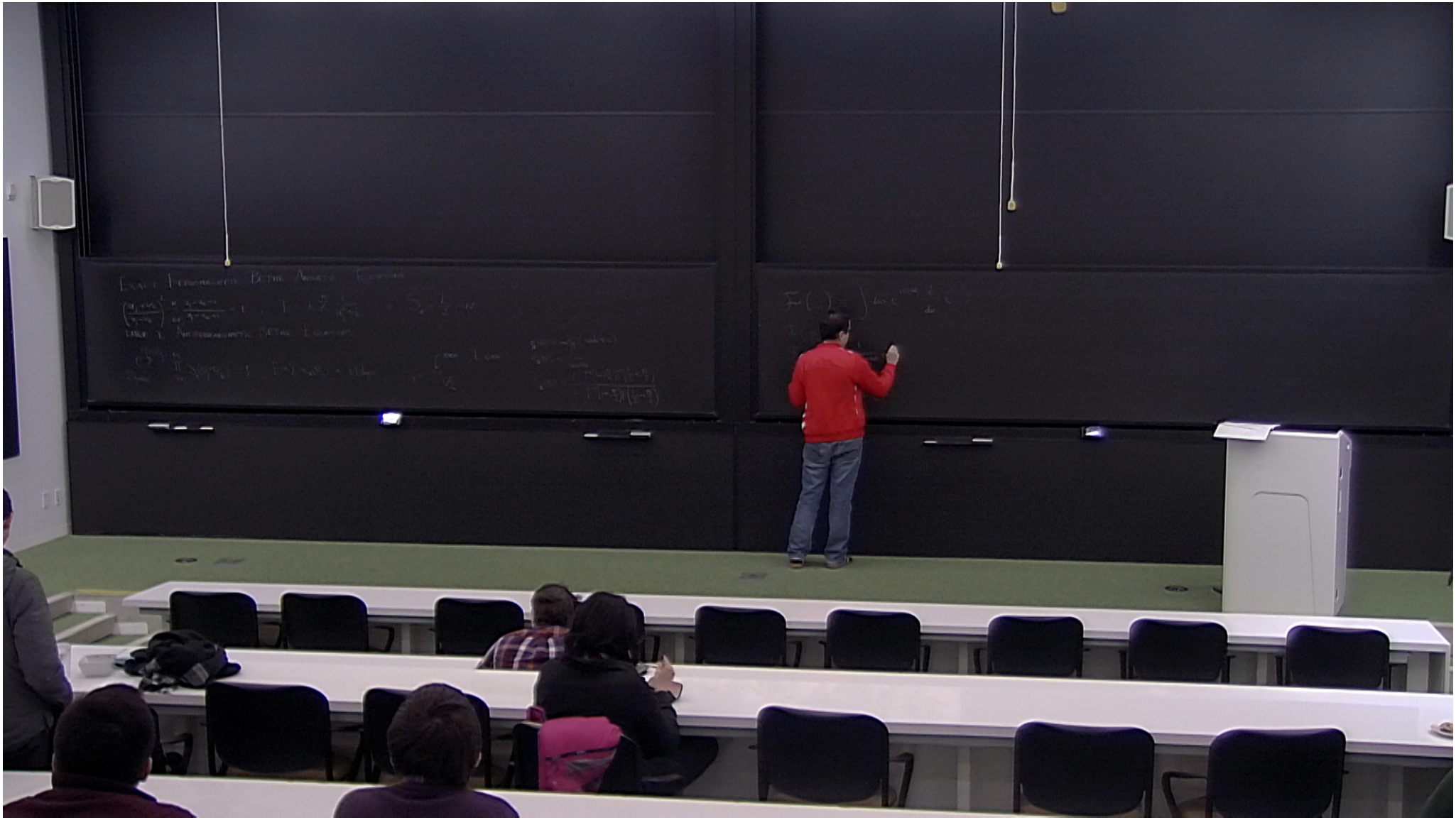
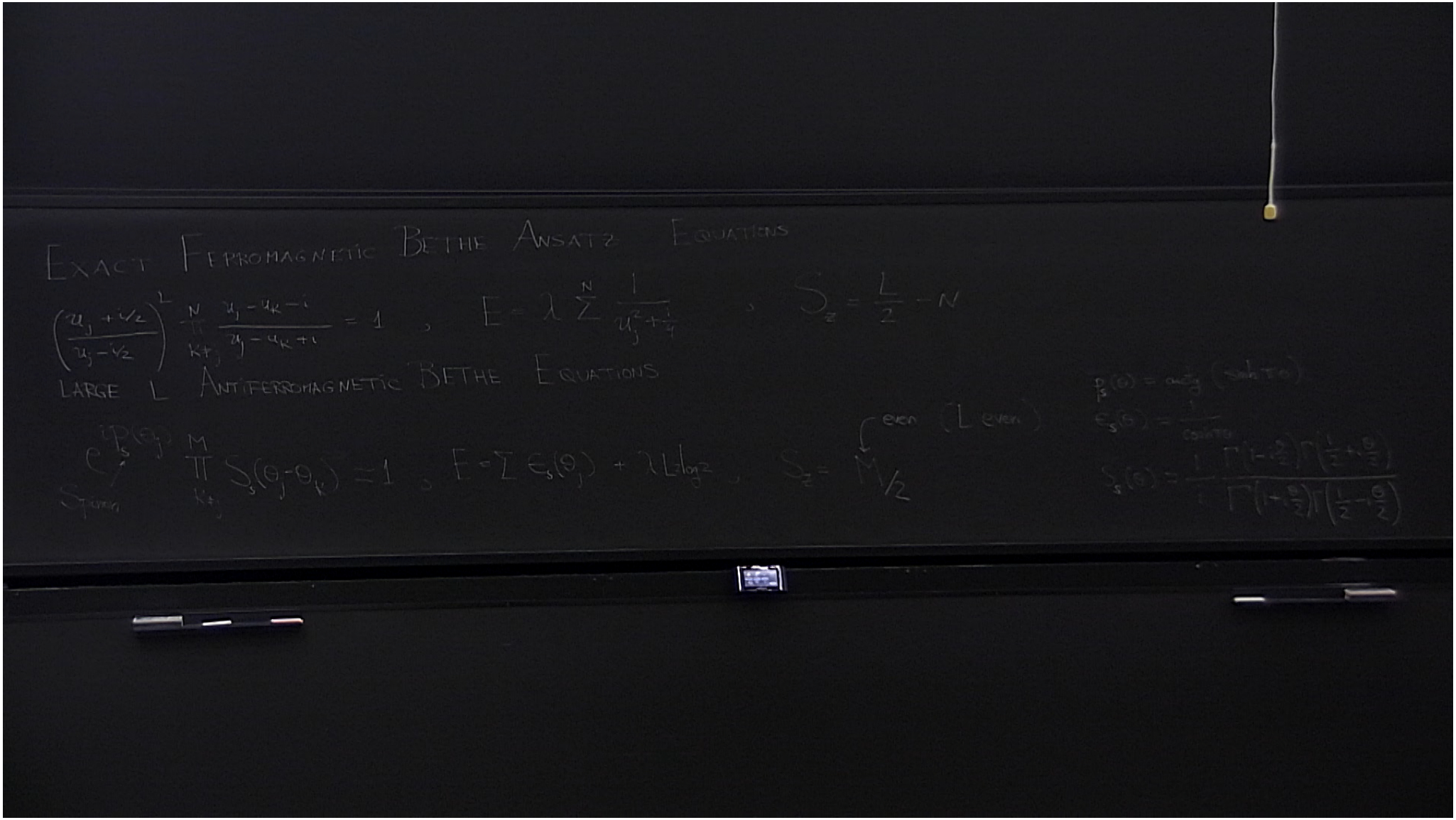
Title: PSI 2016/2017 Introduction to Quantum Integrable Systems - Lecture 5

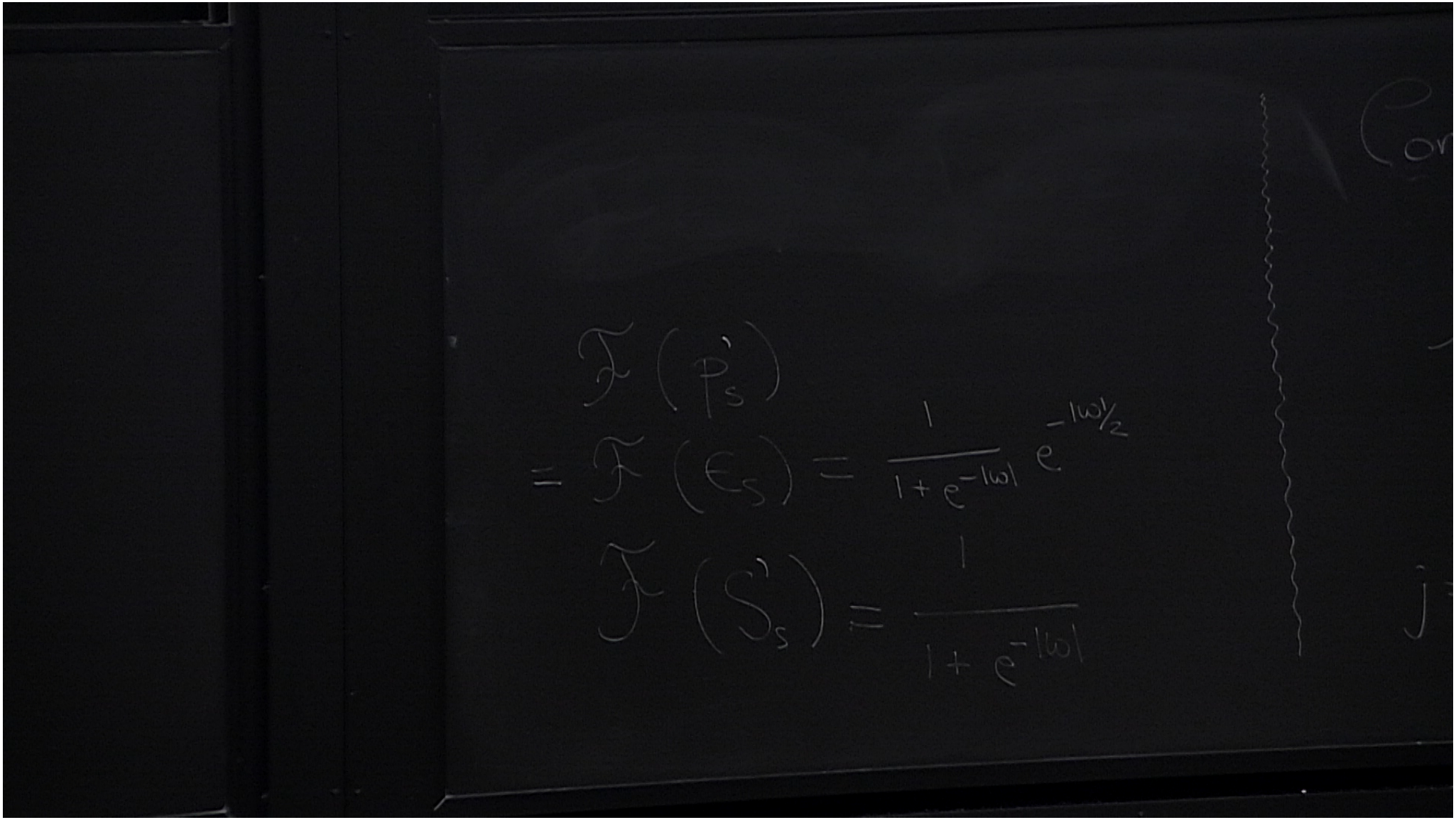
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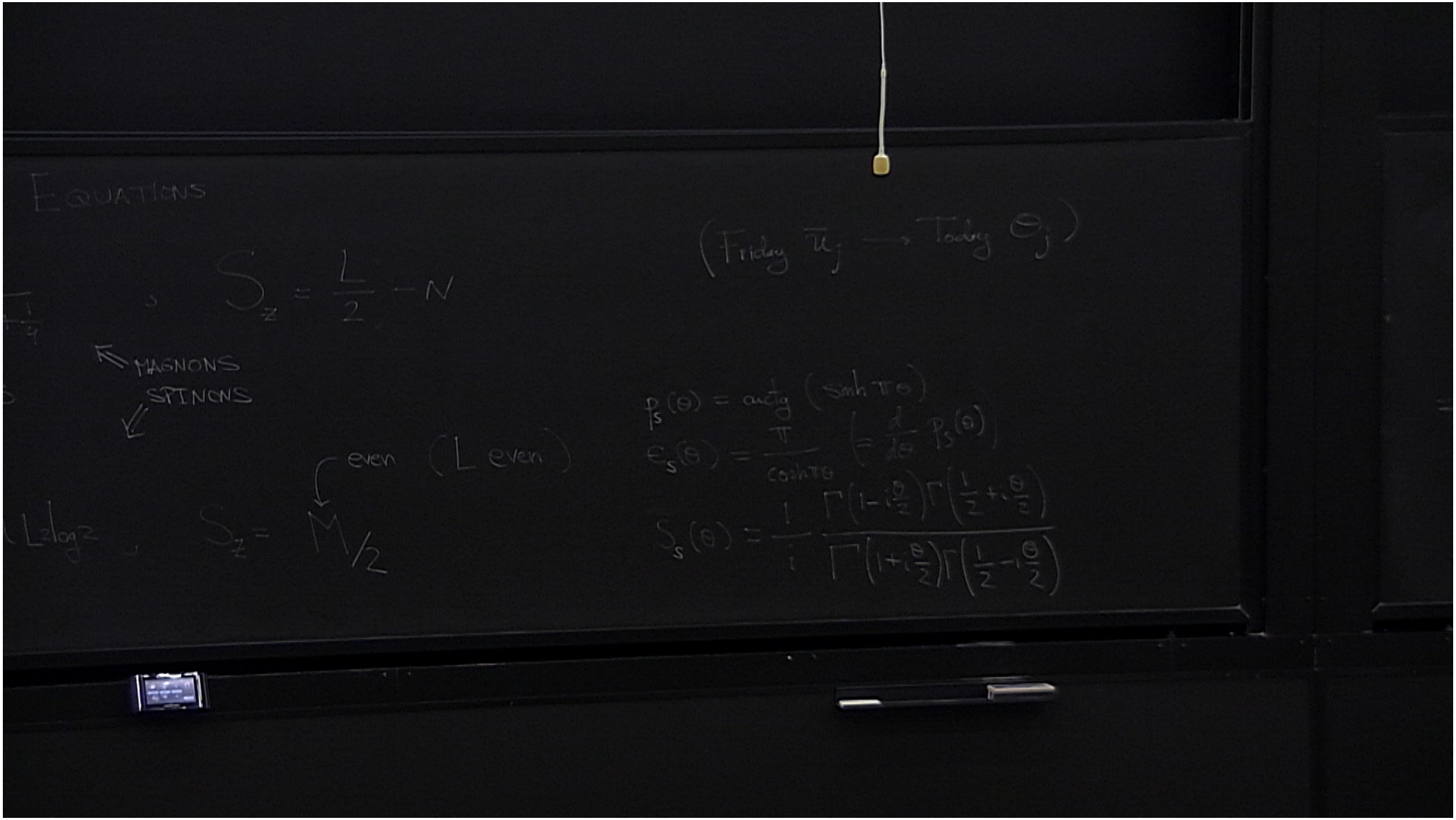
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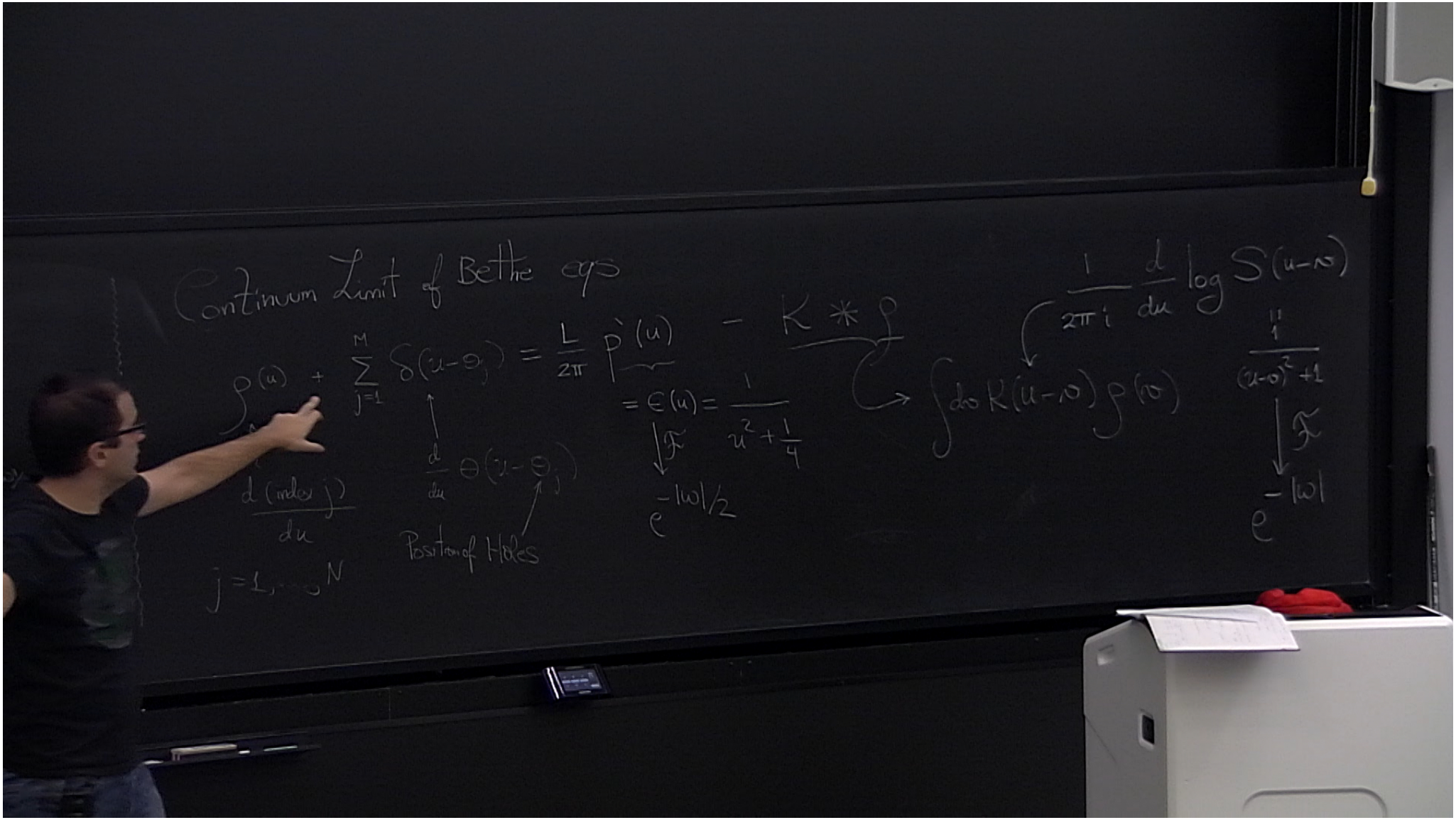
Abstract:

