Title: PSI 2016/2017 Introduction to Quantum Integrable Systems - Lecture 6

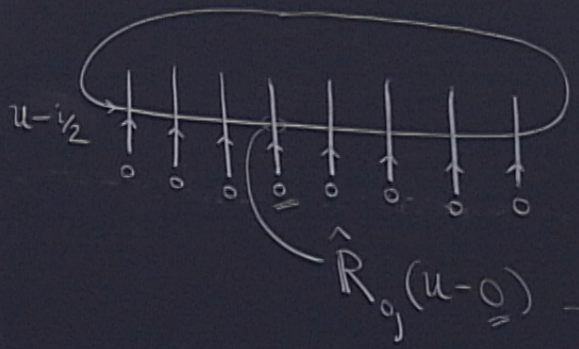
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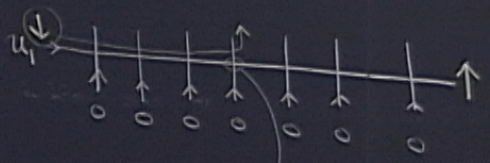
URL: http://pirsa.org/17020123

Abstract:

$$\hat{T}(u-i/2)\,|\Psi\rangle = \left[(u+i/2)^{L}\right.$$

$$\hat{B}(u_1)\cdots\hat{B}(u_N)\,|\uparrow\cdots\uparrow\rangle$$

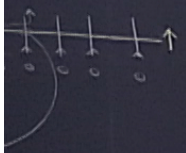




$$\left(\frac{u_K}{u_K}\right.$$

$$= \left[(u+i/2)^L \prod_K \frac{u-u_K-i}{u-u_K} + (u-\tfrac{i}{2})^L \prod_K \frac{u-u_K+i}{u-u_K} \right] |\Psi\rangle$$

$\hat{B}(u_N)|\uparrow \cdots \uparrow\rangle$ Where



$$\left(\frac{u_K+i/2}{u_K-i/2}\right)^L \prod_{j\neq K}^{N} \frac{u_K-u_j-i}{u_K-u_j+i} = 1 \quad \text{and} \quad E = \lambda L + \frac{1}{i}\frac{d}{du}\log T(u)\Big|_{u=0} = \sum_{j=1}^{N} \frac{\lambda}{u_j^2+\frac{1}{4}}$$

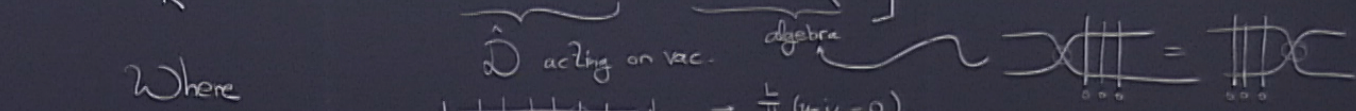
MAGNON BAEs

$$= \left[(u+i/2)^L \prod_K \frac{u-u_K-i}{u-u_K} + \left(u-\frac{i}{2}\right)^L \prod_K \frac{u-u_K+i}{u-u_K} \right] |\Psi\rangle$$

