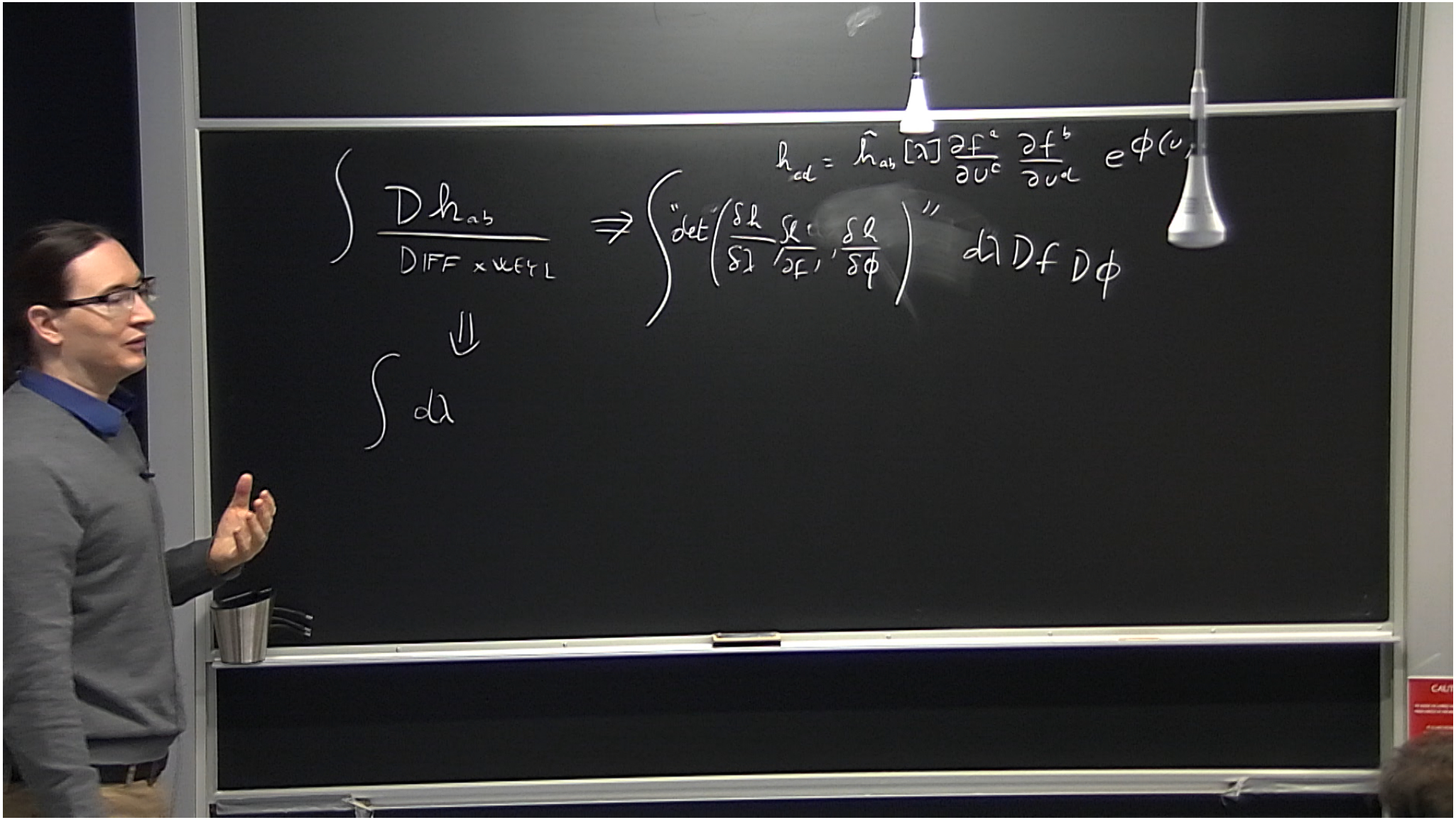
Title: PSI 2016/2017 String Theory (Review) - Lecture 5

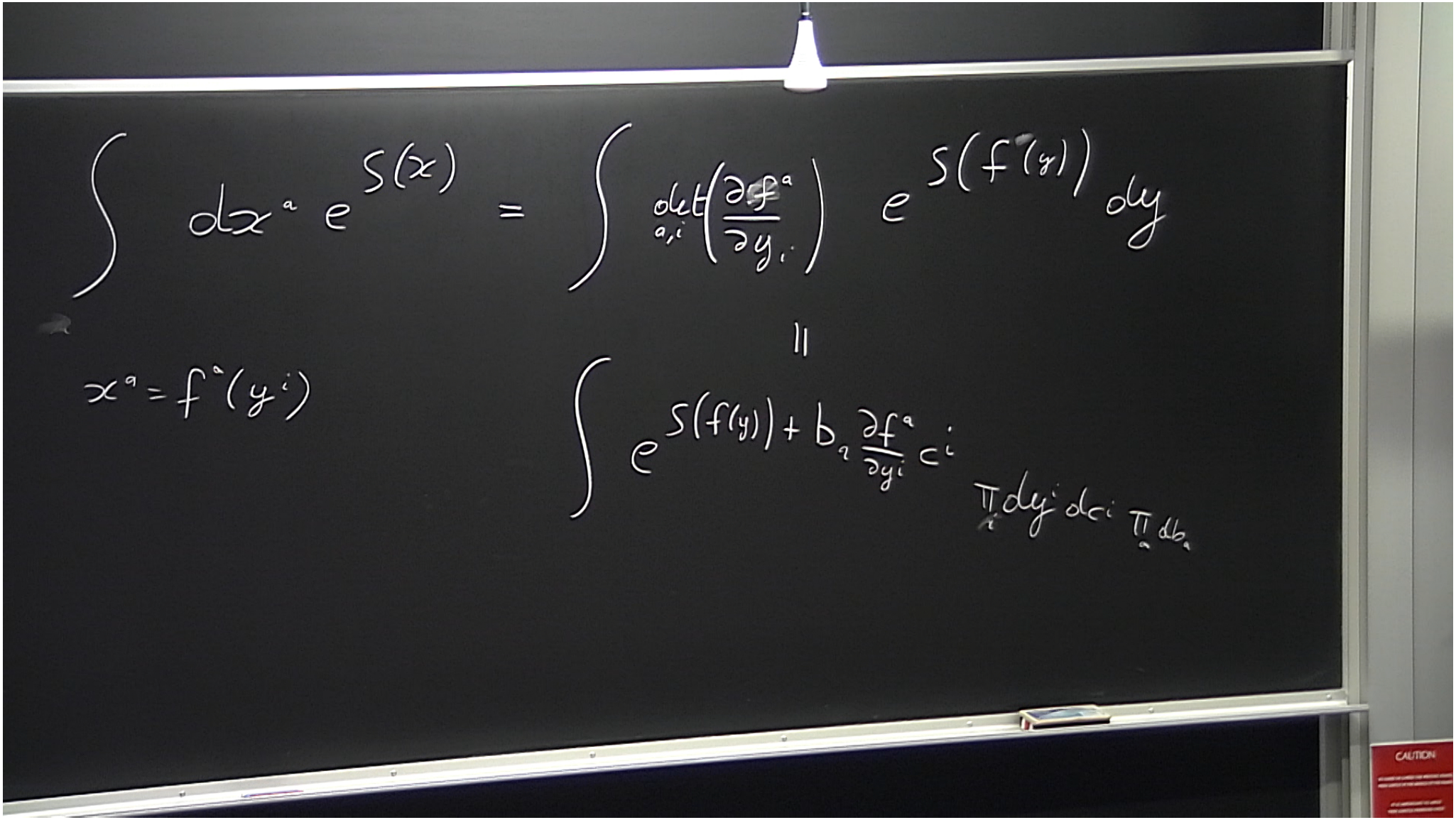
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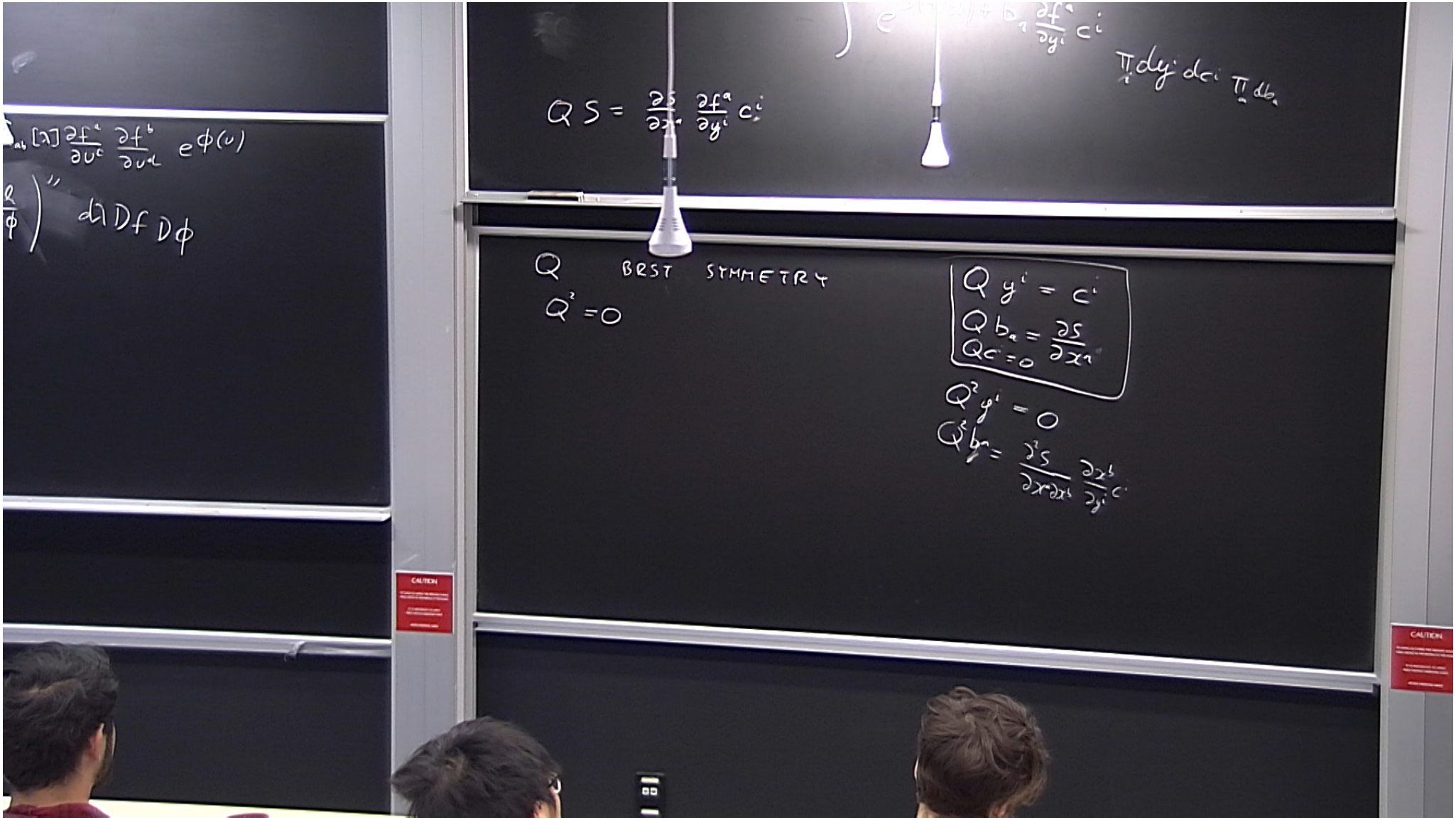
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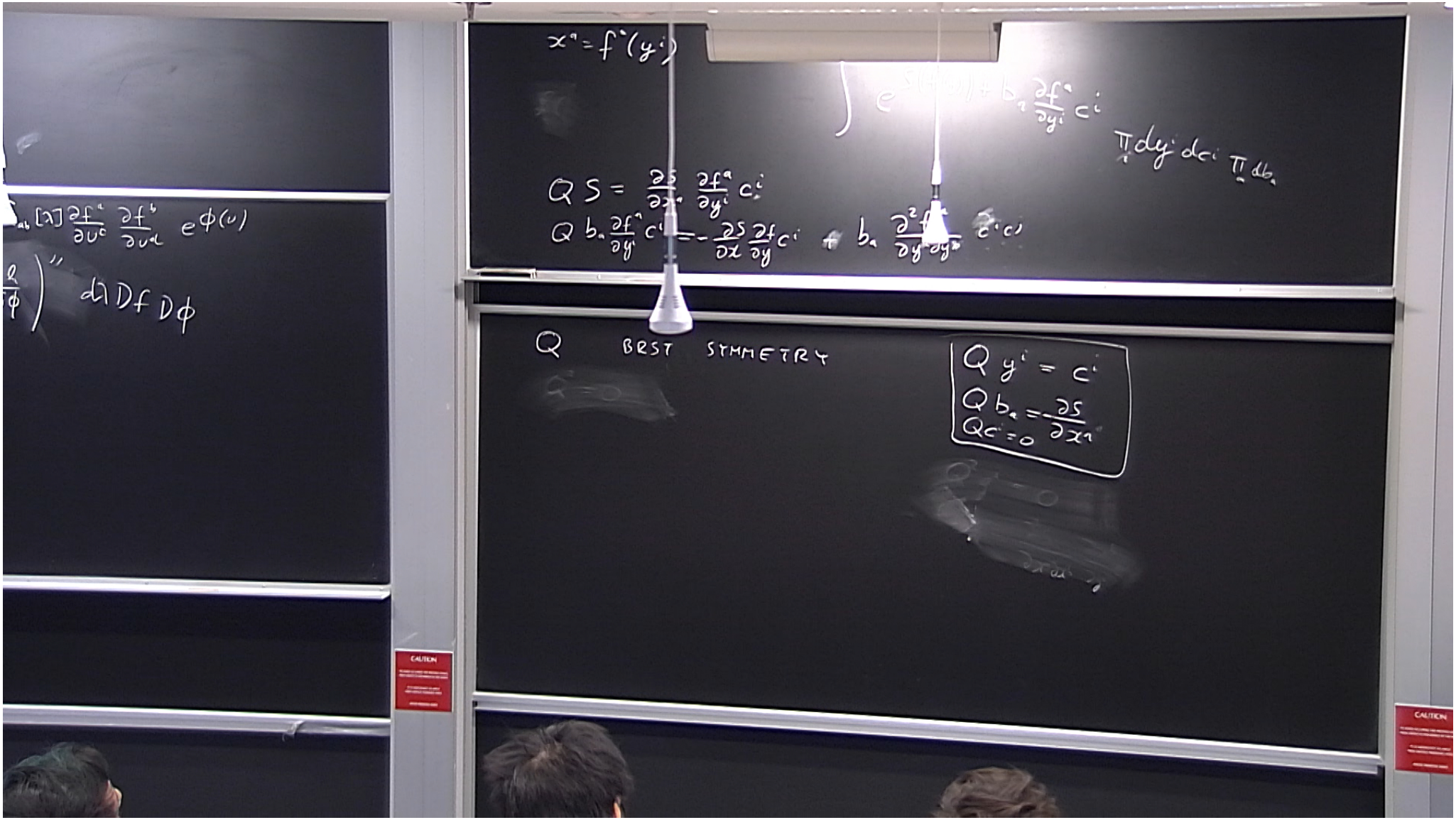
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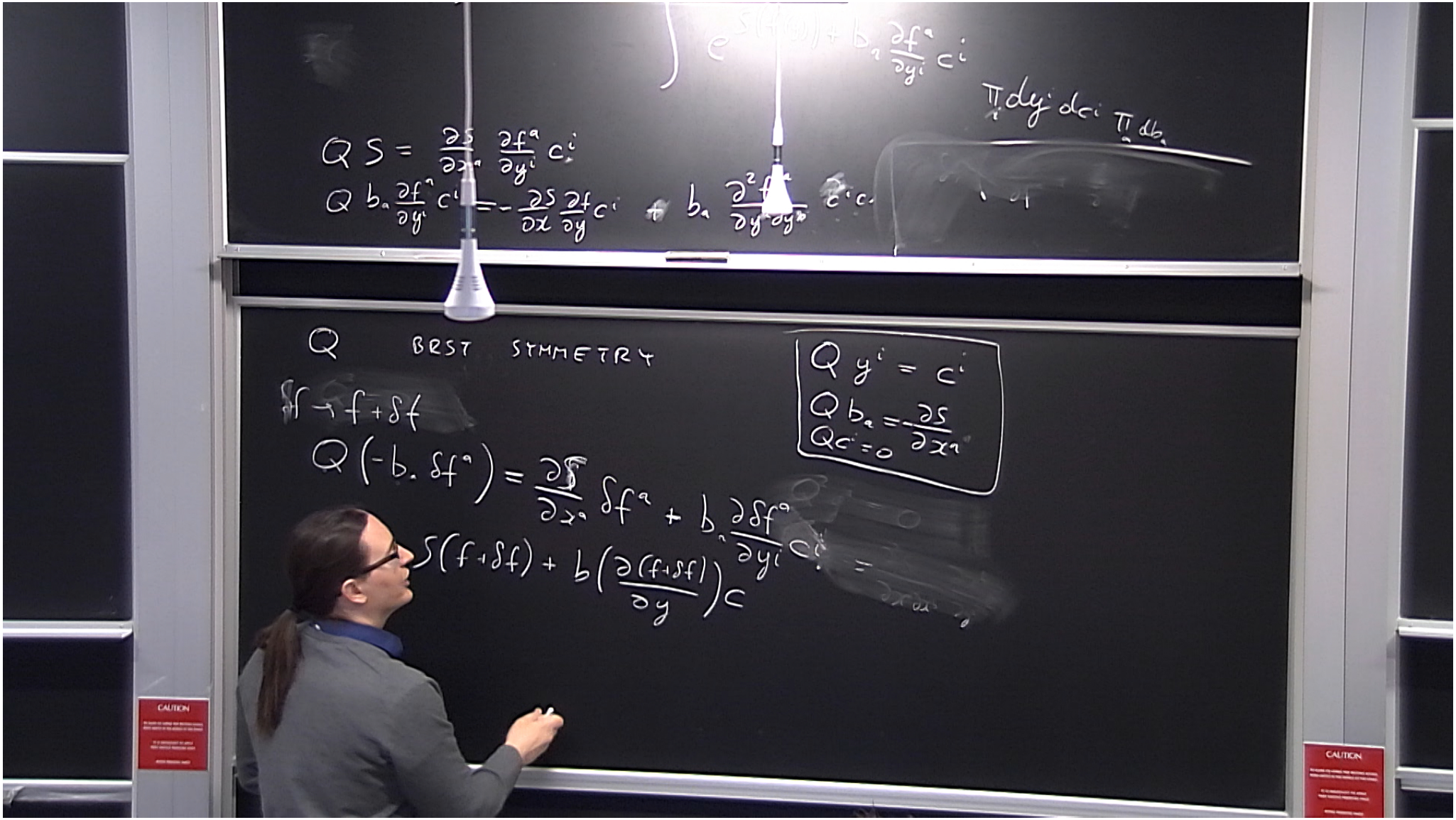


