Title: PSI 2016/2017 Quantum Information (Review) - Lecture 6 (Daniel Gottesman)

Date: Feb 28, 2017 09:00 AM

URL: http://pirsa.org/17020117

Abstract:

$$|\bar{0}\rangle = (|000\rangle + |111\rangle)^{\otimes 3}$$
$$|\bar{1}\rangle = (|000\rangle - |111\rangle)^{\otimes 3}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = iXZ$$
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$Z \otimes Z \otimes I \otimes I \otimes \ldots$

$Z \otimes Z$

$Z \otimes Z$

$Z \otimes Z$

$Z \otimes Z$

$Z \otimes Z$

$X \otimes X \otimes X \otimes X \otimes X \otimes X$

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$S = \{ M \in$

Def: Pauli g

with overall

$|\bar{0}\rangle = (|000\rangle + |111\rangle)^{\otimes 3}$

$|\bar{1}\rangle = (|000\rangle - |111\rangle)^{\otimes 3}$

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[illegible] $Z \otimes I \otimes I \otimes$ [illegible]

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$X \otimes X \otimes X \otimes X \otimes X \otimes X$

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$S = \{ M \in \mathcal{P}_n \mid M|\psi\rangle = |\psi\rangle \; \forall |\psi\rangle \in \text{code} \}$ Stabilizer

Def: Pauli group $\mathcal{P}_n$ = n-qubit tensor products of I, X, Y, Z, with overall phase $\pm 1, \pm i$

$P, Q \in \mathcal{P}_n \Rightarrow$ Either $[P,Q] = 0$ or $\{P,Q\} = 0$ eg $(X \otimes X, Z \otimes Z) = 0$

$P \in \mathcal{P}_n \Rightarrow P^2 = \pm I$

S is a group

$M, N \in S \Rightarrow MN \in S$:

$(MN)|\psi\rangle = M|\psi\rangle = |\psi\rangle$

$-I \notin S$

S is Abelian

$M, N \in S$ $MN|\psi\rangle = |\psi\rangle$

$NM|\psi\rangle = |\psi\rangle$

$[M,N]|\psi\rangle = 0$

$\Rightarrow [M,N] = 0$

Given S s.t. S is an Abelian group and $-I \notin S$, then

$T(S) = \{ |\psi\rangle \mid M|\psi\rangle = |\psi\rangle \; \forall M \in S \}$

Def: For a stabilizer code $C \subset \mathcal{H}$, $T(S(C)) = C$.

Prop: If $|S| = 2^r$, S has r generators and $\dim T(S) = 2^{n-r}$ (n physical qubits)

(i.e. $k = n - r$ logical qubits)

S is Abelian

$M, N \in S \quad MN|\psi\rangle = |\psi\rangle$

$NM|\psi\rangle = |\psi\rangle$

$[M,N]|\psi\rangle = 0$

$\Rightarrow [M,N] = 0$

Def: For a stabilizer code $C \subseteq \mathcal{H}$,

$T(S(C)) = C.$

$M \in S, \; E \in \mathcal{P}_n, \; \{E, M\} = 0$

$M(E|\psi\rangle) = -EM|\psi\rangle = -(E|\psi\rangle)$

$[E, M] = 0$

$M(E|\psi\rangle) = EM|\psi\rangle = E|\psi\rangle$

$N(S) = \{ N \in \mathcal{P}_n \mid [N, M] = 0 \;\; \forall M \in S \}$

$S \subseteq N(S) \qquad |N(S)| = 2^{n+k} \cdot 4$

Undetectable errors $= N(S) \setminus S$.