$\hat{B}(u_N)|\uparrow\ldots\uparrow\rangle$ Where

$\hat{D}$ acting on vac. $\rightarrow \prod_{K=1}^{L}(u-i/2-0)$ $(u-i/2)$

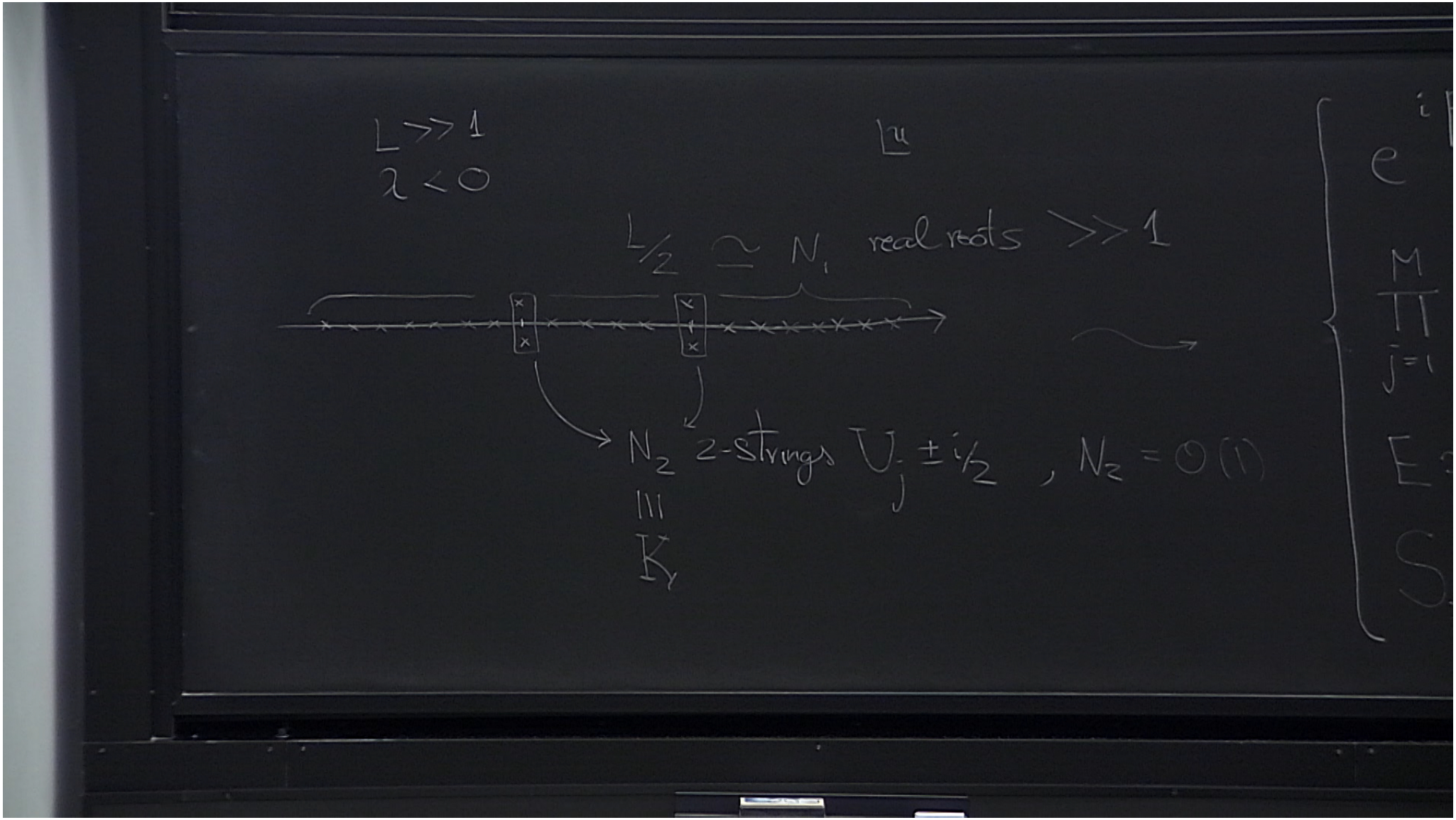
algebra

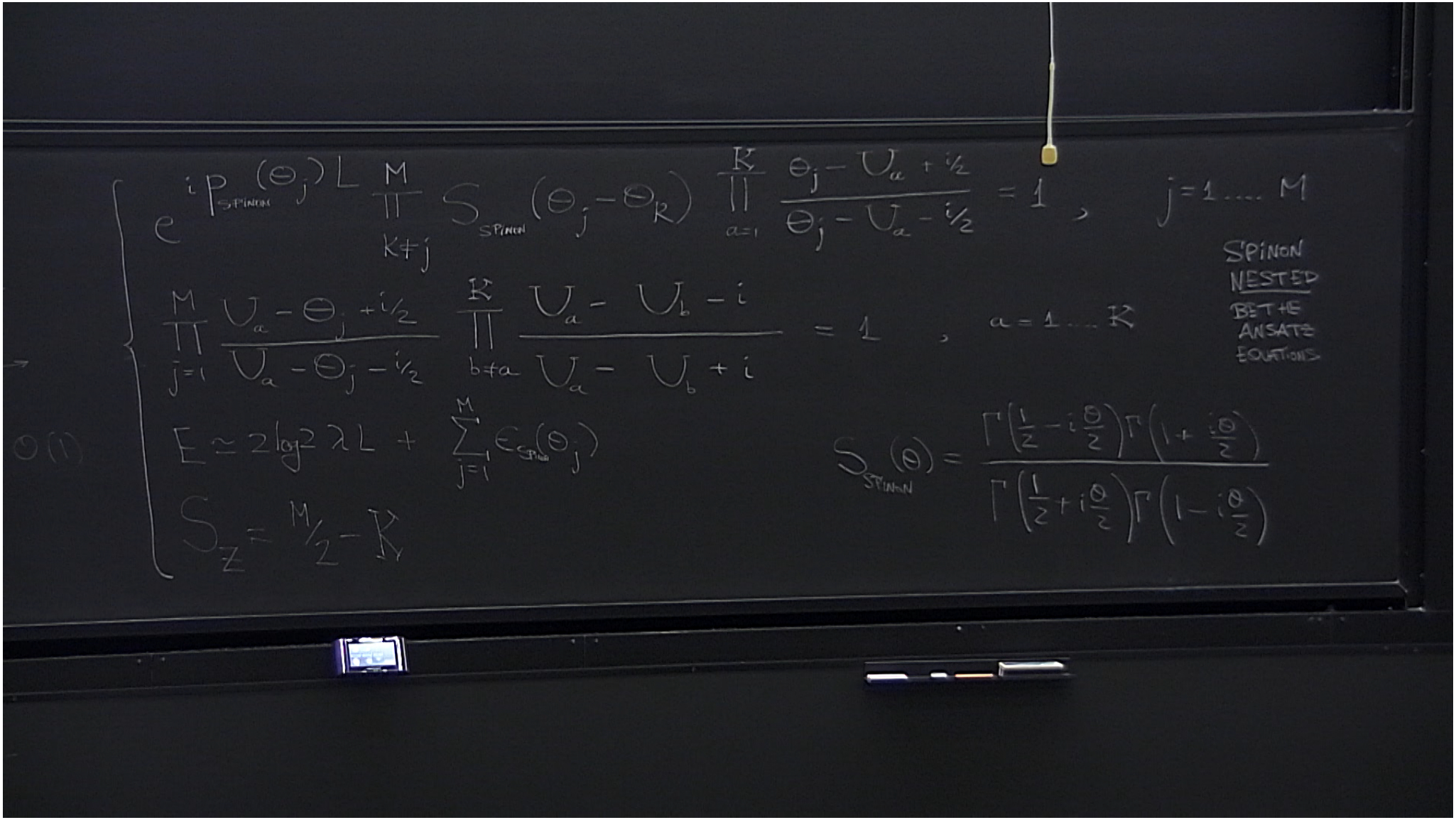


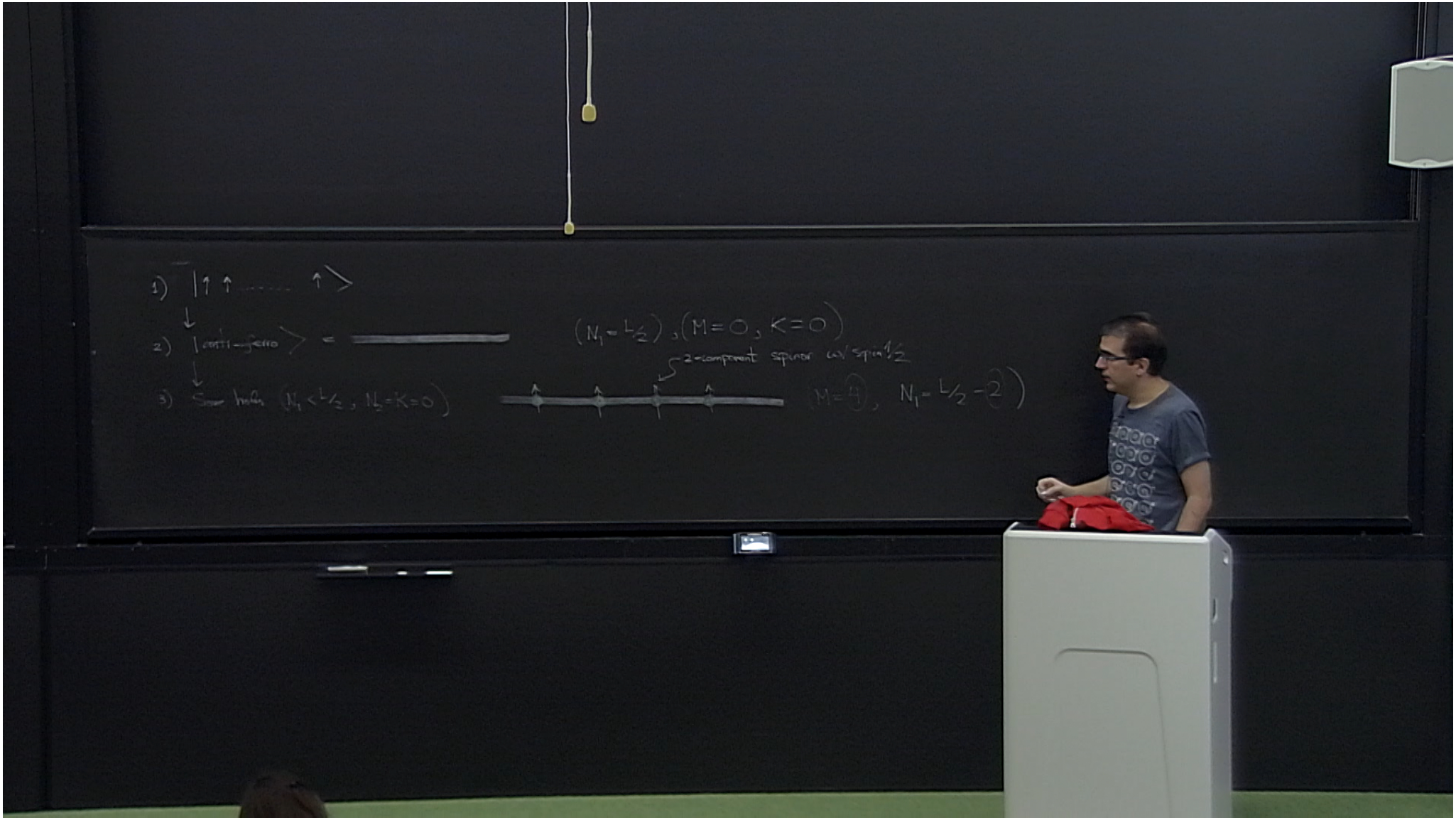
Energy of each magnons

$$\left(\frac{u_K+i/2}{u_K-i/2}\right)^L \prod_{j\neq K}^{N} \frac{u_K-u_j-i}{u_K-u_j+i} = 1 \quad \text{and} \quad E = \lambda L + \frac{1}{i}\frac{d}{du}\log T(u)\Big|_{u=0} = \sum_{j=1}^{N} \frac{\lambda}{u_j^2+\frac{1}{4}}$$

MAGNON BAEs


L >> 1
λ < 0
L/2 ≃ N_1 real roots >> 1
N_2 2-strings U_j ± i/2 , N_2 = 0 (1)
III
K_γ


$e^{iP_{SPINON}(\theta_j)L} \prod_{k\neq j}^{M} S_{SPINON}(\theta_j-\theta_k) \prod_{a=1}^{K} \frac{\theta_j-U_a+i/2}{\theta_j-U_a-i/2} = 1 \quad , \quad j=1\ldots M$
SPINON NESTED BETHE ANSATZ EQUATIONS
$\prod_{j=1}^{M} \frac{U_a-\theta_j+i/2}{U_a-\theta_j-i/2} \prod_{b\neq a}^{K} \frac{U_a-U_b-i}{U_a-U_b+i} = 1 \quad , \quad a=1\ldots K$
$E \simeq 2\log 2\, \lambda L + \sum_{j=1}^{M} \epsilon_{SPINON}(\theta_j)$
$S_z = M/2 - K$
$S_{SPINON}(\theta) = \frac{\Gamma\left(\frac{1}{2}-i\frac{\theta}{2}\right)\Gamma\left(1+i\frac{\theta}{2}\right)}{\Gamma\left(\frac{1}{2}+i\frac{\theta}{2}\right)\Gamma\left(1-i\frac{\theta}{2}\right)}$


1)
2)
3)
$(N_1 = L/2), (M=0, K=0)$
2-component spinor w/ spin 1/2
$M=0, \; N_1 = L/2 - 2$

1) $|\uparrow\uparrow \cdots\cdots \uparrow\rangle$ ← SU(2) is broken

↓

2) $|\text{anti-ferro}\rangle$ =  ← vac preserves SU(2)

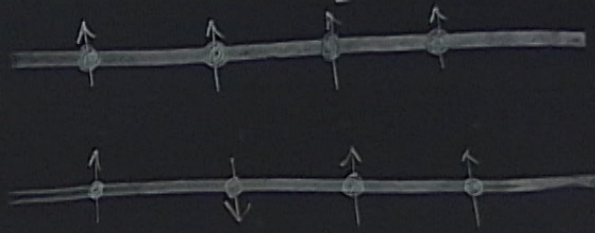
$(N_1 = L/2)$, $(M=0, K=0)$

↓

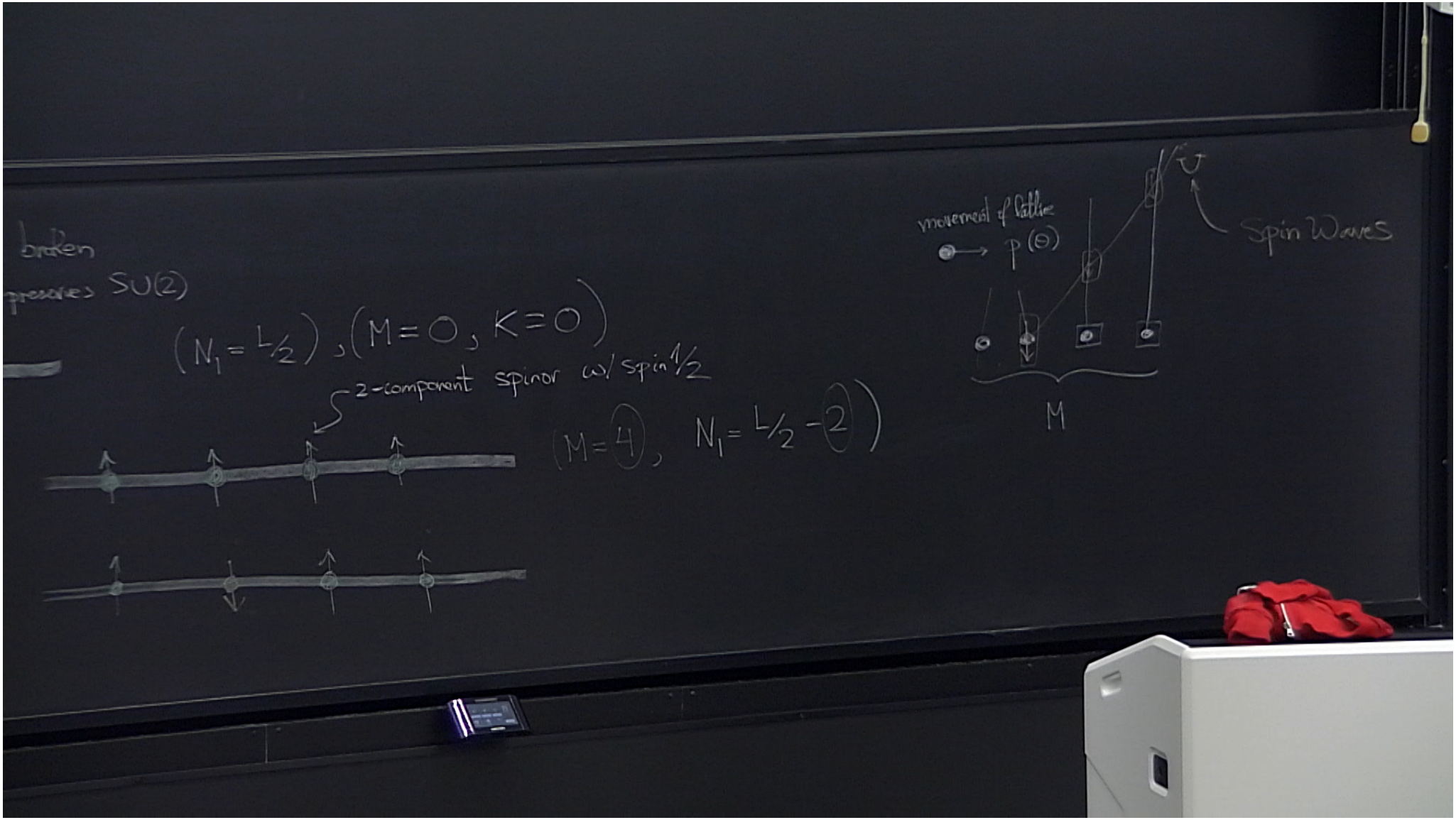
3) Some holes $(N_1 < L/2,\ N_2 = K = 0)$

