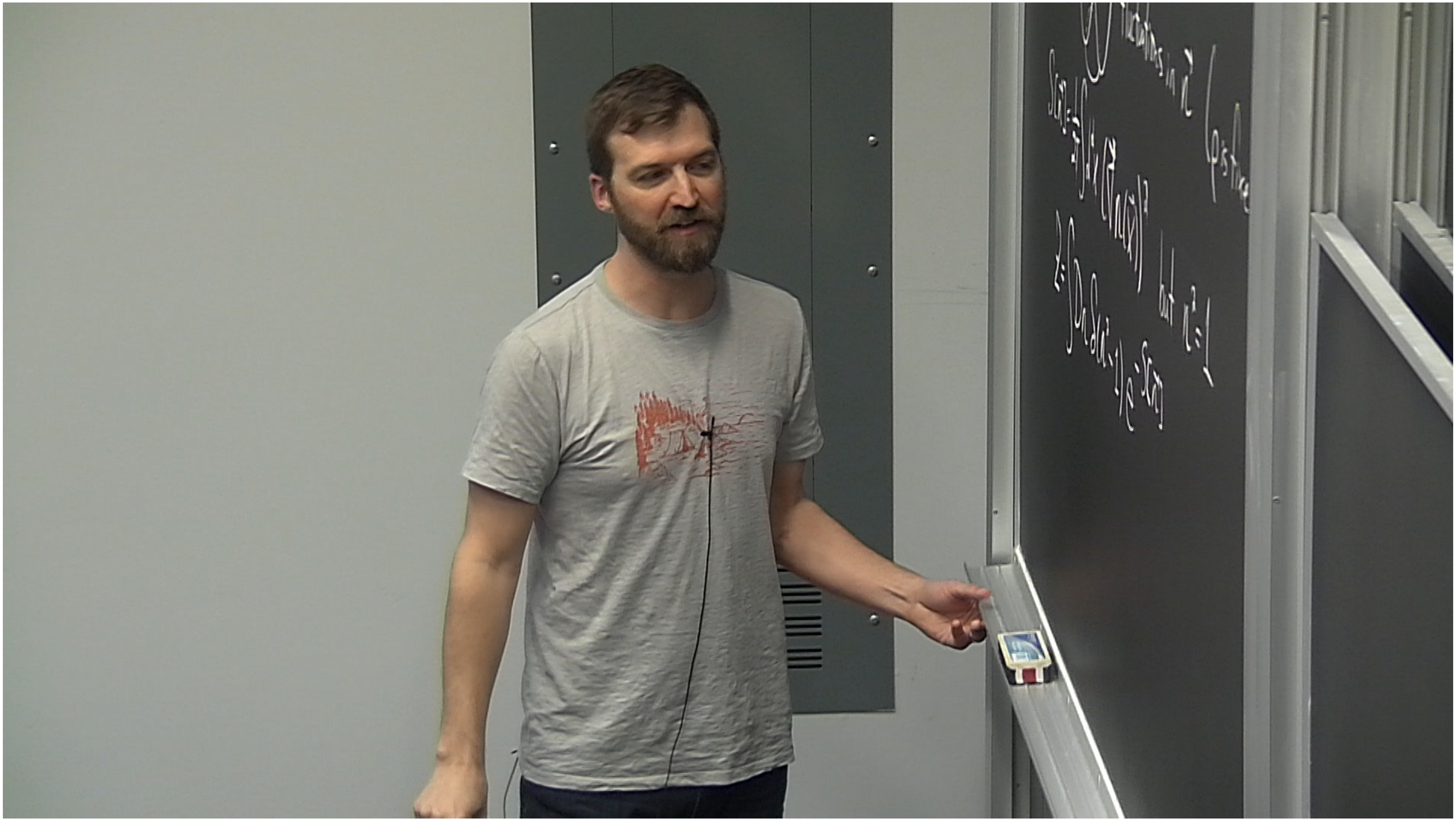
Title: 2016/2017 Statistical Mechanics 2 - Roger Melko - Lecture 23

