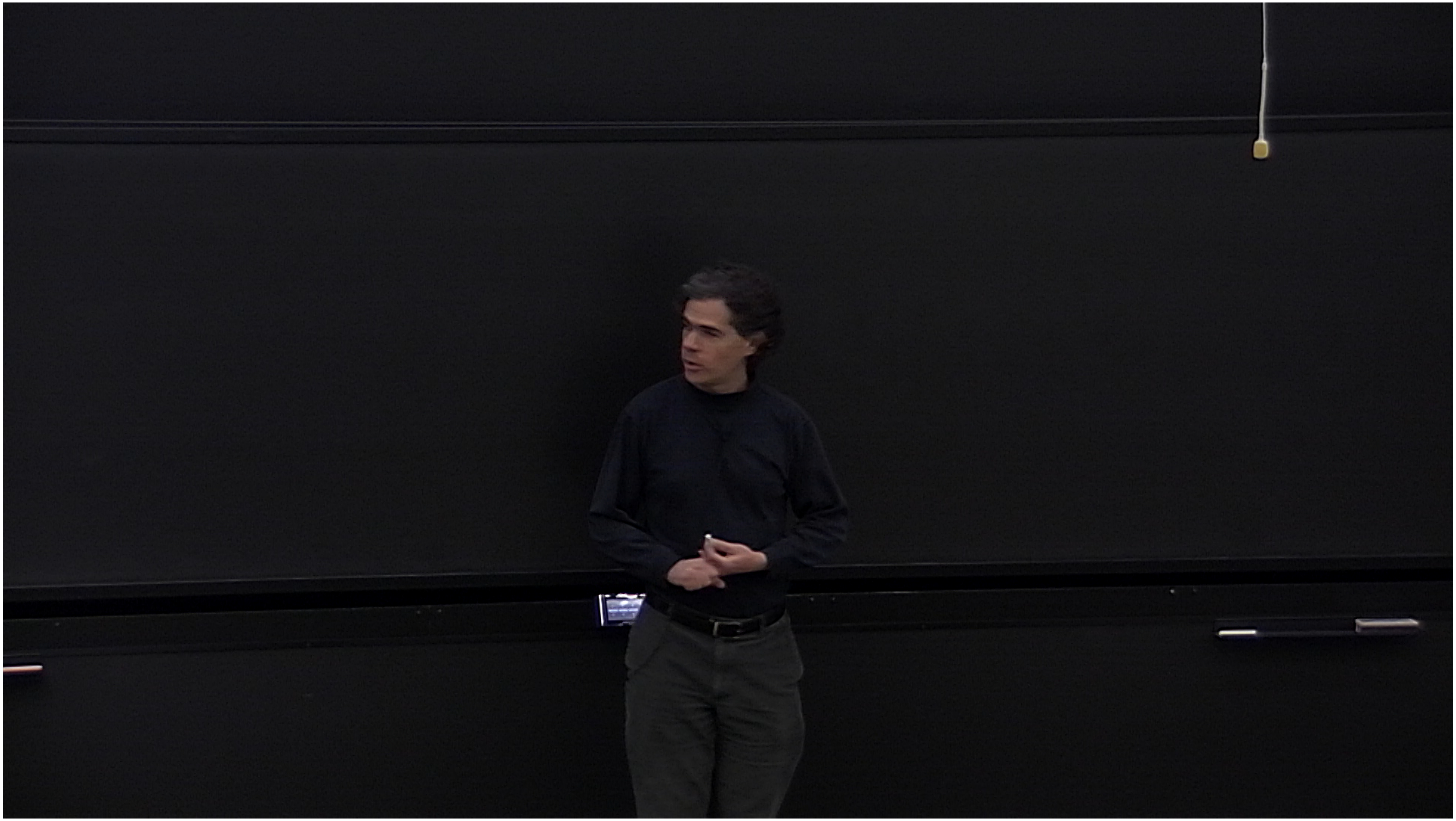
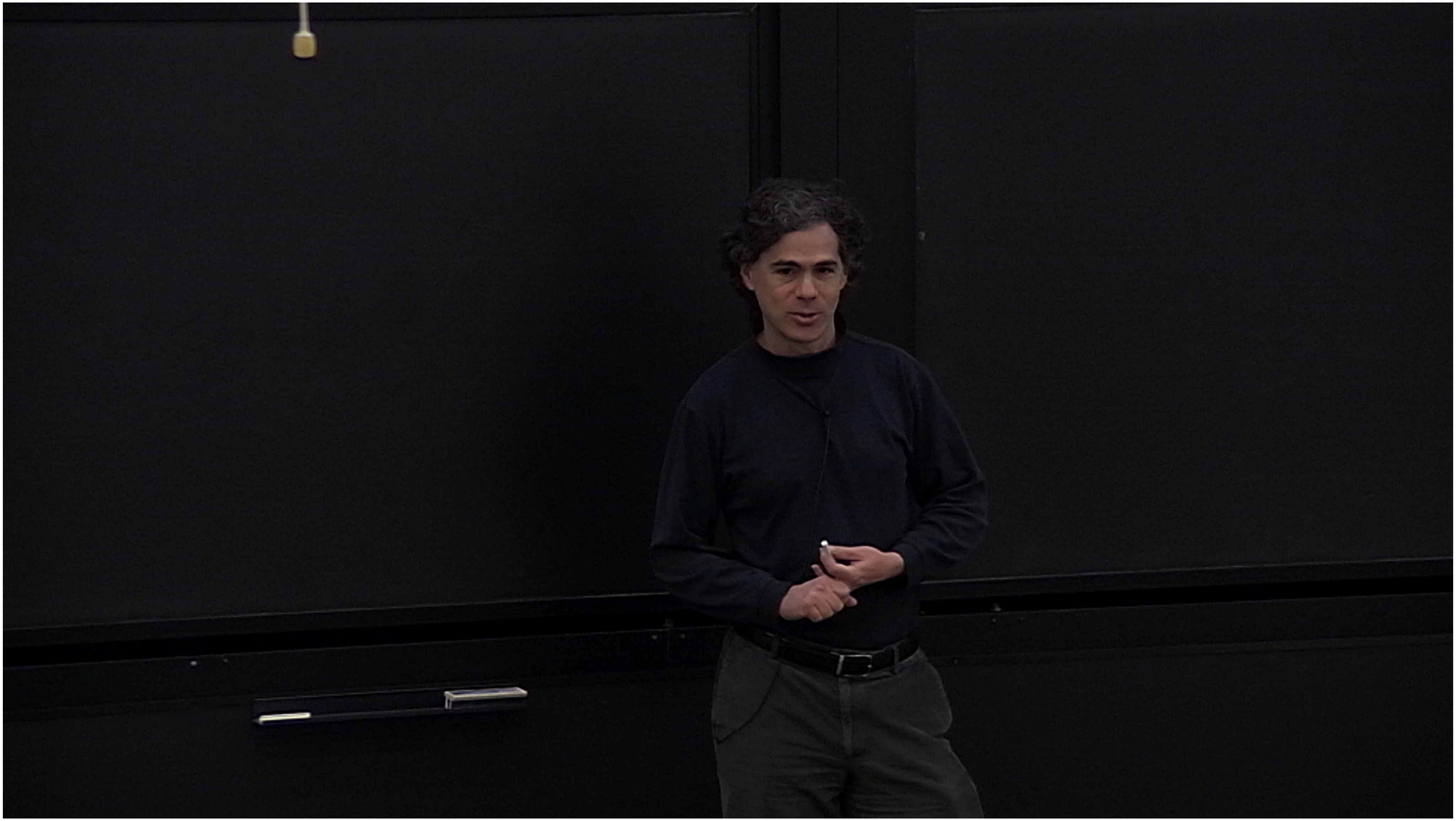
Title: PSI 2016/2017 Quantum Information (Review) - Lecture 5 (Daniel Gottesman)

