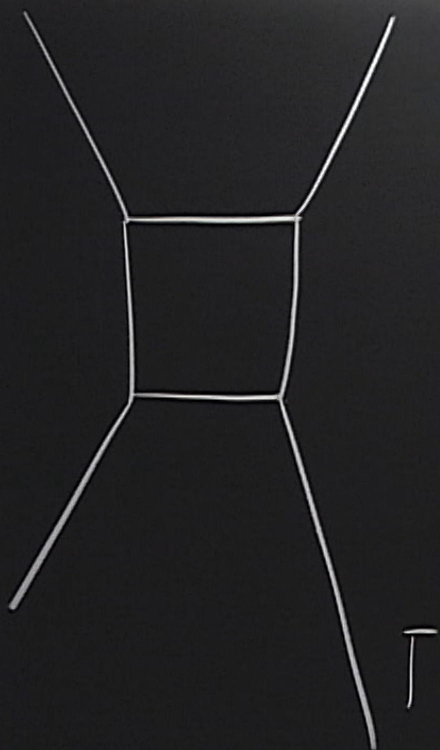
Title: PSI 2016/2017 String Theory (Review) - Lecture 2

Date: Feb 22, 2017 10:15 AM

URL: http://pirsa.org/17020107

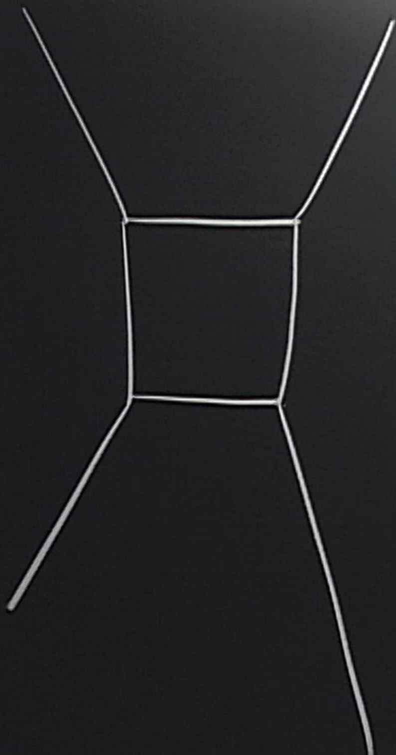
Abstract:



$$S_{cl}[\Gamma \subset \mathbb{R}^{d,1}] = \sum_{i \in \text{edges}} m_i \int ds_i$$

$$A_\Gamma = \int D(\Gamma \subset \mathbb{R}^{d,1})\, e^{\frac{i}{\hbar} S_{cl}}$$

