Title: Supersymmetry and Legendrian Knots

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Abstract: <p>I will discuss a new class of supersymmetric Wilson loop operators in pure N=2 Yang-Mills-Chern-Simons theory. These Wilson loops preserve one supercharge on-shell and wrap arbitrary Legendrian knots in the standard contact R^3. I will also explain a relation, motivated by a global picture of contact three-manifolds, between these loop operators and chiral current algebra in two dimensions. This talk is directly related to, but independent of, my preceding Friday talk in the Mathematical Physics seminar.</p>


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Notices
of the American Mathematical Society
September 2016
Volume 63, Number 8
AMS Fall Eastern Sectional Sampler
page 868
From Mathematical Physics to Analysis: A Walk in Barry Simon's Mathematical Garden, II
page 878
Raleigh Meeting
page 987
AMS
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About the cover: Heisenberg Sculpture (see page 927)

"Supersymmetry and Legendrian Knots"

$d=3$ $\mathcal{N}=2 \supset U(1)$ R-symmetry

$\{Q_\alpha, \tilde{Q}_\alpha\}$, $\alpha=1,2$

"$\frac{1}{4}$-BPS"

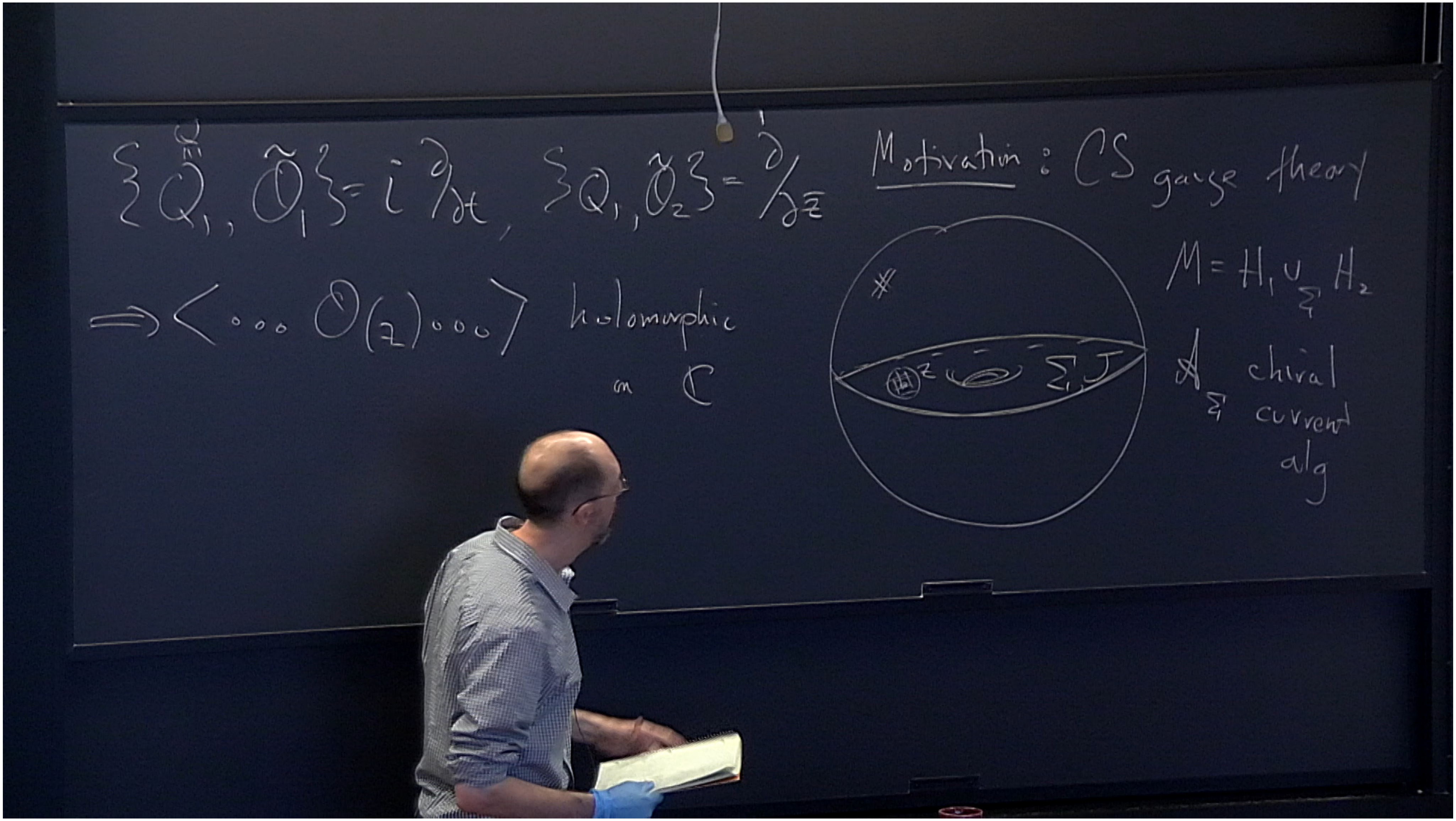
$Q \equiv \xi^\alpha Q_\alpha$ fixed spinor