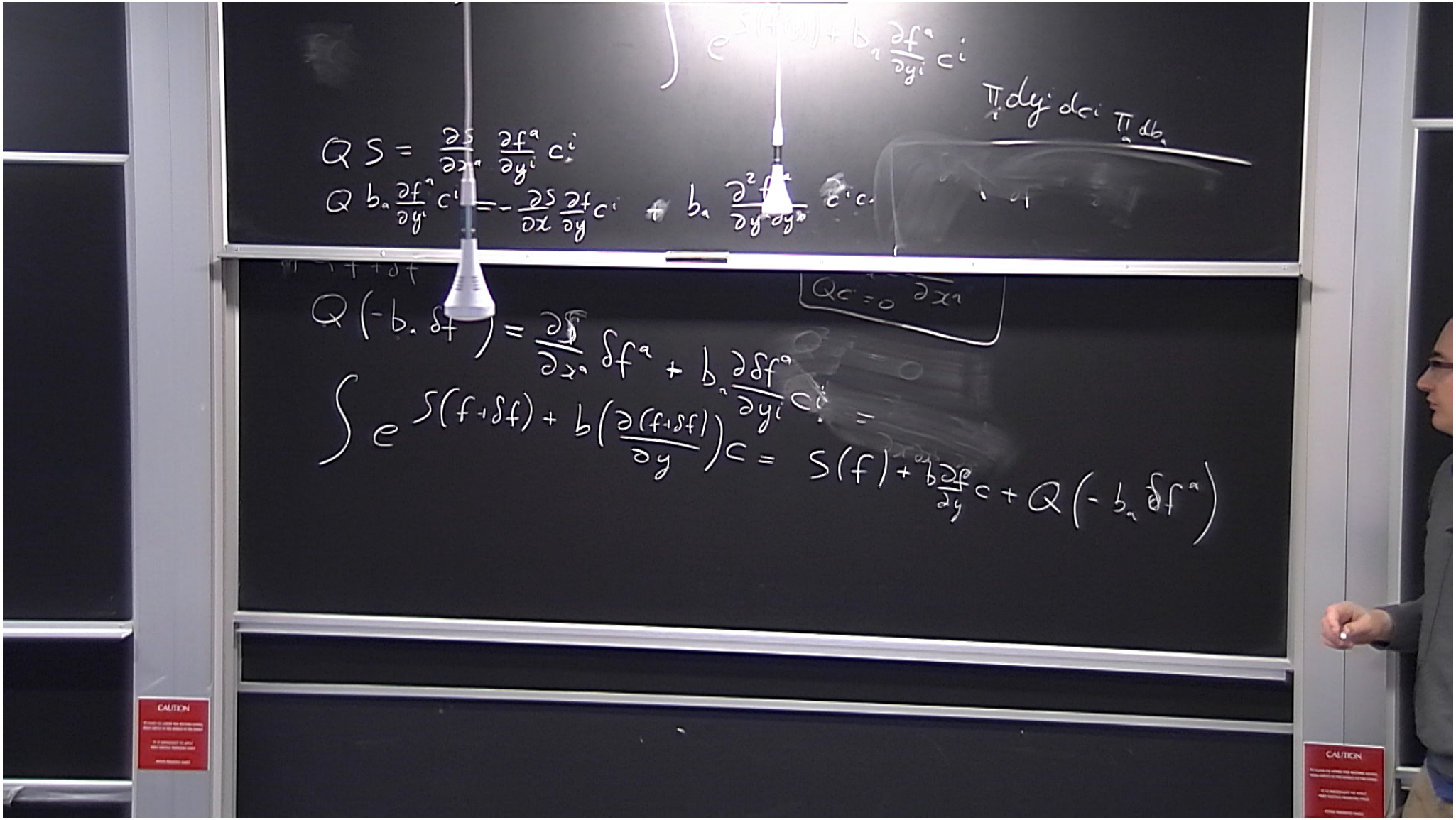

$\int \frac{D h_{ab}}{\text{DIFF} \times \text{WEYL}} \Rightarrow \int "\det\left(\frac{\delta h}{\delta \lambda} \frac{\delta f}{\delta f_i}, \frac{\delta \phi}{\delta \phi}\right)" d\lambda\, Df\, D\phi$
$h_{cd} = \hat{h}_{ab}[\lambda] \frac{\partial f^a}{\partial v^c} \frac{\partial f^b}{\partial v^d}$
$\int d\lambda$

