Title: Quantum Field Theory in the Quantum Information Age

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Abstract: <p>Quantum Field Theories are interacting quantum systems described by an infinite number of degrees of freedom, necessarily living on an infinite-dimensional Hilbert space. Hence, many concepts from Quantum Information Theory have to be adapted before they can be applied to this setting. However, the task is worthwhile as we obtain new tools to understand the entanglement structure of theories describing the fundamental forces of nature. I will outline two approaches along this route, one bottom-down and one bottom-up strategy. First, general Lorentz invariant quantum fields are studied from a quantum information perspective. Second, I discuss the use of tensor networks to model the entanglement structure of quantum fields. The combination of both approaches yields new insights into Quantum Field Theory from an information theoretic point of view. I will end my talk by giving an outlook into future research directions.</p>

Quantum field theory in the Quantum Information age

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Or examining and modelling the entanglement structure of quantum field theories

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My research goals

Developing a new language to understand the entanglement structure of quantum fields.

Two-fold strategy:

- Use quantum information theoretic concepts to examine the entanglement structure of general Lorentz invariant quantum field theories (top-down)
- Apply tools from tensor network theory to rigorously obtain new classes of Ansatz states for specific examples of Quantum field theories (bottom-up)

Other research topics

Parts of my work which are not covered in this talk:

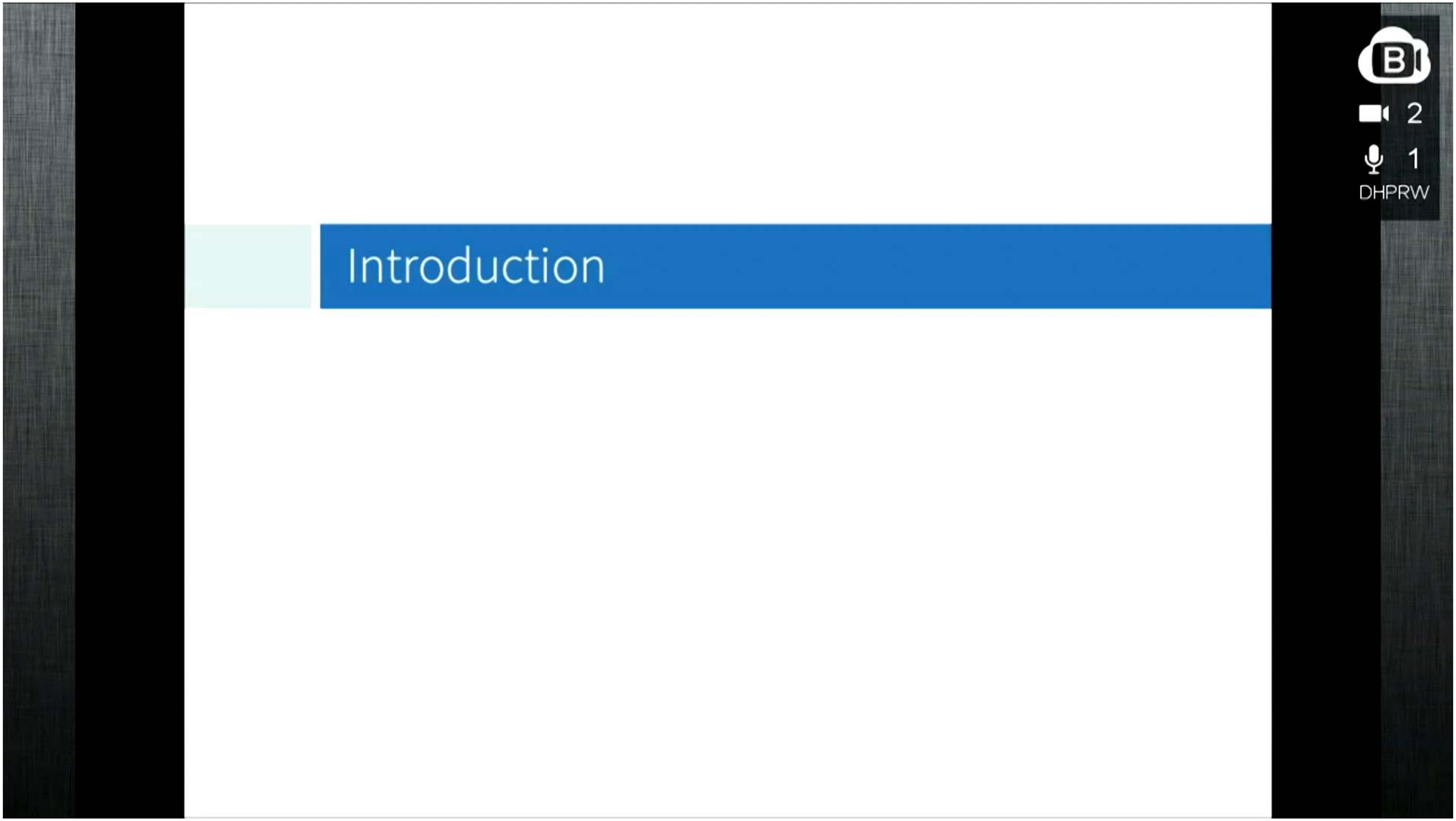
- Tsirelson's problem and connections between operator algebra theory and Quantum Information Theory [JMP 2011, JMP 2015, JMP 2016]
- Quantum cryptography, i.e. the security of randomness extractors against quantum side attacks [IEEE Trans. Inf. Th. 2016]
- Non-commutative optimization theory [SIAM J. Opt. 2016]

