Title: PSI 2016/2017 String Theory (Review) - Lecture 3

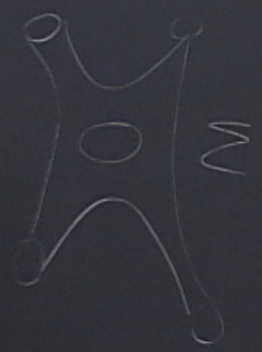
Date: Feb 23, 2017 10:15 AM

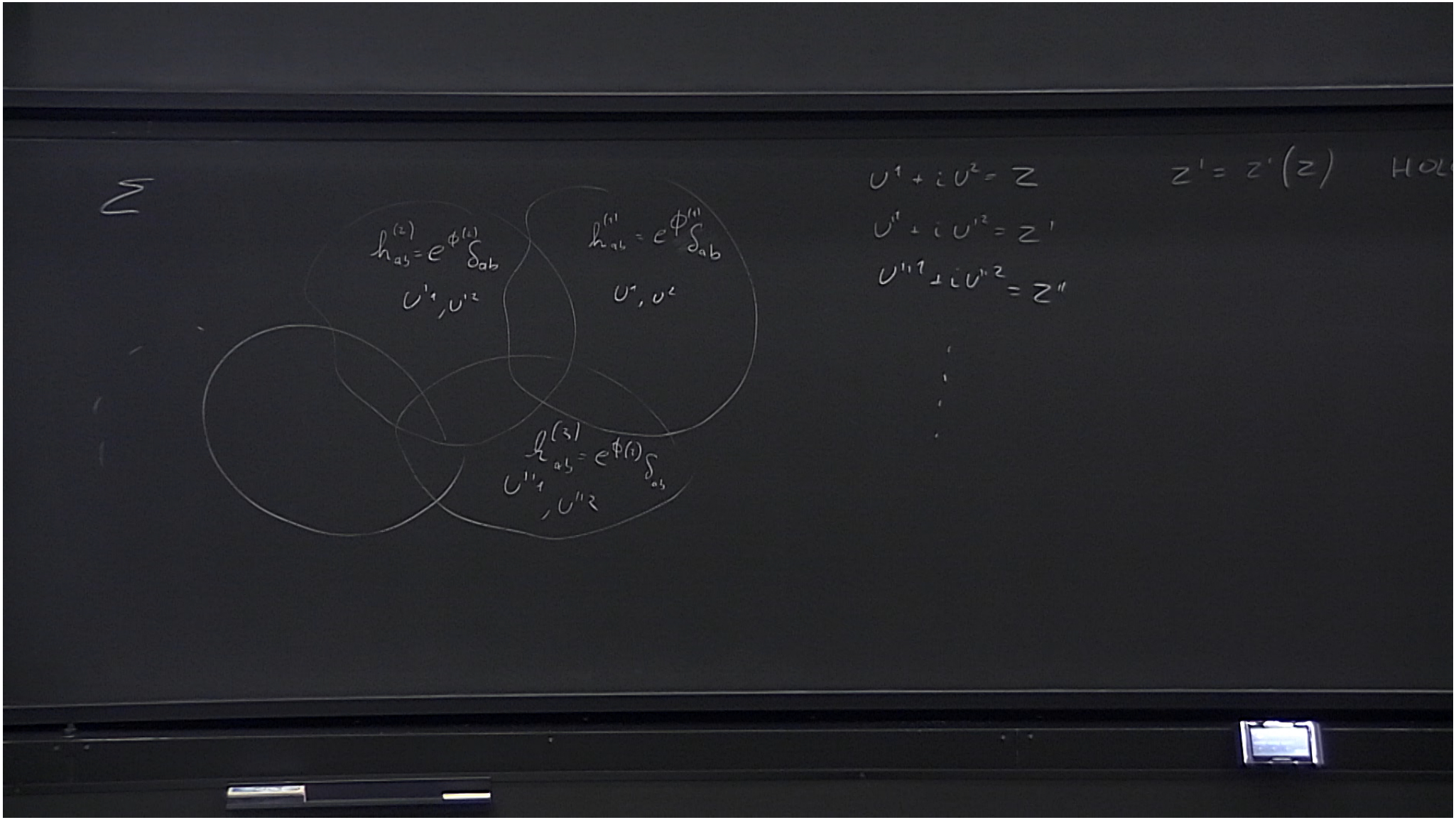
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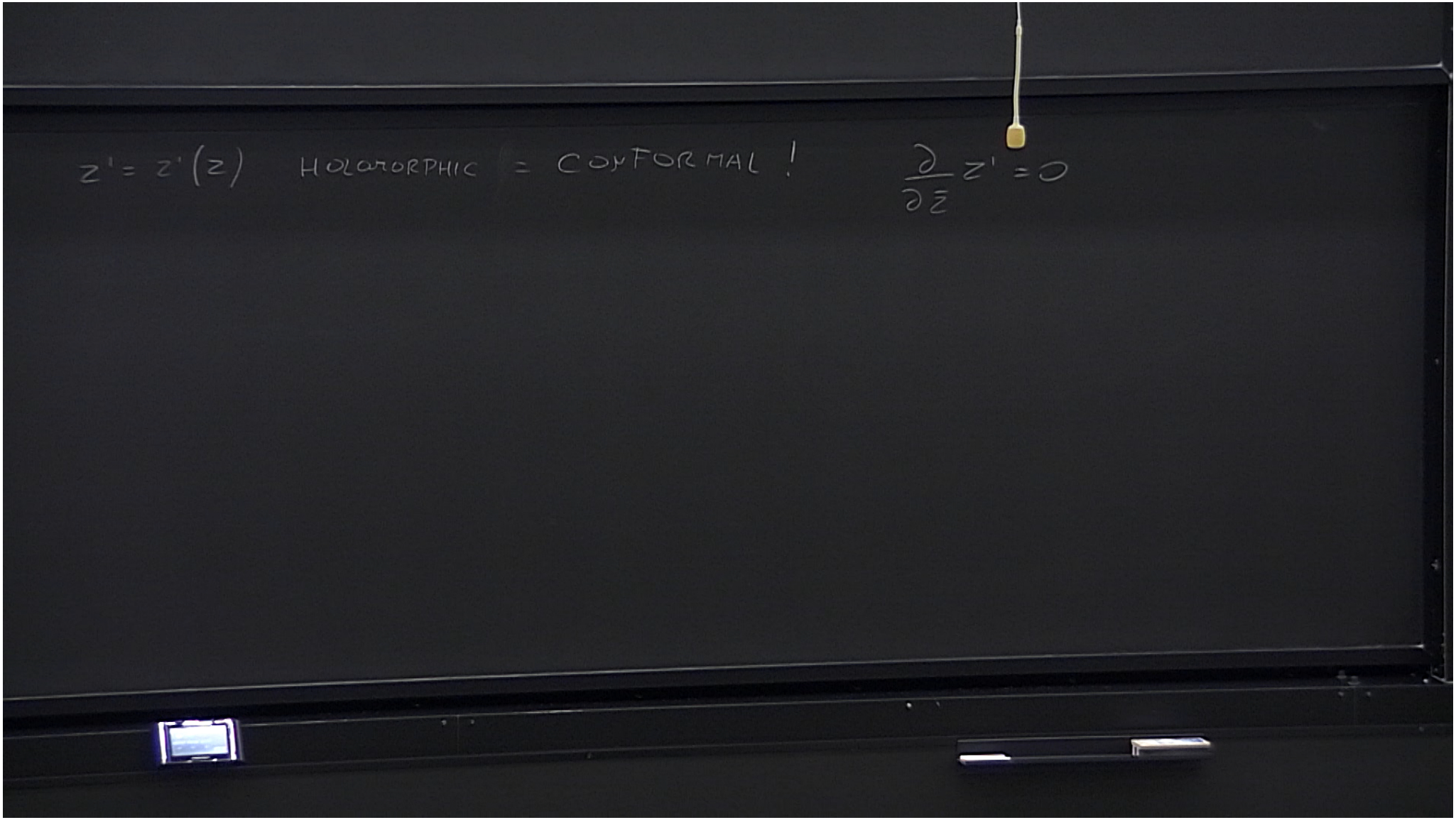
Abstract:

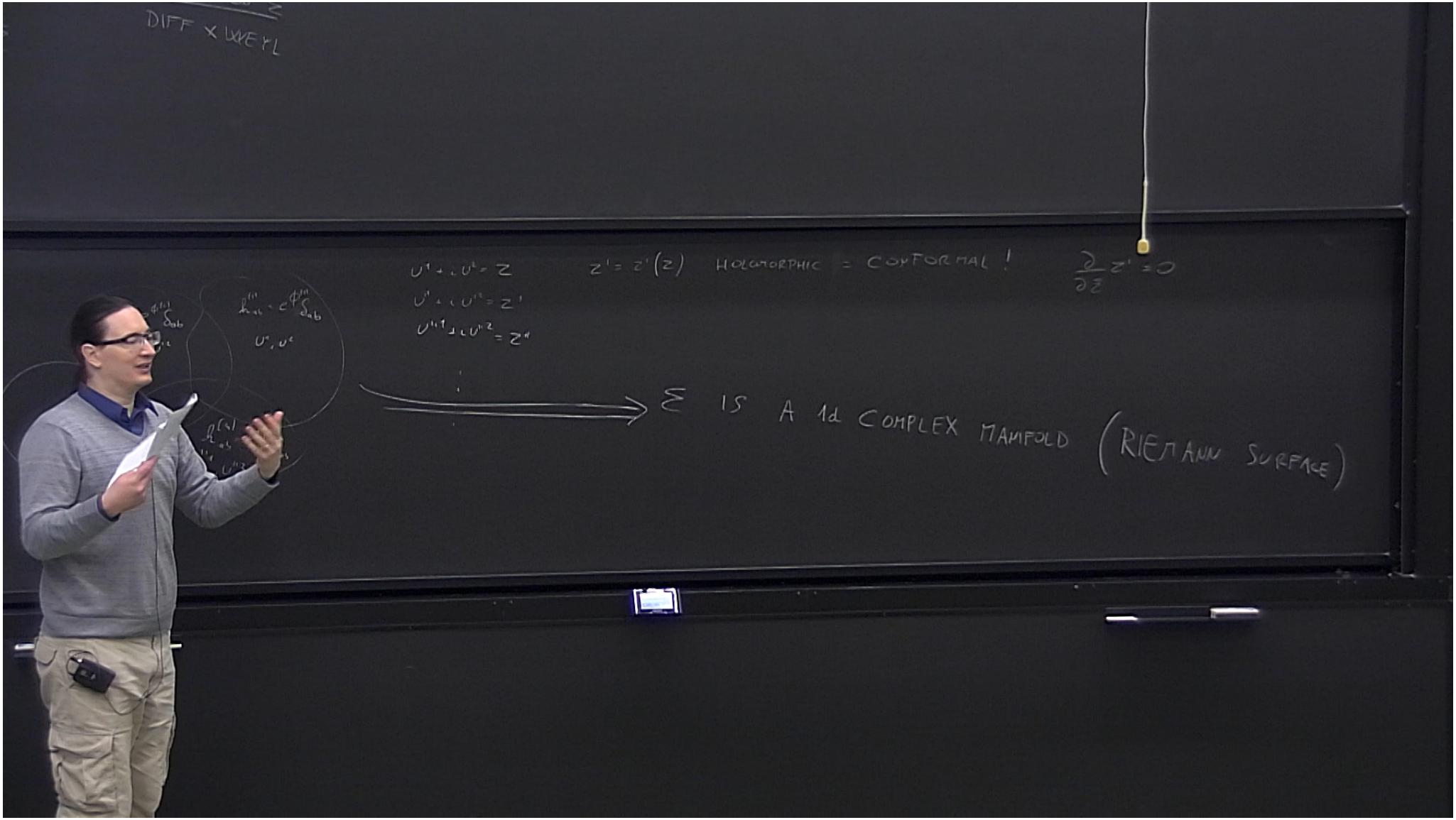
$$A_\Sigma = \int d\lambda \int DX(\sigma,\sigma') \, e^{\int_\Sigma \sqrt{h[\lambda]}\, h^{ab}(\lambda)\, \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}\, d\sigma \cdot d\sigma^2}$$

