Title: Quantum error-correction and black holes

Date: Feb 22, 2017 04:00 PM

URL: http://pirsa.org/17020102

Abstract: <p>It is commonly believed that quantum information is not lost in a black hole. Instead, it is encoded into non-local degrees of freedom in some clever way; like a quantum error-correcting code. In this talk, I will discuss recent attempts to resolve some paradoxes in quantum gravity by using the theory of quantum error-correction. First, I will introduce a simple toy model of the AdS/CFT correspondence based on tensor networks and demonstrate that the correspondence between the AdS gravity and CFT is indeed a realization of quantum codes. I will then show that the butterfly effect in black holes can be interpreted as non-local encoding of quantum information and can be quantitatively measured by out-of-time ordered correlations. Finally, I will discuss how out-of-time ordered correlations, measured outside the black hole horizon, may probe smoothness of the geometry across the horizon.</p>

Collaborators

HEP

Daniel Harlow (Harvard → MIT)

QI

Fernando Pastawski (Caltech → Berlin)

QI

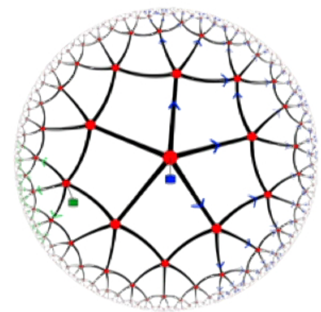
John Preskill (Caltech)

CMT

Xiao-liang Qi (Stanford)

HEP

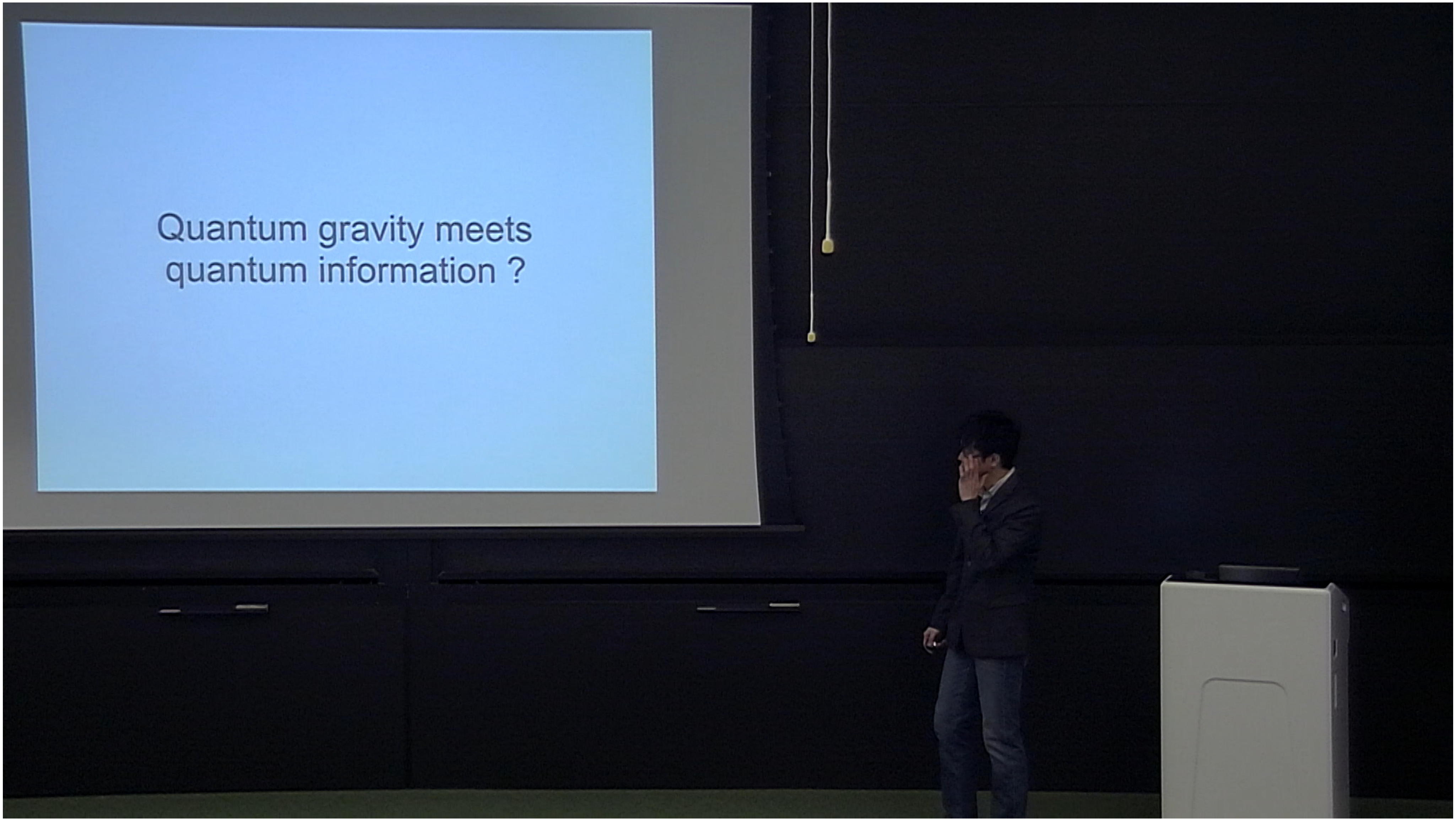
Daniel Roberts (IAS Princeton)