$$A_\Gamma^{QFT}$$

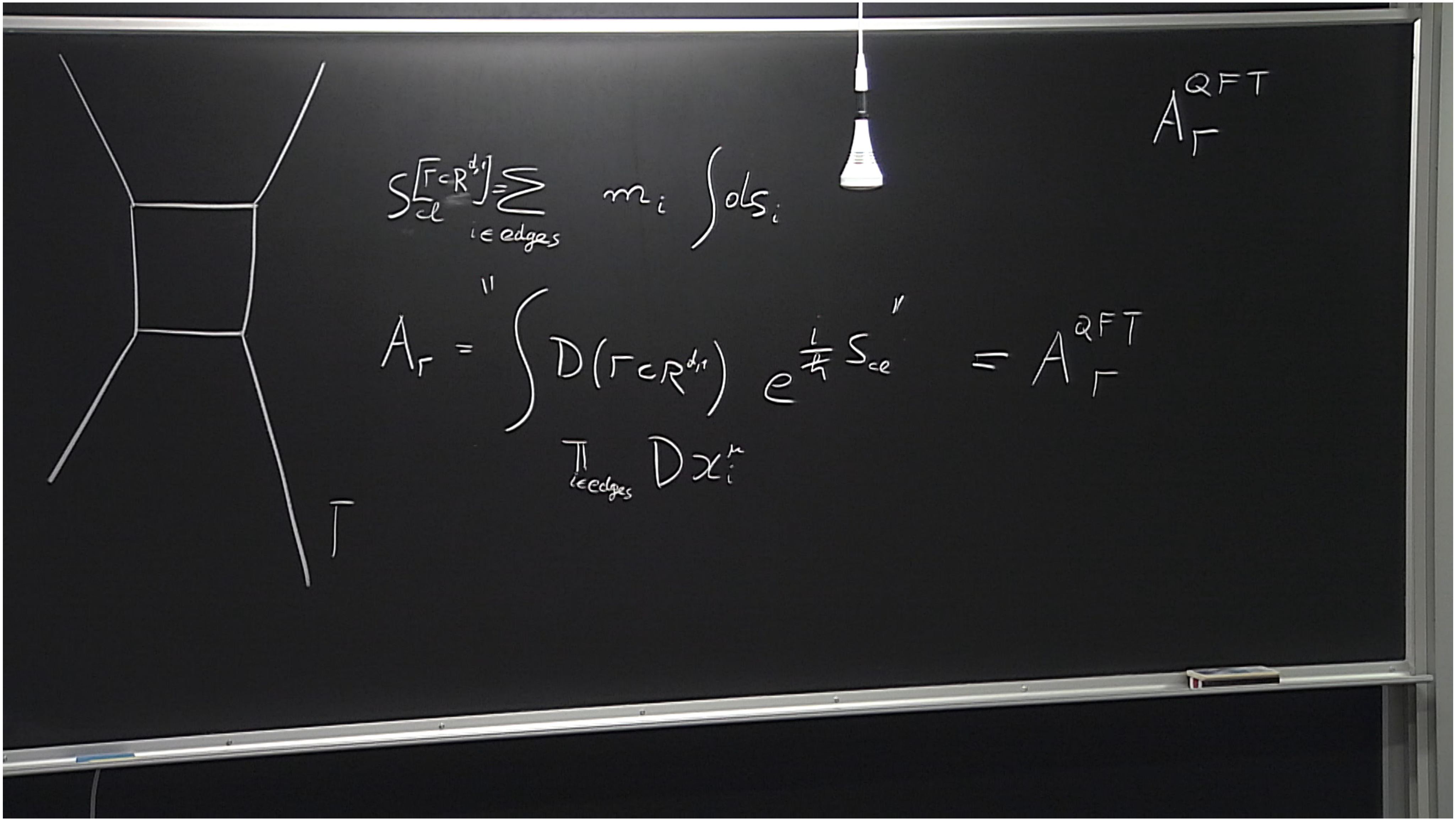


Γ

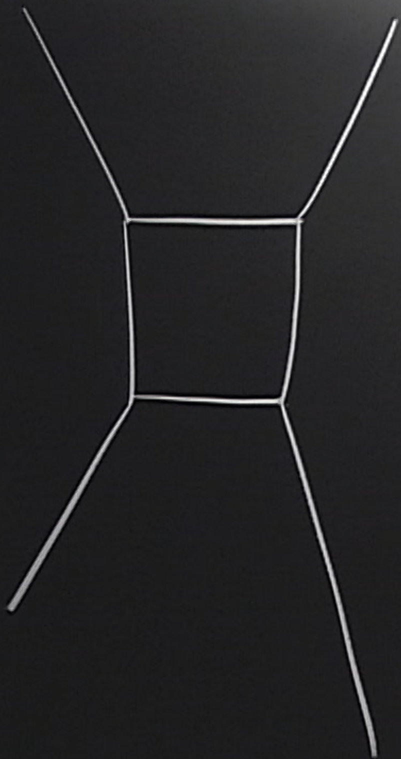
$$S_{cl}[\Gamma \subset \mathbb{R}^{d,1}] = \sum_{i \in \text{edges}} m_i \int ds_i$$

$$A_\Gamma^{QFT}$$

$$A_\Gamma = \int D(\Gamma \subset \mathbb{R}^{d,1})\, e^{\frac{i}{\hbar} S_{cl}} = A_\Gamma^{QFT}$$



A_Γ^{QFT}
$S_{cl}[\Gamma \subset R^{d,1}] = \sum_{i \in edges} m_i \int ds_i$
$A_\Gamma = " \int D(\Gamma \subset R^{d,1}) \, e^{\frac{i}{\hbar} S_{cl}} " = A_\Gamma^{QFT}$
$\prod_{i \in edges} D x_i^\mu$
Γ



Γ

$$A_\Gamma^{QFT}$$

$$S_{cl}[\Gamma \subset \mathbb{R}^{d-1,1}] = \sum_{i \in \text{edges}} m_i \int ds_i$$

$$A_\Gamma \;``=" \int D(\Gamma \subset \mathbb{R}^{d-1,1})\, e^{\frac{i}{\hbar} S_{cl}}\;" = A_\Gamma^{QFT}$$

$$\prod_{i \in \text{edges}} D x_i^\mu$$

$$\int_{\ell=0}^{\infty} e^{-a\ell}\, d\ell = \frac{1}{a}$$

$$A_\Gamma^{QFT} = \int \prod_{\substack{i \in \text{INTERVAL}\\ \text{EDGES}}} dp_i \frac{1}{p_i^2+m_i^2} \prod_{a \in \text{VERTICES}} \delta^{d}\left(\Sigma p\right) = \int \prod_i dp_i \frac{1}{p_i^2+m^2} \int \prod_a dx_a^\mu\, e^{i x_a^\mu (\Sigma p)} =$$

$$= \int \prod_i dp_i\, dL_i\, e^{-L_i (p_i^2+m^2)} \prod_a dx_a^\mu\, e^{i x_a (\Sigma p)}$$

$$\int_{\ell=0}^{\infty} e^{-a\ell}\, d\ell = \frac{1}{a}$$

$$A_\Gamma^{QFT} = \int \prod_{\substack{i \in \text{INTERNAL} \\ \text{EDGES}}} dp_i \frac{1}{p_i^2+m_i^2} \prod_{a \in \text{VERTICES}} \delta^d\left(\Sigma p\right) = \int \prod_i dp_i \frac{1}{p_i^2+m^2} \int \prod_a dx_a^\mu \, e^{i x_a^\mu \left(\Sigma p\right)} =$$

$$= \int \prod_i dp_i \, dL_i \, e^{-L_i\left(p_i^2+m^2\right)} \prod_a dx_a^\mu \, e^{i x_a \left(\Sigma p\right)}$$