$\mathcal{M}_\Sigma = \dfrac{\text{METRICS ON } \Sigma}{\text{DIFF} \times \text{WEYL}}$

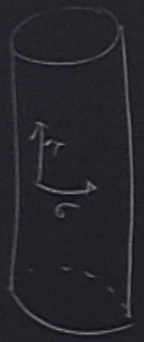



Σ
$h^{(2)}_{ab} = e^{\phi^{(2)}} \delta_{ab}$
U^1, U^2
$h^{(1)}_{ab} = e^{\phi^{(1)}} \delta_{ab}$
U^1, U^2
$h^{(3)}_{ab} = e^{\phi^{(3)}} \delta_{ab}$
U''^1, U''^2
$U^1 + iU^2 = Z$
$z' = z'(z)$
$U'^1 + iU'^2 = Z'$
$U''^1 + iU''^2 = Z''$


$z' = z'(z)$ HOLOMORPHIC = CONFORMAL !
$\frac{\partial}{\partial \bar{z}} z' = 0$


DIFF × WEYL
$U^1 \cap U^2 = Z$
$U^1 \cap U^{1'} = Z'$
$U^{1'} \cap U^{1''} = Z''$
$z' = z'(z)$ HOLOMORPHIC = CONFORMAL !
$\frac{\partial}{\partial \bar{z}} z' = 0$
Σ IS A 1d COMPLEX MANIFOLD (RIEMANN SURFACE)

$\mathbb{R}^2 = \mathbb{C}$



$s = \sigma + i\tau$

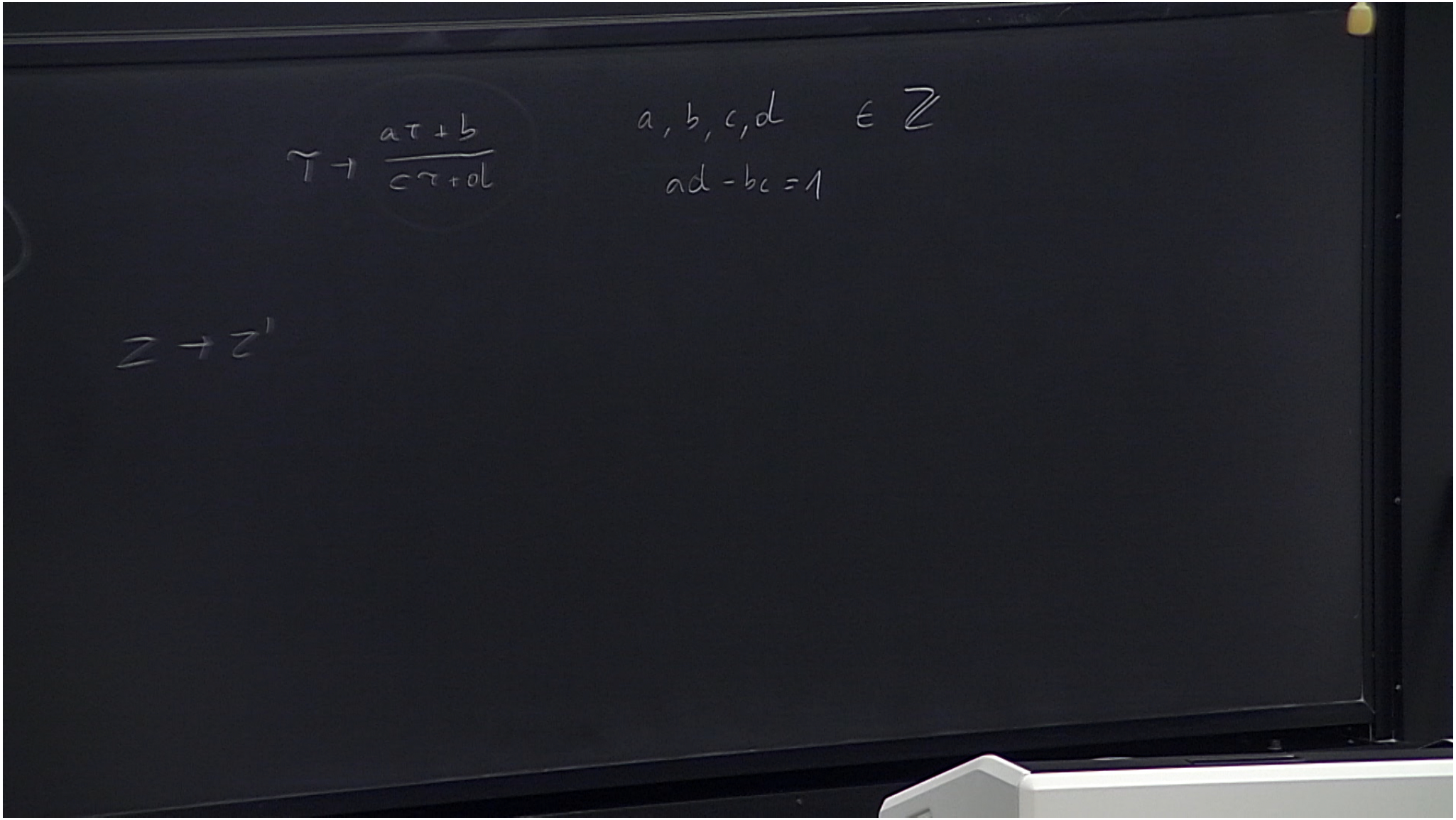
$s \equiv s + 2\pi R$



$z \to z + 2\pi$

$z \to z + 2\pi\tau$

$\tau \to \frac{a\tau + b}{c\tau + d}$


$\tau \to \frac{a\tau + b}{c\tau + d}$
$a, b, c, d \in \mathbb{Z}$
$ad - bc = 1$
$z \to z'$

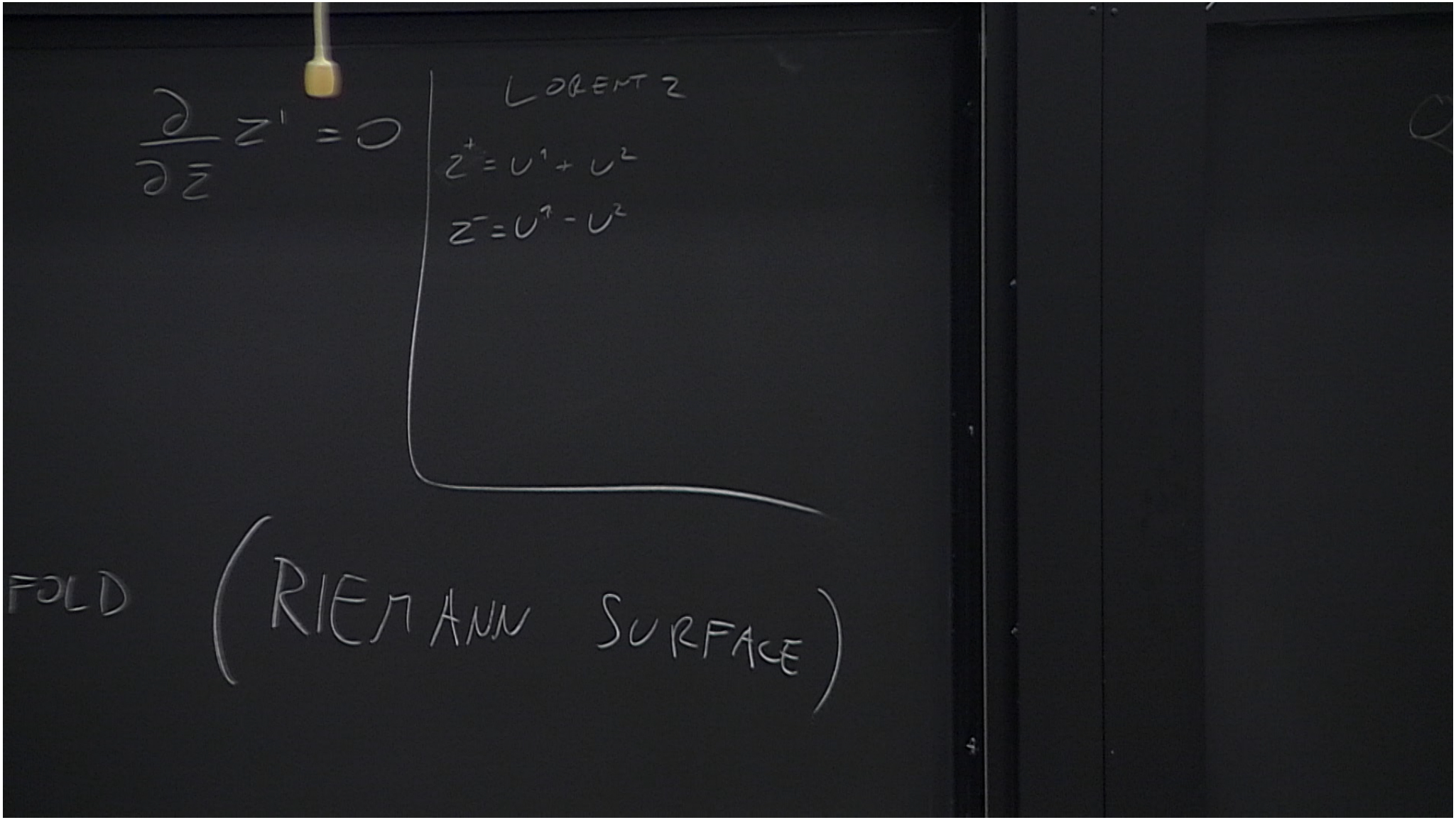
$z' = z'(z)$ HOLOMORPHIC = CONFORMAL !