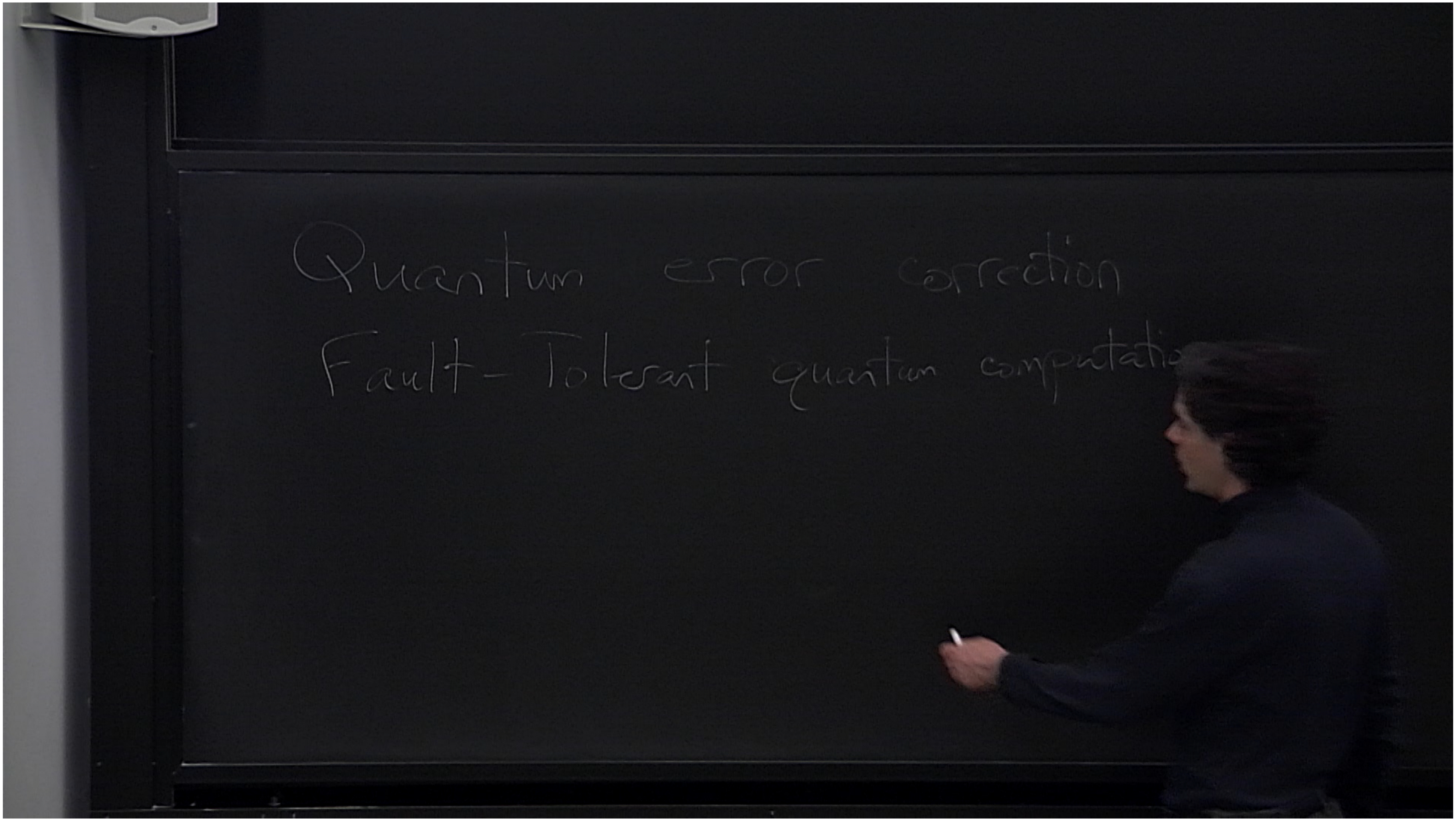
Date: Feb 27, 2017 09:00 AM

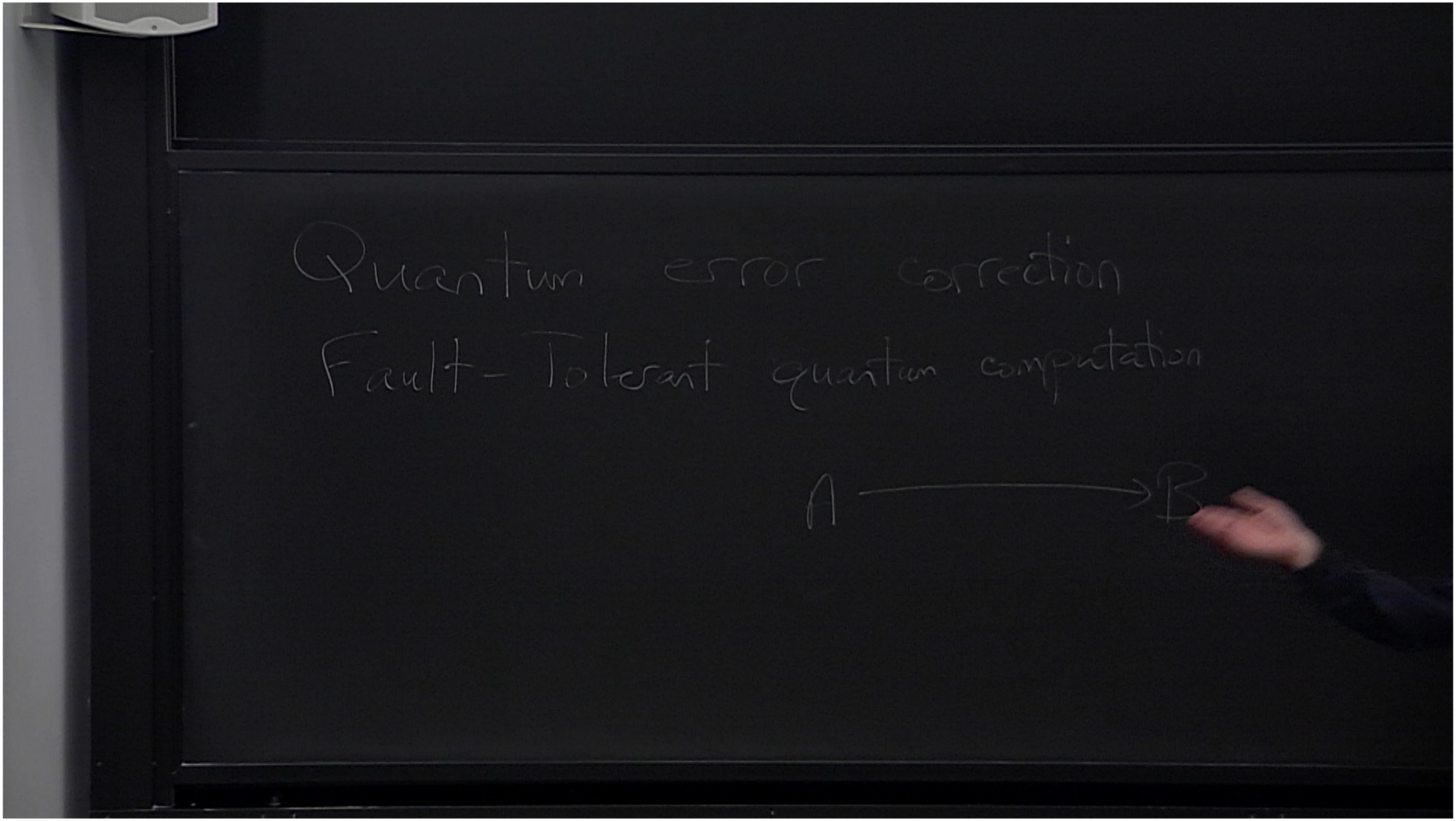