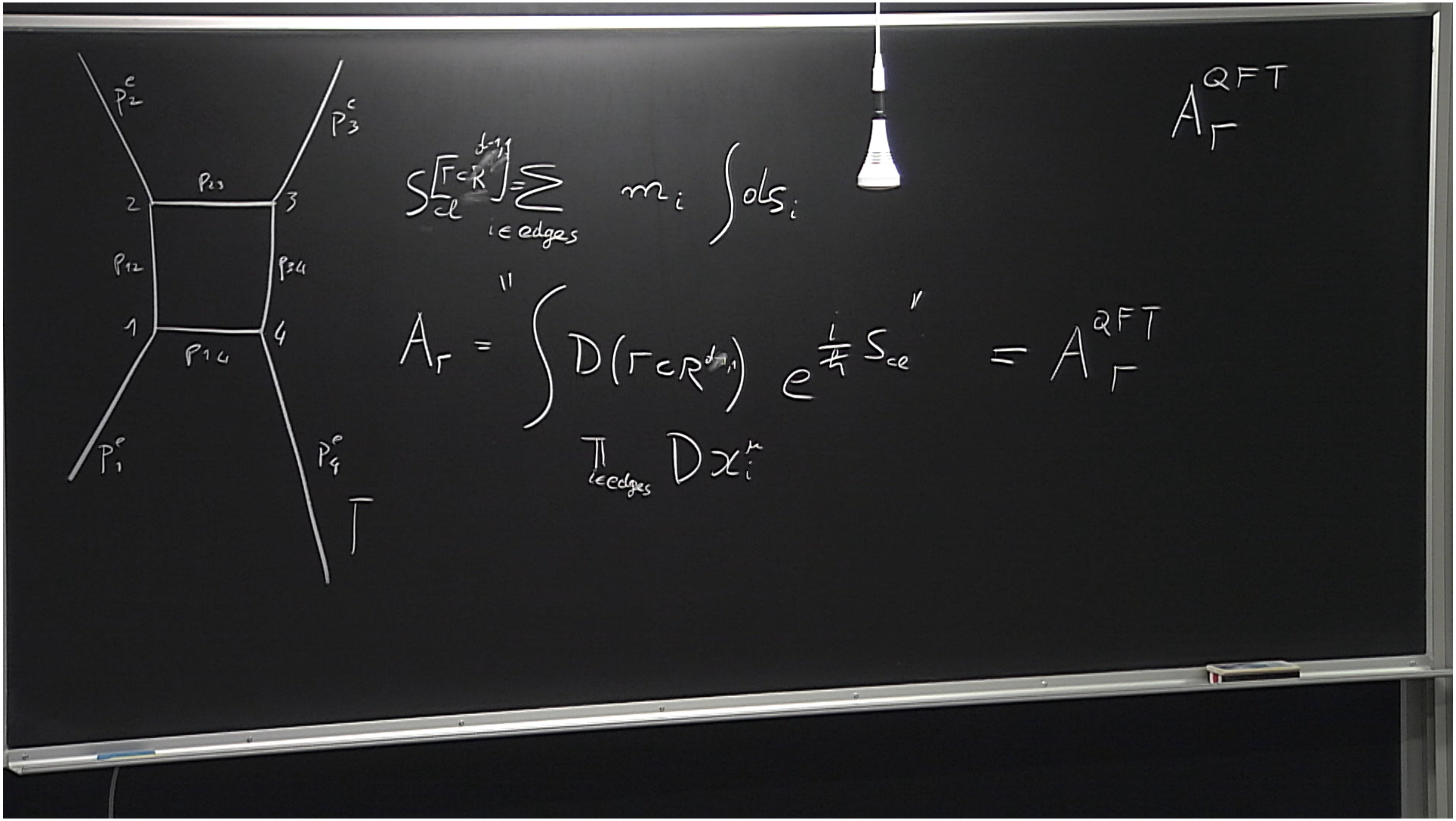
$$= \int_0^\infty dL_{ab} \int \prod_a dx_a^\mu \prod_{\substack{j \in \text{EXTERNAL} \\ \text{EDGES}}} e^{i x_j^\mu p_j} \prod_{\substack{(a,b) \in \text{INTERNAL} \\ \text{EDGES}}} e^{-\frac{(x_a-x_b)^2}{4L_{ab}} - L_{ab} m_{ab}^2}$$

$$\int_{\ell=0}^{\infty} e^{-a\ell}\, d\ell = \frac{1}{a}$$

$$A_\Gamma^{QFT} = \int \prod_{i \in \text{INTERNAL EDGES}} dp_i \, \frac{1}{p_i^2+m_i^2} \prod_{a \in \text{VERTICES}} \delta^d\left(\Sigma p\right) = \int \prod_i dp_i \, \frac{1}{p_i^2+m^2} \int \prod_a dx_a^\mu \, e^{i x_a^\mu (\Sigma p)} =$$

$$= \int \prod_i dp_i \, dL_i \, e^{-L_i (p_i^2+m^2)} \prod_a dx_a^\mu \, e^{i x_a (\Sigma p)}$$

$$= \int_0^\infty dL_{ab} \int \prod_a dx_a^\mu \prod_{j \in \text{EXTERNAL EDGES}} e^{i x_j^\mu p_j} \prod_{(a,b) \in \text{INTERNAL EDGES}} e^{-\frac{(x_a-x_b)^2}{4L_{ab}} - L_{ab} m_{ab}^2}$$

