Title: Loops in AdS from conformal symmetry

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Abstract: <p>In this talk I will discuss a new use for conformal field theory crossing equation in the context of AdS/CFT: the computation of loop amplitudes in AdS, dual to non-planar correlators in holographic CFTs. I will revisit this problem and the dual 1/N expansion of CFTs, in two independent ways. The first is to show how to explicitly solve the crossing equations to the first subleading order in 1/N^2, given a leading order solution. This is done as a systematic expansion in inverse powers of the spin, to all orders. These expansions can be resummed, leading to the CFT data for finite values of the spin. The second approach involves Mellin space. As an example, Iâ€™ll show how the polar part of the four-point, loop-level Mellin amplitudes can be fully reconstructed from the leading-order data. The anomalous dimensions computed with both methods agree. In the case of \phi^4 theory in AdS, the crossing solution reproduces a previous computation of the one-loop bubble diagram. I will end with a discussion on open problems and new developments.</p>

Loops in AdS from CFT

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based on hep-th:1612.0389, with O.Aharony, L.F. Alday and E. Perlmutter

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What and why?

Amplitudes in AdS

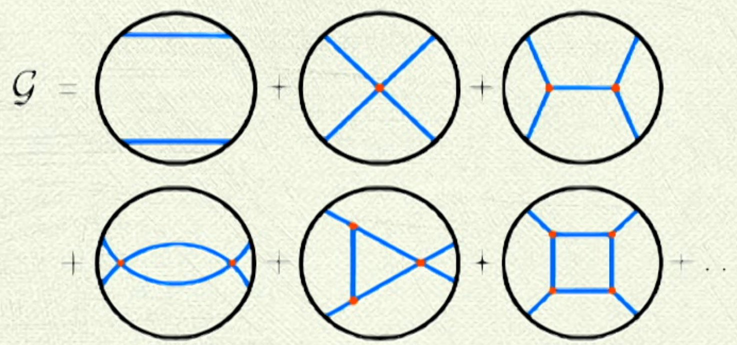
AdS/CFT correspondence

CFT correlation functions at large N

- Understand the structure of amplitudes in curved space
- Difficult to access with usual Feynman diagrams techniques
- Understand better the dynamics of holographic CFTs, beyond leading order

How?

<u>Amplitudes in AdS</u> → Mellin representation



↕ ?

<u>CFT correlation functions at large N</u> → Bootstrap approach

Plan of the talk

- Review of the problem at order $\frac{1}{N^2}$
- Solving crossing equations at order $\frac{1}{N^4}$
    - Using an expansion in inverse powers of the spin, reconstruct the anomalous dimension (also for finite spins)
- Reconstruct the polar part of Mellin amplitude from the leading order data
    - Compute the anomalous dimension
- Discussion and open problems.

Generalities

- Consider the four point function of a generic CFT in d dimensions, with large N expansion and large mass gap.

$$\mathcal{O} \times \mathcal{O} = 1 + \mathcal{O} + T_{\mu\nu} + [\mathcal{O}\mathcal{O}]_{n,\ell} + [TT]_{n,\ell} + [\mathcal{O}T]_{n,\ell}$$

single trace operator of dimension Δ

stress tensor of dimension d and spin 2

double trace operators $\mathcal{O}\Box^n \partial_{\mu_1} \cdots \partial_{\mu_\ell} \mathcal{O} + \cdots$

Generalities

- Consider the four point function of a generic CFT in d dimensions, with large N expansion and large mass gap.

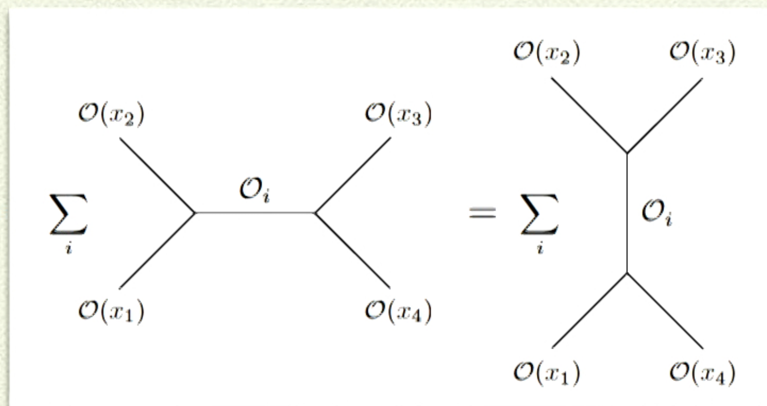
$$\mathcal{O}\times\mathcal{O} = 1 + \xcancel{\mathcal{O}} + \xcancel{T}_{\mu\nu} + [\mathcal{O}\mathcal{O}]_{n,\ell} + [\xcancel{TT}]_{n,\ell} + [\xcancel{\mathcal{O}T}]_{n,\ell}$$

