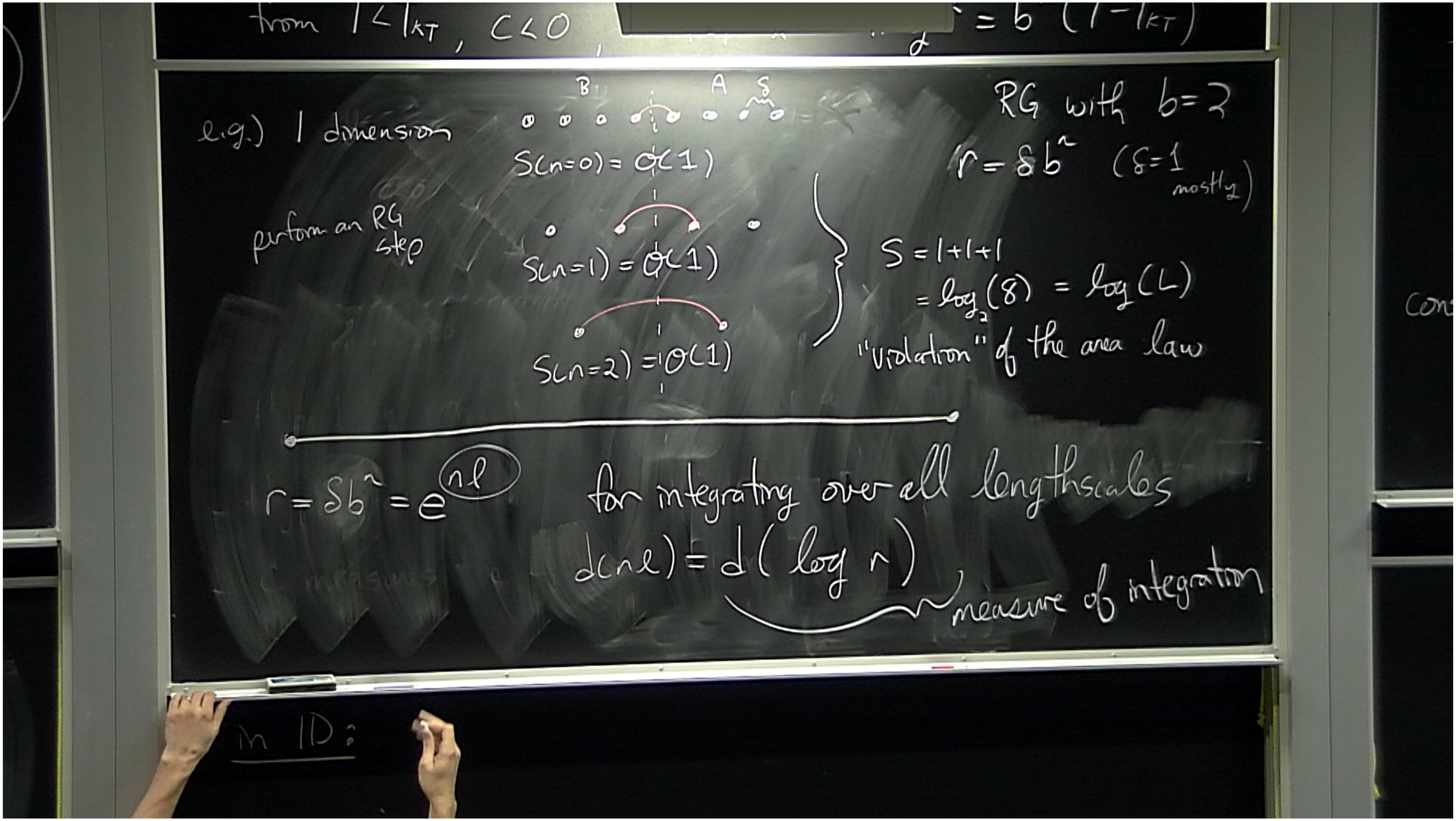
entanglement (entropy) scales as the size of ∂A (size of the boundary)