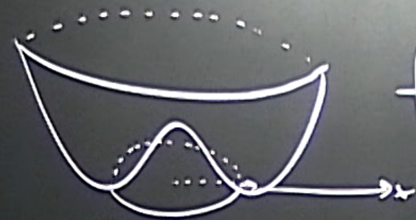
Date: Mar 22, 2017 10:30 AM

URL: http://pirsa.org/17030007

Abstract:



Evaluations: http://evaluate.uwaterloo.ca

NLσM:  fluctuations in $\vec{n}$ (ρ is fixed)

$$S[\vec{n}] = \frac{1}{2T}\int d^dx\, (\vec{\nabla} n(\vec{x}))^2 \quad \text{but} \quad n^2 = 1$$

$$Z = \int Dn\, \delta(n^2 - 1)\, e^{-S[\vec{n}]}$$

define $\vec{n} = (\sigma, \vec{\pi}) \Rightarrow \sigma = \sqrt{1 - \vec{\pi}^2}$

$$Z = \int D\vec{\pi}\, e^{-S[\vec{\pi}]}$$ (constraint is used to eliminate σ)

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$$S[\vec{\pi}] = \frac{1}{2T}\int d^dx \left[(\vec{\nabla}\vec{\pi})^2 + (\vec{\nabla}\sqrt{1-\vec{\pi}^2}) + \frac{T}{a^d}\ln(1-\vec{\pi}^2) \right]$$

We performed an RG analysis: Ft to $\vec{k}$-space $\vec{\pi}(\vec{k})$

- momentum shell: $\vec{\pi}(\vec{k}) = \vec{\pi}_<(\vec{k}) + \vec{\pi}_>(\vec{k})$
- integrate over fast modes (outer shell)
- rescale $\vec{k}' = b\vec{k}$ and $\vec{\pi}'(\vec{k}') = z^{-1}\vec{\pi}_<(\vec{k})$ (renormalize)

$$S[\vec{\pi}] = \frac{1}{2T}\int d^d x \left[(\vec{\nabla}\vec{\pi})^2 + (\vec{\nabla}\sqrt{1-\vec{\pi}^2}) + \frac{T}{a^d}\ln(1-\vec{\pi}^2)\right]$$

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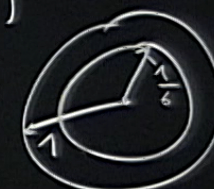

Relationship between this S and S' gives the RG recursion.

$$\frac{1}{T'} = \frac{1}{T} + \frac{1}{T}\left[\varepsilon - (n-2)\,T\Lambda^{\varepsilon}\frac{S_d}{(2\pi)^d}\right]\Delta l$$

where $b = e^{\delta l}$, $\boxed{\varepsilon = d-2}$ (forget $\tilde{T}$), $K_d = \frac{S_d}{(2\pi)^d}$

$$\frac{d}{dl}\left(\frac{1}{T}\right) = \frac{1}{T}\left(\varepsilon - (n-2) T \Lambda^{\varepsilon} K_d\right)$$

if $T = \frac{1}{K}$, $\frac{dT}{dl} = -\frac{1}{K^2}\frac{dK}{dl}$

or $$\boxed{\frac{dT}{dl} = -(d-2)T + (n-2) K_d \Lambda^{\varepsilon} T^2} = \beta_T$$

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"Callan-Symanzik" β-function.