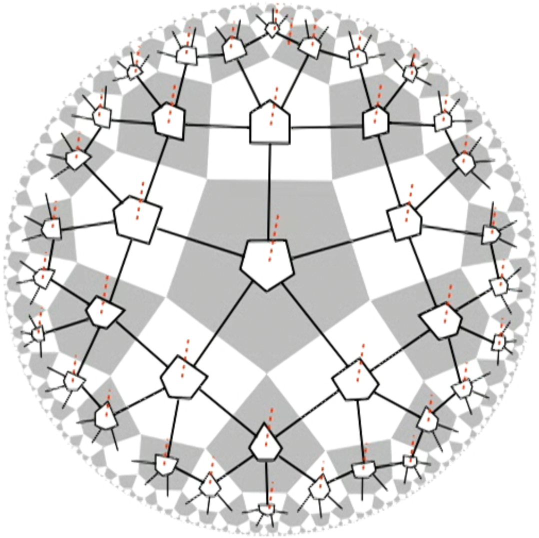
Simons collaborations

Thanks to

Patrick Hayden, Pavan Hosur, Alexei Kitaev, Michael Walter and many others…


Quantum gravity meets
quantum information ?

Quantum error-correction and black holes



Beni Yoshida (Perimeter Institute)

@ Perimeter (Feb 2017)

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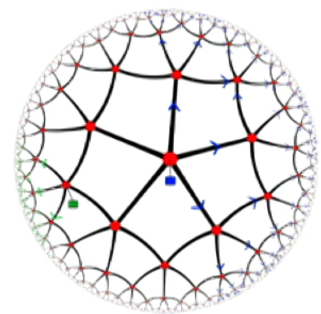

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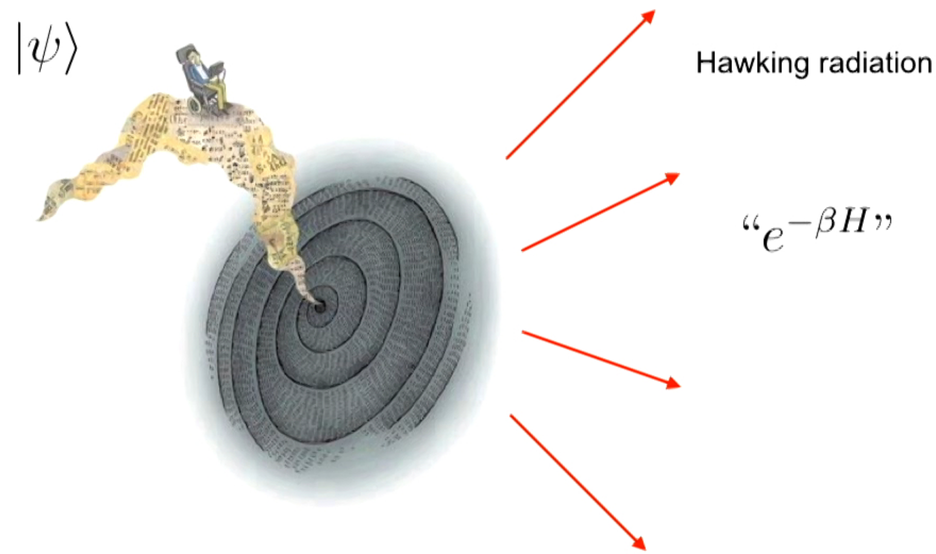
Information loss puzzle