E & F have same error syndrome

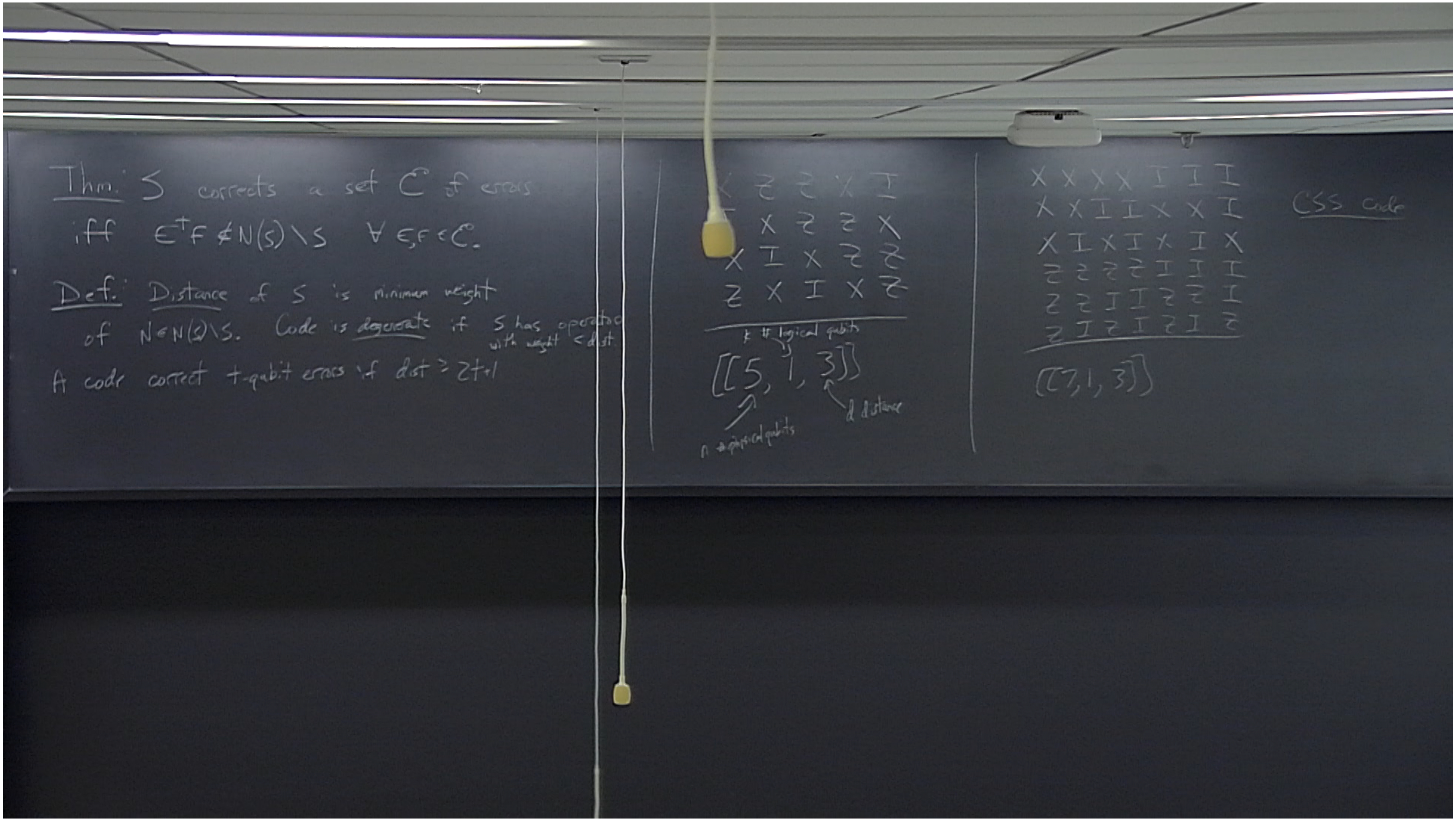
$\Leftrightarrow$ E & F commute with same elements of S