As before - fixed points are determined by

$$\frac{dT^*}{dl} = 0 \Rightarrow \quad 1)\ T^* = 0$$
$$2)\ T^* = \frac{\epsilon}{K_d \Lambda^\epsilon (n-2)}$$

Examine the β-function to determine stability

$$\beta(T) = -\varepsilon T + (n-2) K_d \Lambda^\varepsilon T^2$$

$$\frac{d\beta}{dT} = -\varepsilon + 2(n-2) K_d \Lambda^\varepsilon T$$

Recall: $M = \begin{pmatrix} \frac{\partial \beta}{\partial K_i} & \cdots \\ \vdots & \end{pmatrix}_{K^*}$ gives the y_i

So $\left.\frac{d\beta}{dT}\right|_{T^*=0} = -\varepsilon = 2-d = y_t$ for $T^*=0$

So the flow reverses direction at $d=2$

$$\left.\frac{d\beta}{dT}\right|_{T^*=\frac{\varepsilon}{K_d \Lambda^\varepsilon (n-2)}} = -\varepsilon + 2(n-2)K_d\Lambda^\varepsilon \frac{\varepsilon}{K_d \Lambda^\varepsilon (n-2)}$$

$= \varepsilon = y_t$ for the non-trivial fixed point

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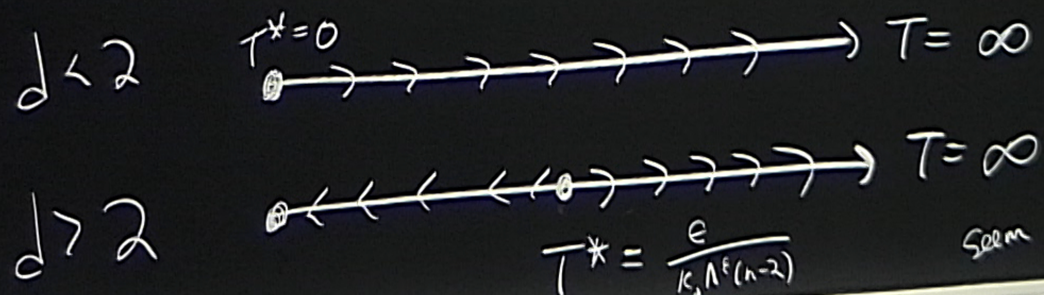