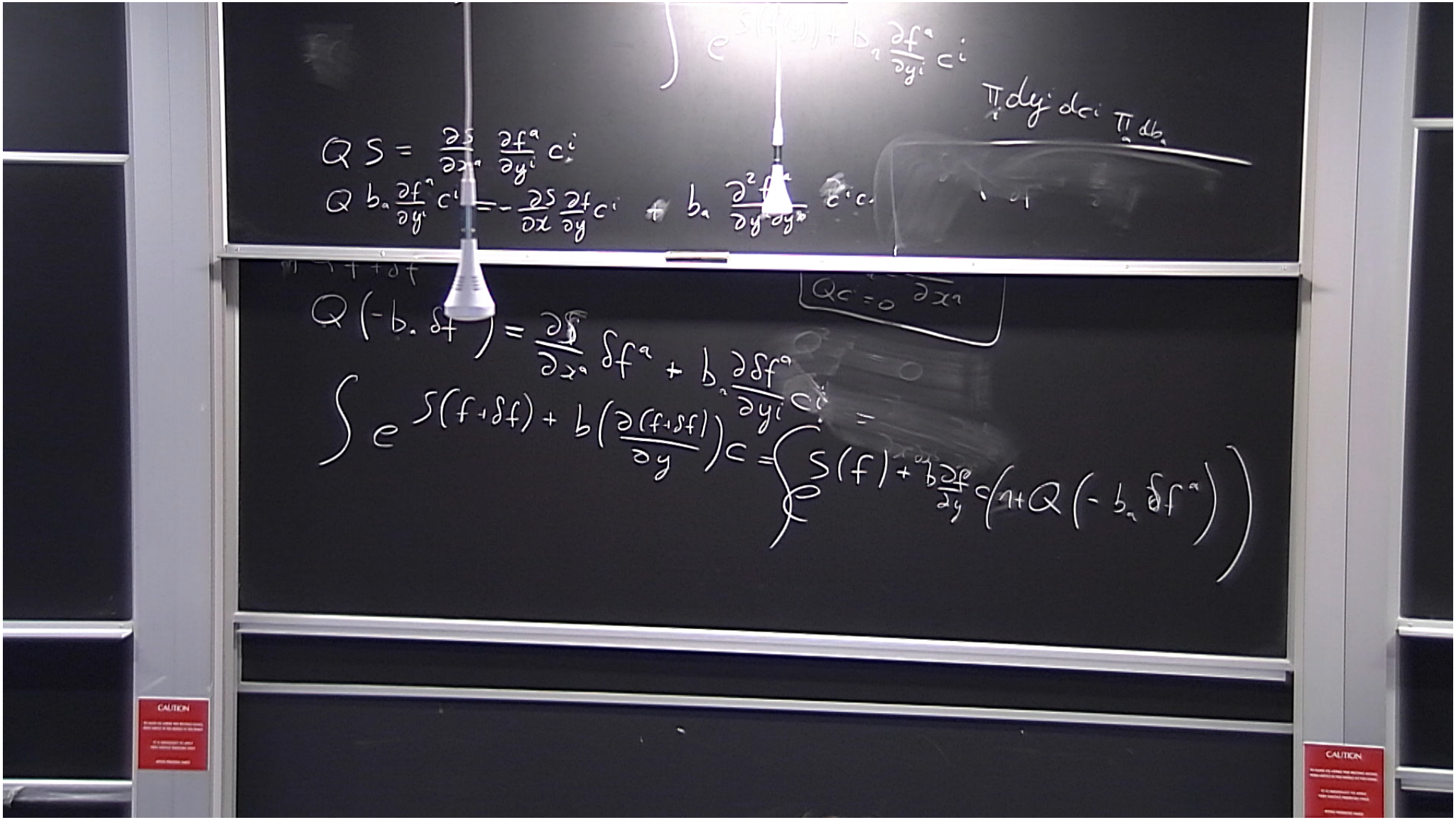

$\int dx^a\, e^{S(x)} = \int \det_{a,i}\left(\frac{\partial f^a}{\partial y_i}\right) e^{S(f(y))}\, dy$
$x^a = f^a(y^i)$
$= \int e^{S(f(y)) + b_a \frac{\partial f^a}{\partial y^i} c^i} \prod_i dy^i\, dc^i \prod_a db_a$

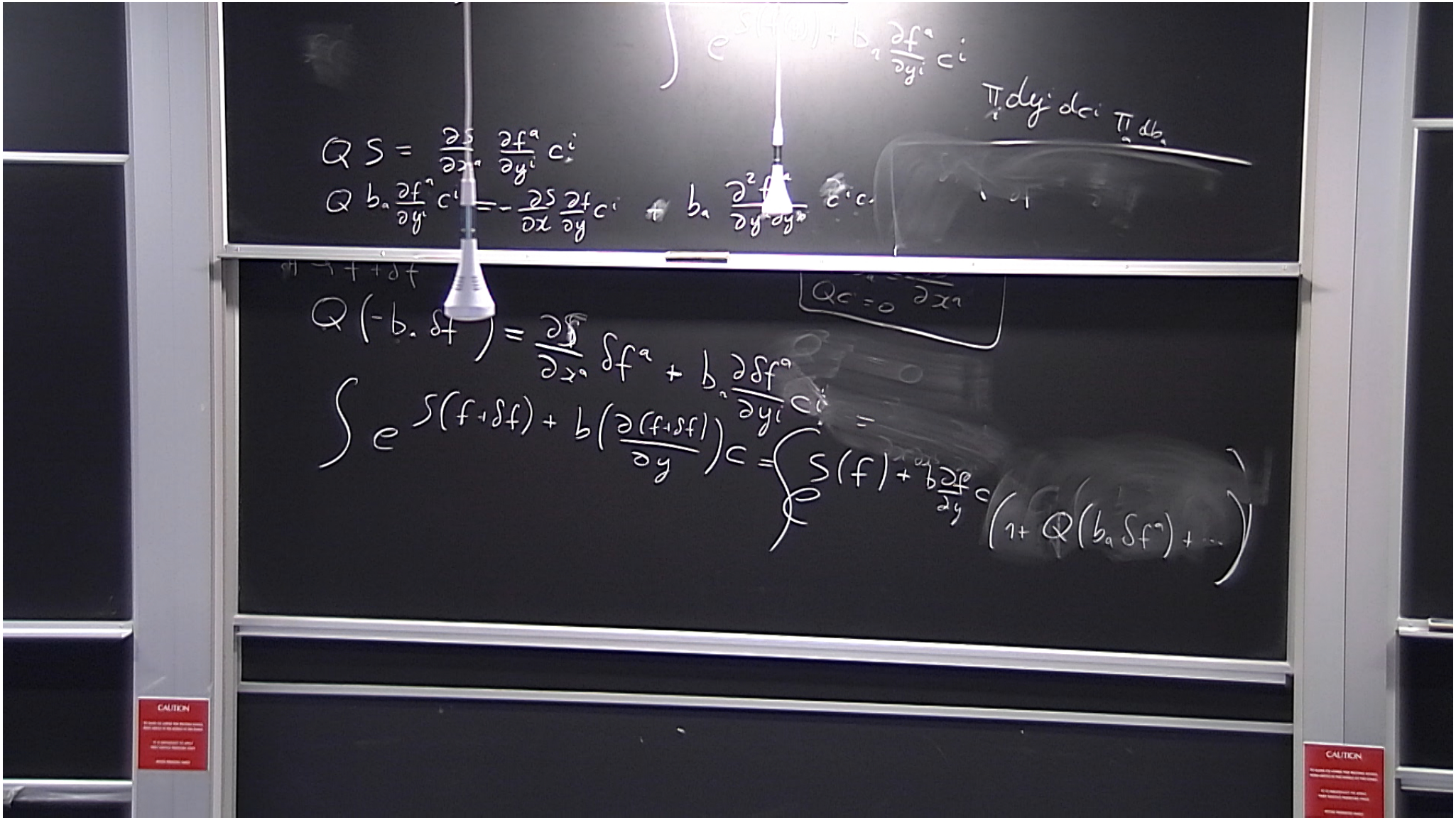

$\int_{ab}[x]\frac{\partial f^a}{\partial v^c}\frac{\partial f^b}{\partial v^d}\, e^{\phi(v)}$
$\left(\frac{\phi}{\phi}\right)''$ $dx\, Df\, D\phi$
$QS = \frac{\partial S}{\partial x^a}\frac{\partial f^a}{\partial y^i}c^i$
$\prod_i dy^i\, dc^i \prod_a db_a$
Q BRST SYMMETRY
$Q^2 = 0$
$Qy^i = c^i$
$Qb_a = \frac{\partial S}{\partial x^a}$
$Qc^i = 0$
$Q^2 y^i = 0$
$Q^2 b_a = \frac{\partial^2 S}{\partial x^a \partial x^b}\frac{\partial x^b}{\partial y^i}c^i$
CAUTION
CAUTION

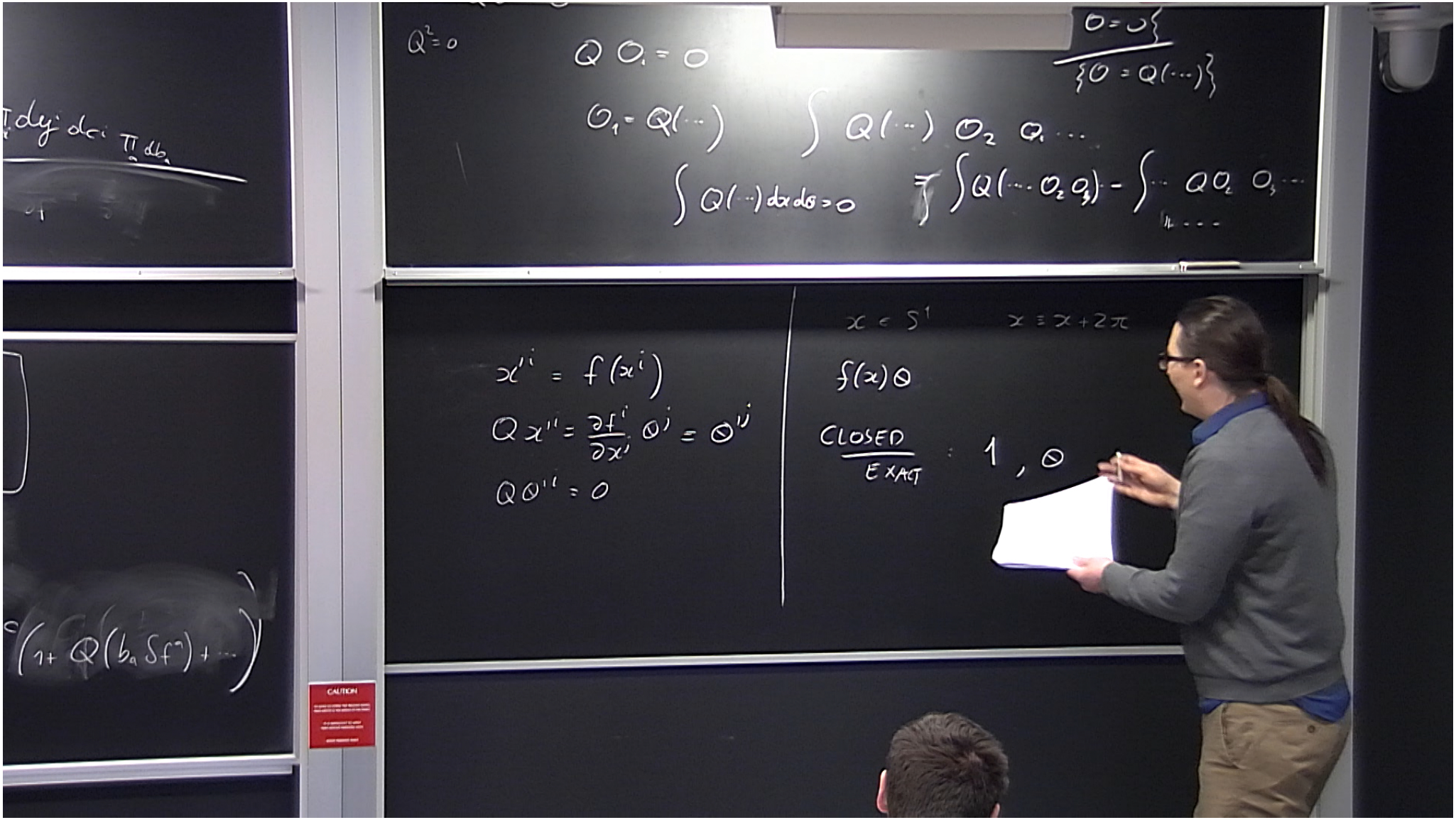

$x^a = f^a(y^i)$
$Q S = \frac{\partial S}{\partial x^a} \frac{\partial f^a}{\partial y^i} c^i$
$Q b_a \frac{\partial f^a}{\partial y^i} c^i = -\frac{\partial S}{\partial x} \frac{\partial f}{\partial y} c^i + b_a \frac{\partial^2 f}{\partial y^i \partial y^j} c^i c^j$
$\int e^{S[\phi]} + b_a \frac{\partial f^a}{\partial y^i} c^i$
$\prod_i dy^i dc^i \prod_a db_a$
Q BRST SYMMETRY
$Q y^i = c^i$
$Q b_a = -\frac{\partial S}{\partial x^a}$
$Q c^i = 0$
$\frac{\partial f^a}{\partial v^b} \frac{\partial f^b}{\partial v^a} e^{\phi(v)}$
$\left(\frac{\partial}{\partial \phi} \right)^n d\lambda Df D\phi$
CAUTION
CAUTION