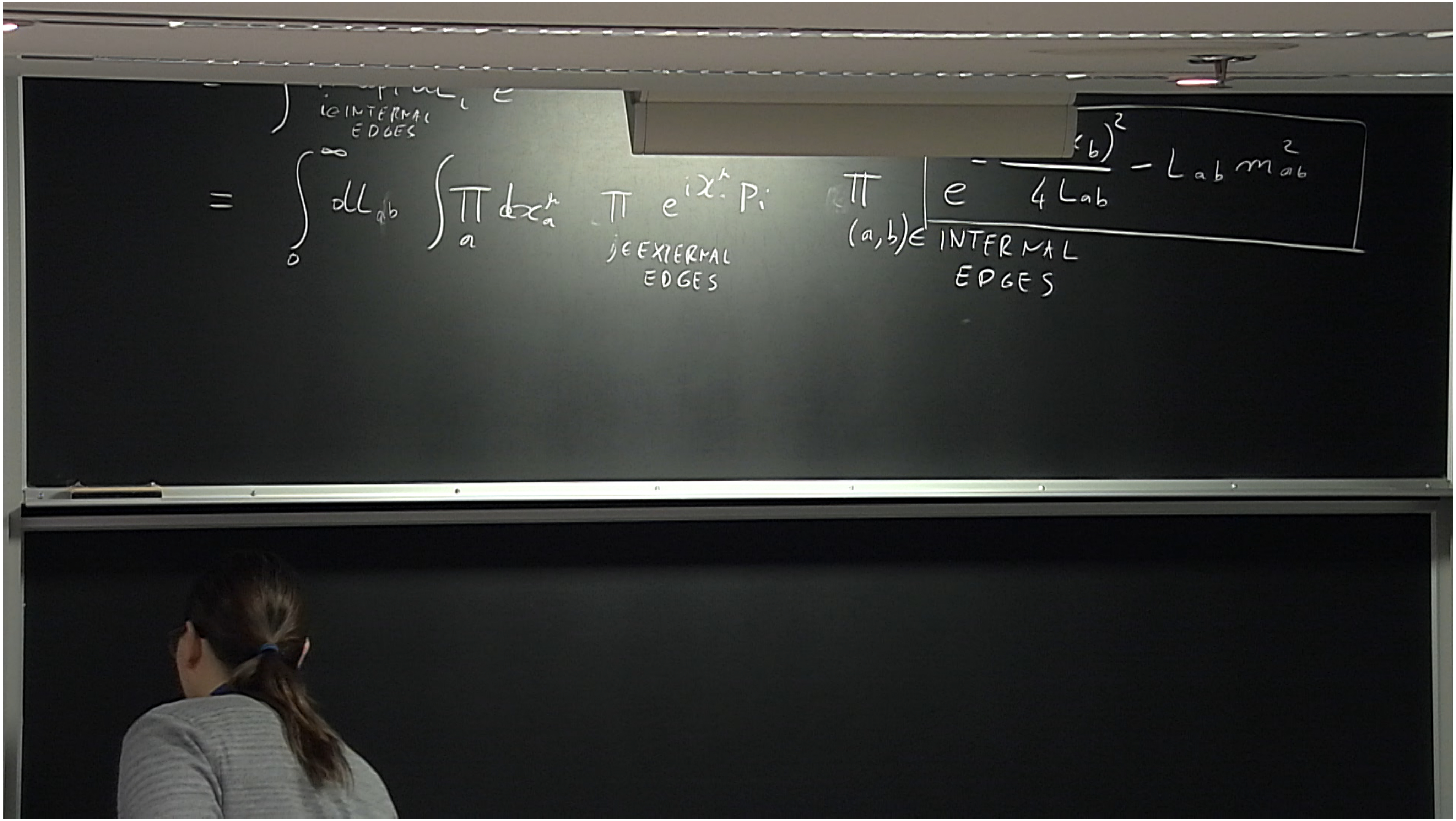

p_2^e
p_3^e
p_{23}
2
3
p_{12}
p_{34}
1
4
p_{14}
p_1^e
p_4^e
Γ
A_Γ^{QFT}
$S_{cl}[\Gamma \subset \mathbb{R}^{d-1,1}] = \sum_{i \in edges} m_i \int ds_i$
A_Γ " $=$ " $\int D(\Gamma \subset \mathbb{R}^{d-1,1}) \, e^{\frac{i}{\hbar} S_{cl}}$ " $= A_\Gamma^{QFT}$
$\prod_{i \in edges} D x_i^\mu$

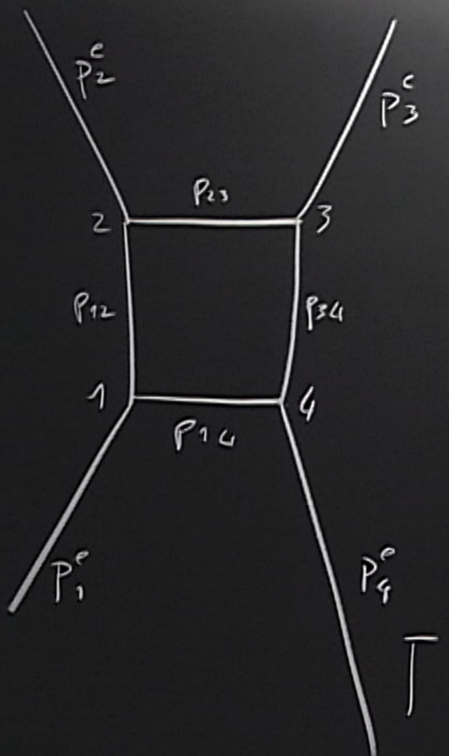
$$\int_{\ell=0}^{\infty} e^{-a\ell}\, d\ell = \frac{1}{a}$$

$$A_\Gamma^{QFT} = \int \prod_{\substack{i \in \text{INTERNAL} \\ \text{EDGES}}} dp_i \frac{1}{p_i^2+m_i^2} \prod_{a \in \text{VERTICES}} \delta^d\left(\Sigma p\right) = \int \prod_i dp_i \frac{1}{p_i^2+m^2} \int \prod_a dx_a^\mu \, e^{i x_a^\mu (\Sigma p)} =$$

$$= \int \prod_i dp_i \, dL_i \, e^{-L_i (p_i^2+m^2)} \prod_a dx_a^\mu \, e^{i x_a (\Sigma p)}$$

$$= \int_0^\infty dL_{ab} \int \prod_a dx_a^\mu \prod_{\substack{j \in \text{EXTERNAL} \\ \text{EDGES}}} e^{i x_j^\mu p_j} \prod_{\substack{(a,b) \in \text{INTERNAL} \\ \text{EDGES}}} \boxed{e^{-\frac{(x_a-x_b)^2}{4L_{ab}} - L_{ab} m_{ab}^2}}$$