- $\mathcal{O}$ forbidden in a theory with $\mathbb{Z}_2$ symmetry
- In the simplest setting, it is possible to ignore T
- Quadruple and higher traces can also appear, but not up to order $\frac{1}{N^4}$

Crossing equation

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^{2\Delta}x_{34}^{2\Delta}}$$

Associativity of the OPE



Crossing equation

$$v^{\Delta}\mathcal{G}(u,v) = u^{\Delta}\mathcal{G}(v,u)$$

$$U = \frac{x_{12}^2 \, x_{34}^2}{x_{13}^2 \, x_{24}^2} \qquad V = \frac{x_{14}^2 \, x_{23}^2}{x_{13}^2 \, x_{24}^2}$$

Crossing equation

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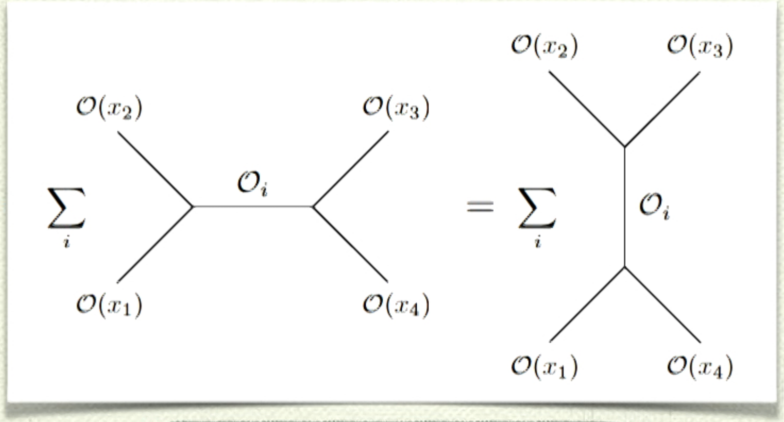
Associativity of the OPE ➡



Crossing equation ➡ $$v^{\Delta}\mathcal{G}(u,v) = u^{\Delta}\mathcal{G}(v,u)$$

Crossing equation

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^{2\Delta}x_{34}^{2\Delta}}$$

$$v^{\Delta}\mathcal{G}(u,v) = u^{\Delta}\mathcal{G}(v,u)$$

Decomposition in conformal blocks

$$\mathcal{G}(u,v) = 1 + \sum_{n,\ell} a_{n,\ell} u^{\frac{\Delta_{n,\ell}-\ell}{2}} g_{\Delta_{n,\ell},\ell}(u,v)$$

OPE coefficients squared

Conformal blocks

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Expansion: CFT data

The four point function and the CFT data can be expanded in inverse powers of N, more specifically:

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \cdots$$

$$\Delta_{n,\ell} = 2\Delta + 2n + \ell + \frac{1}{N^2}\gamma^{(1)}_{n,\ell} + \frac{1}{N^4}\gamma^{(2)}_{n,\ell} + \cdots$$

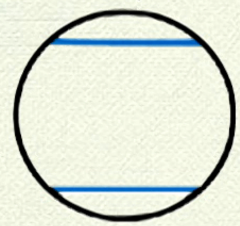
$$a_{n,\ell} = a^{(0)}_{n,\ell} + \frac{1}{N^2}a^{(1)}_{n,\ell} + \frac{1}{N^4}a^{(2)}_{n,\ell} + \cdots$$

Crossing has to be satisfied order by order in N.