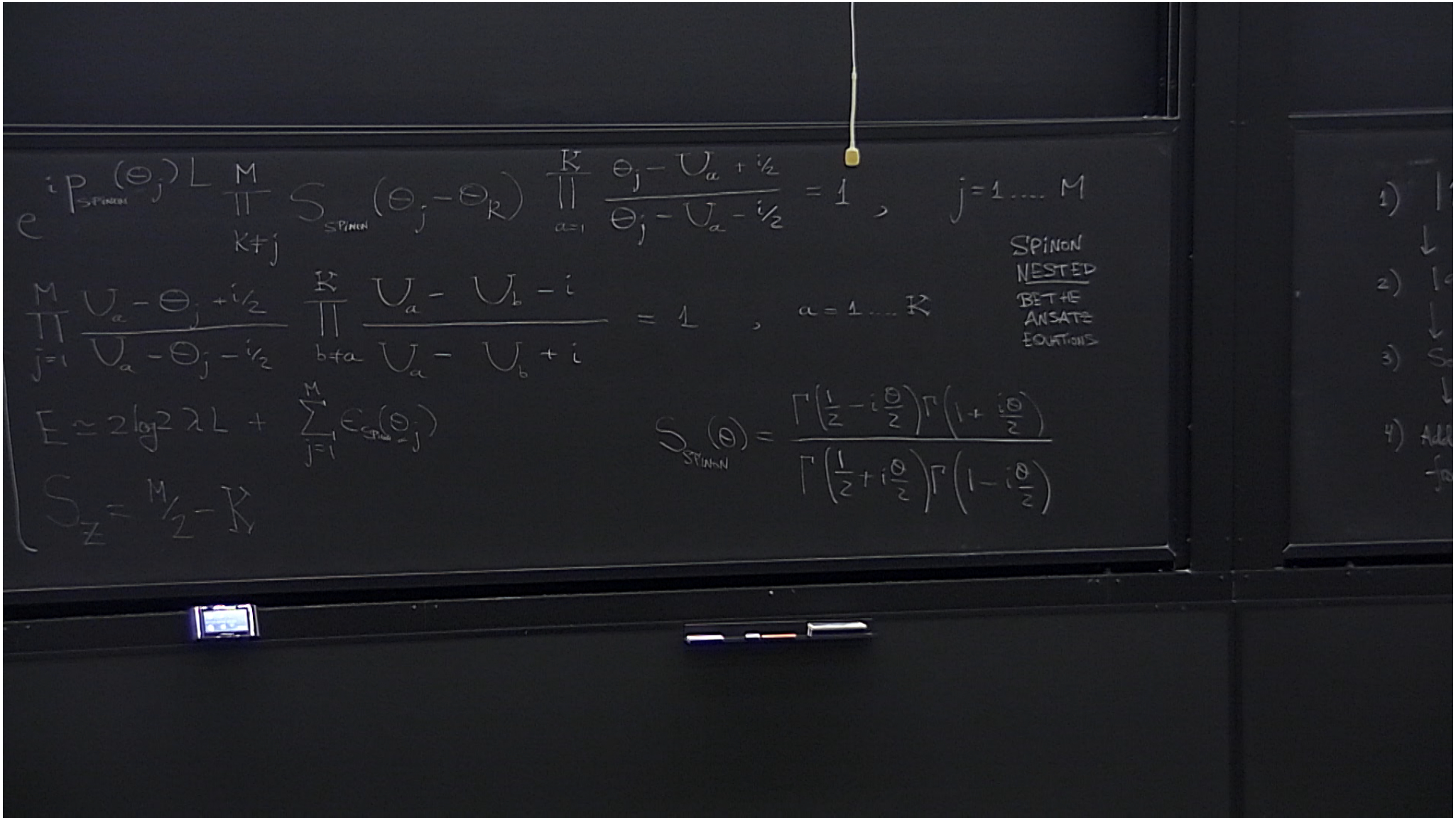
↓

4) Add strings ~ flip K of the spinons from ↑ to ↓.

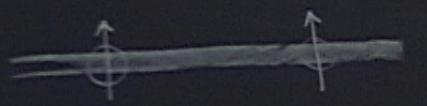


$(M=4,\ N_1 = L/2 - 2)$


broken
preserves SU(2)
$(N_1 = 1/2)$, $(M=0, K=0)$
2-component spinor w/ spin 1/2
$(M=4, N_1 = 1/2 - 2)$
movement of lattice
p(θ)
Spin Waves
M


$e^{iP_{SPINON}(\theta_j)L} \prod_{k\neq j}^{M} S_{SPINON}(\theta_j-\theta_k) \prod_{a=1}^{K} \frac{\theta_j-U_a+i/2}{\theta_j-U_a-i/2} = 1 \quad , \quad j=1\ldots M$
SPINON NESTED BETHE ANSATZ EQUATIONS
$\prod_{j=1}^{M} \frac{U_a-\theta_j+i/2}{U_a-\theta_j-i/2} \prod_{b\neq a}^{K} \frac{U_a-U_b-i}{U_a-U_b+i} = 1 \quad , \quad a=1\ldots K$
$E \simeq 2\log 2\, \lambda L + \sum_{j=1}^{M} \epsilon_{SPINON}(\theta_j)$
$S_{SPINON}(\theta) = \frac{\Gamma\left(\frac{1}{2}-i\frac{\theta}{2}\right)\Gamma\left(1+i\frac{\theta}{2}\right)}{\Gamma\left(\frac{1}{2}+i\frac{\theta}{2}\right)\Gamma\left(1-i\frac{\theta}{2}\right)}$
$S_z = M/2 - K$
1)
2)
3)
4)

from ↑ to ↓.

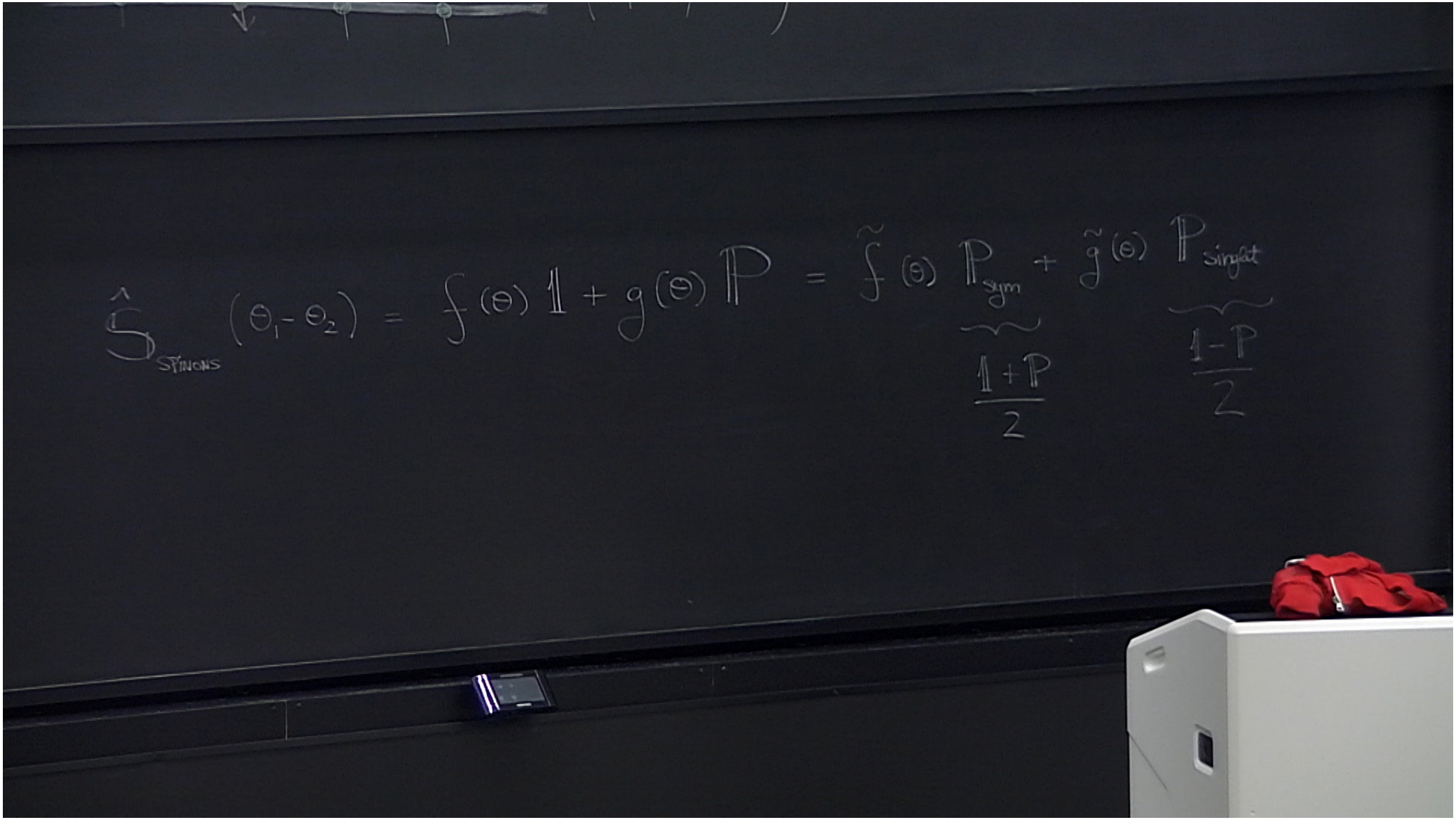


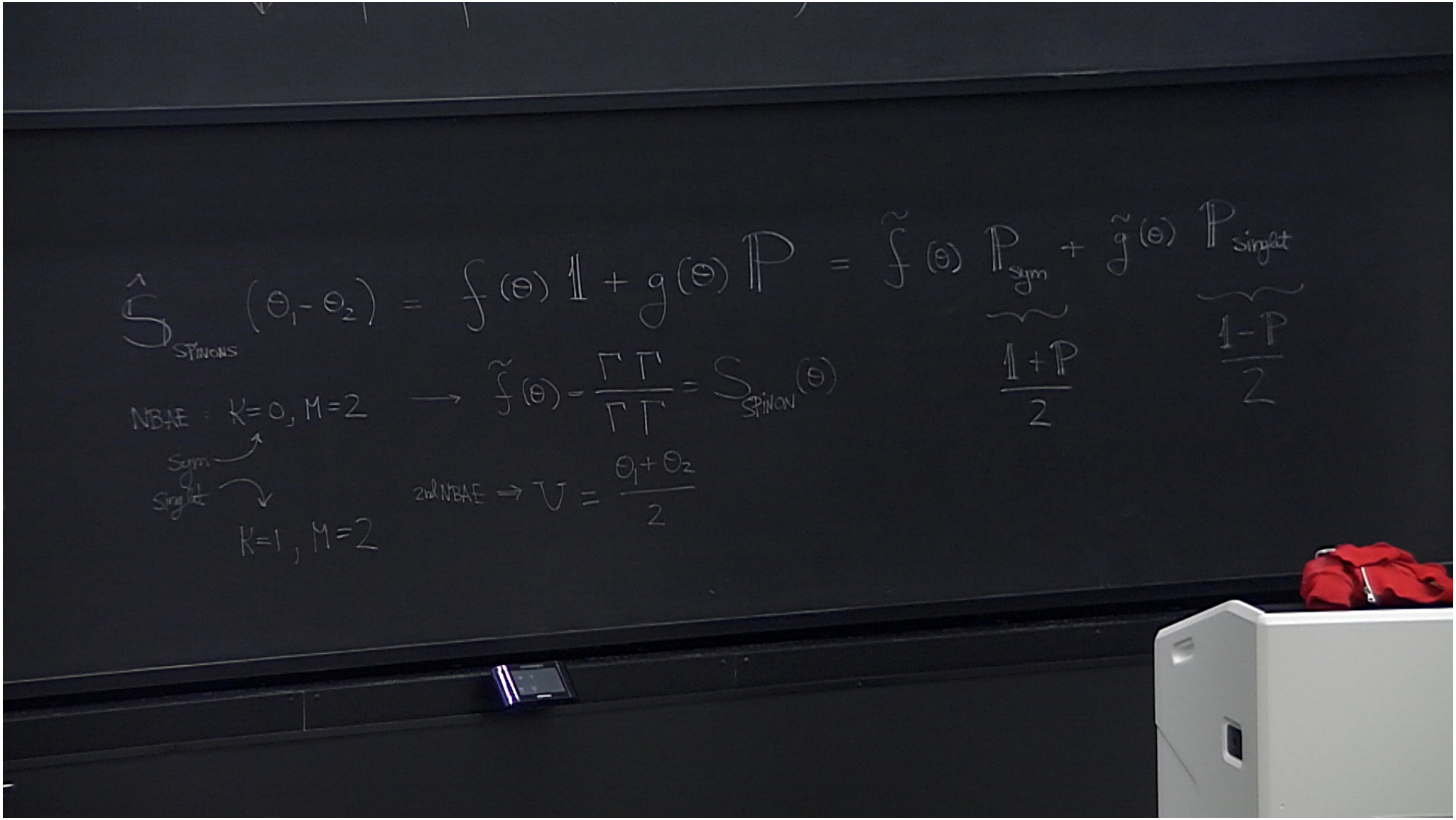
Spin 1 (Sym)