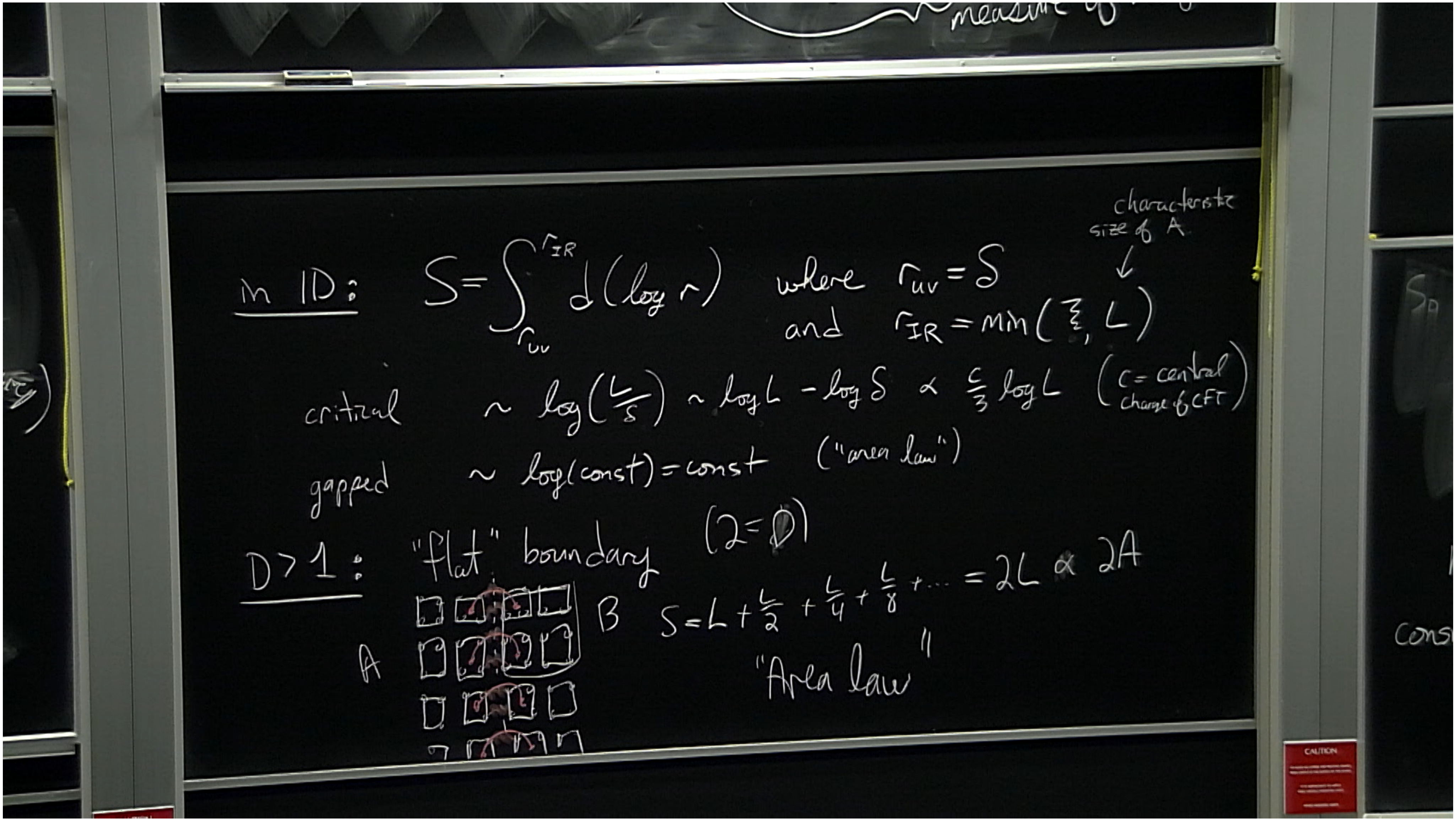
Case 2: what if we're at a critical point
where $\xi \to \infty$ (no lengthscale)