Expansion: 0-th order

At 0-th order the four point function is given by

$$\mathcal{G}^{(0)}(u,v) = 1 + u^{\Delta} + \left(\frac{u}{v}\right)^{\Delta}$$



+ crossed

We can decompose this in conformal blocks, to obtain

$$\Delta^{(0)}_{n,\ell} = 2\Delta + 2n + \ell \qquad a^{(0)}_{n,\ell}$$

Expansion: CFT data

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \cdots$$

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Heemskerk, Penedones, Polchinski, and Sully, '09

Expansion: 1-st order

By expanding the CPW $\sum_{n,\ell} a_{n,\ell} u^{\frac{\Delta_{n,\ell}-\ell}{2}} g_{\Delta_{n,\ell},\ell}(u,v)$, we obtain:

$$\mathcal{G}^{(1)}(u,v) = \sum_{n,\ \ell} u^{\Delta+n} \left(a^{(1)}_{n,\ell} + \frac{1}{2} a^{(0)}_{n,\ell} \gamma^{(1)}_{n,\ell} \left(\underline{\log u} + \frac{\partial}{\partial n} \right) \right) g_{2\Delta+2n+\ell,\ell}(u,v)$$

Note on Conformal Block

- $u \to 0$ limit is explicit from above
- $v \to 0$

$$g_{\Delta_p,\ell}(u,v)\big|_{v\to 0} \sim \tilde{a}_{\Delta_p,\ell}(u,v) + \tilde{b}_{\Delta_p,\ell}(u,v) \underline{\log v}$$

Heemskerk, Penedones, Polchinski, and Sully, '09

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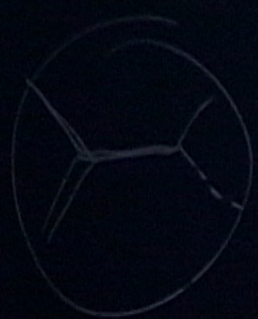
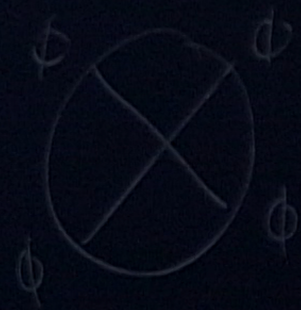
- By studying divergent terms and using crossing, it is possible to find set of solutions $\{\gamma^{(1)}_{n,\ell}, a^{(1)}_{n,\ell}\}$
- $\{\gamma^{(1)}_{n,\ell}, a^{(1)}_{n,\ell}\}$ are different from zero only for a finite range of spin. $\log u \log v$ is reproduced with FINITE num of blocks
- There is a relation between $\gamma^{(1)}_{n,\ell}$ and $a^{(1)}_{n,\ell}$

$$a^{(1)}_{n,\ell} = \frac{1}{2} \frac{\partial (a^{(0)}_{n,\ell} \gamma^{(1)}_{n,\ell})}{\partial n}$$

Heemskerk, Penedones, Polchinski, and Sully, '09

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$





Mellin representation

- A complementary way to study this problem is by using the Mellin representation.
- The main advantage of this approach is that crossing symmetry is manifest and very easy to implement.
- Analytic properties of the Mellin amplitudes, as poles and their residues, are directly related to CFT data.

$$\mathcal{G}(u,v) = \frac{1}{(4\pi i)^2}\int_{-i\infty}^{i\infty} ds\, dt\, M(s,t)\, u^{t/2} v^{(\hat{u}-2\Delta)/2} \Gamma^2\left(\frac{2\Delta-t}{2}\right)\Gamma^2\left(\frac{2\Delta-s}{2}\right)\Gamma^2\left(\frac{2\Delta-\hat{u}}{2}\right)$$

$$\hat{u} \equiv 4\Delta - s - t$$

Crossing is equivalent to: $M(s,t) = M(s,\hat{u}) = M(t,s)$

Poles and residues

- In the t channel, the amplitude has poles in t at the twist $\tau_p = \Delta_p - \ell_p$ of the exchanged operator $\mathcal{O}_p$ and the residues encode the OPE coefficients:

$$M(s,t) = \sum_p C^2_{\mathcal{O}\mathcal{O}\mathcal{O}_p} \sum_{n=0}^{\infty} \frac{\mathcal{Q}_{\ell,n}(s;\tau_p)}{t-(\tau_p+2n)}$$