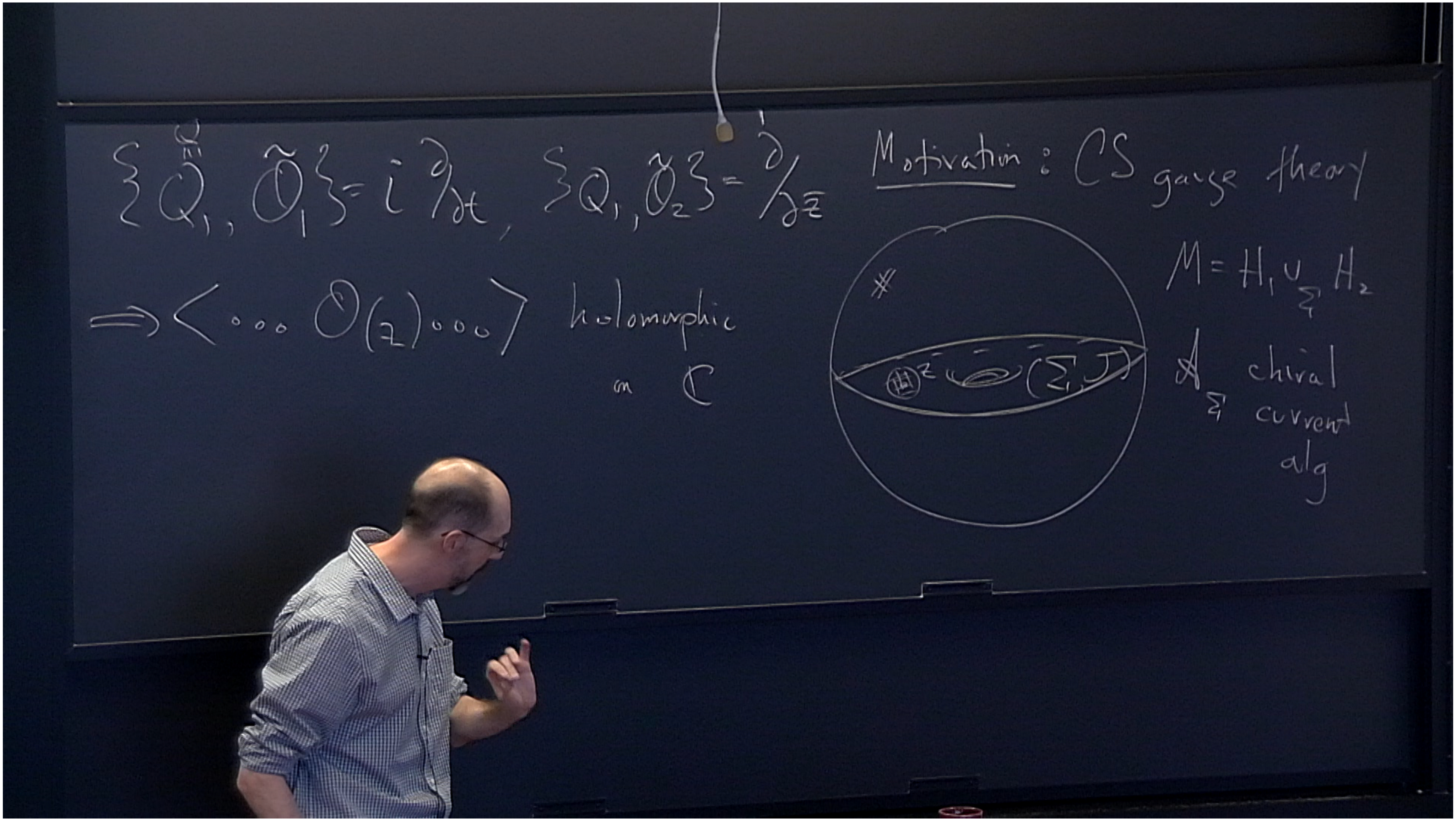
$[Q, \mathcal{O}] = 0$

$\mathbb{R}^3 \overset{\text{reduce}}{\rightsquigarrow} \mathbb{R}_t \times \mathbb{C}_z$

$\mathbb{R}^3$: $\mathrm{Spin}(3) \simeq SU(2)$

$\mathbb{C}_z$: $\mathrm{Spin}(2) \simeq U(1)$

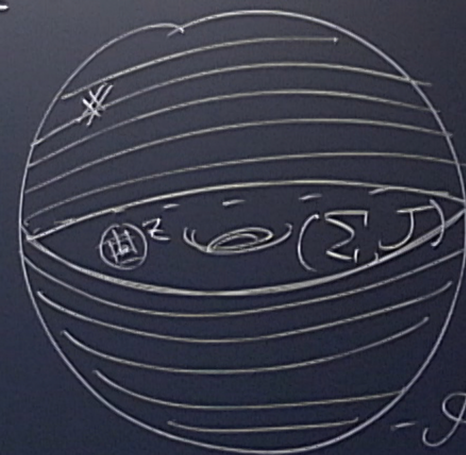

$\{Q_1, \tilde{\mathcal{O}}_1\} = i\,\partial/\partial t$, $\{Q_1, \tilde{\mathcal{O}}_2\} = \partial/\partial \bar{z}$
$\Rightarrow \langle \ldots \mathcal{O}(z) \ldots \rangle$ holomorphic on $\mathbb{C}$
Motivation: CS gauge theory
$M = H_1 \cup_{\Sigma} H_2$
Σ, J
$\mathcal{A}_{\Sigma}$ chiral current alg


{Q_1, Õ_1} = i ∂/∂t , {Q_1, Õ_2} = ∂/∂z̄
⇒ < ... O(z) ... > holomorphic on ℂ
Motivation: CS gauge theory
M = H_1 ∪_Σ H_2
(Σ, J)
A_Σ chiral current alg

$$\{Q_1, \tilde{\mathcal{O}}_1\} = i\,\partial/\partial t, \quad \{Q_1, \tilde{\mathcal{O}}_2\} = \partial/\partial \bar{z}$$

$$\Rightarrow \langle \cdots \mathcal{O}(z) \cdots \rangle \text{ holomorphic on } \mathbb{C}$$