URL: http://pirsa.org/17020116

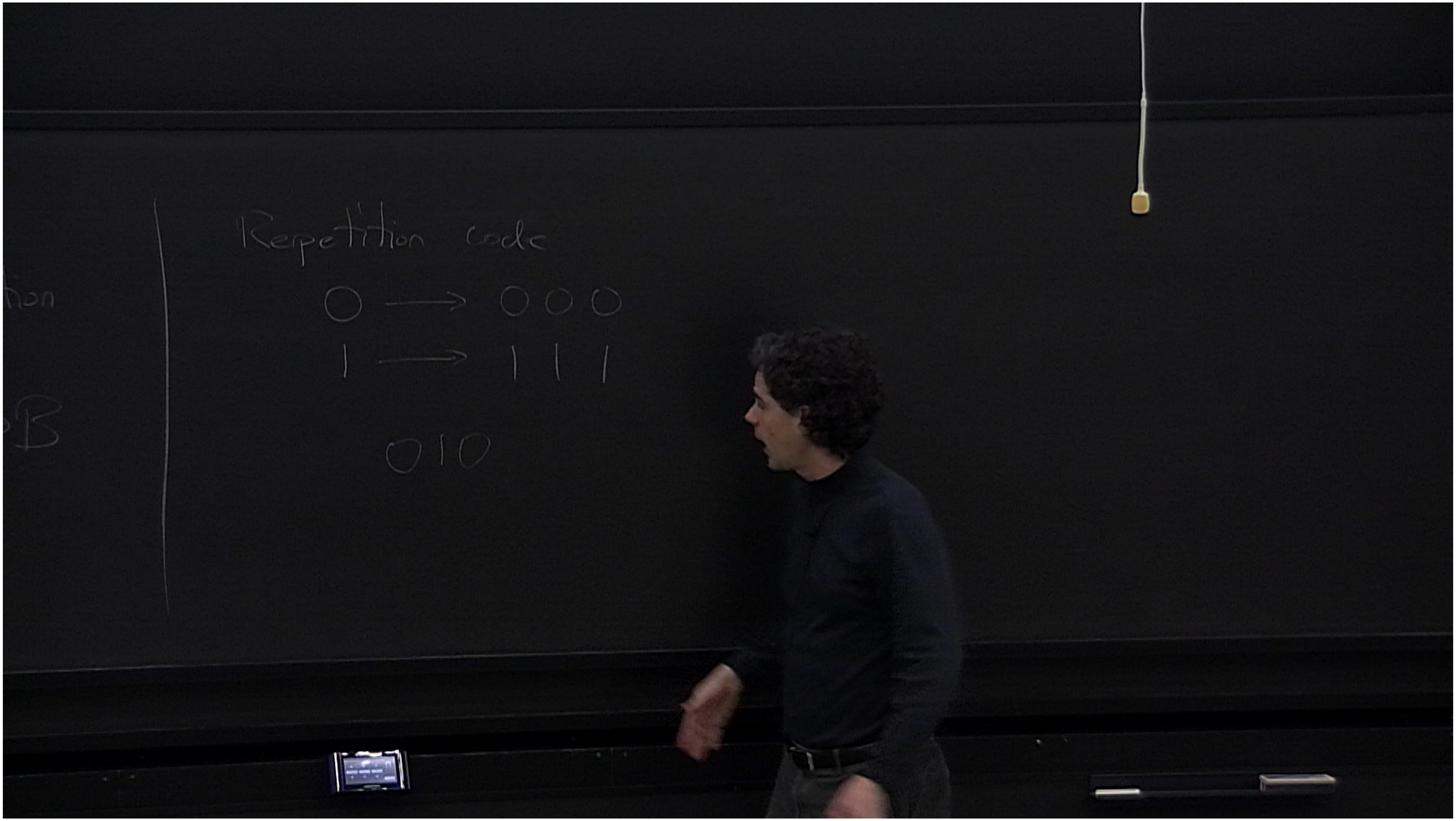
Abstract:

