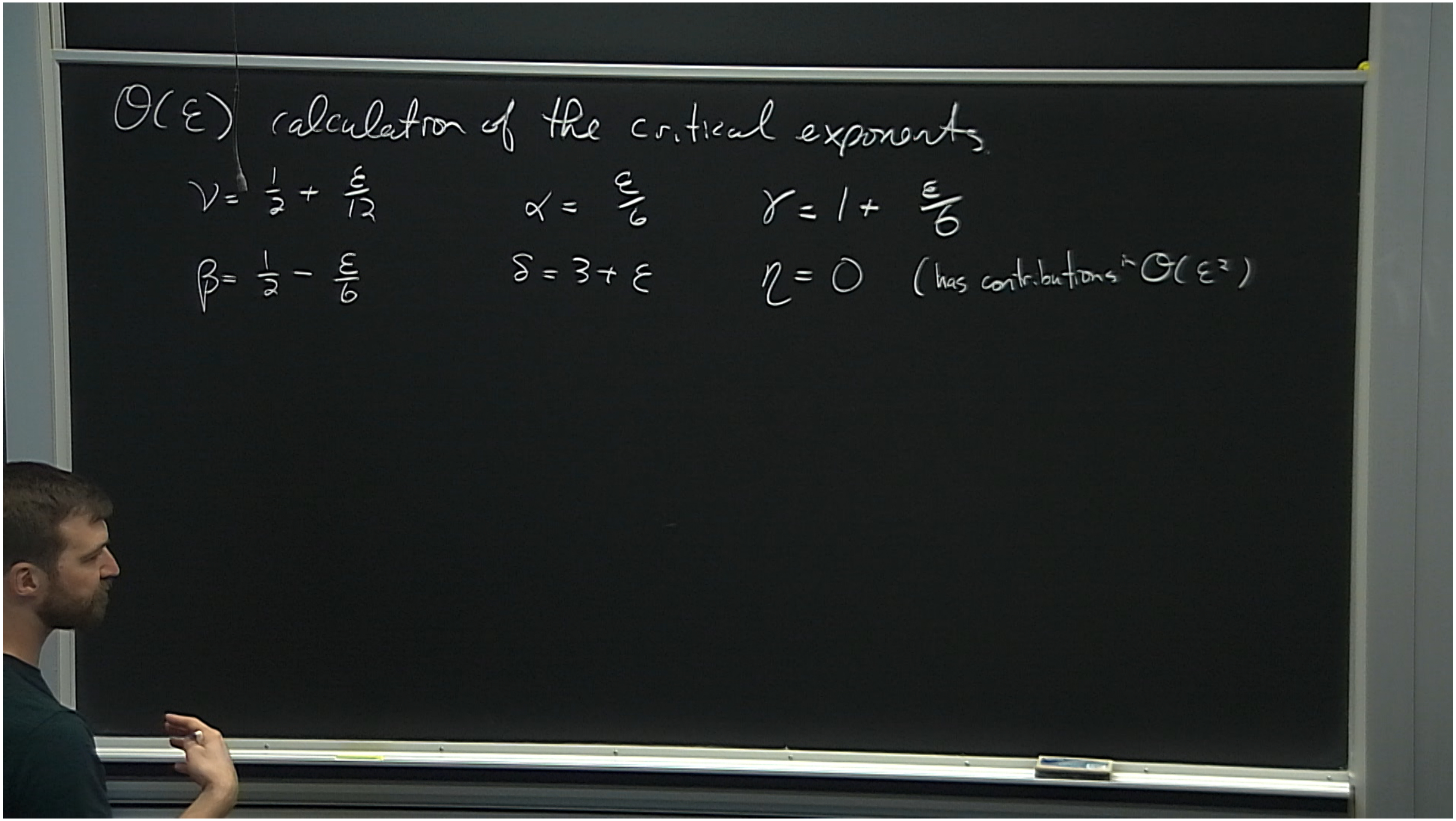
Title: 2016/2017 Statistical Mechanics 2 - Roger Melko - Lecture 18

Date: Mar 03, 2017 10:30 AM

URL: http://pirsa.org/17030002

Abstract:


$\mathcal{O}(\varepsilon)$ calculation of the critical exponents
$\nu = \frac{1}{2} + \frac{\varepsilon}{12}$ $\alpha = \frac{\varepsilon}{6}$ $\gamma = 1 + \frac{\varepsilon}{6}$
$\beta = \frac{1}{2} - \frac{\varepsilon}{6}$ $\delta = 3 + \varepsilon$ $\eta = 0$ (has contributions in $\mathcal{O}(\varepsilon^2)$)

$\mathcal{O}(\varepsilon)$ calculation of the critical exponents

$\nu = \frac{1}{2} + \frac{\varepsilon}{12}$ $\qquad \alpha = \frac{\varepsilon}{6}$ $\qquad \gamma = 1 + \frac{\varepsilon}{6}$

$\beta = \frac{1}{2} - \frac{\varepsilon}{6}$ $\qquad \delta = 3 + \varepsilon$ $\qquad \eta = 0$ (has contributions in $\mathcal{O}(\varepsilon^2)$)

for $\varepsilon = 1$

3D

exponent	$\mathcal{O}(\varepsilon)$	MFT	exact
α	0.167	0	
β	0.333	$\frac{1}{2}$	
γ	1.167	1	
δ	4	3	
ν	0.583	$\frac{1}{2}$	
η	0	0	

$O(\varepsilon)$ calculation of the critical exponents

$\nu = \frac{1}{2} + \frac{\varepsilon}{12}$ $\qquad \alpha = \frac{\varepsilon}{6}$ $\qquad \gamma = 1 + \frac{\varepsilon}{6}$

$\beta = \frac{1}{2} - \frac{\varepsilon}{6}$ $\qquad \delta = 3 + \varepsilon$ $\qquad \eta = 0$ (has contributions in $O(\varepsilon^2)$)

for $\varepsilon = 1$

3D

exponent	$O(\varepsilon)$	MFT	exact
α	0.167	0	0.110
β	0.333	$1/2$	0.325
γ	1.167	1	1.237
δ	4	3	4.82
ν	0.583	$1/2$	0.630
η	0	0	0.032

$O(\epsilon)$ calculation of the critical exponents

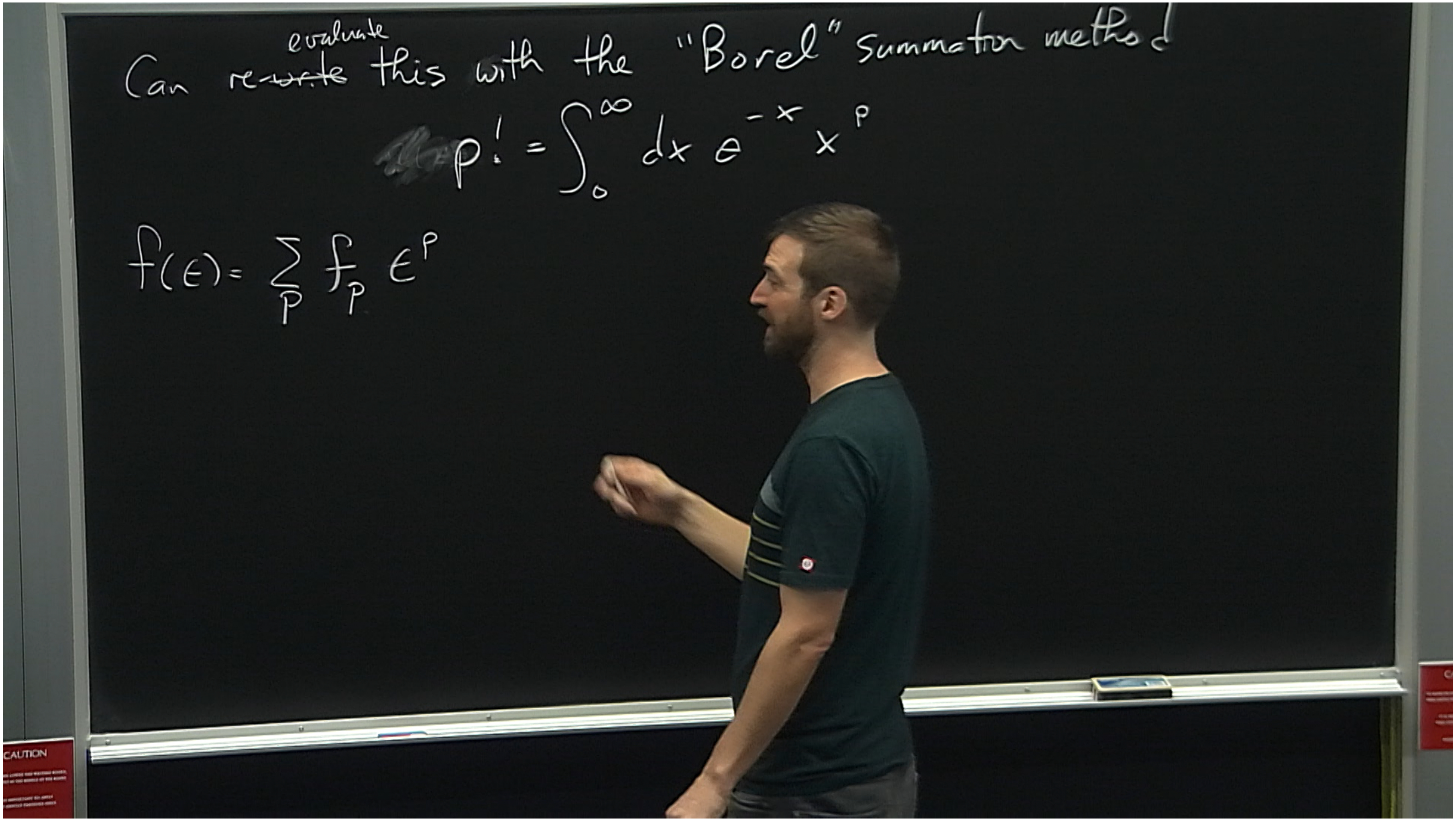
$\nu = \frac{1}{2} + \frac{\epsilon}{12}$ $\qquad \alpha = \frac{\epsilon}{6}$ $\qquad \gamma = 1 + \frac{\epsilon}{6}$