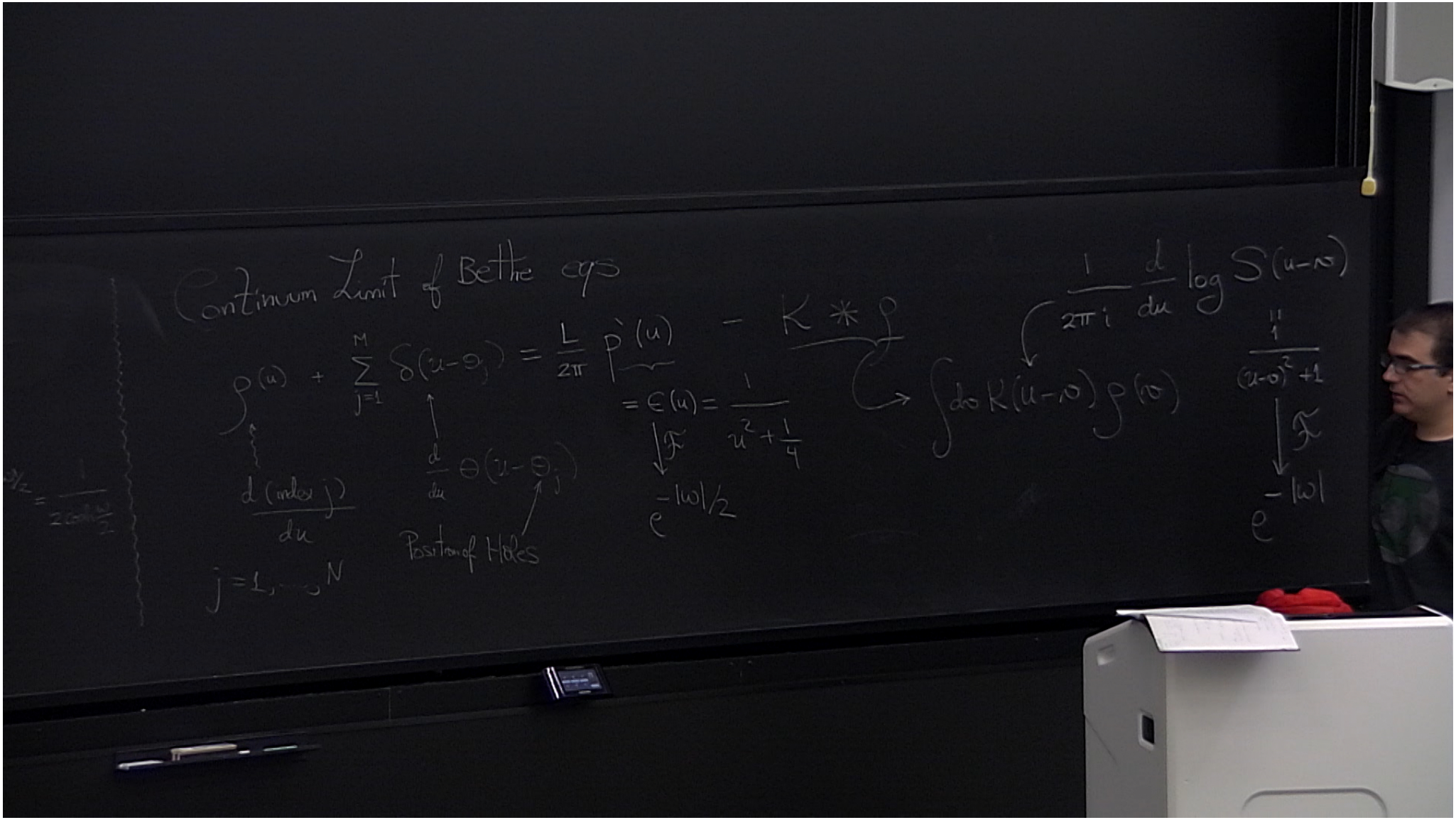


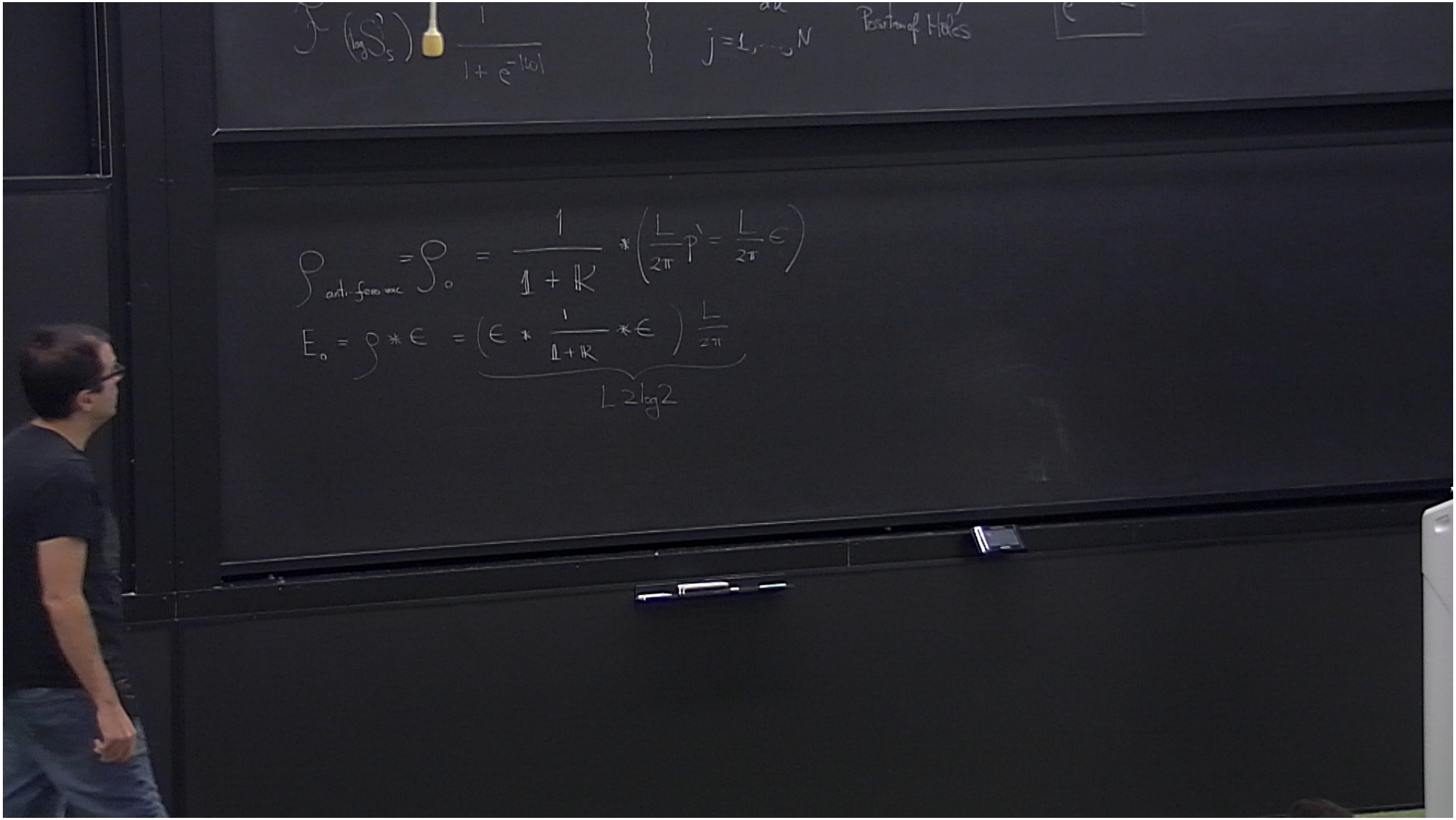

EXACT FERROMAGNETIC BETHE ANSATZ EQUATIONS
LARGE L ANTIFERROMAGNETIC BETHE EQUATIONS

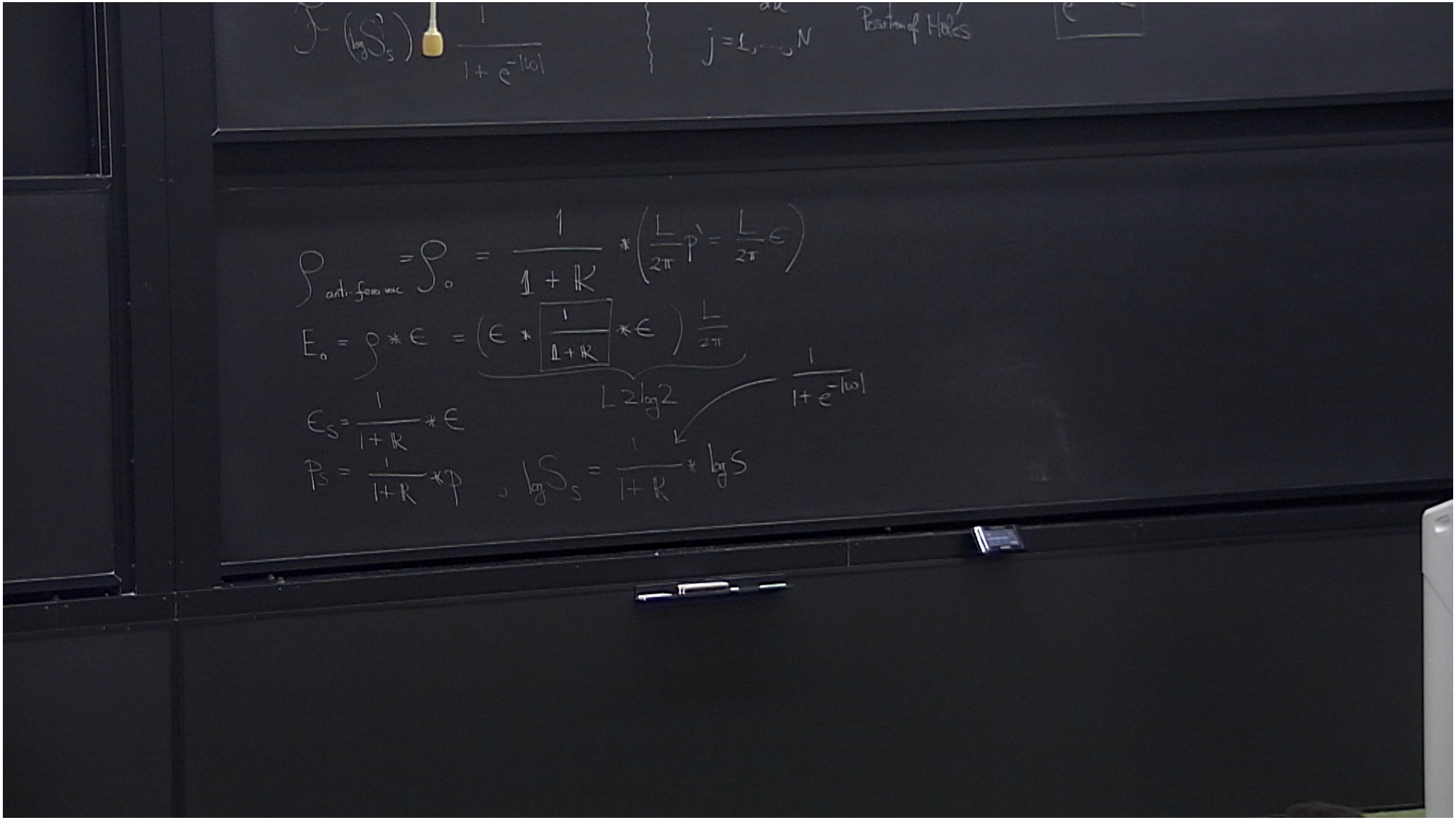

$\mathcal{F}(P_s')$
$= \mathcal{F}(\epsilon_s) = \frac{1}{1+e^{-|\omega|}} e^{-|\omega|/2}$
$\mathcal{F}(S_s') = \frac{1}{1+e^{-|\omega|}}$

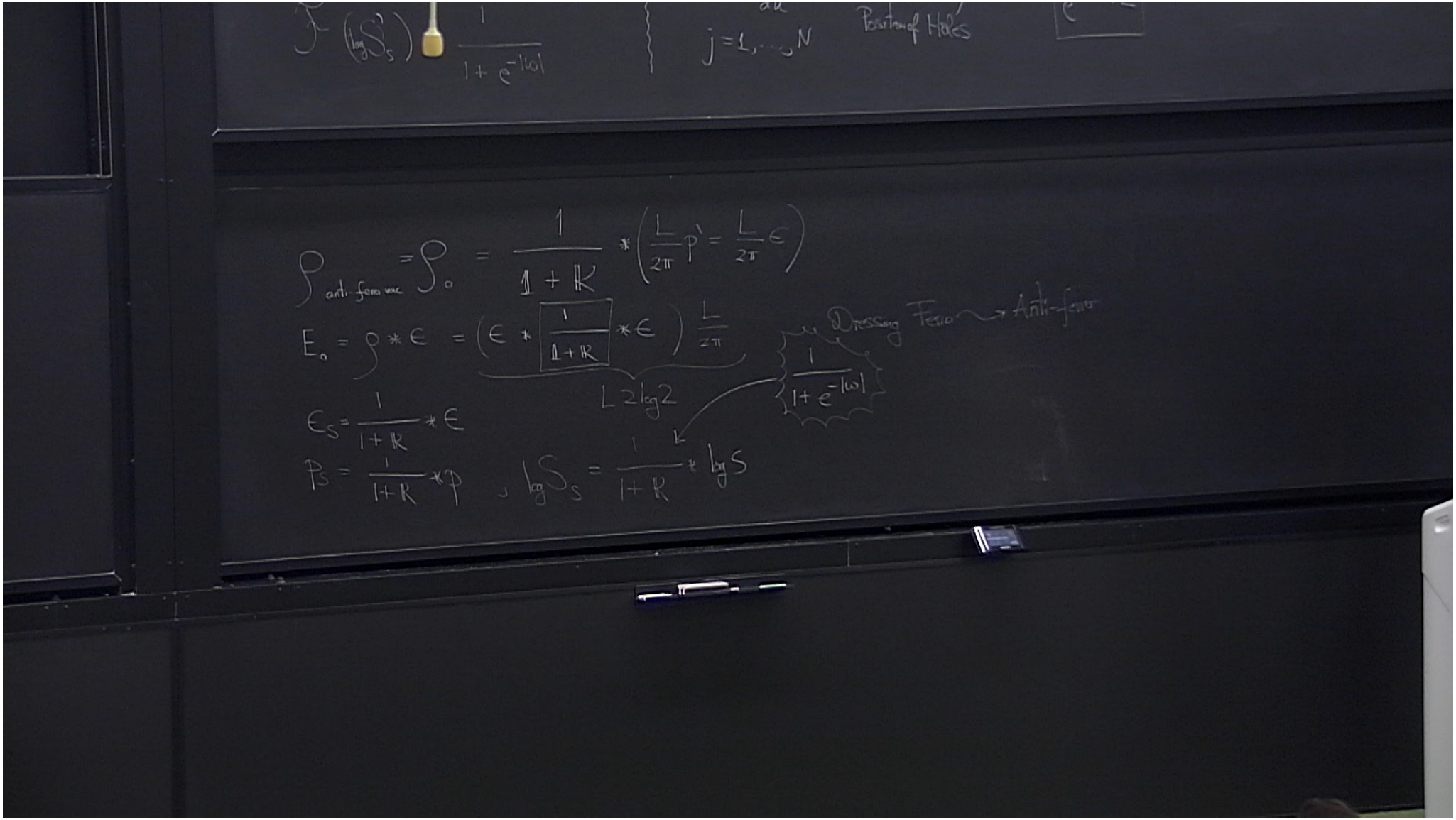

EQUATIONS
$S_z = \frac{L}{2} - N$
(Friday $\bar{u}_j$ ⟶ Today θ_j)
MAGNONS
SPINONS
even (L even)
$S_z = M/2$
$\rho_s(\theta) = \frac{d}{d\theta}\left(\sinh^{-1}\frac{\theta}{2}\right)$
$\epsilon_s(\theta) = \frac{\pi}{\cosh\frac{\theta}{2}}$ $\left(= \frac{d}{d\theta}\rho_s(\theta)\right)$
$S_s(\theta) = \frac{1}{i}\frac{\Gamma\left(1-i\frac{\theta}{2}\right)\Gamma\left(\frac{1}{2}+i\frac{\theta}{2}\right)}{\Gamma\left(1+i\frac{\theta}{2}\right)\Gamma\left(\frac{1}{2}-i\frac{\theta}{2}\right)}$

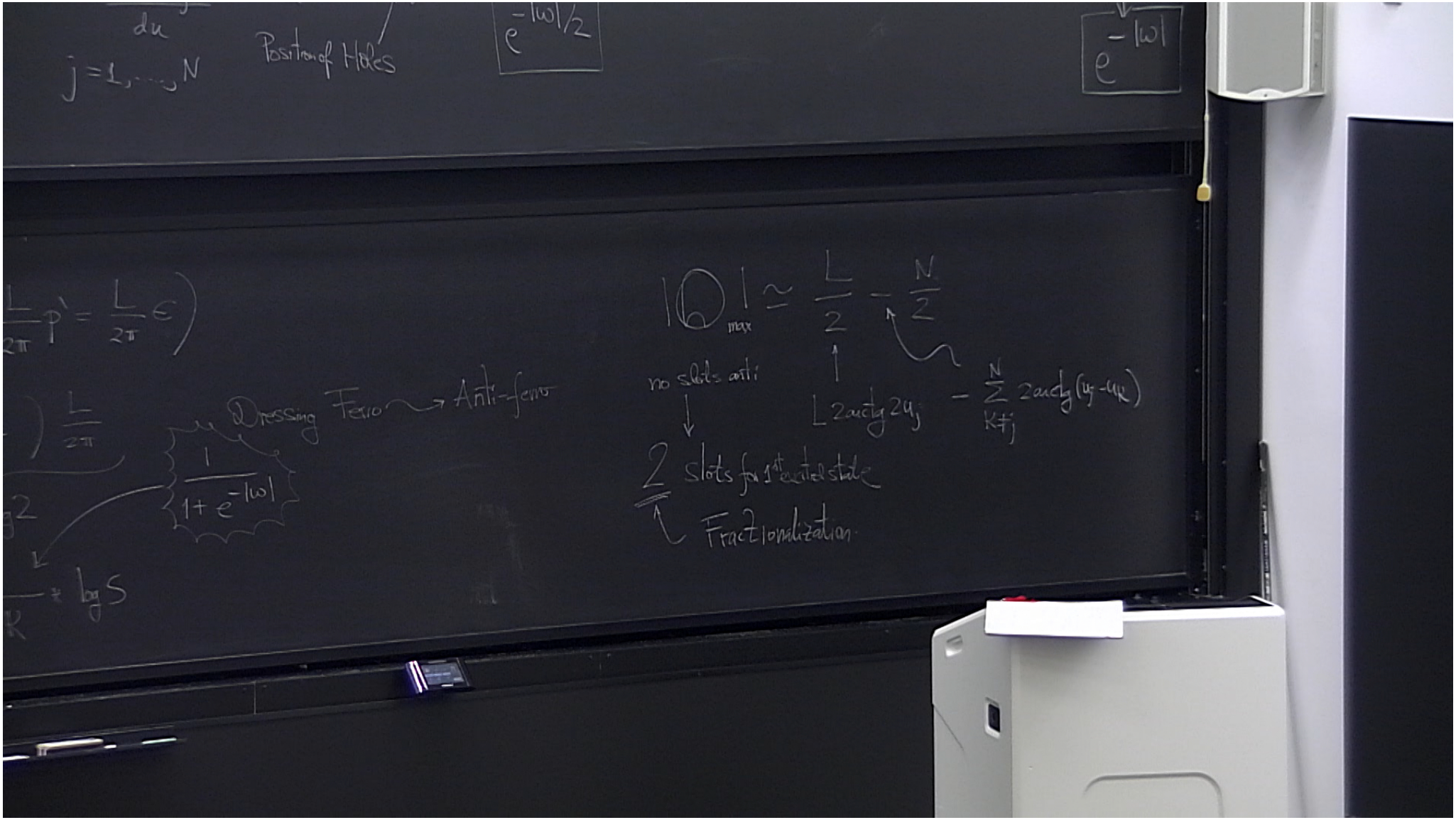

Continuum Limit of Bethe eqs
$\rho(u) + \sum_{j=1}^{M} \delta(u-\theta_j) = \frac{L}{2\pi} p'(u) - K * \rho$
$\frac{d(\text{index } j)}{du}$
$j = 1, \dots, N$
$\frac{d}{du}\Theta(u-\theta_j)$
Position of Holes
$= \epsilon(u) = \frac{1}{u^2+\frac{1}{4}}$
$e^{-|\omega|/2}$
$\frac{1}{2\pi i}\frac{d}{du}\log S(u-v)$
$\int dv\, K(u-v)\rho(v)$
$\frac{1}{(u-v)^2+1}$
$e^{-|\omega|}$

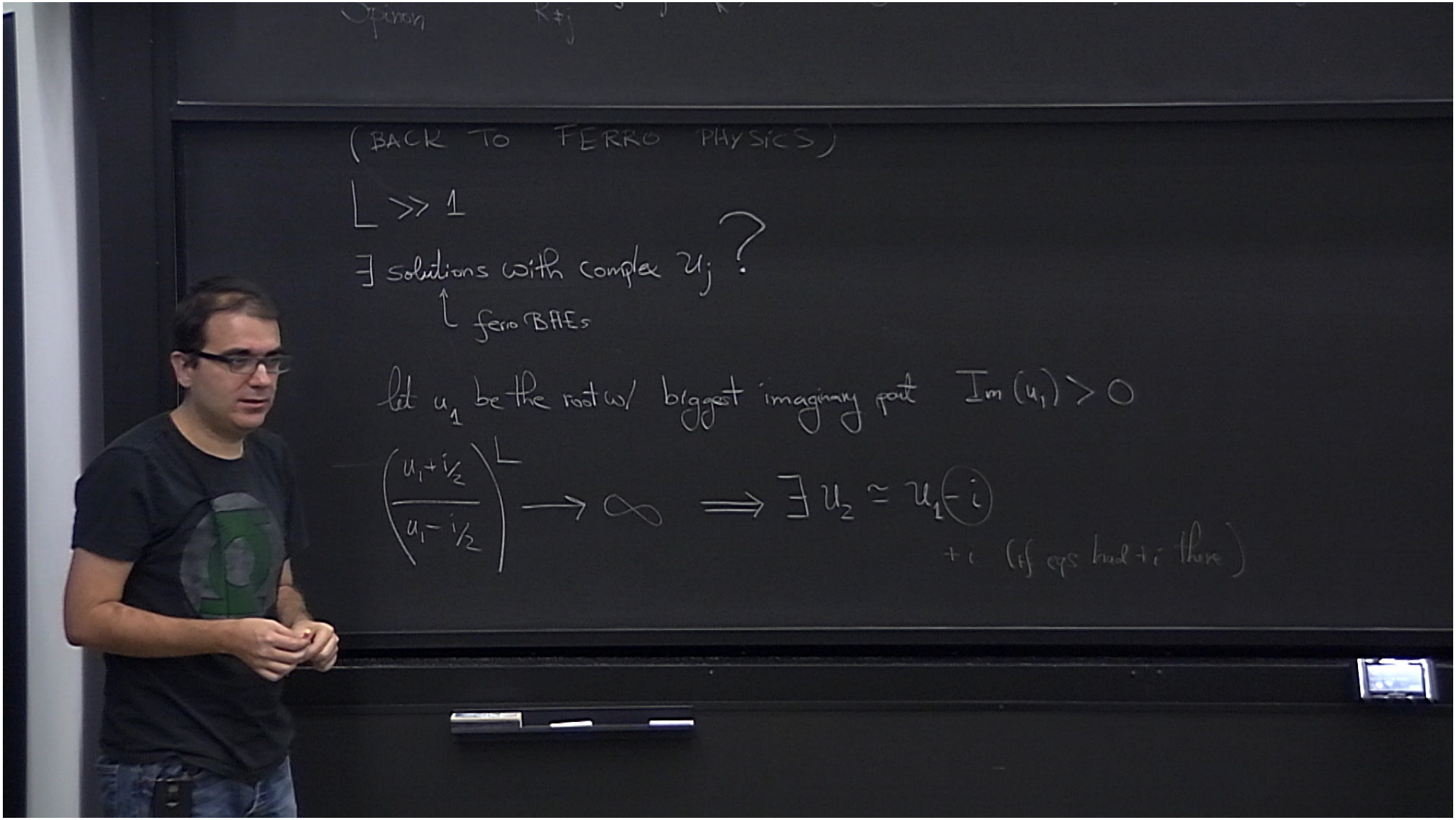

Continuum Limit of Bethe eqs
$\rho(u) + \sum_{j=1}^{M} \delta(u-\theta_j) = \frac{L}{2\pi} p'(u) - K * \rho$
$p'(u) = \epsilon(u) = \frac{1}{u^2+\frac{1}{4}}$
$\mathcal{F}$
$e^{-|\omega|/2}$
$\frac{d(\text{index } j)}{du}$
$\frac{d}{du}\theta(u-\theta_j)$
Position of Holes
$j = 1, \dots, N$
$\int du\, K(u-v)\rho(v)$
$\frac{1}{2\pi i}\frac{d}{du}\log S(u-v)$
$\frac{1}{(u-v)^2+1}$
$\mathcal{F}$
$e^{-|\omega|}$