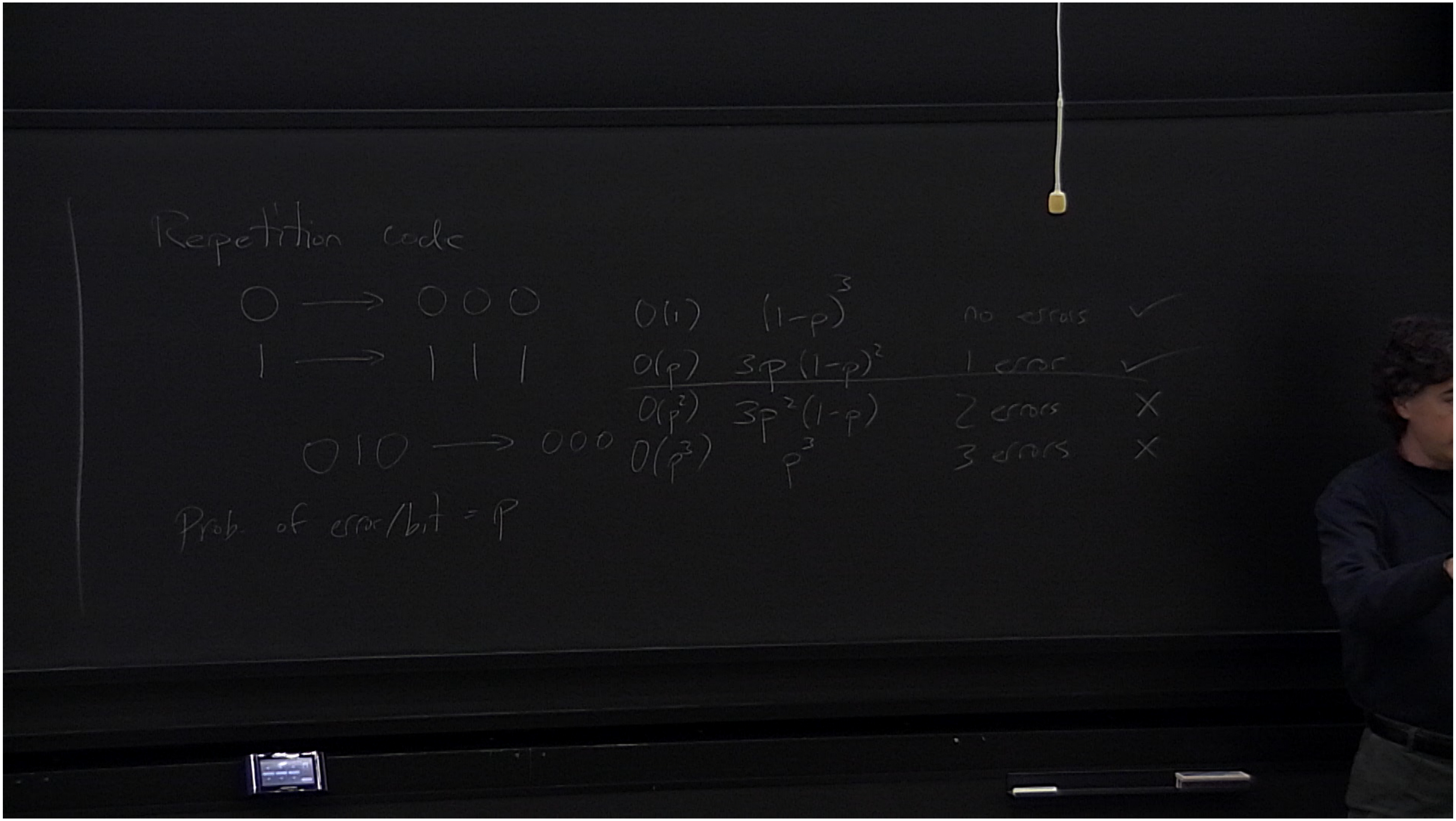





Quantum error correction
Fault-Tolerant quantum computatio


Quantum error correction
Fault-Tolerant quantum computation
A B


Repetition code
0 → 000
1 → 111
010


Repetition code
0 → 000
1 → 111
010 → 000
Prob. of error/bit = p
O(1) (1-p)^3 no errors
O(p) 3p(1-p)^2 1 error
O(p^2) 3p^2(1-p) 2 errors
O(p^3) p^3 3 errors

Problems with QECCs

1. No-cloning thm. prohibits repetition
2. Measuring to find errors collapse superpositions

## Types of quantum errors

bit flip $\quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad |0\rangle \to |1\rangle, \quad |1\rangle \to |0\rangle$

phase flip $\quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad |0\rangle \to |0\rangle, \quad |1\rangle \to -|1\rangle$