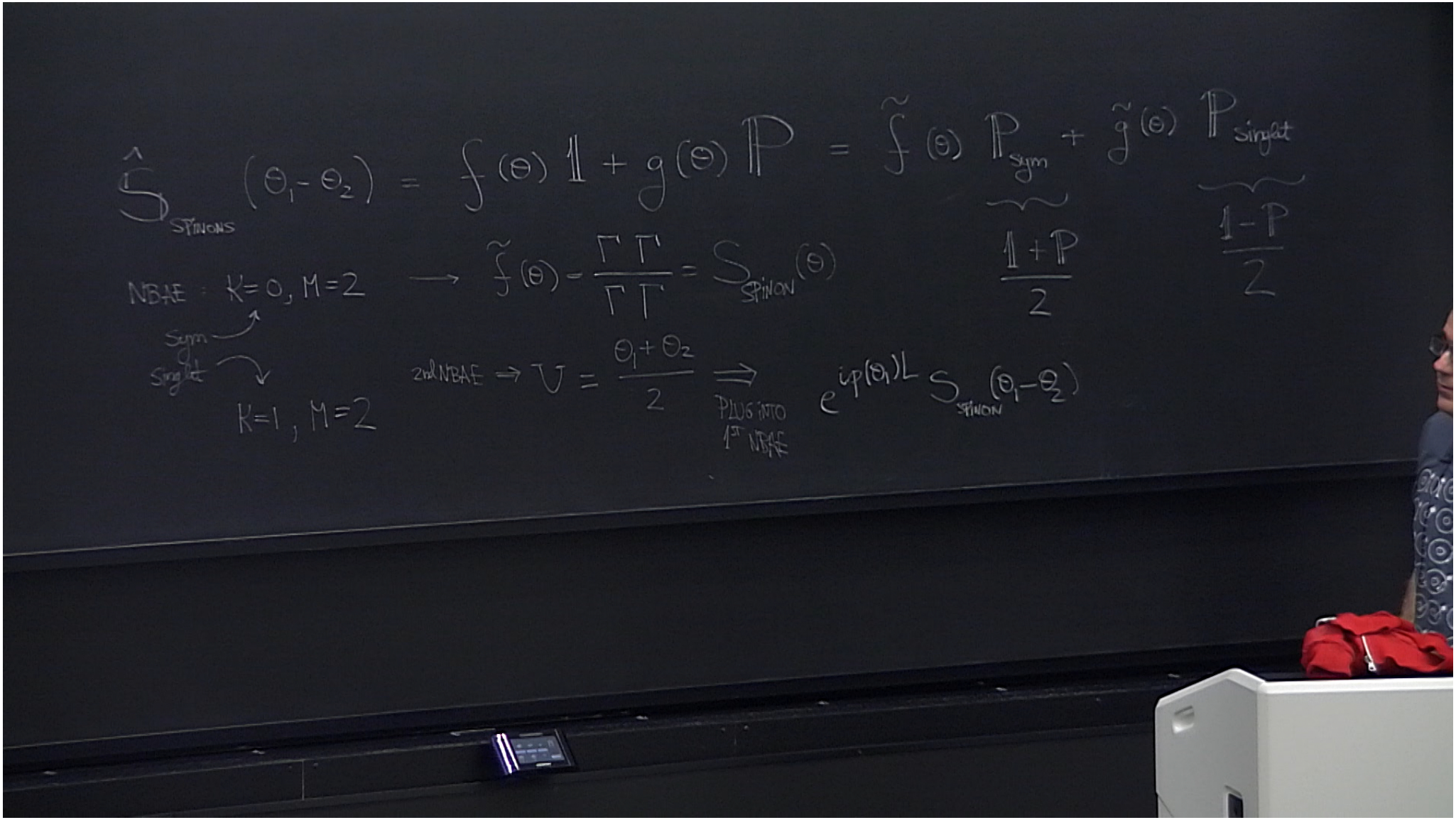


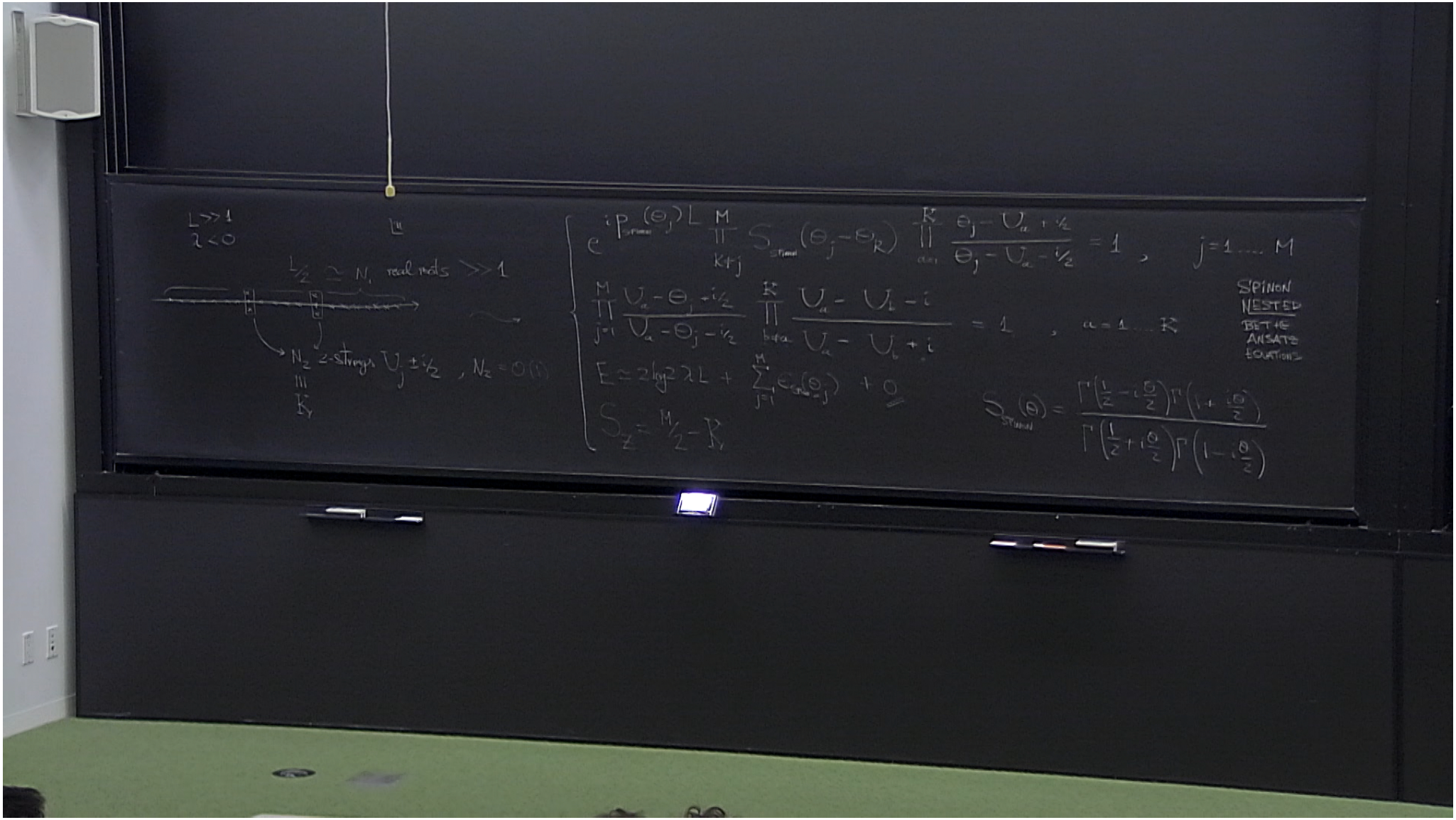
Spin 0 (Singlet)