$\beta = \frac{1}{2} - \frac{\epsilon}{6}$ $\qquad \delta = 3 + \epsilon$ $\qquad \eta = 0$ (has contributions in $O(\epsilon^2)$)

for $\epsilon = 1$

3D

exponent	$O(\epsilon)$	MFT	exact
α	0.167	0	0.110
β	0.333	$\frac{1}{2}$	0.325
γ	1.167	1	1.237
δ	4	3	4.82
ν	0.583	$\frac{1}{2}$	0.630
η	0	0	0.032


Can re-write evaluate this with the "Borel" summation method
$p! = \int_0^\infty dx\, e^{-x} x^p$
$f(\epsilon) = \sum_p f_p \epsilon^p$

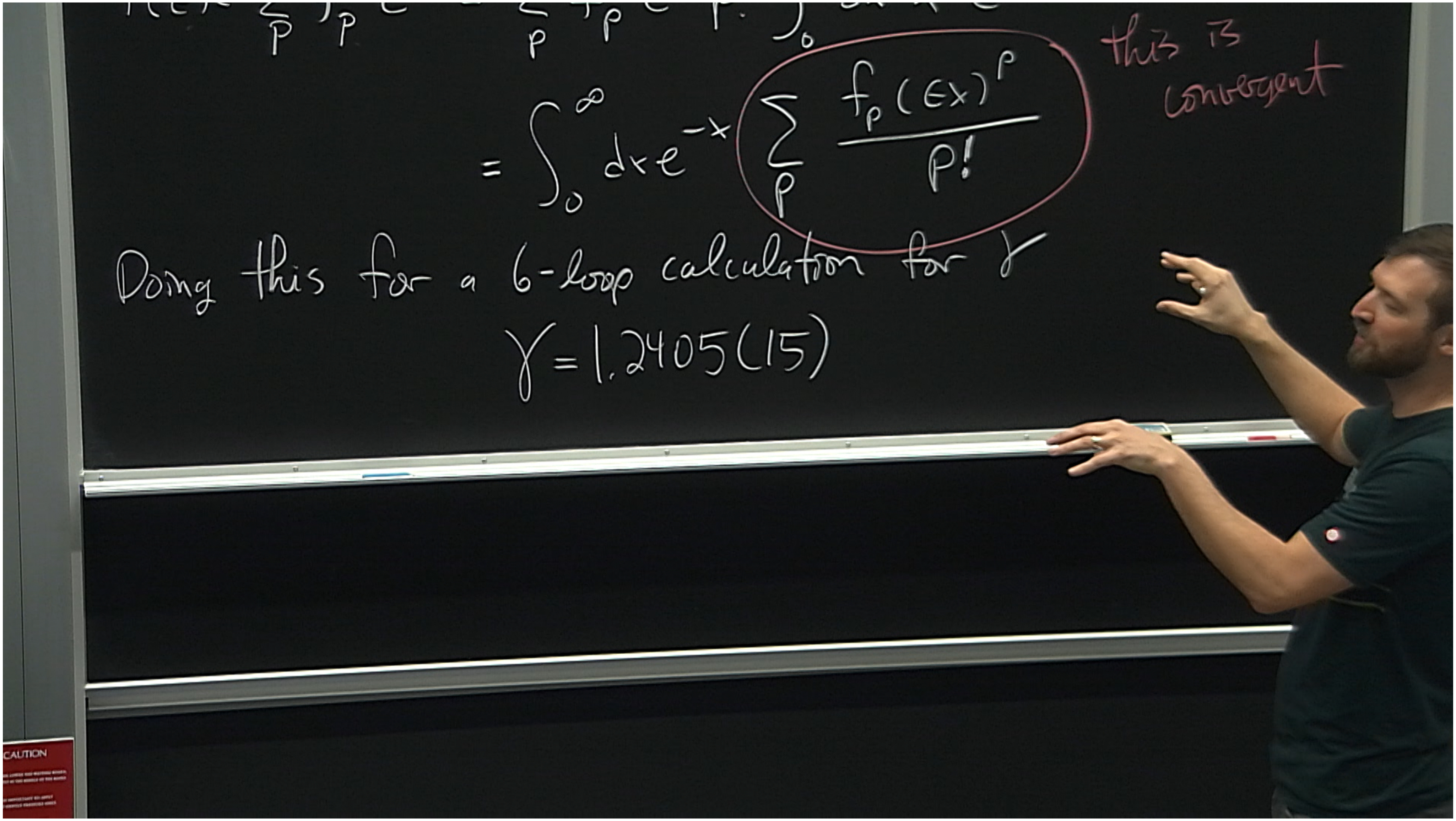
Can re-~~write~~ evaluate this with the "Borel" summation method!

$$p! = \int_0^\infty dx\, e^{-x} x^p$$

$$f(\epsilon) = \sum_p f_p \epsilon^p = \sum_p f_p \epsilon^p \frac{1}{p!} \int_0^\infty dx\, x^p e^{-x}$$

$$= \int_0^\infty dx\, e^{-x} \sum_p \frac{f_p (\epsilon x)^p}{p!}$$

this is convergent


$= \int_0^\infty dx\, e^{-x} \sum_p \frac{f_p (\epsilon x)^p}{p!}$
this is convergent
Doing this for a 6-loop calculation for γ
$\gamma = 1.2405(15)$
CAUTION

Since 2008 or so, the record for calculating these exponents have come from a non-perturbative "meta"-theory called the conformal bootstrap.