General strategy

Combining tools from Quantum Information Theory, Operator Algebra Theory and Quantum Field Theory in order to obtain new insights into either field.

Contents

- Introduction
- Entanglement structure of general Quantum field theories
- Rigorous results for tensor network approximations
- Future direction: tensor network approximations
- Future direction: quantum protocols


Introduction
2
1
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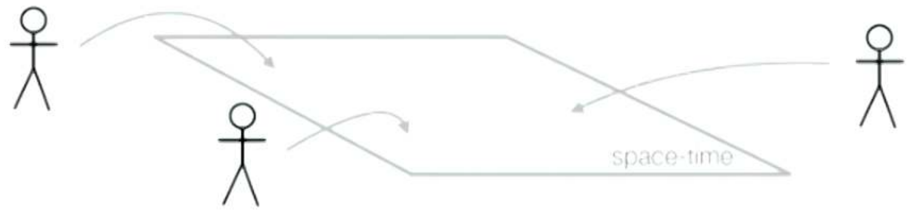
Quantum fields

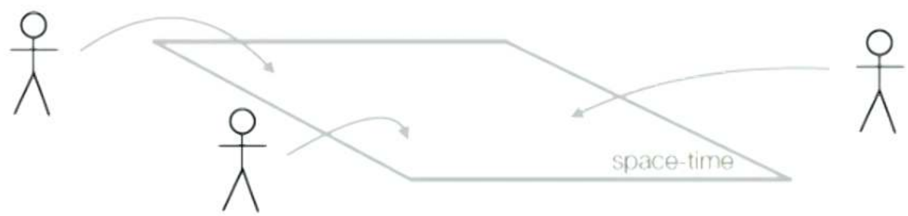


- A quantum theory possessing a continuum of degree of freedoms representing the variables of space and time
- defined in terms of its correlation functions with respect to the vacuum state $|\Omega\rangle$,

$$\langle \Omega | \phi(x_1)\phi(x_2)\cdots\phi(x_n)\Omega \rangle$$

$\phi(x_i)$: measurement performed at point x_i in space-time
- correlations functions are invariant under Lorentz transformations (symmetries of special relativity)

Quantum fields are different



- The mathematical setting of quantum field theory is very different to the usual one of quantum information theory
- The Hilbert space of the theory is necessary infinite-dimensional
- Not all operators acting on the Hilbert space are allowed physical observables (Lorentz symmetry, gauge symmetry)
- Subsystems are not described by a tensor product of Hilbert space