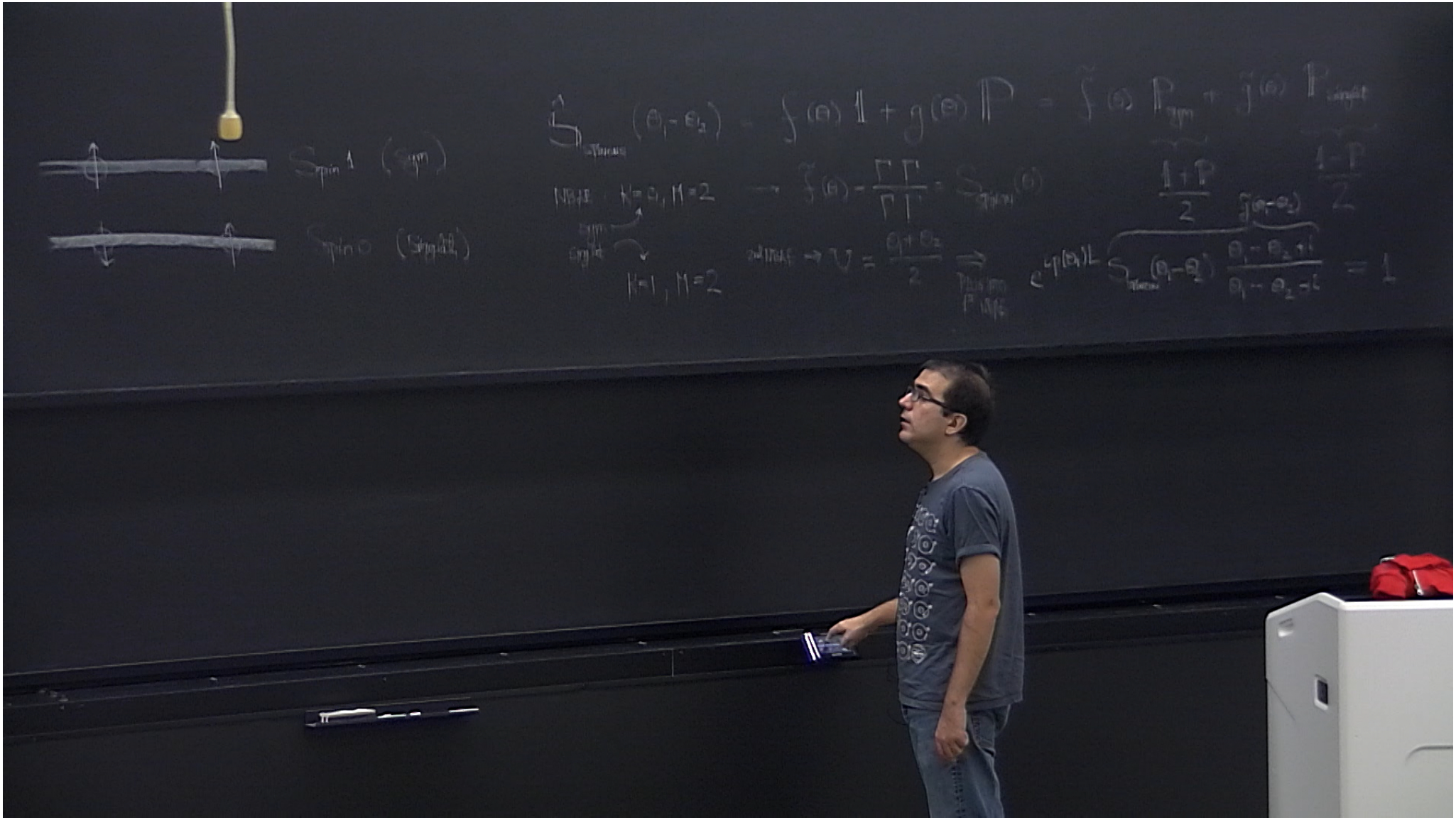
$$\hat{S}_{\text{SPINORS}}(\theta_1 - \theta_2) = f(\theta)\,\mathbb{1} + g(\theta)\,\mathbb{P}$$

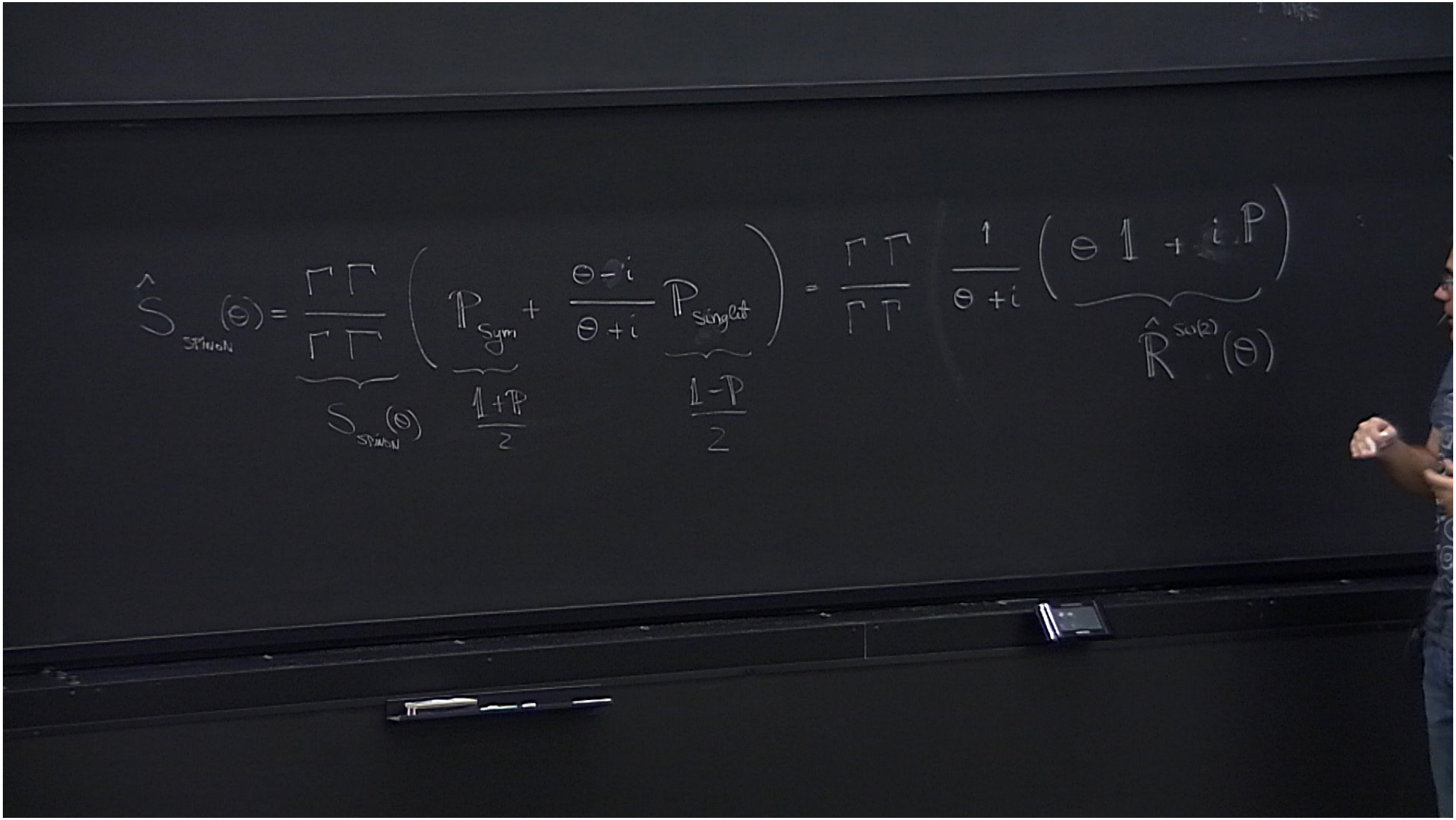

$\hat{S}_{\text{SPINONS}}(\theta_1 - \theta_2) = f(\theta)\,\mathbb{1} + g(\theta)\,\mathbb{P} = \tilde{f}(\theta)\,\underbrace{\mathbb{P}_{\text{sym}}}_{\frac{\mathbb{1}+\mathbb{P}}{2}} + \tilde{g}(\theta)\,\underbrace{\mathbb{P}_{\text{singlet}}}_{\frac{\mathbb{1}-\mathbb{P}}{2}}$

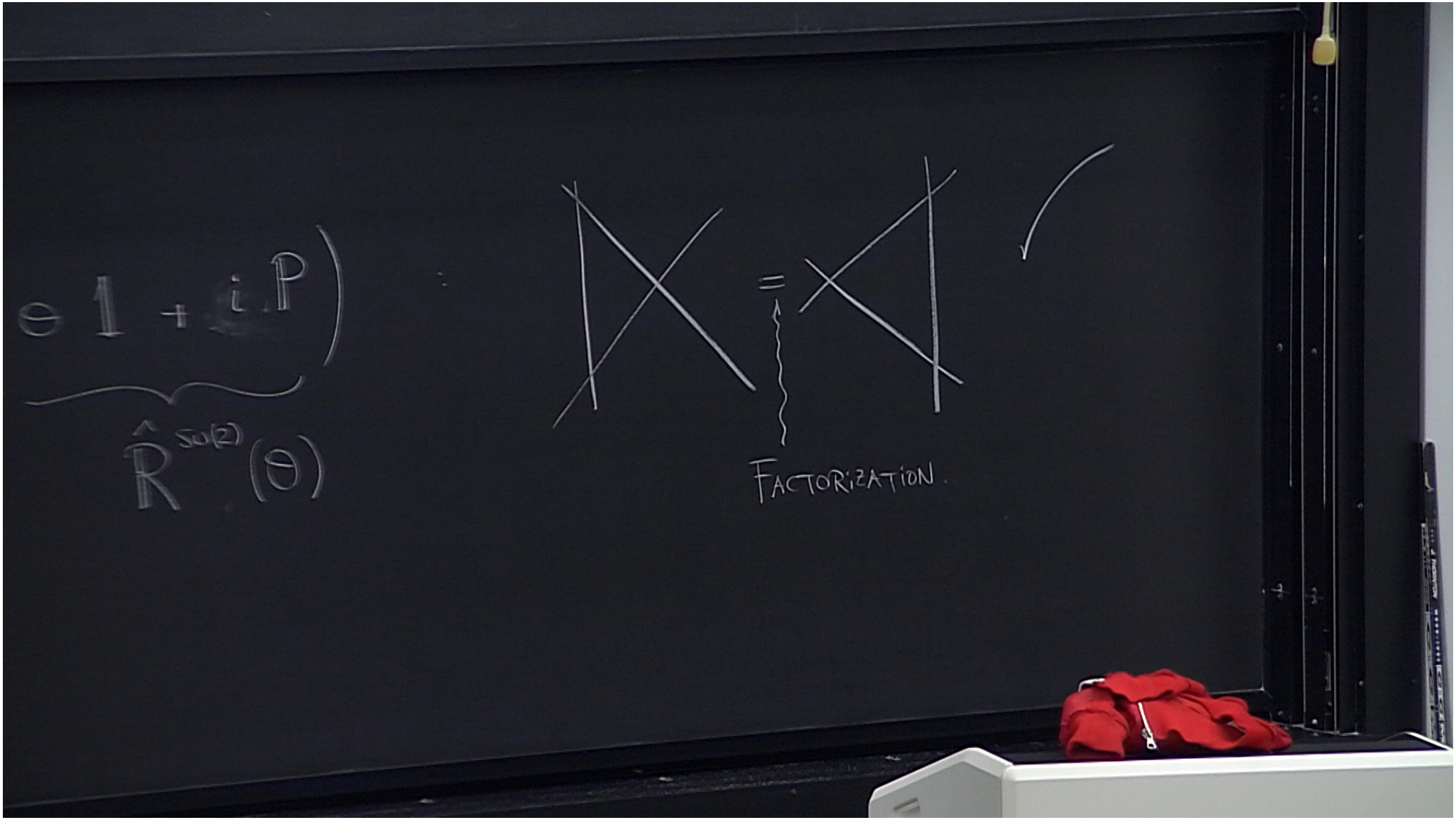

$\hat{S}_{\text{SPINONS}}(\theta_1 - \theta_2) = f(\theta)\,\mathbb{1} + g(\theta)\,\mathbb{P} = \tilde{f}(\theta)\,\mathbb{P}_{\text{sym}} + \tilde{g}(\theta)\,\mathbb{P}_{\text{singlet}}$
$\frac{\mathbb{1}+\mathbb{P}}{2}$
$\frac{\mathbb{1}-\mathbb{P}}{2}$
NBAE : $K=0, M=2 \longrightarrow \tilde{f}(\theta) = \frac{\Gamma\Gamma}{\Gamma\Gamma} = S_{\text{SPINON}}(\theta)$
sym
singlet
$K=1, M=2$
2nd NBAE $\Rightarrow U = \frac{\theta_1+\theta_2}{2}$