<u>Motivation</u>: CS gauge theory



$M = H_1 \cup_\Sigma H_2$

$\mathcal{A}_\Sigma$ chiral current alg

$\mathcal{A}_M$ "3d" chiral alg

$\mathcal{A}_\Sigma$ $\quad$ $\mathcal{A}_M$ $\quad$ $b_1 > 0$

<u>Ex</u>. $\left\{ \Sigma_1 \longrightarrow M \right\}$ monodromy

$\pi \downarrow$ $\quad [\phi] \in MCG(\Sigma_1)$

S^1

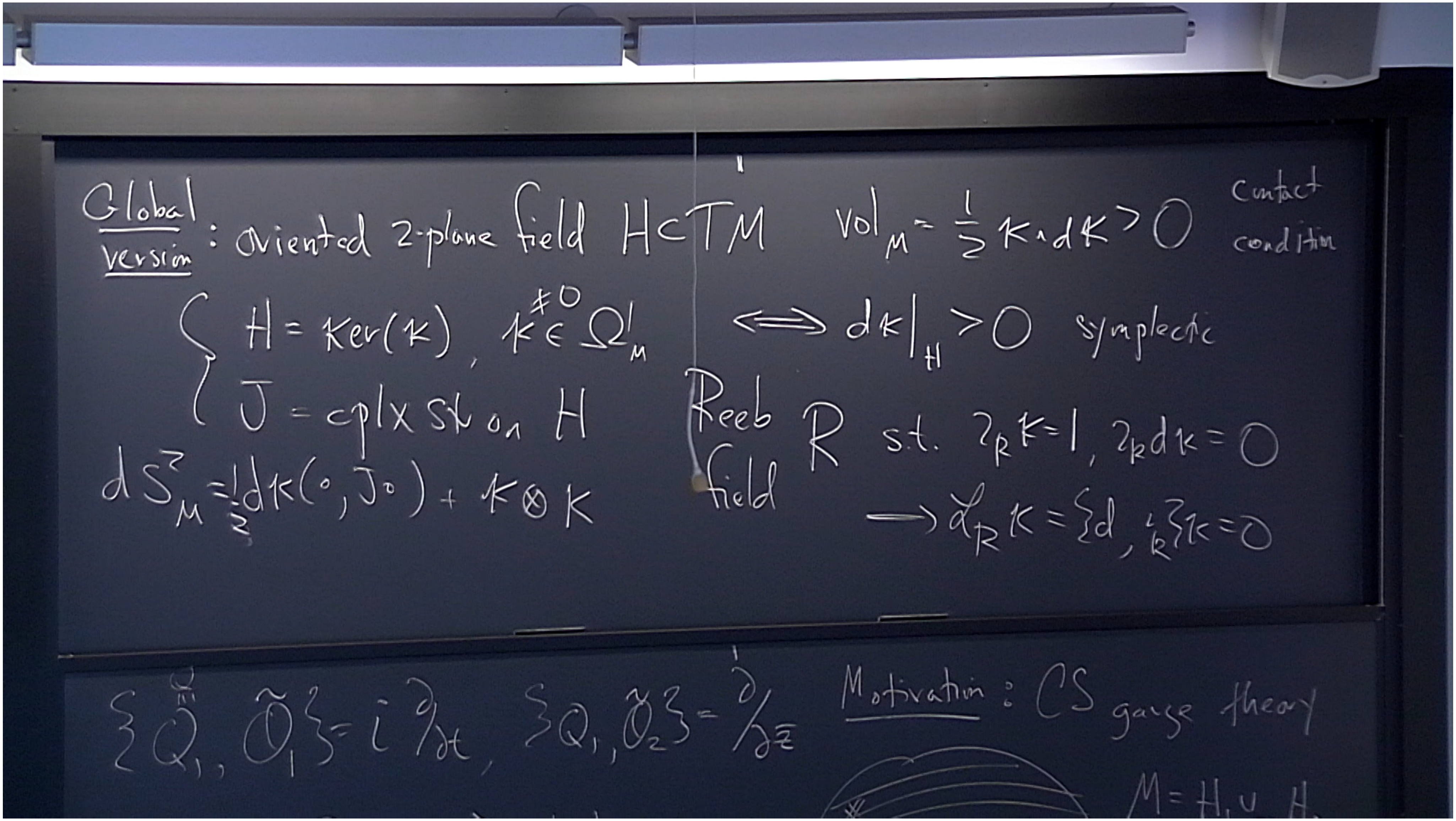
taut

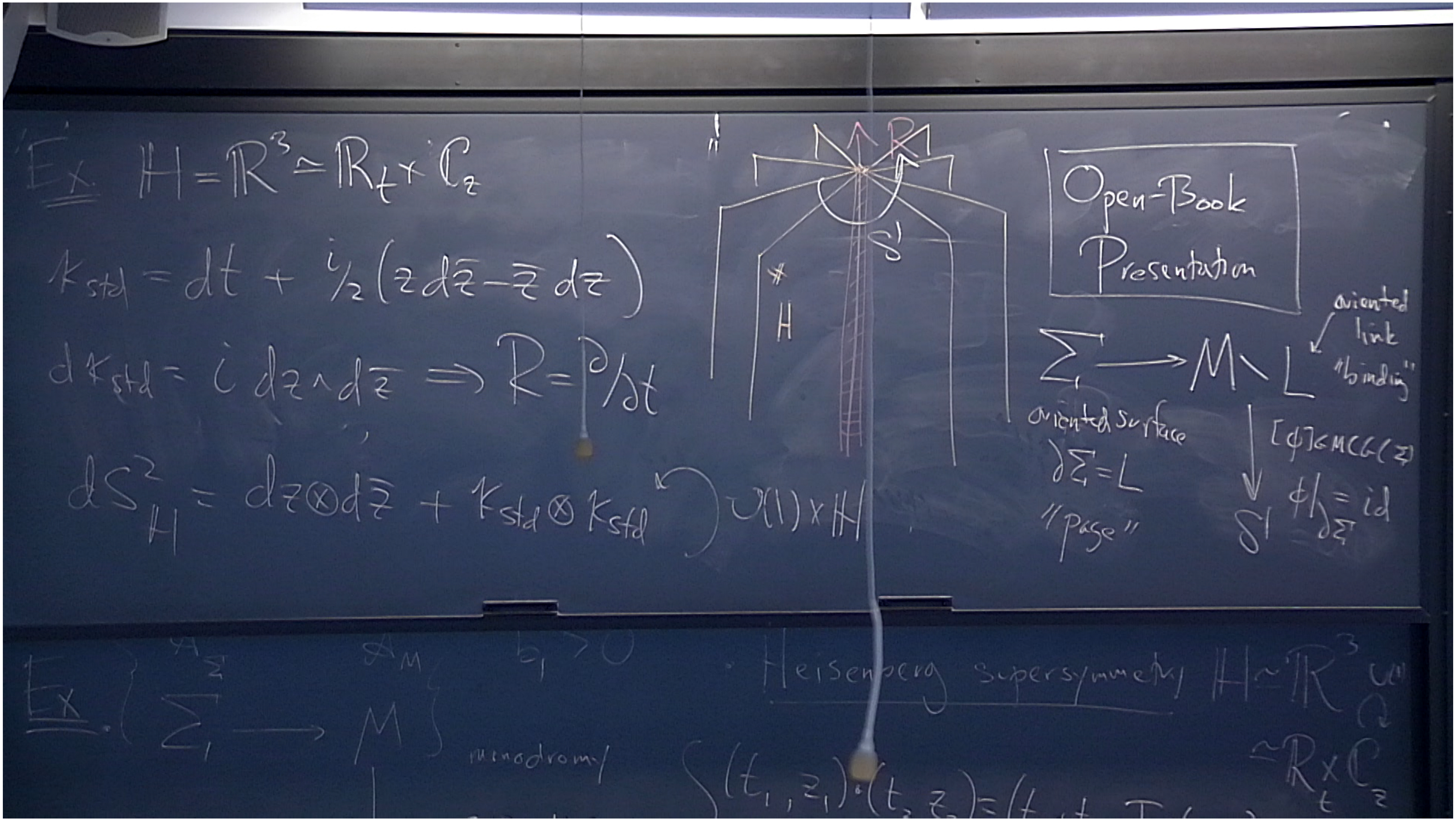
metric: $ds^2_M = \omega(\cdot, J\cdot) + \pi^*(d\theta)^2$

