Title: Cosmology Group Meeting - Hierarchy of varying c theories - theory and observations

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Abstract: <p>Despite varying speed of light theories (VSL) should be considered as another type of alternative gravity theories with an extra scalar degree of freedom, their formulation causes the problems in view of breaking the light principle and relativity principle. Besides, there are a couple of physical contexts in which c plays the crucial role and it is uncertain that it has the same meaning everywhere. During my talk I will discuss some basic theoretical formulations of varying c theories and discuss their benefits as well as problems. Then, I will review some cosmological tests of VSL theories making also a comparative statistical analysis of the basic theoretical frameworks. Among them, the recent Moffat's varying c approach. </p>

Hierarchy of varying c theories - theory and observations.

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Perimeter Institute, Waterloo - 23 February 2017

Plan:

- 1. Introduction - main frameworks of varying constants theories.
- 2. Benefits and problems of varying c theories.
- 3. Redshift drift test of varying c models.
- 4. Cosmological measurement of c with baryon acoustic oscillations (BAO).
- 5. Modelling spatial variations of c.
- 6. Statistical analysis of c-varying models: Barrow-Magueijo (BM), Avelino-Martins (AM) and Moffat (M).
- 7. Conclusions.

Collaborators:

- MPD, K. Marosek, JCAP 02 (2013), 012.
- A. Balcerzak, MPD, PLB **728**, 15 (2014).
- MPD, T. Denkiewicz, C.J.A.P. Martins, P. Vielzeuf - PRD **89**, 123512 (2014).
- V. Salzano, MPD, R. Lazkoz, PRL **114**, 101304 (2015).
- V. Salzano, MPD, R. Lazkoz, PRD **93**, 063521 (2016).
- V. Salzano, A. Balcerzak, MPD - arXiv: 1604.07655.
- V. Salzano, MPD - arXiv: 1612.06367.

1. Introduction - main frameworks of varying constants theories.

Long story of varying constants theories:

H. Weyl (1919): electron radius/its gravitational radius $\sim 10^{40}$

A. Eddington (1935) discussed:

1. proton-to-electron mass $1/\beta = m_p/m_e \sim 1840$
2. an inverse of fine structure constant $1/\alpha = (hc)/(2\pi e^2) \sim 137$
3. electromagnetic to gravitational force between a proton and an electron $e^2/(4\pi\epsilon_0 G m_e m_p) \sim 10^{40}$
4. introduced "Eddington number" $N_{edd} \sim 10^{80}$

P.A.M. Dirac (1937) interesting remarks about the relations between atomic and cosmological quantities: If $G \propto H(t) = (da/dt)/a$, then $a(t) \propto t^{1/3}$ and $G(t) \propto 1/t$ - fundamental constants must evolve in time.

Conclusion: electromagnetic force is strong compared to gravitational since the universe is "old" i.e. $F_e/F_p \propto (e^2/m_e m_p)t \propto t$!!!

varying gravitational constant G theories

First fully quantitative framework: Brans-Dicke scalar-tensor gravity (1961)
The gravitational constant G is associated with an average gravitational potential (scalar field) ϕ surrounding a given particle:
$<\phi>= GM/(c/H_0) \propto 1/G = 1.35 \times 10^{28} g/cm$. The scalar field gives the strength of gravity

$$G = \frac{1}{16\pi\Phi} \tag{1}$$

With the action

$$S = \int d^4x\sqrt{-g}\left(\Phi R - \frac{\omega}{\Phi}\partial_\mu\Phi\partial^\mu\Phi + \Lambda + L_m\right) \tag{2}$$

it relates to low-energy-effective superstring theory for $\omega = -1$
String coupling constant (running) $g_s = \exp(\phi/2)$ changes in time with ϕ - the dilaton and $\Phi = \exp(-\phi)$.

Varying speed of light c (VSL) theories

Attempts: Einstein (1907), Dicke (1957), J.-P. Petit (1988) (Einstein eqs remain same due to fine-tuned change of c and G), first full formulation by Moffat (1993). Albrecht & Magueijo model (1998) (AM model) (Barrow 1999; Magueijo 2003): Introduce a scalar field

$$c^4 = \psi(x^\mu) \tag{3}$$

and so the action is

$$S = \int d^4x \sqrt{-g} \left[\frac{\psi(R + 2\Lambda)}{16\pi G} + L_m + L_\psi \right] \tag{4}$$

AM model breaks Lorentz invariance (relativity principle and light principle) so that there is a preferred frame (cosmological or CMB) in which the field is minimally coupled to gravity. The Riemann tensor is computed in such a frame for a constant $\psi = c^4$ and no additional terms $\partial_\mu \psi$ appear in this frame (though they do in other frames). Einstein eqs remain the same **except** c now varies.

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Varying fine structure constant α theories

Varying fine structure constant α (or charge $e = e_0\epsilon(x^\mu)$ theories (Webb et al. 1999, Sandvik 2002)

$$S = \int d^4x\sqrt{-g}\left(R - \frac{\omega}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{4}f_{\mu\nu}f^{\mu\nu}e^{-2\psi} + L_m\right) \tag{8}$$

with $\psi = \ln\epsilon$ and $f_{\mu\nu} = \epsilon F_{\mu\nu}$.

Can be related with the VSL theories due to the definition of the fine structure constant

$$\alpha = \frac{e^2}{\hbar c} \quad \text{i.e.} \quad \alpha(t) = \frac{e^2}{\hbar c(t)} \tag{9}$$

Assume linear expansion $e^\psi = 1 - 8\pi G\zeta(\psi - \psi_0) = 1 - \Delta\alpha/\alpha$ with the constraint on the local equivalence principle violence $\mid \zeta \mid \leq 10^{-3}$. The relation to dark energy is (e.g. Vielzeuf and Martins 2012):

$$w + 1 = \frac{(8\pi G\frac{d\psi}{d\ln a})^2}{\Omega_\psi} . \tag{10}$$

Varying fine structure constant α theories

The field equations for Friedmann universes are (e.g. Barrow, Kimberly, Magueijo 2004)

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left(\varrho_r + \varrho_\psi\right) - \frac{kc^2}{a^2}, \tag{11}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}\left(\varrho_r + 2\varrho_\psi\right), \tag{12}$$

$$\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} = 0, \tag{13}$$

where $\varrho_r \propto a^{-4}$ stands for the density of radiation while

$$\varrho_\psi = \frac{p_\psi}{c^2} = \frac{\sigma}{2}\dot{\psi}^2 \tag{14}$$

stands for the density of the scalar field ψ (standard with $\sigma = +1$ and phantom with σ = -1) and

$$\alpha = \alpha_0 e^{2\psi}. \tag{15}$$

2. Benefits and problems of varying c and α theories.

In the minimal coupling Barrow-Magueijo (BM) approach the generalized VSL Einstein-Friedmann equations (variation of gravitational constant G was also added, ϱ - mass density; $\varepsilon = \varrho c^2(t)$ - energy density) read as

$$\varrho(t) = \frac{3}{8\pi G(t)}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right), \quad (16)$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2}\right), \quad (17)$$

and the generalized conservation law is obtained from (16) and (17)

$$\dot{\varrho}(t) + 3\frac{\dot{a}}{a}\left(\varrho(t) + \frac{p(t)}{c^2(t)}\right) = -\varrho(t)\frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi G a^2}. \quad (18)$$

Benefits of varying c models

Solves basic problems of standard cosmology: flatness and horizon.

Flatness: inserting this into Friedmann (16) one gets

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G_0 C}{3} a^{-3(w+1)} + \frac{k c_0^2 a^{2n-2}(2n-1)}{2n+3w+1} , \tag{19}$$

and the density term (with C) will dominate the curvature term at large scale factor if

$$2 \geq 2n + 3(w+1) \tag{20}$$

Horizon: For large scale factor the solution is $a(t) = t^{2/3(w+1)}$ and the proper distance to the horizon reads as

$$d_H = c(t)t = c_0 a^n(t) t = c_0 a_0^n t^{(3w+3+2n)/3(w+1)} \tag{21}$$

and the scale factor grows faster than d_H under the same condition as in (20).

varying c (and G) removing or changing singularities.

Varying constants can **remove or change the nature of singularities** (MPD, Marosek 2013).

Type	Name	t sing.	a(t_s)	$\varrho(t_s)$	p(t_s)	$\dot{p}(t_s)$ etc.	w(t_s)	T	I
0	Big-Bang (BB)	0	0	∞	∞	∞	finite	strong	str
I	Big-Rip (BR)	t_s	∞	∞	∞	∞	finite	strong	str
I_l	Little-Rip (LR)	∞	∞	∞	∞	∞	finite	strong	str
I_p	Pseudo-Rip (PR)	∞	∞	finite	finite	finite	finite	weak	we
II	Sudden Future (SFS)	t_s	a_s	ϱ_s	∞	∞	finite	weak	we
II_g	Gen. Sudden Future (GSFS)	t_s	a_s	ϱ_s	p_s	∞	finite	weak	we
III	Finite Scale Factor (FSFS)	t_s	a_s	∞	∞	∞	finite	weak	str
IV	Big-Separation (BS)	t_s	a_s	0	0	∞	∞	weak	we
V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	we

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varying c (and G) removing or changing singularities.