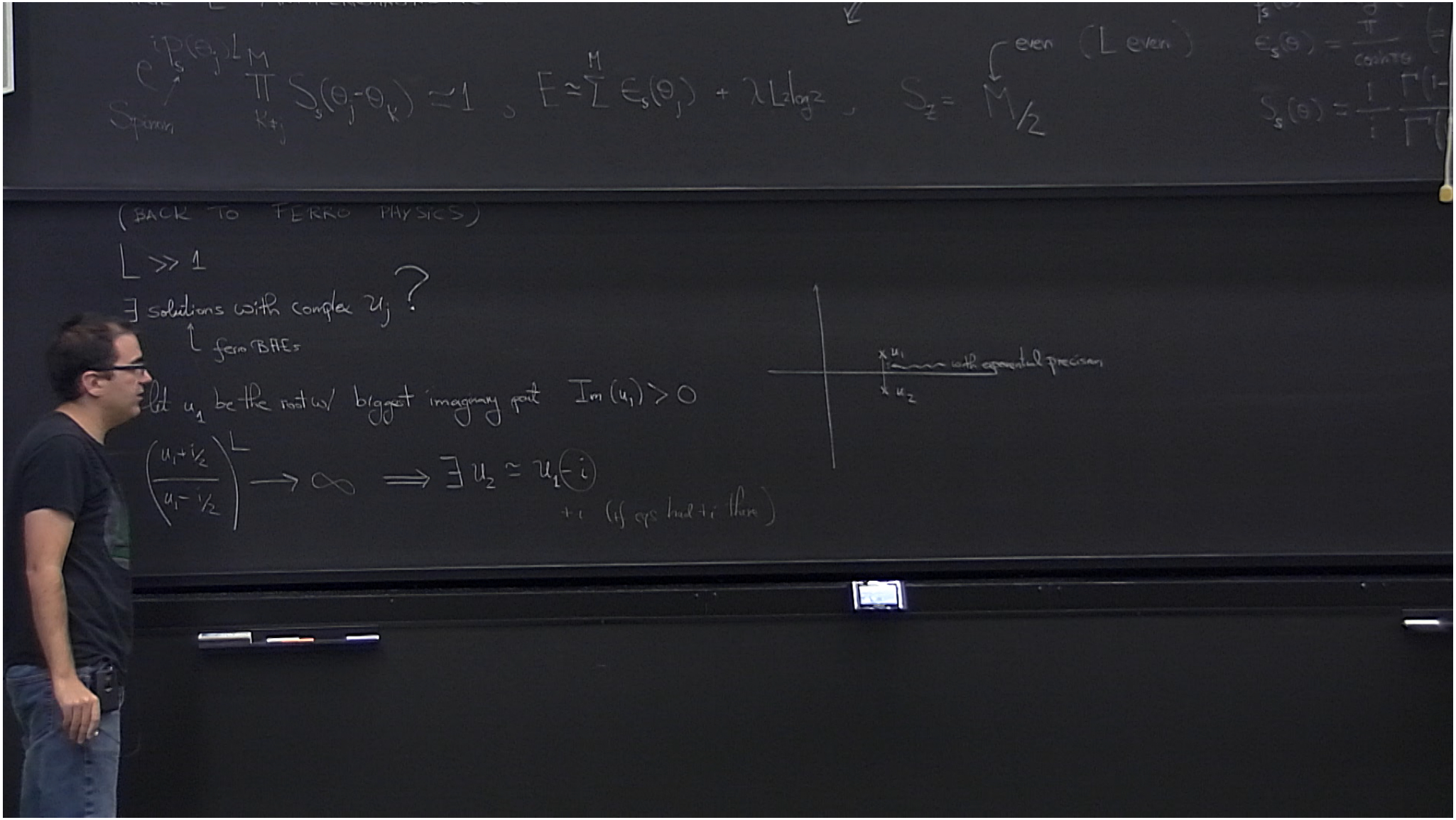

$\mathcal{F}(\log S_s')$
$\frac{1}{1+e^{-|\omega|}}$
$j=1,\dots,N$
Position of Holes
$\rho_{\text{anti-ferro vac}} = \rho_0 = \frac{1}{\mathbb{1}+\mathbb{K}} * \left(\frac{L}{2\pi}p' = \frac{L}{2\pi}\epsilon\right)$
$E_0 = \rho * \epsilon = \underbrace{\left(\epsilon * \frac{1}{\mathbb{1}+\mathbb{K}} * \epsilon\right)\frac{L}{2\pi}}_{L\,2\log 2}$

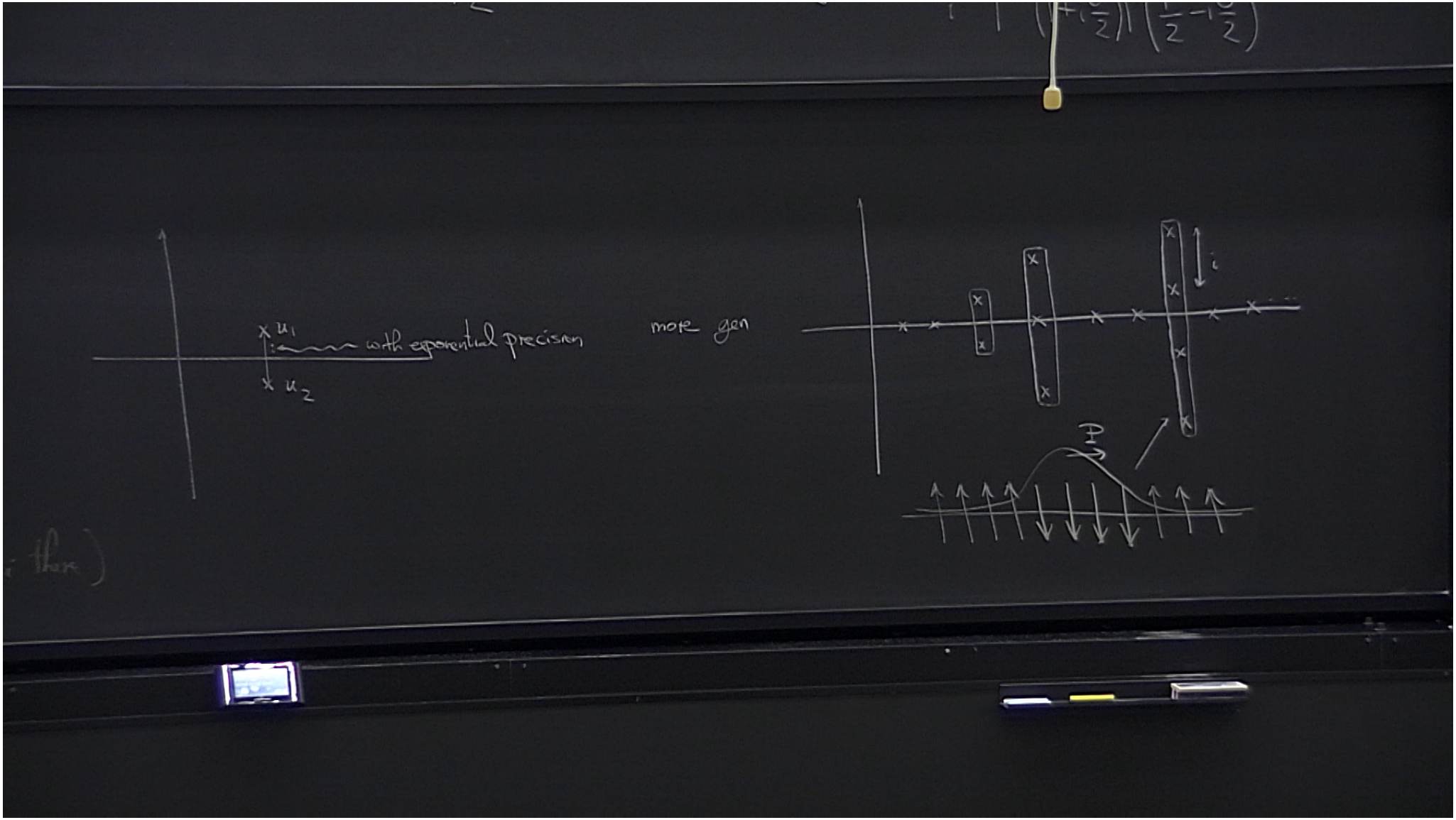

$\mathcal{F}(\log S_s)$
$\frac{1}{1+e^{-|\omega|}}$
$j = 1, \dots, N$
Position of Holes
$\rho_{\text{anti-ferro vac}} = \rho_0 = \frac{1}{1+R} * \left(\frac{L}{2\pi} p' = \frac{L}{2\pi} \epsilon\right)$
$E_0 = \rho * \epsilon = \left(\epsilon * \frac{1}{1+R} * \epsilon\right) \frac{L}{2\pi}$
$L\, 2 \log 2$
$\frac{1}{1+e^{-|\omega|}}$
$\epsilon_s = \frac{1}{1+R} * \epsilon$
$p_s = \frac{1}{1+R} * p$
$\log S_s = \frac{1}{1+R} * \log S$


$\mathcal{F}(\log S_s)$
$\frac{1}{1+e^{-|\omega|}}$
$j = 1, \dots, N$
Position of Holes
$\rho_{\text{anti-ferro vac}} = \rho_0 = \frac{1}{1+R} * \left(\frac{L}{2\pi} p' = \frac{L}{2\pi} \epsilon\right)$
$E_0 = \rho * \epsilon = \left(\epsilon * \frac{1}{1+R} * \epsilon\right) \frac{L}{2\pi}$
$L \, 2 \log 2$
Dressing Ferro $\rightsquigarrow$ Anti-ferro
$\frac{1}{1+e^{-|\omega|}}$
$\epsilon_s = \frac{1}{1+R} * \epsilon$
$\rho_s = \frac{1}{1+R} * \rho$
$\log S_s = \frac{1}{1+R} * \log S$


Position of Holes
Dressing Fero → Anti-fero
no slots anti
2 slots for 1st excited state
Fractionalization


(BACK TO FERRO PHYSICS)
$L \gg 1$
$\exists$ solutions with complex u_j ?
↑
ferro BAEs
let u_1 be the root w/ biggest imaginary part $\mathrm{Im}(u_1) > 0$
$\left(\frac{u_1 + i/2}{u_1 - i/2}\right)^L \longrightarrow \infty \Longrightarrow \exists\, u_2 \simeq u_1 - i$
$+i$ (if eqs had $+i$ there)


(BACK TO FERRO PHYSICS)
$L \gg 1$
$\exists$ solutions with complex u_j ?
from BAEs
Let u_1 be the root w/ biggest imaginary part $\mathrm{Im}(u_1) > 0$
$\left(\frac{u_1+i/2}{u_1-i/2}\right)^L \to \infty \Longrightarrow \exists\, u_2 \simeq u_1 - i$
+ (if eqs hold + if there)
u_1
with exponential precision
u_2


u_1
with exponential precision
u_2
more gen

ANTIFERROMAGNETIC BETHE EQUATIONS

MAGNONS
SPINONS

even (L even)

$$\sum_{k=1}^{M} S_s(\theta_j - \theta_k) \simeq 1 \;, \quad E \simeq \sum_{j=1}^{M} \epsilon_s(\theta_j) + \chi L \log 2 \;, \quad S_z = \frac{L}{2} - M$$

$$p_{ps}(\theta) = \arctan\left(\frac{\sinh \pi\theta}{\cosh \pi\theta}\right)$$

$$\epsilon_s(\theta) = \frac{\pi}{\cosh \pi\theta} \left(= \frac{1}{2\theta} p_{ps}(\theta) \right)$$

$$S_s(\theta) = \frac{1}{i} \frac{\Gamma\left(1 - \frac{i\theta}{2}\right)\Gamma\left(\frac{1}{2} + \frac{i\theta}{2}\right)}{\Gamma\left(1 + \frac{i\theta}{2}\right)\Gamma\left(\frac{1}{2} - \frac{i\theta}{2}\right)}$$

TERRO PHYSICS)

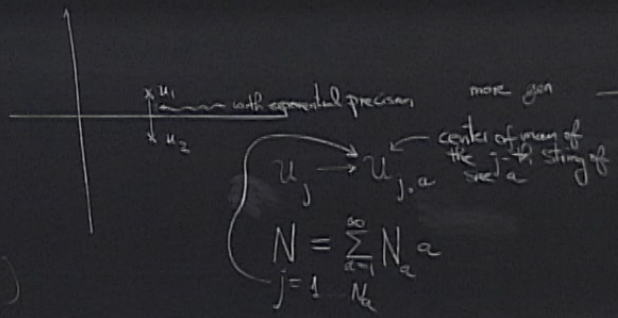
th complex u_j ?

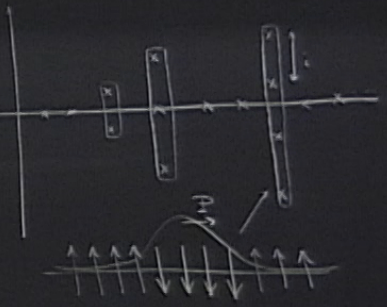
BAEs

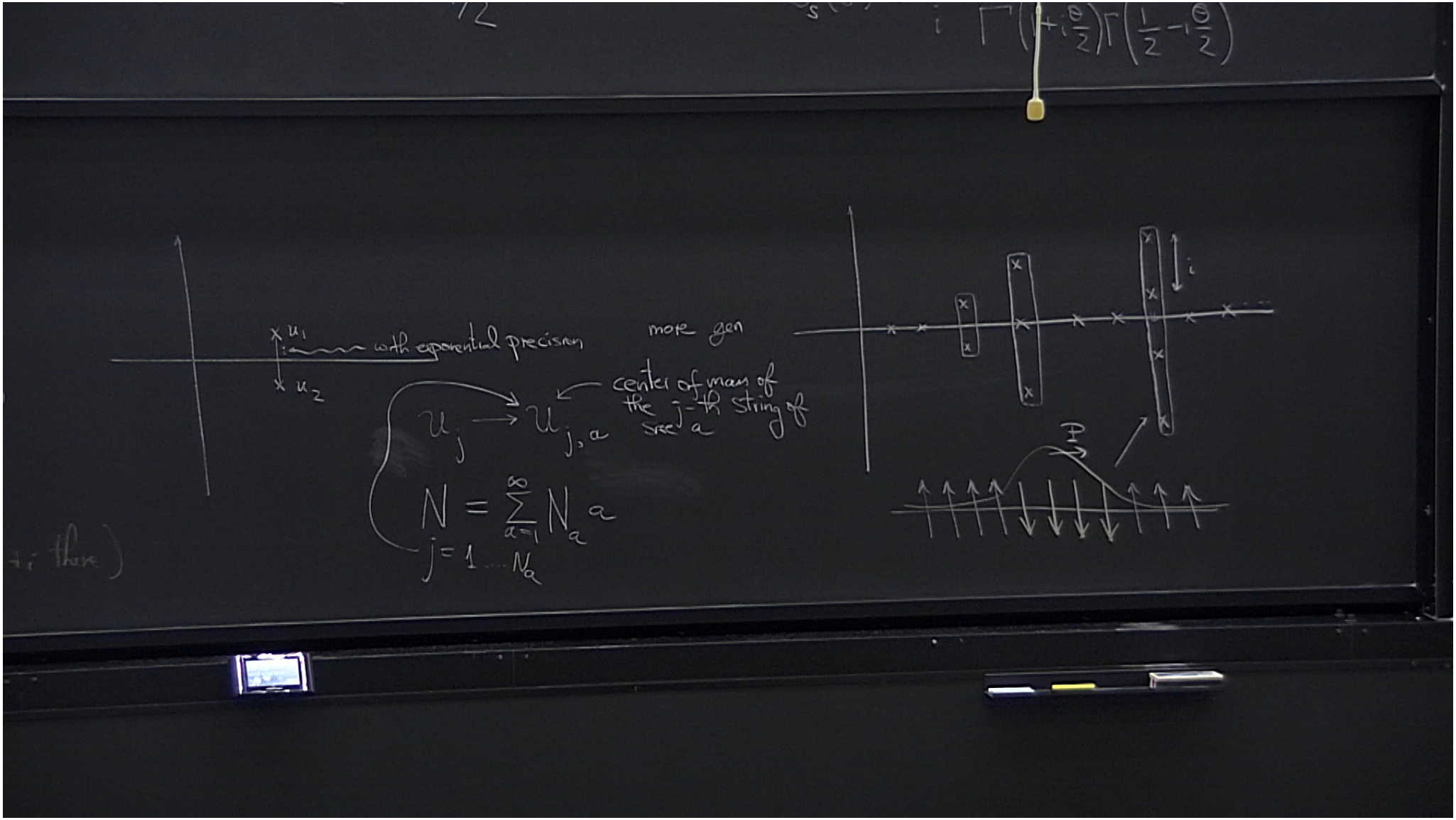
e root w/ biggest imaginary part $\mathrm{Im}(u_1) > 0$

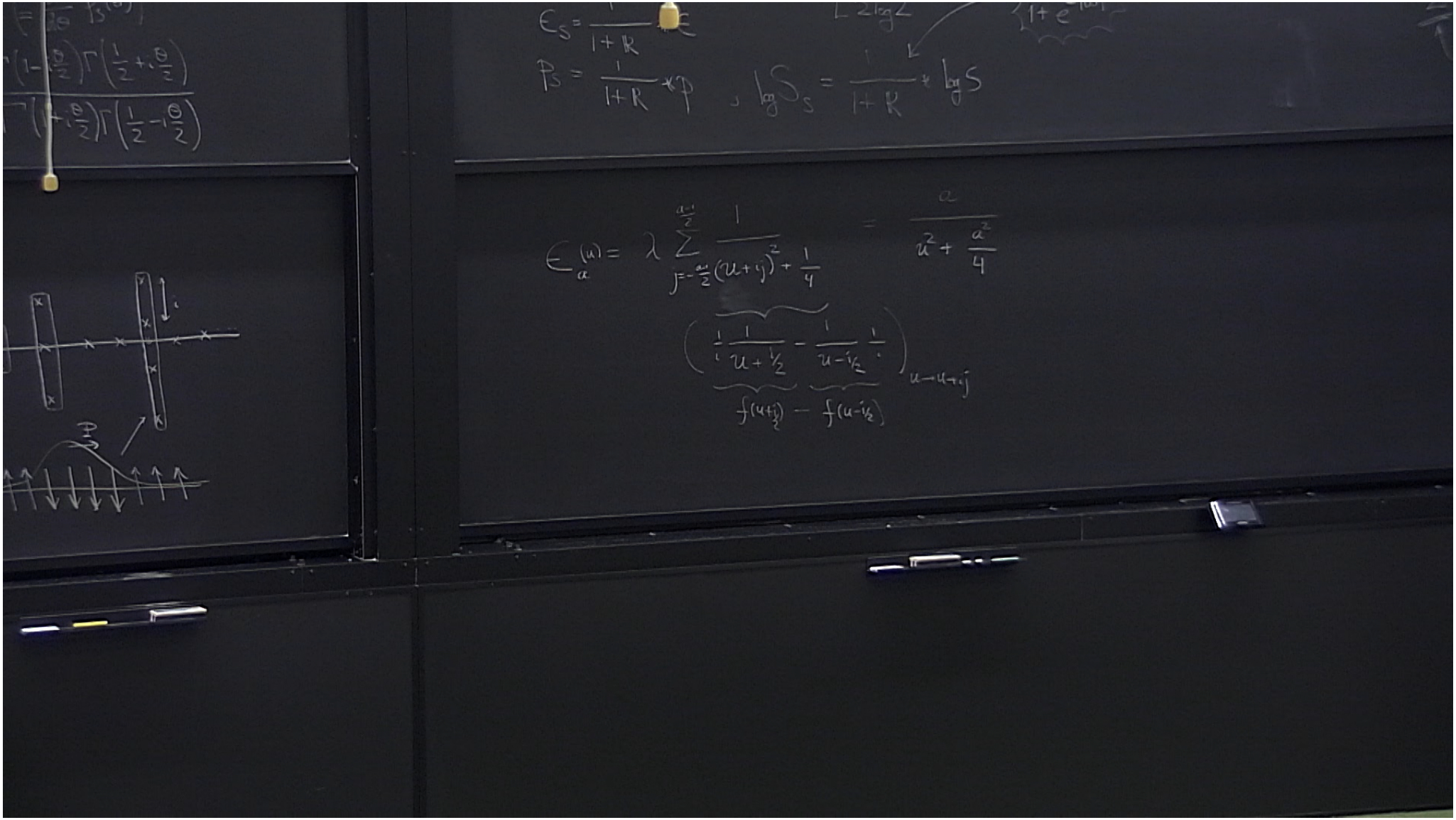
$\to \infty \implies \exists\, u_2 \simeq u_1 + i$