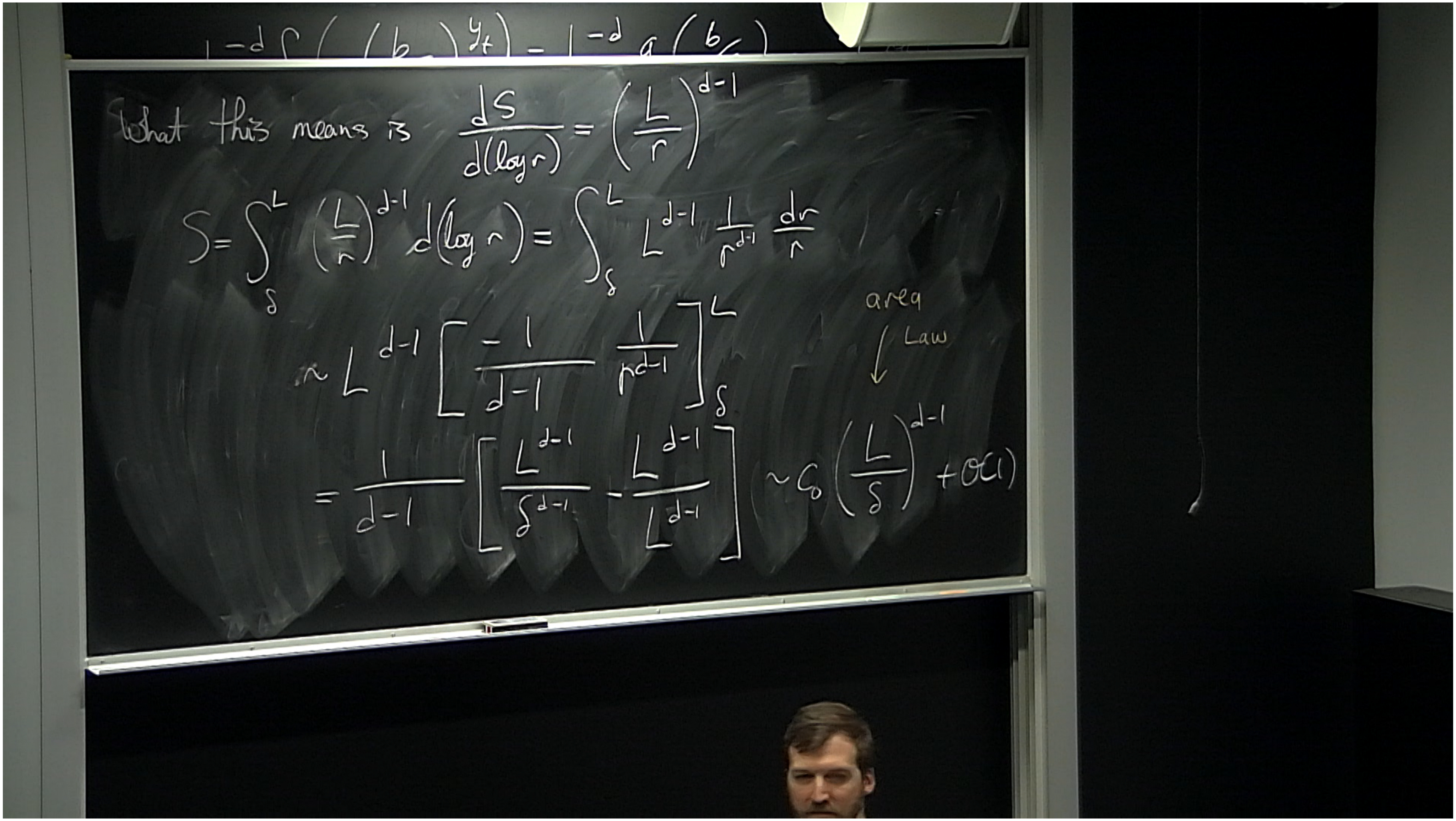
postulates

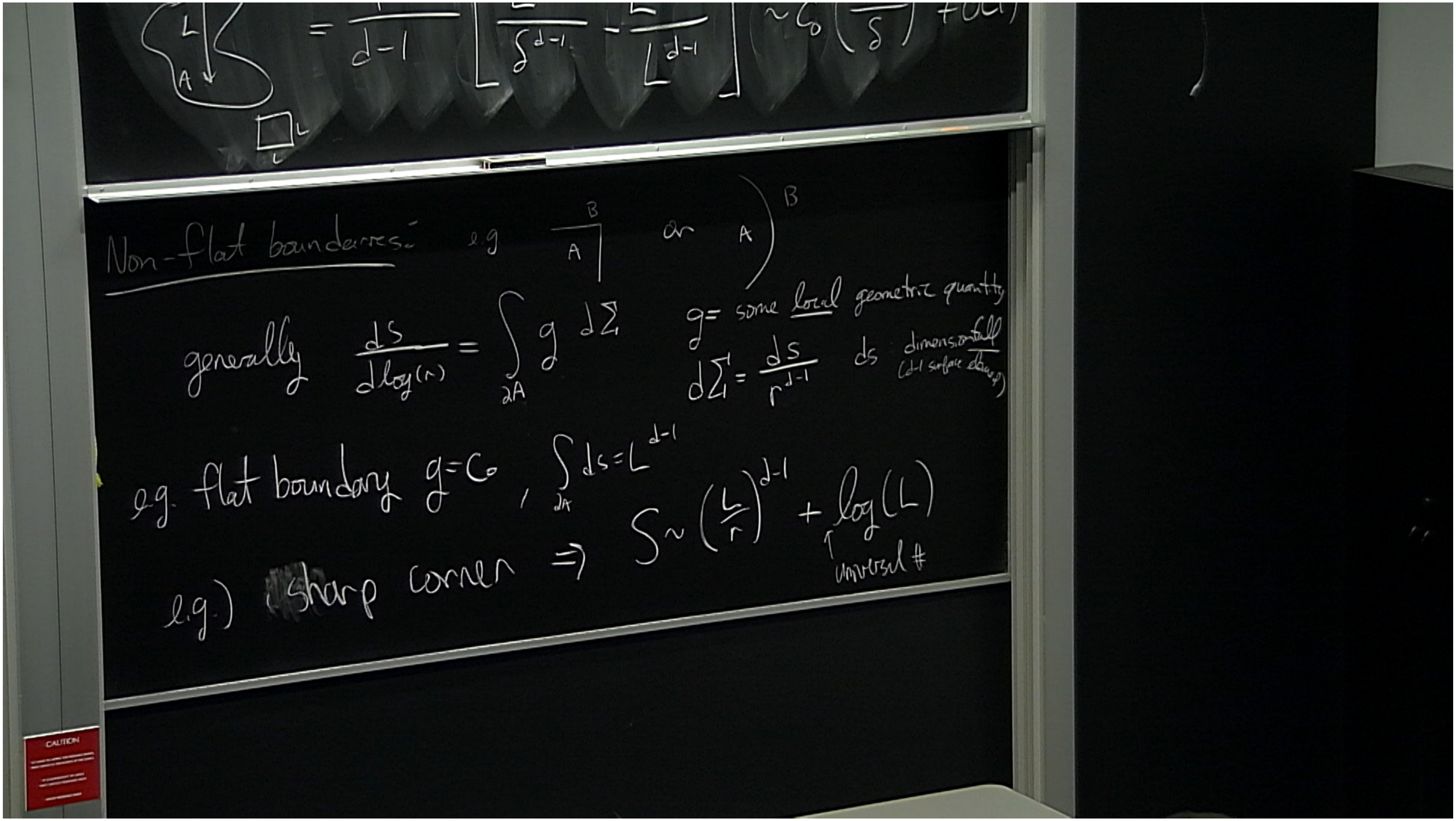
1) S comes from contributions "local" to the boundary