$$Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad |0\rangle \to i|1\rangle, \quad |1\rangle \to -i|0\rangle$$

$010 \longrightarrow 000$ | $O(p^3)$ $p^3$ 3 errors X

Prob. of error/bit = $p$

Types of quantum errors

bit flip $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $|0\rangle \longrightarrow |1\rangle$, $|1\rangle \longrightarrow |0\rangle$

phase flip $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $|0\rangle \longrightarrow |0\rangle$, $|1\rangle \longrightarrow -|1\rangle$

$Y = iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $|0\rangle \rightarrow i|1\rangle$, $|1\rangle \rightarrow -i|0\rangle$

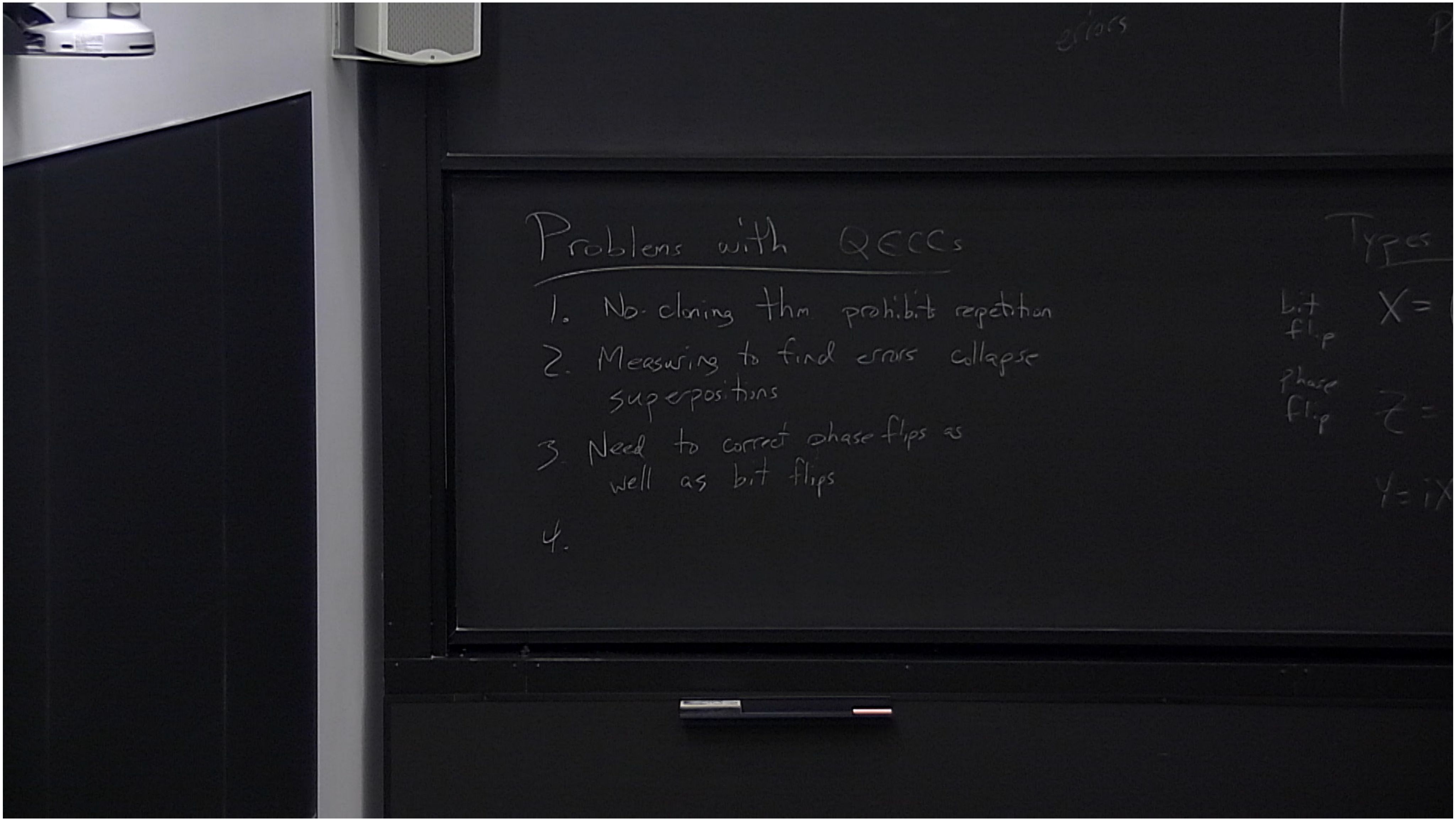
$R_\theta = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix}$ $|0\rangle \rightarrow e^{-i\theta}|0\rangle$, $|1\rangle \rightarrow e^{i\theta}|1\rangle$