Mack polynomials:

- They are closely related to the Mellin transform of conformal blocks
- They satisfy orthogonality relations, known explicitly only for n=0

Tree-level example

- At tree level, the only poles come from single-trace operators.
- Double-trace exchanges are accounted for by the explicit gamma functions in the definition of the Mellin transform.
- Using the orthogonality properties of Mack polynomials, it possible to write the anomalous dimension as:

$$\gamma^{(1)}_{0,\ell} = -\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} ds\, M_{\text{tree}}(s, 2\Delta)\Gamma^2\left(\frac{s}{2}\right)\Gamma^2\left(\frac{2\Delta - s}{2}\right) {}_3F_2(-\ell, \ell + 2\Delta - 1, \frac{s}{2}; \Delta, \Delta; 1)$$

- In our simple setup, the Mellin transform is a polynomial. This reflect the fact that there is no exchange of single trace operators.

Tree-level example

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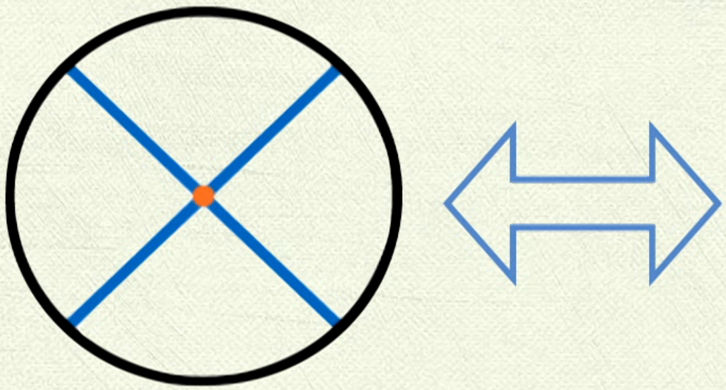
- <u>WARNING</u>: There can be REGULAR terms!

Example: ϕ^4



$$\gamma^{(1)}_{0,n} \neq 0 \quad a^{(1)}_{0,n} \neq 0$$

$$a^{(1)}_{\ell,n} = \gamma^{(1)}_{\ell,n} = 0 \;\; \forall \ell \neq 0$$

Mellin amplitude is a constant

How to proceed further?

$$\mathcal{G}(u,v) = \mathcal{G}^{(0)}(u,v) + \frac{1}{N^2}\mathcal{G}^{(1)}(u,v) + \frac{1}{N^4}\mathcal{G}^{(2)}(u,v) + \cdots$$

$$\Delta_{n,\ell} = 2\Delta + 2n + \ell + \frac{1}{N^2}\gamma^{(1)}_{n,\ell} + \frac{1}{N^4}\gamma^{(2)}_{n,\ell} + \cdots$$

$$a_{n,\ell} = a^{(0)}_{n,\ell} + \frac{1}{N^2}a^{(1)}_{n,\ell} + \frac{1}{N^4}a^{(2)}_{n,\ell} + \cdots$$

Expand at next order

Let's expand the CPW at order $\dfrac{1}{N^4}$

$$\mathcal{G}^{(2)}(u,v) = \sum_{n,\,\ell} u^{\Delta+n} \Bigg(a^{(2)}_{n,\ell} + \frac{1}{2} a^{(0)}_{n,\ell}\gamma^{(2)}_{n,\ell}\left(\log u + \frac{\partial}{\partial n}\right) + \frac{1}{2} a^{(1)}_{n,\ell}\gamma^{(1)}_{n,\ell}\left(\log u + \frac{\partial}{\partial n}\right) + \frac{1}{8} a^{(0)}_{n,\ell}(\gamma^{(1)}_{n,\ell})^2\left(\log^2(u) + 2\log u\frac{\partial}{\partial n} + \frac{\partial^2}{\partial n^2}\right)\Bigg) g_{2\Delta+2n+\ell,\ell}(u,v)$$

only part where $a^{(2)}_{n,\ell}$ and $\gamma^{(2)}_{n,\ell}$ appear

- under crossing this term produces a $\log^2 v$
- as already noticed, each CB diverges at most as $\log v$

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What can we learn from crossing?