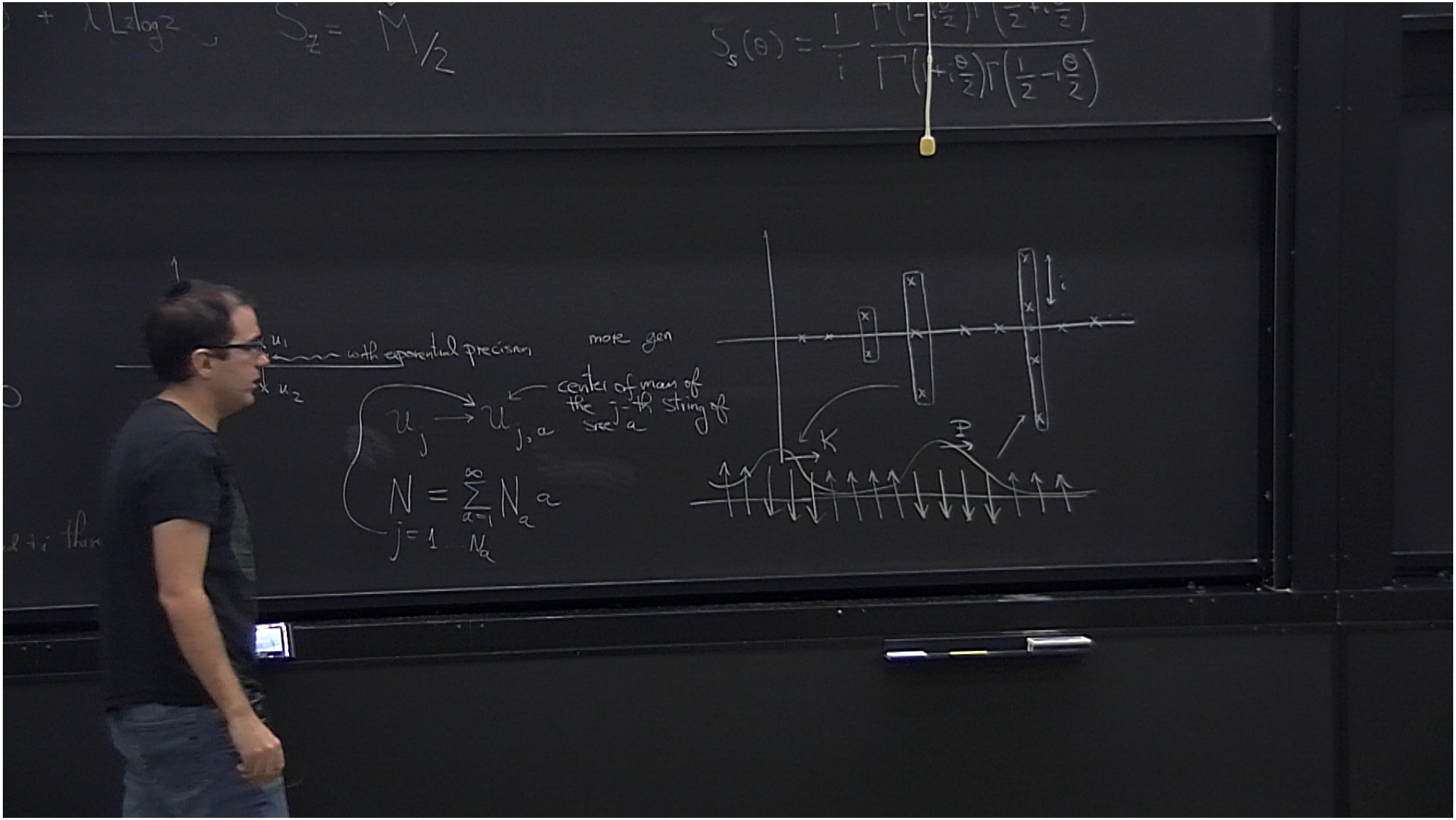
+ i (if eqs hold + there)

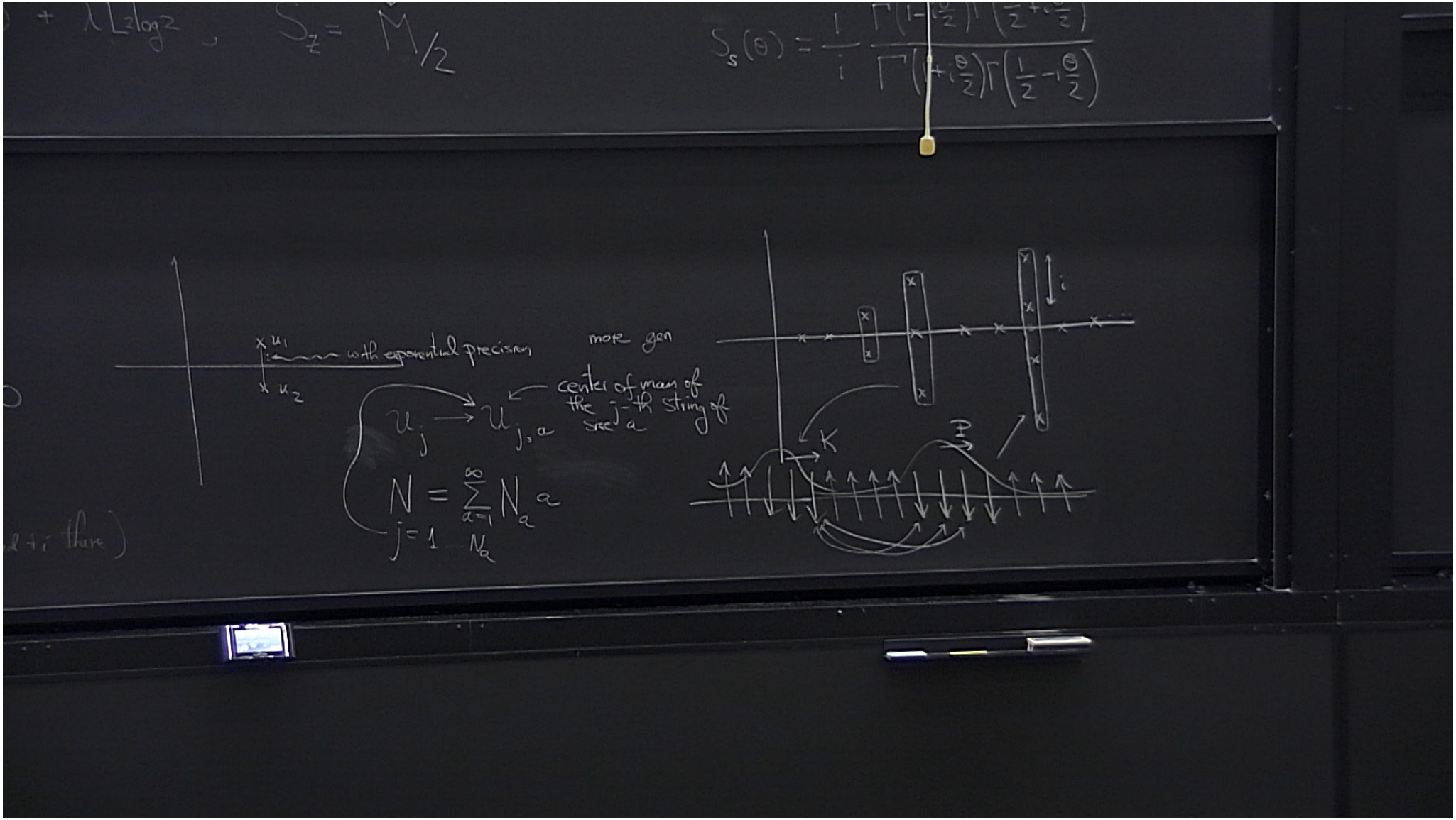





u_1
u_2
with exponential precision
more gen
$u_j \to u_{j,a}$
center of mass of the j-th string of size a
$N = \sum_{a=1}^{\infty} N_a \, a$
$j = 1 \dots N_a$


$\epsilon_S = \frac{1}{1+R}\,\epsilon$
$P_S = \frac{1}{1+R}\,P$
$\log S_S = \frac{1}{1+R}\,\log S$
$\epsilon_a^{(u)} = \lambda \sum_{j=-\frac{a-1}{2}}^{\frac{a-1}{2}} \frac{1}{(u+j)^2+\frac{1}{4}} = \frac{a}{u^2+\frac{a^2}{4}}$
$\left(\frac{1}{i}\frac{1}{u+\frac{1}{2}} - \frac{1}{u-\frac{1}{2}}\frac{1}{i}\right)_{u\to u+ij}$
$f(u+\frac{i}{2}) - f(u-\frac{i}{2})$


$S_z = M/2$
$S_s(\theta) = \frac{1}{i} \frac{\Gamma(1-\frac{\theta}{2})\Gamma(\frac{1}{2}+\frac{\theta}{2})}{\Gamma(1+\frac{\theta}{2})\Gamma(\frac{1}{2}-\frac{\theta}{2})}$
u_1
u_2
with exponential precision
more gen
$u_j \to u_{j,a}$
center of mass of the j-th string of size a
$N = \sum_{a=1}^{\infty} N_a \, a$
$j = 1 \dots N_a$
i
K
P


$S_z = M/2$
$S_s(\theta) = \frac{1}{i} \frac{\Gamma(1-\frac{\theta}{2})\Gamma(\frac{1}{2}+\frac{\theta}{2})}{\Gamma(1+\frac{\theta}{2})\Gamma(\frac{1}{2}-\frac{\theta}{2})}$
u_1
u_2
with exponential precision
more gen
$u_j \to u_{j,a}$
center of mass of the j-th string of size a
$N = \sum_{a=1}^{\infty} N_a \, a$
$j = 1 \dots N_a$
i
K

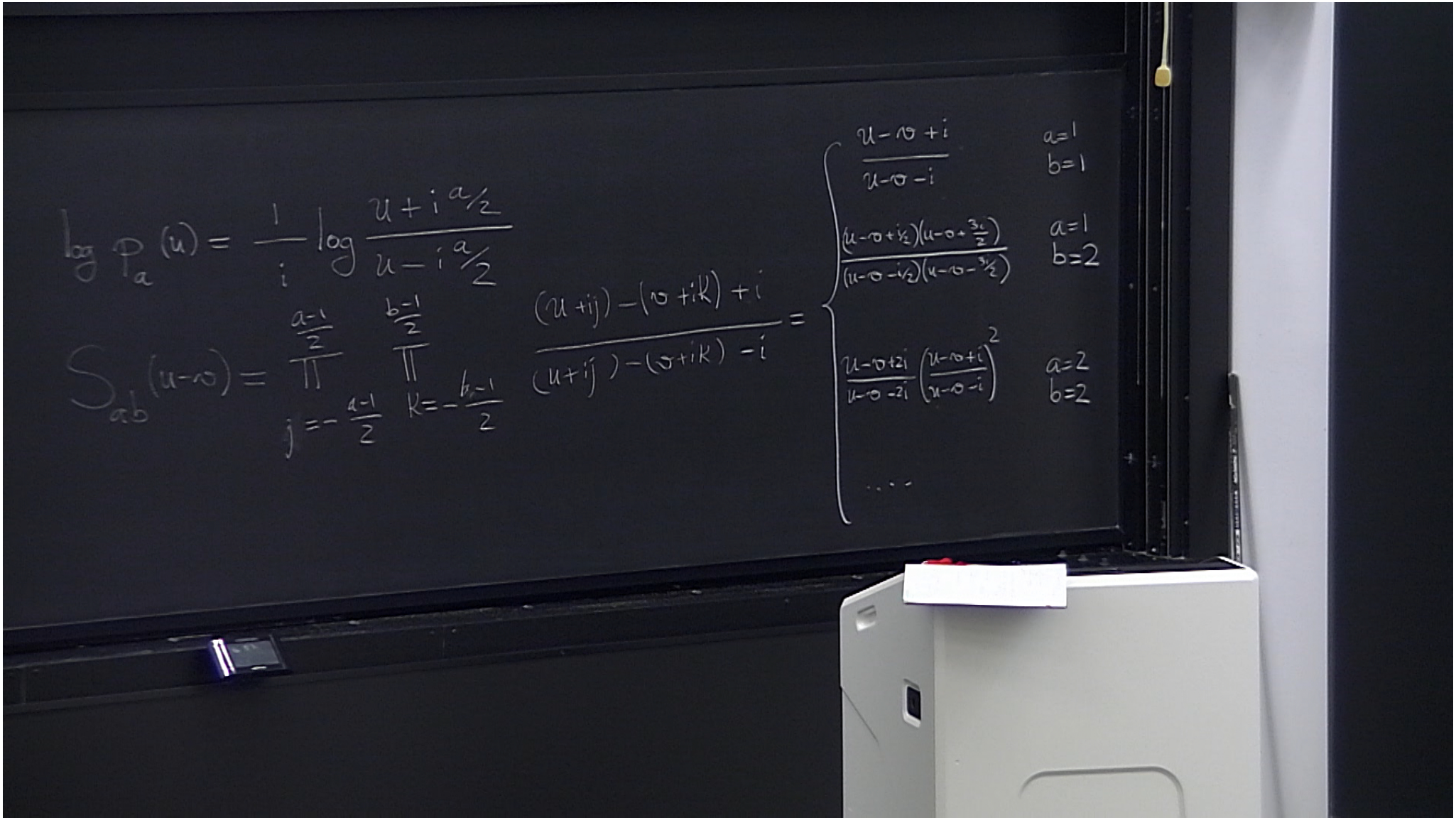
$$\frac{1}{(u+ij)^2+\frac{1}{4}} = \frac{a}{u^2+\frac{a^2}{4}}$$

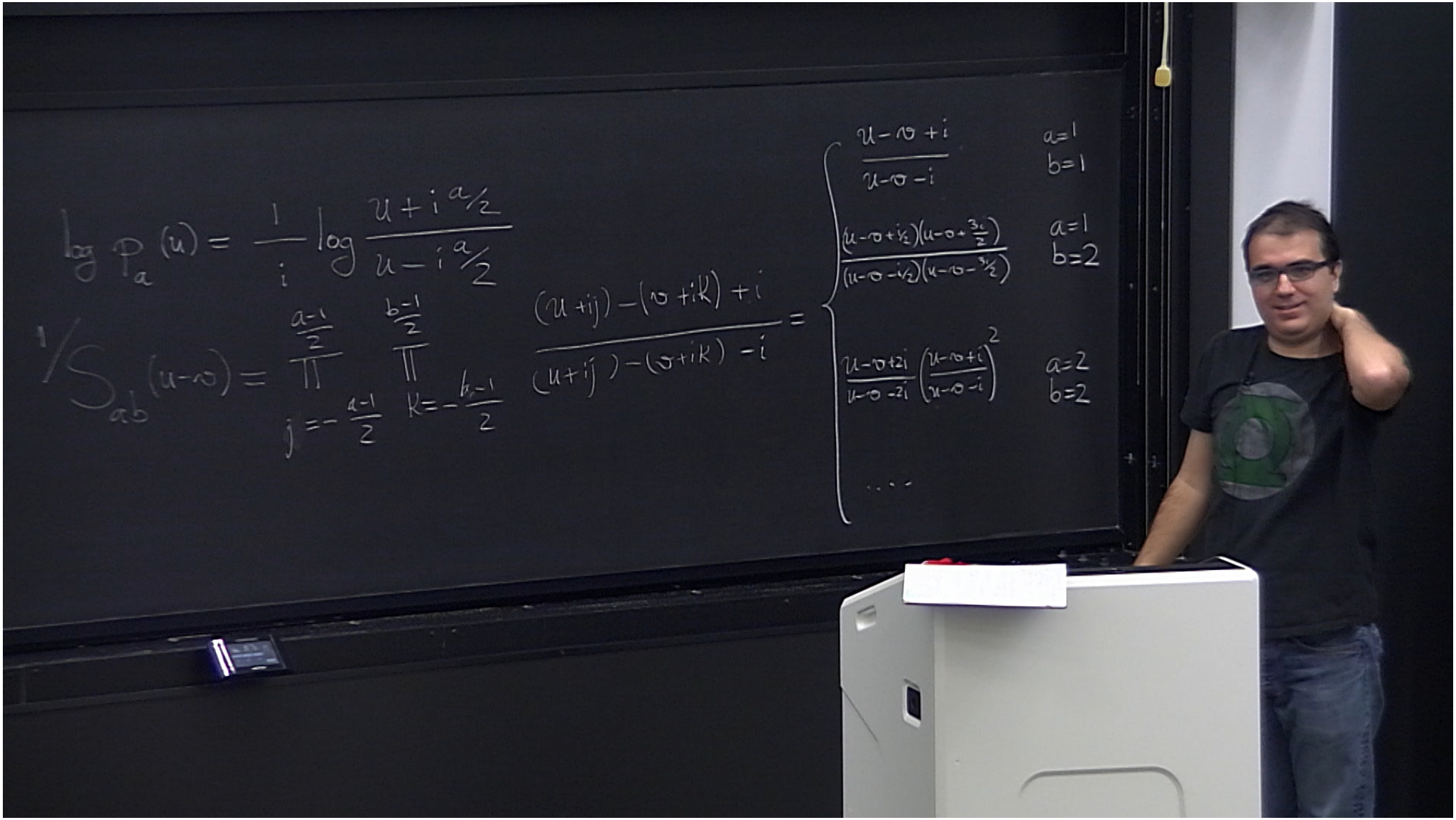
$$\log P_a(u) = \frac{1}{i}\log\frac{u+i\,a/2}{u-i\,a/2}$$

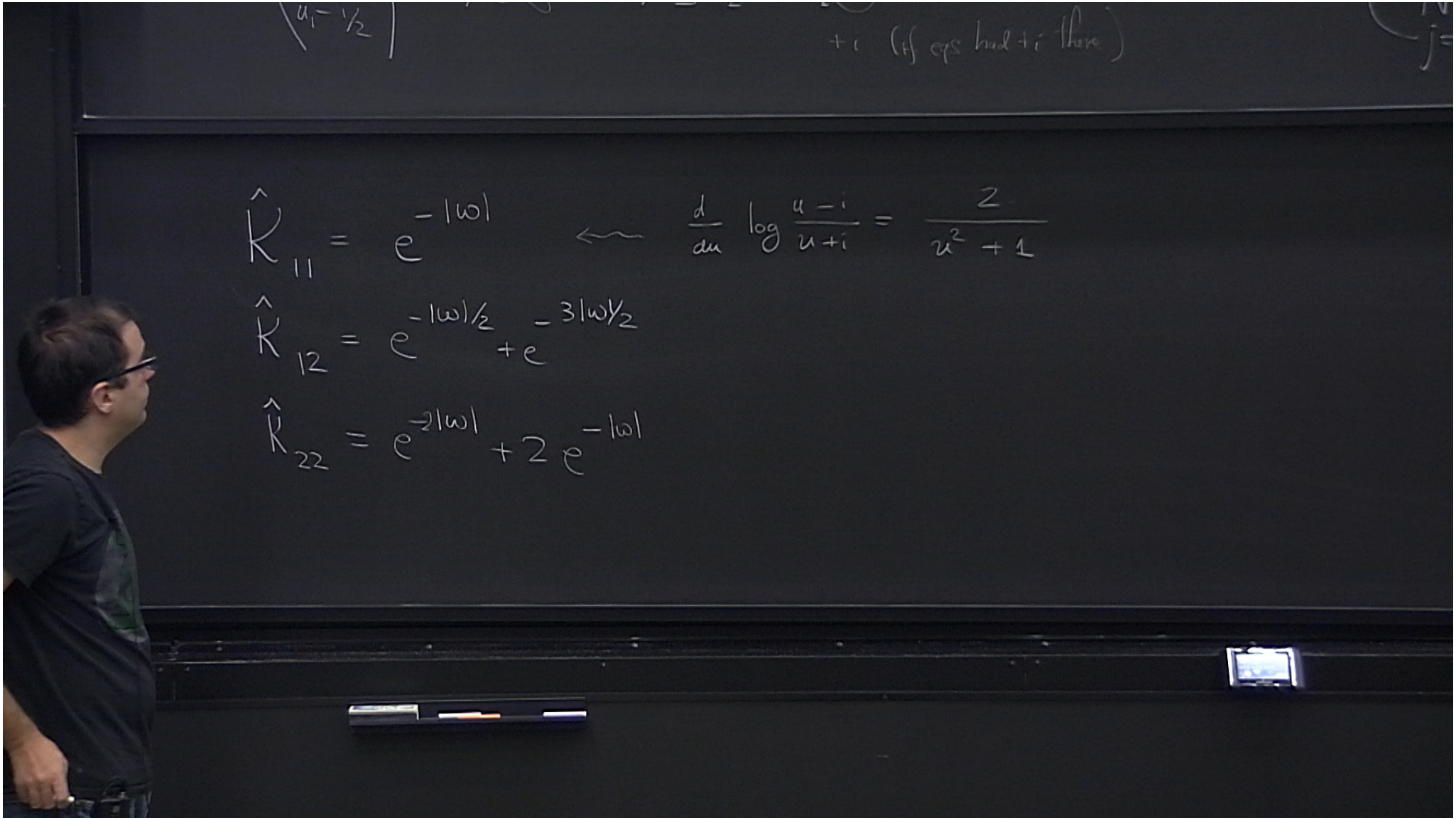
$$\left(\frac{1}{u+\frac{i}{2}} - \frac{1}{u-\frac{i}{2}}\right)\frac{1}{i} \qquad u \to u+ij$$