seem to need $n>2$

for the non-trivial fixed point $(d>2,\ n>2)$

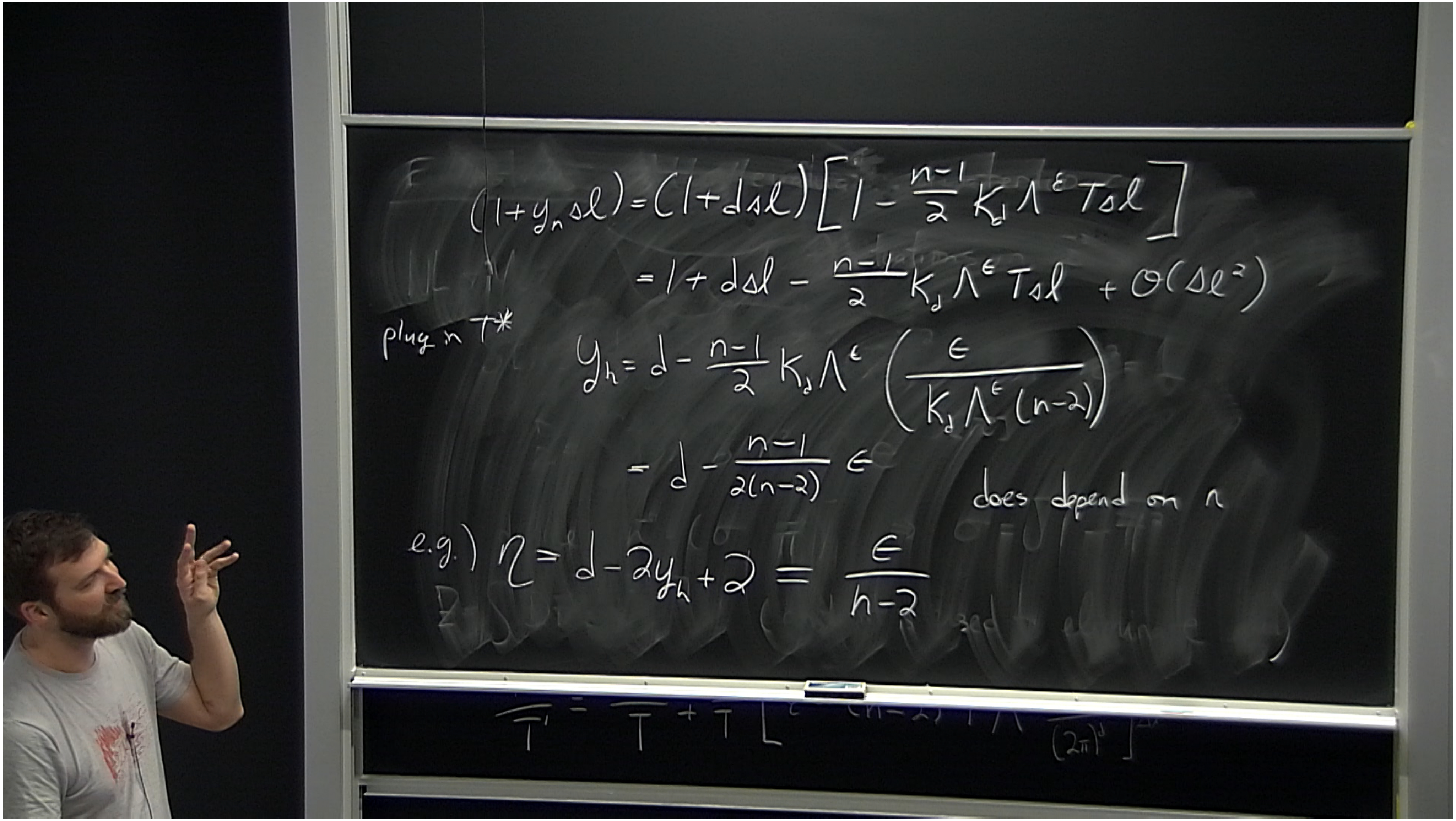
$$y_t = \epsilon, \quad \nu = \frac{1}{\epsilon}, \quad \alpha = 2 - \frac{d}{y_t} = 2 - \frac{2+\epsilon}{\epsilon}$$