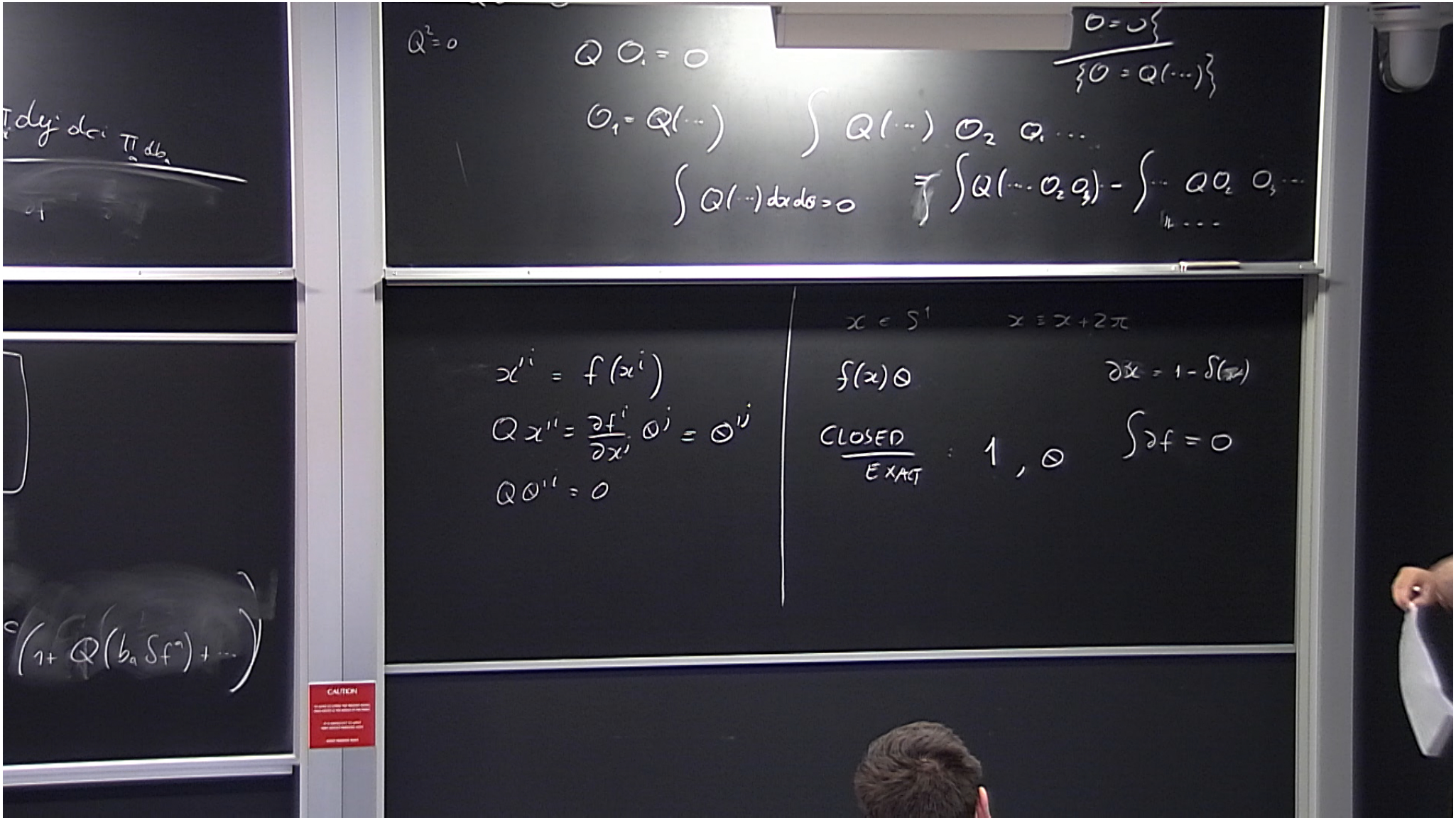

$\int e^{S(f+\delta f)+b_a \frac{\partial f^a}{\partial y^i} c^i}$
$\prod_i dy^i dc^i \prod_a db_a$
$Q S = \frac{\partial S}{\partial x^a} \frac{\partial f^a}{\partial y^i} c^i$
$Q b_a \frac{\partial f^a}{\partial y^i} c^i = -\frac{\partial S}{\partial x} \frac{\partial f}{\partial y} c^i + b_a \frac{\partial^2 f^a}{\partial y^i \partial y^b} c^i c$
Q BRST SYMMETRY
$f \to f + \delta f$
$Q(-b_a \delta f^a) = \frac{\partial S}{\partial x^a} \delta f^a + b_a \frac{\partial \delta f^a}{\partial y^i} c^i$
$S(f+\delta f) + b\left(\frac{\partial (f+\delta f)}{\partial y}\right) c$
$Q y^i = c^i$
$Q b_a = -\frac{\partial S}{\partial x^a}$
$Q c^i = 0$
CAUTION
CAUTION

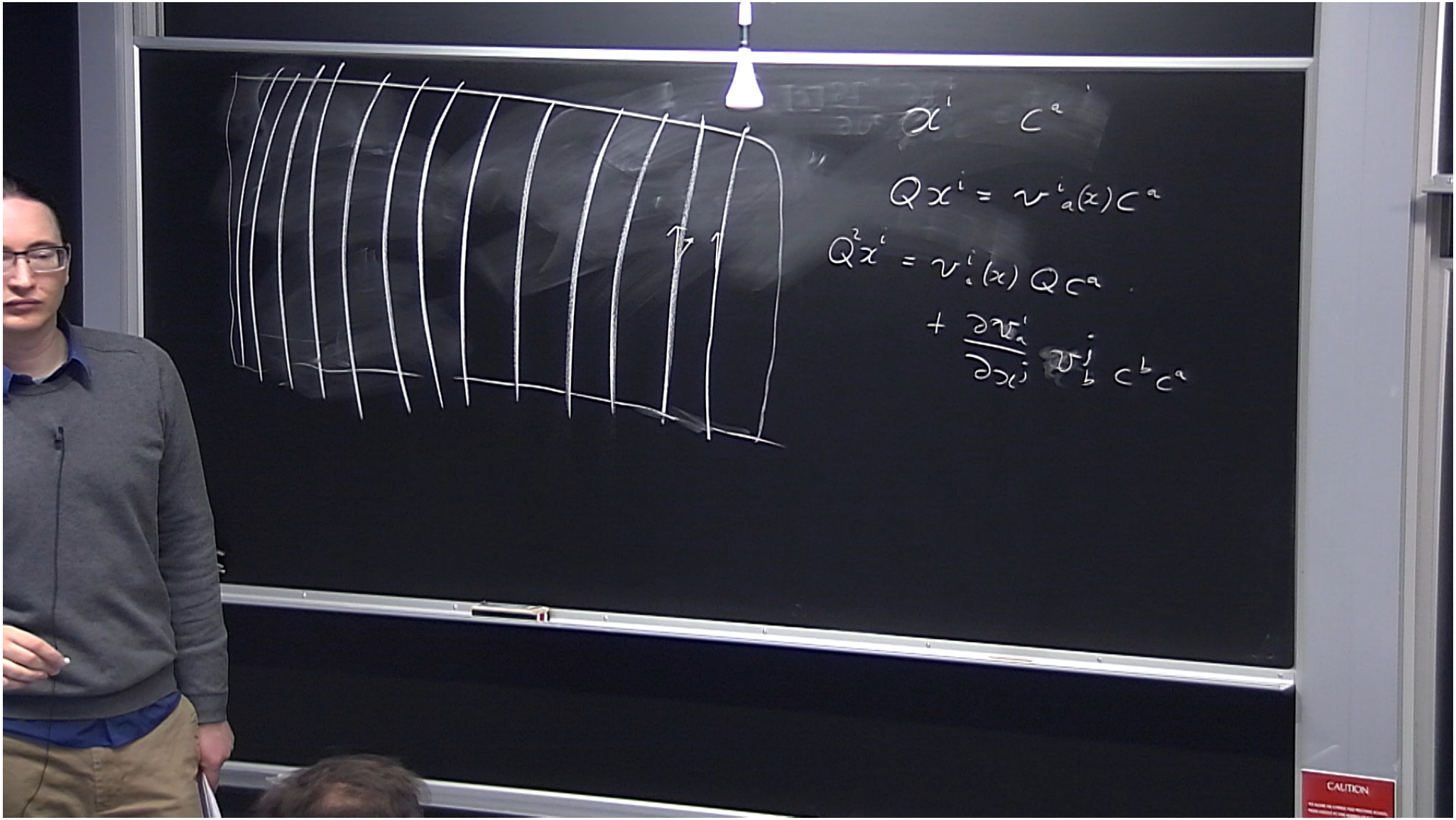

$\int e^{S(f)+b_a \frac{\partial f^a}{\partial y^i} c^i}$
$\prod_i dy^i \, dc^i \prod_a db_a$
$QS = \frac{\partial S}{\partial x^a} \frac{\partial f^a}{\partial y^i} c^i$
$Q\, b_a \frac{\partial f^a}{\partial y^i} c^i = -\frac{\partial S}{\partial x} \frac{\partial f}{\partial y} c^i + b_a \frac{\partial^2 f^a}{\partial y^i \partial y^b} c^i c^b$
$Qc = 0 \quad \frac{\partial}{\partial x^a}$
$Q(-b_a \delta f^a) = \frac{\partial S}{\partial x^a} \delta f^a + b_a \frac{\partial \delta f^a}{\partial y^i} c^i$
$\int e^{S(f+\delta f) + b\left(\frac{\partial (f+\delta f)}{\partial y}\right) c} = S(f) + b \frac{\partial f}{\partial y} c + Q(-b_a \delta f^a)$
CAUTION
CAUTION