Since 2008 or so, the record for calculating these exponents have come from a non-perturbative "meta"-theory called the conformal bootstrap.

$$\gamma = 1.237075(10)$$ (conjectured conformal invariance)

<u>see</u>: David Simmons-Duffin PIRSA 17010082

Since 2008 or so, the record for calculating these exponents have come from a non-perturbative "meta"-theory called the conformal bootstrap.

$$\gamma = 1.237075(10) \quad \left(\begin{array}{l}\text{conjectured} \\ \text{conformal invariance}\end{array}\right)$$

see: David Simmons-Duffin PIRSA 17010082

CAUTION

have come from a non-perturbative "meta"-theory called the conformal bootstrap.

$$\gamma = 1.237075(10) \quad \left(\text{conjectured conformal invariance}\right).$$

<u>see</u>: David Simmons-Duffin PIRSA 17010082

Irrelevance of other interactions

have come from a non-perturbative [illegible]

the conformal bootstrap.

$$\gamma = 1.237075(10) \quad \left(\begin{array}{l}\text{conjectured} \\ \text{conformal invariance}\end{array}\right)$$

see: David Simmons-Duffin PIRSA 17010082

Irrelevance of other interactions

→ just by symmetry arguments the LGW action needs other terms

$$S[\varphi] = S_0[\varphi] + S_{int}[\varphi]$$

$$S_0[\varphi] = \int d^d x \left[\frac{K}{2}(\nabla\varphi)^2 + \frac{r}{2}\varphi^2 + \frac{L}{2}(\nabla^2\varphi)^2 + \cdots \right]$$

$$S_{int}[\varphi] = \int d^d x \left[\frac{u}{4}\varphi^4 + v\,\varphi^2(\nabla\varphi)^2 + \cdots + \frac{u_6}{6}\varphi^6 + \frac{u_8}{8}\varphi^8 + \cdots \right]$$

$L' = L b \Rightarrow \frac{dL}{dl} = -2L$

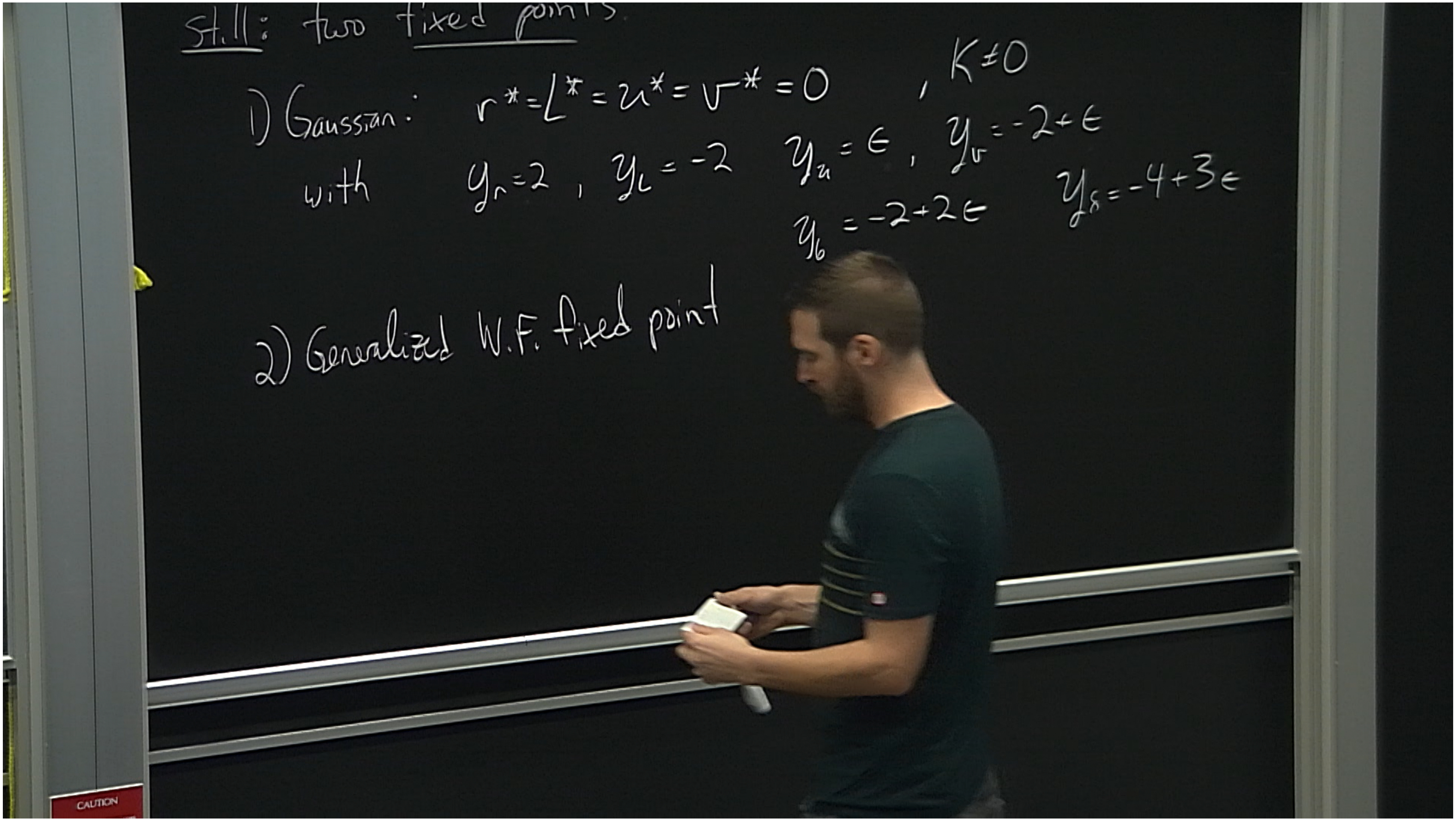
$u_8' = u_8 b^{8-3d} \Rightarrow \frac{du_8}{dl} = (-4+3\epsilon)\, u_8$

Still: two fixed points

1) Gaussian: $r^* = L^* = u^* = v^* = 0$, $K \neq 0$

with $y_r = 2$, $y_L = -2$, $y_u = \epsilon$, $y_v = -2+\epsilon$

$y_6 = -2+2\epsilon$, $y_8 = -4+3\epsilon$


Still: two fixed points:
1) Gaussian: $r^* = L^* = u^* = v^* = 0$, $K \neq 0$
with $y_r = 2$, $y_L = -2$ $y_u = \epsilon$, $y_v = -2+\epsilon$
$y_6 = -2+2\epsilon$ $y_8 = -4+3\epsilon$
2) Generalized W.F. fixed point
CAUTION

1) Gaussian: $r^* = L^* = u^* = v^* = 0$, $K \neq 0$

with $y_r = 2$, $y_L = -2$ $y_u = \epsilon$, $y_v = -2+\epsilon$

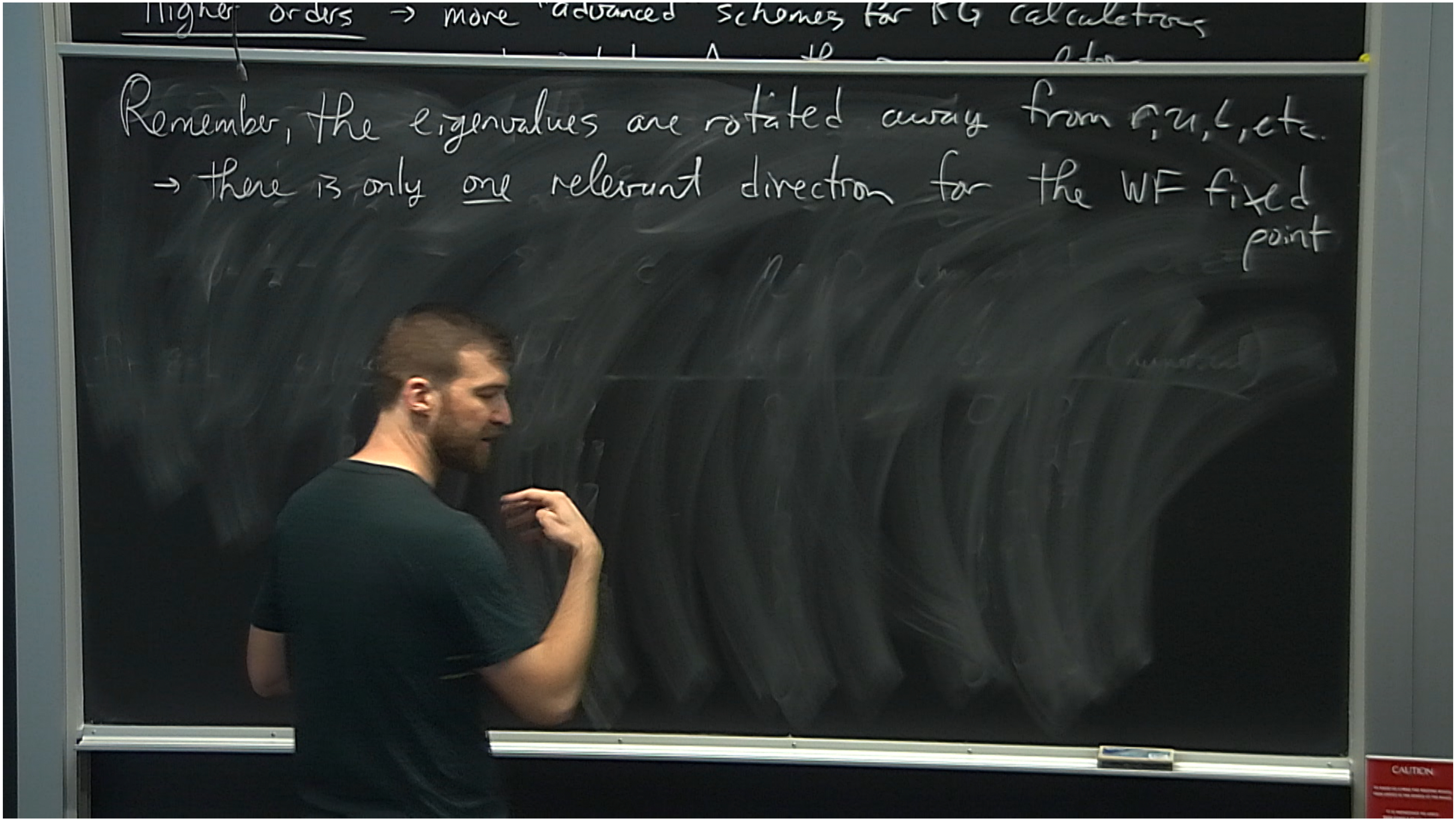
$y_6 = -2+2\epsilon$ $y_8 = -4+3\epsilon$