Some of these can be regularized (removed by variable constants):

- In order to regularize an SFS or an FSF singularity by varying $c(t)$, the **light should slow and eventually stop propagating** at a singularity (strong coupling regime of an appropriate field). In loop quantum cosmology (LQC): anti-newtonian limit $c = c_0\sqrt{1 - \varrho/\varrho_c} \to 0$ for $\varrho \to \varrho_c$ with ϱ_c being the critical density (Cailettau et al. 2012). The low-energy limit $\varrho \ll \varrho_0$ gives the standard limit $c \to c_0$.)
- To regularize an SFS, FSF by varying gravitational constant $G(t)$ - **the strength of gravity has to become infinite** at an initial (curvature) singularity. Effectively, a new singularity - **of strong coupling** for a physical field such as $G \propto 1/\Phi$ appears. Such problems were already dealt with in superstring and brane cosmology where both the curvature singularity and a strong coupling singularity show up (choice of coupling, quantum corrections).

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Type	Name	t sing.	a(t$_s$)	$\varrho(t_s)$	p(t$_s$)	$\dot{p}(t_s)$ etc.	w(t$_s$)	T	[illegible]
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I_l	Little-Rip (LR)	∞	∞	∞	∞	∞	finite	strong	str
I_p	Pseudo-Rip (PR)	∞	∞	finite	finite	finite	finite	weak	we
II	Sudden Future (SFS)	t_s	a_s	ϱ_s	∞	∞	finite	weak	we
II_g	Gen. Sudden Future (GSFS)	t_s	a_s	ϱ_s	p_s	∞	finite	weak	we
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V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	we

Problems of varying c models

Main problem: to obtain the field equations out of any action (cf. also quantum cosmology)? (Ellis, Uzan 2003)

Equations (16)-(18) have just been obtained in a special frame - the one in which c is a constant and does not lead to any extra boundary terms (apart from standard ones). Einstein equations were simply generalized:

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\psi} T_{\mu\nu} \tag{22}$$

while the action (4) varied in a standard way leads to different field equations

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\psi} T_{\mu\nu} - \frac{1}{\psi}\psi_{;\nu;\mu} + \frac{1}{\psi}\Box\psi. \tag{23}$$

The application of Bianchi identity to (22) gives a conservation equation with dynamical ψ

$$T^{\mu\nu}_{;\mu} = -T^{\mu\nu}\psi_{;\mu} \tag{24}$$

Problems of varying c quantum cosmology

If ψ was supposed to be a dynamical matter field, then one could get the evolution equation using the Lagrangian

$$L_\psi = -\frac{\omega}{16\pi G\psi}\dot{\psi}^2, \tag{25}$$

but working only in a preferred frame and with ψ not coupled to $\sqrt{-g}$.
Treating $\psi = c^4$ as constant in a preferred frame also requires special treatment of the boundary terms in c-varying quantum cosmology. As mentioned, we vary the action in the special frame where c is constant which means that we drop c-induced boundary terms, but recover the time dependence of c again to proceed towards WdW equation (V_3 is a 3-volume)

$$L = \frac{3V_3 c^3(x^0)}{8\pi G(x^0)}\left(ka - a_{,0}^{\,2}\, a - \frac{\Lambda}{3}a^3 - \frac{8\pi G(x^0)}{3c^2}\varrho a^3\right) \tag{26}$$

Benefits of varying α cosmology

Since one does not brake Lorentz invariance in varying fine structure constant α theories, then there are no such problems in these models - the standard variational principle applies and the dynamical equation for the scalar field is given!
According to the definition, any variability of c $(e, \hbar)$ can be related to the variability of α:

$$\frac{\Delta\alpha}{\alpha} = -\frac{\Delta c}{c}. \tag{27}$$

The best constraints on $\Delta\alpha$ are:

- Oklo natural nuclear reactor: $\Delta\alpha/\alpha = (0.15 \pm 1.05) \cdot 10^{-7}$ at z = 0.14
- VLT/UVES quasars: $\Delta\alpha/\alpha = (0.15 \pm 0.43) \cdot 10^{-5}$ at 1.59 < z < 2.92
- SDSS quasars: $\Delta\alpha/\alpha = (1.2 \pm 0.7) \cdot 10^{-4}$ at 0.16 < z < 0.8 .

α dipole

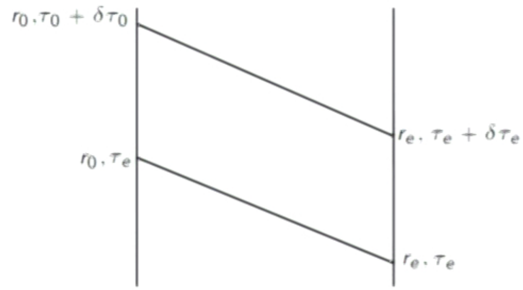
By Webb et al. (PRL 107, 191101 (2011)) (α-dipole $R.A. 17.4 \pm 0.9h$, $\delta = -58 \pm 9$: Keck ($\Delta\alpha < 0$) and VLT) as well as other specific measurements of α given in the table below (in parts per million):

Object	z	$\Delta\alpha/\alpha$	Spectrograph	Ref.
HE0515−4414	1.15	-0.1 ± 1.8	UVES	Molaro et al. (2008)
HE0515−4414	1.15	0.5 ± 2.4	HARPS/UVES	Chand et al. (2006)
HE0001−2340	1.58	-1.5 ± 2.6	UVES	Agafonowa et al. (2011)
HE2217−2818	1.69	1.3 ± 2.6	UVES–LP	Molaro et al. (2013)
Q1101−264	1.84	5.7 ± 2.7	UVES	Molaro et al. (2008)

UVES - Ultraviolet and Visual Echelle Telescope
HARPS - High Accuracy Radial velocity Planet Searcher
LP - Large Program measurement

3. Redshift drift test of varying c models.

Redshift drift (Sandage 1962, Loeb 1998) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.



There is a relation between the times of emission of light by the source τ_e and $\tau_e + \Delta\tau_e$ and times of their observation at τ_o and $\tau_o + \Delta\tau_o$:

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e+\Delta\tau_e}^{\tau_o+\Delta\tau_o} \frac{d\tau}{a(\tau)} , \qquad (28)$$

which for small $\Delta\tau_e$ and $\Delta\tau_o$ reads as $\frac{\Delta\tau_e}{a(\tau_e)} = \frac{\Delta\tau_o}{a(\tau_o)}$.

Redshift drift test.

The redshift drift is defined as ($\tau \to t$ here)

$$\Delta z = z_e - z_0 = \frac{a(t_0 + \Delta t_0)}{a(t_e + \Delta t_e)} - \frac{a(t_0)}{a(t_e)} \;, \qquad (29)$$

which can be expanded in series and to first order in Δt as

$$\Delta z = \frac{a(t_0) + \dot{a}(t_0)\Delta t_0}{a(t_e) + \dot{a}(t_e)\Delta t_e} - \frac{a(t_0)}{a(t_e)} \approx \frac{a(t_0)}{a(t_e)}\left[\frac{\dot{a}(t_0)}{a(t_0)}\Delta t_0 - \frac{\dot{a}(t_e)}{a(t_e)}\Delta t_e\right] \;. \qquad (30)$$

Using above relations we have

$$\Delta z = \Delta t_0 \left[H_0(1+z) - H(t(z))\right] = (1+z)\frac{\Delta v}{c} \;, \qquad (31)$$

where Δv is the velocity shift and $H(t(z))$ is given in a standard way.

Redshift drift in varying c theory.