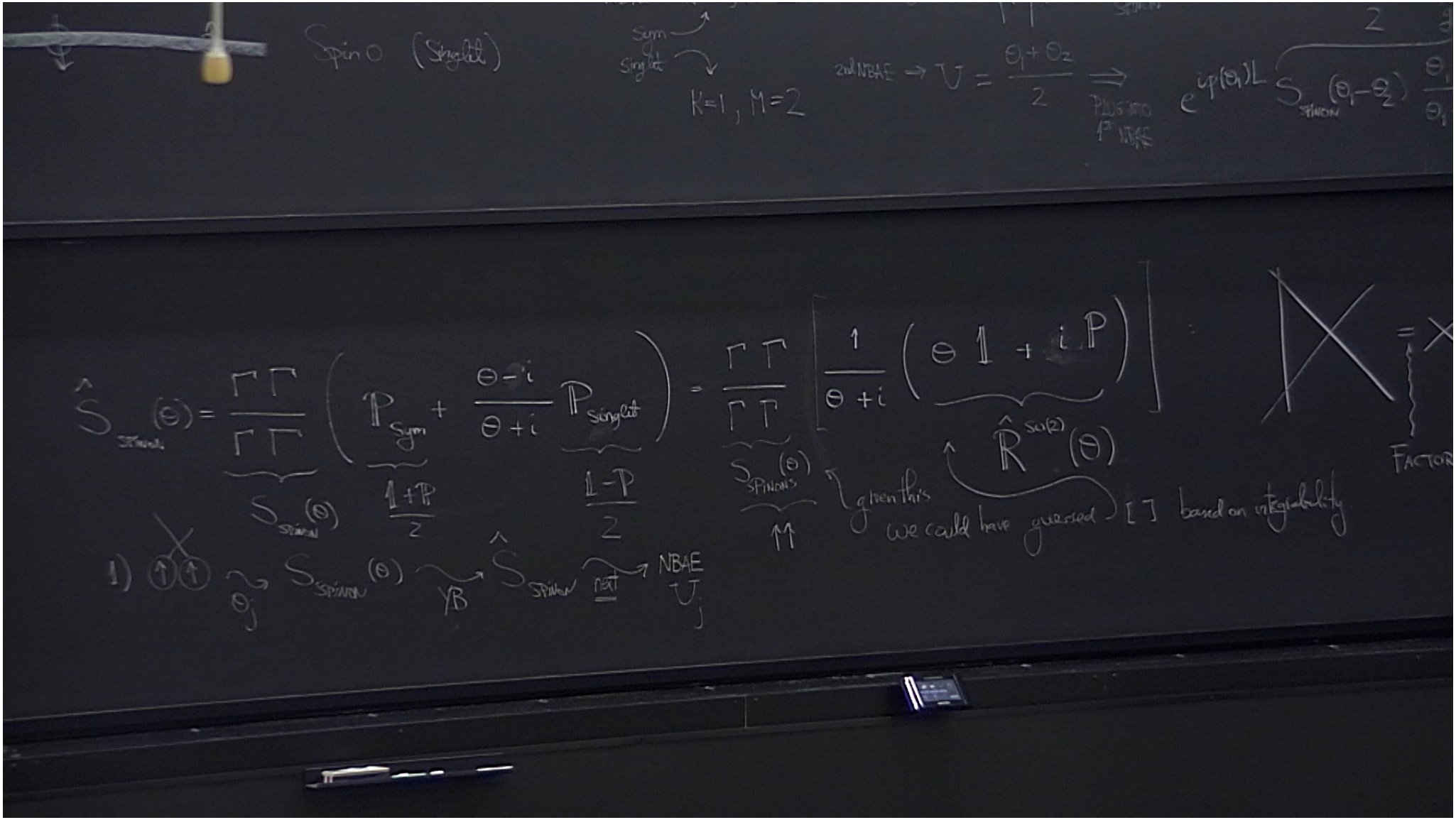

$\hat{S}_{\text{SPINONS}}(\theta_1-\theta_2) = f(\theta)\,\mathbb{1} + g(\theta)\,\mathbb{P} = \tilde{f}(\theta)\,\mathbb{P}_{\text{sym}} + \tilde{g}(\theta)\,\mathbb{P}_{\text{singlet}}$
$\frac{\mathbb{1}+\mathbb{P}}{2}$
$\frac{\mathbb{1}-\mathbb{P}}{2}$
NBAE : $K=0, M=2 \longrightarrow \tilde{f}(\theta) = \frac{\Gamma\Gamma}{\Gamma\Gamma} = S_{\text{SPINON}}(\theta)$
Sym
Singlet
$K=1, M=2$
2nd NBAE $\Rightarrow U = \frac{\theta_1+\theta_2}{2} \Rightarrow$
PLUG INTO 1ST NBAE
$e^{ip(\theta_1)L}\, S_{\text{SPINON}}(\theta_1-\theta_2)$

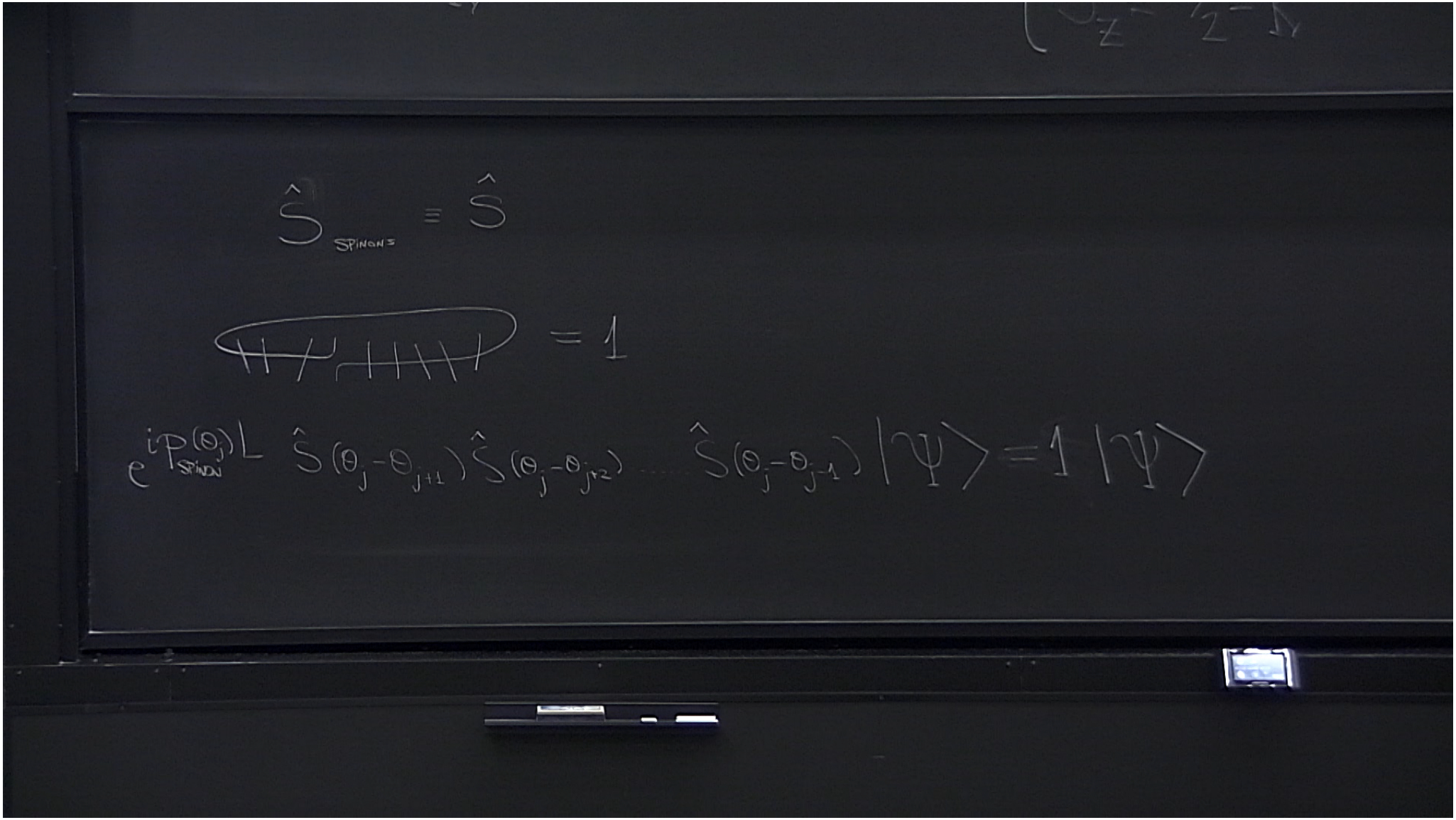

L → ∞
λ < 0
L/2 ≃ N_1 real rods ≫ 1
N_2 2-strings U_j ± i/2 , N_2 = O(1)
K_r
SPINON NESTED BETHE ANSATZ EQUATIONS