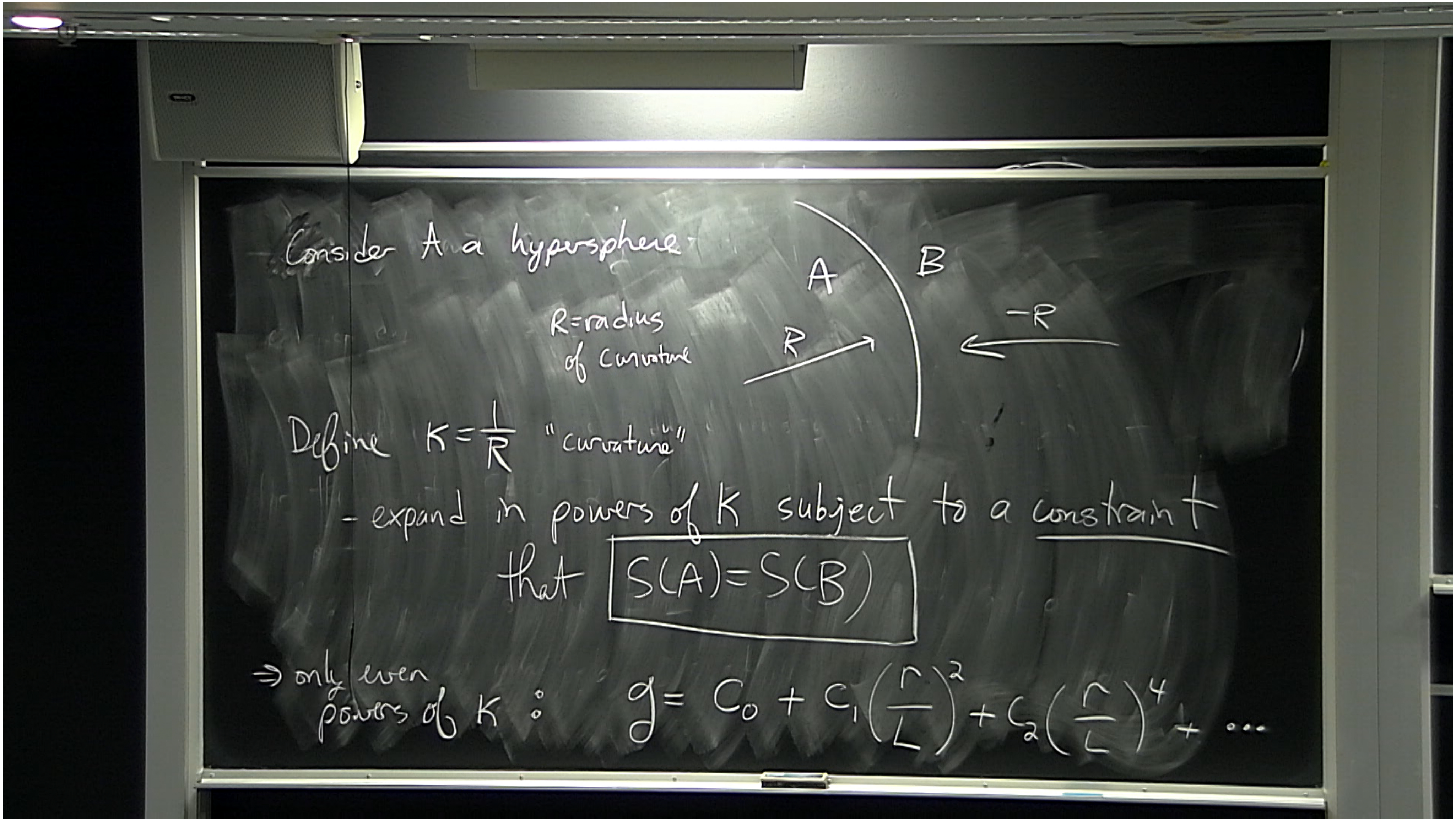
2) S picks up these local contributions at every lengthscale