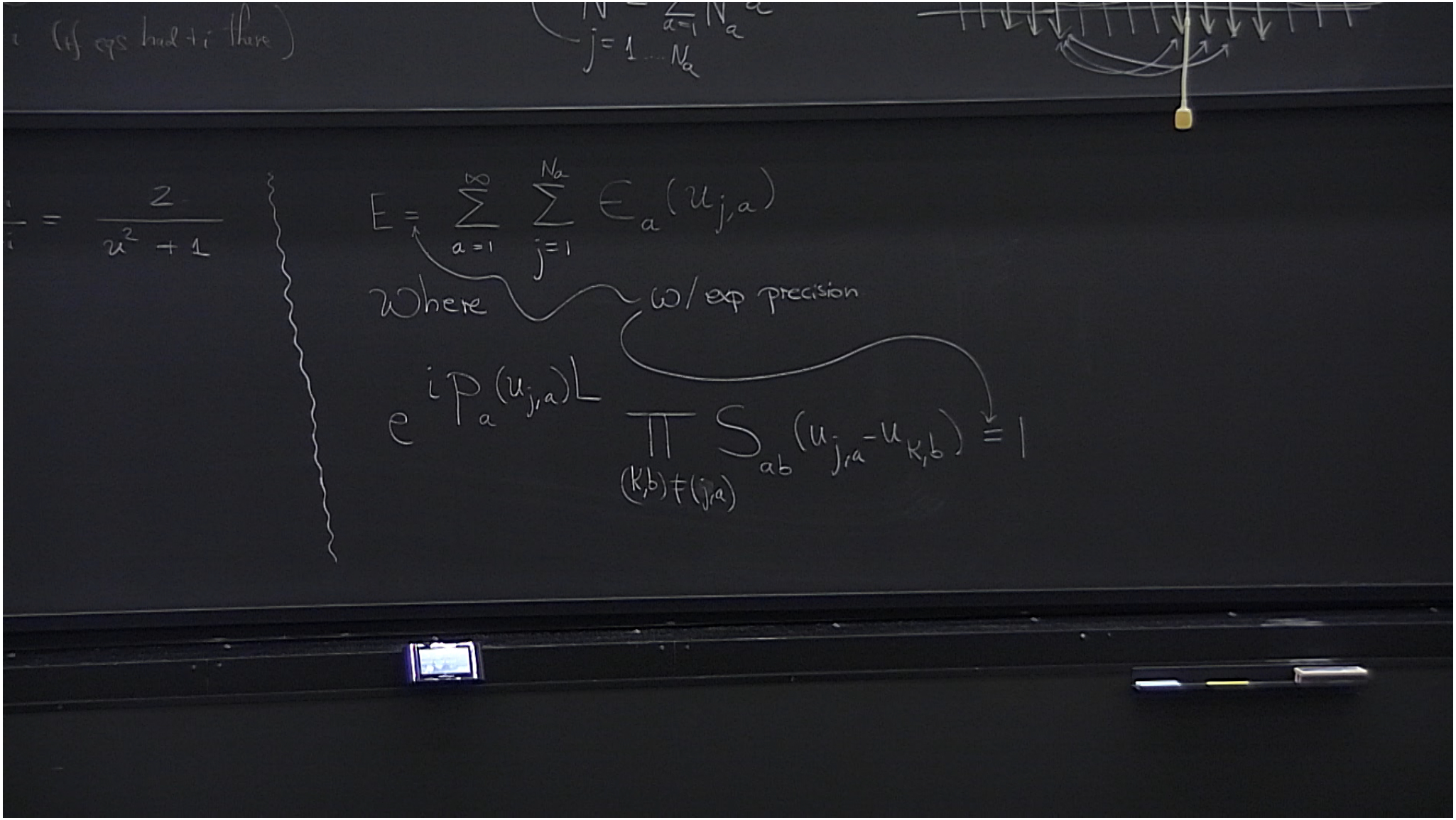
$$f\left(u+\tfrac{i}{2}\right) - f\left(u-\tfrac{i}{2}\right)$$

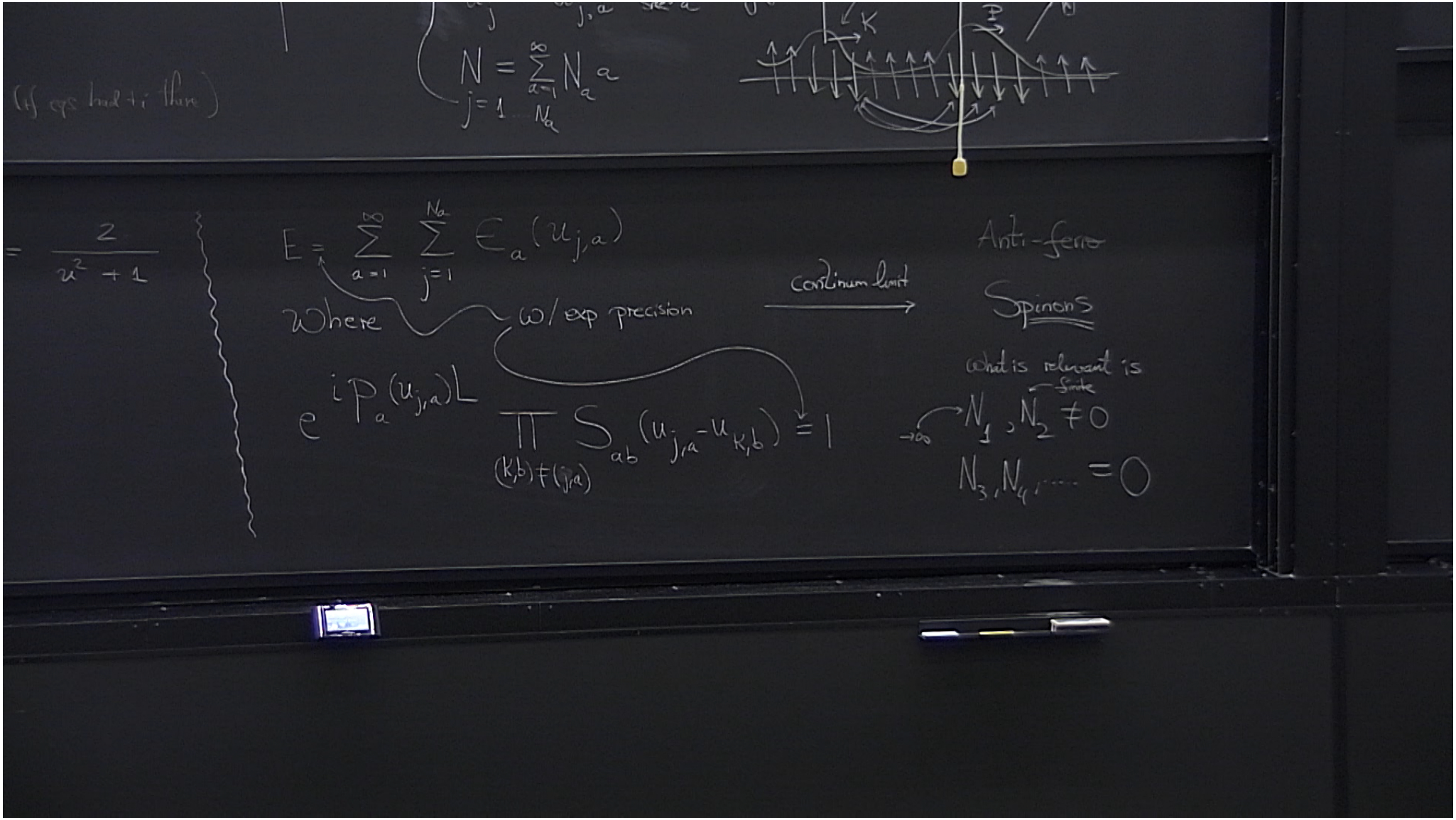
$$S_{ab}(u-v) = \prod_{j=-\frac{a-1}{2}}^{\frac{a-1}{2}} \prod_{K=-\frac{b-1}{2}}^{\frac{b-1}{2}} \frac{(u+ij)-(v+iK)+i}{(u+ij)-(v+iK)-i}$$


$\log P_a(u) = \frac{1}{i} \log \frac{u + i\,a/2}{u - i\,a/2}$
$S_{ab}(u-v) = \prod_{j=-\frac{a-1}{2}}^{\frac{a-1}{2}} \prod_{k=-\frac{b-1}{2}}^{\frac{b-1}{2}} \frac{(u+ij)-(v+ik)+i}{(u+ij)-(v+ik)-i} =$
$\frac{u-v+i}{u-v-i}$ a=1 b=1
$\frac{(u-v+i/2)(u-v+3i/2)}{(u-v-i/2)(u-v-3i/2)}$ a=1 b=2
$\frac{u-v+2i}{u-v-2i}\left(\frac{u-v+i}{u-v-i}\right)^2$ a=2 b=2
....


$\log P_a(u) = \frac{1}{i} \log \frac{u + i\,a/2}{u - i\,a/2}$
$1/S_{ab}(u-v) = \prod_{j=-\frac{a-1}{2}}^{\frac{a-1}{2}} \prod_{k=-\frac{b-1}{2}}^{\frac{b-1}{2}} \frac{(u+ij)-(v+ik)+i}{(u+ij)-(v+ik)-i} =$
$\frac{u-v+i}{u-v-i}$ $a=1$ $b=1$
$\frac{(u-v+i/2)(u-v+\frac{3i}{2})}{(u-v-i/2)(u-v-\frac{3i}{2})}$ $a=1$ $b=2$
$\frac{u-v+2i}{u-v-2i}\left(\frac{u-v+i}{u-v-i}\right)^2$ $a=2$ $b=2$
. . .


$u_1 - 1/2$
+i (if eqs had +i there)
$\hat{K}_{11} = e^{-|\omega|}$
$\frac{d}{du} \log \frac{u-i}{u+i} = \frac{2}{u^2+1}$
$\hat{K}_{12} = e^{-|\omega|/2} + e^{-3|\omega|/2}$
$\hat{K}_{22} = e^{-2|\omega|} + 2e^{-|\omega|}$


(if eqs had +i there)
$j = 1 \dots N_a$
$= \frac{2}{u^2 + 1}$
$E = \sum_{a=1}^{\infty} \sum_{j=1}^{N_a} \epsilon_a(u_{j,a})$
where
w/ exp precision
$e^{iP_a(u_{j,a})L}$
$\prod_{(k,b) \neq (j,a)} S_{ab}(u_{j,a} - u_{k,b}) = 1$

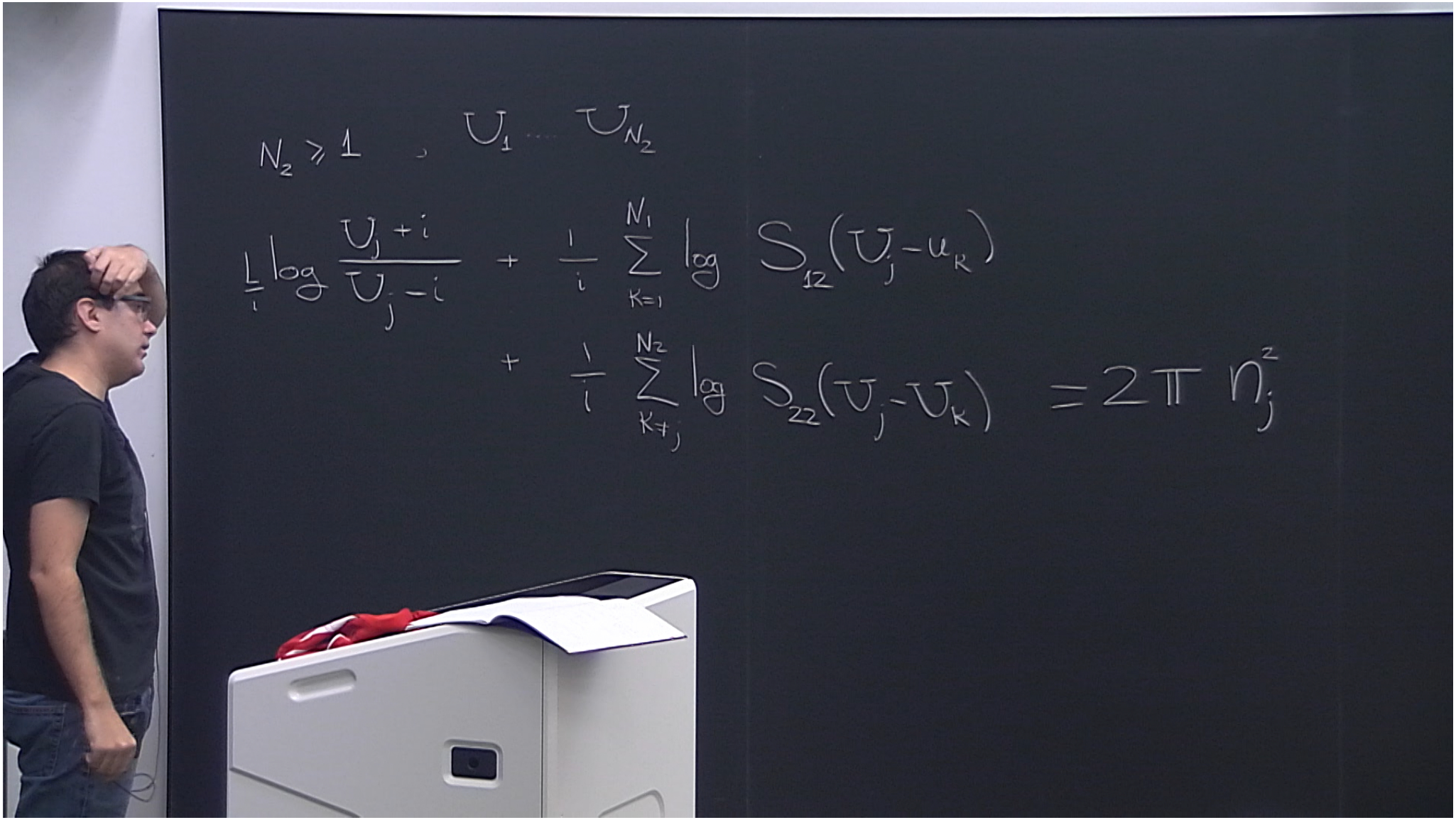

$N = \sum_{a=1}^{\infty} N_a$
$j = 1 \dots N_a$
$= \frac{2}{u^2 + 1}$
$E = \sum_{a=1}^{\infty} \sum_{j=1}^{N_a} \epsilon_a(u_{j,a})$
where
w/ exp precision
Continuum limit
Anti-ferro
Spinons
$e^{iP_a(u_{j,a})L}$
$\prod_{(k,b) \neq (j,a)} S_{ab}(u_{j,a} - u_{k,b}) = 1$
what is relevant is
finite
$\to \infty$
$N_1, N_2 \neq 0$
$N_3, N_4, \dots = 0$

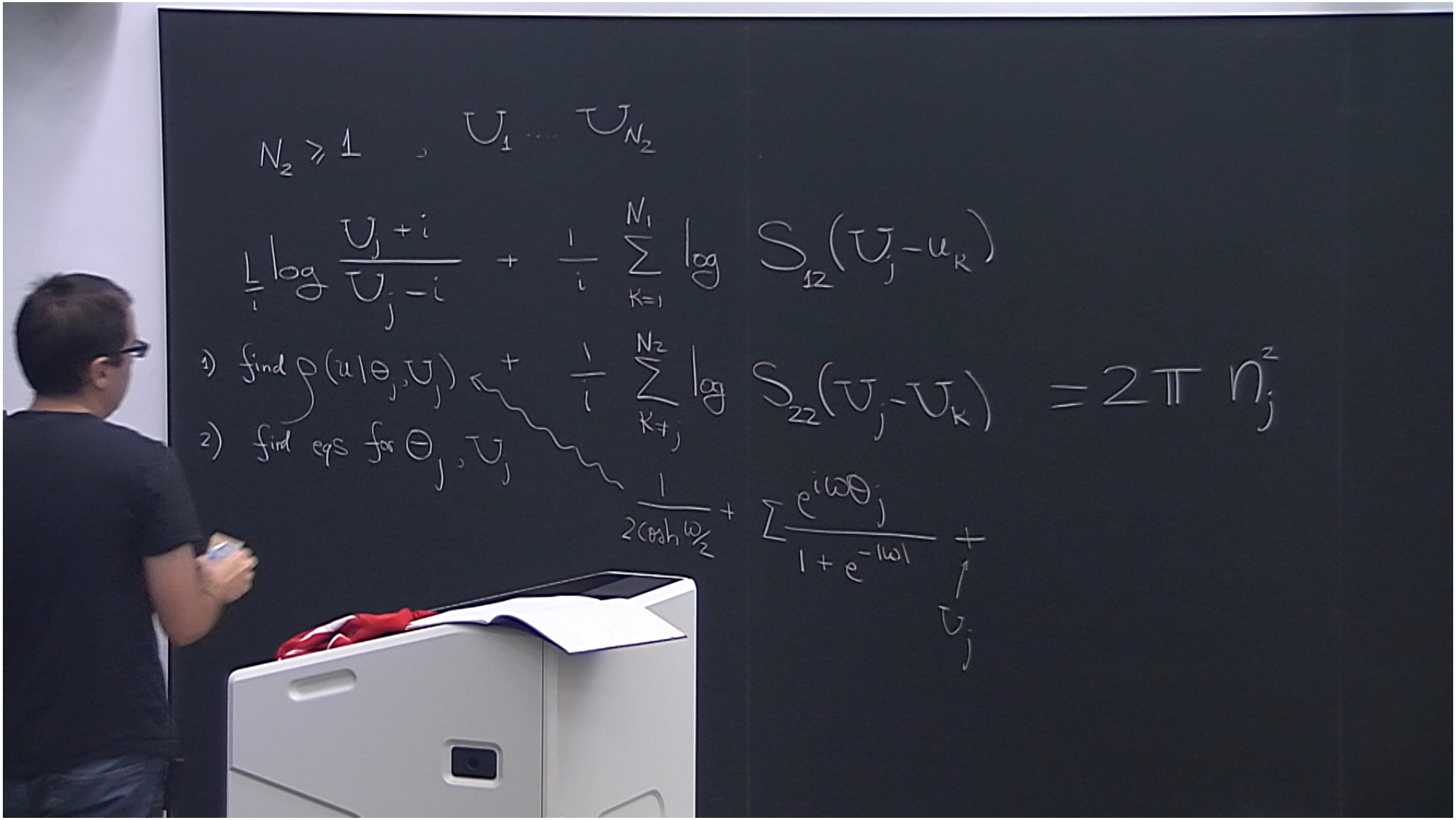
$N_1 = L/2$
$N_2 = 0$
} vac $\quad Q_j = \left\{ -\frac{L}{4} + \frac{1}{2}, \cdots, \frac{L}{4} - \frac{1}{2} \right\}$

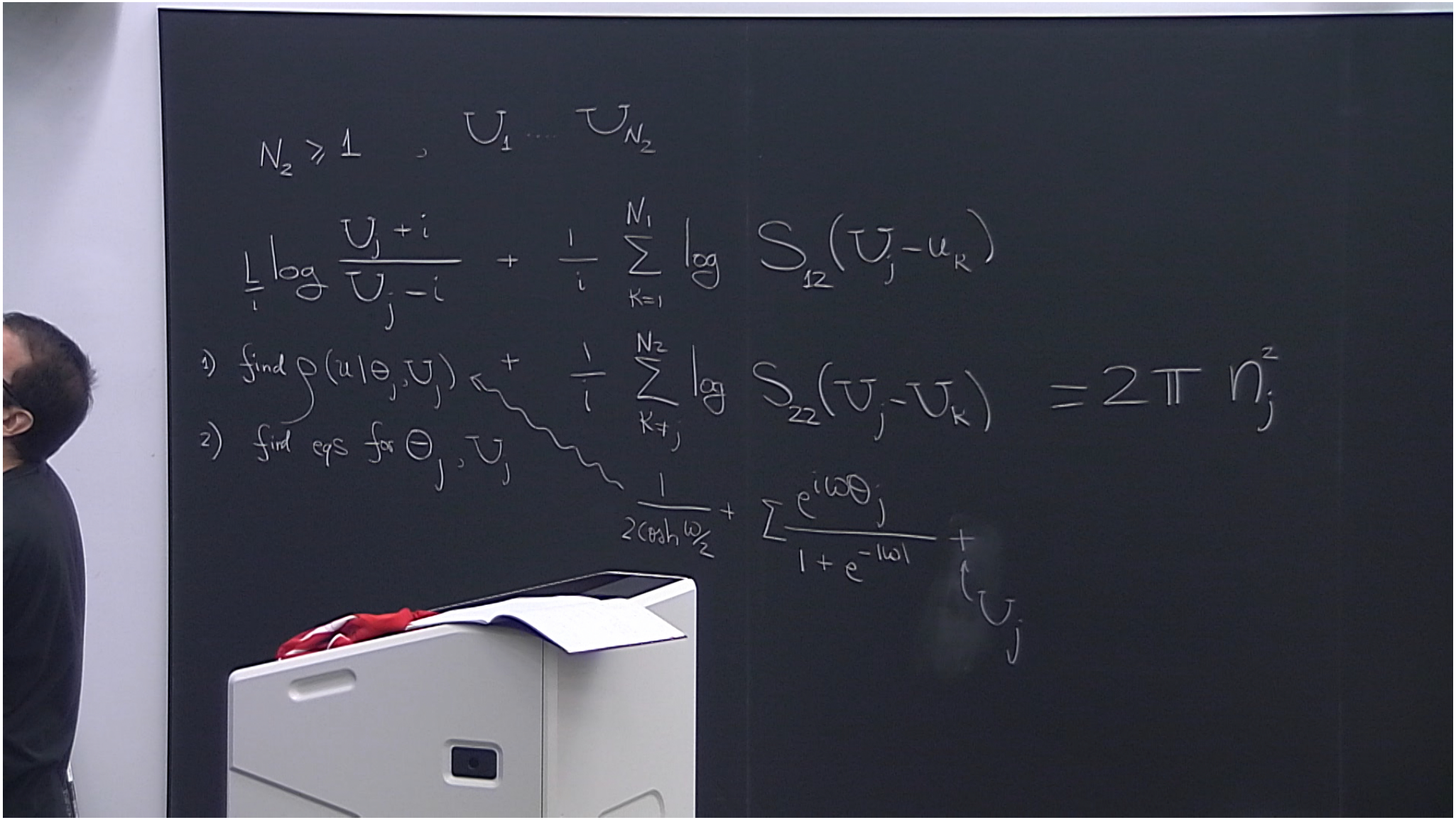
$\#\text{slot} = 2\left(\frac{L}{4} - \frac{1}{2}\right) + 1 = \frac{L}{2}$ ✓