solutions at order N^{-2}

truncate at spin L

$$\mathcal{G}^{(2)}(u,v)\Big|_{\log^2(u)} = \sum_n \sum_{\ell=0}^{L} u^{\Delta+n}\frac{1}{8}a^{(0)}_{n,\ell}(\gamma^{(1)}_{n,\ell})^2 g_{2\Delta+2n+\ell,\ell}(u,v)$$

$$\equiv u^{\Delta}\left(f(u,v)\log v + g(u,v)\right)$$

depend only on CFT data that we have from N^0 and N^{-2}

What can we learn from crossing?

solutions at order N^{-2}
truncate at spin L

$$\mathcal{G}^{(2)}(u,v)\Big|_{\log^2(u)} = \sum_n \sum_{\ell=0}^{L} u^{\Delta+n}\frac{1}{8}a^{(0)}_{n,\ell}(\gamma^{(1)}_{n,\ell})^2 g_{2\Delta+2n+\ell,\ell}(u,v)$$

$$\equiv u^{\Delta}\left(f(u,v)\log v + g(u,v)\right)$$

Recall that crossing implies $\mathcal{G}^{(2)}(u,v) = \left(\frac{u}{v}\right)^{\Delta}\mathcal{G}^{(2)}(v,u)$, thus there $\mathcal{G}^{(2)}(u,v)$ should contain

$$\boxed{\mathcal{G}^{(2)}(u,v) = u^{\Delta}\log^2(v)\left(f(v,u)\log u + g(v,u)\right) + \cdots}$$

From which term of the decomposition can come?

An equation for $\gamma^{(2)}_{n,\ell}$

$$\mathcal{G}^{(2)}(u,v) = \sum_{n,\ \ell} u^{\Delta+n}\Bigg(a^{(2)}_{n,\ell} + \frac{1}{2} a^{(0)}_{n,\ell}\gamma^{(2)}_{n,\ell}\left(\log u + \frac{\partial}{\partial n}\right)$$

$$+ \frac{1}{2} a^{(1)}_{n,\ell}\gamma^{(1)}_{n,\ell}\left(\log u + \frac{\partial}{\partial n}\right)$$

$$+ \frac{1}{8} a^{(0)}_{n,\ell}(\gamma^{(1)}_{n,\ell})^2\left(\log^2(u) + 2\log u\frac{\partial}{\partial n} + \frac{\partial^2}{\partial n^2}\right)\Bigg) g_{2\Delta+2n+\ell,\ell}(u,v)$$

are truncated sum, cannot produce a $\log^2 v$

Provides an equation for $\gamma^{(2)}_{n,\ell}$

$$\sum_{n,\ell} u^n \frac{1}{2} a^{(0)}_{n,\ell}\gamma^{(2)}_{n,\ell} g_{2\Delta+2n+\ell,\ell}(u,v)\Big|_{\log^2(v)} = f(v,u)$$

How to solve this equation

$$\sum_{n,\ell} u^n \frac{1}{2} a^{(0)}_{n,\ell} \gamma^{(2)}_{n,\ell} g_{2\Delta+2n+\ell,\ell}(u,v) \Big|_{\log^2(v)} = f(v,u)$$

- To obtain $\log^2 v$ we need to sum over an infinite number of spin
- The divergence comes from the tail of the sum, from the region of large spin
- The solution has the structure

all the coefficients are computable!

$$\gamma^{(2)}_{n,\ell} = \frac{c^{(0)}_n}{\ell^{2\Delta}} \left(1 + \frac{b^{(1)}_n}{\ell} + \frac{b^{(2)}_n}{\ell^2} + \cdots \right)$$

- In some cases, for instance ϕ^4 we manage to re-sum the series and extrapolate results for finite spins.