e.g.) 1 dimension
B A S
S(n=0) = O(1)
perform an RG step
S(n=1) = O(1)
S(n=2) = O(1)
RG with b=2
r = δb^n (δ=1 mostly)
S = 1+1+1
= log_2(8) = log(L)
"violation" of the area law
r = δb^n = e^{nl}
for integrating over all lengthscales
d(nl) = d(log r)
measure of integration
in 1D:


characteristic size of A
in 1D: S = ∫ from r_UV to r_IR d(log r) where r_UV = δ
and r_IR = min(ξ, L)
critical ~ log(L/δ) ~ log L − log δ ∝ c/3 log L (c = central charge of CFT)
gapped ~ log(const) = const ("area law")
D > 1: "flat" boundary (2 = D)
B S = L + L/2 + L/4 + L/8 + ... = 2L ∝ 2A
A
"Area law"


What this means is $\frac{dS}{d(\log r)} = \left(\frac{L}{r}\right)^{d-1}$
$S = \int_\delta^L \left(\frac{L}{r}\right)^{d-1} d(\log r) = \int_\delta^L L^{d-1} \frac{1}{r^{d-1}} \frac{dr}{r}$
$\sim L^{d-1} \left[\frac{-1}{d-1} \frac{1}{r^{d-1}} \right]_\delta^L$
$= \frac{1}{d-1} \left[\frac{L^{d-1}}{\delta^{d-1}} - \frac{L^{d-1}}{L^{d-1}} \right] \sim c_0 \left(\frac{L}{\delta}\right)^{d-1} + O(1)$
area Law


Non-flat boundaries: e.g. or
generally dS/dlog(r) = ∫_∂A g dΣ
g = some local geometric quantity
dΣ = ds/r^(d-1)
ds: dimensionfull
(d-1 surface element)
e.g. flat boundary g = C0 , ∫_∂A ds = L^(d-1)
S ~ (L/r)^(d-1) + log(L)
universal #
e.g.) sharp corner ⇒


Consider A a hypersphere
A
B
R=radius
of Curvature
R
−R
Define K = 1/R "curvature"
- expand in powers of K subject to a constraint
that S(A)=S(B)
⇒ only even
powers of K :
g = C₀ + C₁(r/L)² + C₂(r/L)⁴ + ...

that $\boxed{S(A)=S(B)}$

⇒ only even powers of r: $g = c_0 + c_1\left(\frac{r}{L}\right)^2 + c_2\left(\frac{r}{L}\right)^4 + \cdots$