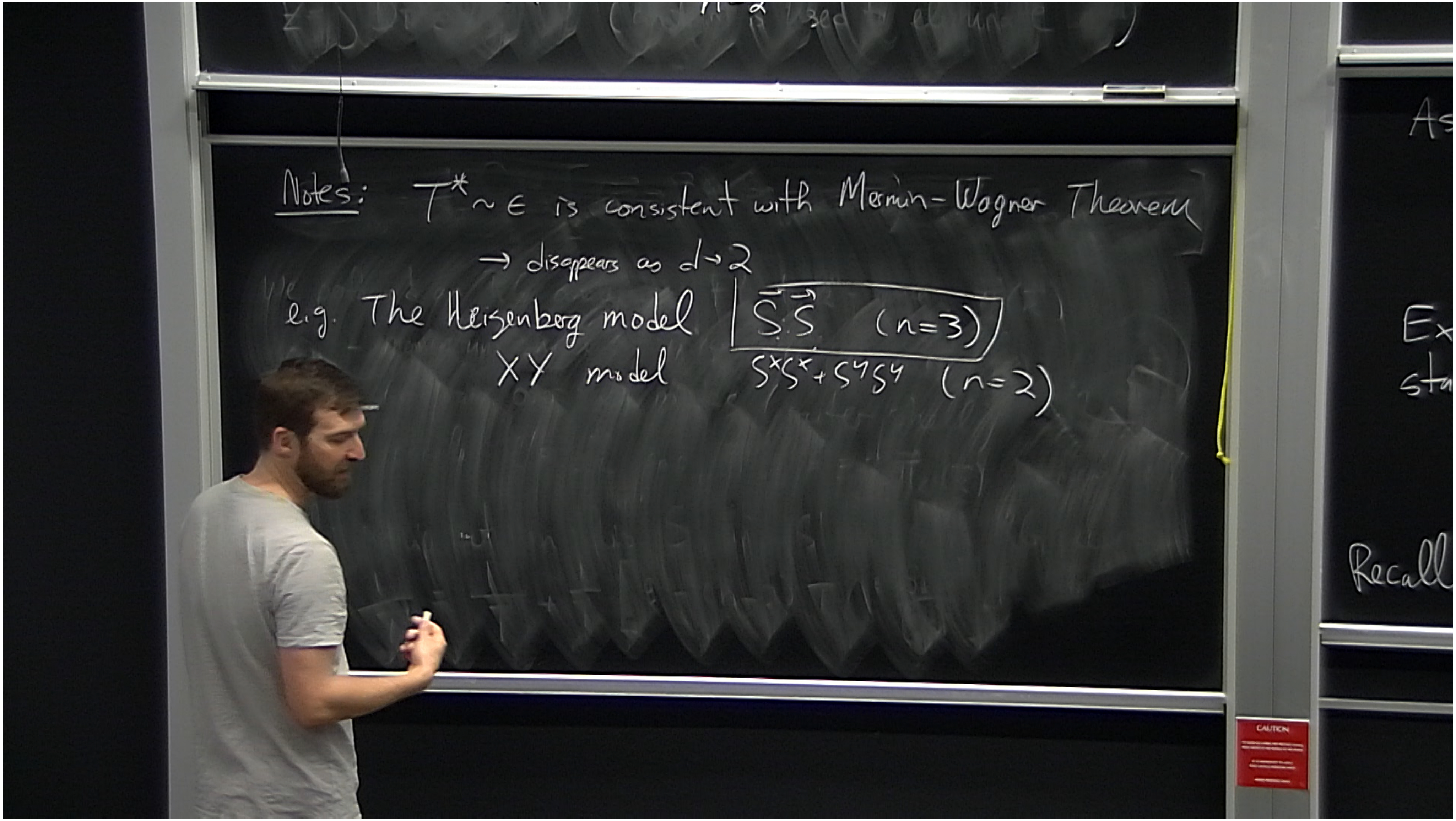
To get the other exponents you need y_h

add a term $\quad -\vec{h}\cdot\int d^dx\, \vec{n}(\vec{x})$

Under the RG: $h' = b^{y_h} h$ (check)

find $\quad b^{y_h} = b^d\left[1 - \frac{n-1}{2} I_1\right]$


$(1+y_h \delta l) = (1+d\,\delta l)\left[1 - \frac{n-1}{2} K_d \Lambda^\epsilon T \delta l\right]$
$= 1 + d\,\delta l - \frac{n-1}{2} K_d \Lambda^\epsilon T \delta l + O(\delta l^2)$
plug in T^*
$y_h = d - \frac{n-1}{2} K_d \Lambda^\epsilon \left(\frac{\epsilon}{K_d \Lambda^\epsilon (n-2)}\right)$
$= d - \frac{n-1}{2(n-2)}\epsilon$
does depend on n
e.g.) $\eta = d - 2y_h + 2 = \frac{\epsilon}{n-2}$


Notes: T* ~ ε is consistent with Mermin-Wagner Theorem
→ disappears as d → 2
e.g. The Heisenberg model S⃗·S⃗ (n=3)
XY model S^x S^x + S^y S^y (n=2)
Recall

Notes: $T^* \sim \epsilon$ is consistent with Mermin-Wagner Theorem

$\rightarrow$ disappears as $d \to 2$

e.g. The Heisenberg model $\boxed{\vec{S}\cdot\vec{S} \quad (n=3)}$

XY model $S^x S^x + S^y S^y \quad (n=2)$

- have finite T transitions.