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Summary and strategy

Let's summarise what we have discussed so far:

1. Given a solution $\{\gamma^{(1)}, a^{(1)}\}$, there is a term proportional to $\log^2 u$ which is fully specified by such solution:

$$\sum_n \sum_{\ell=0}^{L} u^{\Delta+n} \frac{1}{8} a^{(0)}_{n,\ell} (\gamma^{(1)}_{n,\ell})^2 g_{2\Delta+2n+\ell,\ell}(u,v) = u^{\Delta} \left(f(u,v)\log v + \cdots\right)$$

2. Using crossing, we identified an equation for $\gamma^{(2)}_{n,\ell}$

$$\left. \sum_{n,\ell} u^n \frac{1}{2} a^{(0)}_{n,\ell} \gamma^{(2)}_{n,\ell} g_{2\Delta+2n+\ell,\ell}(u,v) \right|_{\log^2(v)} = f(v,u)$$

3. This eq. admits a solution of the form

$$\gamma^{(2)}_{n,\ell} = \frac{c^{(0)}_n}{\ell^{2\Delta}} \left(1 + \frac{b^{(1)}_n}{\ell} + \frac{b^{(2)}_n}{\ell^2} + \cdots\right)$$

Summary and strategy

What's next?

A. Find an algebraic method to compute the coefficients, given a $f(u,v)$:

$$\gamma^{(2)}_{n,\ell} = \frac{c^{(0)}_n}{\ell^{2\Delta}}\left(1 + \frac{b^{(1)}_n}{\ell} + \frac{b^{(2)}_n}{\ell^2} + \cdots\right)$$

B. Find the contribution to $\gamma^{(2)}_{0,\ell}$ due to a single conformal block in

$$\sum_n \sum_{\ell=0}^{L} u^{\Delta+n}\frac{1}{8}a^{(0)}_{n,\ell}(\gamma^{(1)}_{n,\ell})^2 g_{2\Delta+2n+\ell,\ell}(u,v) = u^{\Delta}\left(f(u,v)\log v + \cdots\right)$$

C. Specify the solution $\{\gamma^{(1)}, a^{(1)}\}$ and sum over n

Casimir approach

- In the small u limit, the blocks can be written as

$$g_{\Delta_p,\ell}(u,v) = \sum_{m=0}^{\infty} u^m g^{(m)}_{\Delta_p,\ell}(v)$$

and we define the collinear block as $g^{(0)}_{\Delta_p,\ell}(v) \equiv g^{\text{coll}}_{\Delta_p,\ell}(v)$

- Collinear blocks are eigenfunctions of an operator, closely related to the quadratic Casimir of the conformal group

$$\mathcal{C} g^{\text{coll}}_{2\Delta+\ell,\ell}(v) = J^2 g^{\text{coll}}_{2\Delta+\ell,\ell}(v)$$

with $J^2 = (\ell+\Delta)(\ell+\Delta-1)$

Casimir approach

In the small v limit,

$$\sum_{\ell \text{ even}} \frac{1}{2} a^{(0)}_{0,\ell} \gamma^{(2)}_{0,\ell} g^{\text{coll}}_{2\Delta+\ell,\ell}(v) = f(v) \log^2(v) + \cdots$$

The idea is to act on both sides of this equation with the operator $\mathcal{C}$:

- increase the divergence on the rhs
- explore more and more terms in the large ℓ expansion.

Note: By acting a finite number of times with the operator on $v^n \log^2 v$ produces a negative power of v

Algebraic problem

Let's start by considering

$$\sum_{\ell \text{ even}} a^{(0)}_{0,\ell} g^{\text{coll}}_{2\Delta+\ell,\ell}(v) = \frac{1}{v^{\Delta}} + \cdots$$

Now we can consider insertion of this form

$$\sum_{\ell \text{ even}} \left(\frac{1}{J^2}\right)^{\Delta+n} a^{(0)}_{0,\ell} g^{\text{coll}}_{2\Delta+\ell,\ell}(v) = h^{(n)}(v) \log^2(v) + \cdots$$

Assuming an expansion for $\gamma^{(2)}$ as $\gamma^{(2)}_{0,J} = \frac{2}{J^{2\Delta}}\left(b_0 + \frac{b_1}{J^2} + \frac{b_2}{J^4} + \frac{b_3}{J^6} + \cdots\right)$

we can rephrase the problem as

$$b_0 h^{(0)}(v) + b_1 h^{(1)}(v) + b_2 h^{(2)}(v) + \cdots = f(v)$$

Find the basis

$$b_0 h^{(0)}(v) + b_1 h^{(1)}(v) + b_2 h^{(2)}(v) + \cdots = f(v)$$

We have a recurrence relation of the form

$$\mathcal{C} h^{(n)}(v) = h^{(n-1)}(v)$$

and for integer Δ

$$\mathcal{C}^{\Delta}(h^{(0)}(v) \log^2(v)) = \frac{1}{v^{\Delta}} + \cdots$$

Finding the basis in this language is complicated, but there is a way out.