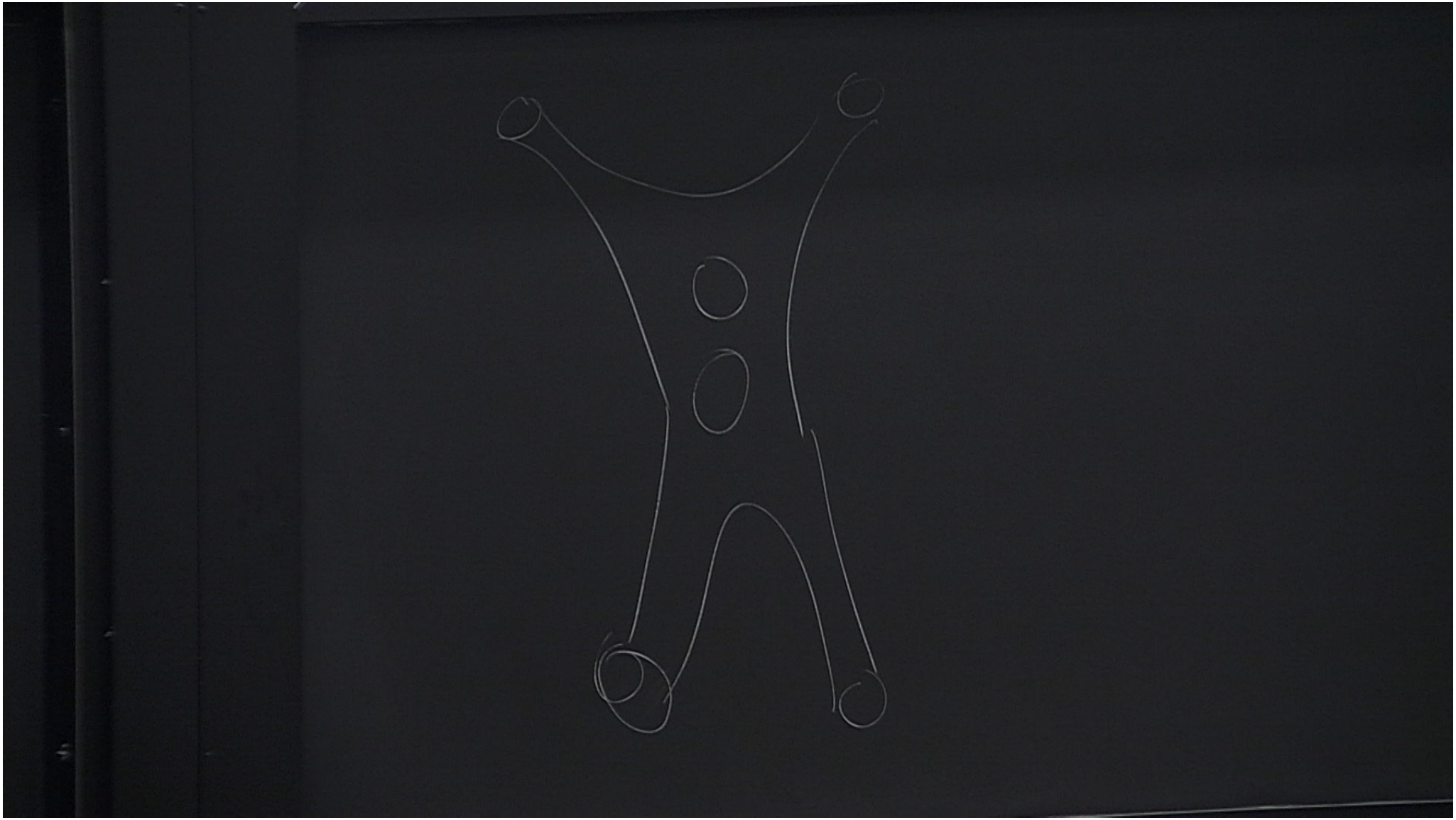
$\bar{z}' = \bar{z}'(z)$

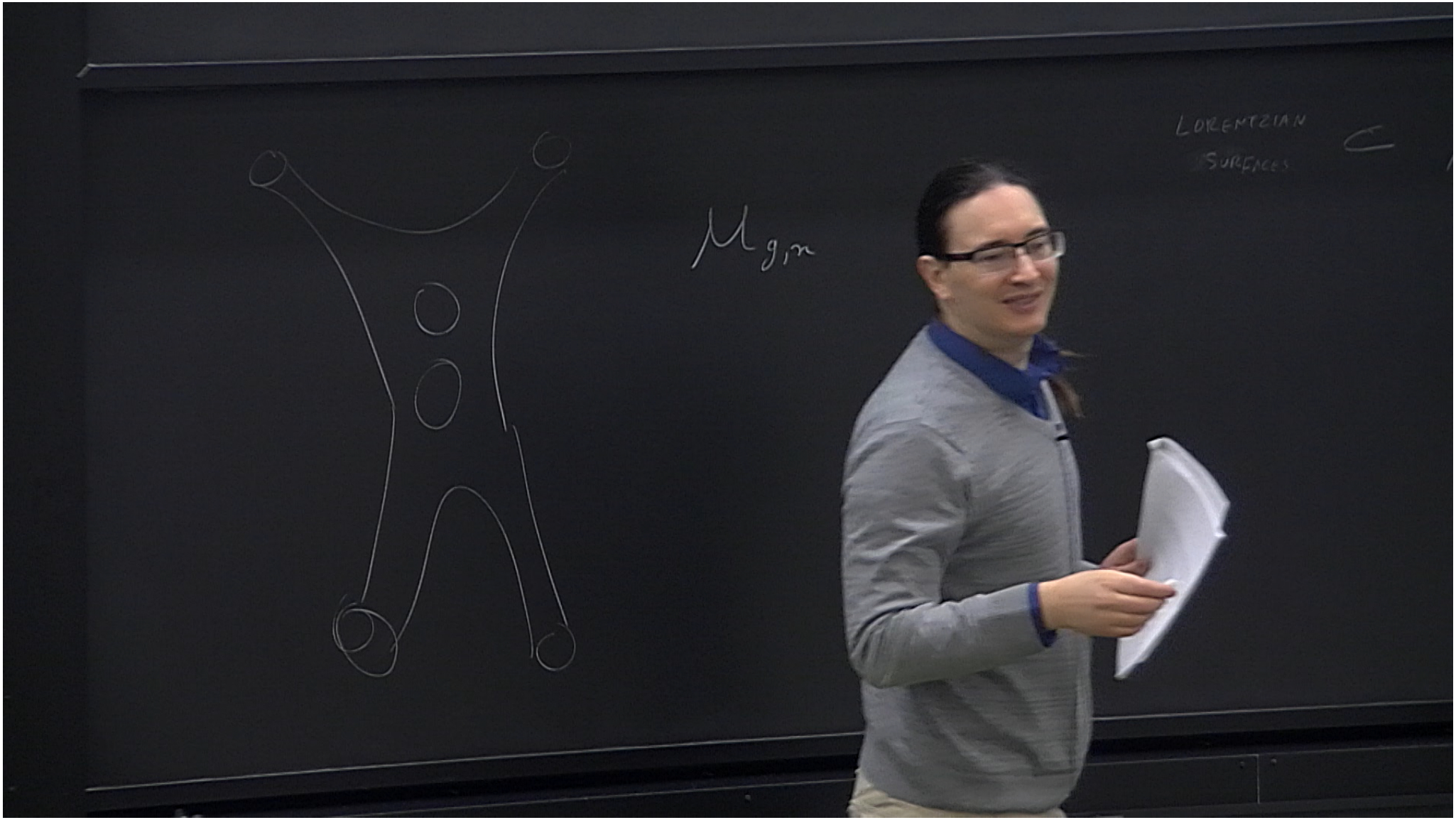
$\bar{z} = (z)^*$

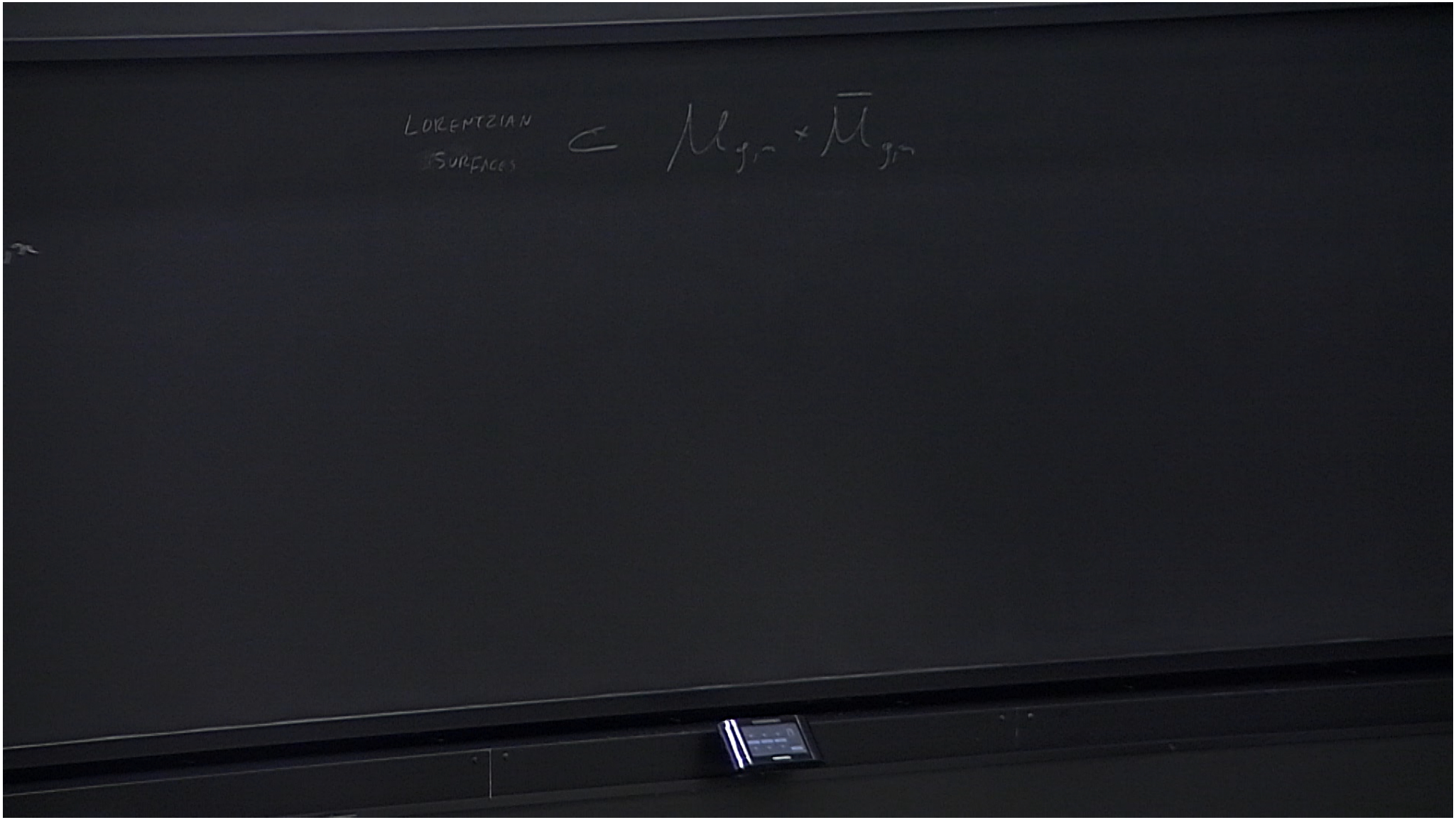
$\bar{z}' = (z')^*$

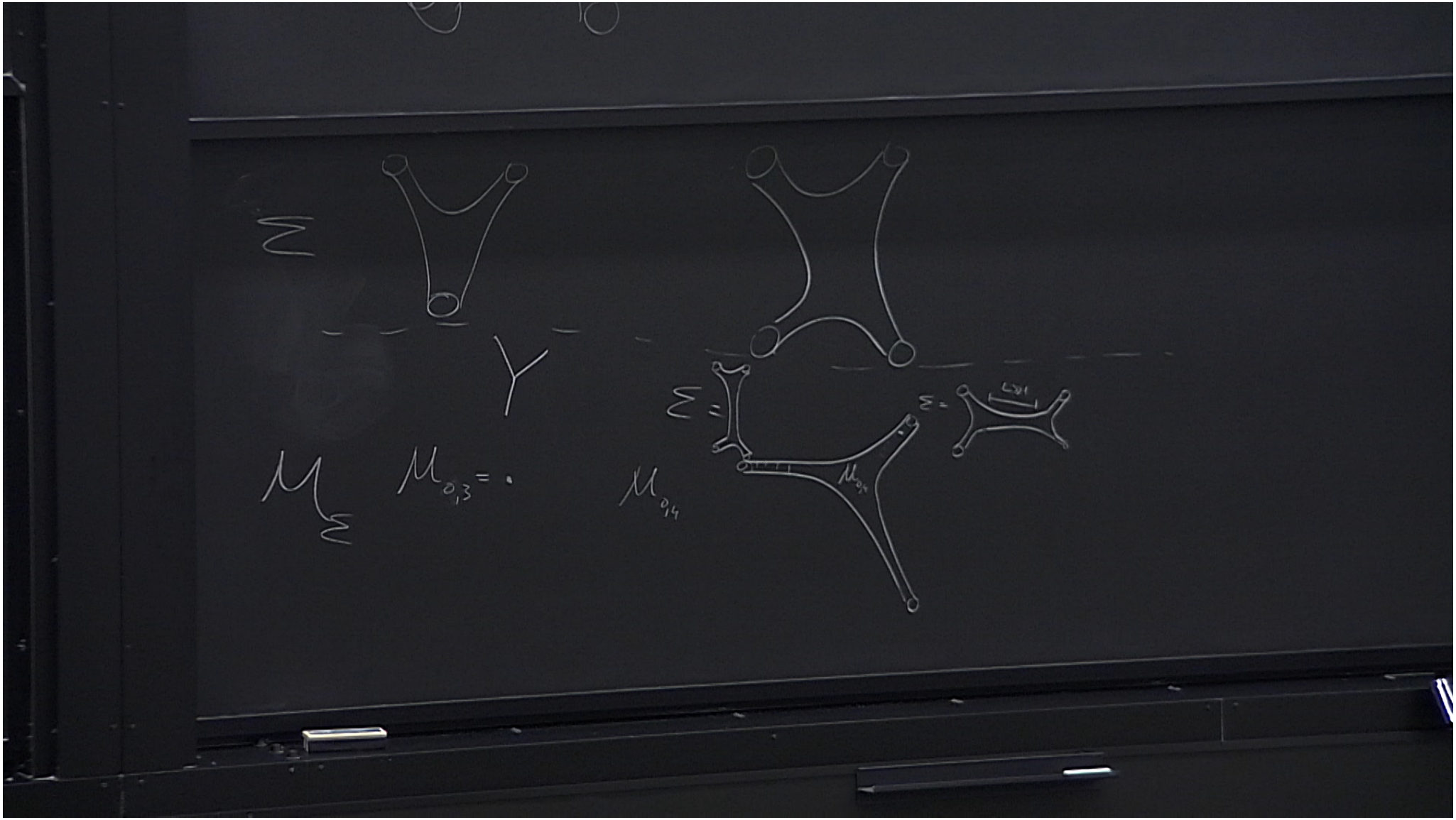