- Quantum mechanics and general relativity are in serious conflicts !

(a) Quantum mechanics says that information is never lost.

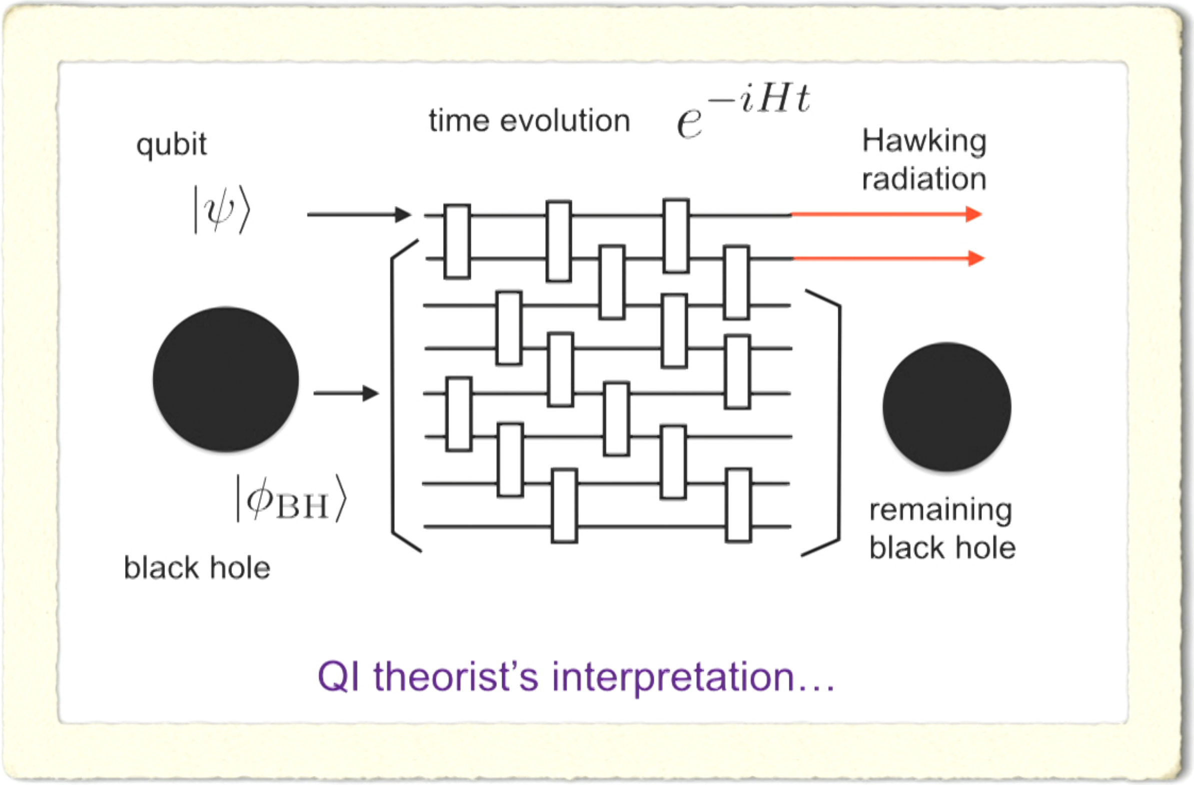
$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

(b) General relativity says information is lost in black holes.



