

Title: PSI 2016/2017 String Theory (Review) - Lecture 4

Date: Feb 24, 2017 10:15 AM

URL: <http://pirsa.org/17020109>

Abstract:

$$[a_n, a_{-n}] = n \quad [\bar{a}_n, \bar{a}_{-n}] = n \quad [x, p] = i \quad a_0 = -\bar{a}_0 = p$$

$$i\partial_s X = \sum_{n=-\infty}^{\infty} a_n e^{-ns}$$

$$-i\partial_{\bar{s}} X = \sum_{n=-\infty}^{\infty} \bar{a}_n e^{-n\bar{s}}$$

$$S = \frac{T}{2} \int \partial_s X \partial_{\bar{s}} X$$

$$T = -\frac{1}{2} \lim_{s' \rightarrow s} \left[\partial_s X \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$$

$$|p\rangle \quad p \in \mathbb{R}$$

$$a_{-1}|p\rangle \quad \bar{a}_{-1}|p\rangle$$

$$\begin{array}{cccc} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{array}$$

$$[a_n, a_{-n}] = n \quad [\bar{a}_n, \bar{a}_{-n}] = n \quad [x, p] = i \quad a_0 = -\bar{a}_0 = p$$

$$i\partial_s X = \sum_{n=-\infty}^{\infty} a_n e^{-ns}$$

$$-i\partial_{\bar{s}} X = \sum_{n=-\infty}^{\infty} \bar{a}_n e^{-n\bar{s}}$$

$$S = \frac{T}{2} \int \partial_s X \partial_{\bar{s}} X$$

$$T_{ss} = -\frac{1}{2} \lim_{s' \rightarrow s} \left[\partial_s X(s) \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$$

$$|p\rangle \quad p \in \mathbb{R}$$

$$a_{-1}|p\rangle \quad \bar{a}_{-1}|p\rangle$$

$$\begin{array}{ccc} - & - & - \\ - & - & - \\ - & - & - \\ - & - & - \end{array}$$

$$T_{s\bar{s}} = 0$$

$$\begin{aligned}
 & -\frac{1}{(s-s')^2} + e^{s'-s} + 2e^{2(s'-s)} \dots \\
 & = \lim_{s' \rightarrow s} \left[\frac{e^{s+s'}}{(e^s - e^{s'})^2} - \frac{1}{(s-s')^2} \right] = -\frac{1}{24}
 \end{aligned}$$

$$T_{ss} = -\frac{1}{2} \lim_{s' \rightarrow s} \left[\partial_s X(s) \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$$

$$= -\frac{1}{2} \sum_n \sum_m :a_n a_m: e^{-(n+m)s} - \frac{1}{24} = \sum_n L_n e^{-ns} - \frac{1}{24}$$

$$T_{ss} = 0$$

$$L_n = \frac{1}{2}$$

$$s = \gamma - i\sigma$$

$$\sigma \rightarrow \sigma + 2\pi$$

$$P = \int_{-\infty}^{\infty} T_{T\sigma} = \int_0^{2\pi} T_{ss} - T_{\bar{s}\bar{s}}$$

$$T = -\frac{1}{2} \lim_{s' \rightarrow s} \left[- \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} 2n a_m e^{-(ns-m)s'} - \frac{1}{(s-s')^2} \right]$$

$$= -\frac{1}{2} \lim_{s' \rightarrow s} \left[\sum_n \sum_m : a_n a_m : e^{-ns-ms'} - \frac{1}{(s-s')^2} \right] + \sum_{n>0} n a_n e^{-ns+ms'}$$

$$\frac{(e^s - e^{s'})^2}{(s-s')^2} = \frac{1}{4}$$

$$= -\frac{1}{2} \lim_{s' \rightarrow s} \left[-\sum_n \sum_m : a_n a_m : e^{-ns - ms'} + \frac{1}{(s-s')^2} \right] + \sum_{n>0} \left[\cancel{a_n a_{-n}} \right] e^{-ns + ms'}$$

$$1/2 + e^{s'-s} + 2e^{2(s'-s)} \dots = \lim_{s' \rightarrow s} \left[\frac{e^{s+s'}}{(e^s - e^{s'})^2} - \frac{1}{(s-s')^2} \right] = \frac{1}{24}$$

$$L_n = \frac{1}{2} \sum_{m=-\infty}^{\infty} : a_{n-m} a_m :$$

$$[L_n, a_m] = m a_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_s (e^{ns} \partial_s X)$$

$$[L_n, X(s)] = e^{ns} \partial_s X$$

$$a_n |p\rangle = 0 \quad n > 0$$

$$L_n |p\rangle = 0 \quad n > 0$$

$$L_0 |p\rangle = \frac{p^2}{2} |p\rangle$$

$$L_{-1} |p\rangle = p a_{-1} |p\rangle$$

$$\alpha_n |P\rangle = 0 \quad n > 0$$

$$L_n |P\rangle = 0 \quad n > 0$$

$$L_0 |P\rangle = \frac{p^2}{2} |P\rangle$$

$$[L_n, \partial_s X \partial_{s'} X] = \partial_s (e^{ns} \partial_{s'} X) + \partial_{s'} (\partial_s X(s) e^{ns'} \partial_{s'} X)$$