$\Longrightarrow$ Σ IS A 1d COMPLEX MANIFOLD

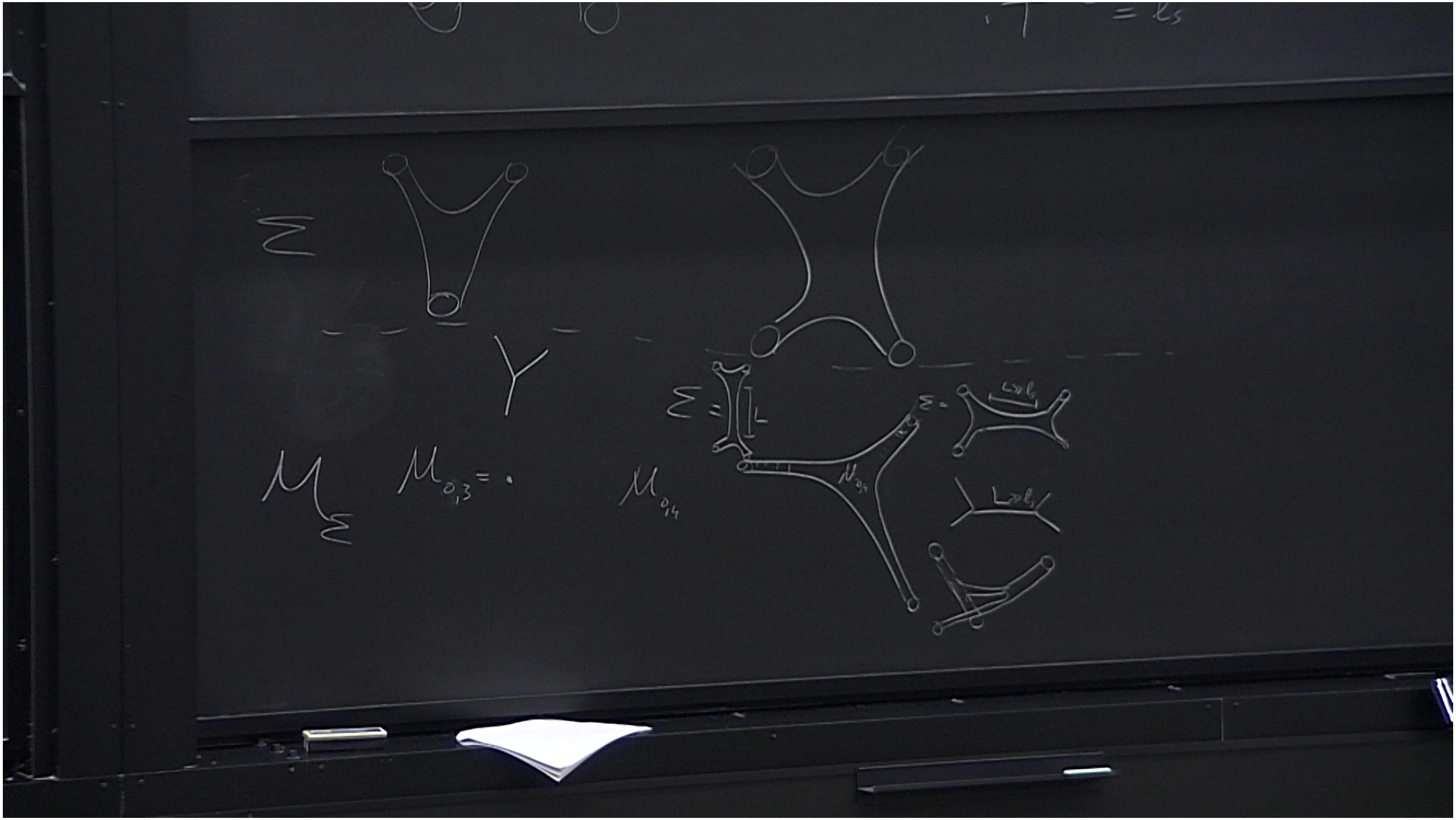

$\frac{\partial}{\partial \bar{z}} z^1 = 0$
LORENTZ
$z^+ = u^1 + u^2$
$z^- = u^1 - u^2$
FOLD (RIEMANN SURFACE)