2) Generalized W.F. fixed point

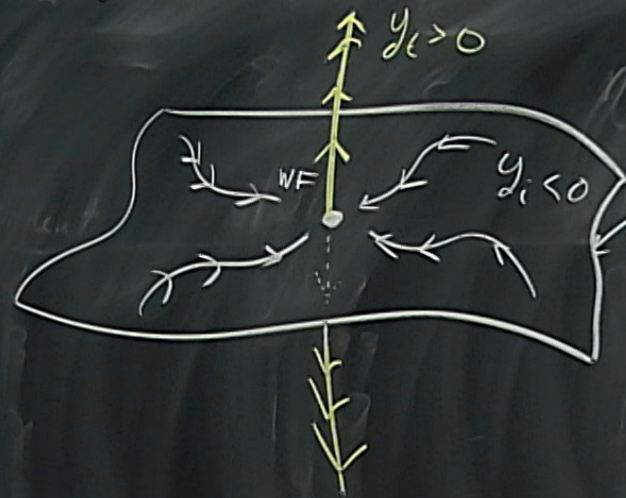
$r^* \sim u^* \sim O(\epsilon)$, $L^* \sim v^* \sim O(\epsilon^2)$ $u_6^* \sim O(\epsilon^3)$

$y_t = 2 - \frac{1}{3}\epsilon$, $y_L = -2 + O(\epsilon)$ $y_6 = -2 + O(\epsilon)$

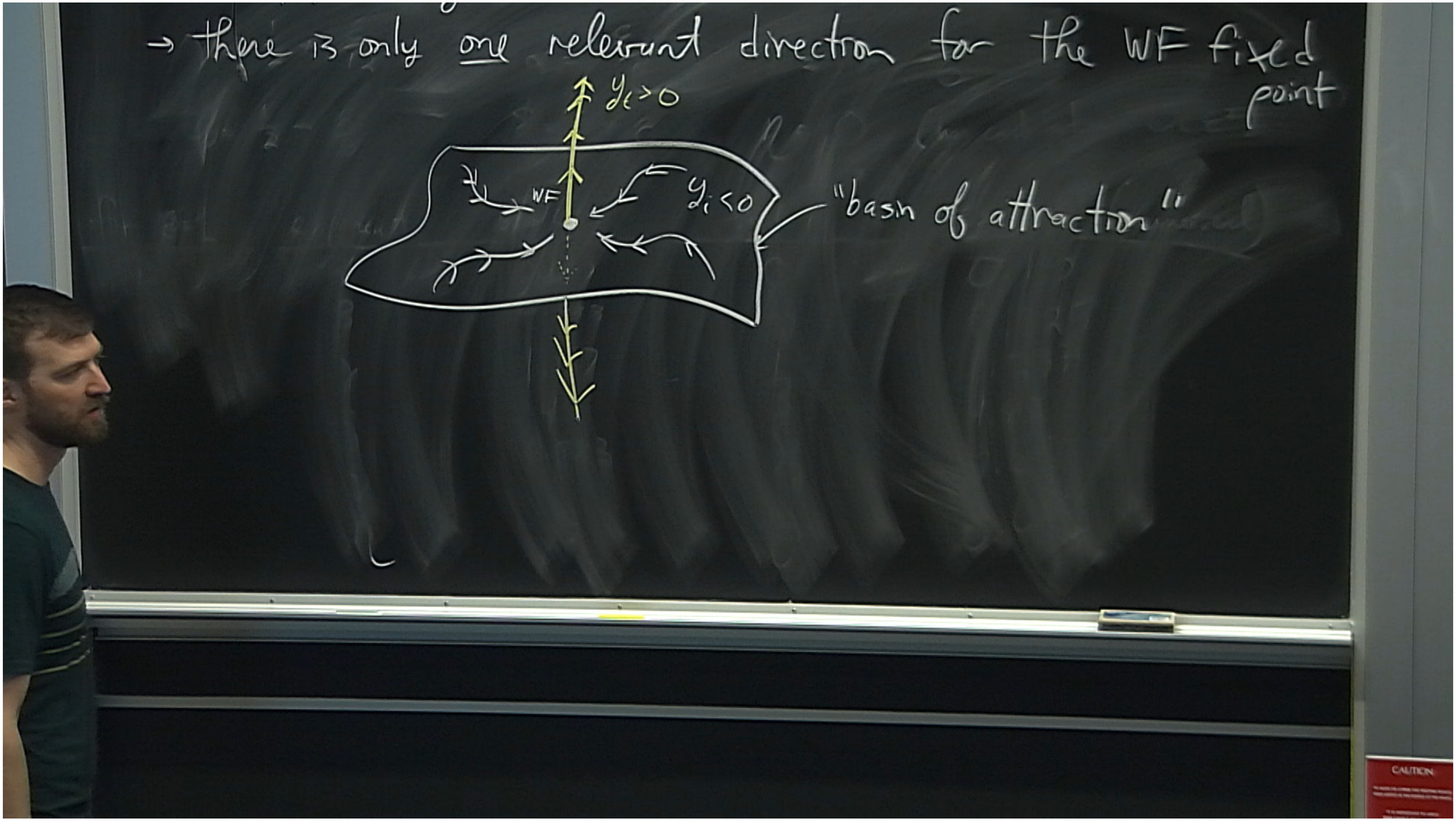
$y_u = -\epsilon + O(\epsilon^2)$, $y_v = -2 + O(\epsilon)$, $y_8 = -4 + O(\epsilon)$


Higher orders → more "advanced" schemes for RG calculations
Remember, the eigenvalues are rotated away from r, u, λ, etc.
→ there is only one relevant direction for the WF fixed point

→ there is only <u>one</u> relevant direction for the WF fixed point



"basin of attraction"


→ there is only one relevant direction for the WF fixed point
$y_i > 0$
WF
$y_i < 0$
"basin of attraction"