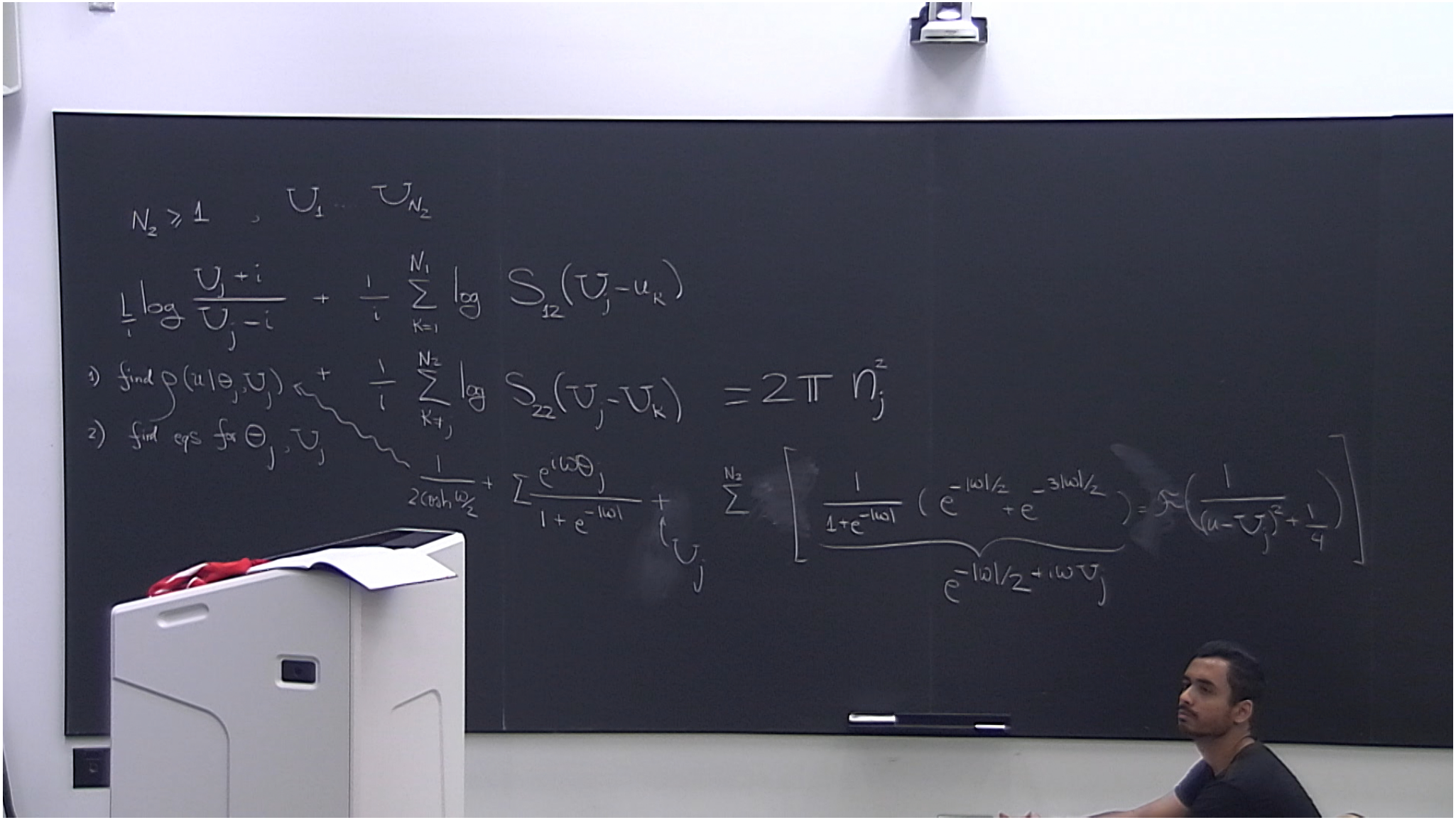
$N_1 = L/2 - 1$
$N_2 = 0$
$\Big\}, \; Q_j \in \left\{ -\frac{L}{4}, \cdots, \frac{L}{4} \right\} \Rightarrow \#\text{slots} = \frac{L}{2} + 1 \Rightarrow$ 2 Holes

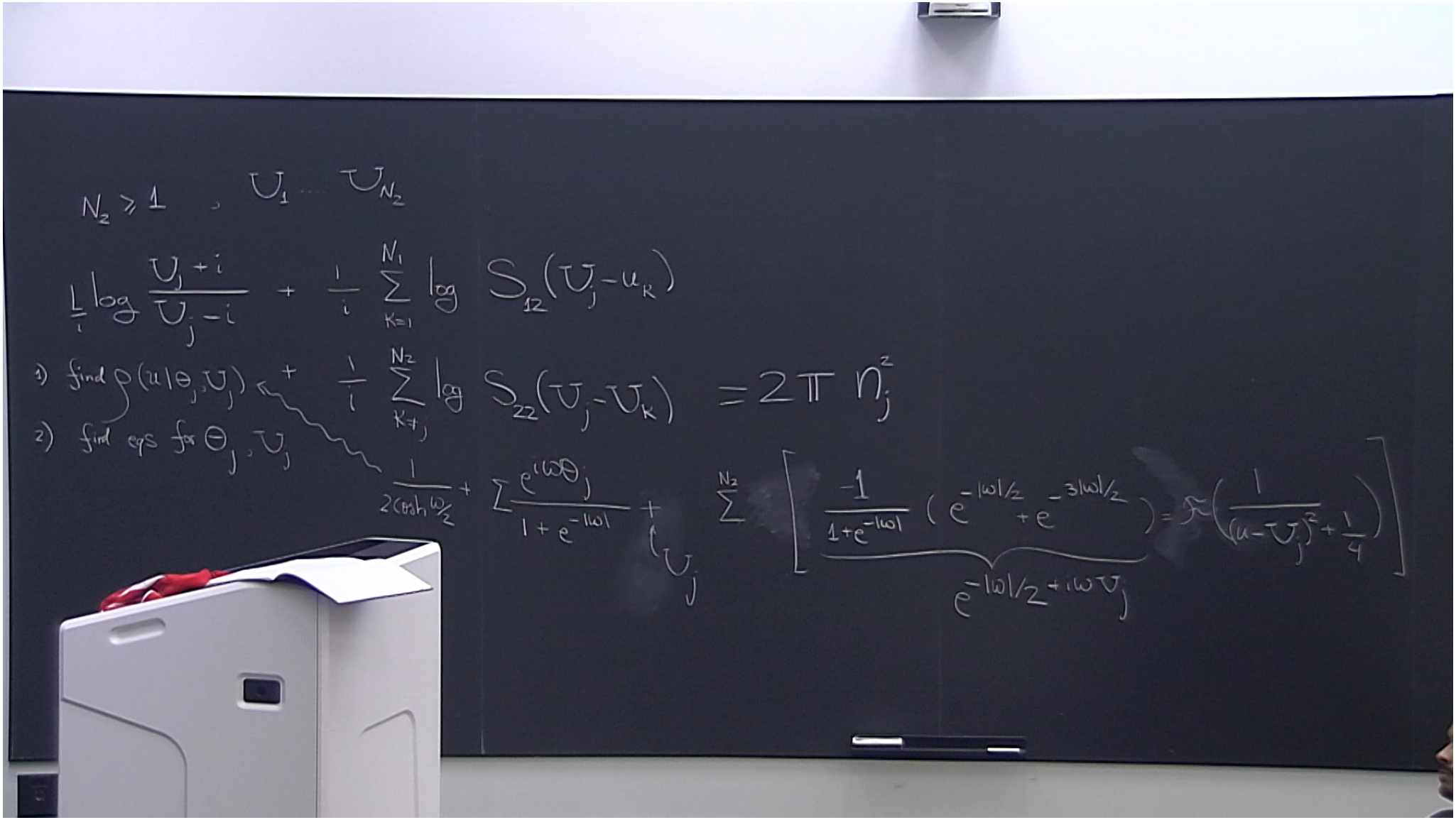
$N_1 = L/2 - 2$
$N_2 = 1$

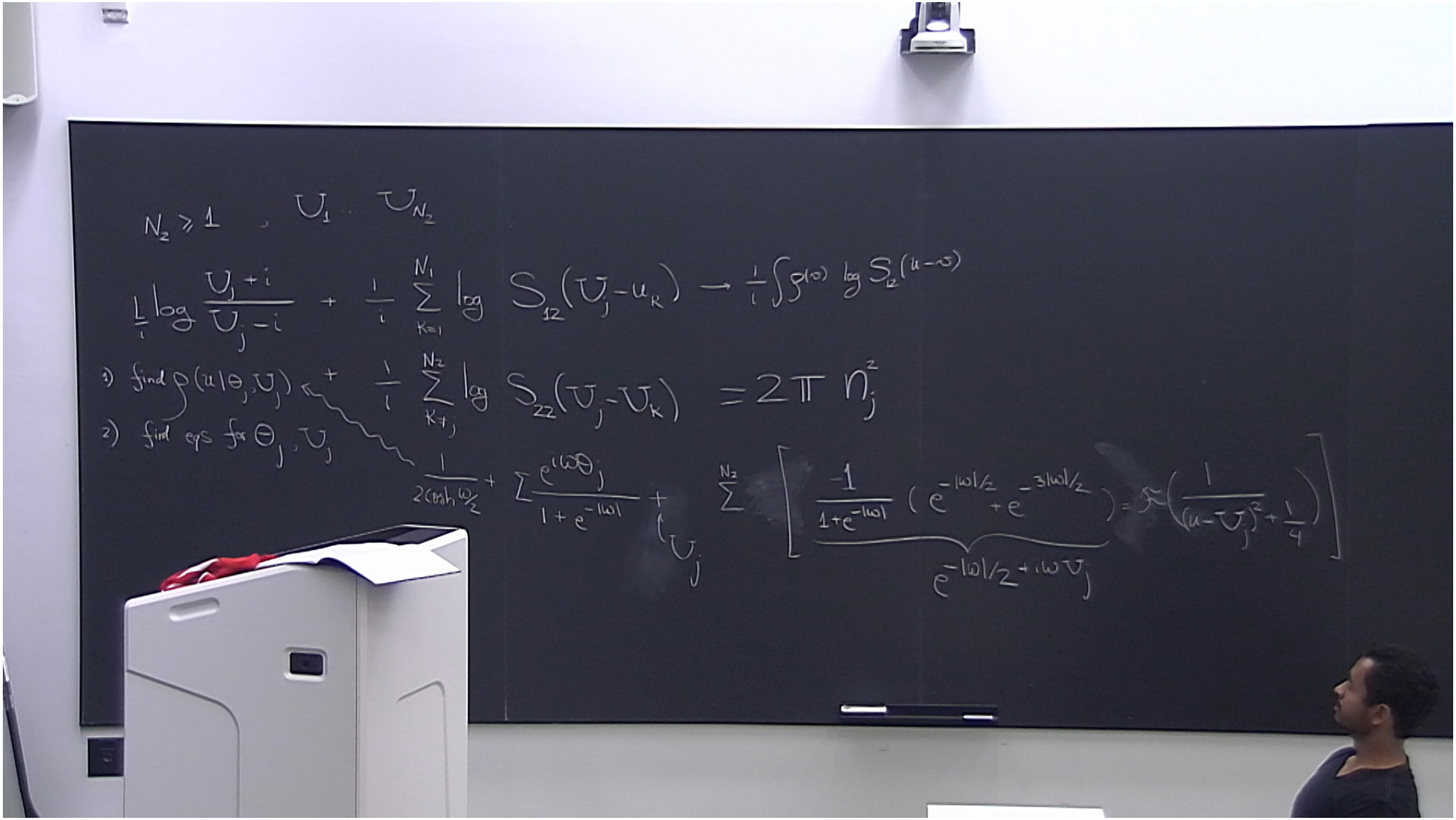

$N_2 \geq 1$, $u_1 \cdots u_{N_2}$
$\frac{1}{i} \log \frac{u_j + i}{u_j - i} + \frac{1}{i} \sum_{k=1}^{N_1} \log S_{12}(u_j - u_k)$
$+ \frac{1}{i} \sum_{k \neq j}^{N_2} \log S_{22}(u_j - u_k) = 2\pi n_j$

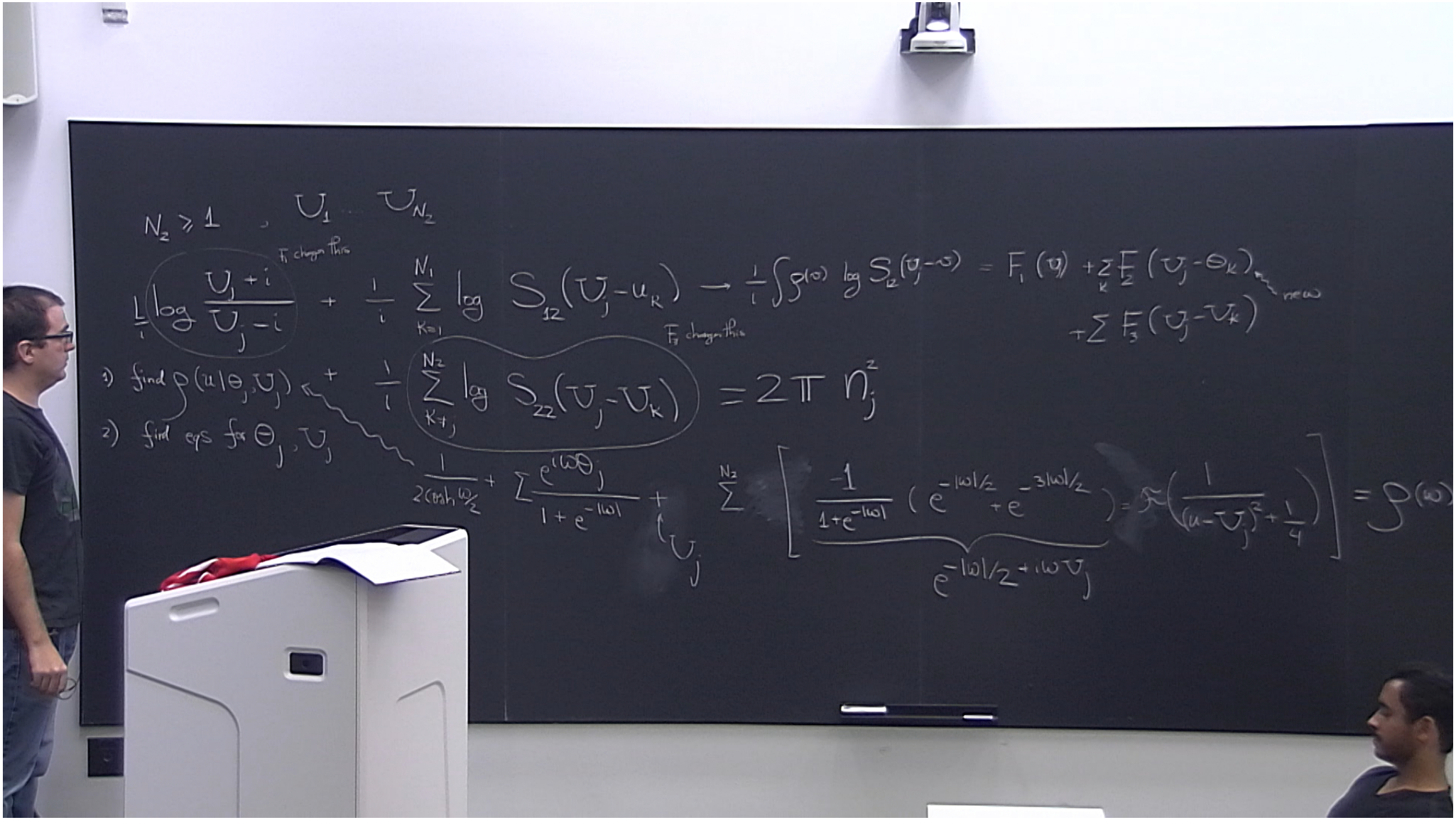

$N_2 \geq 1$, $v_1 \cdots v_{N_2}$
$\frac{L}{i}\log\frac{v_j+i}{v_j-i} + \frac{1}{i}\sum_{k=1}^{N_1}\log S_{12}(v_j-u_k)$
1) find $\rho(u|\theta_j, v_j)$
2) find eqs for θ_j, v_j
$+\frac{1}{i}\sum_{k\neq j}^{N_2}\log S_{22}(v_j-v_k) = 2\pi n_j$
$\frac{1}{2\cosh\omega/2} + \sum\frac{e^{i\omega\theta_j}}{1+e^{-|\omega|}}$
v_j

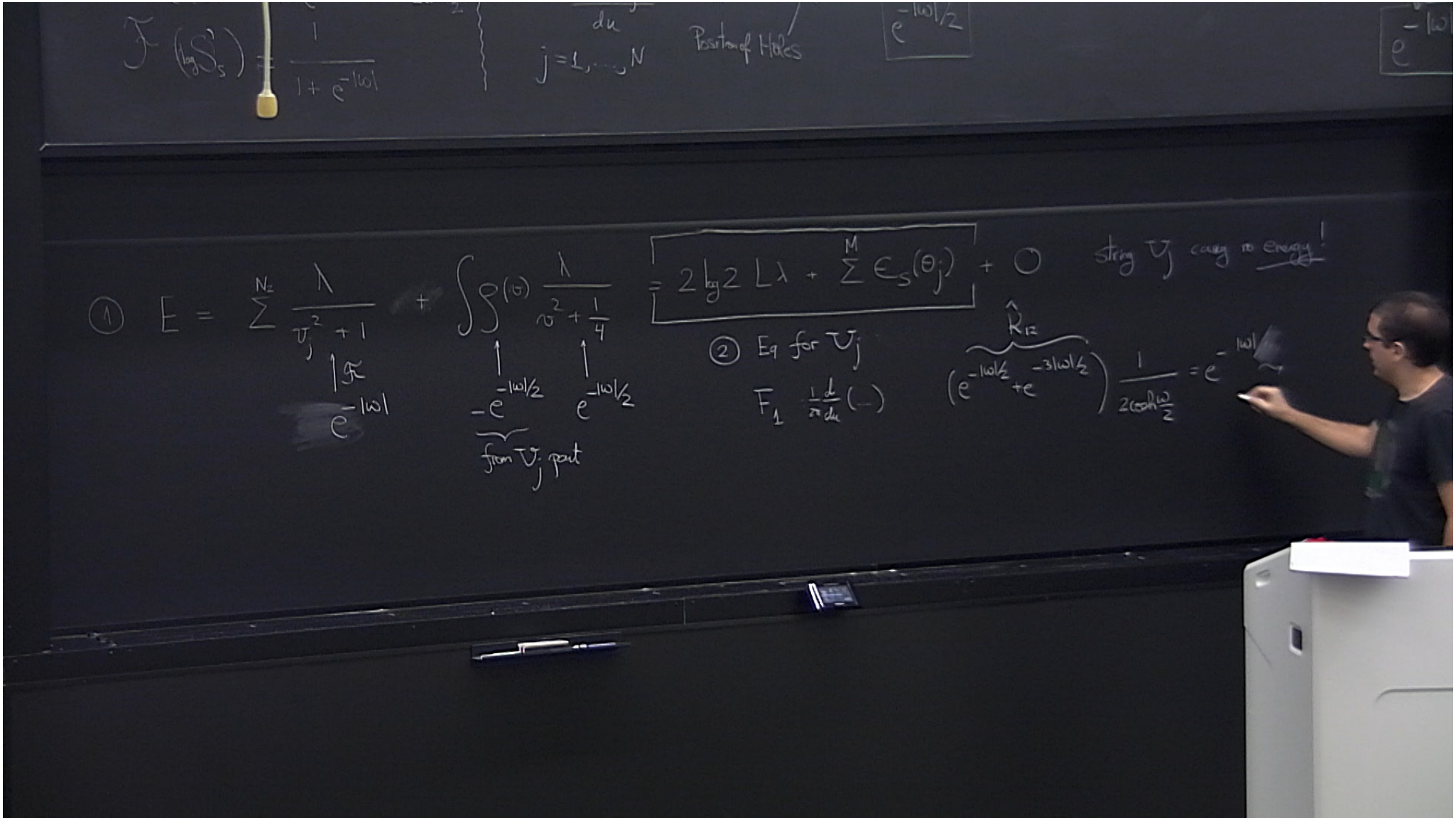

$N_2 \geq 1$, $v_1 \cdots v_{N_2}$
$\frac{L}{i} \log \frac{v_j + i}{v_j - i} + \frac{1}{i} \sum_{k=1}^{N_1} \log S_{12}(v_j - u_k)$
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2) find eqs for θ_j, v_j
$+ \frac{1}{i} \sum_{k \neq j}^{N_2} \log S_{22}(v_j - v_k) = 2\pi n_j$
$\frac{1}{2\cosh \omega/2} + \sum \frac{e^{i\omega\theta_j}}{1 + e^{-|\omega|}} + \ldots v_j$