In VSL theory the relation (28) generalizes into

$$\int_{t_e}^{t_o} \frac{c(t)dt}{a(t)} = \int_{t_e+\Delta t_e}^{t_o+\Delta t_o} \frac{c(t)dt}{a(t)} , \tag{32}$$

which for small Δt_e and Δt_o transforms into

$$\frac{c(t_e)\Delta t_e}{a(t_e)} = \frac{c(t_0)\Delta t_o}{a(t_o)} . \tag{33}$$

The definition of redshift in VSL theories remains the same as in standard Einstein relativity and reads as (Barrow, Magueijo 1999)

$$1 + z = \frac{a(t_0)}{a(t_e)} . \tag{34}$$

Redshift drift - varying c

Using (33) we have

$$\Delta z = \Delta t_0 \left[H_0(1+z) - H(t_e) \frac{c(t_0)}{c(t_e)} \right] \quad , \tag{35}$$

which after applying the ansatz

$$c(t) = c_0 a^n(t) \tag{36}$$

gives

$$\frac{\Delta z}{\Delta t_0} = \frac{\Delta z}{\Delta t_0}(z,n) = H_0(1+z) - H(z)(1+z)^n \quad . \tag{37}$$

Redshift drift - varying c

In the limit $n \to 0$ the formula (37) reduces to (31) for standard Friedmann universe. Bearing in mind definitions Ω's, and assuming $K = 0$ we have

$$H^2(z) = H_0^2 \left[\Omega_{m0}(1+z)^3 + \Omega_\Lambda\right] \tag{38}$$

and so (37) gives

$$\begin{aligned} \frac{\Delta z}{\Delta t_0} &= H_0 \left[1 + z - (1+z)^n \sqrt{\Omega_{m0}(1+z)^3 + \Omega_\Lambda}\right] \\ &= H_0 \left[1 + z - \sqrt{\Omega_{m0}(1+z)^{3+2n} + \Omega_\Lambda (1+z)^{2n}}\right] \end{aligned} \tag{39}$$

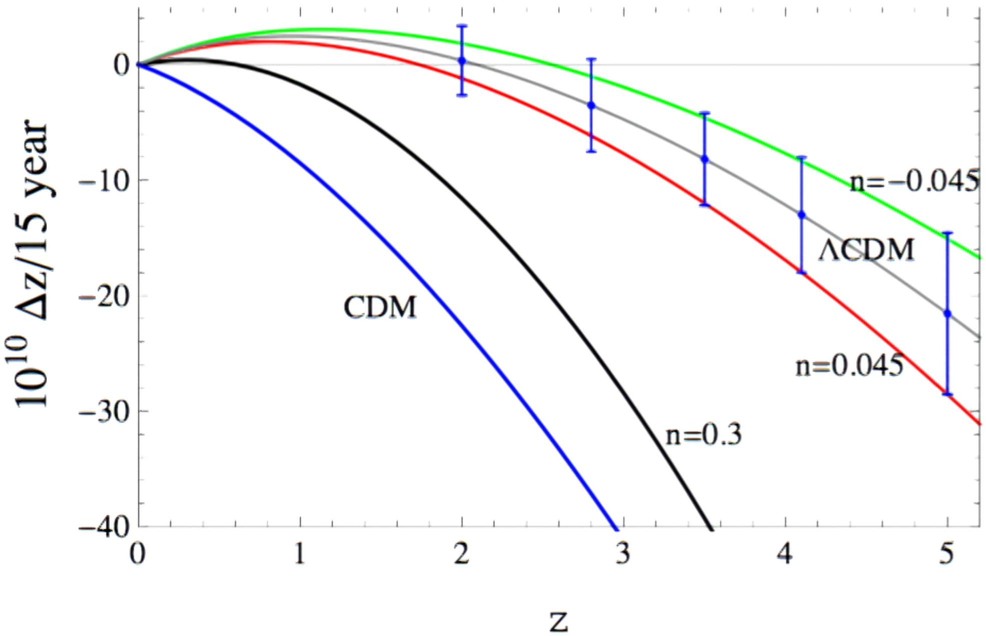
which can further be rewritten to define new redshift function

$$\tilde{H}(z) \equiv (1+z)^n H(z) = H_0 \sqrt{\sum_{i=1}^{i=k} \Omega_{wi} (1+z)^{3(w_{eff}+1)}} \;, \tag{40}$$

where $w_{eff} = w_i + \frac{2}{3}n$.

Redshift drift test - varying c

The VSL redshift drift effect for 15 year period of observations.



Redshift drift test - varying c

- If $n < 0$ (c decreases) then dust matter becomes little negative pressure matter and the cosmological constant became phantom. Both components can mimic dark energy.
- If $n > 0$ then (growing $c(t)$) VSL model becomes more like Cold Dark Matter (CDM) model.
- Theoretical error bars are taken from Quercellini et al. 2012 and presumably show that for $|n| < 0.045$ – **one cannot distinguish between VSL models and ΛCDM models**.
- In other words, by measuring redshift drift, **bounds** on the variability of c can be given from European Extremely Large Telescope (EELT) (with its spectrograph CODEX (COsmic Dynamics EXperiment)); Thirty Meter Telescope (TMT), the Giant Magellan Telescope (GMT). Also from gravitational wave interferometers DECIGO/BBO (DECi-hertz Interferometer Gravitational Wave Observatory/Big Bang Observer).

4. Measuring c with baryon acoustic oscillations (BAO)

Speed of light c appears in many observational quantities.
Among them in the angular diameter distance

$$D_A = \frac{D_L}{(1+z)^2} = \frac{a_0}{1+z} \int_{t_1}^{t_2} \frac{c(t)dt}{a(t)} \tag{41}$$

where D_L is the luminosity distance, a_0 present value of the scale factor (normalized to $a_0 = 1$ later), and we have taken the spatial curvature $k = 0$ (otherwise there would be $\sin$ or $\sinh$ in front of the integral). Using the definition of redshift and the dimensionless parameters Ω_i we have

$$D_A = \frac{1}{1+z} \int_0^z \frac{c(z)dz}{H(z)}, \tag{42}$$

where

$$H(z) = \sqrt{\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_\Lambda}. \tag{43}$$

Angular diameter distance maximum.

Due to the expansion of the universe, there is a maximum of the distance at

$$D_A(z_m) = \frac{c(z_m)}{H(z_m)}. \quad (44)$$

which can be obtained by simple differentiating (42) with respect to z:

$$\frac{\partial D_A}{\partial z} = -\frac{1}{(1+z)^2}\int_0^z \frac{c(z)dz}{H(z)} + \frac{1}{1+z}\frac{c(z)}{H(z)} = 0 \quad (45)$$

In a flat $k = 0$ cold dark matter CDM model

$$z_m = 1.25 \quad \text{and} \quad D_A \approx 1230 \;\; \text{Mpc} \quad (46)$$

For standard ΛCDM model of our interest:

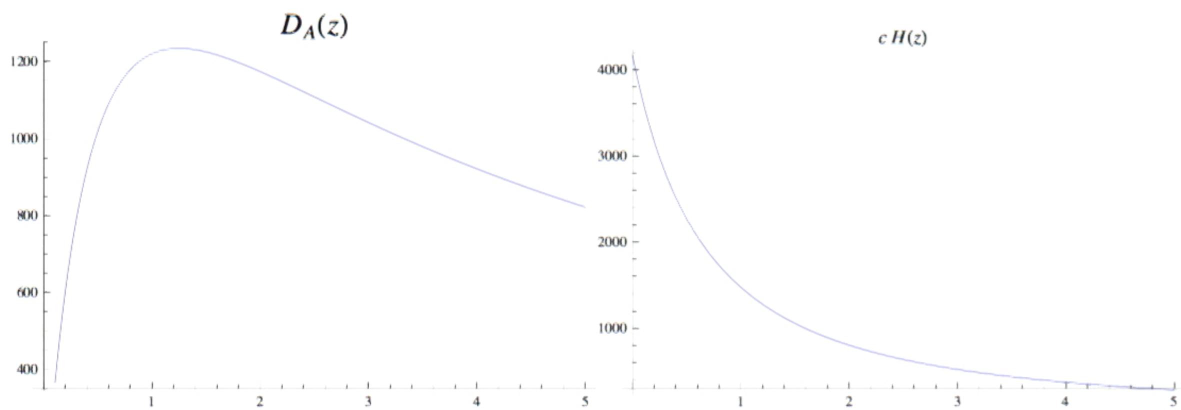
$$1.4 < z_m < 1.8. \quad (47)$$

D_A versus $H(z)$

The point: The product of D_A and H gives **exactly** the speed of light c at maximum (the curves intersect at z_m):

$$D_A(z_m)H(z_m) = c_0 \equiv 299792.458 \;\; \mathrm{kms}^{-1} \tag{48}$$

if we believe it is constant! (defined officially www.bipm.org; a relative error 10^{-9} by Evenson et al. 1972)



Measuring z_m

Measuring z_m problematic if one uses D_A only (large plateau around z_m makes it difficult to avoid errors from small sample of data – besides, one has binned data, observational errors, and instrinsic dispersion).
However, one can appeal to an independent measurement of $c_0/H(z)$ which is the radial (line-of-sight) mode of the baryon acoustic oscillations surveys (BAO) for which $D_A(z)$ is the tangential mode (e.g. Nesseris et al. 2006). In other words, we have both tangential and horizontal modes as

$$y_t = \frac{D_A}{r_s} \qquad y_r = \frac{c}{Hr_s}, \tag{49}$$

where

$$r_s = \int_{z_{dec}}^{\infty} \frac{cc_s(z)dz}{H(z)} \tag{50}$$

is the sound horizon size at decoupling and c_s the speed of sound.

Baryon acoustic oscillations.