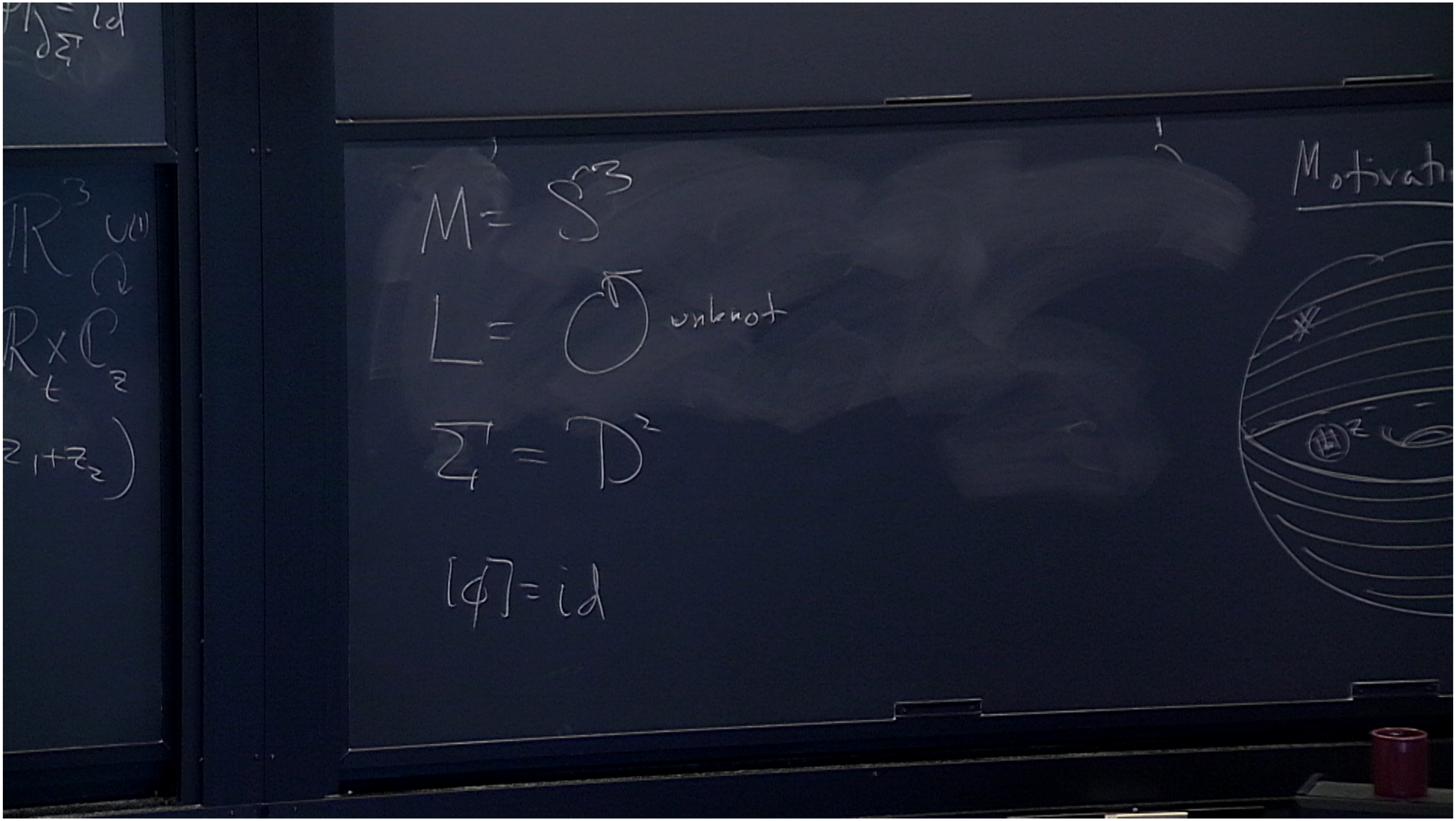
· <u>Heisenberg supersymmetry</u> $\mathbb{H} \simeq \mathbb{R}^3 \circlearrowleft U(1)$