These give us important experimental tests of QFT.

Lipa et. al. 2003 Measured specific heat in microgravity

$$\alpha = -0.0127(3)$$

Theoretical predictions:

For ^{4}He the order parameter is the phase of the complex wavefunction – $O(2)$

4-loops (continuum) $\alpha = -0.01126(10)$

lattice Monte Carlo $\alpha = -0.0146(8)$

lattice variational RG $\alpha = -0.0125(39)$

Theoretical predictions:

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lattice Monte Carlo $\alpha = -0.0146(8)$ 3D

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$\boxed{d=2}$

$$\beta_T = \frac{dT}{dl} = (n-2) K_d \Lambda^{\epsilon} T^2 + O(T^3)$$

(This is a toy model for 4d QCD - quantum chromodynamics)

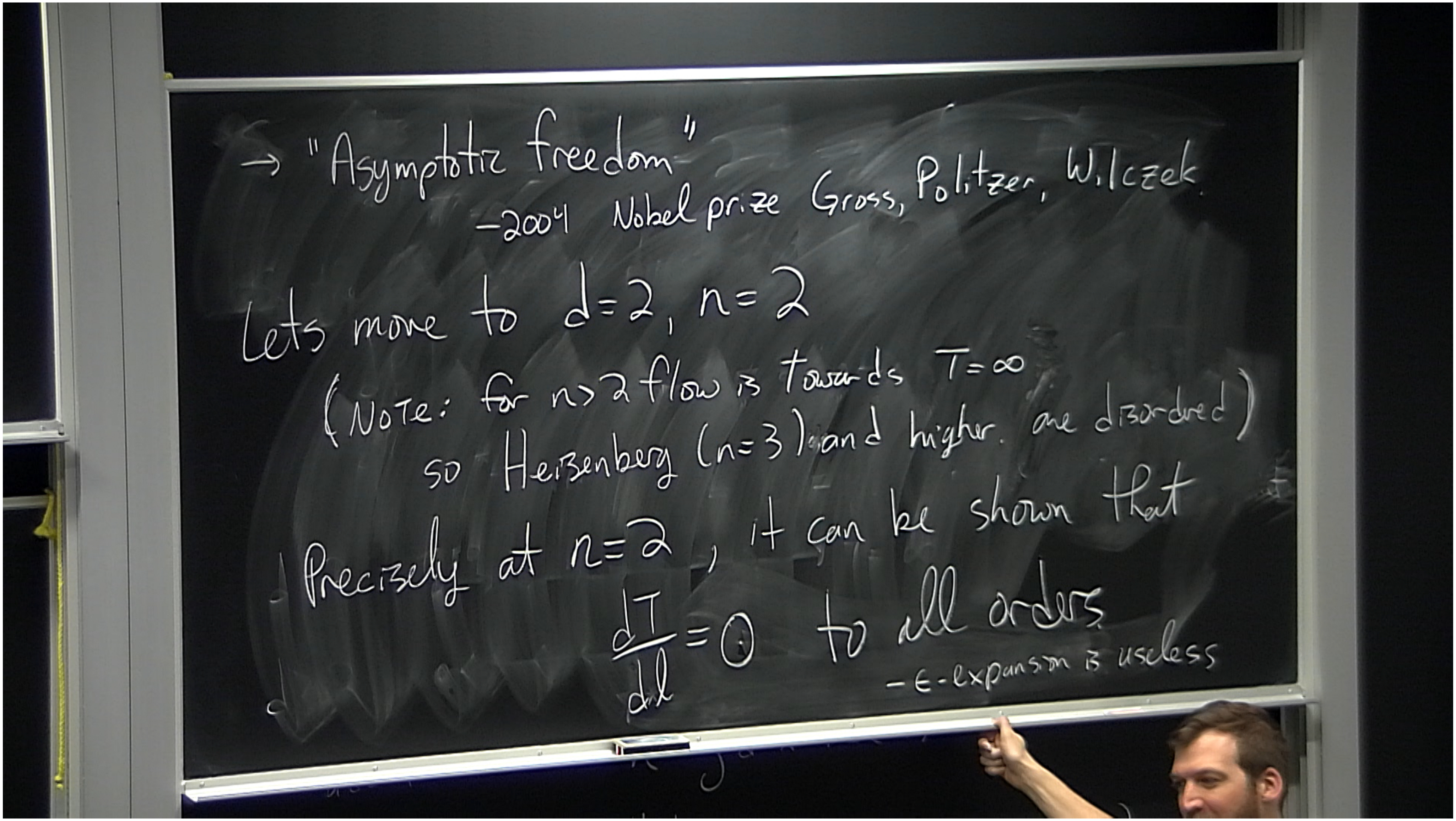
→ from β-function you could calculate the spin-spin c.f.