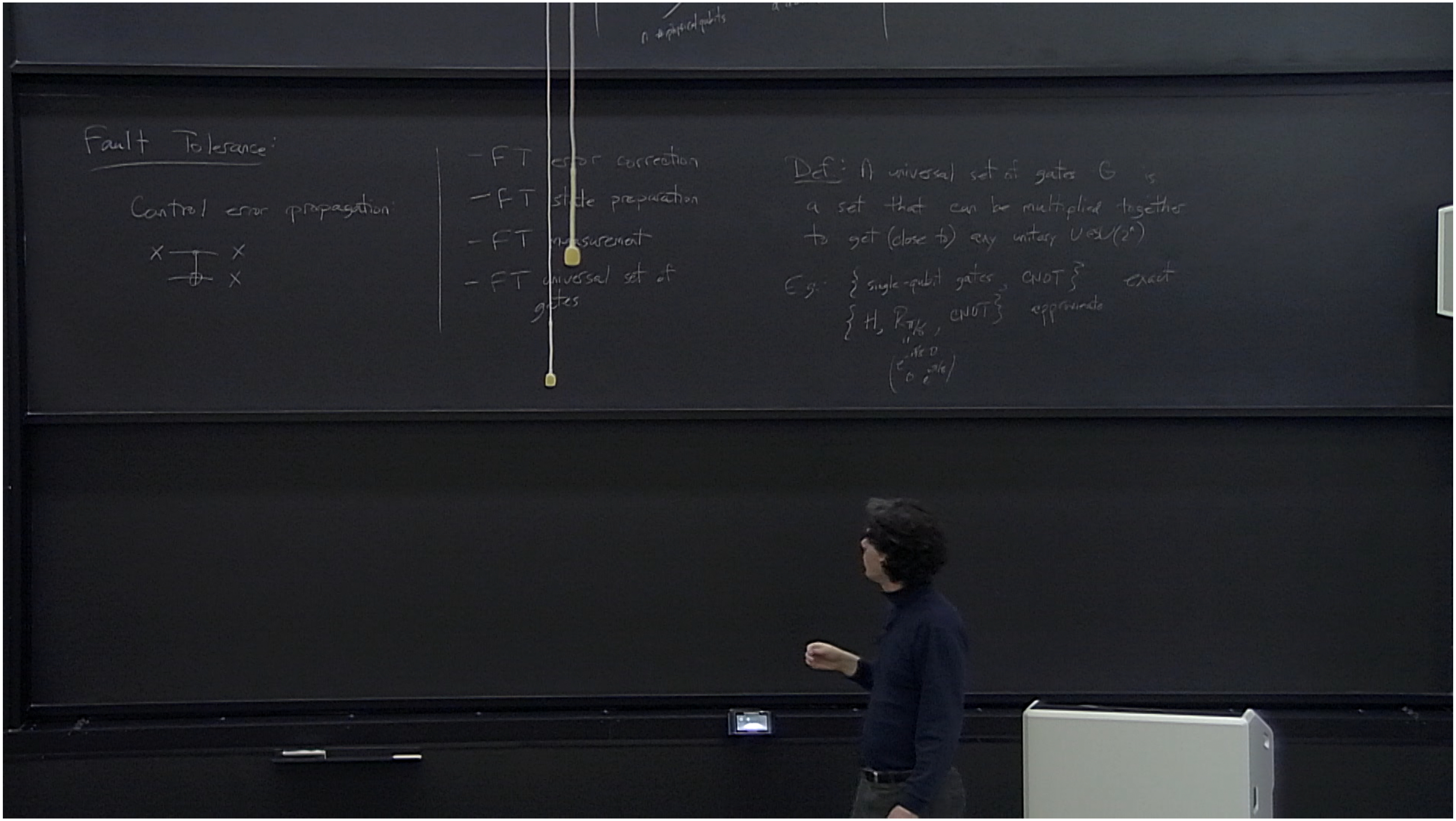
$\Leftrightarrow [E^\dagger F, M] = 0 \quad \forall M \in S$