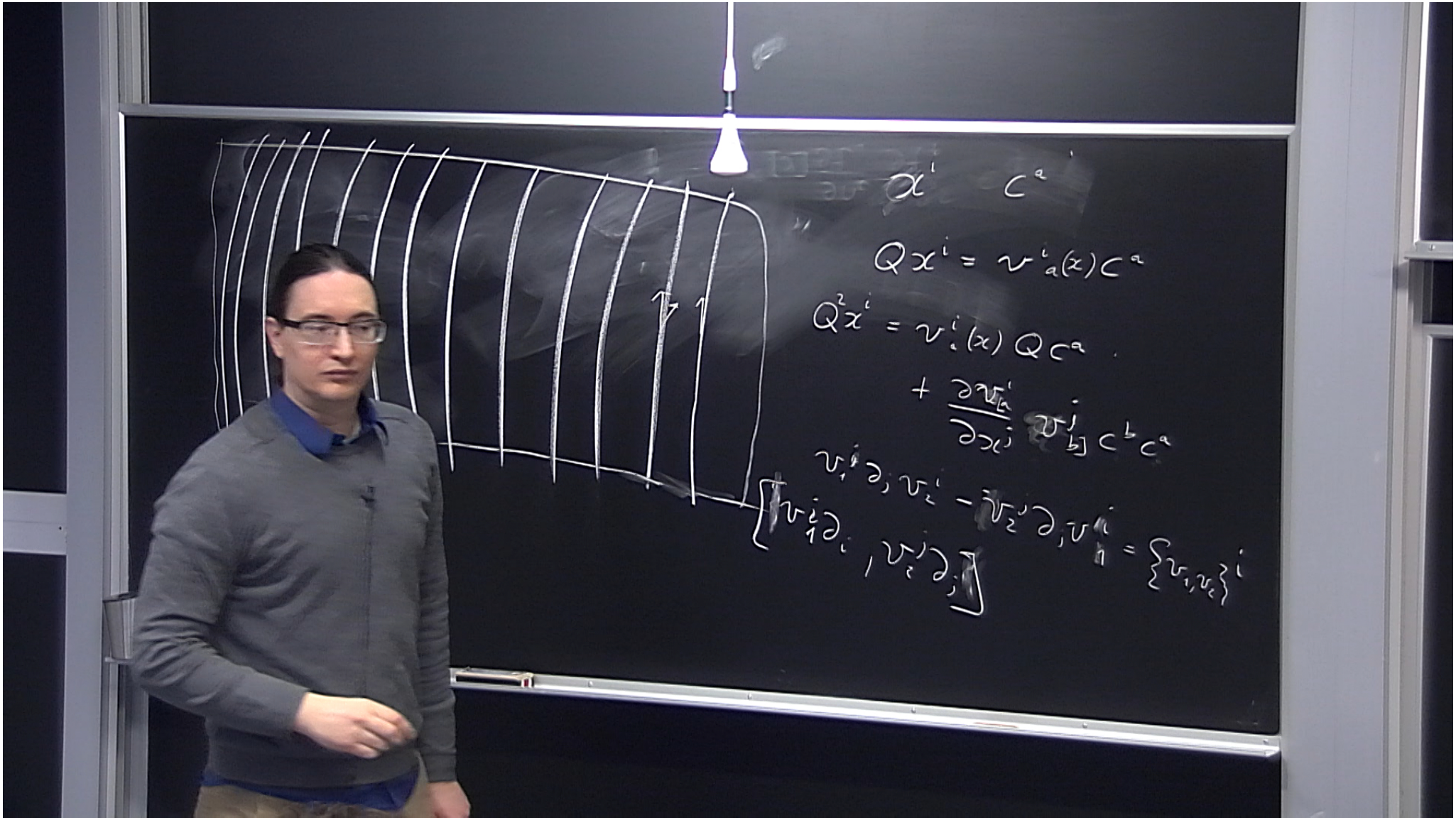

$\int e^{S(f+\delta f)+b_a \frac{\partial f^a}{\partial y^i} c^i}$
$\prod_i dy^i\, dc^i \prod_a db_a$
$Q\,S = \frac{\partial S}{\partial x^a}\frac{\partial f^a}{\partial y^i} c^i$
$Q\, b_a \frac{\partial f^a}{\partial y^i} c^i = -\frac{\partial S}{\partial x}\frac{\partial f}{\partial y} c^i + b_a \frac{\partial^2 f^a}{\partial y^i \partial y^k} c^k c^i$
$Qc = 0$
∂x^a
$Q(-b_a \delta f^a) = \frac{\partial S}{\partial x^a}\delta f^a + b_a \frac{\partial \delta f^a}{\partial y^i} c^i$
$\int e^{S(f+\delta f) + b\left(\frac{\partial (f+\delta f)}{\partial y}\right)c} =$
$\int e^{S(f) + b\frac{\partial f}{\partial y} c}\left(1 + Q(-b_a \delta f^a)\right)$
CAUTION
CAUTION

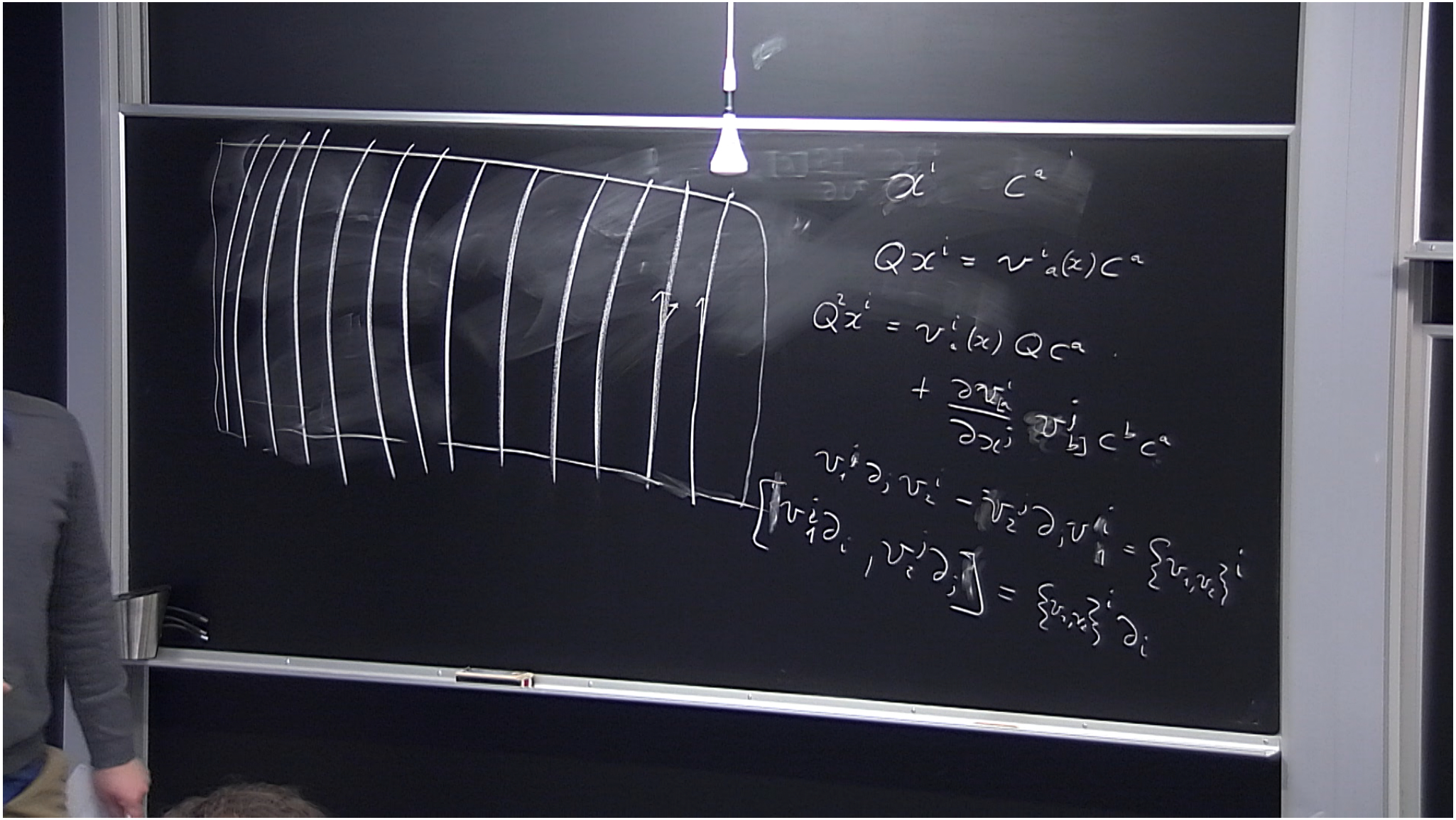

$Q S = \frac{\partial S}{\partial x^a} \frac{\partial f^a}{\partial y^i} c^i$
$Q b_a \frac{\partial f^a}{\partial y^i} c^i = - \frac{\partial S}{\partial x} \frac{\partial f}{\partial y} c^i + b_a \frac{\partial^2 f^a}{\partial y^i \partial y^k} c^i c^k$
$Q c = 0$
$Q(-b_a \delta f^a) = \frac{\partial S}{\partial x^a} \delta f^a + b_a \frac{\partial \delta f^a}{\partial y^i} c^i$
$\int e^{S(f+\delta f) + b\left(\frac{\partial (f+\delta f)}{\partial y}\right) c} = \int e^{S(f) + b \frac{\partial f}{\partial y} c} \left(1 + Q(b_a \delta f^a) + \dots\right)$
CAUTION
CAUTION

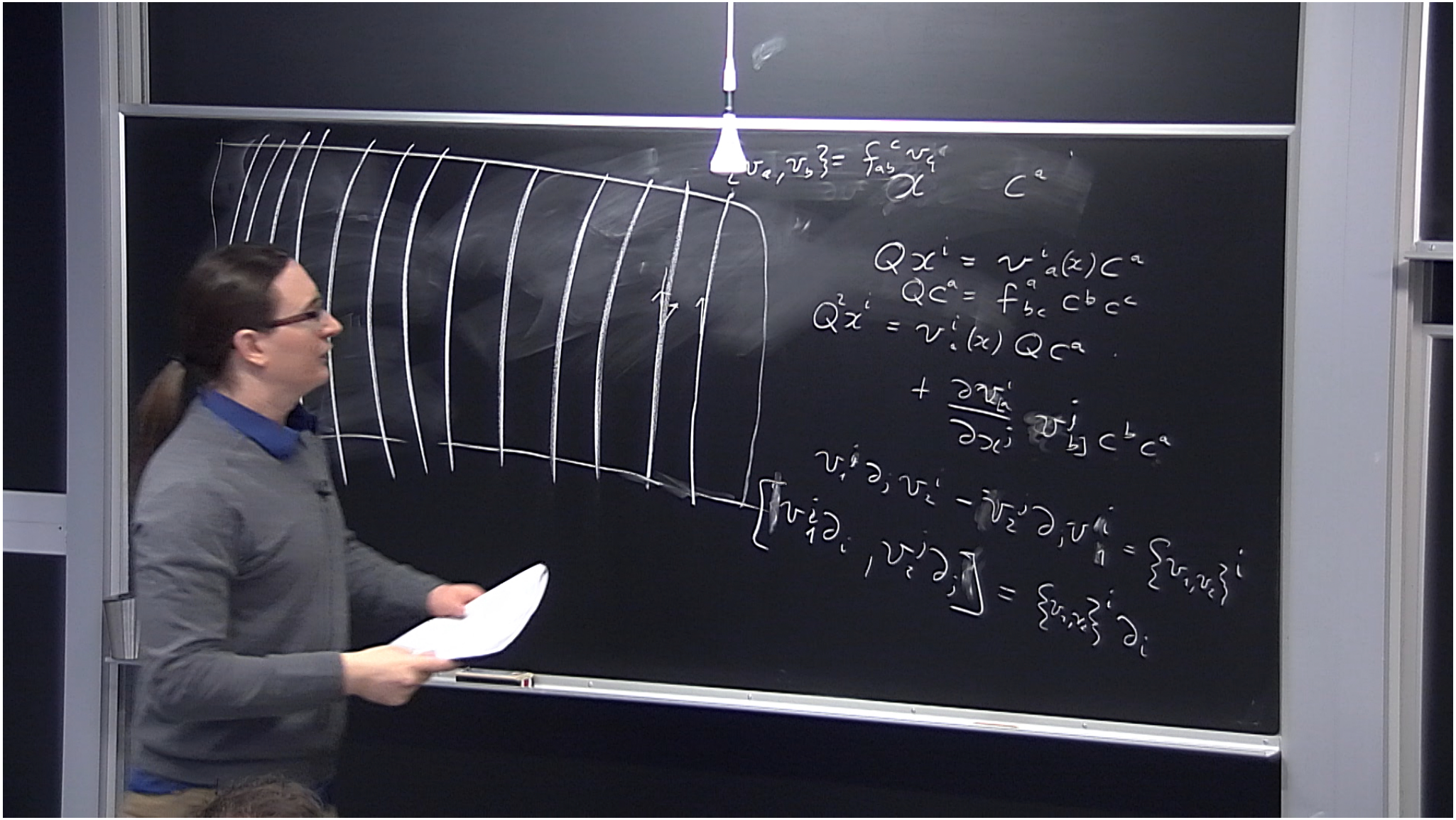

$Q^2 = 0$
$Q\,O_1 = 0$
$O = \cdots$
$\{O = Q(\cdots)\}$
$O_1 = Q(\cdots)$
$\int Q(\cdots)\, O_2\, O_1 \cdots$
$\int Q(\cdots)\, dx\, d\sigma = 0$
$\int Q(\cdots O_2 O_3) - \int \cdots Q O_2 O_3 \cdots$
$x'^i = f(x^i)$
$Q\,x'^i = \frac{\partial f^i}{\partial x^j}\,\theta^j = \theta'^i$
$Q\,\theta'^i = 0$
$x \in S^1$
$x \equiv x + 2\pi$
$f(x)\,\theta$
CLOSED / EXACT : 1, θ
CAUTION