$$[L_n, T(s)] = e^{ns} (2n + \partial_s) \left(T(s) + \frac{1}{24} \right) + \frac{1}{12} n (n^2 - 1) e^{ns}$$

$$L_n = \frac{1}{2} \sum_{m=-\infty}^{\infty} : a_{n-m} a_m :$$

$$[L_n, X(s)] = e^{ns} \partial_s X$$

$$[L_n, a_m] = m a_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_s (e^{ns} \partial_s X)$$

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{1}{12} n(n^2-1) \delta_{n+m,0}$$

$$L_n |P\rangle = 0 \quad n > 0$$

$$L_0 |P\rangle = \frac{p^2}{2} |P\rangle$$

$$[L_n, \partial_s X \partial_{s'} X] = \partial_s (e^{ns} \partial_s X \partial_{s'} X) + \partial_{s'} (\partial_s X(s) e^{ns'} \partial_{s'} X)$$

$$[L_n, T(s)] = e^{ns} (2n + \partial_s) \left(T(s) + \frac{1}{24} \right) + \frac{1}{12} n (n^2 - 1) e^{ns}$$

$$\int T_{ss} \delta h^{ss} + c.c.$$

$$a_n |p\rangle = 0 \quad n > 0$$

$$L_n |p\rangle = 0 \quad n > 0$$

$$L_0 |p\rangle = \frac{p^2}{2} |p\rangle$$

$$[L_n, \partial_s X \partial_{s'} X] = i \partial_s (e^{ns} \partial_s X \partial_{s'} X) + \partial_{s'} (\partial_s X(s) e^{ns'} \partial_{s'} X)$$

$$[L_n, T(s)] = e^{ns} (2n + \partial_s) \left(T(s) + \frac{1}{24} \right) + \frac{1}{12} n(n^2 - 1) e^{ns}$$

$$L_n = \frac{1}{2} \sum_{m=-\infty}^{\infty} : a_{n-m} a_m :$$

$$[L_n, X(s)] = e^{ns} \partial_s X$$

$$[L_n, a_m] = m a_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_s (e^{ns} \partial_s X)$$

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{\kappa}{12} n(n^2-1) \delta_{n+m,0}$$

$$c_X = 1$$

$$\partial X(z) \partial X(u) \sim \frac{1}{(z-u)^2}$$

$$T T = \partial X(z) \partial X(z) \partial X(u) \partial X(u) \sim \frac{1}{(z-u)^4} + \frac{T}{(z-u)^2} + \frac{2\partial T}{z-u} + \dots$$

$$\int_{\Sigma} \sqrt{d\sigma} \Rightarrow \int \sqrt{h} \eta^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

$$\Rightarrow \int \partial X^\mu \bar{\partial} X_\mu$$

$$\partial \bar{\partial} X^\mu = 0, \quad \partial X^\mu \partial X_\mu = 0, \quad \bar{\partial} X^\mu \bar{\partial} X_\mu = 0$$

$$= \frac{1}{2} \sum_n \sum_m :a_m a_n: e^{-(n+m)s} - \frac{1}{24} = \sum_n \frac{1}{2} a_n e^{-ns} - \frac{1}{24}$$

$$[a_n^\mu, a_m^\nu] = \eta^{\mu\nu} n \delta_{n+m}$$

$$\langle P | a_1^\mu, a_{-1}^\nu | P' \rangle = \eta^{\mu\nu} \delta(P-P')$$

$$|P^\mu\rangle$$

$$a_{-1}^\mu |P^\nu\rangle$$

...

$$T_{ss} = -\frac{1}{2} \lim_{s' \rightarrow s} \left[\partial_s X^\mu \partial_{s'} X_\mu + \frac{1}{(s-s')^2} \right]$$

$$= \frac{1}{2} \sum_n \sum_m :a_n a_m: e^{-(n+m)s} - \frac{1}{24} = \sum_n \alpha_n e^{-ns} - \frac{1}{24}$$

$$T_{ss} = 0$$

$$[a_n^\mu, a_m^\nu] = \eta^{\mu\nu} n \delta_{n+m,0}$$

$$|p^\mu\rangle$$

$$a_{-1}^\mu |p^\mu\rangle$$

...