$\Rightarrow$ **The usual tools of quantum information theory have to be adopted to this new setting.**

Known facts: entanglement in quantum fields

- Lorentz invariant quantum field theory: The vacuum is maximally entangled between any region and its complement [Summers & Werner]
- Bell inequalities are maximally violated [Summers & Werner]

$$\beta = |\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle| = 2\sqrt{2}$$

- Conformal Field Theories in 1+1D: Entanglement Entropy of various regions are very well understood [Holzhey et. al., Cardy & Calabrese]

$$S(L) \sim c \log \frac{L}{\varepsilon}$$

- Entanglement Entropy of bounded regions in 3+1D: many special cases known [Casini & Huerta, Myers, ...]

Results: Top-down

Top-down Strategy

Research Strategy

Apply operational concepts from Quantum Information Theory to Quantum Field Theory in order to learn more about the structure of quantum correlations.

Results presented:

- Measuring entanglement in gauge theories
- Characterizing entanglement in general Lorentz invariant theories

Distillable Entanglement of Gauge theories

- Gauge theories are quantum field theories which have additional local symmetries implemented by unitary gauge transformations $U_g(x)$

$$|\psi\rangle \in \mathcal{H}_{phys} \Leftrightarrow U_g(x)|\psi\rangle = |\psi\rangle \quad \forall U_g(x)$$

- These additional local symmetries imply that the Hilbert space cannot be decomposed into local Hilbert spaces, not even in an approximate sense → reduced density matrices not well-defined

How to measure entanglement?

How many EPR pairs can we extract by acting with local gauge invariant operations and allowing classical communication (LOCC) in a region A and its complement $\bar{A}$ on many copies of some state $|\psi\rangle$?

Distillable Entanglement of Gauge theories

Lemma

The symmetry constraints imply that the observable algebras $\mathcal{A}$ and its commutant $\mathcal{A}'$ have a joint center $Z = \text{span}\{P_r\}$, where P_r are orthogonal projections localised at the boundary. Moreover, each $P_r\mathcal{H}_{phys}$ splits in a well-defined manner into a tensor product Hilbert space.

$$X \in \mathcal{A} \otimes \mathcal{A}' : \langle \psi | X\psi \rangle = \langle \psi | \sum_r P_r X \sum_s P_s \psi \rangle = \text{Tr}[\sum_r P_r |\psi\rangle\langle\psi| P_r X]$$

$\Rightarrow$ the state $|\psi\rangle$ is physically indistinguishable from the depolarized mixed state $\sigma^\psi = \sum_r P_r |\psi\rangle\langle\psi| P_r$.

Distillable Entanglement of Gauge theories

The state $|\psi\rangle$ is physically indistinguishable from the depolarized mixed state $\sigma^{\psi} = \sum_r P_r|\psi\rangle\langle\psi|P_r$.

Theorem (K.v.Acoleyen, M.Marien, N. Bultinck, J. Haegeman, VBS, F. Verstraete, PRL 2016, QIP plenary 2017)

The distillable Entanglement of the state $|\psi\rangle$ equals the entanglement of the mixed state σ^{ψ},

$$E_D(|\psi\rangle) = \sum_r p_r S(\mathrm{Tr}_{\bar{A}}[P_r|\psi\rangle\langle\psi|P_r])$$

where $p_r = \langle\psi|P_r\psi\rangle$.

Entanglement between two wedges

Example of a wedge: $W = \{ x \in \mathbb{R}^{3,1} : |x_0| < x_1 \}$ (can be related to the Unruh setting of an accelerated observer).

Entanglement in Lorentz invariant QFTs

Can we operationally characterize the entanglement in vacuum state $|\Omega\rangle$ of general Lorentz invariant quantum field theories between spatially separated wedges?

Entanglement primitive: embezzling

Hayden and van Dam introduced a new class of highly entangled bipartite states. They are characterized by the property that we can "steal" entanglement from them without noticing.