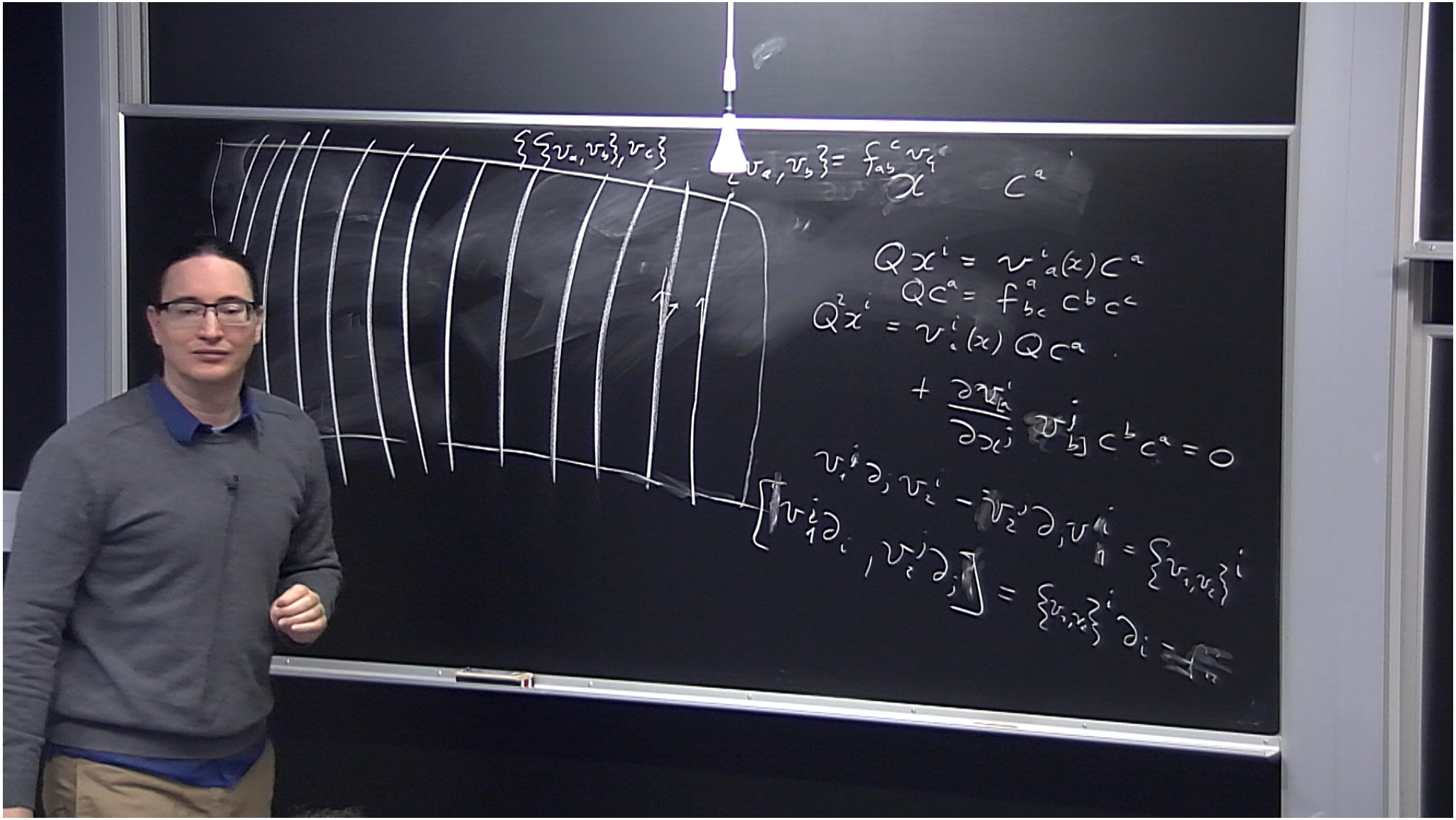

$Q^2 = 0$
$Q\,O_1 = 0$
$O = O$
$\{O = Q(\cdots)\}$
$O_1 = Q(\cdots)$
$\int Q(\cdots)\, O_2\, O_1 \cdots$
$\int Q(\cdots)\, dx\, d\sigma = 0$
$= \int Q(\cdots O_2 O_3) - \int \cdots Q O_2 O_3 \cdots$
$x'^i = f(x^i)$
$Q\,x'^i = \frac{\partial f^i}{\partial x^j}\, \Theta^j = \Theta'^i$
$Q\,\Theta'^i = 0$
$x \in S^1$
$x \equiv x + 2\pi$
$f(x)\,\Theta$
$\partial x = 1 - \delta(\pi)$
CLOSED / EXACT : 1, 0
$\int \partial f = 0$
$\int dy\, dc\, \Pi\, db$
$\left(1 + Q(b_a\, \delta f^a) + \cdots\right)$
CAUTION

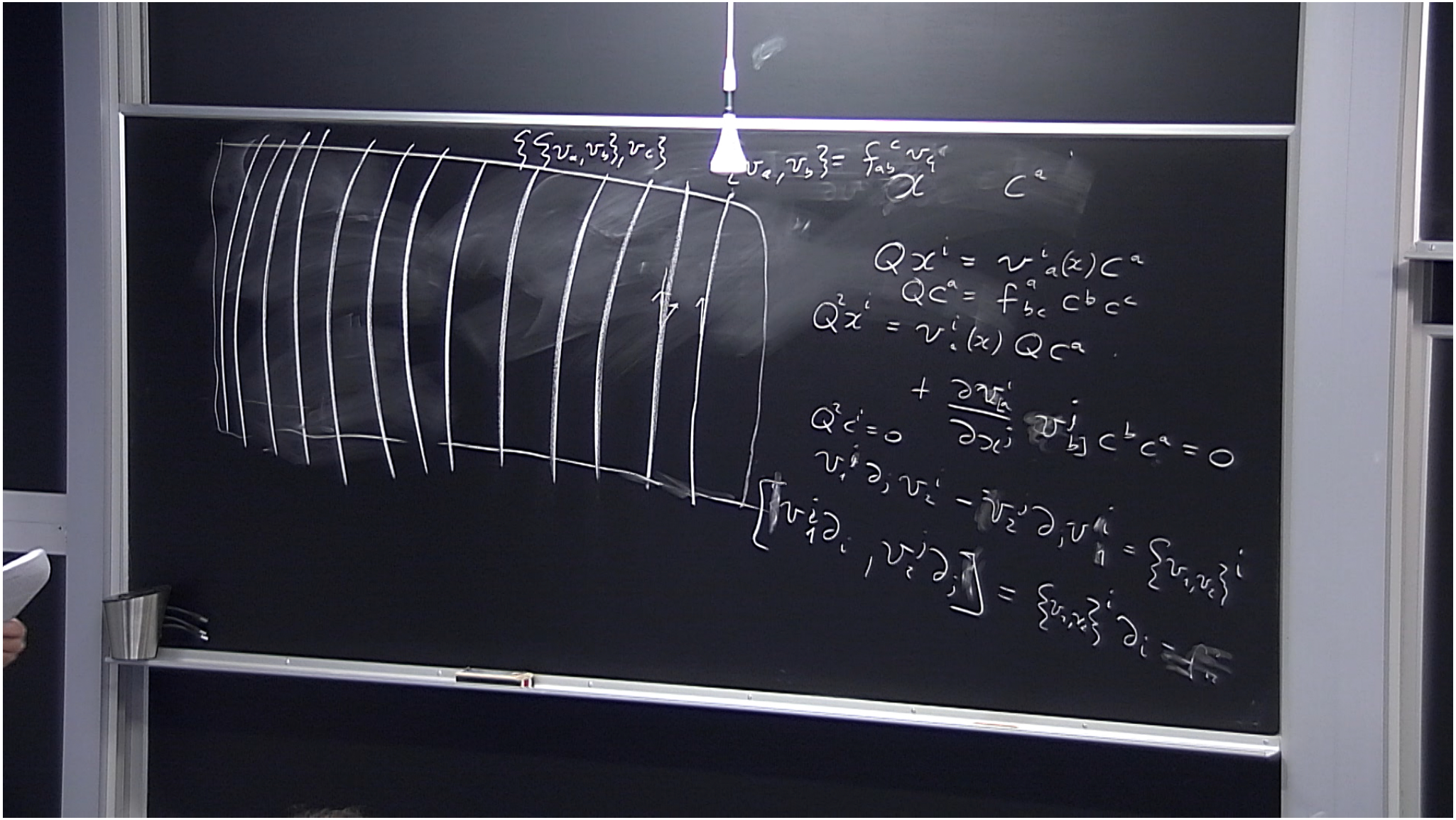

$x^i \quad c^a$
$Qx^i = v^i_a(x) c^a$
$Q^2 x^i = v^i_a(x) Q c^a$
$+ \frac{\partial v^i_a}{\partial x^j} v^j_b c^b c^a$
CAUTION

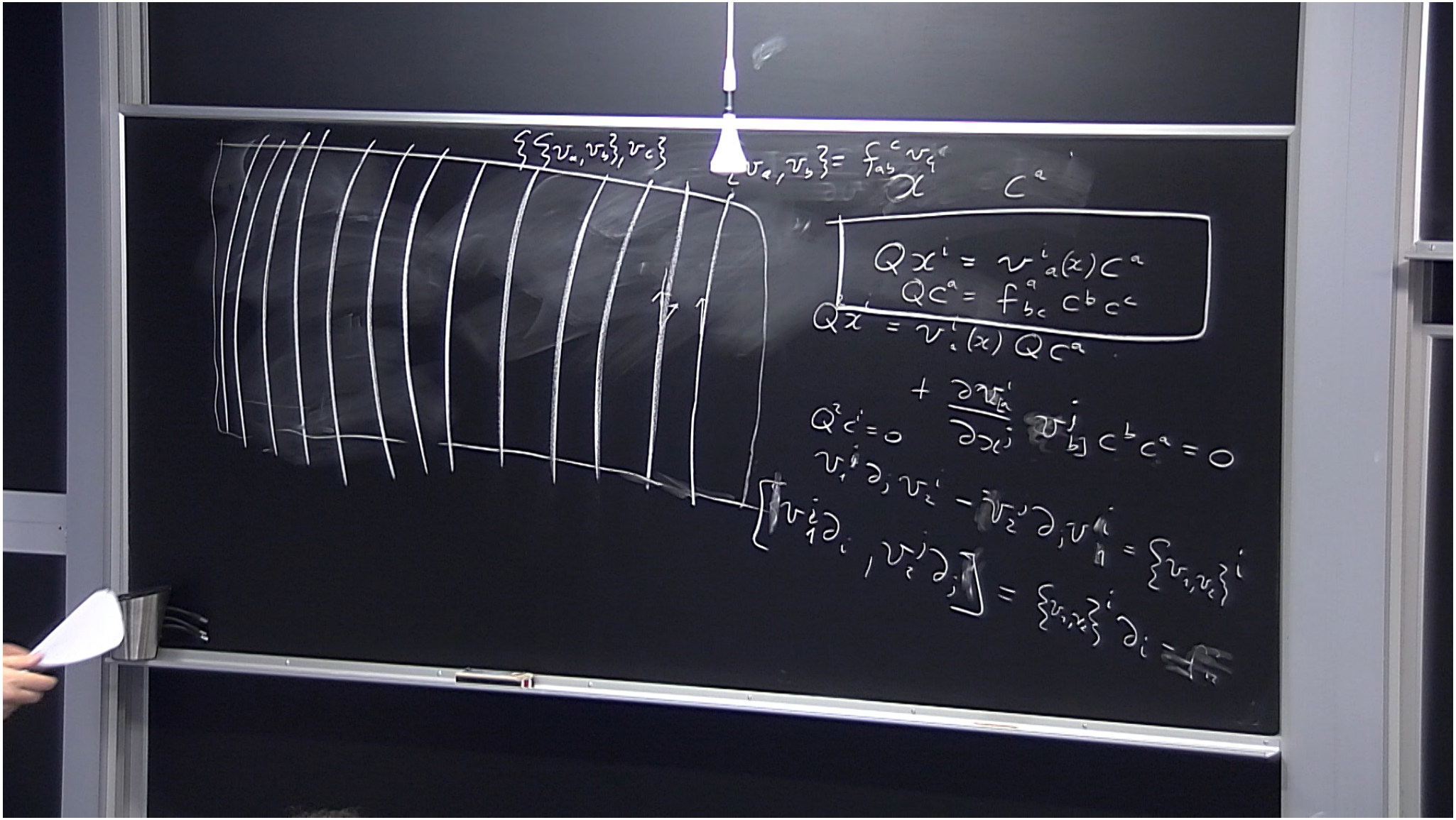

$x^i \quad c^a$
$Qx^i = v^i_a(x) c^a$
$Q^2 x^i = v^i_a(x) Q c^a$
$+ \frac{\partial v^i_a}{\partial x^j} v^j_b c^b c^a$
$v_1^i \partial_i v_2^j - v_2^i \partial_i v_1^j = \{v_1, v_2\}^i$
$[v_1^i \partial_i, v_2^j \partial_j]$