$\simeq \mathbb{R}_t \times \mathbb{C}_z$

$$\begin{cases} (t_1, z_1)\cdot(t_2, z_2) = (t_1 + t_2 + \mathrm{Im}(z_1 \bar{z}_2), z_1 + z_2) \\ (t, z)^{-1} = (-t, -z) \end{cases}$$


Global version : oriented 2-plane field $H \subset TM$
$\mathrm{vol}_M = \frac{1}{2} \kappa \wedge d\kappa > 0$
contact condition
$\begin{cases} H = \ker(\kappa), \ \overset{\neq 0}{\kappa} \in \Omega^1_M \\ J = \text{cplx str on } H \end{cases}$
$\Longleftrightarrow d\kappa|_H > 0$ symplectic
Reeb field R s.t. $\iota_R \kappa = 1$, $\iota_R d\kappa = 0$
$\longrightarrow \mathcal{L}_R \kappa = \{d, \iota_R\}\kappa = 0$
$dS^2_M = \frac{1}{2} d\kappa(\cdot, J\cdot) + \kappa \otimes \kappa$
$\{Q_1, \tilde{Q}_1\} = i\,\partial/\partial t$, $\{Q_1, \tilde{Q}_2\} = \partial/\partial \bar{z}$
Motivation : CS gauge theory
$M = H_1 \cup H_2$


Ex. $\mathbb{H} = \mathbb{R}^3 \simeq \mathbb{R}_t \times \mathbb{C}_z$
$\kappa_{std} = dt + \frac{i}{2}(z\,d\bar{z} - \bar{z}\,dz)$
$d\kappa_{std} = i\,dz \wedge d\bar{z} \Rightarrow R = \partial/\partial t$
$dS^2_{\mathbb{H}} = dz \otimes d\bar{z} + \kappa_{std} \otimes \kappa_{std}$
$U(1) \times \mathbb{H}$
R
S^1
H
Open-Book Presentation
$\Sigma_1 \longrightarrow M \setminus L$
oriented link
"binding"
oriented surface
$\partial\Sigma_1 = L$
"page"
$[\phi] \in MCG(\Sigma)$
S^1
$\phi|_{\partial\Sigma} = id$
Ex. $\Sigma_1 \longrightarrow M$
monodromy
$b_1 > 0$
Heisenberg supersymmetry $\mathbb{H} \simeq \mathbb{R}^3$
$\simeq \mathbb{R}_t \times \mathbb{C}_z$


M = S^3
L = unknot
Σ = D^2
[φ] = id
Motivati

$M = S^3$

$L = $ ○ unknot

$\Sigma = D^2$

$[\phi] = id$

• Supersymmetry algebra: Assume $\mathcal{L}_R J = 0$.

$\{Q, \tilde{Q}\} = i\mathcal{L}_R$, $Q^2 = \tilde{Q}^2 = 0$

Sps $\mathcal{O}_C$ is a loop operator for $C \hookrightarrow M$ s.t.