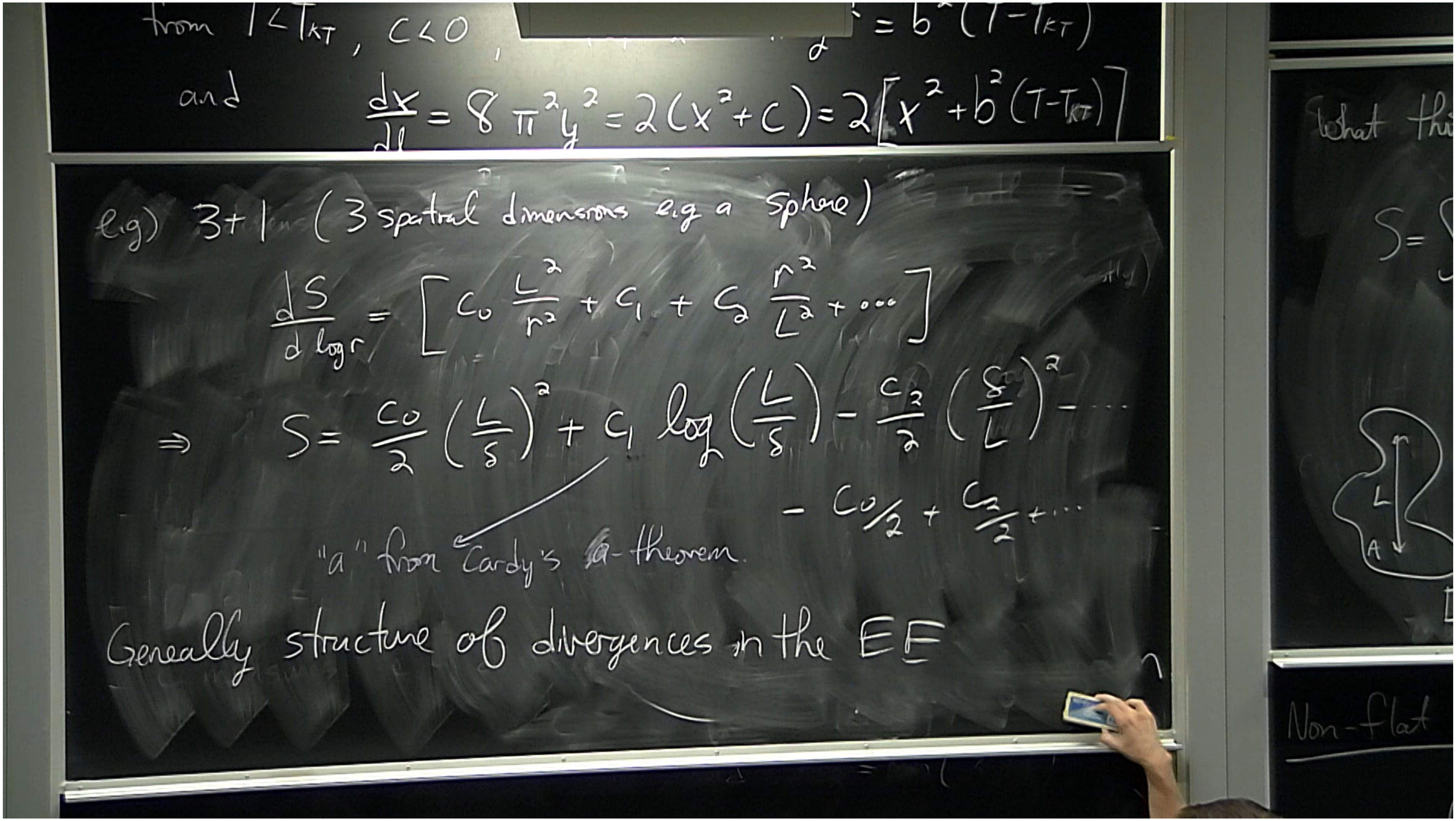
where $L \sim R$. calculate S in general d.