From BOSS DR11 CMASS (Samushia et al. 2014)

$$\frac{D_V}{r_s(z_d)} = 13.85 \pm 0.17 \quad \text{at} \quad \bar{z} = 0.57, \tag{51}$$

where the volume-averaged distance is

$$D_V = \left[(1+z)^2 cz \frac{D_A^2}{H}\right]^{\frac{1}{3}}, \tag{52}$$

while from BOSS DR11 LOWZ (Tojeiro et al. 2014)

$$D_V = (1264 \pm 25)\left(\frac{r_s(z_d)}{r_{s,fid}(z_d)}\right) \quad \text{at} \quad \bar{z} = 0.32. \tag{53}$$

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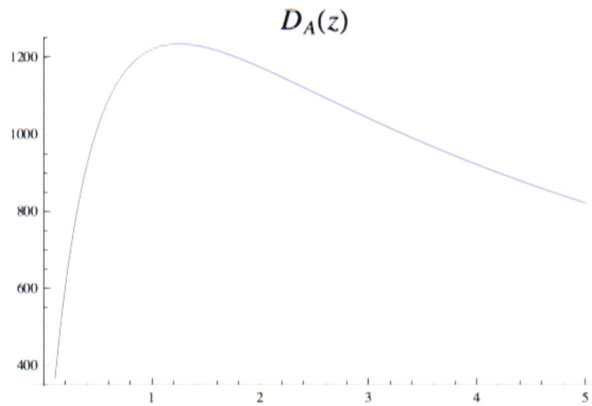
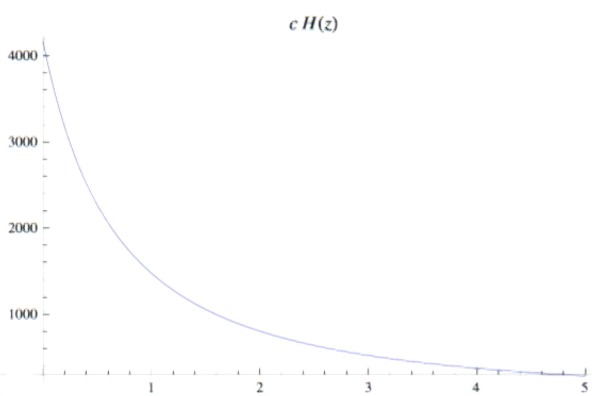
$$D_A(z_m)H(z_m) = c_0 \equiv 299792.458 \ \ \mathrm{kms}^{-1} \tag{48}$$

if we believe it is constant! (defined officially www.bipm.org; a relative error 10^{-9} by Evenson et al. 1972)





The method to measure c.

(Salzano, MPD, Lazkoz 2015)

- Measure independently $D_A(z)$ and $H(z)$.
- Calculate z_m.
- The product $D_A(z_m)H(z_m) = c(z_m)$.
- But $c(z_m)$ may not be equal to c_0, so that we can measure $\Delta c = c(z_m) - c_0$.
- This would determine possible variability of c.

The scenarios.

Take background ΛCDM model with an ansatz (Magueijo 2003)

$$c(a) \propto c_0 \left(\frac{1+a}{a_c} \right)^n \tag{54}$$

where a_c is the scale factor at the transition epoch from some $c(a) \neq c_0$ (at early times) to $c(a) \to c_0$ (at late times to now).

Three scenarios (Salzano, MPD, Lazkoz 2015):

1) standard case $c = c_0$;

2) $a_c = 0.005$, $n = -0.01 \to \Delta c/c \approx 1\%$ at $z \propto 1.5$;

3) $a_c = 0.005$, $n = -0.001 \to \Delta c/c \approx 0.1\%$ at $z \propto 1.5$.

The results.

Based on 10^3 Euclid project (Laureijs et al. 0912.0914) mock data simulations (Font-Ribeira et al. 2014):

1) $z_m = 1.592^{+0.043}_{-0.039}$ (fiducial model input $z_m = 1.596$) and $c/c_0 = 1 \pm 0.009$

2) $z_m = 1.528^{+0.038}_{-0.036}$ (fiducial $z_m = 1.532$) and $c(z_m)/c_0 = 1.00925 \pm 0.00831$

and

$$< c(z_m)/c_0 - 1\sigma_{c(z_m)/c_0} >= 1.00094^{+0.00014}_{-0.00033} \quad (55)$$

so that **a detection by Euclid of 1% variation at 1σ-level will be possible**.

3) $z_m = 1.584^{+0.042}_{-0.039}$ (fiducial $z_m = 1.589$) and $c(z_m)/c_0 = 1.00095 \pm 0.00852$

and

$$< c(z_m)/c_0 - 1\sigma_{c(z_m)/c_0} >= 0.99243^{+0.00016}_{-0.00013} \quad (56)$$

so that **a detection by Euclid of 1% variation at 1σ-level will not be possible**.

Perspectives.

- Euclid will have $1/10$ of the errors of the current missions like WiggleZ Dark Energy Survey (e.g. Blake et al. 2011, 2012).
- Other missions which will be competitive to Euclid and useful for our task will be:
- Dark Energy Spectroscopic Instrument (DESI) (Levi et al. 1308.0847)
- Square Kilometer Array (SKA) (Bull et al. 1405.1452)
- Wide-Field Infrared Survey Telescope (WFIRST) (Spergel et al. 1305.5425) (esp. having largest sensitivity at potential z_m region i.e. $1.5 < z < 1.6$).

5. Modelling spatial variations of c.

The $\alpha-$dipole reported at the Right Ascension $R.A. = 17.4 \pm 0.9$ h and declination $\delta = -58^\circ \pm 9^\circ$ or $(l, b) = (320^\circ, -11^\circ)$ can be related to a possible c-dipole. This was modelled within the framework of an inhomogeneous pressure Stephani-type model of the universe which is complementary to an LTB inhomogeneous density model (Balcerzak, MPD, Salzano 2016) with energy and pressure

$$\varrho(t) = \frac{3}{8\pi G}\left[\frac{\dot{a}^2(t)}{a^2(t)} + \frac{k(t)c_0^2}{a^2(t)}\right], \quad (57)$$

$$p(t,r) = w_{eff}(t,r)\varrho(t)c_0^2 \quad (58)$$

$$\equiv \left[-1 + \frac{1}{3}\frac{\dot{\varrho}(t)}{\varrho(t)}\frac{\left[\frac{V(t,r)}{a(t)}\right]}{\left[\frac{V(t,r)}{a(t)}\right]^{\bullet}}\right]\varrho(t)c_0^2,$$

where

$$V(t,r) = 1 + \frac{1}{4}k(t)r^2, \quad (\ldots)^{\bullet} \equiv \partial/\partial t. \quad (59)$$

Spatial variations of c

Here $k(t)$ is time-dependent curvature index. The radial dependence of the effective barotropic index $w_{eff}(r,t)$ is due to the radial dependence of the fluid pressure and means that a comoving observer **does not follow a geodesic**. In fact, a comoving observer has a four-velocity with a non vanishing radial component and **moves in the radial direction** in addition to its movement due to the expansion. Extra radial force pushes him out of a geodesic.
A specific model is with $k(t) = \beta a(t)$ and $\beta = const.$ and gives the simple metric (MPD 1993)

$$ds^2 = -\frac{c_0^2}{V^2} dt^2 + \frac{a^2(t)}{V^2} \left(dr^2 + r^2 d\Omega^2 \right) . \tag{60}$$

This metric can be considered as **defining spatially dependent effective speed of light** $c(t,r) = c_0/V(t,r)$ (provided we work in a special frame in which the Einstein field equations (57)-(58) are valid - BM approach) or still can mimic the spatial dependence of the speed of light provided we take $c_0 \to c = c(t)$ in (60) and make an appropriate ansatz.