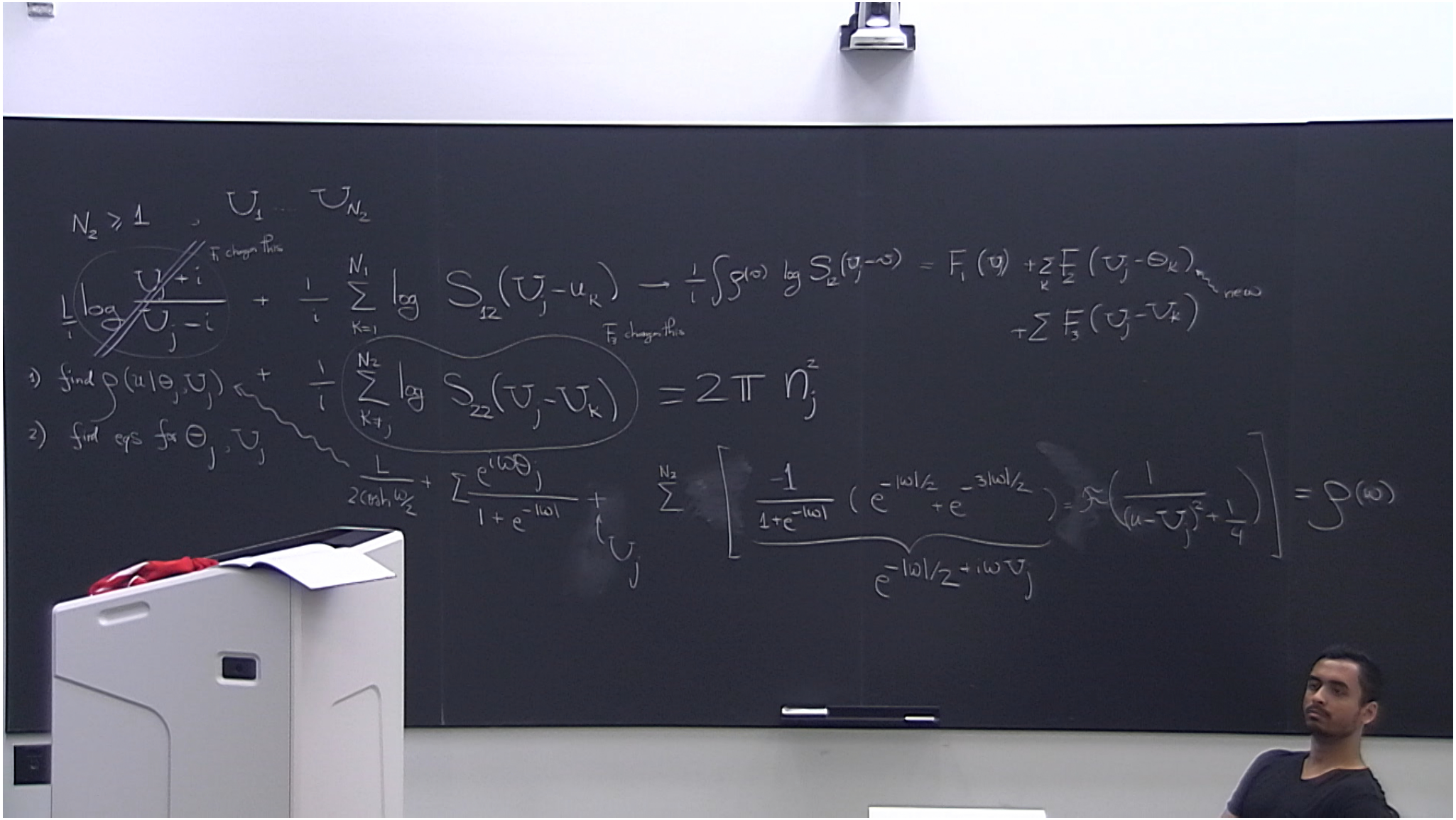

$N_2 \geq 1$, $v_1 \dots v_{N_2}$
$\frac{1}{i}\log\frac{v_j+i}{v_j-i} + \frac{1}{i}\sum_{k=1}^{N_1}\log S_{12}(v_j-u_k)$
$+ \frac{1}{i}\sum_{k\neq j}^{N_2}\log S_{22}(v_j-v_k) = 2\pi n_j^2$
1) find $\rho(u|\theta_j, v_j)$
2) find eqs for θ_j, v_j
$\frac{1}{2\cosh\omega/2} + \sum\frac{e^{i\omega\theta_j}}{1+e^{-|\omega|}} + \dots v_j$
$\sum^{N_2}\left[\frac{1}{1+e^{-|\omega|}}\left(e^{-|\omega|/2} + e^{-3|\omega|/2}\right) = \pi\left(\frac{1}{(u-v_j)^2+\frac{1}{4}}\right)\right]$
$e^{-|\omega|/2+i\omega v_j}$

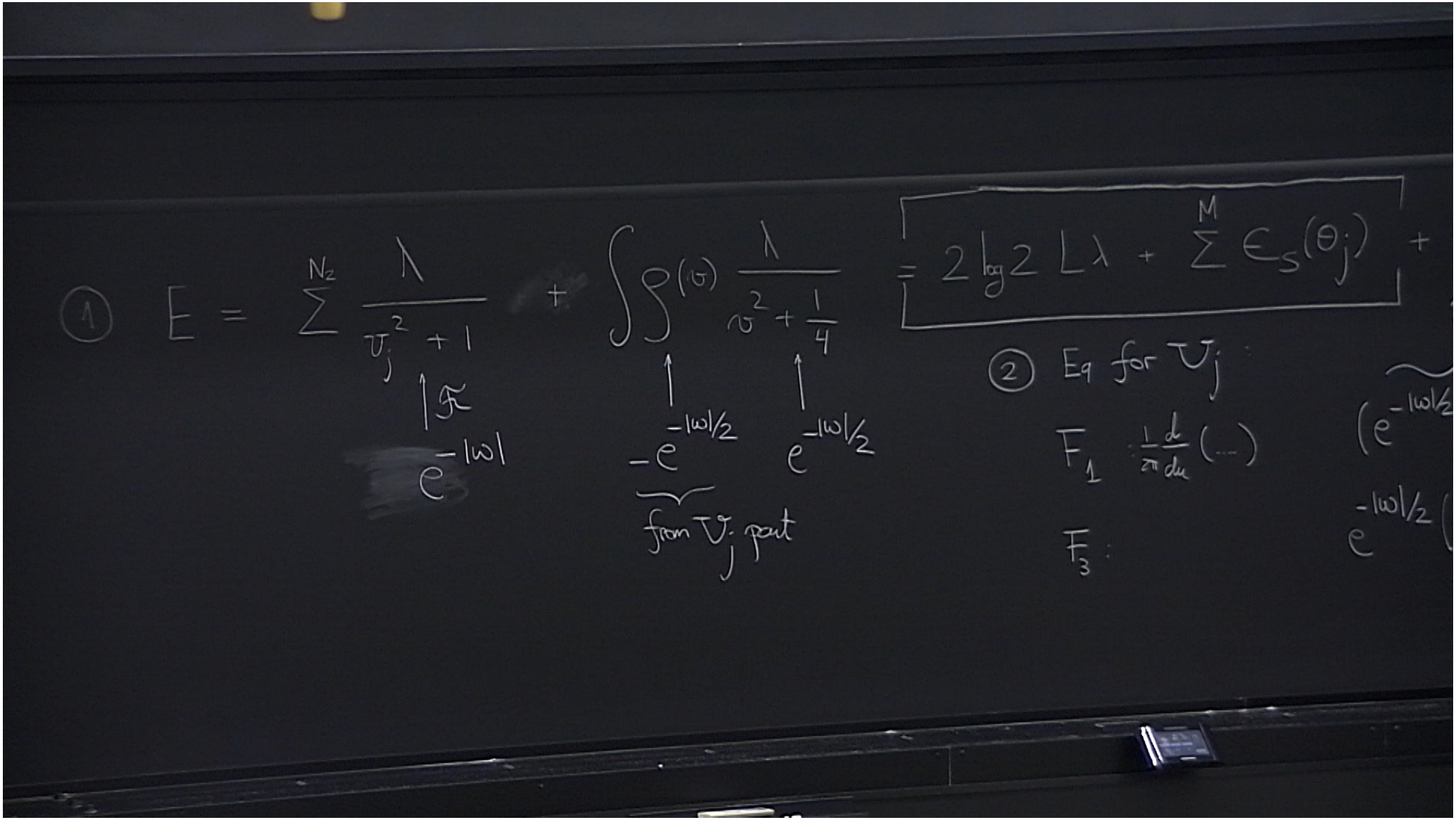

$N_2 \geq 1$ $v_1 \dots v_{N_2}$
$\frac{1}{i}\log\frac{v_j+i}{v_j-i} + \frac{1}{i}\sum_{k=1}^{N_1}\log S_{12}(v_j-u_k)$
$+ \frac{1}{i}\sum_{k\neq j}^{N_2}\log S_{22}(v_j-v_k) = 2\pi n_j^2$
1) find $\rho(u|\theta_j, v_j)$
2) find eqs for θ_j, v_j
$\frac{1}{2\cosh\omega/2} + \sum \frac{e^{i\omega\theta_j}}{1+e^{-|\omega|}} + \{v_j$
$\sum^{N_2}\left[\frac{-1}{1+e^{-|\omega|}}\left(e^{-|\omega|/2} + e^{-3|\omega|/2}\right)\right] = \Re\left(\frac{1}{(u-v_j)^2+\frac{1}{4}}\right)$
$e^{-|\omega|/2+i\omega v_j}$

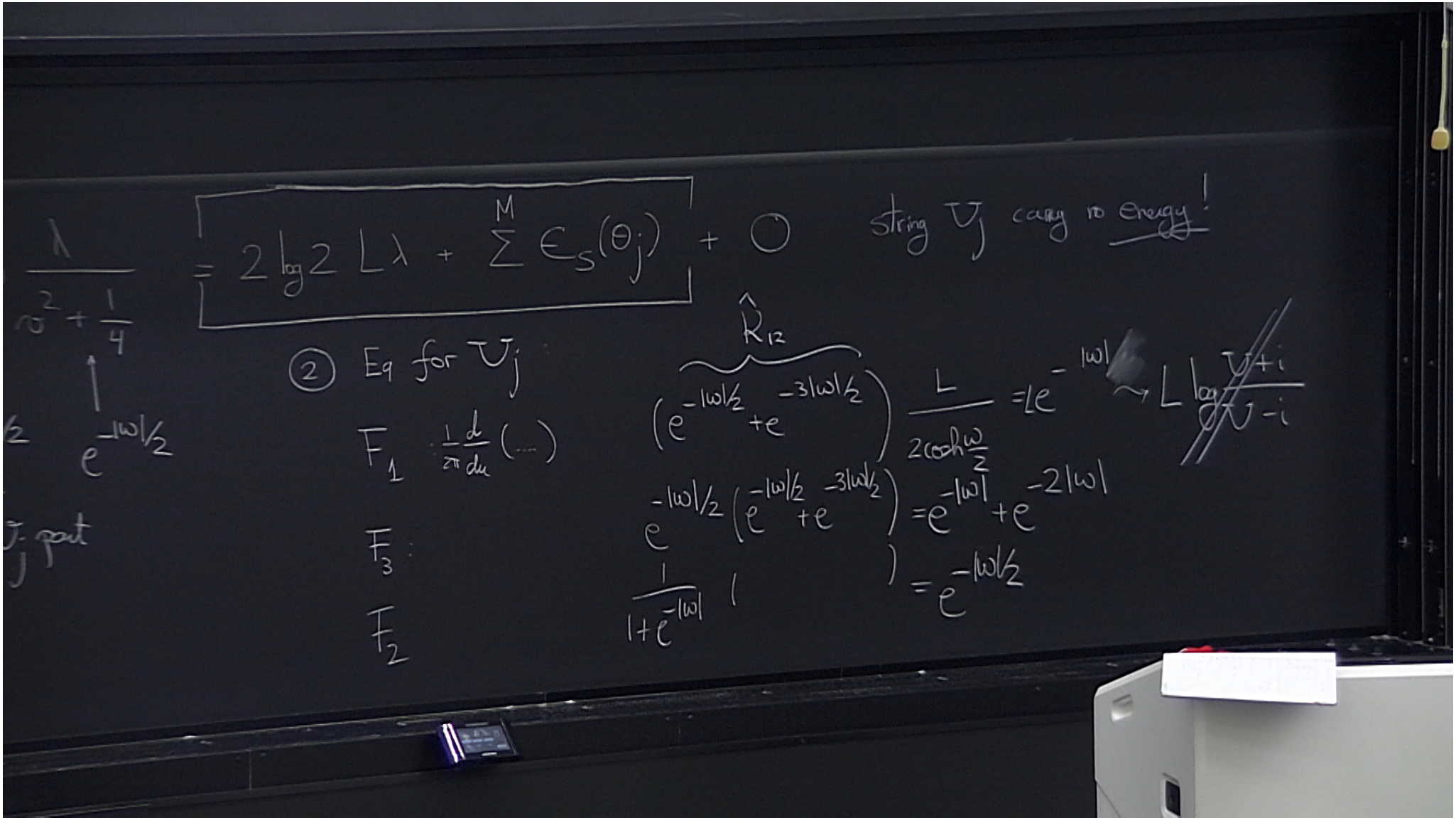

$N_2 \geq 1$ v_1 v_{N_2}
$\frac{1}{i}\log\frac{v_j+i}{v_j-i} + \frac{1}{i}\sum_{k=1}^{N_1}\log S_{12}(v_j-u_k) \to \frac{1}{i}\int\rho(\omega)\log S_{12}(u-\omega)$
1) find $\rho(u|\theta_j, v_j)$
2) find eqs for θ_j, v_j
$+\frac{1}{i}\sum_{k\neq j}^{N_2}\log S_{22}(v_j-v_k) = 2\pi n_j^2$
$\frac{1}{2\cosh\omega/2} + \sum\frac{e^{i\omega\theta_j}}{1+e^{-|\omega|}} + \ldots v_j$
$\sum^{N_2}\left[\frac{-1}{1+e^{-|\omega|}}\left(e^{-|\omega|/2}+e^{-3|\omega|/2}\right)\right] = \pi\left(\frac{1}{(u-v_j)^2+\frac{1}{4}}\right)$
$e^{-|\omega|/2+i\omega v_j}$

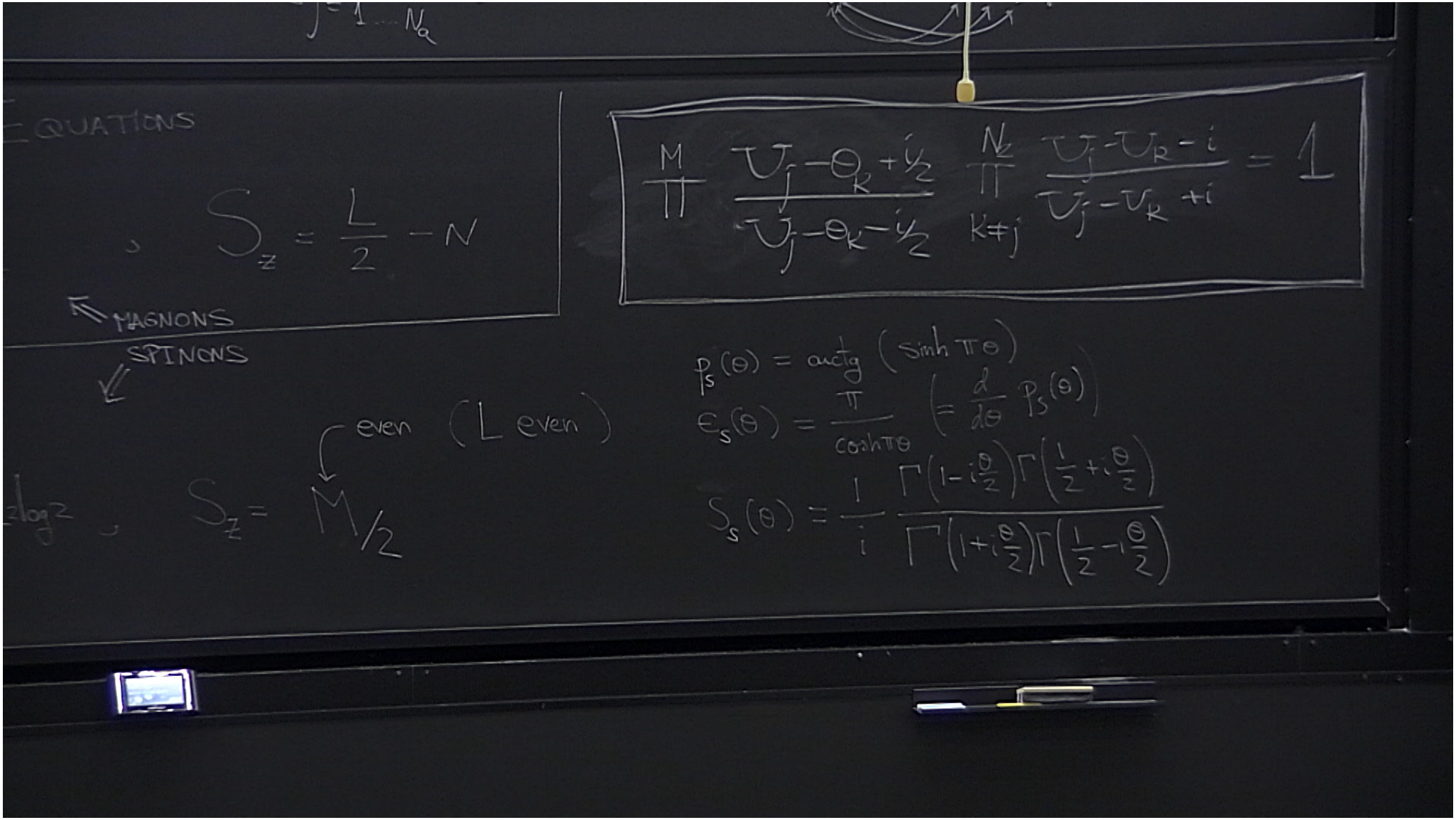

$N_2 \geq 1$
$v_1 \ldots v_{N_2}$
F, changes this
$\frac{L}{i}\log\frac{v_j+i}{v_j-i} + \frac{1}{i}\sum_{k=1}^{N_1}\log S_{12}(v_j-u_k) \to \frac{1}{i}\int\rho(\omega)\log S_{12}(v_j-\omega) = F_1(v_j) + \sum_k F_2(v_j-\theta_k) + \sum F_3(v_j-v_k)$
new
F, changes this
1) find $\rho(u|\theta_j, v_j)$
2) find eqs for θ_j, v_j
$\frac{1}{i}\sum_{k\neq j}^{N_2}\log S_{22}(v_j-v_k) = 2\pi n_j^2$
$\frac{1}{2\cosh\omega/2} + \sum\frac{e^{i\omega\theta_j}}{1+e^{-|\omega|}} + \ldots v_j$
$\sum^{N_2}\left[\frac{-1}{1+e^{-|\omega|}}\left(e^{-|\omega|/2} + e^{-3|\omega|/2}\right) = 2\pi\left(\frac{1}{(u-v_j)^2+\frac{1}{4}}\right)\right] = \rho(\omega)$
$e^{-|\omega|/2+i\omega v_j}$