A sequence of bipartite states $|\mu_n\rangle \in \mathbb{C}^n \otimes \mathbb{C}^n$ is called embezzling, if for any state $|\varphi\rangle \in \mathbb{C}^d \otimes \mathbb{C}^d$ and $\varepsilon > 0$ there exist an n and unitaries U, V such that

$$\|U \otimes V |\mu_n\rangle|00\rangle - |\mu_n\rangle \otimes |\varphi\rangle\| \leq \varepsilon\,.$$

Example: $|\mu_n\rangle = \frac{1}{H_n}\sum_{i=1}^{n}\frac{1}{\sqrt{i}}|ii\rangle$, H_n: harmonic sum.

The vacuum is embezzling

Theorem (VBS, Werner, in preparation)

For any finite dimensional entangled state $|\varphi\rangle \in \mathbb{C}^d \otimes \mathbb{C}^d$, and any $\varepsilon > 0$, there exists local unitaries U, V, depending on ε and $|\varphi\rangle$ and spatially located in one wedge or its complement such that

$$\| U \cdot V |\Omega\rangle |00\rangle - |\Omega\rangle \otimes |\varphi\rangle \| \leq \varepsilon \, .$$

Remarks:

- Result holds for general Lorentz invariant quantum field theories
- Energy constraints lead to precise bounds between the available energy and what d and ε are possible

Bottom-up Strategy

Research strategy

Develop new classes of Ansatz states which model the entanglement structure of quantum fields. We are thereby interested in analytic solutions with rigorous error bounds.

Result presented:

Tensor network approximations to conformal field theories.

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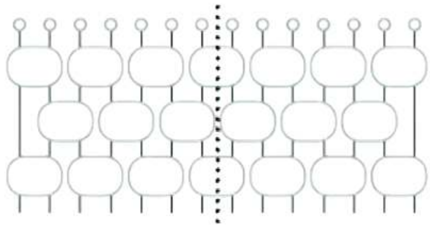
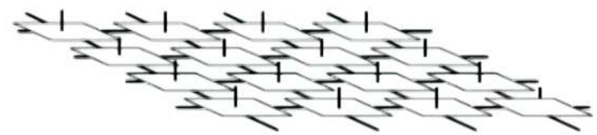
Why tensor network states?





- Tensor network states model the entanglement structure of quantum many body states in a geometrically local way [Fannes, Werner, Nachtergaele, Vidal, Verstraete, Cirac,...]
- Successfully used both analytically and numerically, i.e. for the classification of topological phases of matter or to obtain better ground state energies than existing techniques [Verstraete, Schuch, Cirac,...]
- Their application to Quantum Field Theory opens up a whole new world of Ansatz states, characterized by local objects (tensors)

Approximating quantum field theories

What does approximate mean?

How to approximate a continuous quantum field theory by objects on discrete spatial structures?

Idea:

If tensor networks can approximately reproduce correlation functions of quantum field theories, then we can use them to understand the entanglement structure of quantum field theories.

Start with the simplest case: Matrix product states and Conformal Field Theories.

Matrix product states

- Maximally entangled pairs of (bond) dimension D are placed between the physical particles (physical dimension d)
- At each physical lattice site, the ends of the maximally entangled pairs are contracted by an isometry $V : \mathbb{C}^D \to \mathbb{C}^D \otimes \mathbb{C}^d$ (tensor)
- Various numerical studies and heuristic arguments support the conjecture that Matrix product states can be used to model the entanglement structure of Conformal Field Theories.

Approximating conformal field theories

Theorem (R.Koenig, VBS, PRL 2016)

Correlation functions of Conformal Field Theories can be arbitrarily well approximated by correlations functions of Matrix Product tensor networks,

$$|\langle\Omega|\phi(x_1)\cdots\phi(x_n)\Omega\rangle - \langle\psi^D_{MPS}|\hat{\phi}(x_1)\cdots\hat{\phi}(x_n)\psi^D_{MPS}\rangle| \leq f(D, n, c_{CFT})\,.$$

The function $f(D, n, c_{CFT})$ leads to the following scaling of the bond dimension:

	fixed n, $d = \min\lvert x_i - x_j\rvert$	fixed ε
Scaling of D	$\sim \left(\frac{1}{\varepsilon}\right)^{\kappa c_{CFT}\frac{n}{d}}$	$\sim \gamma(\varepsilon)e^{2\pi\sqrt{\frac{1}{6}c_{CFT}n}}$

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Approximating conformal field theories

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Remarks:

- Holds for general CFTs, proof uses the theory of Vertex operator algebras
- It is rigorous and constructive, allows to quantify entanglement and takes additional physical symmetries into account
- Generalizations possible (MERA?)