Find the basis

Let's introduce $\mathcal{C} = (1-v)^{-\Delta}\hat{\mathcal{C}}(1-v)^{\Delta}$, $\quad h^{(n)}(v) = \dfrac{\hat{h}^{(n)}(v)}{(1-v)^{\Delta}}$

$$\zeta = \frac{v}{1-v}$$

Then it is possible to reformulate the problem and write the solution, for $\Delta > 1$ and integer, as

$$\hat{h}^{(0)}_{\Delta}(\zeta) = \alpha_0 H_{\vec{\rho}_0}(\zeta) + \alpha_1 H_{\vec{\rho}_1}(\zeta) + \cdots + \alpha_{\Delta-2} H_{\vec{\rho}_{\Delta-2}}(\zeta)$$

Harmonic polylogs

$$H_0(\zeta) = \log(\zeta) \qquad H_{\vec{0}_n}(\zeta) = \frac{1}{n!}\log^n(\zeta)$$

$$H_1(\zeta) = \log(1+\zeta) \qquad \vec{\rho}_\omega = (\ldots, 1, 0, 1)$$

$$H_{a\vec{\omega}}(\zeta) = \int_0^\zeta \frac{1}{a+x} H_{\vec{\omega}}(x)\,dx$$

Solution

Now we have found a method to compute all the coefficients, for a given $f(v)$.

How can we use this procedure?

- Find the term proportional to $\log v$ for each block $f_{n,s}(u)$
- Compute the coefficients in the large spin expansion in the anomalous dimension

$$f_{n,s}(v) \to \gamma^{(2)}_{0,\ell}\Big|_{(n,s)} = -\frac{c^{(0)}_{n,s}}{J^{2\Delta+2n}}\left(1+\frac{c^{(1)}_{n,s}}{J^2}+\cdots\right) \equiv -\frac{c^{(0)}_{n,s}}{J^{2\Delta+2n}}\,\hat{\gamma}^{(2)}_{0,\ell}\Big|_{(n,s)}$$

- Sum over all the conformal blocks

Example of ϕ^4

With some manipulations, we can find analytic results for

$$\gamma^{(2)}_{0,\ell} = \frac{1}{8}\sum_n a^{(0)}_{n,0}\left(\gamma^{(1)}_{n,0}\right)^2 \gamma^{(2)}_{0,\ell}\Big|_{(n,0)}$$

$d=4$ Very similar, the main differences are in the degree of the polynomials and in the dependence with n of $\gamma^{(1)}_{n,\ell}$ and $a^{(0)}_{n,\ell}$

$\gamma^{(1)}_{0,0} = \alpha$

$d=2$

$$\gamma^{(2)}_{0,0} = \frac{15-4\pi^2}{10}\alpha^2$$

$$\gamma^{(2)}_{0,2} = \frac{455-48\pi^2}{420}\alpha^2$$

$$\gamma^{(2)}_{0,4} = \frac{5863-600\pi^2}{8580}\alpha^2$$

$d=4$

$\gamma^{(2)}_{0,0} \rightarrow$ divergent

$$\gamma^{(2)}_{0,2} = \frac{2(174\pi^2-1925)}{3465}\alpha^2$$

$$\gamma^{(2)}_{0,4} = \frac{150600\pi^2-1520519}{2252250}\alpha^2$$

Example of ϕ^4

For concreteness let us study the case of $\Delta = 2$, in $d = 2$ and $d = 4$. In both cases $J^2 = (\ell+1)(\ell+2)$.

In this case, the relevant conformal block is $s = 0$

$d = 2$

$$f_{n,0}(v) \to \hat{\gamma}^{(2)}_{0,\ell}\Big|_{(n,0)} = J^{2n+2} P_1^{2+2n}(\ell) + J^{2n+6} P_2^{2n}(\ell)\psi_2(\ell+1)$$

$P_1^{2+2n}(\ell)$ ↓ polynomial in ℓ of degree $2+2n$

$P_2^{2n}(\ell)$ ↓ polynomial in ℓ of degree $2n$

Mellin at one loop

Now we turn to studying the same problem in Mellin space.