i∈INTERNAL
EDGES
j∈EXTERNAL
EDGES
(a,b)∈ INTERNAL
EDGES

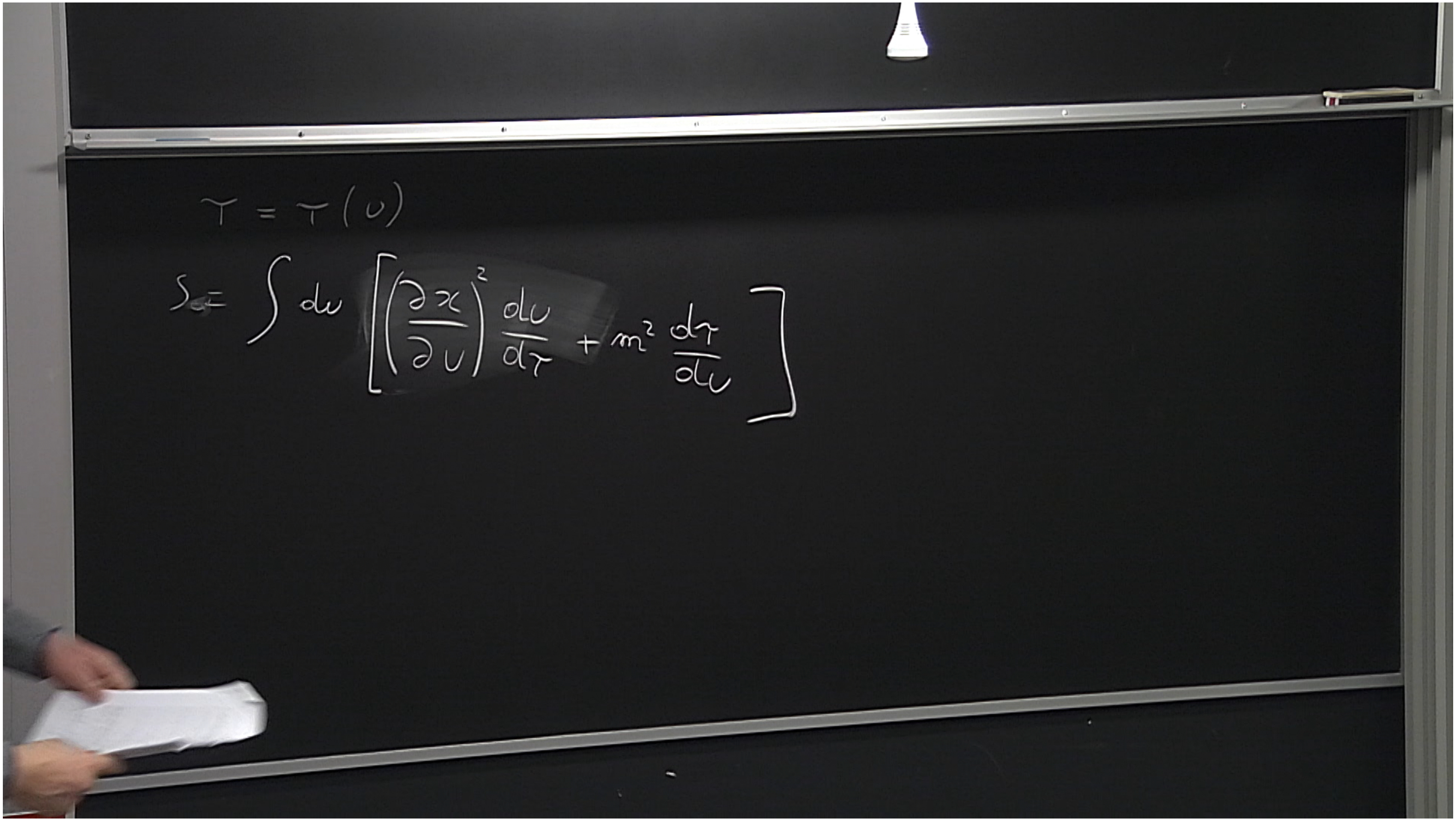


$$A_\Gamma^{QFT}$$

$$S_{cl}[\Gamma \subset \mathbb{R}^{d-1,1}] = \sum_{i \in edges} m_i \int ds_i$$

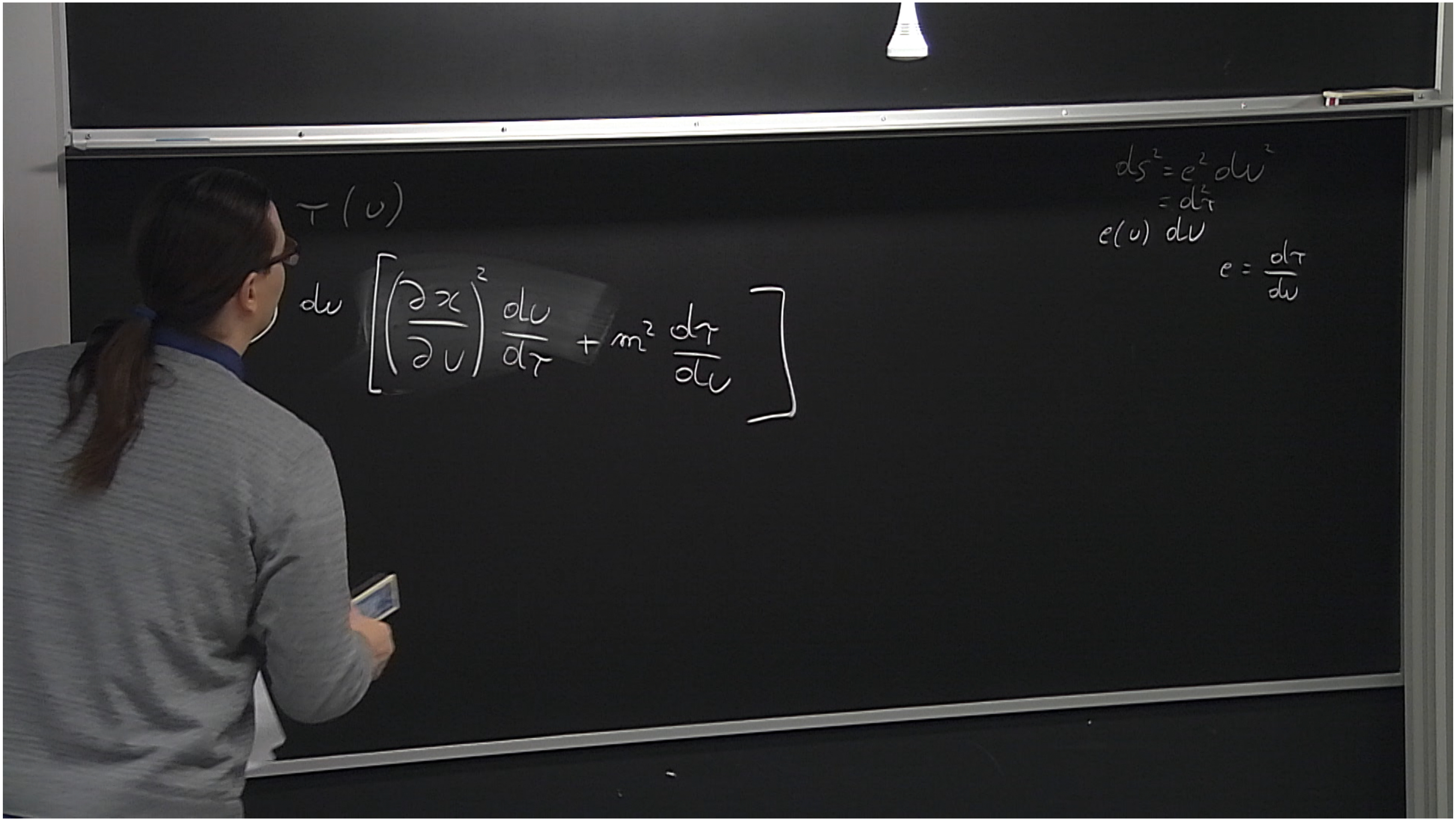
$$A_\Gamma = " \int D(\Gamma \subset \mathbb{R}^{d-1,1})\, e^{\frac{i}{\hbar} S_{cl}} " = A_\Gamma^{QFT}$$

$$\prod_{i \in edges} Dx_i^\mu$$


$\tau = \tau(u)$
$S = \int du \left[\left(\frac{\partial x}{\partial u}\right)^2 \frac{du}{d\tau} + m^2 \frac{d\tau}{du} \right]$

$$\tau = \tau(u)$$

$$S = \int du \left[\left(\frac{\partial x}{\partial u} \right)^2 \frac{du}{d\tau} + m^2 \frac{d\tau}{du} \right]$$