Spatial variations of c

Redshift is different from Friedmann models and reads as

$$1 + z = \frac{a_0}{a_e} \frac{V_e}{V_0} \quad , \tag{61}$$

and the radial distance r can be calculated from the condition of taking the null geodesic $ds^2 = 0$ in (60) (replacing $c_0 \to c = c(t)$ (MPD, Balcerzak 2014)), i.e.

$$r = \int_{t_e}^{t_0} \frac{c(t)dt}{a(t)}. \tag{62}$$

The Friedmann equation reads as

$$H^2(a) = H_0^2 \left[\frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^{3+3(1+w)}} + \frac{\Omega_{\beta,0}}{a} f_\beta(a) \right] \quad , \tag{63}$$

where the density of inhomogeneity is

$$\Omega_{\beta,0} = -\frac{\beta\, c_0^2}{a_0\, H^2(0)} \quad , \tag{64}$$

Angular diameter distance maximum - inhomogeneous modelling

while

$$f_\beta(a) = \begin{cases} 1 & \text{for } c(t) = c_0 = const. \\ a^{2n}(t) & \text{for } c(t) = c_0 a^n(t) \\ \frac{1}{V^2(t,r)} & \text{for } c(t,r) \equiv \frac{c_0}{V(t,r)} \end{cases} \tag{65}$$

for a standard no-varying c ansatz, BM ansatz, and an inhomogeneous ansatz. The angular diameter distance for the model (60) reads as

$$D_A = \frac{a(t)}{V(t,r)} r = \frac{a_0}{V_0(1+z)} r \tag{66}$$

and the condition for the maximum is

$$\frac{\partial D_A}{\partial t} = \frac{\dot{a}V - \dot{V}a}{V^2} \int_{t_e}^{t_0} \frac{c(t)dt}{a(t)} - \frac{c}{V} = 0. \tag{67}$$

Angular diameter distance maximum - inhomogeneous modelling

This gives the relation which can be used to evaluate the timely and spatial dependence of the speed of light

$$D_A(t,r) = \frac{c(t,r)}{HV - \dot{V}}, \tag{68}$$

which allows to relate the inhomogeneity with the variability of the speed of light c. In other words, **variability of c can be mimicked by spatial inhomogeneity, and vice versa, the inhomogeneity can be mimicked by the variability of c.** The expression for the maximum in the angular diameter distance can be finally written down from (42) as:

$$c(a) = \frac{D_A(a)H(a)}{1 + \frac{\Omega_{\beta,0}}{2}\, a\, r^2(a)}\ , \tag{69}$$

Angular diameter distance maximum - inhomogeneous modelling

We implicitly assume that the relations (70)-(72) are evaluated at the maximum $a = a_M$. In (Salzano, MPD, Lazkoz 2015) we found that for homogeneous models we have:

$$D_A(a)H(a) = c(a) \;, \tag{73}$$

but with the assumption of no spatial curvature. In (Salzano, MPD, Lazkoz 2016) we have shown that this relation is valid, to some order, even for $k \neq 0$, because contributions derived from present bounds on curvature are ~ 2 order smaller than a VSL signal. Clearly, in a standard scenario of constant speed of light, this relation converts in:

$$\frac{D_A(a)H(a)}{c_0} = 1. \tag{74}$$

Angular diameter distance maximum - inhomogeneous modelling

Considering the curvature from the beginning, the maximum relation is changed into:

$$\Delta_c = \frac{D_A(a)H(a)}{c_0} \tag{75}$$

$$= \begin{cases} 1 + \frac{\Omega_{\beta,0}}{2}\, a\, r^2(a) & \text{for } c(t) = const. \\ a^n \left(1 + \frac{\Omega_{\beta,0}}{2}\, a\, r^2(a)\right) & \text{for } c(t) = c_0 a^n(t) \\ \left[\frac{1+\frac{\Omega_{\beta,0}}{2}\, a\, r^2(a)}{1-\frac{\Omega_{\beta,0}}{4}\, a\, r^2(a)}\right] & \text{for } c(t,r) = \frac{c_0}{V(r,t)} \end{cases}$$

- Even if $c_0 = 0$ an inhomogeneity may play the role of an effective VSL - mimics timely and spatial variations of c

Data analysis

Used type Ia Supernovae (SNeIa), Baryon Acoustic Oscillations (BAO), Cosmic Microwave Data (CMB) and a prior on the Hubble constant parameter, H_0.

	H_0	Ω_β	w	n	z_M	Δ_c
$c(t) = c_0 = const.$	$69.6^{+0.7}_{-0.7}$	$0.682^{+0.022}_{-0.023}$	$-0.014^{+0.004}_{-0.004}$	–	1.553 ± 0.026	1.140 ± 0.011
$c(t) = c_0 a^n(t)$	$69.6^{+0.7}_{-0.6}$	$0.638^{+0.031}_{-0.029}$	$-0.139^{+0.047}_{-0.045}$	$-0.083^{+0.034}_{-0.034}$	1.816 ± 0.132	1.281 ± 0.074
$c(t,r) = c_0/V(r,t)$	$69.6^{+0.7}_{-0.7}$	$0.669^{+0.022}_{-0.022}$	$0.003^{+0.003}_{-0.003}$	–	1.708 ± 0.042	1.200 ± 0.015

- as mentioned already, SKA will be able to detect a 1% deviation Δ_c from constant speed of light at 3σ confidence level at the maximum redshift. Here we have variations which are fully detectable, being of the order of 10%.
- Inhomogeneous c completely falsifiable.
- VSL still be possible even without spatial inhomogeneity.

6. Statistical analysis of c-varying models: Barrow-Magueijo (BM), Avelino-Martins (AM) and Moffat (M).

Barrow-Magueijo model has been already presented by eqs. (16), (17), (18). The Avelino-Martins model is given by the set of equations

$$H^2(t) = \frac{8\pi G}{3}\rho(t) - \frac{k\,c^2(t)}{a^2(t)}\ , \tag{76}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}\left(\rho(t) + 3\frac{p(t)}{c^2(t)}\right) + H(t)H_c(t)\ . \tag{77}$$

The continuity equation can be obtained from the combination of the above

$$\dot{\rho}(t) + 3H(t)\left(\rho(t) + \frac{p(t)}{c^2(t)}\right) = 2\rho(t)H_c(t)\ , \tag{78}$$

and we have defined

$$H_c(t) \equiv \frac{\dot{c}(t)}{c(t)};\ \ q_c(t) \equiv -\frac{\ddot{c}(t)c(t)}{\dot{c}^2(t)}. \tag{79}$$

Hierarchy of c-varying models - Moffat model

In M model the action is made of up to four terms,

$$S = S_G + S_\psi + S_\phi + S_M \ , \qquad (80)$$

where: S_G is the usual gravitational action, with the speed of light promoted to a field, $\Phi(x) = c^4(x)$, and no minimal coupling requirement is assumed,

$$S_G = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[\Phi(R + 2\Lambda) - \frac{\kappa}{\Phi} \partial^\sigma \Phi \partial_\sigma \Phi \right] \ , \qquad (81)$$

κ= constant (dimensionless). S_ψ is the action of a vector field ψ_μ driving a spontaneous violation of $SO(3,1)$ Lorentz invariance to $O(3)$, and is given by

$$S_\psi = \int d^4x \sqrt{-g} \left[-\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - W(\psi_\mu) \right] \ , \qquad (82)$$

$B_{\mu\nu} = \partial_\mu \psi_\nu - \partial_\nu \psi_\mu$, W - a potential.

Hierarchy of c-varying models - Moffat model

For the (80) we obtain set of Einstein eqs

$$H^2(t) = \frac{8\pi G}{3}\rho(t) - \frac{k\,c^2(t)}{a^2(t)} - 4H(t)H_c(t) + \frac{8\kappa}{3}H_c^2(t)\ , \tag{83}$$

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}\left(\rho(t) + 3\frac{p(t)}{c^2(t)}\right) - 2H(t)H_c(t) - 2\left(3 + \frac{8\kappa}{3} - q_c\right)H_c^2(t), \tag{84}$$

plus the equation of motion for the field Φ,

$$\nabla^\alpha\nabla_\alpha\Phi = \frac{8\pi G}{3+2\kappa}T \tag{85}$$

which further give continuity equation (same as in BM model)

$$\dot{\rho}(t) + 3H(t)\left[\rho(t) + \frac{p(t)}{c^2(t)}\right] = \frac{3kc^2(t)}{4\pi G a^2(t)}H_c(t)\ . \tag{86}$$

Hierarchy of c-varying models

We have considered the following cosmological fluid contents

- $w_{DE} = -1$, (when non-dynamical dark energy, plus matter),
- $w_{DE} = w_0 + w_1\,(1-a)$ - Chevallier-Polarski-Linder (CPL) model (2001, 2003) as reference model for dynamical dark energy models,

and three different ansätze for $c(t)$ or $c(a)$ functions

- $c(a) = c_0\,a^n$ or $H_c = nH$ (named "c-cl" in tables), the classical and most general ansatz (cf. Barrow, Magueijo '99);
- $c(a) = c_0\left[1 + \left(\frac{a}{a_c}\right)^n\right]$ (named "c-Mag" in tables), proposed by Magueijo (2000) and here applied for the first time to cosmological data.
- $c(a) = c_0\,[1 + n\,(1-a)]$ (named "c-CPL" in tables), a linear (in scale factor) VSL *à-la CPL.*

Hierarchy of c-varying models

For BM and M model one can make a split of interacting fluids as follows **(variability of c is influencing the dark energy fluid - matter and radiation - only)**

$$\rho_i'(a) + \frac{3}{a}\left[1 + w_i(a)\right]\rho_i(a) = 0 \tag{87}$$

for the dark energy

$$\rho_{DE}'(a) + \frac{3}{a}\left[1 + w_{DE}(a)\right]\rho_{DE}(a) = \frac{3k\,c^2(a)}{4\pi G\,a^2}\frac{c'(a)}{c(a)}. \tag{88}$$

For AM model c has to be directly coupled to all fluids

$$\rho_i'(a) + \frac{3}{a}\left[1 + w_i(a)\right]\rho_i(a) = 2\rho_i(a)\frac{c'(a)}{c(a)}\ . \tag{89}$$