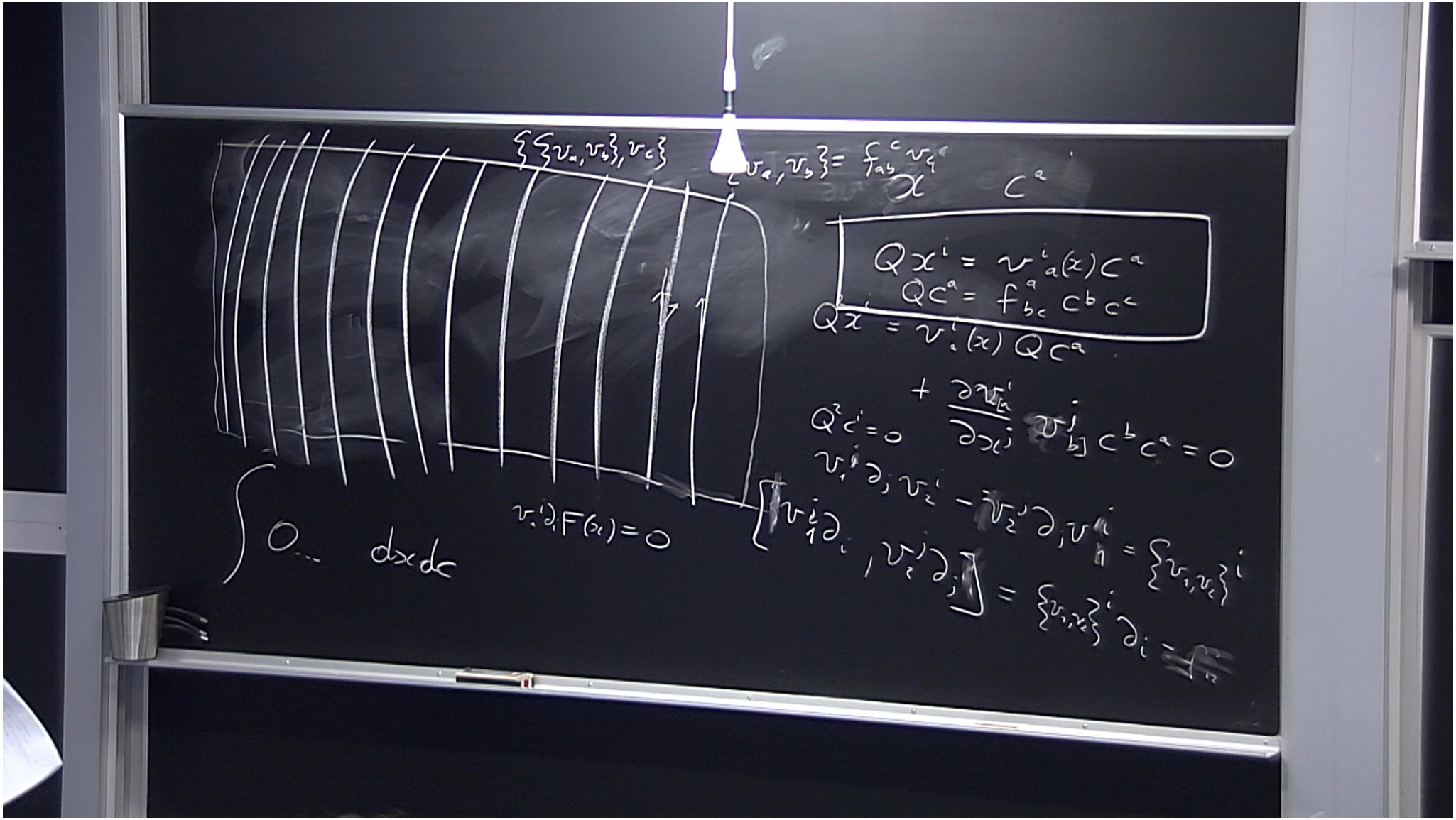

$x^i \quad c^a$
$Qx^i = v^i_a(x) c^a$
$Q^2 x^i = v^i_a(x) Q c^a$
$+ \frac{\partial v^i_a}{\partial x^j} v^j_b c^b c^a$
$v_1^i \partial_i v_2^j - v_2^i \partial_i v_1^j = \{v_1, v_2\}^i$
$[v_1^i \partial_i, v_2^j \partial_j] = \{v_1, v_2\}^i \partial_i$

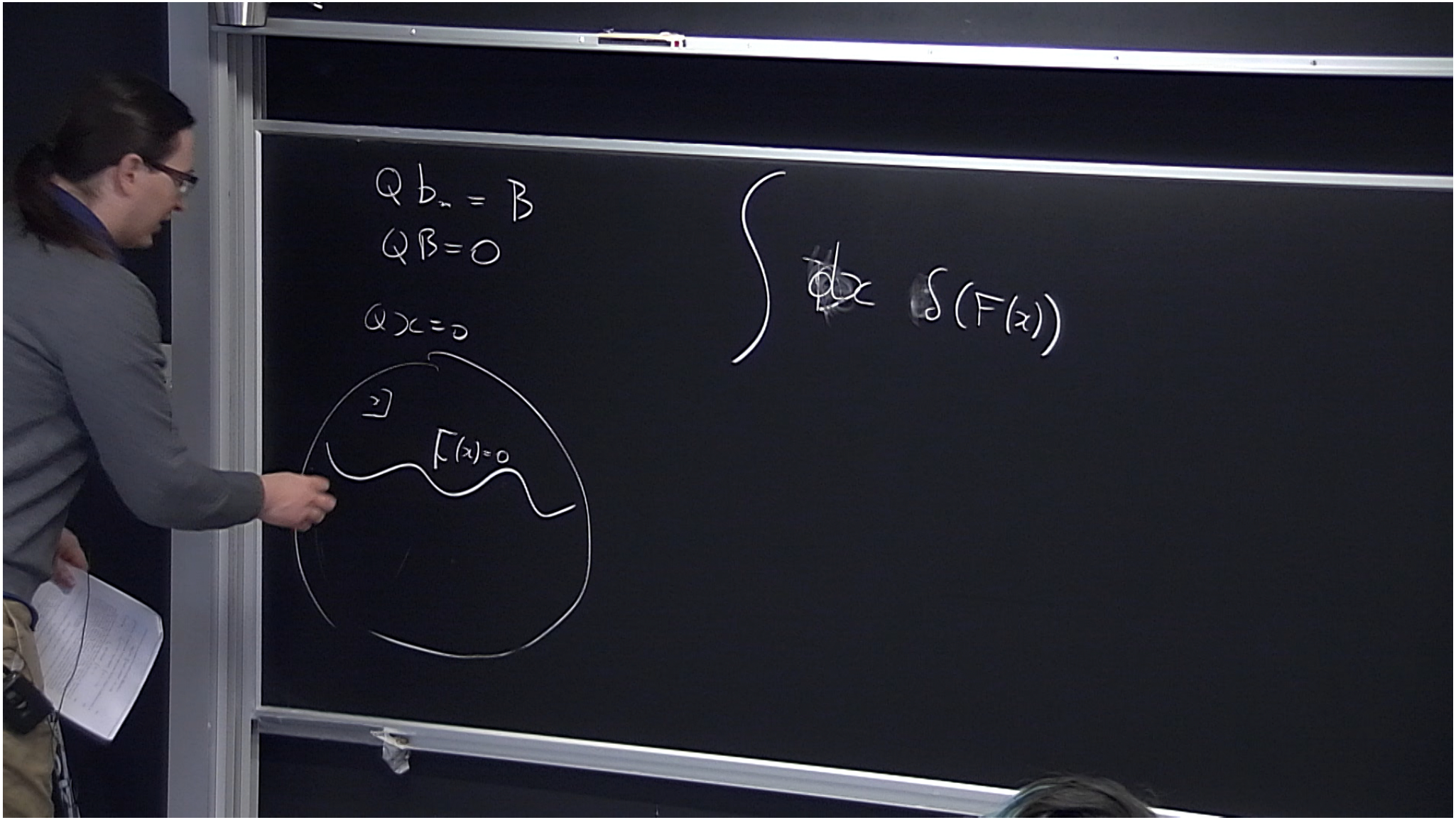

$\{v_a, v_b\} = f_{ab}^{\ c} v_c$
$Qx^i = v^i_{\ a}(x) c^a$
$Qc^a = f^a_{\ bc} c^b c^c$
$Q^2 x^i = v^i_{\ a}(x) Qc^a + \frac{\partial v^i_a}{\partial x^j} v^j_b c^b c^a$
$v_1^i \partial_i v_2^j - v_2^i \partial_i v_1^j = \{v_1, v_2\}^i$
$[v_1^i \partial_i, v_2^j \partial_j] = \{v_1, v_2\}^i \partial_i$

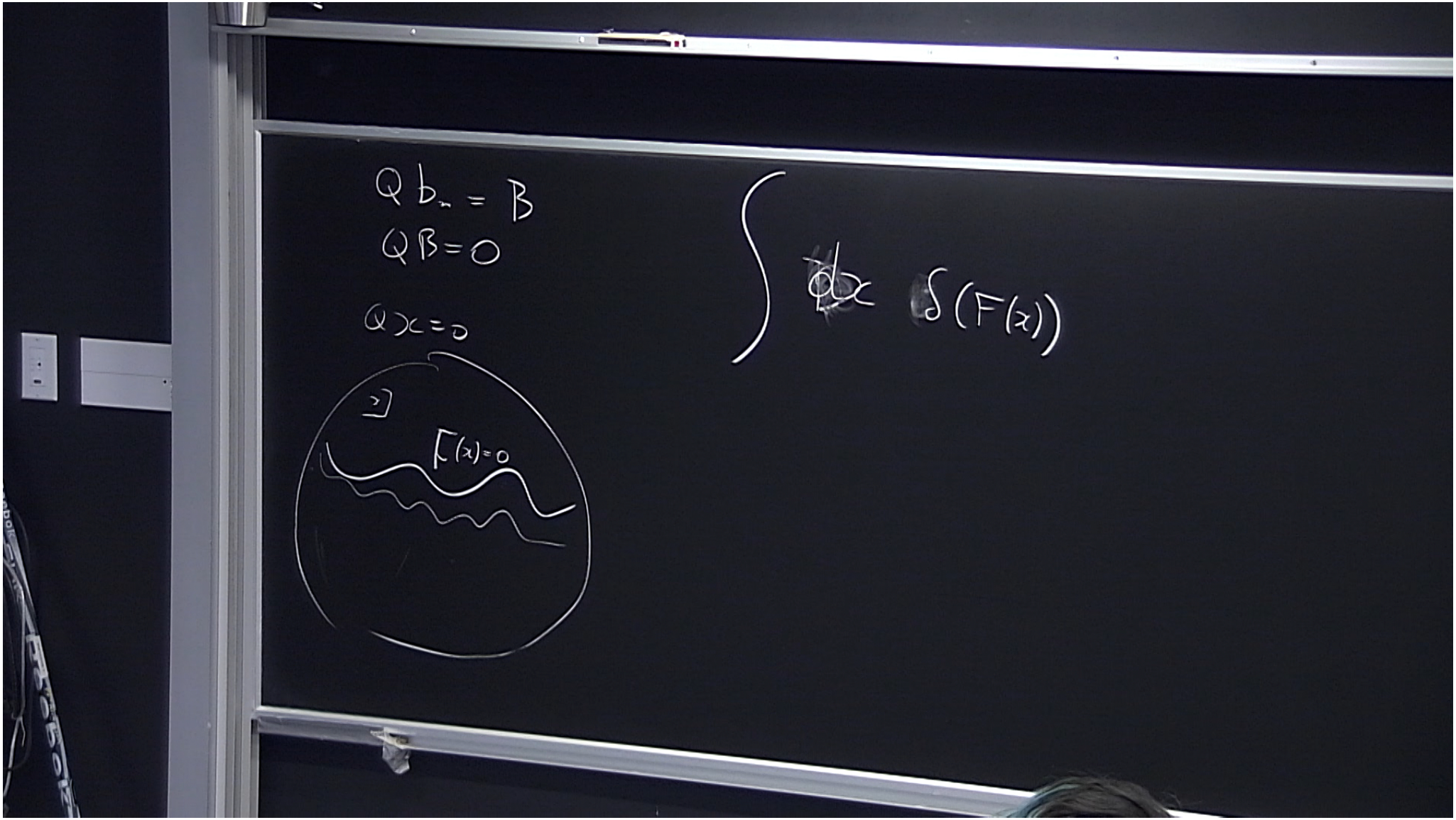

$\{\{v_a, v_b\}, v_c\}$
$\{v_a, v_b\} = f_{ab}^{c} v_c$
$Qx^i = v^i_a(x) c^a$
$Qc^a = f^a_{bc} c^b c^c$
$Q^2 x^i = v^i_a(x) Qc^a + \frac{\partial v^i_a}{\partial x^j} v^j_b c^b c^a = 0$
$v_1^i \partial_i v_2^j - v_2^i \partial_i v_1^j = \{v_1, v_2\}^j$
$[v_1^i \partial_i, v_2^j \partial_j] = \{v_1, v_2\}^i \partial_i$