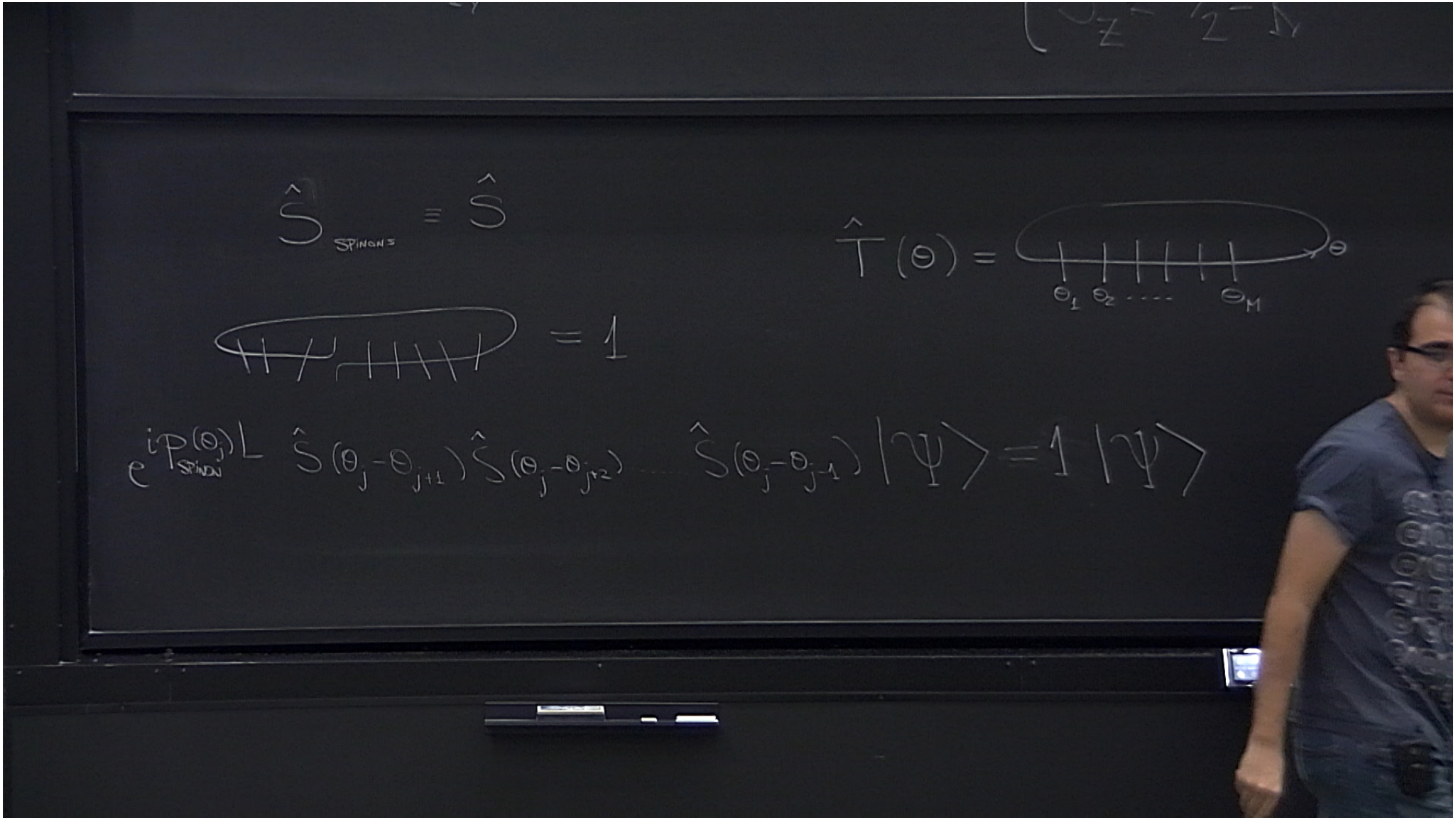


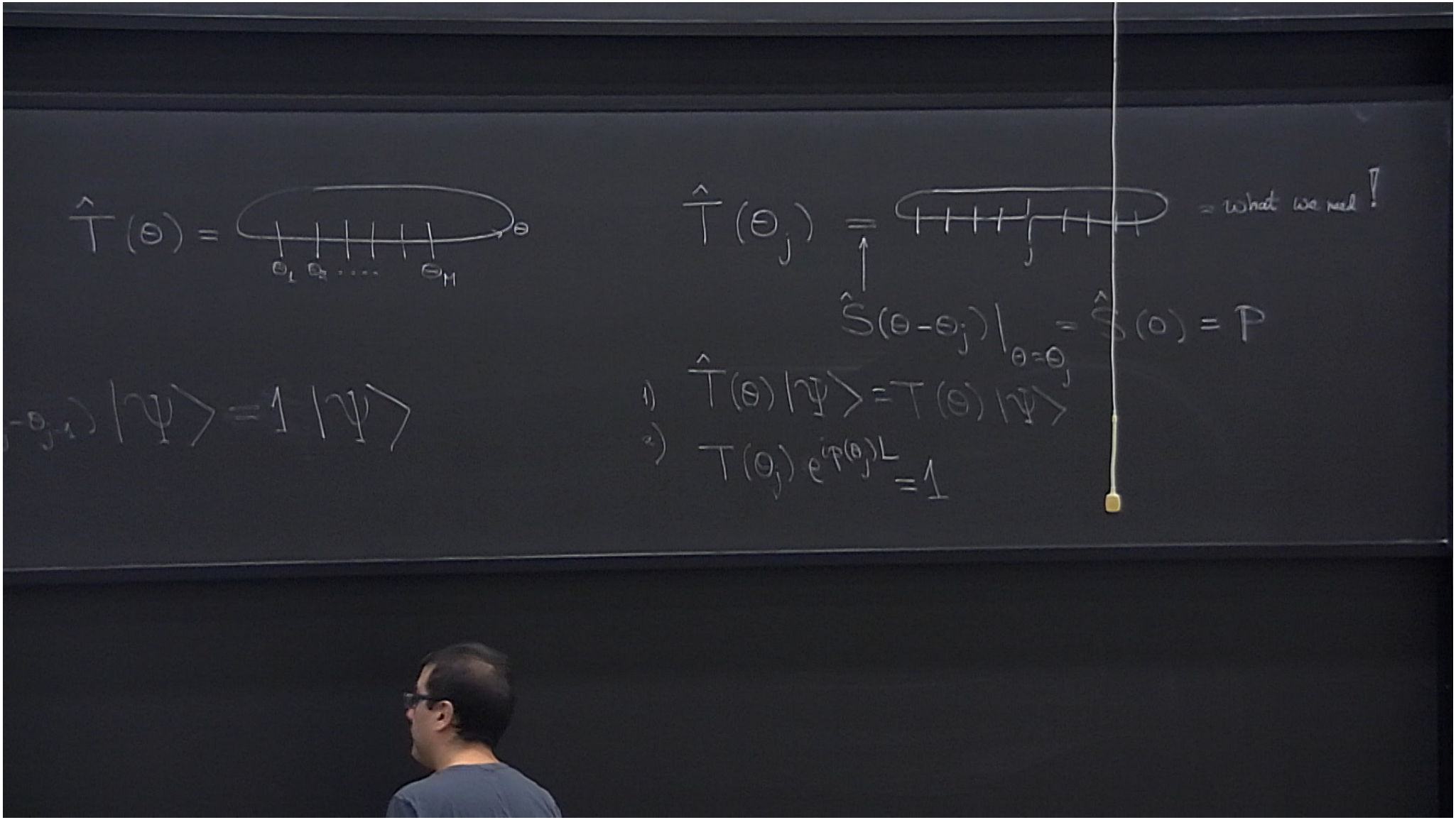

$\hat{S}_{\text{SPIN-SU}}(\theta) = \frac{\Gamma\Gamma}{\Gamma\Gamma}\left(\mathbb{P}_{\text{Sym}} + \frac{\theta - i}{\theta + i}\mathbb{P}_{\text{singlet}}\right) = \frac{\Gamma\Gamma}{\Gamma\Gamma}\frac{1}{\theta + i}\left(\theta\mathbb{1} + i\mathbb{P}\right)$
$S_{\text{SPIN-SU}}(\theta)$
$\frac{\mathbb{1}+\mathbb{P}}{2}$
$\frac{\mathbb{1}-\mathbb{P}}{2}$
$\hat{R}^{SU(2)}(\theta)$


$\hat{R}^{SO(2)}(\theta)$
FACTORIZATION.


Spin 0 (Singlet)
Sym
Singlet
K=1, M=2
2nd NBAE
PLUG INTO 1st NBAE
FACTOR
we could have guessed
based on integrability
given this
NEXT
NBAE
YB


$\hat{S}_{\text{SPINONS}} = \hat{S}$
$= 1$
$e^{iP_{\text{SPINON}}^{(\theta_j)}L}\, \hat{S}(\theta_j - \theta_{j+1})\, \hat{S}(\theta_j - \theta_{j+2}) \cdots \hat{S}(\theta_j - \theta_{j-1})\, |\Psi\rangle = 1\, |\Psi\rangle$


$\hat{S}_{\text{SPINONS}} \equiv \hat{S}$
$\hat{T}(\theta) =$
θ_1 θ_2 .... θ_M θ
$= 1$
$e^{iP^{(\theta_j)}_{\text{SPINON}} L} \hat{S}(\theta_j - \theta_{j+1}) \hat{S}(\theta_j - \theta_{j+2}) \cdots \hat{S}(\theta_j - \theta_{j-1}) |\Psi\rangle = 1 |\Psi\rangle$

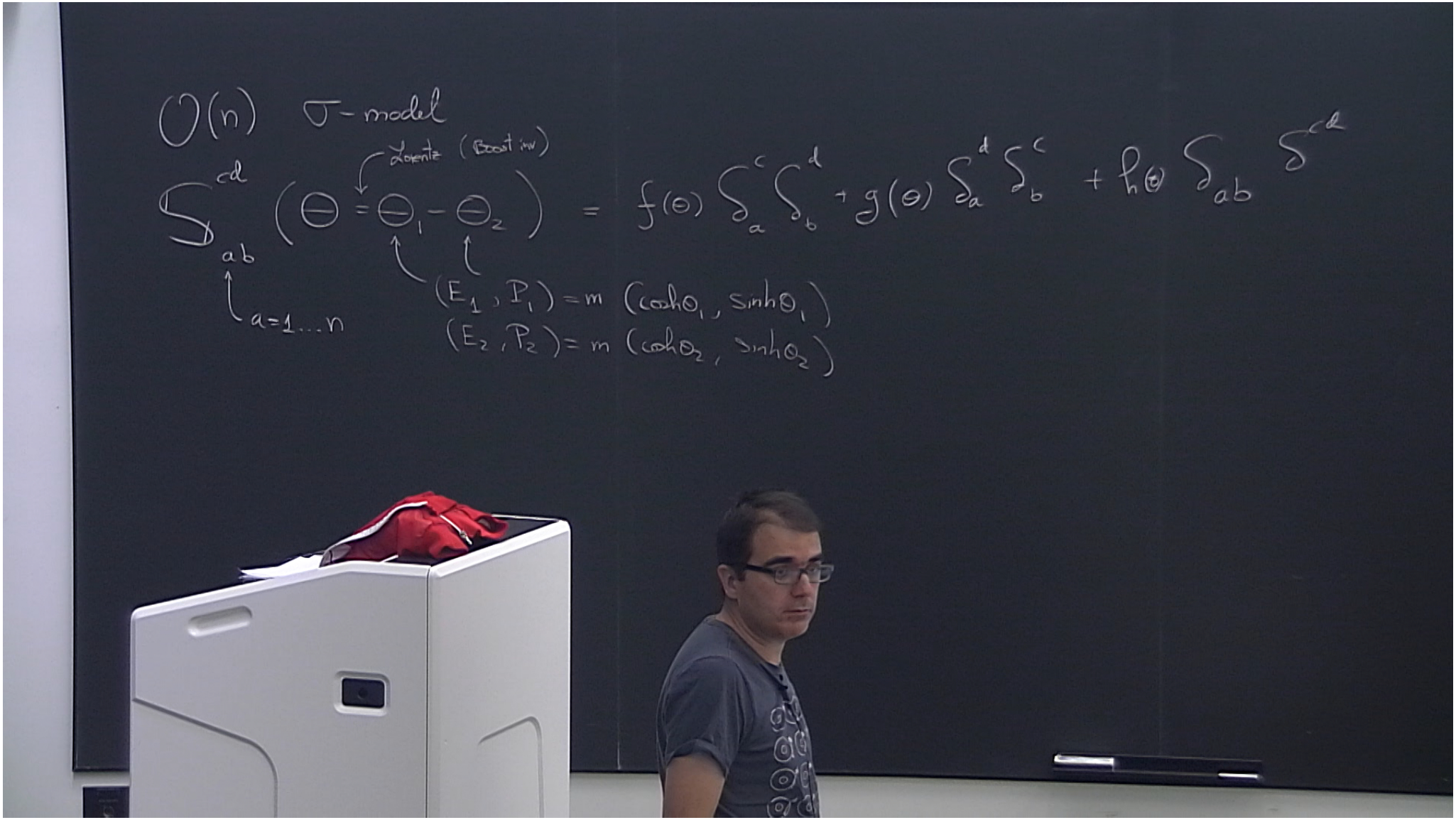

$\hat{T}(\theta) =$
θ
θ_1 θ_2 ..... θ_M
$\hat{T}(\theta_j) =$
j
= what we need !
$\hat{S}(\theta - \theta_j)\big|_{\theta=\theta_j} = \hat{S}(0) = P$
$-\theta_{j-1}) |\psi\rangle = 1 |\psi\rangle$
1) $\hat{T}(\theta)|\psi\rangle = T(\theta)|\psi\rangle$
2) $T(\theta_j)\, e^{i\varphi(\theta_j)L} = 1$