Hierarchy of c-varying models

One may check degeneracy between VSL and curvature:

$$\frac{D_A(z_M)H(z_M)}{c_0} = \Delta_c(z_M) \cdot \Delta_k(z_M) \,, \tag{90}$$

where z_M is the redshift at which the angular diameter distance reaches its maximum, and $\Delta_c(z_M)$ and $\Delta_k(z_M)$ are defined as

$$\Delta_c(z_M) = \frac{c(z_M)}{c_0} \,, \tag{91}$$

and

$$\Delta_k(z_M) = \begin{cases} \cosh\left(\sqrt{\Omega_k}\frac{D_C(z_M)}{D_H}\right) & \text{for } \Omega_k > 0 \\ 1 & \text{for } \Omega_k = 0 \\ \cos\left(\sqrt{|\Omega_k|}\frac{D_C(z_M)}{D_H}\right) & \text{for } \Omega_k < 0 \,. \end{cases} \tag{92}$$

Cosmological data

We apply the following data:

- expansion rate from early-type galaxies (ETG) as cosmic chronometers (Moresco 2015)
- type Ia supernovae - JLA (Joint-Light curve Analysis - Betoule et al. 2014) - 740 SnIa from SDSS-II and SNLS
- BAO - WiggleZ Dark Energy Survey (Blake et al. 2012); SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS) DR12 (Alam et al. 1607.03155); SDS-III BOSS DR11 (Font-Ribera et al. 2014)
- CMB shift parameter (Wand, Dai 2016)
- prior on the Hubble constant parameter $H_0 = 69.6 \pm 0.7$ (Bennett et al. 2014)

Information criteria - comparison

- We apply Bayesian Information Criterion (BIC) to select best-fit c-varying models.
- Other options: Akaike Information Criterion (AIC), Residual Information Criterion (RIC), Deviance Information Criterion (DIC) can also be used but still they (except DIC) refer to χ^2 minimum.
- They always favour models with less number of parameters.
- Bayesian Evidence - obtained by Bayes factor defined as the ratio of evidences of two models, M_i and M_j,

$$\mathcal{B}^i_j = \mathcal{E}_i/\mathcal{E}_j \tag{93}$$

If $\mathcal{B}^i_j > 1$, model M_i is preferred over M_j, given the data.

- We have used, separately, the cosmological constant and the CPL model, both with constant speed of light and null spatial curvature, as reference models M_j.

Information criteria - comparison

Used Jeffrey's scale (1998):

- if $\ln \mathcal{B}^i_j < 1$, the evidence in favor of model M_i is not significant;
- if $1 < \ln \mathcal{B}^i_j < 2.5$, the evidence is substantial;
- if $2.5 < \ln \mathcal{B}^i_j < 5$, is strong;
- if $\mathcal{B}^i_j > 5$, is decisive.
- Negative values of $\ln \mathcal{B}^i_j$ can be easily interpreted as evidence against model M_i (or in favor of model M_j).

Fitting the data - ΛCDM as DE

id.	Ω_m	Ω_k	h	n	$\mathcal{B}^i_{\Lambda CDM}$	$\ln \mathcal{B}^i_{\Lambda CDM}$	z_M	$\Delta_c\Delta_k$	Δ_c	Δ_k
					ΛCDM					
no Ω_k	$0.309^{+0.005}_{-0.005}$	*0*	$0.679^{+0.004}_{-0.004}$	–	1	0	$1.594^{+0.007}_{-0.007}$	*1*	–	–
Ω_k	$0.309^{+0.005}_{-0.005}$	$0.0008^{+0.0017}_{-0.0016}$	$0.681^{+0.005}_{-0.005}$	–	0.79	-0.24	$1.594^{+0.007}_{-0.007}$	$1.0005^{+0.0010}_{-0.0009}$	–	$1.0005^{+0.0010}_{-0.0009}$
					Barrow & Magueijo					
c-cl.	$0.307^{+0.005}_{-0.005}$	$0.0008^{+0.0010}_{-0.0006}$	$0.688^{+0.006}_{-0.006}$	$0.0007^{+0.0005}_{-0.0004}$	2.26	0.81	$1.594^{+0.007}_{-0.003}$	$0.9999^{+0.0007}_{-0.0006}$	$0.9994^{+0.0004}_{-0.0004}$	$1.0005^{+0.0006}_{-0.0004}$
c-Mag.	$0.307^{+0.005}_{-0.005}$	$0.0008^{+0.0010}_{-0.0006}$	$0.689^{+0.006}_{-0.006}$	$0.0014^{+0.0009}_{-0.0008}$	2.40	0.88	$1.594^{+0.007}_{-0.003}$	$0.9999^{+0.0007}_{-0.0006}$	$0.9993^{+0.0004}_{-0.0004}$	$1.0005^{+0.0005}_{-0.0004}$
c-CPL	$0.302^{+0.006}_{-0.006}$	$0.0008^{+0.0009}_{-0.0006}$	$0.693^{+0.006}_{-0.006}$	$-0.018^{+0.008}_{-0.008}$	12.13	2.50	$1.594^{+0.007}_{-0.007}$	$0.989^{+0.005}_{-0.005}$	$0.989^{+0.005}_{-0.005}$	$1.0003^{+0.0003}_{-0.0002}$
					Avelino & Martins					
c-cl.	$0.322^{+0.014}_{-0.013}$	$0.006^{+0.006}_{-0.005}$	$0.683^{+0.005}_{-0.005}$	$-0.0017^{+0.0016}_{-0.0016}$	0.89	-0.11	$1.577^{+0.017}_{-0.020}$	$1.005^{+0.004}_{-0.005}$	$1.002^{+0.002}_{-0.002}$	$1.003^{+0.003}_{-0.003}$
c-Mag.	$0.322^{+0.014}_{-0.014}$	$0.006^{+0.006}_{-0.006}$	$0.683^{+0.005}_{-0.005}$	$-0.003^{+0.003}_{-0.003}$	0.94	-0.06	$1.577^{+0.017}_{-0.020}$	$1.005^{+0.004}_{-0.004}$	$1.002^{+0.002}_{-0.002}$	$1.003^{+0.003}_{-0.003}$
c-CPL	$0.318^{+0.013}_{-0.013}$	$0.004^{+0.005}_{-0.005}$	$0.682^{+0.005}_{-0.005}$	$0.008^{+0.011}_{-0.011}$	0.72	-0.33	$1.581^{+0.020}_{-0.016}$	$1.007^{+0.009}_{-0.010}$	$1.005^{+0.007}_{-0.007}$	$1.002^{+0.003}_{-0.003}$
					Moffat					
c-cl.	$0.307^{+0.005}_{-0.005}$	$0.0008^{+0.0011}_{-0.0007}$	$0.686^{+0.005}_{-0.005}$	$0.0004^{+0.0004}_{-0.0003}$	1.29	0.26	$1.597^{+0.007}_{-0.007}$	$1.0000^{+0.0007}_{-0.0005}$	$0.9996^{+0.0003}_{-0.0004}$	$1.0005^{+0.0006}_{-0.0004}$
c-Mag.	$0.307^{+0.005}_{-0.005}$	$0.0009^{+0.0011}_{-0.0006}$	$0.686^{+0.005}_{-0.005}$	$0.0009^{+0.0007}_{-0.0006}$	1.27	0.24	$1.597^{+0.007}_{-0.007}$	$1.0000^{+0.0007}_{-0.0005}$	$0.9996^{+0.0003}_{-0.0004}$	$1.0005^{+0.0006}_{-0.0004}$
c-CPL	$0.305^{+0.006}_{-0.005}$	$0.0008^{+0.0010}_{-0.0006}$	$0.690^{+0.006}_{-0.006}$	$-0.008^{+0.005}_{-0.005}$	3.51	1.26	$1.597^{+0.007}_{-0.007}$	$0.996^{+0.003}_{-0.003}$	$0.995^{+0.003}_{-0.003}$	$1.0005^{+0.0005}_{-0.0003}$