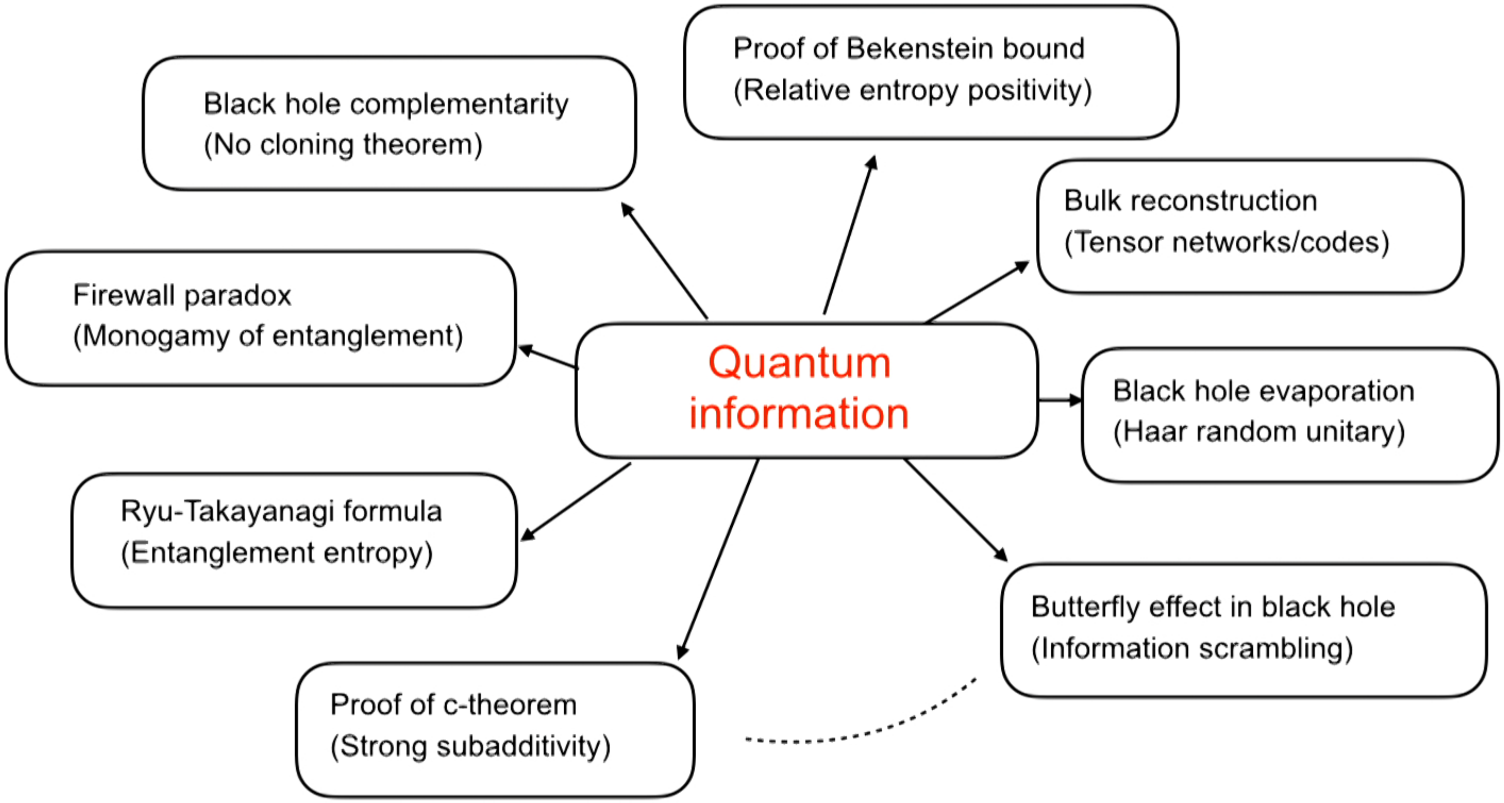
Why quantum information ?

- Information loss puzzle = "Information problem in a quantum system"



Quantum information and HEP

- Many breakthrough ideas in quantum gravity come from quantum information theory



* Definitely there are many other examples.