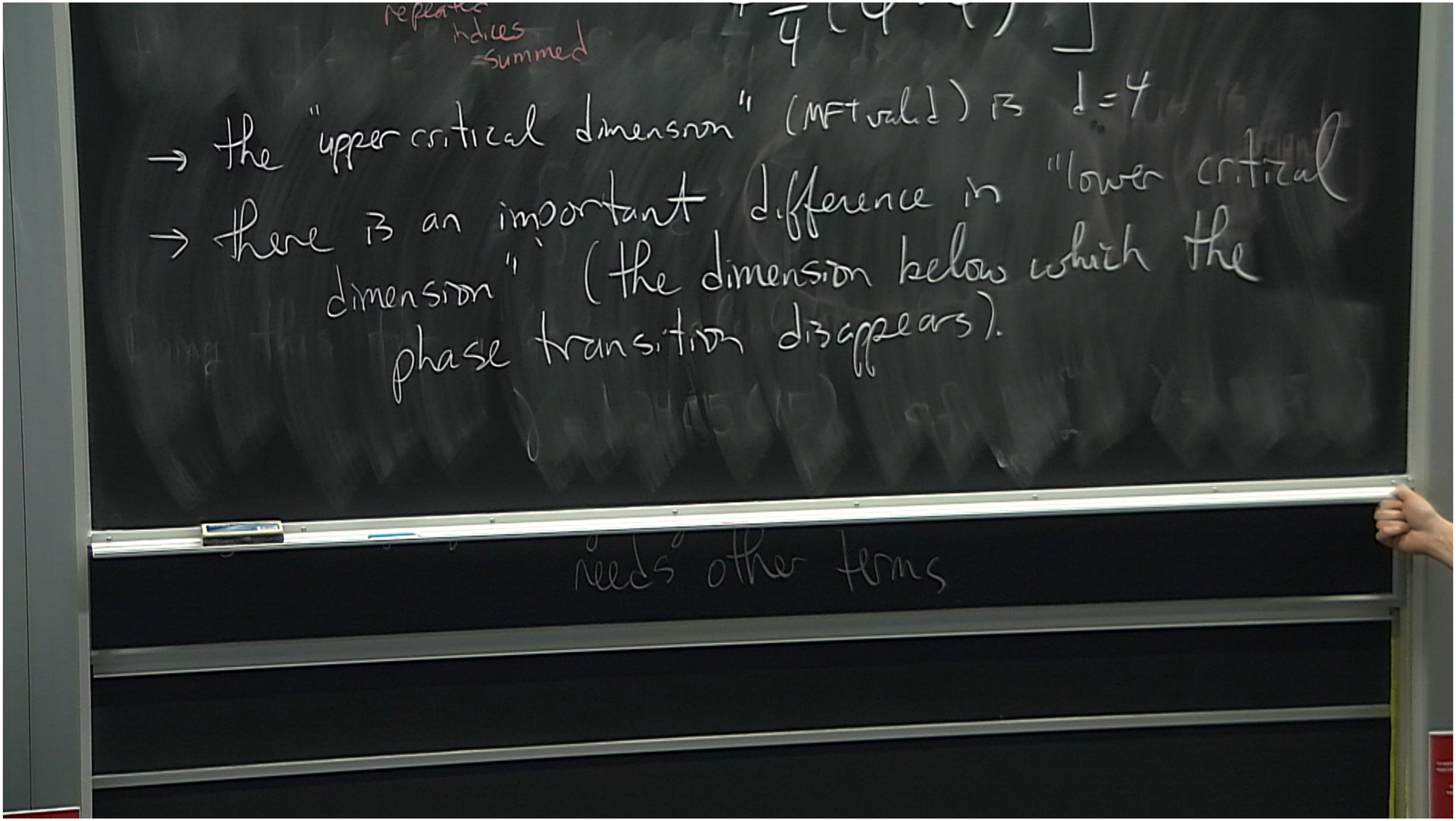
Continuous symmetry:

Move beyond "single component" φ^4 theories to spins with higher symmetries. (where spins are vectors).

e.g. "Heisenberg" $H = -\sum_{ij} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$ $\quad O(3)$

i.e. $\vec{S} = (S_x, S_y, S_z)$

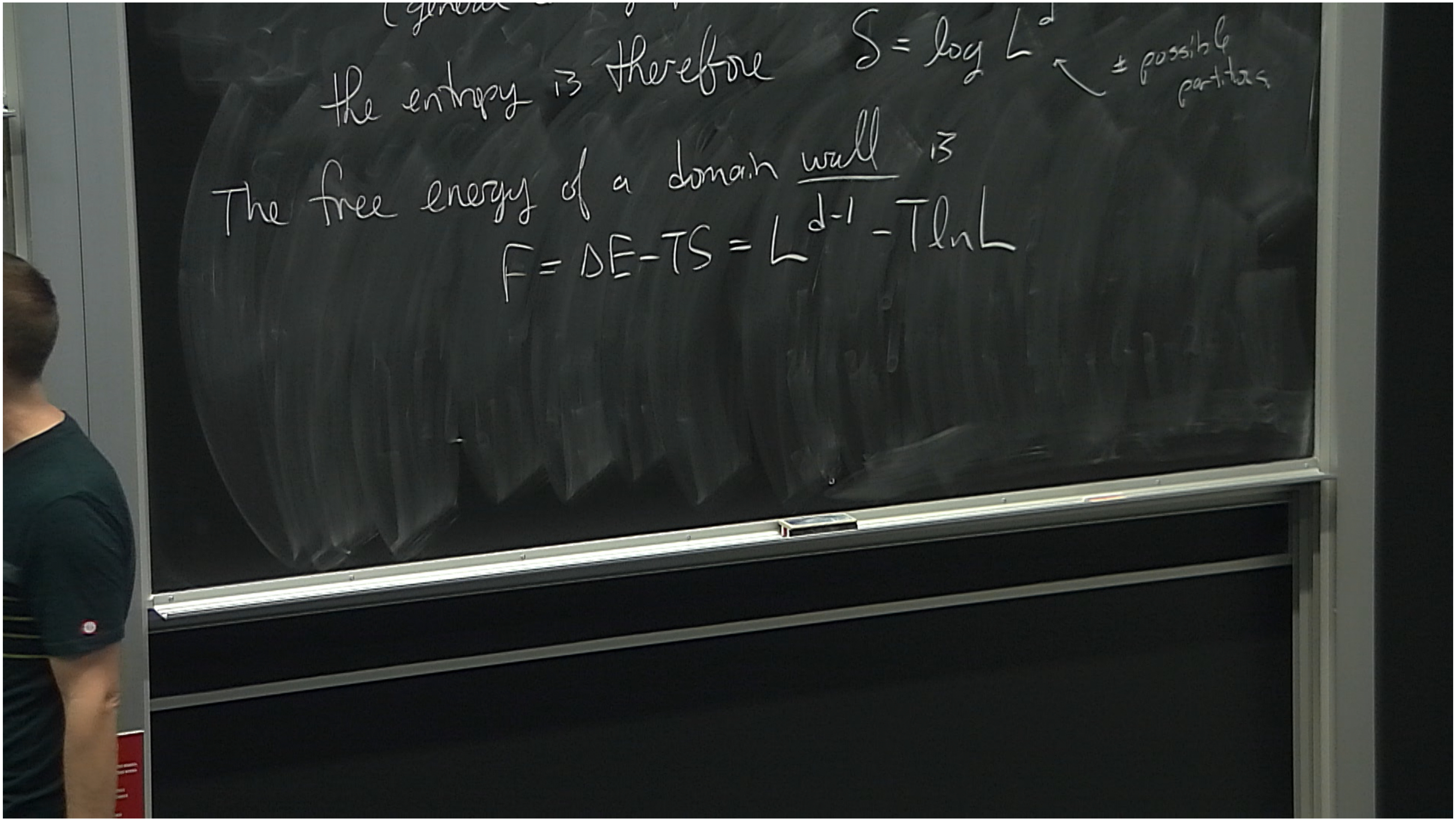
"XY" special case of $\vec{S} = (S_x, S_y)$ $\quad O(2)$