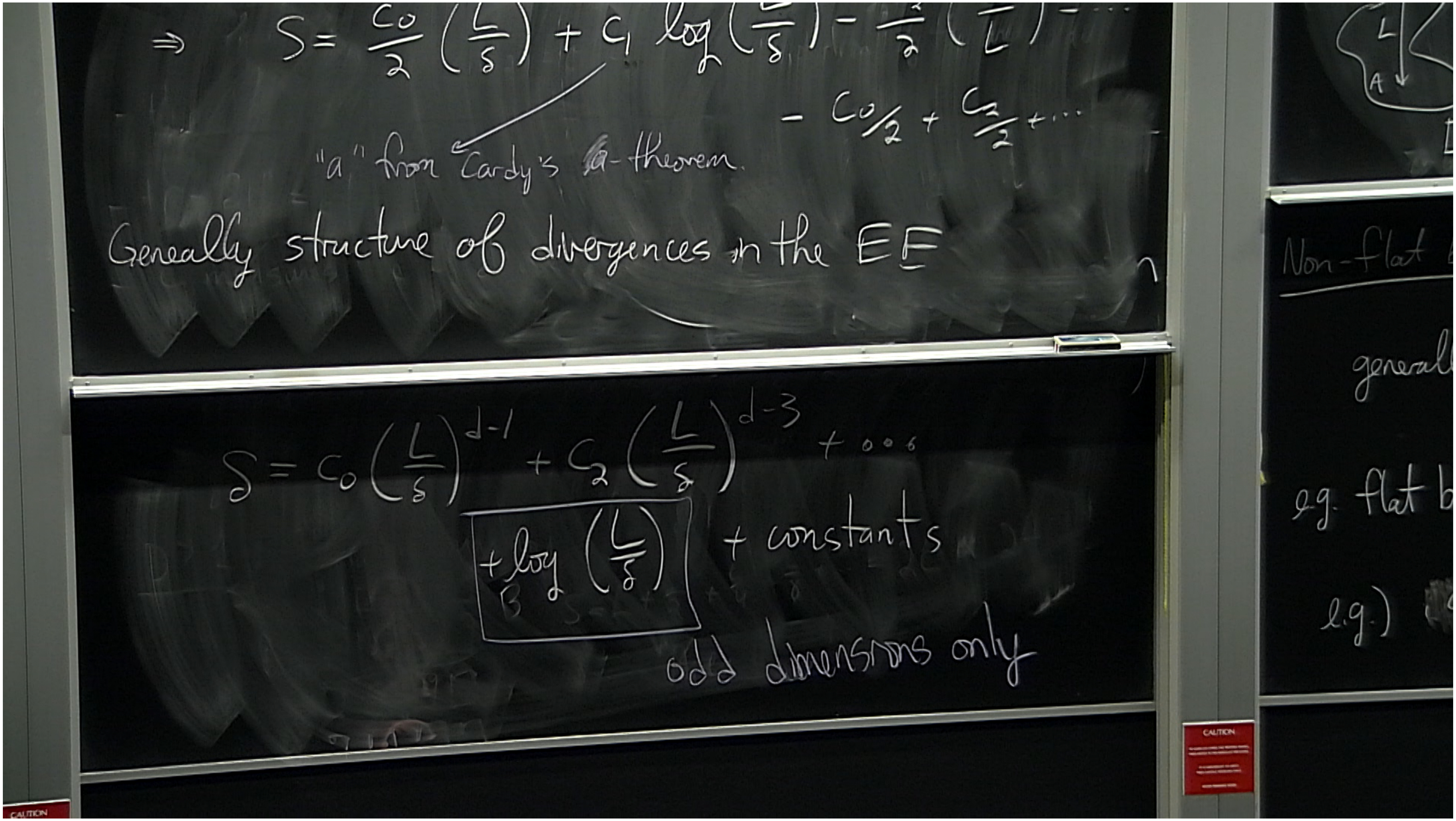
e.g.) 2+1 $\quad \frac{dS}{d\log r} = \left[c_0 \frac{L}{r} + c_1 \frac{r}{L} + c_2 \frac{r^3}{L^3} + \cdots \right]$

$$S = \int_\delta^L \frac{dr}{r}\left[\quad\right] = c_0 \frac{L}{\delta} - c_1 \frac{\delta}{L} - \frac{c_2}{3}\left(\frac{\delta}{L}\right)^3 - \cdots$$

$$\boxed{- c_0 + c_1 + \frac{c_2}{3} + \cdots}$$

for a circle, these are "F" $\quad S = c_0 \frac{L}{\delta} - F$


from $T < T_{KT}$, $c < 0$
and $\frac{dx}{dl} = 8\pi^2 y^2 = 2(x^2 + c) = 2[x^2 + b^2(T - T_{KT})]$
e.g.) 3+1 (3 spatial dimensions e.g a sphere)
$\frac{dS}{d\log r} = \left[c_0 \frac{L^2}{r^2} + c_1 + c_2 \frac{r^2}{L^2} + \cdots \right]$
$\Rightarrow S = \frac{c_0}{2}\left(\frac{L}{\delta}\right)^2 + c_1 \log\left(\frac{L}{\delta}\right) - \frac{c_2}{2}\left(\frac{\delta}{L}\right)^2 - \cdots$
$- c_0/2 + c_2/2 + \cdots$
"a" from Cardy's a-theorem
Generally structure of divergences in the EE
What
Non-flat
L
A


"a" from Cardy's a-theorem.
Generally structure of divergences in the EE
+ constants
odd dimensions only
Non-flat