This talk :

A black hole is a quantum error-correcting code

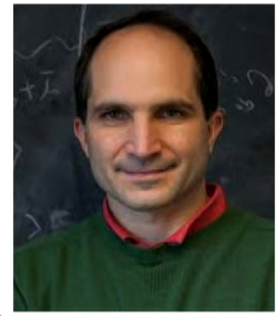
Part 1 :

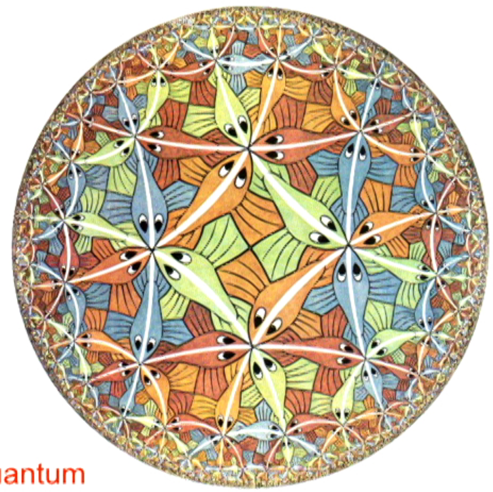
Simple toy model of the AdS/CFT correspondence

(joint with Harlow, Pastawski and Preskill 2015)

Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence

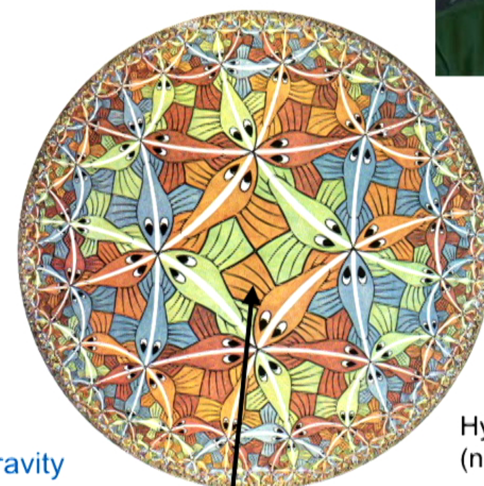
- [Conjecture] Equivalence between string (gravity) theory in bulk and (certain types of) CFT on boundary (Maldacena 1997)





Quantum

Boundary D-dimensional conformal field theory (without gravity)



Gravity

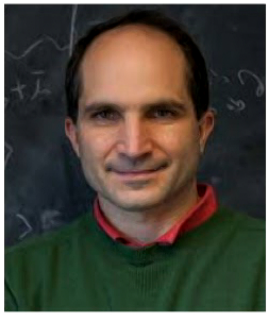
Hyperbolic space (negatively curved)