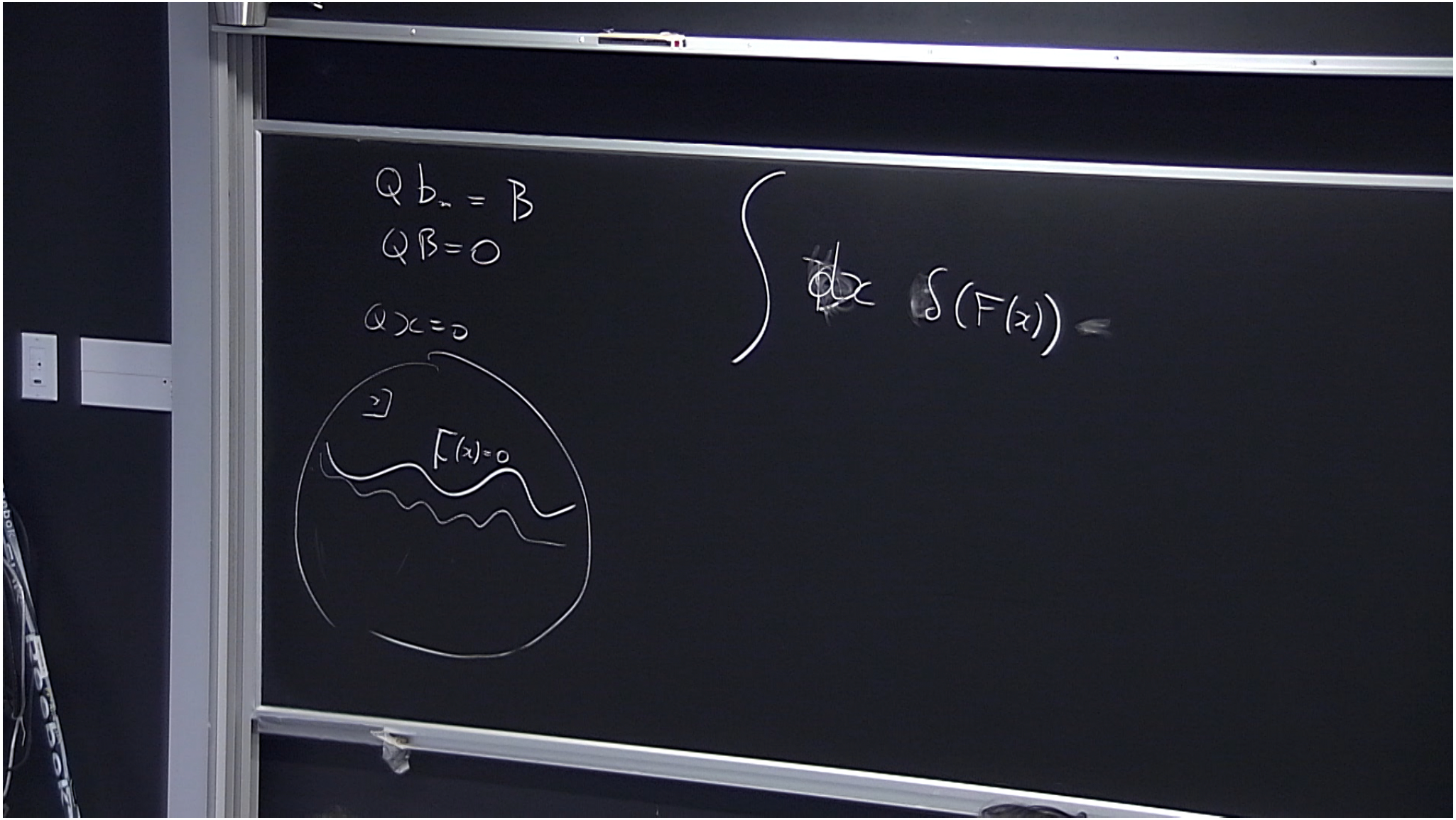

$\{\{v_a, v_b\}, v_c\}$
$\{v_a, v_b\} = f_{ab}^{\ c} v_c^i$
$Qx^i = v^i_a(x) c^a$
$Qc^a = f^a_{bc} c^b c^c$
$Q^2 x^i = v^i_a(x) Qc^a + \frac{\partial v^i_a}{\partial x^j} v^j_b c^b c^a$
$Q^2 c^i = 0$
$v_1^j \partial_j v_2^i - v_2^j \partial_j v_1^i = \{v_1, v_2\}^i$
$[v_1^j \partial_j, v_2^j \partial_j] = \{v_1, v_2\}^i \partial_i$

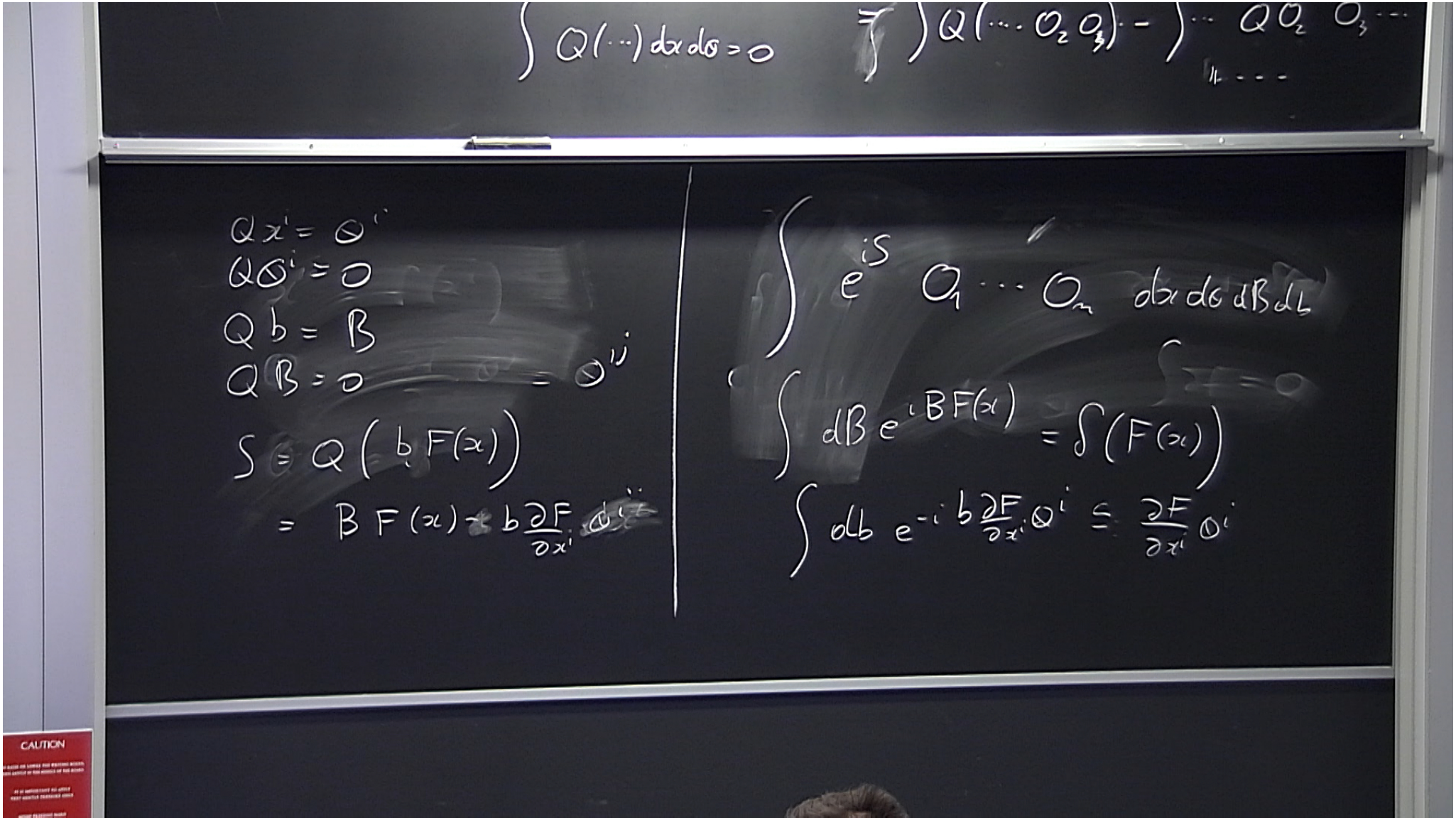

$\{\{v_a, v_b\}, v_c\}$
$\{v_a, v_b\} = f_{ab}^{\ c} v_c$
x^i
c^a
$Qx^i = v^i{}_a(x) c^a$
$Qc^a = f^a_{bc} c^b c^c$
$Q^2 x^i = v^i{}_a(x) Qc^a + \frac{\partial v^i_a}{\partial x^j} v^j_b c^b c^a = 0$
$Q^2 c^i = 0$
$v_1^j \partial_j v_2^i - v_2^j \partial_j v_1^i = \{v_1, v_2\}^i$
$[v_1^j \partial_j, v_2^j \partial_j] = \{v_1, v_2\}^i \partial_i$


$\{\{v_a, v_b\}, v_c\}$
$\{v_a, v_b\} = f_{ab}^{\ c} v_c$
$Qx^i = v^i_{\ a}(x) c^a$
$Qc^a = f^a_{\ bc} c^b c^c$
$Qx^i = v^i_{\ a}(x)\, Qc^a$
$+ \frac{\partial v^i_a}{\partial x^j} v^j_b c^b c^a = 0$
$Q^2 c^i = 0$
$v_1^j \partial_j v_2^i - v_2^j \partial_j v_1^i = \{v_1, v_2\}^i$
$[v_1^j \partial_j, v_2^j \partial_j] = \{v_1, v_2\}^i \partial_i$
$\int \mathcal{O} \ldots dx\, dc$
$v_1^j \partial_j F(x) = 0$


$Qb_n = B$
$QB = 0$
$Qx = 0$
$F(x) = 0$
$\int dx\ \delta(F(x))$


$Q b_n = B$
$QB = 0$
$Qx = 0$
$F(x) = 0$
$\int dx\ \delta(F(x))$


$Q b_m = B$
$QB = 0$
$Qx = 0$
$F(x) = 0$
$\int dx\ \delta(F(x))$


$\int Q(\cdots)\,dx\,d\theta = 0$
$Q x^i = \theta^i$
$Q\theta^i = 0$
$Q b = B$
$Q B = 0$
$S = Q\left(b_i F(x)\right)$
$= B F(x) + b \frac{\partial F}{\partial x^i} \theta^i$
$\int e^{iS} O_1 \cdots O_n\, dx\, d\theta\, dB\, db$
$\int dB\, e^{iBF(x)} = \delta(F(x))$
$\int db\, e^{-ib\frac{\partial F}{\partial x^i}\theta^i} = \frac{\partial F}{\partial x^i}\theta^i$