LORENTZIAN
SURFACES
$\mathcal{M}_{g,n}$

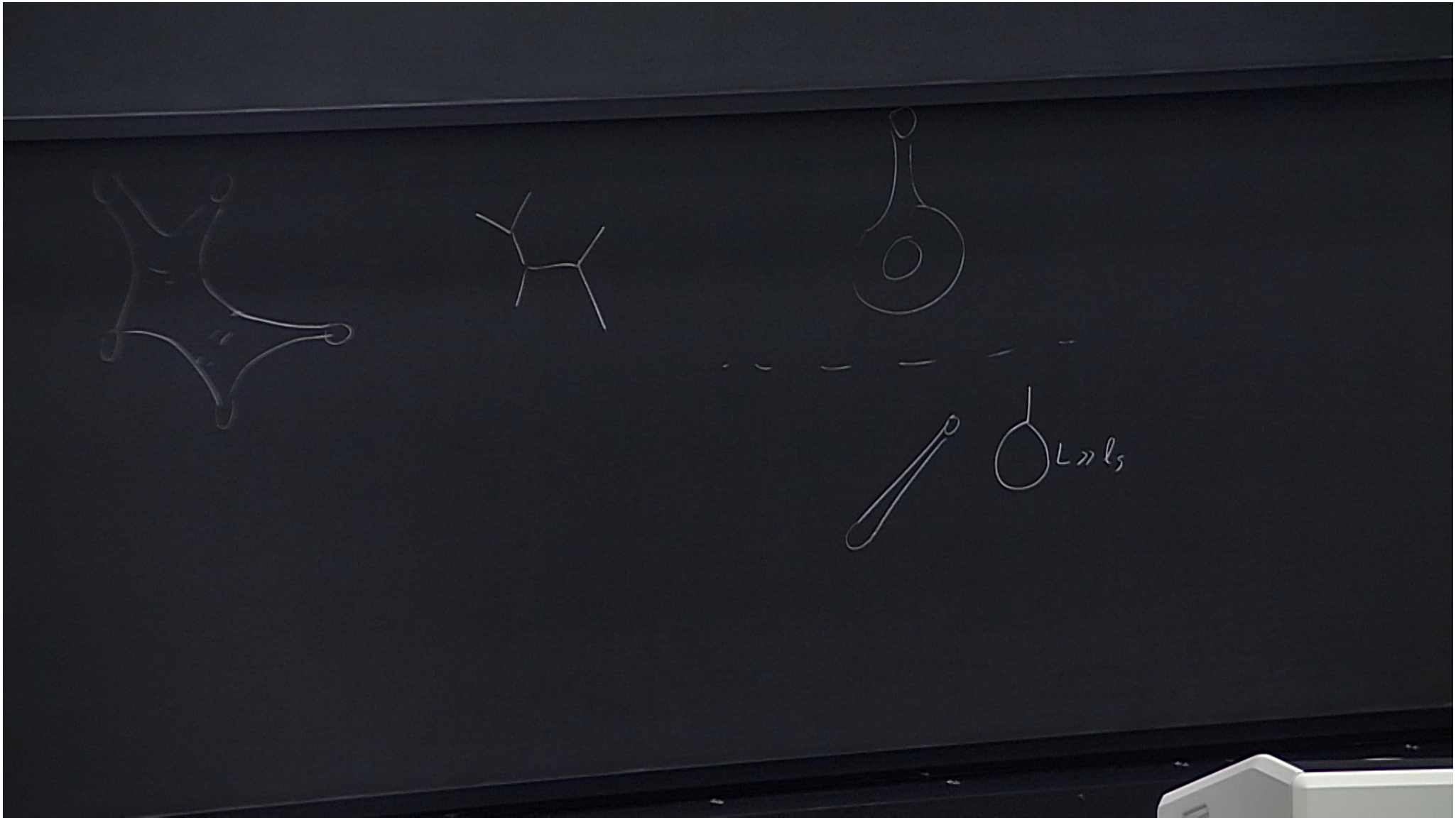

LORENTZIAN
SURFACES


Σ
Y
$\mathcal{M}_\Sigma$
$\mathcal{M}_{0,3} = \cdot$
$\mathcal{M}_{0,4}$
Σ =
Σ =
$\mathcal{M}_{0,4}$


Σ
Y
$\mathcal{M}_\Sigma$
$\mathcal{M}_{0,3} = \cdot$
$\mathcal{M}_{0,4}$
$\Sigma =$
$\Sigma =$