$\tau(\upsilon)$
$d\upsilon \left[\left(\frac{\partial x}{\partial \upsilon}\right)^2 \frac{d\upsilon}{d\tau} + m^2 \frac{d\tau}{d\upsilon} \right]$
$ds^2 = e^2 d\upsilon^2$
$= d\tau^2$
$e(\upsilon)\, d\upsilon$
$e = \frac{d\tau}{d\upsilon}$

$$\tau = \tau(\upsilon)$$

$$S_E = \int d\upsilon \left[\left(\frac{\partial x}{\partial \upsilon} \right)^2 e^{-1} + m^2 e \right]$$

$$ds^2 = e^2 d\upsilon^2$$
$$= d\tau^2$$

$$e(\upsilon)\, d\upsilon$$

$$e = \frac{d\tau}{d\upsilon}$$

$\tau = \tau(v)$

$v' = f(v)$

$e' = (f')^{-1} e$

$ds^2 = e^2 dv^2$

$= d\tau^2$

$e(v)\, dv$

$e = \frac{d\tau}{dv}$

$S = \int dv \left[\left(\frac{\partial x}{\partial v}\right)^2 e^{-1} + m^2 e \right]$

$L = \int_0^L d\tau = \int_0^1 e(v)\, dv$

$$\prod_{i \in \text{edges}} \mathcal{D}x_i^\mu$$

$$A_\Gamma = \int dx_a^\mu \int_{x_{ab}(0)=x_a}^{x_{ab}(1)=x_b} \frac{\mathcal{D}X_{ab}\,\mathcal{D}e_{ab}}{\text{DIFF}}\, e^{\sum_{ab} S[x_{ab},\, e_{ab}]}$$

$$\tau = \tau(v)$$

$$S = \int_0^1 dv \left[\left(\frac{\partial x}{\partial v}\right)^2 e^{-1} + m^2 e\right]$$