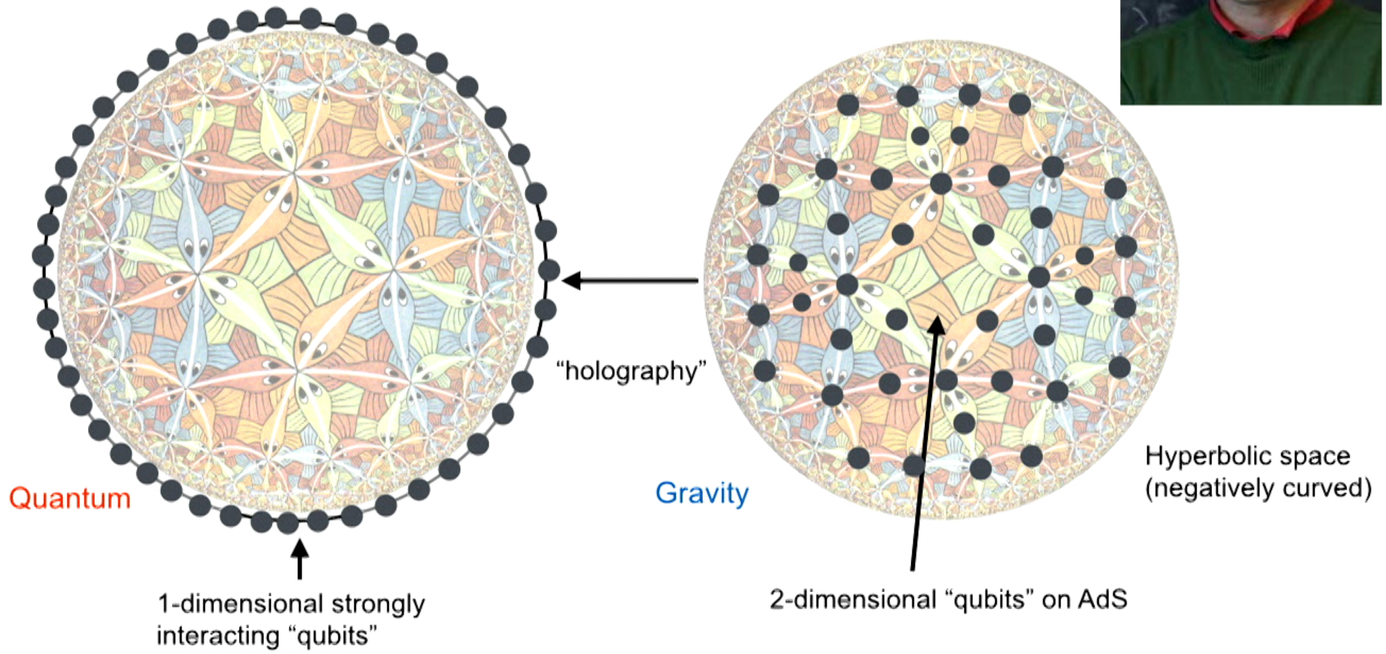
Bulk (D+1)-dimensional theory with gravity on AdS space

Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence

- [Conjecture] Equivalence between string (gravity) theory in bulk and (certain types of) CFT on boundary (Maldacena 1997)

- [Holography] Bulk degrees of freedom are encoded in boundary, like a hologram.





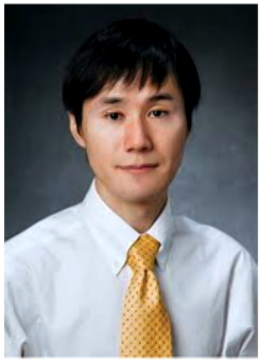
* Finite-dimensional Hilbert space cartoon picture.

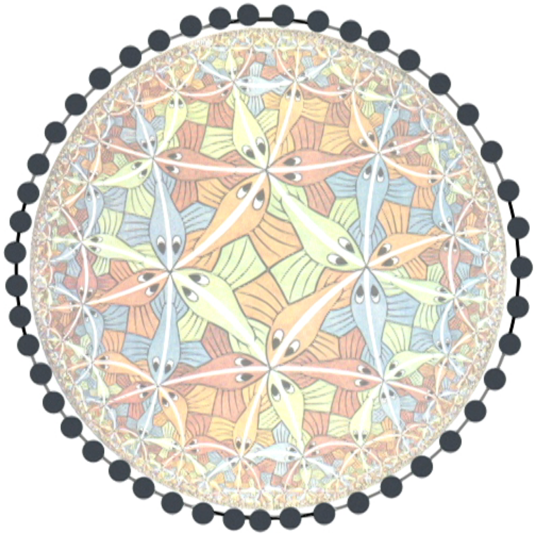
Quantum entanglement in AdS/CFT

- [Ryu-Takayanagi formula 06]

Quantum → $$S(A) = \frac{1}{4G_N} \min_{\gamma_A}(\text{area}(\gamma_A))$$ ← Gravity