$$\langle p | a_1^\mu a_{-1}^\nu | p' \rangle = \eta^{\mu\nu} \delta(p-p')$$

$$\mathcal{H}_{\text{STRING}} = \frac{\mathcal{H}_{\text{PHYSICAL}}}{\mathcal{H}_{\text{PHYSICAL}} \wedge \mathcal{H}_{\text{NULL}}}$$

$$L_n | \text{PHYSICAL} \rangle = 0 \quad n > 0$$

$$(L_0 - 1) | \text{PHYSICAL} \rangle = 0$$

$$(L_0 - 1) | \dots \rangle = 0$$

$$\langle \text{PHYSICAL} | L_n = 0 \quad n < 0$$

$$\langle \text{PHYSICAL} | (L_0 - 1) = 0$$

$$|P^\mu\rangle \quad [a_n^\mu, a_{-n}^\nu] = \eta^{\mu\nu} n$$

$$| \text{NULL} \rangle = L_{-1} | \dots \rangle$$

$$\langle \text{PHYSICAL} | \text{NULL} \rangle = 0$$

$|P\rangle$

PHYSICAL IF

$P=2$

NOT NULL

$$\epsilon_{\mu\nu} a_{-1}^{\mu} \bar{a}_{-1}^{\nu} |P\rangle$$

$$\frac{P^2}{2} + 1 - 1 = 0$$

$$\boxed{P^2 = 0}$$

$$\epsilon_{\mu\nu}^{ST} = 0$$

$g_{\mu\nu}$

$$\epsilon_{\mu\nu}^A$$

$B_{\mu\nu}$

$$\epsilon_{\mu\nu} = \delta_{\mu\nu}$$

$$L_{-1}(\epsilon_{\mu\nu} a_{-1}^{\mu} \bar{a}_{-1}^{\nu} |P\rangle) = (p_{\mu} \epsilon_{\nu} a_{-1}^{\mu} \bar{a}_{-1}^{\nu} |P\rangle)$$

CAUTION

BE AWARE OF LAMPS AND BRIGHT LIGHTS
WHEN MOVING IN THE AREA OF THE BOARD

IF AN OBSTACLE OR A PERSON
IS IN THE AREA, STOP IMMEDIATELY

THANK YOU FOR YOUR ATTENTION

$$[L_n, \partial_s X \partial_s X] = 12_s (e^{ns} \partial_s X \partial_s X - \partial_s X \partial_s X e^{ns})$$

$$[L_n, T(s)] = e^{ns} (2n+2s) \left(T(s) + \frac{1}{24} \right) + \frac{1}{12} n(n^2-1) e^{ns}$$

$$\left(B_{\mu\nu} \frac{\partial X^\mu}{\partial \tau} \frac{\partial X^\nu}{\partial \sigma} \right) \int A_{\mu\nu} \frac{\partial X^\mu}{\partial \tau} \frac{\partial X^\nu}{\partial \sigma} d\tau$$

$|P\rangle$

PHYSICAL IF $P^2 = 2$

NOT NULL

$$\epsilon_{\mu\nu} a_{-1}^\mu \bar{a}_{-1}^\nu |P\rangle$$

$$\frac{P^2}{2} + 1 - 1 = 0$$

$$\boxed{P^2 = 0}$$

$$\epsilon_{\mu\nu}^{ST} \epsilon$$

$$\int g_{\mu\nu}$$

$$\epsilon_{\mu\nu}^A$$

$$\int B_{\mu\nu}$$

$$\epsilon_{\mu\nu} = \delta_{\mu\nu}$$

$$L_{-1} (e_\nu \bar{a}_{-1}^\nu |P\rangle) = (p_\nu e_\nu a_{-1}^\mu \bar{a}_{-1}^\nu |P\rangle)$$

$$|P\rangle \leftrightarrow \begin{cases} \phi_{TAG} \\ \delta g_{\mu\nu} \end{cases} \quad \begin{cases} \nabla_\mu^2 \phi = -2\phi \\ \nabla_\mu \phi = 0 \end{cases} \quad \left(B_{\mu\nu} \frac{\partial x^\mu}{\partial \tau} \frac{\partial x^\nu}{\partial \sigma} \right) \quad A_{\mu\nu} \frac{\partial x^\mu}{\partial \tau} \frac{\partial x^\nu}{\partial \sigma}$$

$|P\rangle$

PHYSICAL IF $P = 2T$

NOT NULL

$$\epsilon_{\mu\nu} a_{-1}^\mu \bar{a}_{-1}^\nu |P\rangle$$

$$\frac{P^2}{2} + 1 - 1 = 0$$

$$\boxed{P^2 = 0}$$

$$\epsilon_{\mu\nu}^{ST} \quad \epsilon$$

$$\delta g_{\mu\nu}$$

$$\epsilon_{\mu\nu}^A$$

$$\delta B_{\mu\nu}$$

$$\epsilon_{\mu\nu} = \eta_{\mu\nu}$$

$$\delta \phi$$

$$L_{-1}(\epsilon_{\mu\nu} e_\nu^\mu \bar{a}_{-1}^\nu |P\rangle) = (P_\mu e_\nu^\mu a_{-1}^\mu \bar{a}_{-1}^\nu |P\rangle)$$

$$\epsilon_{\mu\nu} a_{-2}^\mu \bar{a}_{-2}^\nu |P\rangle \quad P^2 = -2T$$