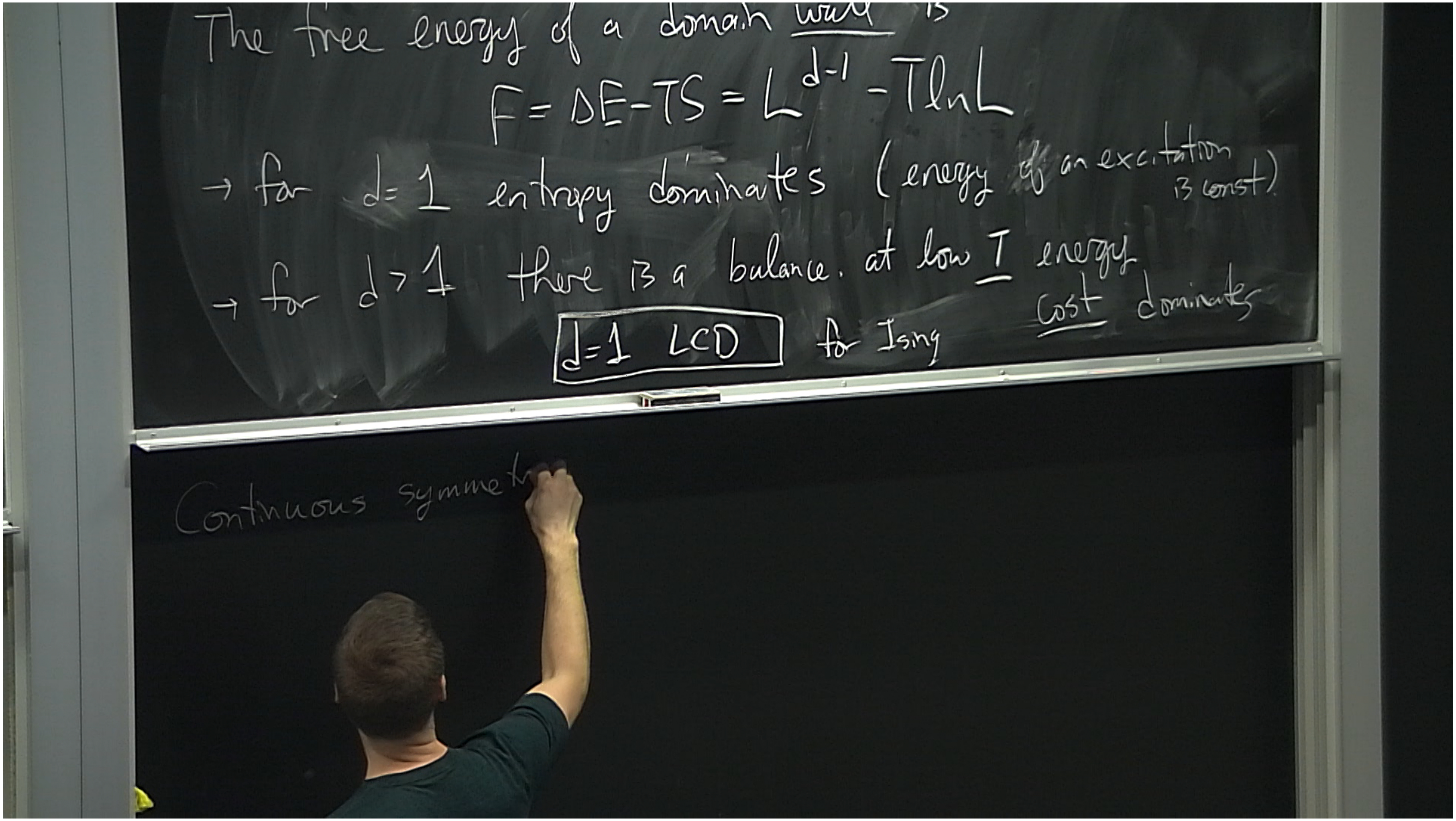

indices
summed
→ the "upper critical dimension" (MFT valid) is d = 4
→ there is an important difference in "lower critical dimension" (the dimension below which the phase transition disappears).
needs other terms

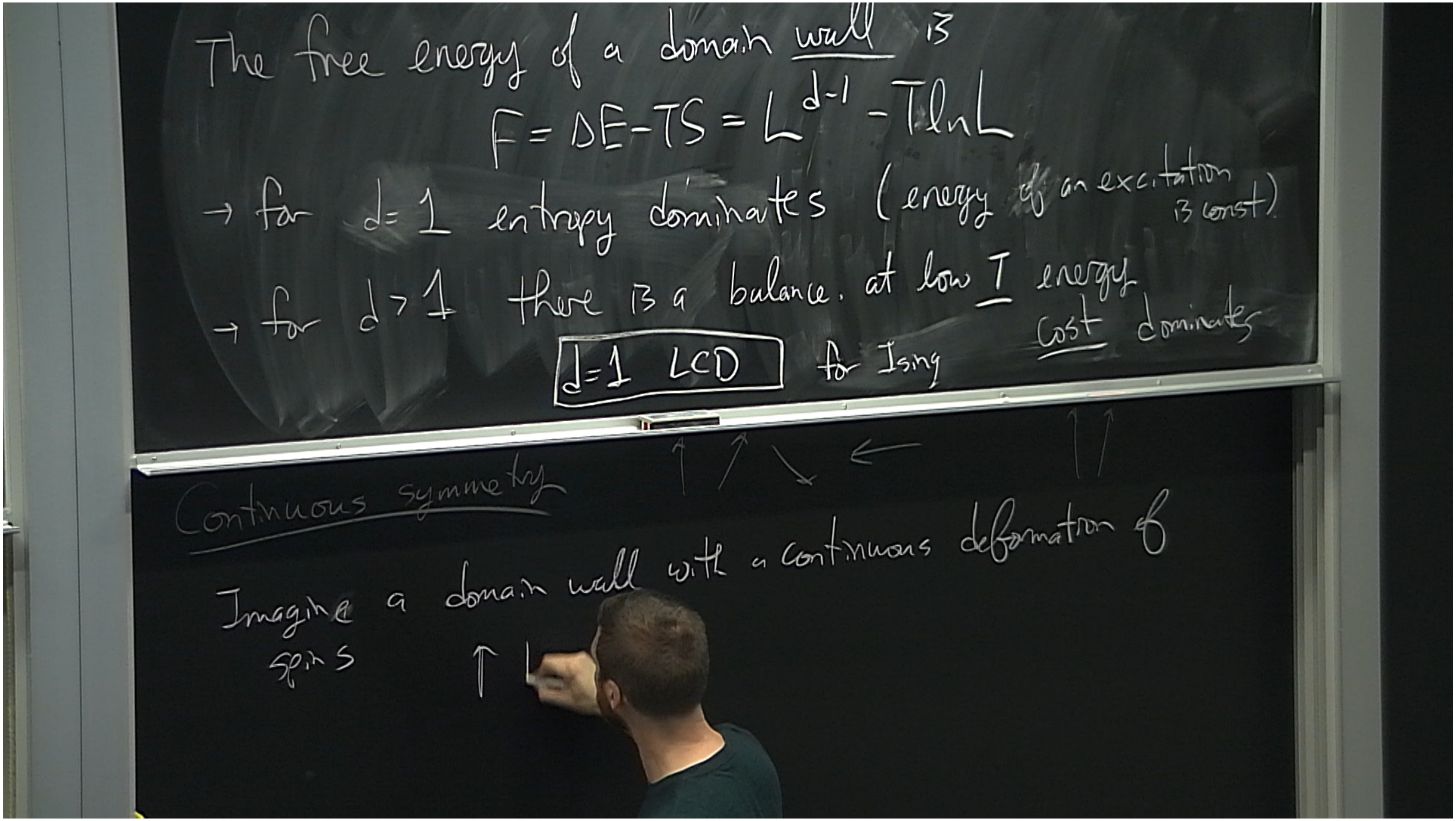
the entropy is therefore $S = \log L^d$ ± possible partitions

The free energy of a domain <u>wall</u> is

$$F = \Delta E - TS = L^{d-1} - T\ln L$$


the entropy is therefore $S = \log L^d$
± possible partitions
The free energy of a domain wall is
$F = \Delta E - TS = L^{d-1} - T \ln L$


The free energy of a domain wall is
F = ΔE − TS = L^{d-1} − T ln L
→ for d=1 entropy dominates (energy of an excitation is const)
→ for d>1 there is a balance. at low T energy cost dominates
d=1 LCD for Ising
Continuous symmet


The free energy of a domain wall is
F = ΔE − TS = L^{d-1} − T ln L
→ for d = 1 entropy dominates (energy of an excitation is const)
→ for d > 1 there is a balance. at low T energy cost dominates
d=1 LCD for Ising
Continuous symmetry
Imagine a domain wall with a continuous deformation of spins