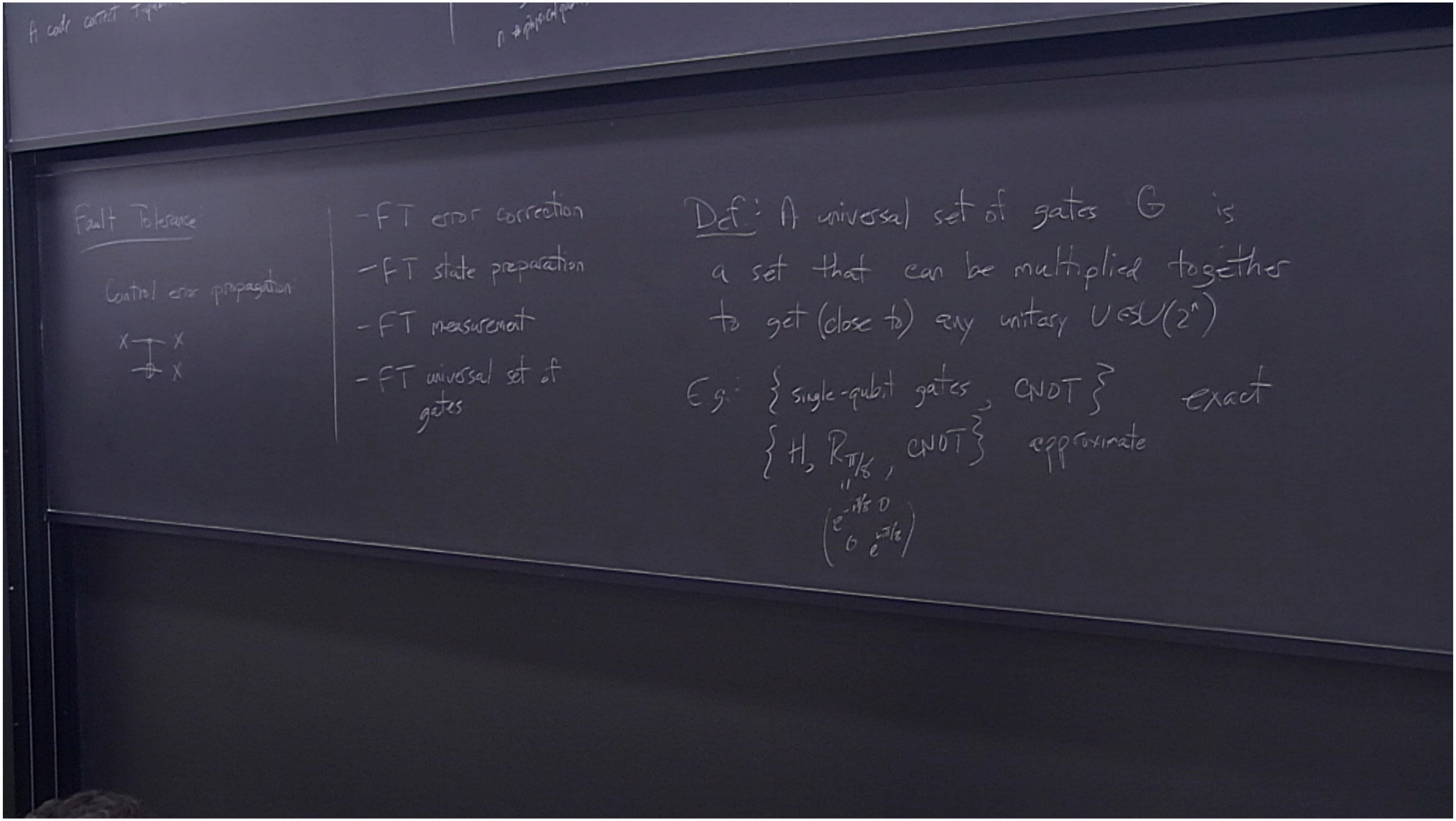
$\Leftrightarrow E^\dagger F \in N(S)$

$E^\dagger F \in S \qquad E^\dagger F|\psi\rangle = |\psi\rangle \quad \forall |\psi\rangle \in T(S)$

$\Leftrightarrow F|\psi\rangle = E|\psi\rangle$


Thm: S corrects a set ℰ of errors
iff E†F ∉ N(S)\S ∀ E,F ∈ ℰ.
Def: Distance of S is minimum weight
of N ∈ N(S)\S. Code is degenerate if S has operator with weight < dist.
A code correct t-qubit errors if dist ≥ 2t+1
X Z Z X I
I X Z Z X
X I X Z Z
Z X I X Z
k # logical qubits
[[5, 1, 3]]
n # physical qubits
d distance
X X X X I I I
X X I I X X I
X I X I X I X
Z Z Z Z I I I
Z Z I I Z Z I
Z I Z I Z I Z
[[7, 1, 3]]
CSS code


Fault Tolerance:
Control error propagation
- FT error correction
- FT state preparation
- FT measurement
- FT universal set of gates
Def: A universal set of gates G is
a set that can be multiplied together
to get (close to) any unitary U∈SU(2^n)
E.g. {single-qubit gates, CNOT} exact
{H, R_π/8, CNOT} approximate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