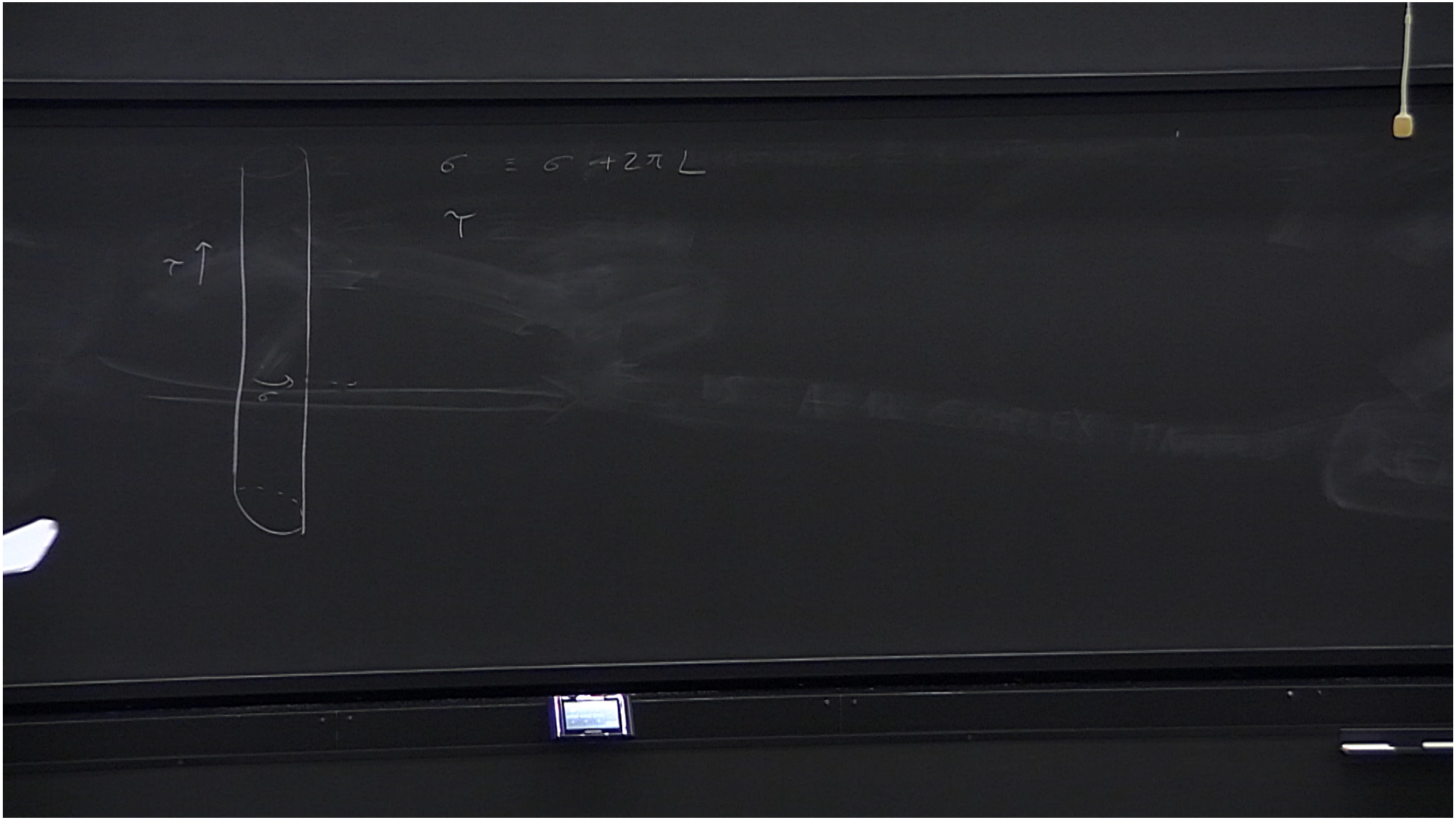
$$\dim \mathcal{M}_{g,n} = 3g + n - 3$$

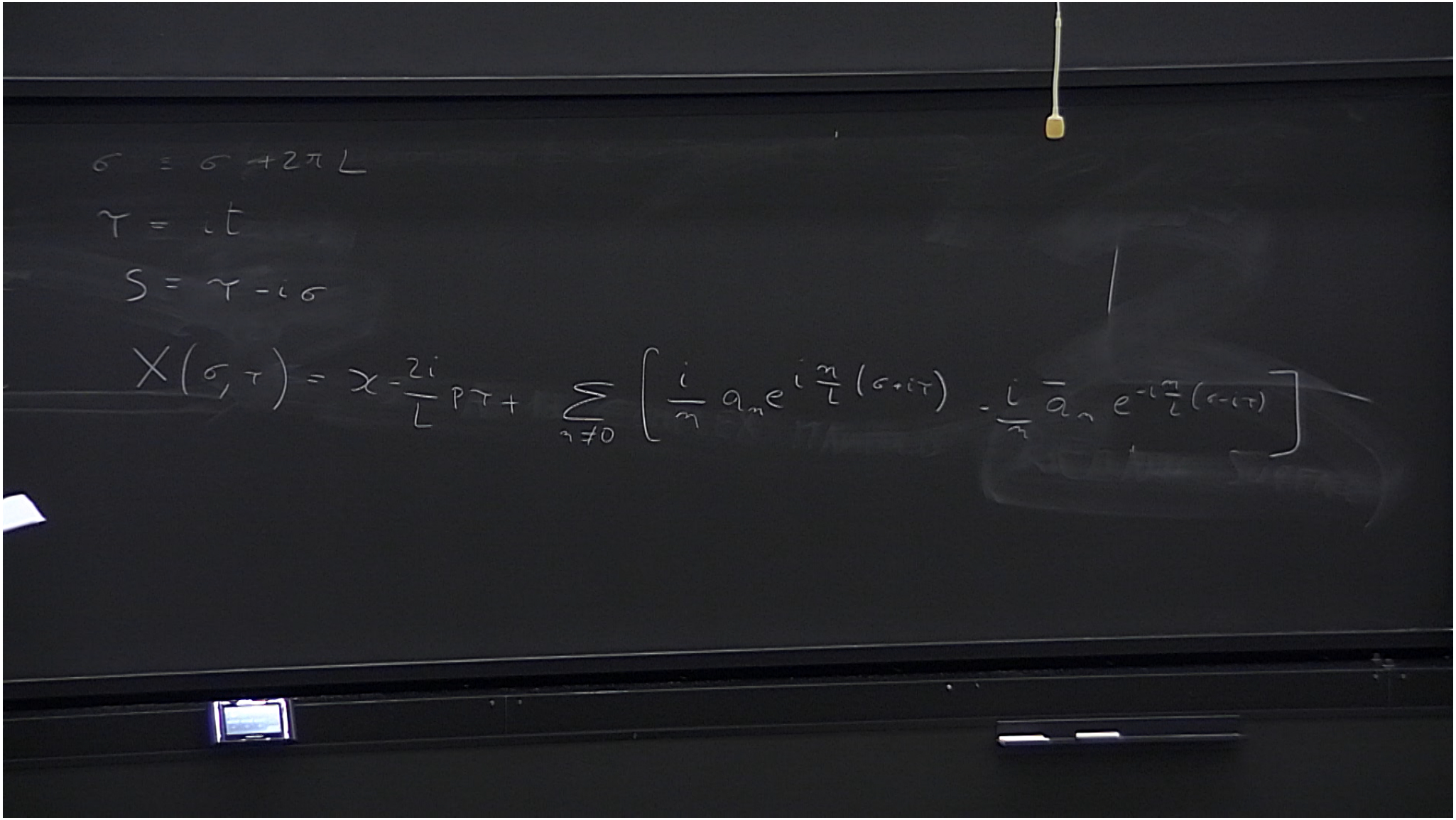
$$T^{-\frac{1}{2}} =$$

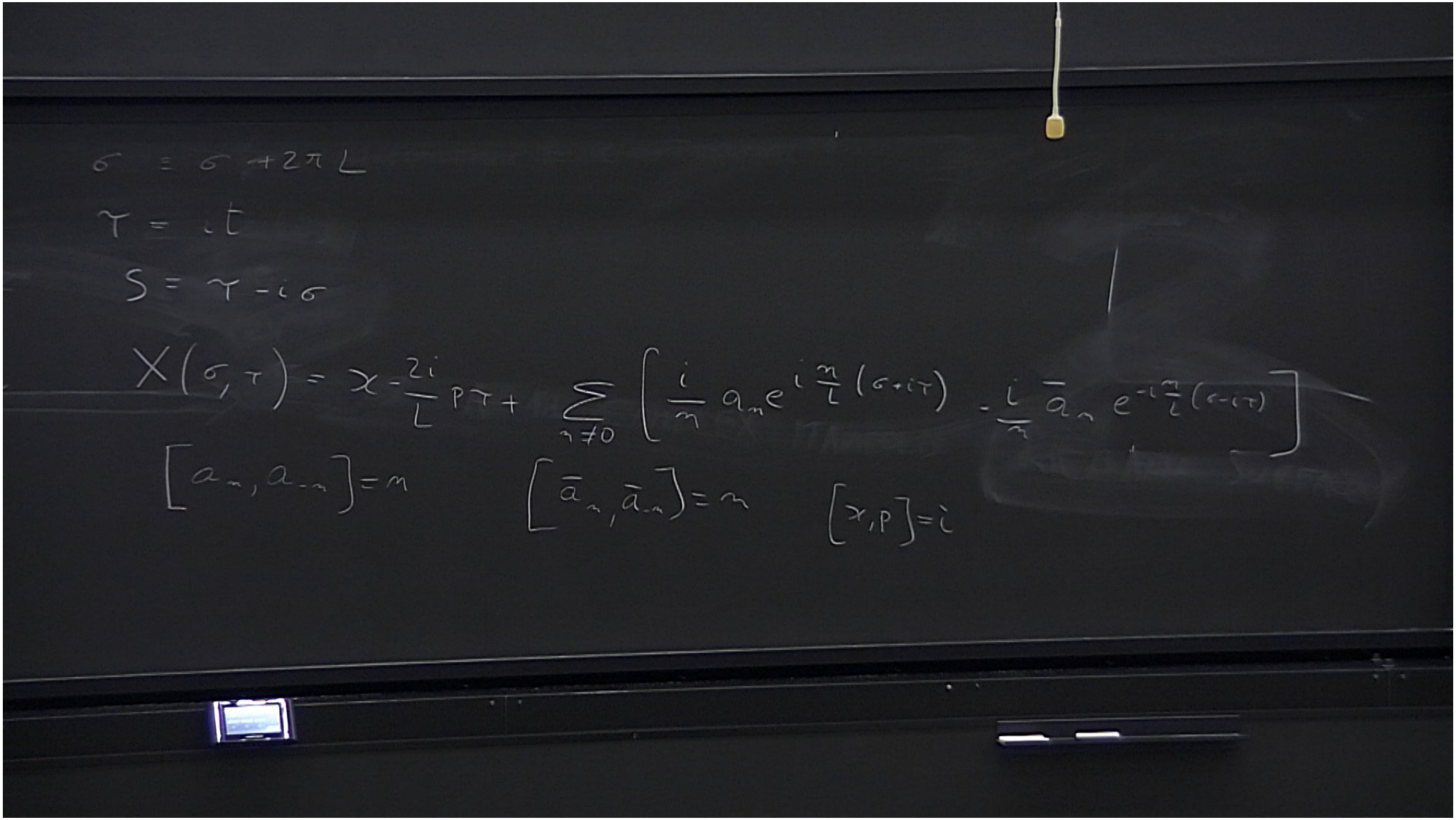

L >> ls

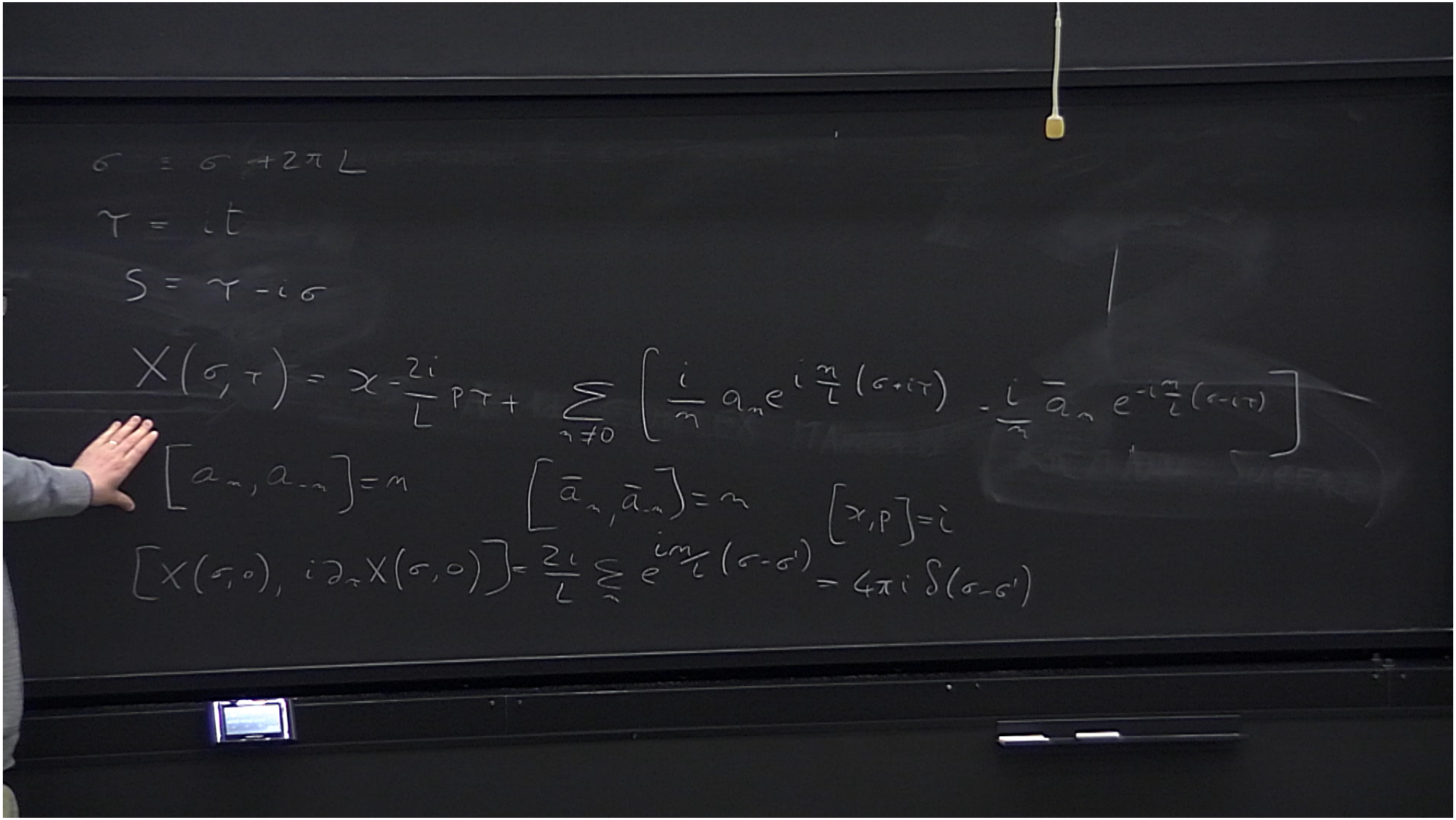
$$S(X) = \frac{T}{2} \int \frac{\partial X}{\partial u^a} \frac{\partial X}{\partial u^a} du^1 du^2$$




$\sigma \equiv \sigma + 2\pi L$
τ
τ
σ


$\sigma = \sigma + 2\pi L$
$\tau = it$
$S = \tau - i\sigma$
$X(\sigma,\tau) = x - \frac{2i}{L} p\tau + \sum_{n\neq 0} \left[\frac{i}{n} a_n e^{i\frac{n}{L}(\sigma+i\tau)} - \frac{i}{n} \bar{a}_n e^{-i\frac{n}{L}(\sigma-i\tau)} \right]$