$$x(v') \to x(f(v))$$

$$e(v') \to f'(v)\, e(v)$$

$$\left(\frac{\partial x}{\partial v}\right)^2 = m^2 e^2$$

SOLVE FOR e

$$S = \int_0^1 \sqrt{\left(\frac{\partial x}{\partial v}\right)^2}\, dv$$

$$v' = f(v)$$

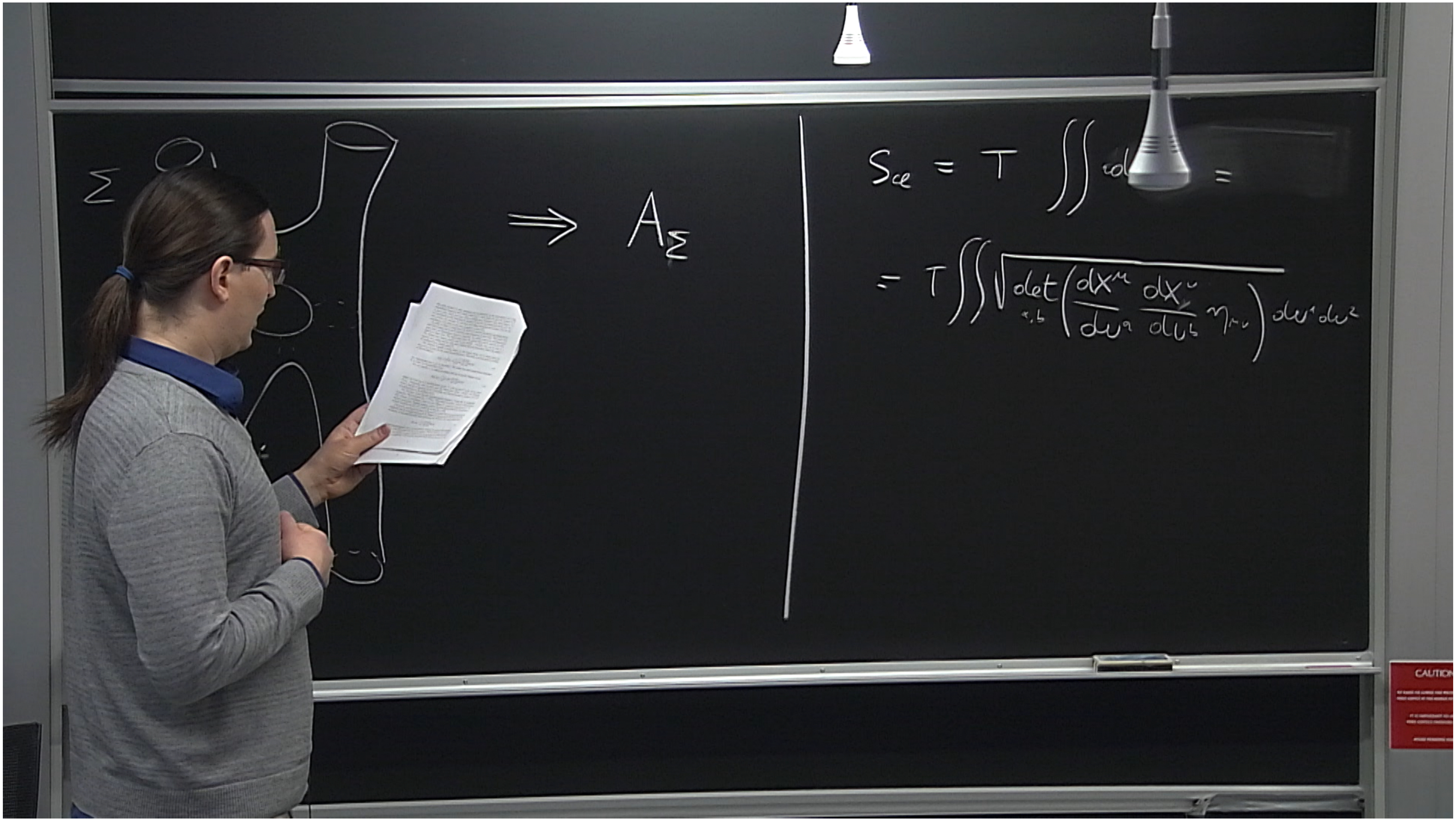
$$e' = (f')^{-1} e$$

$$ds^2 = e^2 dv^2 = d\tau^2$$

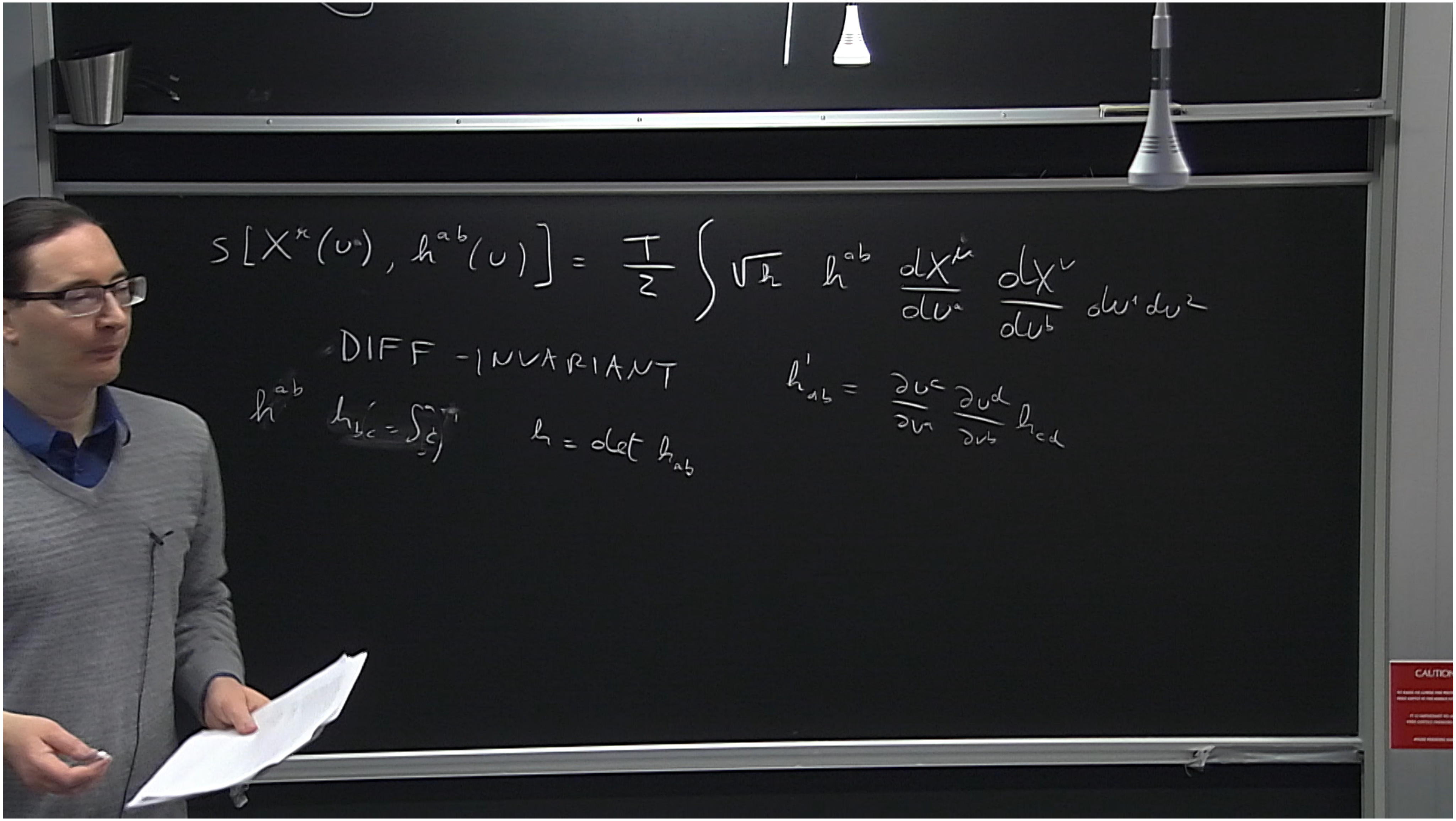
$$e(v)\, dv$$

$$e = \frac{d\tau}{dv}$$

$$L = \int_0^L d\tau = \int_0^1 e(v)\, dv$$



Σ
$\Longrightarrow A_\Sigma$
$S_{cl} = T \iint d$
$= T \iint \sqrt{\det_{a,b}\left(\frac{dX^\mu}{du^a}\frac{dX^\nu}{du^b}\eta_{\mu\nu}\right)} du^1 du^2$


$S[X^\mu(u), h^{ab}(u)] = \frac{T}{2}\int \sqrt{h}\, h^{ab} \frac{dX^\mu}{du^a}\frac{dX^\nu}{du^b} du^1 du^2$
DIFF-INVARIANT
$h'_{ab} = \frac{\partial u^c}{\partial u^a}\frac{\partial u^d}{\partial u^b} h_{cd}$
h^{ab}
$h = \det h_{ab}$
CAUTION

$$S[X^\mu(u), h^{ab}(u)] = \frac{1}{2}\int \sqrt{h}\; h^{ab}\, \frac{dX^\mu}{du^a}\, \frac{dX^\nu}{du^b}\, du^1 du^2$$