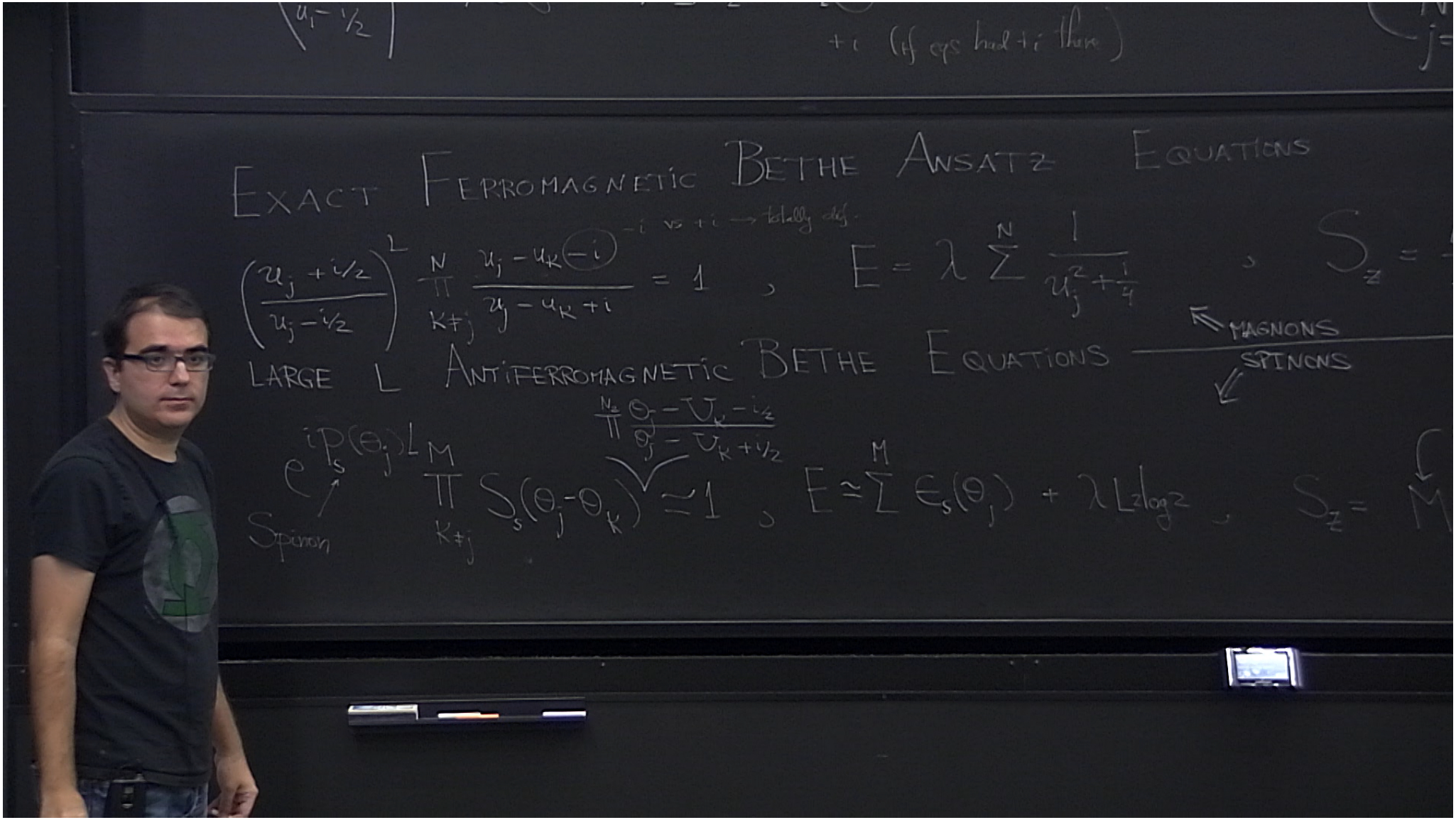

$\mathcal{F}(\log S_s) = \frac{1}{1+e^{-|\omega|}}$
$j = 1, \dots, N$
Position of Holes
$e^{-|\omega|/2}$
① $E = \sum^{N_-} \frac{\lambda}{v_j^2+1} + \int\int \rho(v) \frac{\lambda}{v^2+\frac{1}{4}} = 2\log 2\, L\lambda + \sum^{M} \epsilon_s(\theta_j) + O$
string v_j carry no energy!
$\mathfrak{F}$
$e^{-|\omega|}$
$-e^{-|\omega|/2}$
$e^{-|\omega|/2}$
from v_j part
② Eq for v_j
$F_1 \frac{1}{2\pi}\frac{d}{dv}(-)$
$\hat{R}_{12}$
$(e^{-|\omega|/2} + e^{-3|\omega|/2}) \frac{1}{2\cosh\frac{\omega}{2}} = e^{-|\omega|}$

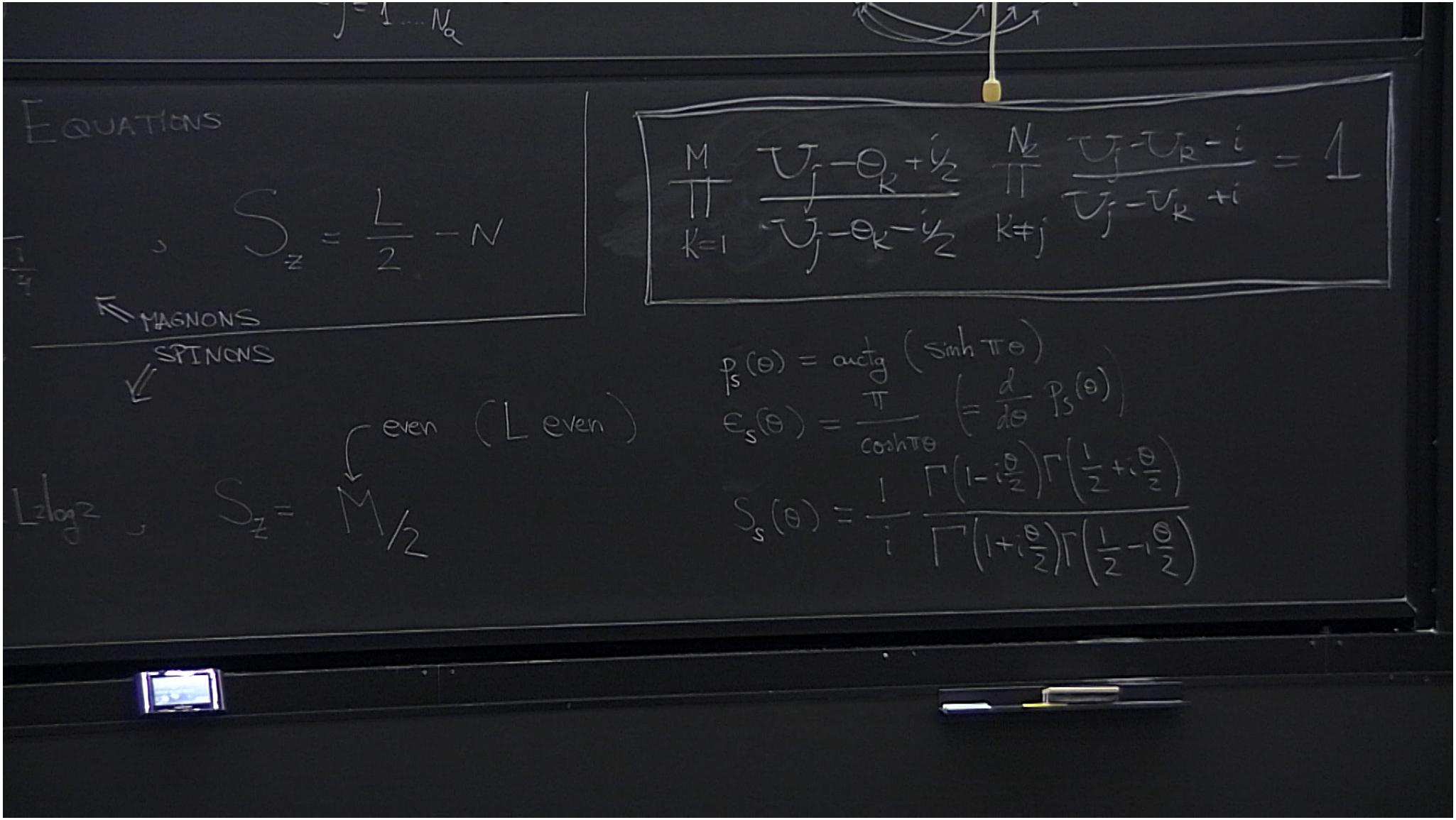

$N_2 \geq 1$
$v_1 \dots v_{N_2}$
F_1 changes this
$\frac{L}{i}\log\frac{v_j+i}{v_j-i} + \frac{1}{i}\sum_{k=1}^{N_1}\log S_{12}(v_j-u_k) \to \frac{1}{i}\int\rho(\omega)\log S_{12}(v_j-\omega) = F_1(v_j) + \sum_k F_2(v_j-\theta_k)$ new
$+\sum F_3(v_j-v_k)$
F_3 changes this
$+\frac{1}{i}\sum_{k\neq j}^{N_2}\log S_{22}(v_j-v_k) = 2\pi n_j^2$
1) find $\rho(u|\theta_j,v_j)$
2) find eqs for θ_j, v_j
$\frac{L}{2\cosh\omega/2} + \sum\frac{e^{i\omega\theta_j}}{1+e^{-|\omega|}} + t(v_j) + \sum_{N_2}\left[\frac{-1}{1+e^{-|\omega|}}\underbrace{\left(e^{-|\omega|/2}+e^{-3|\omega|/2}\right)}_{e^{-|\omega|/2}+i\omega v_j}\mathcal{F}\left(\frac{1}{(u-v_j)^2+\frac{1}{4}}\right)\right] = \rho(\omega)$

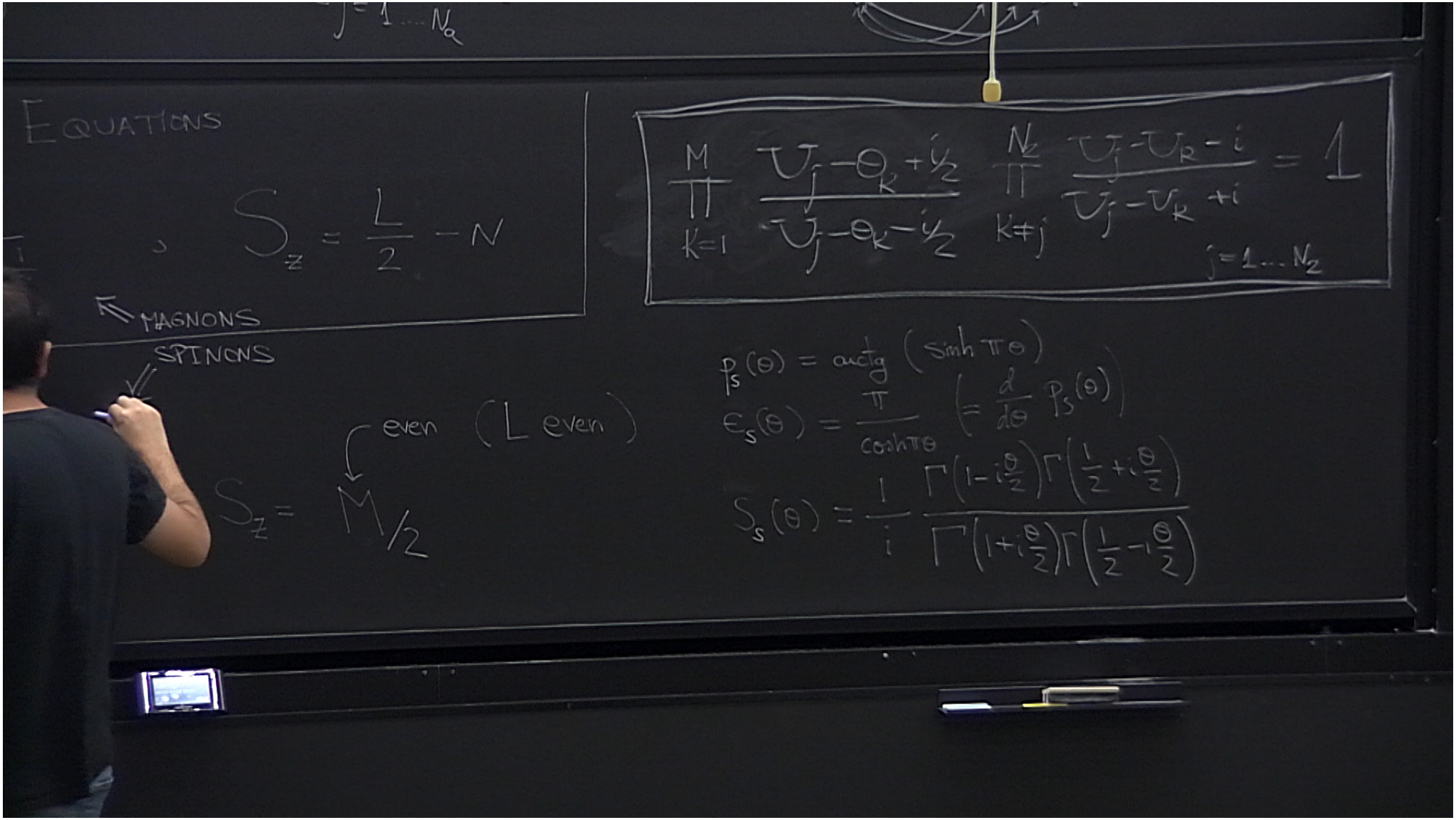

① $E = \sum^{N_2} \frac{\lambda}{v_j^2 + 1} + \int \rho(v) \frac{\lambda}{v^2 + \frac{1}{4}} = 2\log 2\, L\lambda + \sum^{M} \epsilon_s(\theta_j) +$
$\mathcal{F}$
$e^{-|\omega|}$
$-e^{-|\omega|/2}$
$e^{-|\omega|/2}$
from v_j part
② Eq for v_j
F_1 $\frac{1}{\pi}\frac{d}{dv}(\ldots)$
F_3
$(e^{-|\omega|/2}$
$e^{-|\omega|/2}$

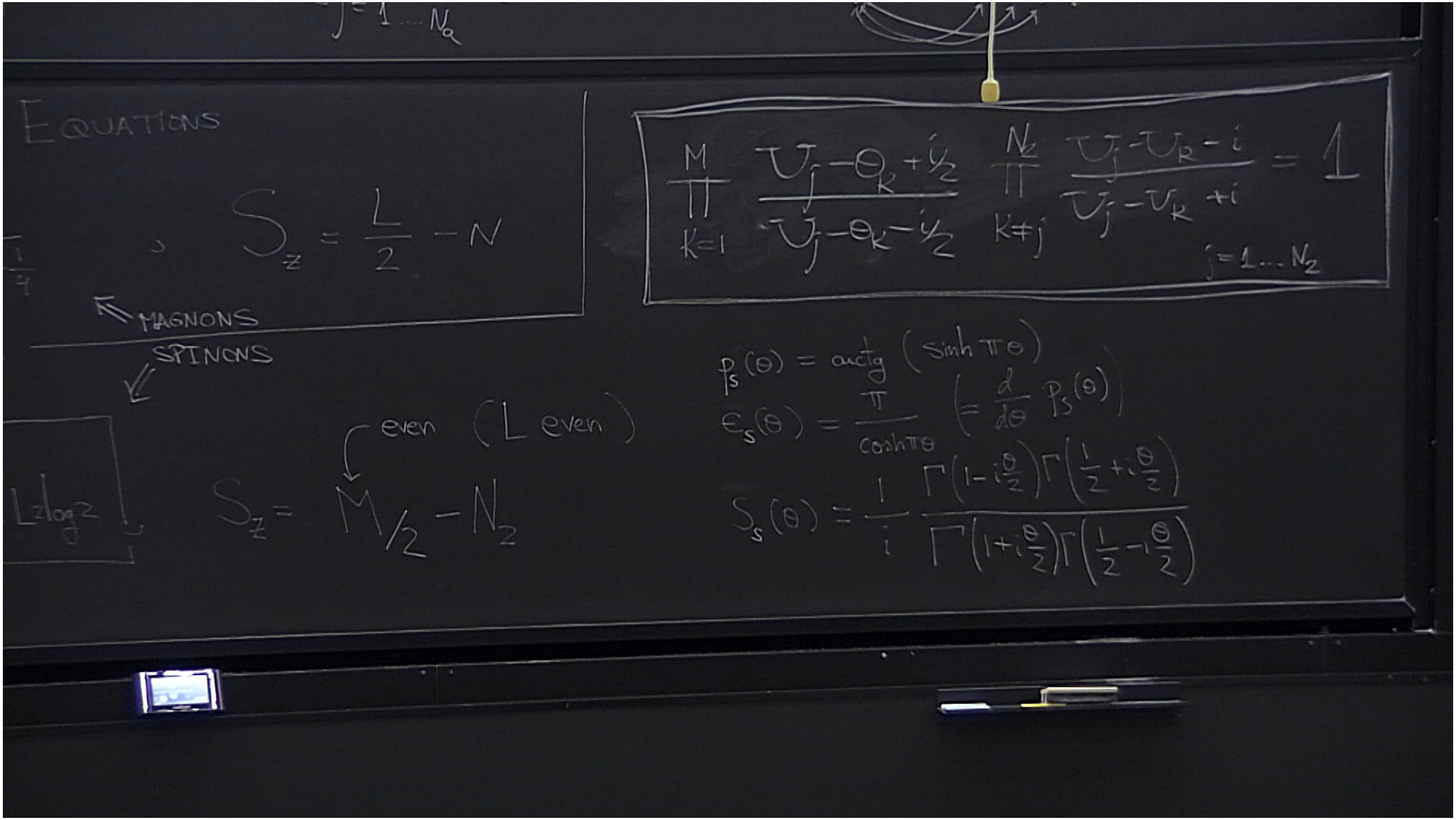

$= 2\log 2\, L\lambda + \sum_{j}^{M} \epsilon_s(\theta_j) + \mathcal{O}$
string v_j carry no energy!
$\frac{\lambda}{u^2 + \frac{1}{4}}$
$e^{-|\omega|/2}$
② Eq for v_j
F_1 $\frac{1}{2\pi i}\frac{d}{du}(\ldots)$
F_3
F_2
$\hat{R}_{12}$
$(e^{-|\omega|/2} + e^{-3|\omega|/2})\frac{L}{2\cosh\frac{\omega}{2}} = Le^{-|\omega|}$
$e^{-|\omega|/2}(e^{-|\omega|/2} + e^{-3|\omega|/2}) = e^{-|\omega|} + e^{-2|\omega|}$
$\frac{1}{1+e^{-|\omega|}}(\quad) = e^{-|\omega|/2}$
$\sim L\log\frac{v+i}{v-i}$


QUATIONS
$S_z = \frac{L}{2} - N$
MAGNONS
SPINONS
even (L even)
$S_z = M/2$
$\frac{M}{\Pi} \frac{v_j - \theta_k + i/2}{v_j - \theta_k - i/2} \frac{N}{\Pi} \frac{v_j - v_k - i}{v_j - v_k + i} = 1$
$k \neq j$
$\rho_s(\theta) = \text{arctg}(\sinh \pi\theta)$
$\epsilon_s(\theta) = \frac{\pi}{\cosh \pi\theta} \left(= \frac{d}{d\theta} \rho_s(\theta)\right)$
$S_s(\theta) = \frac{1}{i} \frac{\Gamma(1 - i\frac{\theta}{2})\Gamma(\frac{1}{2} + i\frac{\theta}{2})}{\Gamma(1 + i\frac{\theta}{2})\Gamma(\frac{1}{2} - i\frac{\theta}{2})}$


EXACT FERROMAGNETIC BETHE ANSATZ EQUATIONS
LARGE L ANTIFERROMAGNETIC BETHE EQUATIONS
MAGNONS
SPINONS
Spinon


EQUATIONS
$S_z = \frac{L}{2} - N$
MAGNONS
SPINONS
even (L even)
$L = 1, 2, \ldots$
$S_z = M/2$
$\prod_{k=1}^{M} \frac{v_j - \theta_k + i/2}{v_j - \theta_k - i/2} \prod_{k \neq j}^{N} \frac{v_j - v_k - i}{v_j - v_k + i} = 1$
$p_s(\theta) = \text{arctg}\left(\sinh \pi\theta\right)$
$\epsilon_s(\theta) = \frac{\pi}{\cosh \pi\theta} \left(= \frac{d}{d\theta} p_s(\theta)\right)$
$S_s(\theta) = \frac{1}{i} \frac{\Gamma\left(1 - i\frac{\theta}{2}\right)\Gamma\left(\frac{1}{2} + i\frac{\theta}{2}\right)}{\Gamma\left(1 + i\frac{\theta}{2}\right)\Gamma\left(\frac{1}{2} - i\frac{\theta}{2}\right)}$


EQUATIONS
$S_z = \frac{L}{2} - N$
MAGNONS
SPINONS
even (L even)
$S_z = M/2$
$\prod_{k=1}^{M} \frac{v_j - \theta_k + i/2}{v_j - \theta_k - i/2} \prod_{k \neq j}^{N_2} \frac{v_j - v_k - i}{v_j - v_k + i} = 1$
$j = 1 \dots N_2$
$p_s(\theta) = \text{arctg}\left(\sinh \pi\theta\right)$
$\epsilon_s(\theta) = \frac{\pi}{\cosh \pi\theta} \left(= \frac{d}{d\theta} p_s(\theta)\right)$
$S_s(\theta) = \frac{1}{i} \frac{\Gamma\left(1 - i\frac{\theta}{2}\right)\Gamma\left(\frac{1}{2} + i\frac{\theta}{2}\right)}{\Gamma\left(1 + i\frac{\theta}{2}\right)\Gamma\left(\frac{1}{2} - i\frac{\theta}{2}\right)}$


EQUATIONS
$S_z = \frac{L}{2} - N$
MAGNONS
SPINONS
$L \geq \log 2$
$S_z = M/2 - N_2$ (even (L even))
$\frac{M}{\Pi}_{k=1} \frac{v_j - \theta_k + i/2}{v_j - \theta_k - i/2} \frac{N_2}{\Pi}_{k \neq j} \frac{v_j - v_k - i}{v_j - v_k + i} = 1$
$j = 1 \dots N_2$
$p_s(\theta) = \text{arctg}(\sinh \pi\theta)$
$\epsilon_s(\theta) = \frac{\pi}{\cosh \pi\theta}$ $\left(= \frac{d}{d\theta} p_s(\theta)\right)$
$S_s(\theta) = \frac{1}{i} \frac{\Gamma(1 - i\frac{\theta}{2})\Gamma(\frac{1}{2} + i\frac{\theta}{2})}{\Gamma(1 + i\frac{\theta}{2})\Gamma(\frac{1}{2} - i\frac{\theta}{2})}$