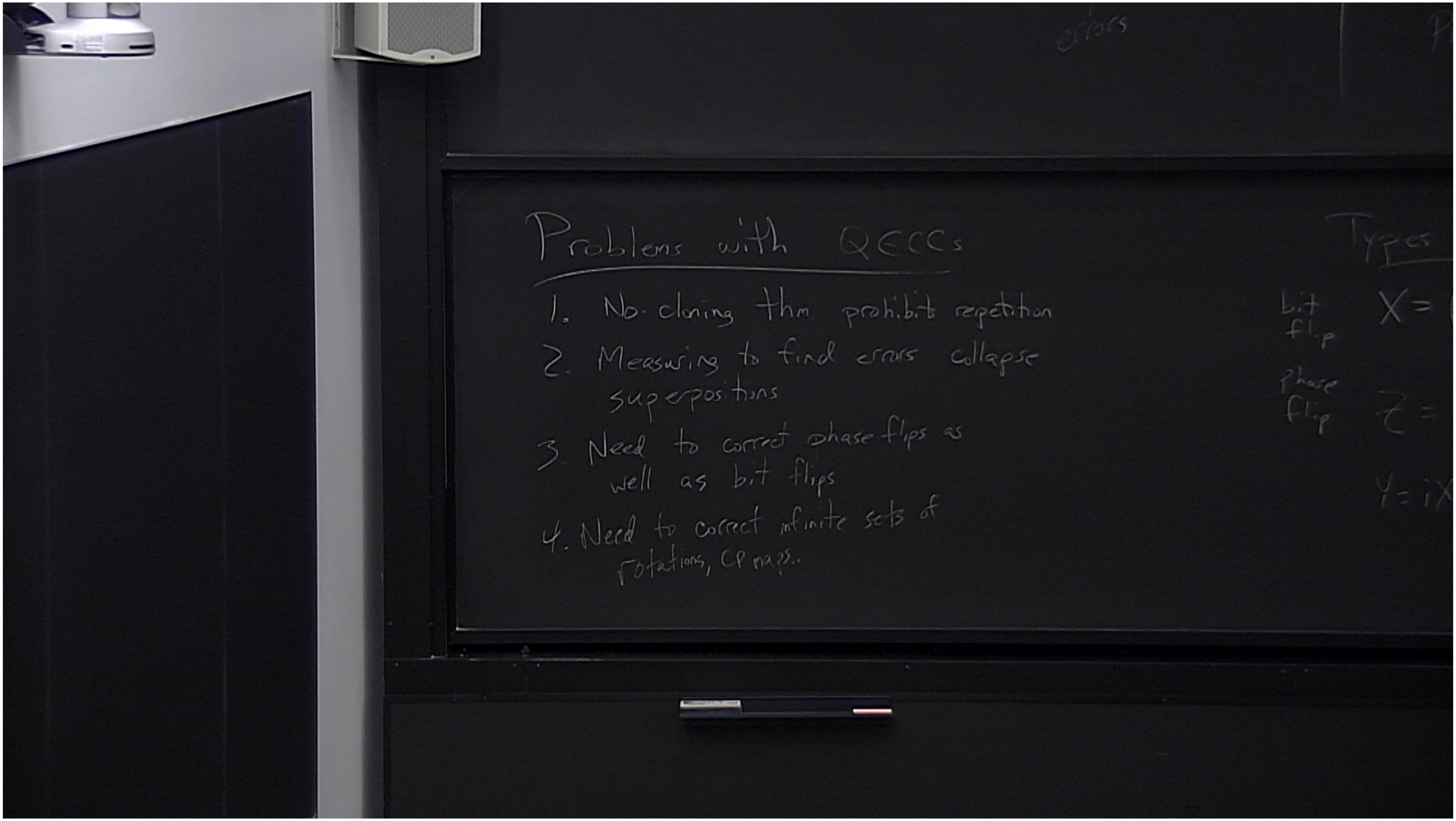
$\rho \Rightarrow (1-p)\rho + p Z \rho Z$

$p = \frac{1}{2}$ $(\alpha|0\rangle + \beta|1\rangle)(\alpha\langle 0| + \beta\langle 1|)$

$\longrightarrow |\alpha|^2 |0\rangle\langle 0| + |\beta|^2 |1\rangle\langle 1|$

$\rho \mapsto \sum_k A_k \rho A_k^\dagger$


Problems with QECCs
1. No-cloning thm prohibits repetition
2. Measuring to find errors collapse superpositions
3. Need to correct phase flips as well as bit flips
4.
Types
bit flip
phase flip


Problems with QECCs
1. No-cloning thm prohibits repetition
2. Measuring to find errors collapse superpositions
3. Need to correct phase flips as well as bit flips
4. Need to correct infinite sets of rotations, CP maps..
bit flip
phase flip

well as bit flips

4. Need to correct infinite sets of rotations, CP maps..

$Y = iXZ = ( i \;\; 0 )$ $\quad |1\rangle \to -i|0\rangle$

$$|0\rangle \longrightarrow |000\rangle$$

$$|1\rangle \longrightarrow |111\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|000\rangle + \beta|111\rangle$$

$$X_2(\alpha|000\rangle + \beta|111\rangle)$$

$$= \alpha|010\rangle + \beta|101\rangle$$

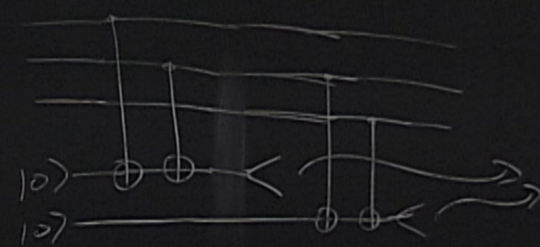
$Y = iXZ = \begin{pmatrix} & \\ i & 0 \end{pmatrix}$ $\quad |1\rangle \to -i|0\rangle$