$[Q, \mathcal{O}_C] = 0 = [\tilde{Q}, \mathcal{O}_C] \Longrightarrow \mathcal{L}_R \mathcal{O} = 0$, C is Reeb orbit

Global version: oriented 2-plane field $H \subset TM$ $\quad vol_M = \frac{1}{2} \kappa \wedge d\kappa > 0$ contact condition

$H = \ker(\kappa)$, $\kappa \in \Omega^1_M$, $\kappa \neq 0$ $\iff d\kappa|_H > 0$ symplectic

$\mathcal{N}=2$ YM-CS theory

Compactify on $M=\Sigma_g\times S^1$ $\rightsquigarrow$ $G=U(1)$

$$I_{YMCS} = -\frac{1}{8\pi e^2}\int_M \mathrm{Tr}\left(F_A\wedge * F_A\right) + ik\,CS(A) + \dots, \quad k\in\mathbb{Z}$$

$\mathcal{N}=(2,2)$: $\delta A_z=\lambda_z$, $\delta A_{\bar z}=0$

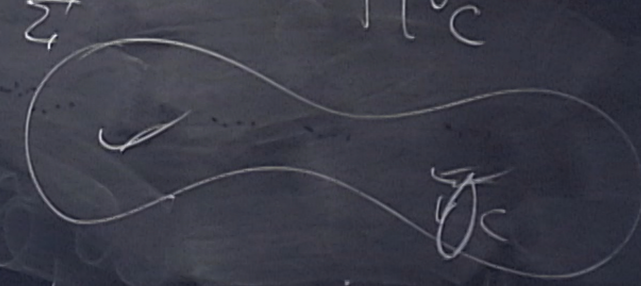
$\sigma\in\mathbb{C}^{\times}$

$\sigma\sim\sigma+2\pi$: $\delta\sigma=0$, $\delta\bar\sigma=\alpha$

$\underline{\text{EoM}}$: $\frac{1}{e^2}\partial_z \alpha + \frac{k}{2\pi}\lambda_z = 0$ 1/4-BPS

Set $\mathcal{J}_z = A_z + \frac{2\pi}{e^2 k}\partial_z \bar{\sigma}$ $W_C = \exp\left[\oint_C \left(A + \frac{2\pi}{e^2 k}\partial_z\bar{\sigma}\right)\right]$

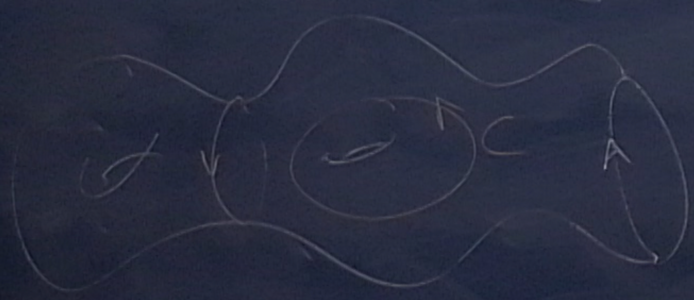
$\Longrightarrow [Q, \mathcal{J}_z] = 0$ on-shell



$\underline{\text{Global version}}$: oriented 2-plane field $H \subset TM$ $\mathrm{vol}_M = \frac{1}{2}\kappa \wedge d\kappa > 0$ contact condition

$H = \ker(\kappa)$, $\kappa \overset{\neq 0}{\in} \Omega^1_M \iff d\kappa|_H > 0$ symplectic

"Lift" to (M, H)



$$H = \ker(\kappa) = T\Sigma + O(\varepsilon)$$

Req'd: $T_p C \subset H \quad \forall p \in C$

$\iff \kappa|_C = 0$

$\iff C$ is Legendrian.

1/4-BPS Wilson loop

$$\mathcal{A} = A - \frac{2\pi}{e^2 k}\left(2_R F_A^{(1,0)} + i\, d_A^{(1,0)} \sigma\right)$$

$\mathfrak{g}$-valued scalar

$$W_C = \mathrm{Tr}_V \, P\exp\left[\oint_C \mathcal{A}\right]$$

* Claim: $[Q, W_C] = 0$

on-shell if C is Legendrian.

EoM: $-\frac{1}{e^2}\partial_2 \alpha + \frac{k}{2\pi}\lambda_2 = 0$ 1/4-BPS