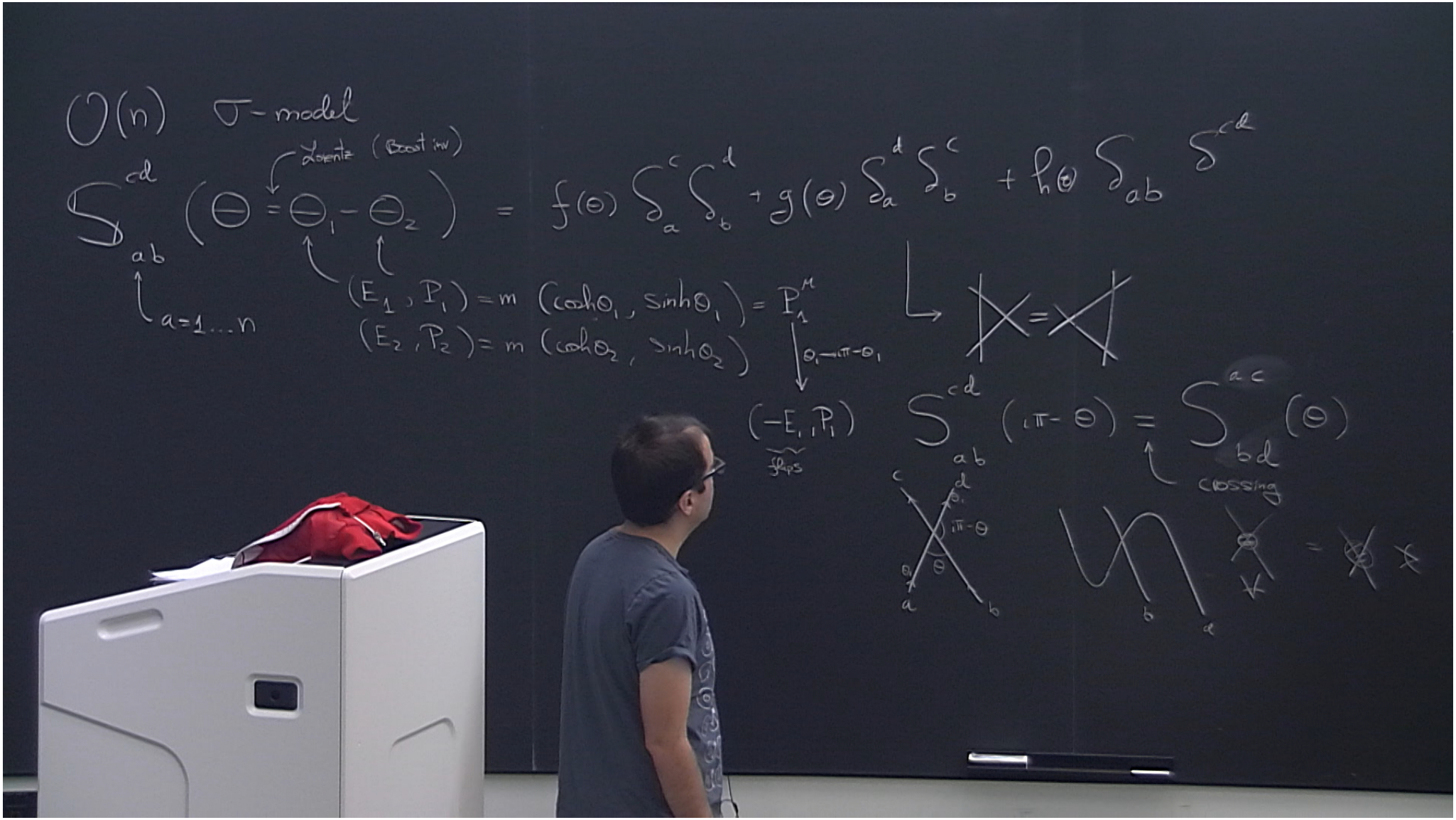
$$T(\theta_j)\, e^{ip(\theta_j)L} = 1$$

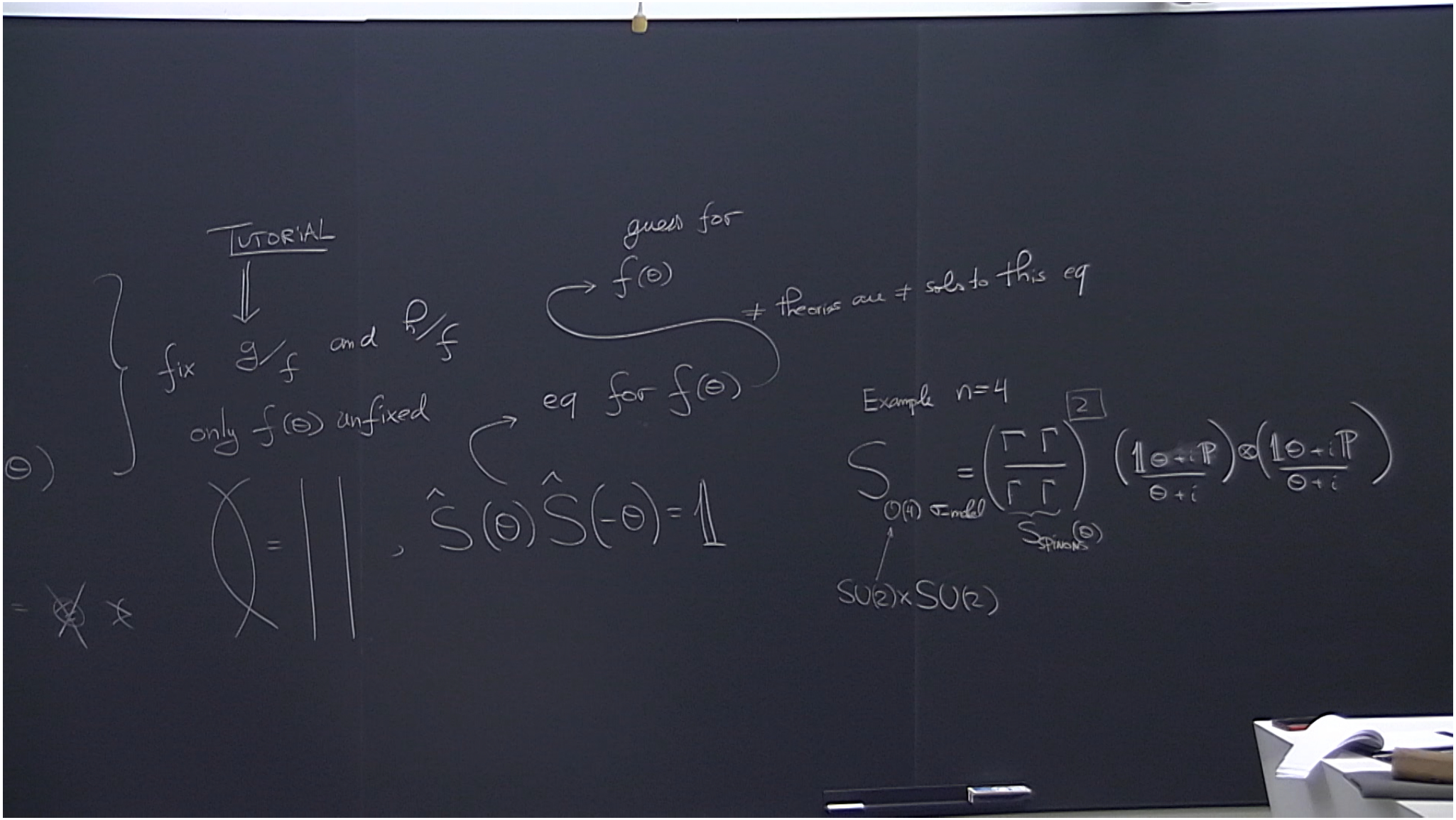
$$\hat{T}(\theta) = \prod_{\text{SPINONS}} S(\theta-\theta_j) \left[\prod_{k=1}^{K} \frac{\theta - v_k - i/2}{\theta - v_k + i/2} + \prod_{k=1}^{M} \frac{\theta-\theta_k}{\theta-\theta_k+i} \prod_{k=1}^{K} \frac{\theta - v_k + i/2}{\theta - v_k + i/2} \right]$$

$T(\theta_j)\, e^{ip(\theta_j)L} = 1 \longrightarrow 1^{st}$ NBAE ✓

$T(\theta)$ is pol $\longrightarrow 2^{nd}$ NBAE


O(n) σ-model
Lorentz (Boost inv)
$\Sigma^{cd}_{ab}\left(\theta = \theta_1 - \theta_2\right) = f(\theta)\,\delta^c_a\,\delta^d_b + g(\theta)\,\delta^d_a\,\delta^c_b + h(\theta)\,\delta_{ab}\,\delta^{cd}$
$a = 1 \ldots n$
$(E_1, P_1) = m\,(\cosh\theta_1, \sinh\theta_1)$
$(E_2, P_2) = m\,(\cosh\theta_2, \sinh\theta_2)$


O(n) σ-model
Lorentz (Boost inv)
a=1...n
(E1, P1) = m (coshθ1, sinhθ1) = P1^μ
(E2, P2) = m (coshθ2, sinhθ2)
θ1 → iπ - θ1
(-E1, P1)
flips
crossing


TUTORIAL
fix g/f and h/f
only f(θ) unfixed
guess for f(θ)
eq for f(θ)
≠ theories are ≠ sols to this eq
Ŝ(θ)Ŝ(−θ) = 𝟙
Example n=4
S = (Γ Γ / Γ Γ)^2 ((1θ+iP)/(θ+i)) ⊗ ((1θ+iP)/(θ+i))
O(4) σ-model
S_SPINONS(θ)
SU(2)×SU(2)