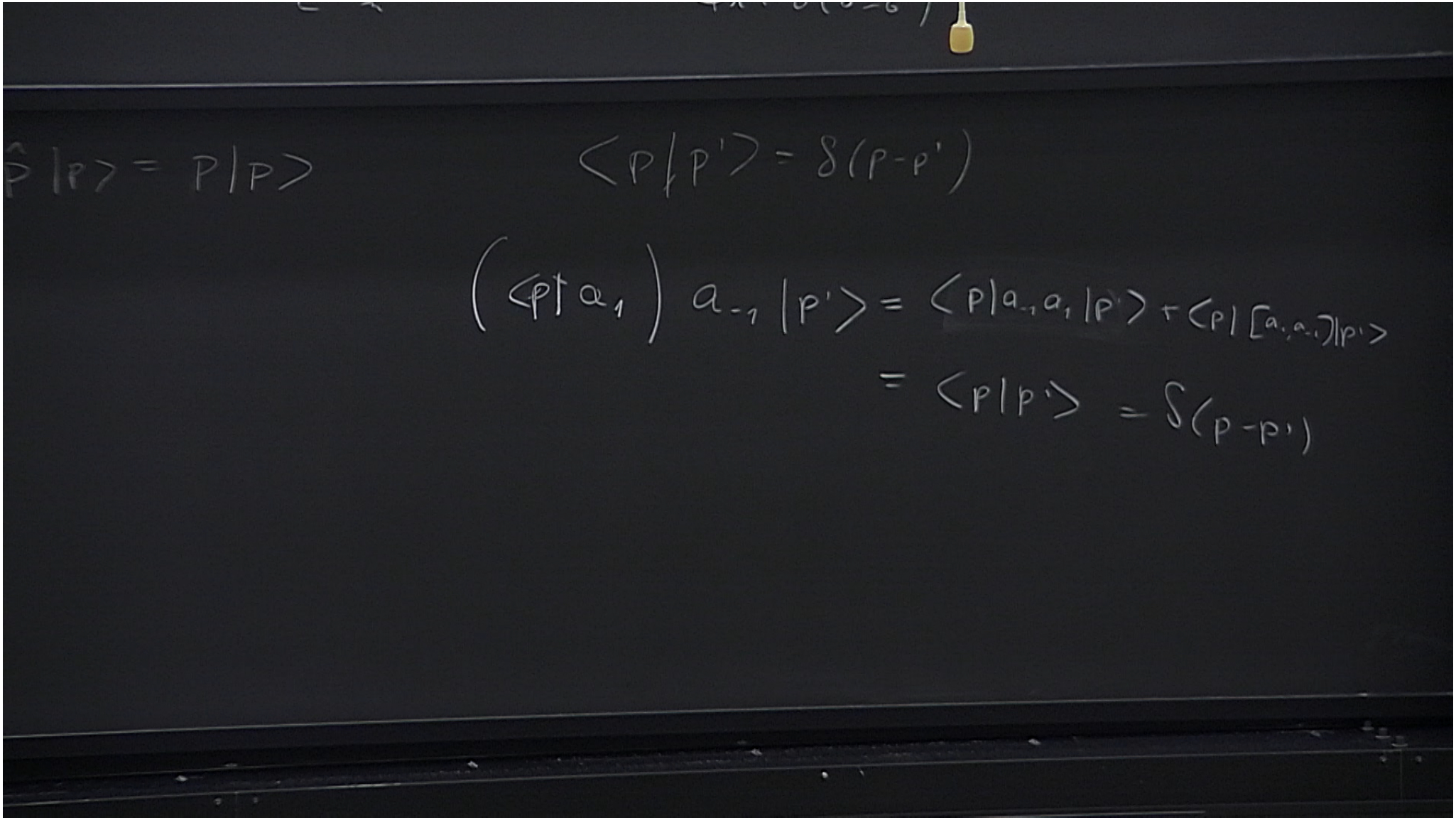

$\sigma \equiv \sigma + 2\pi L$
$\tau = it$
$S = \tau - i\sigma$
$X(\sigma,\tau) = x - \frac{2i}{L} p\tau + \sum_{n\neq 0} \left[\frac{i}{n} a_n e^{i\frac{n}{L}(\sigma+i\tau)} - \frac{i}{n} \bar{a}_n e^{-i\frac{n}{L}(\sigma-i\tau)} \right]$
$[a_m, a_{-n}] = m$
$[\bar{a}_m, \bar{a}_{-n}] = m$
$[x, p] = i$


$\sigma \equiv \sigma + 2\pi L$
$\tau = it$
$S = \tau - i\sigma$
$X(\sigma,\tau) = x - \frac{2i}{L} p\tau + \sum_{n\neq 0} \left[\frac{i}{n} a_n e^{i\frac{n}{L}(\sigma+i\tau)} - \frac{i}{n} \bar{a}_n e^{-i\frac{n}{L}(\sigma-i\tau)} \right]$
$[a_n, a_{-n}] = n$
$[\bar{a}_n, \bar{a}_{-n}] = n$
$[x, p] = i$
$[X(\sigma,0), i\partial_\tau X(\sigma,0)] = \frac{2i}{L} \sum_n e^{\frac{in}{L}(\sigma-\sigma')} = 4\pi i \delta(\sigma-\sigma')$

$|p\rangle$ $\quad a_n|p\rangle = 0 \quad n>0 \quad \bar{a}_n|p\rangle = 0 \quad n>0$

$a_{-1}|p\rangle \quad \bar{a}_{-1}|p\rangle$

$a_{-2}|p\rangle \quad a_{-1}^2|p\rangle \quad a_{-1}\bar{a}_{-1}|p\rangle \quad \bar{a}_{-1}^2|p\rangle \quad \bar{a}_{-2}|p\rangle$


$\hat{p}|p\rangle = p|p\rangle$
$\langle p|p'\rangle = \delta(p-p')$
$(\langle p|a_1)\, a_{-1}|p'\rangle = \langle p|a_{-1}a_1|p'\rangle + \langle p|[a_1,a_{-1}]|p'\rangle$
$= \langle p|p'\rangle = \delta(p-p')$

$$S(X) = \frac{T}{2} \int \frac{\partial X}{\partial v^a} \frac{\partial X}{\partial v^a} \, dv^1 dv^2 = \frac{T}{2} \int \partial_s X \, \partial_{\bar{s}} X$$

$\tau \uparrow$

$X \to X + c$ $\qquad J_a = i \partial_a X$

$$J_s = i \partial_s X = \frac{1}{L} p + \sum_{n \neq 0} \frac{1}{L} a_n e^{-\frac{n}{L} s}$$

$$J_{\bar{s}} = i \partial_{\bar{s}} X = \frac{1}{L} p - \sum_{n \neq 0} \frac{1}{L} \bar{a}_n e^{-\frac{n}{L} \bar{s}}$$

$\sigma \to$ . . .

$\partial_{\bar{s}} J_s + \partial_s J_{\bar{s}} = 0$

$\partial_{\bar{s}} J_s = 0 \qquad \partial_s J_{\bar{s}} = 0$

$\partial_{\bar{s}} (f(s) J_s) = 0$

$J^f = f(z) \partial_s X$

$J^{\bar{f}} = \bar{f}(\bar{z}) \partial_{\bar{s}} X$

$$S(X) = \frac{T}{2} \int \frac{\partial X}{\partial \sigma^a} \frac{\partial X}{\partial \sigma^a} d\sigma^1 d\sigma^2 = \frac{T}{2} \int \partial_s X \partial_{\bar{s}} X$$

$\tau \uparrow$

$\sigma \circlearrowright$

$X \to X + c$ $\qquad J_a = i \partial_a X$

$X \to X + c(z) + \bar{c}(\bar{z})$ $\qquad J_s = i \partial_s X = \frac{1}{L} p + \sum_{n \neq 0} \frac{1}{L} a_n e^{-\frac{n}{L} s}$

$J_{\bar{s}} = i \partial_{\bar{s}} X = \frac{1}{L} p - \sum_{n \neq 0} \frac{1}{L} \bar{a}_n e^{-\frac{n}{L} \bar{s}}$

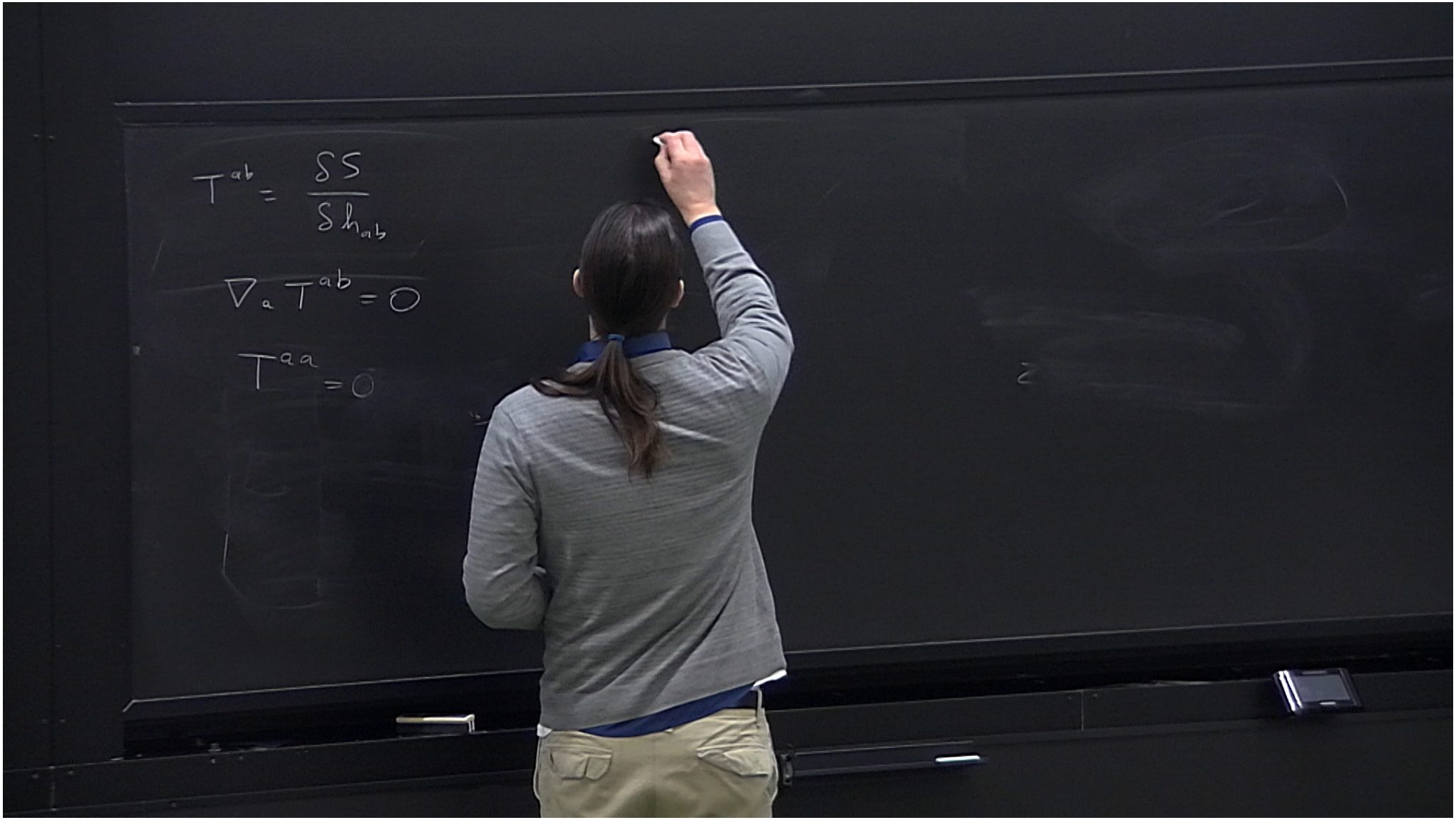
$\partial_{\bar{s}} J_s + \partial_s J_{\bar{s}} = 0$