- In the cases considered in this talk (no single trace in the OPE) the full one loop amplitude is completely determined by the double trace exchanges! $M_{\text{1-loop}}(s,t) = M_{\text{1-loop}}^{[\mathcal{O}\mathcal{O}]}(s,t)$
- All poles and residues of the amplitude are completely fixed by tree level data. **Why?**

$$\mathcal{G}^{(2)}(u,v) = \sum_{n,\ell} u^{\Delta+n} \left(\frac{\log^2(u)}{8} a_{n,\ell}^{(0)} (\gamma_{n,\ell}^{(1)})^2 g_{2\Delta+2n+\ell,\ell}(u,v) + O(\log u) \right)$$

Fully fixed by tree level data!

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Mellin at one loop

In order to reproduce each power of u

1. $M_{\text{1-loop}}$ must acquire simple poles at $\tau = 2\Delta + 2n$
2. Residues are fixed by $\gamma^{(1)}_{n,\ell}$

$$M_{\text{1-loop}}(s,t) = \sum_{n=0}^{\infty} \frac{R_n(s)}{t-(2\Delta+2n)} + f_{\text{reg}}(s,t) + (\text{crossed})$$

f_{reg} → cannot be fixed by tree level data

Mellin at one loop

In order to reproduce each power of u

1. $M_{\text{1-loop}}$ must acquire simple poles at $\tau = 2\Delta + 2n$
2. Residues are fixed by $\gamma_{n,\ell}^{(1)}$

$$M_{\text{1-loop}}(s,t) = \sum_{n=0}^{\infty} \frac{R_n(s)}{t-(2\Delta+2n)} + f_{\text{reg}}(s,t) + (\text{crossed})$$

cannot be fixed by tree level data

Example of ϕ^4

$$M_{\text{1-loop}}(s,t) = \sum_{n=0}^{\infty} R_n \left(\frac{1}{s-(2\Delta+2n)} + \frac{1}{t-(2\Delta+2n)} + \frac{1}{\hat{u}-(2\Delta+2n)} \right)$$

with constant, in s and t, residues R_n . This is a consequence of the fact that there is no single-trace operator in the OPE.

We can also compute the anomalous dimension

$$\gamma^{(2)}_{0,\ell>0} = -\frac{1}{2\pi i}\int_{-i\infty}^{i\infty} ds\, M'_{\text{1-loop}}(s,2\Delta)\Gamma^2\left(\frac{s}{2}\right)\Gamma^2\left(\frac{2\Delta-s}{2}\right) {}_3F_2(-\ell,\ell+2\Delta-1,\frac{s}{2};\Delta,\Delta;1)$$

↓

one loop amplitude minus the pole in $t=2\Delta$

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- To compute this integral, we need to regularise the sum over n
- We computed $\gamma^{(2)}_{0,2}$ and $\gamma^{(2)}_{0,4}$ (and $\gamma^{(2)}_{0,0}$ in $d=2$) and they agree with the CFT results!

Comments on divergences

- The divergence in the spin zero anomalous dimension at one loop in four dimensions is consistent with bulk expectations
- This is a UV divergence, due to the fact that AdS_5 requires a counterterm ϕ^4.
- This is explicit in the fact that we get a finite result for spins greater than zero.

- On the other hand, we do not expect any divergence in two dimensions since AdS_3 is finite.

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What have we learnt?

- ✓ Using the analytic bootstrap, we found solutions of crossing equations to order $\frac{1}{N^4}$
- ✓ The method we used is by resumming the full expansion in inverse powers of the spin, up to finite spin.
- ✓ We computed the anomalous dimensions for several cases, for low values of the spin

- ✓ We constructed the polar part of the Mellin amplitude at one loop
- ✓ We computed the anomalous dimensions associated to these amplitudes, finding agreement with the CFT results.

Open problems

* Can we do the same in the case of single trace in the OPE, as for instance the stress tensor?

  Yes! in progress

* Can we apply this ideas to supersymmetric field theories?

  Yes/No

* Can we push it further? To higher orders in 1/N?

  Maybe