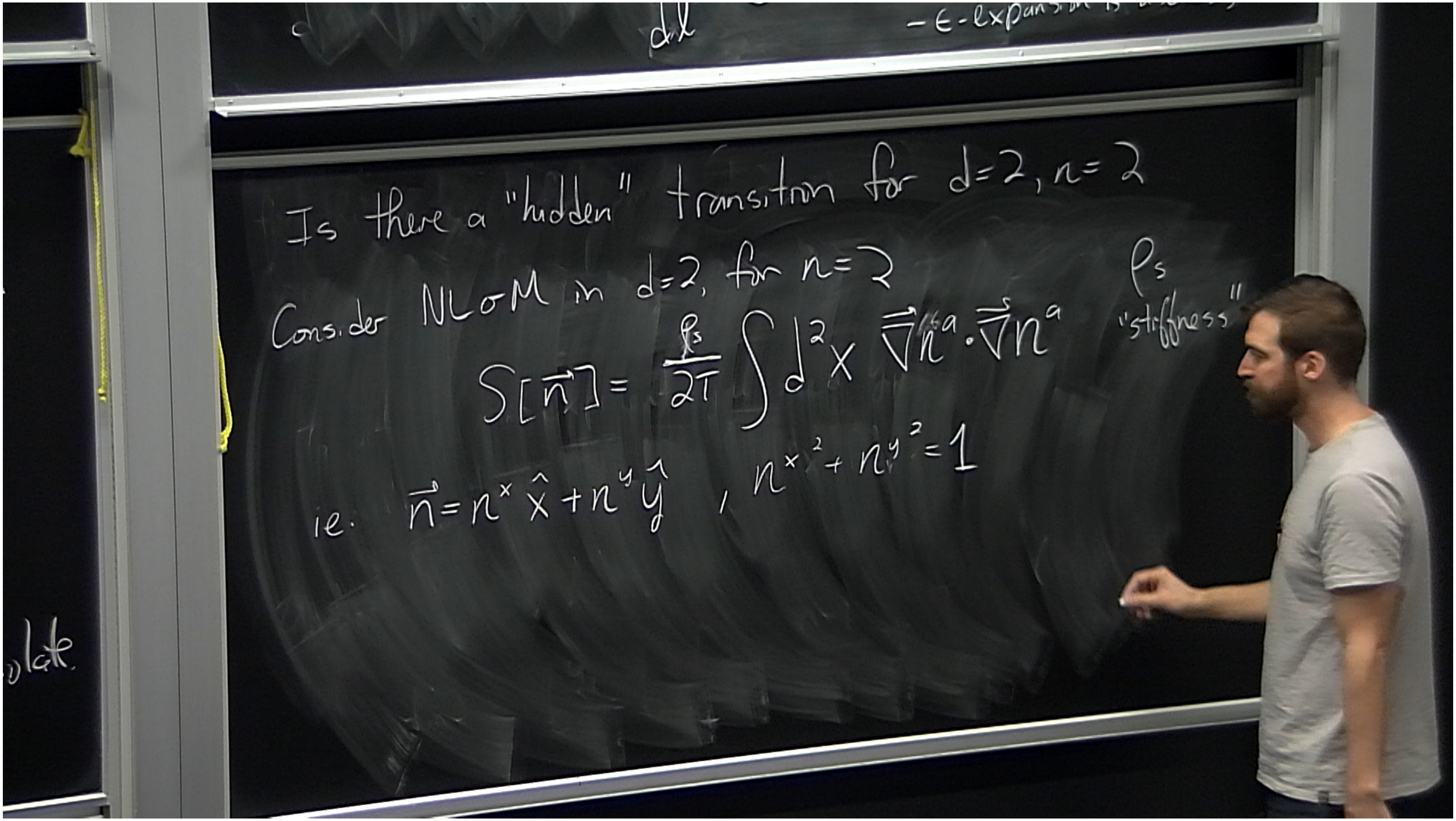
→ "method of characteristics" → introduce a parameter "t"