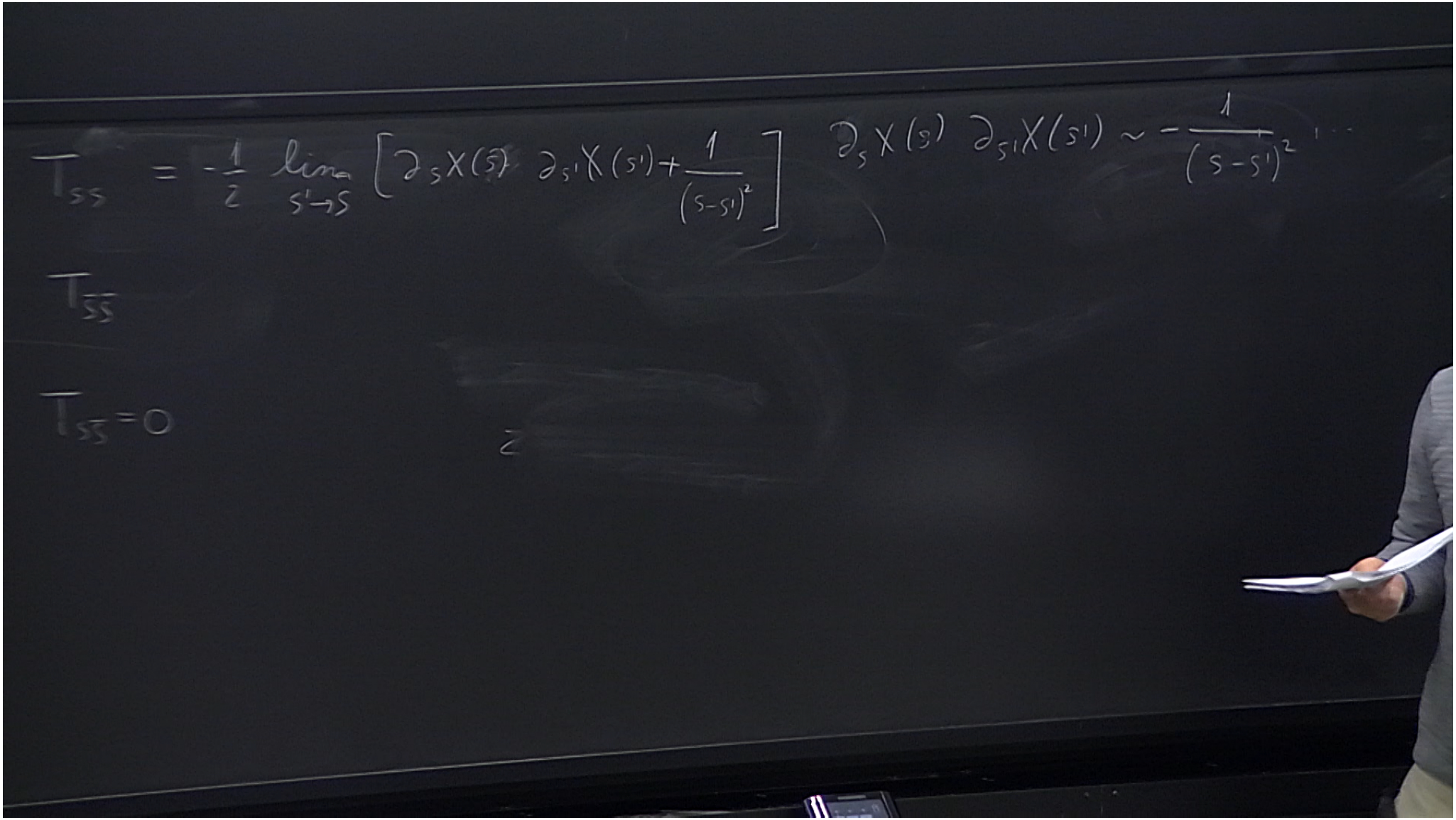
$\partial_{\bar{s}} J_s = 0 \qquad \partial_s J_{\bar{s}} = 0$

$\partial_{\bar{s}} (f(s) J_s) = 0$

$J^f = f(z) \partial_s X$

$J^{\bar{f}} = \bar{f}(\bar{z}) \partial_{\bar{s}} X$

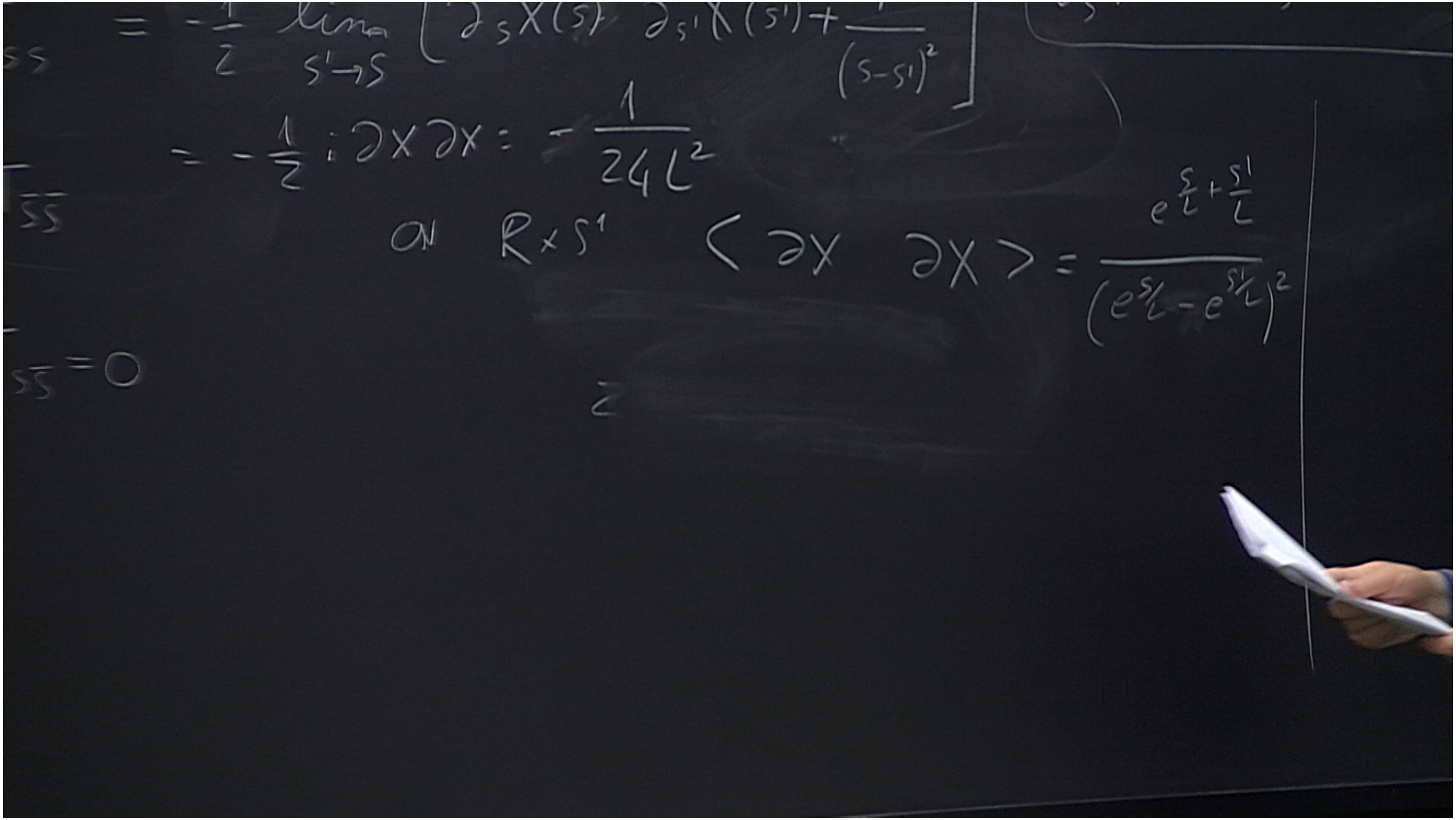

$T^{ab} = \frac{\delta S}{\delta h_{ab}}$
$\nabla_a T^{ab} = 0$
$T^{aa} = 0$


$T_{ss} = -\frac{1}{2} \lim_{s' \to s} \left[\partial_s X(s) \, \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$
$\partial_s X(s) \, \partial_{s'} X(s') \sim -\frac{1}{(s-s')^2}$
T_{ss}
$T_{ss} = 0$
2

$$\partial_s X(s)\,\partial_{s'} X(s') \sim -\frac{1}{(s-s')^2}$$

ON $R^2 \equiv \mathbb{C}$ $\qquad \langle \partial X(s)\,\partial X(s') \rangle_{\mathbb{C}} = -\frac{1}{(s-s')^2}$

$$T_{ss} = -\frac{1}{2} : \partial_s X\, \partial_s X :$$


$= -\frac{1}{2} \lim_{s' \to s} \left[\partial_s X(s) \, \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$
$= -\frac{1}{2} : \partial X \partial X : = -\frac{1}{24 L^2}$
on $R \times S^1$
$\langle \partial X \, \partial X \rangle = \frac{e^{\frac{s}{L} + \frac{s'}{L}}}{(e^{\frac{s}{L}} - e^{\frac{s'}{L}})^2}$
$= 0$