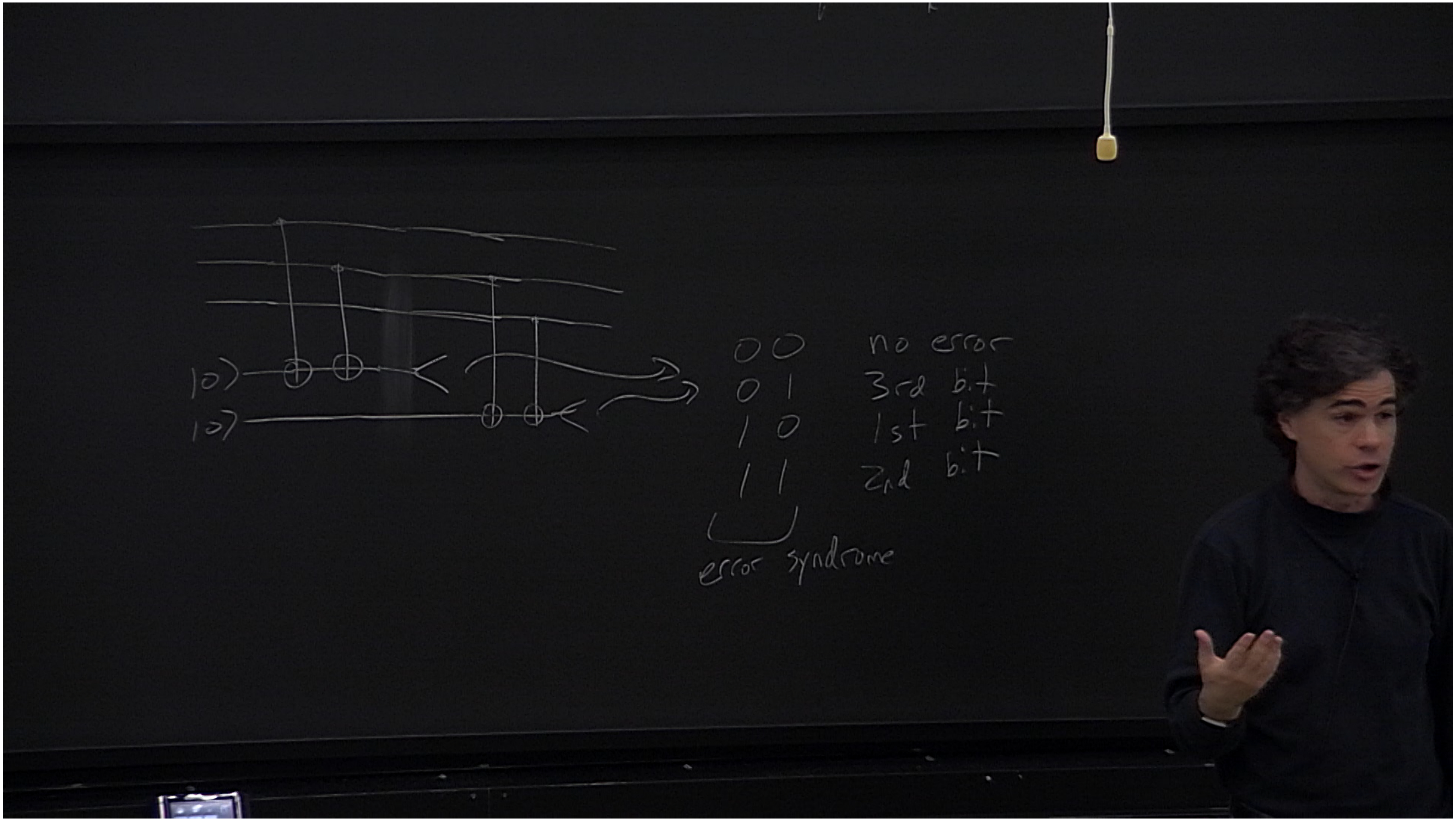
$\rho \mapsto \sum_k A_k \rho A_k^\dagger$

$X_2(\alpha|000\rangle + \beta|111\rangle)$

$= \alpha|010\rangle + \beta|101\rangle$

$|111\rangle$




|0⟩
|0⟩
00 no error
01 3rd bit
10 1st bit
11 2nd bit
error syndrome

Hadamard

$$|0\rangle \to |0\rangle + |1\rangle = |+\rangle \qquad Z|+\rangle = |-\rangle$$

$$|1\rangle \to |0\rangle - |1\rangle = |-\rangle \qquad Z|-\rangle = |+\rangle$$

$$|0\rangle \to |+\rangle|+\rangle|+\rangle$$

$$|1\rangle \to |-\rangle|-\rangle|-\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \to \alpha|+++\rangle + \beta|---\rangle$$

$$Z_1 : \to \alpha|-++\rangle + \beta|+--\rangle$$

9-qubit code

$$|\overline{0}\rangle = \big(|000\rangle+|111\rangle\big)\big(|000\rangle+|111\rangle\big)\big(|000\rangle+|111\rangle\big)$$

$$|\overline{1}\rangle = \big(|000\rangle-|111\rangle\big)\big(|000\rangle-|111\rangle\big)\big(|000\rangle-|111\rangle\big)$$