Tensor networks for quantum fields

Research goals:

- Detailed understanding of entanglement structure
- New insights into the renormalization flow
- New Ansatz states for numerical simulations
- Quantitative tests of qualitative results in the AdS/CFT correspondence, i.e. examine whether the low energy subspace of a 1+1 dimensional CFT constitutes a quantum error correcting code as suggested in the AdS/CFT setup (Harlow et. al.)

Approach: rendering the observable set finite

- Construct a finite-dimensional subspace F_λ of smoothed observables of a given scale of resolution λ which is sufficient to approximate all correlation functions at this scale,

$$\langle \Omega | \phi(f_1) \cdots \phi(f_n) \Omega \rangle \approx_{\exists g_1, \ldots, g_n \in F_\lambda} \langle \Omega | \phi(g_1) \cdots \phi(g_n) \Omega \rangle$$

$$\phi(f) = \int_{\mathbb{R}^{3,1}} dx\, f(x) \phi(x) \,, \qquad \forall f \text{ with } \|\vec{\nabla} f\| \leq \mathrm{poly}(\lambda)$$

- Construct tensor network approximations to correlation functions depending on F_λ,

$$\langle \Omega | \phi(g_1) \cdots \phi(g_n) \Omega \rangle \approx \langle \Omega_{TNS(\lambda)} | \hat{\phi}(g_1) \cdots \hat{\phi}(g_n) \Omega_{TNS(\lambda)} \rangle$$

- The application of these ideas to free theories is ongoing

Approach: exact renormalization flow equations

- Develop renormalization schemes for tensor networks approximating correlation functions of quantum field theories (connection to MERA will be explored)
- Formulate the exact renormalization flow equations [Polchinski, Keller et. al.] for correlations functions in terms of tensor networks; add interactions perturbatively, corresponding to filtering in the tensor network

$$\partial_\lambda \left\langle \Omega_{TNS(\lambda,n)} \,|\, \ldots \Omega_{TNS(\lambda,n)} \right\rangle = L\left(\left\langle \Omega_{TNS(\lambda,n-1)} \,|\, \ldots \Omega_{TNS(\lambda,n-1)} \right\rangle \right) ,$$

L : linear expression , λ : scale , n : perturbation order .

- Different renormalization schemes will lead to different tensor network approximations (like MERA compared to PEPS)

Future direction: top-down

Communication protocols using quantum fields

Research goals:

- Examine basic quantum information theoretic meta-protocols in the setup of QFTs, e.g. quantum hypothesis testing, quantum state merging
- Develop bipartite and multipartite coding schemes for information transmission over Gaussian quantum channels with a continuum of modes
- Apply the results to improve our understanding of the Black-Hole firewall paradox, which is based on heuristic applications of concepts from QIT to QFT

Approach: Direct and converse part

The construction of quantum communication protocols is split in two parts: direct and converse. In the direct part, a coding scheme is developed. The converse part then shows the optimality of this procedure.

- The direct part of quantum communication protocols for quantum fields, such as state merging, will be developed using the tensor network approximations
- The converse part relies on the results extending entropic quantities to operator algebras.

QFT in the quantum information age

Extending concepts from Quantum Information Theory to Quantum Field Theory leads to new insights concerning the structure of quantum correlations of theories describing the fundamental forces of nature

My research strategy is two-fold:

- Top-down approach: obtain insights for general quantum field theories based on information theoretic primitives
- Bottom-up approach: model the entanglement structure of quantum fields using tensor networks

Thank you!