$\boxed{d=1 \text{ LCD}}$ for Ising

$\uparrow \; \nearrow \; \searrow \; \leftarrow \qquad \uparrow\uparrow$

Continuous symmetry

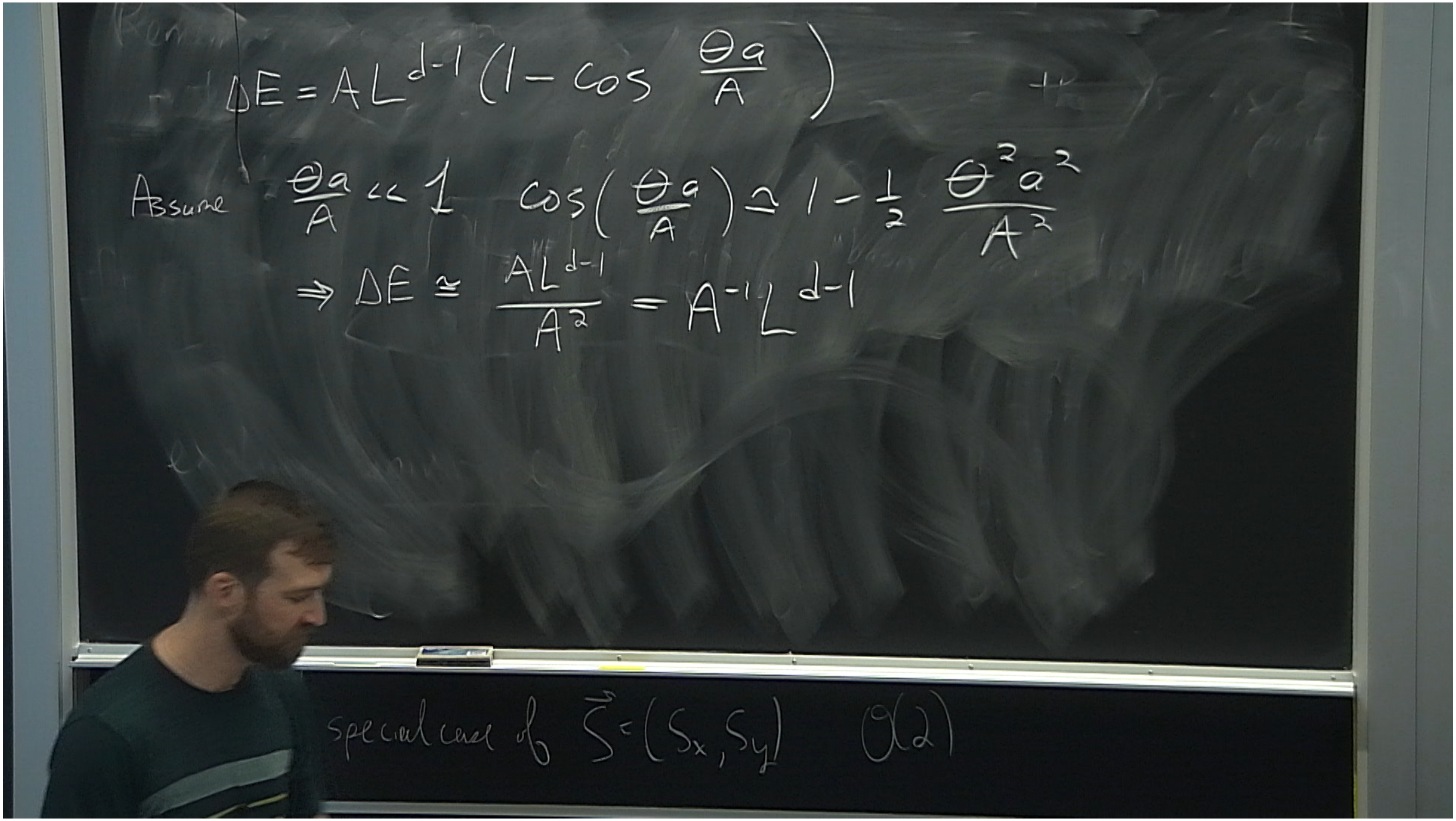
Imagine a domain wall with a continuous deformation of spins

$$\uparrow\;\uparrow\;\underbrace{\uparrow\;\nearrow\;\nearrow\;\rightarrow\;\rightarrow\;\searrow}_{A}\;\downarrow\;\downarrow\;\downarrow\;\underset{a}{\downarrow\;\downarrow}$$

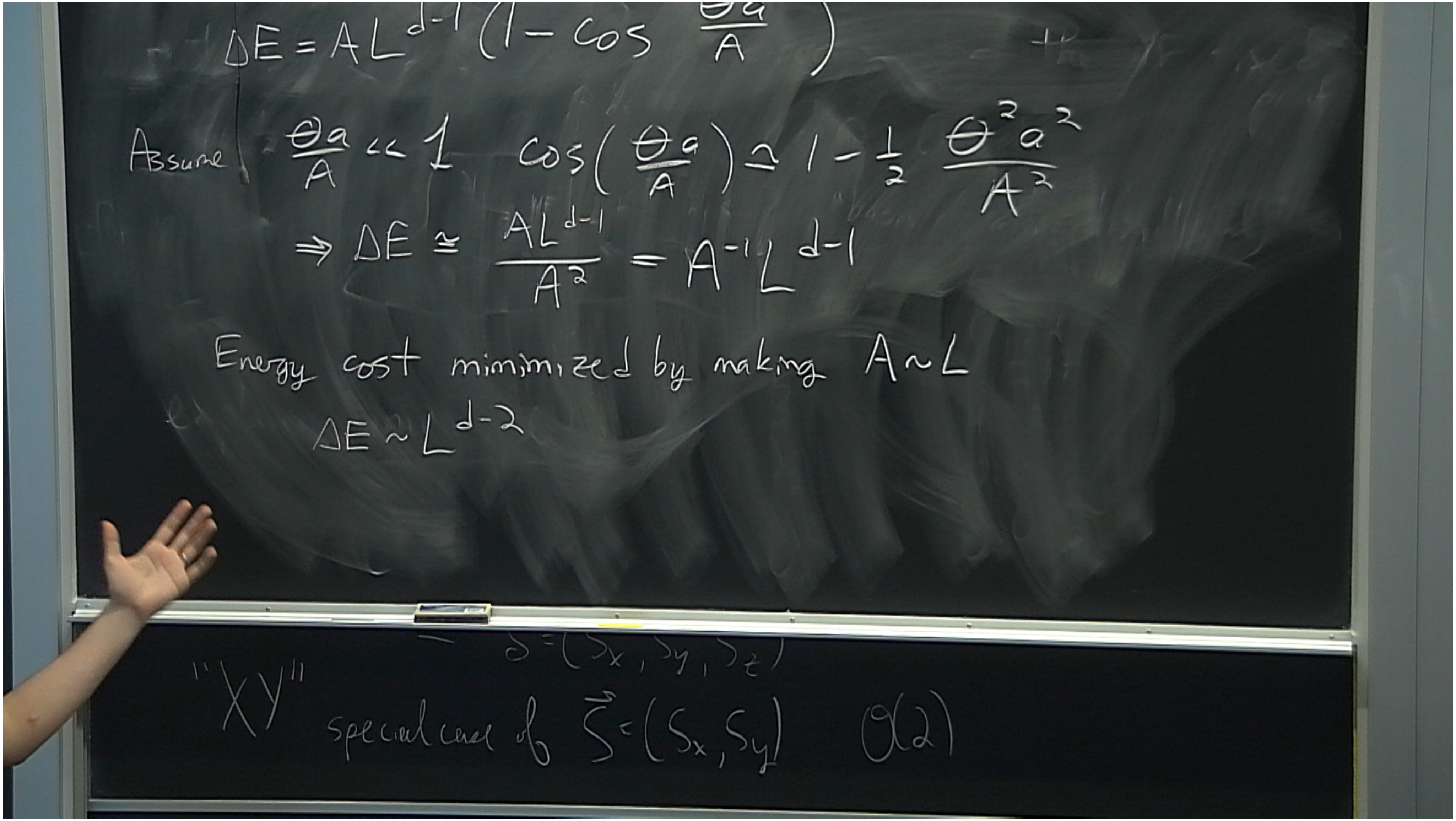
Cost in energy:

$$\Delta E = E_{ex} - E_{gs}$$
$$= AL^{d-1}\left(-\cos\left(\frac{\Theta}{A}a\right)\right) - (-1)AL^{d-1}$$

where Θ is the total angle by which spins rotate



$\Delta E = A L^{d-1} \left(1 - \cos \frac{\theta a}{A}\right)$
Assume $\frac{\theta a}{A} \ll 1$ $\cos\left(\frac{\theta a}{A}\right) \simeq 1 - \frac{1}{2} \frac{\theta^2 a^2}{A^2}$
$\Rightarrow \Delta E \cong \frac{A L^{d-1}}{A^2} = A^{-1} L^{d-1}$
special case of $\vec{S} = (S_x, S_y)$ $O(2)$