Fitting the data - CPL as DE

id.	Ω_m	Ω_k	h	n	w_0	w_1	$\mathcal{B}^i_{CPL}$	$\ln \mathcal{B}^i_{CPL}$	z_M	$\Delta_c\Delta_k$	Δ_c	Δ_k
						CPL						
no Ω_k	$0.302^{+0.006}_{-0.006}$	*0*	$0.689^{+0.006}_{-0.006}$	–	$-1.15^{+0.09}_{-0.08}$	$0.35^{+0.29}_{-0.32}$	1	0	$1.584^{+0.013}_{-0.013}$	*1*	–	–
Ω_k	$0.302^{+0.006}_{-0.006}$	$-0.003^{+0.004}_{-0.003}$	$0.689^{+0.006}_{-0.006}$	–	$-1.11^{+0.11}_{-0.11}$	$0.07^{+0.52}_{-0.61}$	0.75	-0.29	$1.594^{+0.017}_{-0.017}$	$0.998^{+0.002}_{-0.002}$	–	$0.998^{+0.002}_{-0.002}$
						Barrow & Magueijo						
c-cl.	$0.301^{+0.006}_{-0.005}$	$-0.002^{+0.004}_{-0.004}$	$0.695^{+0.007}_{-0.007}$	$0.003^{+0.002}_{-0.002}$	$-1.14^{+0.08}_{-0.08}$	$0.74^{+0.20}_{-0.17}$	4.04	1.40	$1.564^{+0.010}_{-0.010}$	$0.997^{+0.003}_{-0.004}$	$0.997^{+0.002}_{-0.002}$	$0.999^{+0.002}_{-0.002}$
c-Mag.	$0.301^{+0.006}_{-0.005}$	$-0.005^{+0.005}_{-0.004}$	$0.696^{+0.006}_{-0.007}$	$0.008^{+0.003}_{-0.003}$	$-1.09^{+0.08}_{-0.08}$	$0.57^{+0.23}_{-0.16}$	8.44	2.13	–	–	–	–
c-CPL	$0.296^{+0.006}_{-0.006}$	$0.001^{+0.004}_{-0.003}$	$0.696^{+0.006}_{-0.007}$	$-0.031^{+0.016}_{-0.016}$	$-1.14^{+0.07}_{-0.08}$	$0.64^{+0.24}_{-0.16}$	3.55	1.27	$1.561^{+0.010}_{-0.013}$	$0.982^{+0.010}_{-0.010}$	$0.981^{+0.010}_{-0.009}$	$1.001^{+0.002}_{-0.002}$
						Avelino & Martins						
c-cl.	$0.324^{+0.016}_{-0.014}$	$0.004^{+0.006}_{-0.005}$	$0.693^{+0.006}_{-0.006}$	$-0.003^{+0.002}_{-0.002}$	$-1.05^{+0.13}_{-0.11}$	$-0.38^{+0.62}_{-0.79}$	1.27	0.24	$1.574^{+0.020}_{-0.020}$	$1.005^{+0.005}_{-0.004}$	$1.003^{+0.002}_{-0.002}$	$1.003^{+0.003}_{-0.003}$
c-Mag.	$0.325^{+0.015}_{-0.014}$	$0.005^{+0.006}_{-0.005}$	$0.693^{+0.006}_{-0.007}$	$-0.006^{+0.003}_{-0.003}$	$-1.05^{+0.13}_{-0.12}$	$-0.38^{+0.66}_{-0.78}$	1.36	0.31	$1.574^{+0.020}_{-0.020}$	$1.006^{+0.004}_{-0.004}$	$1.003^{+0.001}_{-0.002}$	$1.003^{+0.003}_{-0.003}$
c-CPL	$0.325^{+0.016}_{-0.015}$	$0.004^{+0.005}_{-0.005}$	$0.693^{+0.006}_{-0.007}$	$0.021^{+0.012}_{-0.012}$	$-1.05^{+0.14}_{-0.12}$	$-0.44^{+0.66}_{-0.86}$	1.38	0.32	$1.574^{+0.020}_{-0.020}$	$1.016^{+0.009}_{-0.010}$	$1.013^{+0.007}_{-0.007}$	$1.002^{+0.003}_{-0.003}$
						Moffat						
c-cl.	$0.288^{+0.010}_{-0.013}$	$-0.011^{+0.010}_{-0.011}$	$0.694^{+0.006}_{-0.006}$	$0.006^{+0.004}_{-0.003}$	$-1.08^{+0.09}_{-0.09}$	$0.72^{+0.09}_{-0.09}$	9.87	2.29	$1.584^{+0.020}_{-0.017}$	$0.990^{+0.008}_{-0.008}$	$0.995^{+0.003}_{-0.003}$	$0.995^{+0.005}_{-0.005}$
c-Mag.	$0.301^{+0.006}_{-0.006}$	$0.001^{+0.003}_{-0.003}$	$0.691^{+0.006}_{-0.006}$	$0.002^{+0.003}_{-0.003}$	$-1.15^{+0.10}_{-0.07}$	$0.71^{+0.32}_{-0.97}$	0.35	0.78	$1.577^{+0.023}_{-0.023}$	$0.9997^{+0.0008}_{-0.0015}$	$1.000^{+0.001}_{-0.002}$	$1.0000^{+0.0001}_{-0.0002}$
c-CPL	$0.296^{+0.006}_{-0.006}$	$-0.001^{+0.004}_{-0.004}$	$0.696^{+0.007}_{-0.007}$	$-0.033^{+0.016}_{-0.015}$	$-1.10^{+0.07}_{-0.08}$	$0.59^{+0.21}_{-0.14}$	6.42	1.86	$1.574^{+0.010}_{-0.016}$	$1.016^{+0.021}_{-0.022}$	$0.979^{+0.009}_{-0.009}$	$1.038^{+0.016}_{-0.016}$

Results:

- Most of the models have no statistical evidence - $\ln \mathcal{B}^i_j <| 1 |$
- **BM and M models have substantial evidence** - $\ln \mathcal{B}^i_j > 1$ for linear VSL signal
- most of the VSL scenarios have higher Bayes Factors than curvature-free classical scenario
- most favoured statistical scenarios point toward phantom DE (see the values of w_1)
- BM and M models with a CPL and a classical VSL ansatz can be made falsifiable by Square Kilometer Array (SKA) which will be able to detect a total signal $\Delta_c \Delta_k \approx 1.01$

Fitting the data - CPL as DE

id.	Ω_m	Ω_k	h	n	w_0	w_1	$\mathcal{B}^i_{CPL}$	$\ln \mathcal{B}^i_{CPL}$	z_M	$\Delta_c \Delta_k$	Δ_c	Δ_k
						CPL						
no Ω_k	$0.302^{+0.006}_{-0.006}$	*0*	$0.689^{+0.006}_{-0.006}$	–	$-1.15^{+0.09}_{-0.08}$	$0.35^{+0.29}_{-0.32}$	1	0	$1.584^{+0.013}_{-0.013}$	*1*	–	–
Ω_k	$0.302^{+0.006}_{-0.006}$	$-0.003^{+0.004}_{-0.003}$	$0.689^{+0.006}_{-0.006}$	–	$-1.11^{+0.11}_{-0.11}$	$0.07^{+0.52}_{-0.61}$	0.75	-0.29	$1.594^{+0.017}_{-0.017}$	$0.998^{+0.002}_{-0.002}$	–	$0.998^{+0.002}_{-0.002}$
						Barrow & Magueijo						
c-cl.	$0.301^{+0.006}_{-0.005}$	$-0.002^{+0.004}_{-0.004}$	$0.695^{+0.007}_{-0.007}$	$0.003^{+0.002}_{-0.002}$	$-1.14^{+0.08}_{-0.08}$	$0.74^{+0.20}_{-0.17}$	4.04	1.40	$1.564^{+0.010}_{-0.010}$	$0.997^{+0.003}_{-0.004}$	$0.997^{+0.002}_{-0.002}$	$0.999^{+0.002}_{-0.002}$
c-Mag.	$0.301^{+0.006}_{-0.005}$	$-0.005^{+0.005}_{-0.004}$	$0.696^{+0.006}_{-0.007}$	$0.008^{+0.003}_{-0.003}$	$-1.09^{+0.08}_{-0.08}$	$0.57^{+0.23}_{-0.16}$	8.44	2.13	–	–	–	–
c-CPL	$0.296^{+0.006}_{-0.006}$	$0.001^{+0.004}_{-0.003}$	$0.696^{+0.006}_{-0.007}$	$-0.031^{+0.016}_{-0.016}$	$-1.14^{+0.07}_{-0.08}$	$0.64^{+0.24}_{-0.16}$	3.55	1.27	$1.561^{+0.010}_{-0.013}$	$0.982^{+0.010}_{-0.010}$	$0.981^{+0.010}_{-0.009}$	$1.001^{+0.002}_{-0.002}$
						Avelino & Martins						
c-cl.	$0.324^{+0.016}_{-0.014}$	$0.004^{+0.006}_{-0.005}$	$0.693^{+0.006}_{-0.006}$	$-0.003^{+0.002}_{-0.002}$	$-1.05^{+0.13}_{-0.11}$	$-0.38^{+0.62}_{-0.79}$	1.27	0.24	$1.574^{+0.020}_{-0.020}$	$1.005^{+0.005}_{-0.004}$	$1.003^{+0.002}_{-0.002}$	$1.003^{+0.003}_{-0.003}$
c-Mag.	$0.325^{+0.015}_{-0.014}$	$0.005^{+0.006}_{-0.005}$	$0.693^{+0.006}_{-0.007}$	$-0.006^{+0.003}_{-0.003}$	$-1.05^{+0.13}_{-0.12}$	$-0.38^{+0.66}_{-0.78}$	1.36	0.31	$1.574^{+0.020}_{-0.020}$	$1.006^{+0.004}_{-0.004}$	$1.003^{+0.001}_{-0.002}$	$1.003^{+0.003}_{-0.003}$
c-CPL	$0.325^{+0.016}_{-0.015}$	$0.004^{+0.005}_{-0.005}$	$0.693^{+0.006}_{-0.007}$	$0.021^{+0.012}_{-0.012}$	$-1.05^{+0.14}_{-0.12}$	$-0.44^{+0.66}_{-0.86}$	1.38	0.32	$1.574^{+0.020}_{-0.020}$	$1.016^{+0.009}_{-0.010}$	$1.013^{+0.007}_{-0.007}$	$1.002^{+0.003}_{-0.003}$
						Moffat						
c-cl.	$0.288^{+0.010}_{-0.013}$	$-0.011^{+0.010}_{-0.011}$	$0.694^{+0.006}_{-0.006}$	$0.006^{+0.004}_{-0.003}$	$-1.08^{+0.09}_{-0.09}$	$0.72^{+0.09}_{-0.09}$	9.87	2.29	$1.584^{+0.020}_{-0.017}$	$0.990^{+0.008}_{-0.008}$	$0.995^{+0.003}_{-0.003}$	$0.995^{+0.005}_{-0.005}$
c-Mag.	$0.301^{+0.006}_{-0.006}$	$0.001^{+0.003}_{-0.003}$	$0.691^{+0.006}_{-0.006}$	$0.002^{+0.003}_{-0.003}$	$-1.15^{+0.10}_{-0.07}$	$0.71^{+0.32}_{-0.97}$	0.35	0.78	$1.577^{+0.023}_{-0.023}$	$0.9997^{+0.0008}_{-0.0015}$	$1.000^{+0.001}_{-0.002}$	$1.0000^{+0.0001}_{-0.0002}$
c-CPL	$0.296^{+0.006}_{-0.006}$	$-0.001^{+0.004}_{-0.004}$	$0.696^{+0.007}_{-0.007}$	$-0.033^{+0.016}_{-0.015}$	$-1.10^{+0.07}_{-0.08}$	$0.59^{+0.21}_{-0.14}$	6.42	1.86	$1.574^{+0.010}_{-0.016}$	$1.016^{+0.021}_{-0.022}$	$0.979^{+0.009}_{-0.009}$	$1.038^{+0.016}_{-0.016}$