Minimize over spatial bulk surfaces homologous to A





* Leading order in 1/G. * Casini-Huerta-Myers11, Lewkowycz-Maldacena13

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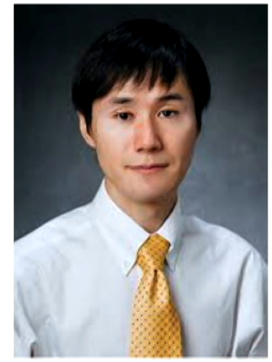
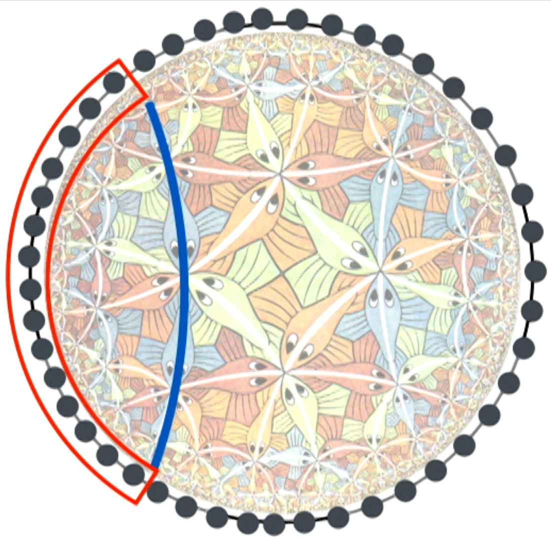


A



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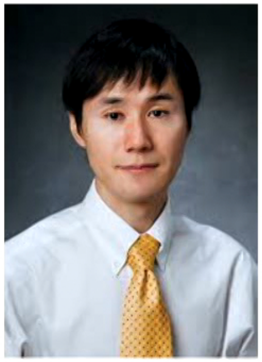

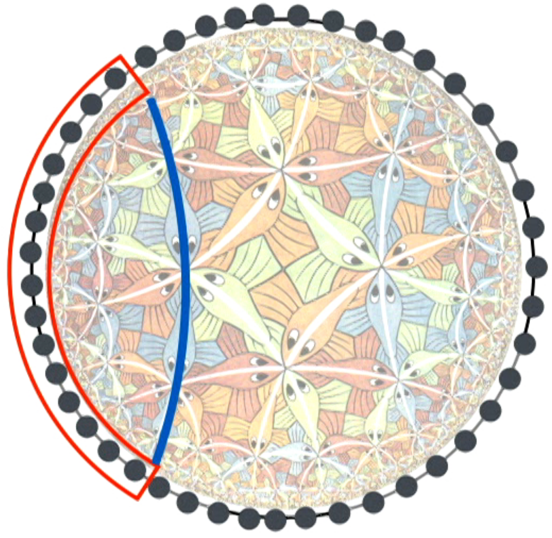

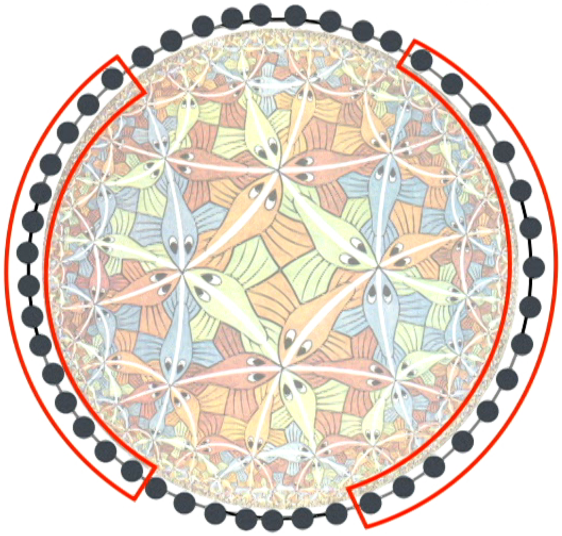

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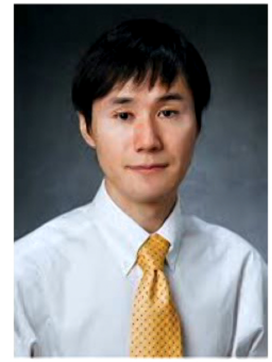