Corrects single-qubit
X, Y, Z errors.

$$R_\theta = \begin{pmatrix} e^{-i\theta} & 0 \\ 0 & e^{i\theta} \end{pmatrix} = \cos\theta\, I - i\sin\theta\, Z$$

$$R_\theta^{(k)}|\psi\rangle = \cos\theta\, I\,|\psi\rangle - i\sin\theta\, Z^{(k)}|\psi\rangle$$

$$\Big\downarrow \text{ error correction}$$

$$\cos\theta\, I|\psi\rangle|\text{no error}\rangle - i\sin\theta\, Z^{(k)}|\psi\rangle|Z^{(k)}\text{error}\rangle$$

$$\Big\downarrow \text{ measure ancilla}$$

Prob. $\cos^2\theta$ : $I|\psi\rangle|\text{no error}\rangle$

Prob. $\sin^2\theta$ : $Z^{(k)}|\psi\rangle|Z^{(k)}\text{error}\rangle$

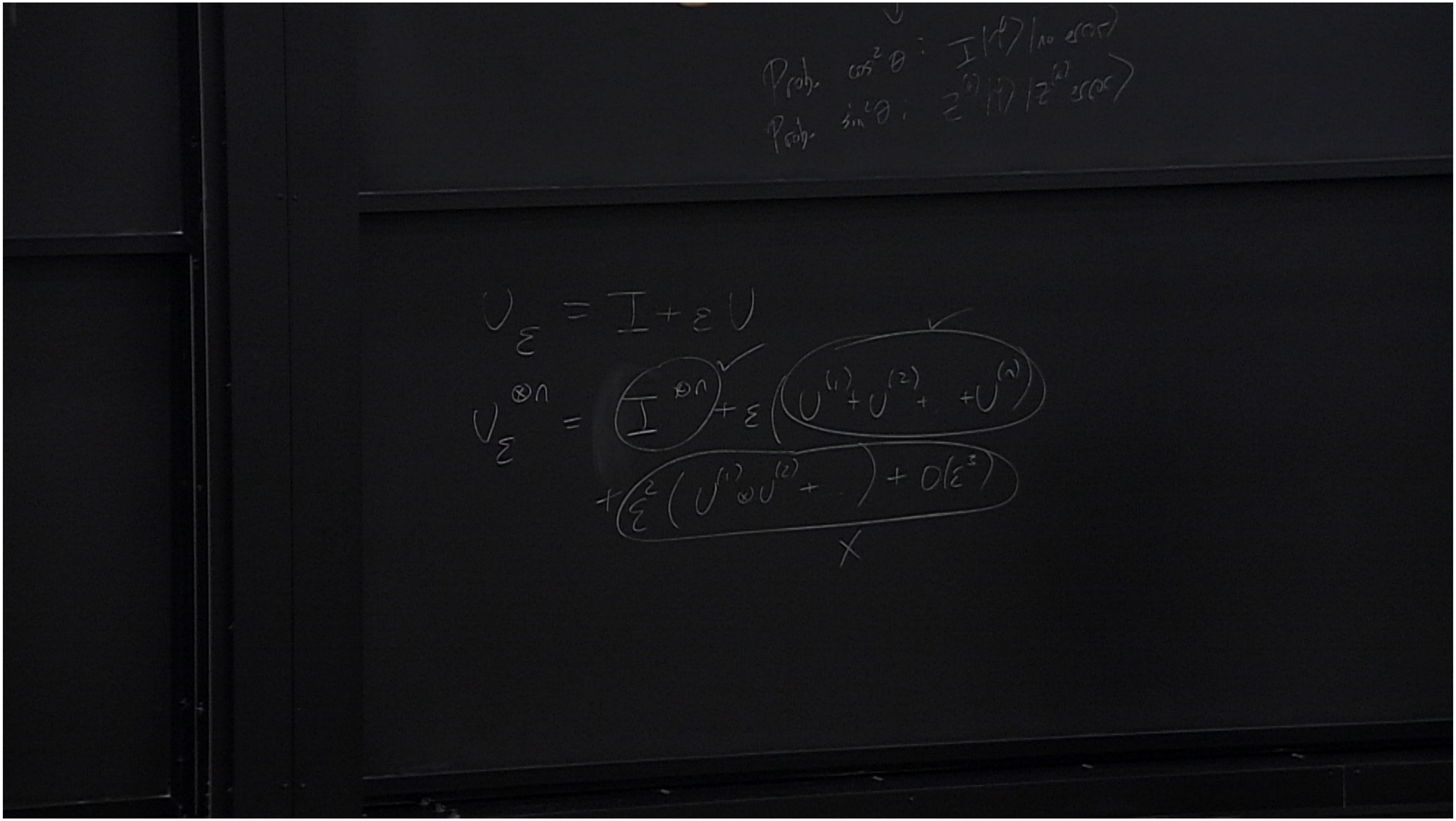
Thm: If

Cor:

Thm: If a QECC corrects $A$ & $B$, it also corrects $\alpha A + \beta B$

Cor: If a QECC corrects $I, X, Y, Z$ on single qubits, it corrects all one-qubit errors

$Z^{(k)}$ error)


Prob. $\cos^2\theta$ : $I|\psi\rangle|\text{no error}\rangle$
Prob. $\sin^2\theta$ : $Z^{(1)}|\psi\rangle|Z^{(1)}\text{ error}\rangle$
$U_\varepsilon = I + \varepsilon U$
$U_\varepsilon^{\otimes n} = I^{\otimes n} + \varepsilon\left(U^{(1)} + U^{(2)} + \dots + U^{(n)}\right)$
$+ \varepsilon^2\left(U^{(1)} \otimes U^{(2)} + \dots\right) + O(\varepsilon^3)$