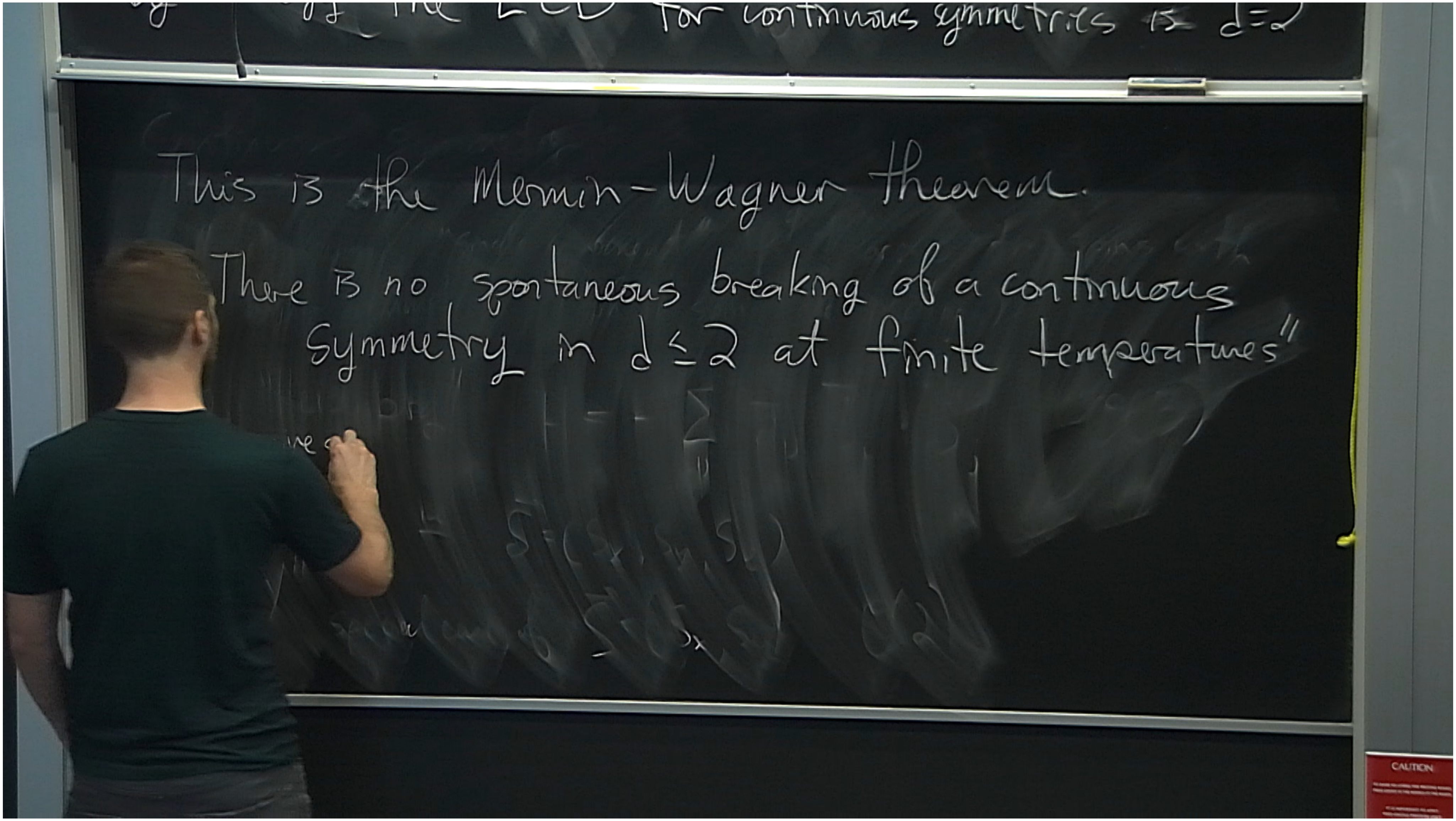

$\Delta E = A L^{d-1} \left(1 - \cos \frac{\theta a}{A}\right)$
Assume $\frac{\theta a}{A} \ll 1$ $\cos\left(\frac{\theta a}{A}\right) \simeq 1 - \frac{1}{2} \frac{\theta^2 a^2}{A^2}$
$\Rightarrow \Delta E \cong \frac{A L^{d-1}}{A^2} = A^{-1} L^{d-1}$
Energy cost minimized by making $A \sim L$
$\Delta E \sim L^{d-2}$
$\vec{S} = (S_x, S_y, S_z)$
"XY" special case of $\vec{S} = (S_x, S_y)$ $O(2)$

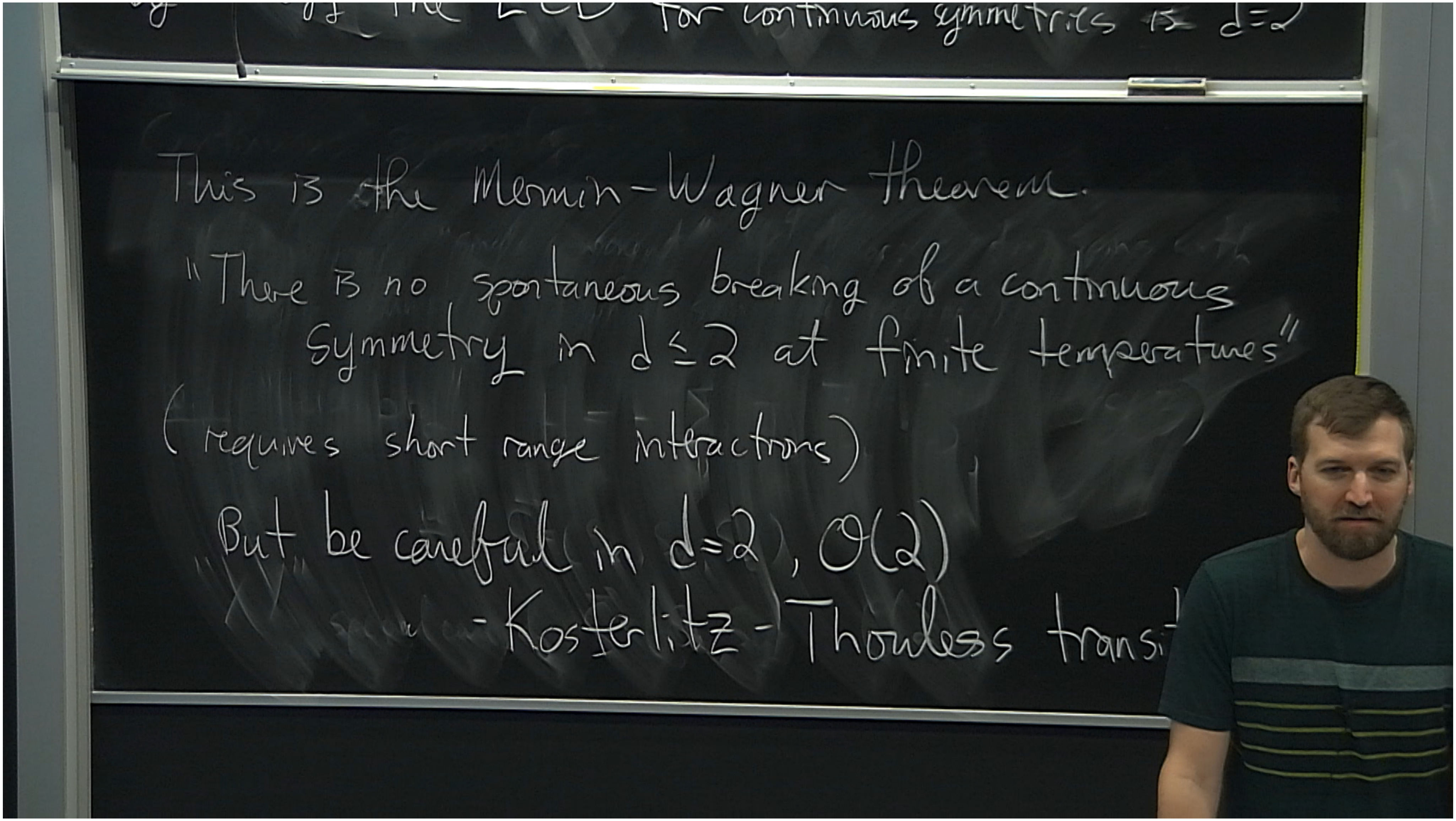
Energy cost minimized by making $A \sim L$

$$\Delta E \sim L^{d-2} \Rightarrow F = L^{d-2} - T \log L$$

By analogy the LCD for continuous symmetries is $d=2$

This is the Mermin-Wagner theorem.


for continuous symmetries is d=2
This is the Mermin-Wagner theorem.
"There is no spontaneous breaking of a continuous symmetry in d≤2 at finite temperatures"
CAUTION


for continuous symmetries is $d=2$
This is the Mermin-Wagner theorem.
"There is no spontaneous breaking of a continuous symmetry in $d \leq 2$ at finite temperatures"
(requires short range interactions)
But be careful in $d=2$, $O(2)$
- Kosterlitz - Thouless transi