DIFF-INVARIANT

h^{ab} $\quad h_{bc}' = \delta_c^a$ $\qquad h = \det h_{ab}$

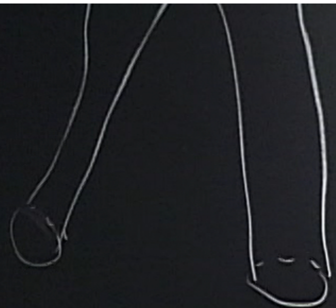
$$h'_{ab} = \frac{\partial u^c}{\partial u'^a}\frac{\partial u^d}{\partial u'^b} h_{cd}$$

$$S[X, h_{ab} e^f] = S[X, h_{ab}]$$

WEYL-INVARIANT

DIFFS $\quad h_{ab}\, du^a du^b \longrightarrow e^{\phi(u)}\left((du^1)^2 + (du^2)^2\right)$

$\qquad h_{ab} \longrightarrow e^{\phi}\delta_{ab}$



DIFF - INVARIANT ?

$$A_g = \int \frac{DX\, Dh}{\text{DIFF} \times \text{WEYL}} e^{-S[X,h]}$$

$$= \int d\lambda \int DX\, e^{-S[X,\, h_{ab}[\lambda]]}$$

$$S[X^\mu(v), h^{ab}(v)] = \frac{1}{2}\int \sqrt{h}\, h^{ab} \frac{dX^\mu}{dv^a}\frac{dX^\nu}{dv^b}\, dv^1 dv^2$$

DIFF - INVARIANT

h^{ab} $h_{bc} = \delta^a_c$ $\quad h = \det h_{ab}$

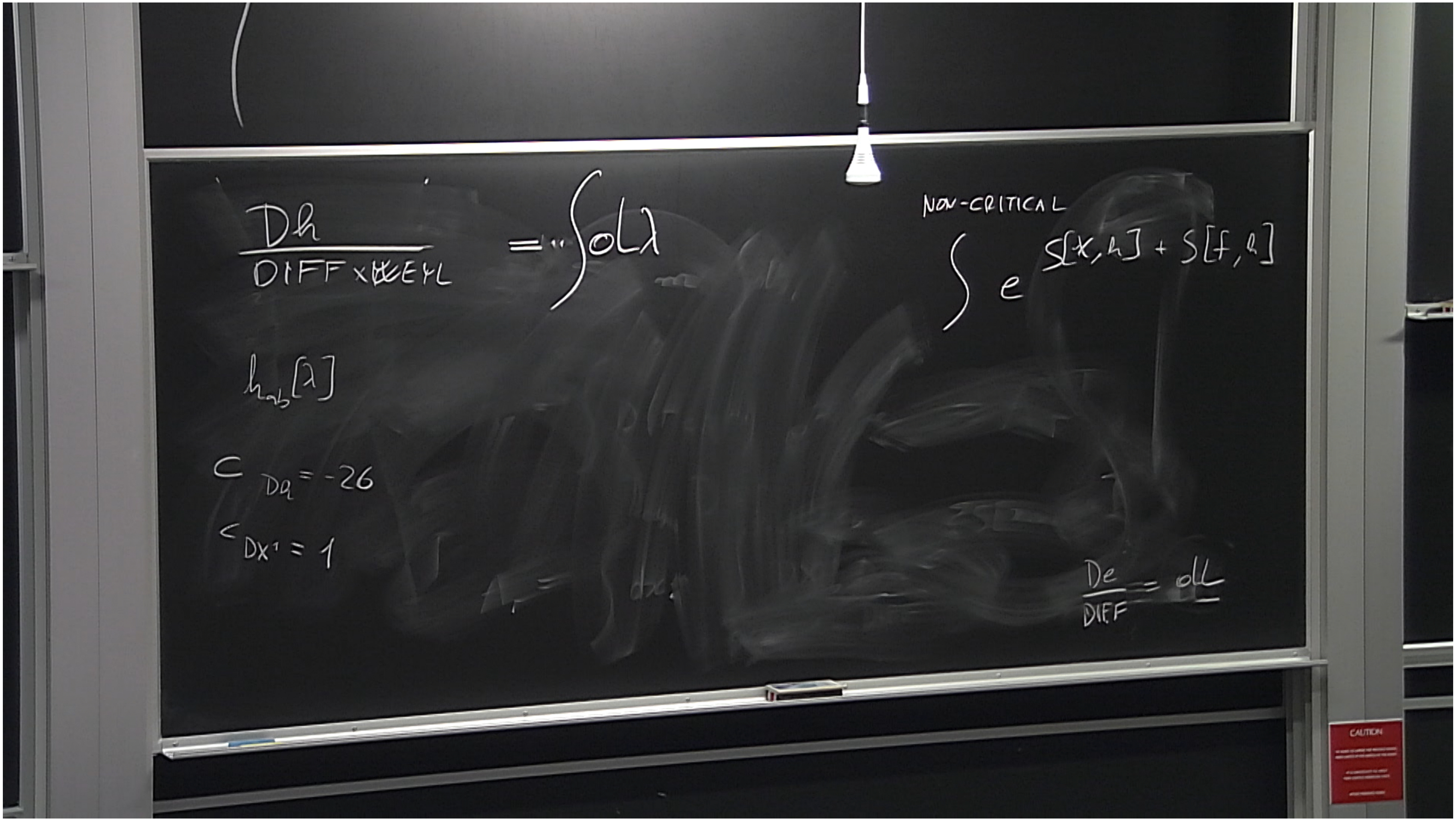
$$h'_{ab} = \frac{\partial v^c}{\partial v'^a}\frac{\partial v^d}{\partial v'^b} h_{cd}$$

$$S[X, h_{ab} e^f] = S[X, h_{ab}]$$

WEYL - INVARIANT

DIFFS $\quad h_{ab}\, dv^a dv^b \longrightarrow e^{\phi(v)}\left((dv^1)^2 + (dv^2)^2\right)$

$$h_{ab} \longrightarrow e^{\phi} \delta_{ab}$$


Dh / DIFF × WEYL = ∫ dλ
NON-CRITICAL
∫ e^{S[x,h] + S[f,h]}
h_ab[λ]
c_Dq = -26
c_Dx = 1
De / DIFF = dL
CAUTION