$\underline{G(\Delta t)}$ – small at short "times"
– large at long "times"

This gives the explanation for quark confinement.

- Quarks inside hadrons are free
- When you try to pull them apart - impossible to isolate


→ "Asymptotic freedom"
– 2004 Nobel prize Gross, Politzer, Wilczek
Lets move to d=2, n=2
(NOTE: for n>2 flow is towards T=∞
so Heisenberg (n=3) and higher are disordered)
Precisely at n=2, it can be shown that
dT/dl = 0 to all orders
– ε-expansion is useless


Is there a "hidden" transition for $d=2$, $n=2$
Consider NLσM in $d=2$, for $n=2$
ρ_s
$S[\vec{n}] = \frac{\rho_s}{2T} \int d^2x \, \vec{\nabla} n^a \cdot \vec{\nabla} n^a$
"stiffness"
i.e. $\vec{n} = n^x \hat{x} + n^y \hat{y}$, ${n^x}^2 + {n^y}^2 = 1$

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Consider NLσM in $d=2$, for $n=2$ $\qquad \rho_s$ "stiffness"

$$S[\vec{n}] = \frac{\rho_s}{2T} \int d^2x \, \vec{\nabla} n^a \cdot \vec{\nabla} n^a$$

i.e. $\vec{n} = n^x \hat{x} + n^y \hat{y}$, ${n^x}^2 + {n^y}^2 = 1$ $\qquad$ replace with $\vec{\theta}$

$\vec{\theta}(\vec{x})$ is a real variable , $n^x = \cos\theta$, $n^y = \sin\theta$

then $\vec{\nabla} n^a \cdot \vec{\nabla} n^a = \sin^2\theta \, (\vec{\nabla}\theta)^2 + \cos^2\theta \, (\vec{\nabla}\theta)^2 = (\vec{\nabla}\theta)^2$

olate.

Consider the continuum limit: expand cosine to 2^{nd} order

(assume $\boxed{|\theta_i - \theta_j| << 2\pi}$)

$$H \cong -2JN + \frac{J}{2}\sum_{\langle ij\rangle}(\theta_i - \theta_j)^2$$

$$= E_0 + J/2 \sum_{\langle ij\rangle_x}(\theta_i - \theta_j)^2 + \frac{J}{2}\sum_{\langle ij\rangle_y}(\theta_i - \theta_j)^2$$

in the continuum

$$\theta_i - \theta_j = \frac{\partial}{\partial x}\theta \text{ or } \frac{\partial}{\partial y}\theta$$

Replace sum w/ integral

$$H = E_0 + \frac{J}{2} \int d^2x \, (\vec{\nabla}\theta)^2$$

the assumption that is missing interesting physics is

$$|\theta_i - \theta_j| \ll 2\pi$$