Fitting the data - ΛCDM as DE

id.	Ω_m	Ω_k	h	n	$\mathcal{B}^i_{\Lambda CDM}$	$\ln \mathcal{B}^i_{\Lambda CDM}$	z_M	$\Delta_c\Delta_k$	Δ_c	Δ_k
					ΛCDM					
no Ω_k	$0.309^{+0.005}_{-0.005}$	*0*	$0.679^{+0.004}_{-0.004}$	–	1	0	$1.594^{+0.007}_{-0.007}$	*1*	–	–
Ω_k	$0.309^{+0.005}_{-0.005}$	$0.0008^{+0.0017}_{-0.0016}$	$0.681^{+0.005}_{-0.005}$	–	0.79	-0.24	$1.594^{+0.007}_{-0.007}$	$1.0005^{+0.0010}_{-0.0009}$	–	$1.0005^{+0.0010}_{-0.0009}$
					Barrow & Magueijo					
c-cl.	$0.307^{+0.005}_{-0.005}$	$0.0008^{+0.0010}_{-0.0006}$	$0.688^{+0.006}_{-0.006}$	$0.0007^{+0.0005}_{-0.0004}$	2.26	0.81	$1.594^{+0.007}_{-0.003}$	$0.9999^{+0.0007}_{-0.0006}$	$0.9994^{+0.0004}_{-0.0004}$	$1.0005^{+0.0006}_{-0.0004}$
c-Mag.	$0.307^{+0.005}_{-0.005}$	$0.0008^{+0.0010}_{-0.0006}$	$0.689^{+0.006}_{-0.006}$	$0.0014^{+0.0009}_{-0.0008}$	2.40	0.88	$1.594^{+0.007}_{-0.003}$	$0.9999^{+0.0007}_{-0.0006}$	$0.9993^{+0.0004}_{-0.0004}$	$1.0005^{+0.0005}_{-0.0004}$
c-CPL	$0.302^{+0.006}_{-0.006}$	$0.0008^{+0.0009}_{-0.0006}$	$0.693^{+0.006}_{-0.006}$	$-0.018^{+0.008}_{-0.008}$	12.13	2.50	$1.594^{+0.007}_{-0.007}$	$0.989^{+0.005}_{-0.005}$	$0.989^{+0.005}_{-0.005}$	$1.0003^{+0.0003}_{-0.0002}$
					Avelino & Martins					
c-cl.	$0.322^{+0.014}_{-0.013}$	$0.006^{+0.006}_{-0.005}$	$0.683^{+0.005}_{-0.005}$	$-0.0017^{+0.0016}_{-0.0016}$	0.89	-0.11	$1.577^{+0.017}_{-0.020}$	$1.005^{+0.004}_{-0.005}$	$1.002^{+0.002}_{-0.002}$	$1.003^{+0.003}_{-0.003}$
c-Mag.	$0.322^{+0.014}_{-0.014}$	$0.006^{+0.006}_{-0.006}$	$0.683^{+0.005}_{-0.005}$	$-0.003^{+0.003}_{-0.003}$	0.94	-0.06	$1.577^{+0.017}_{-0.020}$	$1.005^{+0.004}_{-0.004}$	$1.002^{+0.002}_{-0.002}$	$1.003^{+0.003}_{-0.003}$
c-CPL	$0.318^{+0.013}_{-0.013}$	$0.004^{+0.005}_{-0.005}$	$0.682^{+0.005}_{-0.005}$	$0.008^{+0.011}_{-0.011}$	0.72	-0.33	$1.581^{+0.020}_{-0.016}$	$1.007^{+0.009}_{-0.010}$	$1.005^{+0.007}_{-0.007}$	$1.002^{+0.003}_{-0.003}$
					Moffat					
c-cl.	$0.307^{+0.005}_{-0.005}$	$0.0008^{+0.0011}_{-0.0007}$	$0.686^{+0.005}_{-0.005}$	$0.0004^{+0.0004}_{-0.0003}$	1.29	0.26	$1.597^{+0.007}_{-0.007}$	$1.0000^{+0.0007}_{-0.0005}$	$0.9996^{+0.0003}_{-0.0004}$	$1.0005^{+0.0006}_{-0.0004}$
c-Mag.	$0.307^{+0.005}_{-0.005}$	$0.0009^{+0.0011}_{-0.0006}$	$0.686^{+0.005}_{-0.005}$	$0.0009^{+0.0007}_{-0.0006}$	1.27	0.24	$1.597^{+0.007}_{-0.007}$	$1.0000^{+0.0007}_{-0.0005}$	$0.9996^{+0.0003}_{-0.0004}$	$1.0005^{+0.0006}_{-0.0004}$
c-CPL	$0.305^{+0.006}_{-0.005}$	$0.0008^{+0.0010}_{-0.0006}$	$0.690^{+0.006}_{-0.006}$	$-0.008^{+0.005}_{-0.005}$	3.51	1.26	$1.597^{+0.007}_{-0.007}$	$0.996^{+0.003}_{-0.003}$	$0.995^{+0.003}_{-0.003}$	$1.0005^{+0.0005}_{-0.0003}$

7. Conclusions

- The advantages of varying c theories are: solution of the flatness and horizon problems; possibly also the singularity problem.
- Violation of Lorentz invariance in c-varying theories leads to a choice of a preferred frame and a drop of standard variational principle.
- Proper formulation of c-varying theories should be in field-theoretical approach in a similar fashion as Brans-Dicke theory
- α-varying theories have better formulation - variability of α is related to variability of c.
- New tests to check variability of c in future telescope/space missions have been proposed.
- Redshift drift test which give clear prediction for redshift drift effect which can potentially be measured by future telescopes (E-ELT, TMT, GMT, DECIGO/BBO).

Conclusions contd.

- Angular diameter distance maximum z_m test based on independent measurement of the radial D_A and tangential mode c/H of the volume distance was proposed.
- In simple terms it is a "cosmic" measurement of the speed of light c with D_A giving the dimension of length being a "cosmic ruler" and $1/H$ giving the dimension of time being a "cosmic clock" i.e.

$$c = \frac{D_A}{\left(\frac{1}{H}\right)}. \tag{94}$$

- Future observational missions (DESI, SKA, WFIRST,...) can test 1% variability of c at 1σ level - both timely and spatial variation of c can be measured.
- Statistical analysis (Bayesian Evidence) of the varying-c models (Barrow-Magueijo, Avelino-Martins, Moffat) shows that the most favourable models are BM and M models.