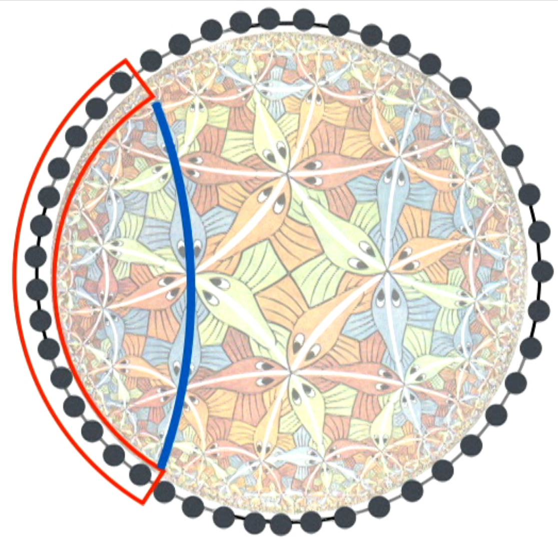

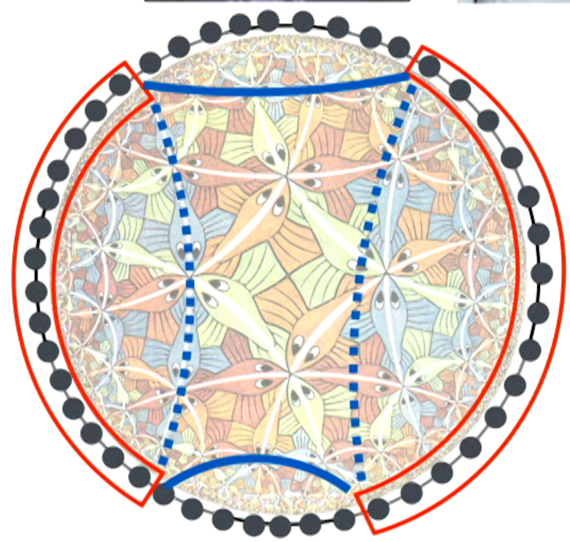

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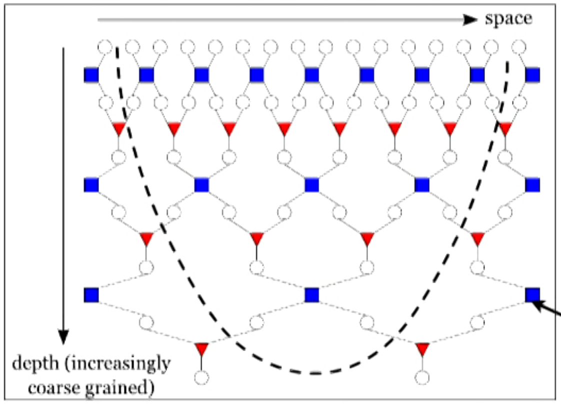
• Powerful numerical method to study strongly-correlated systems. (Vidal 07)



remove short-range entanglement

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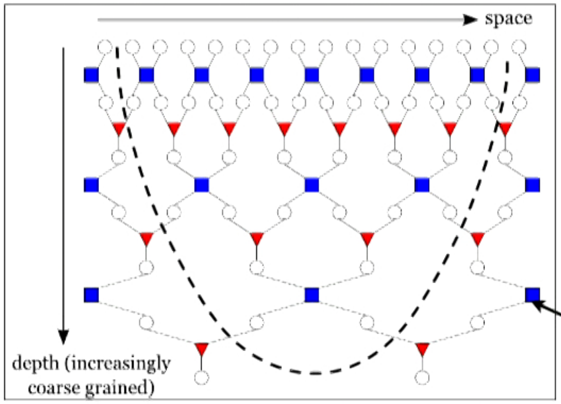
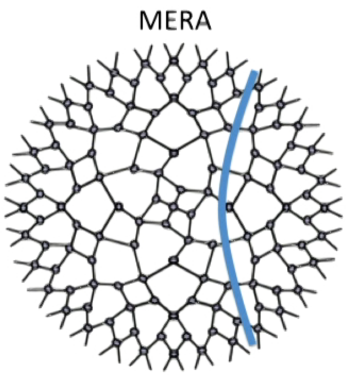
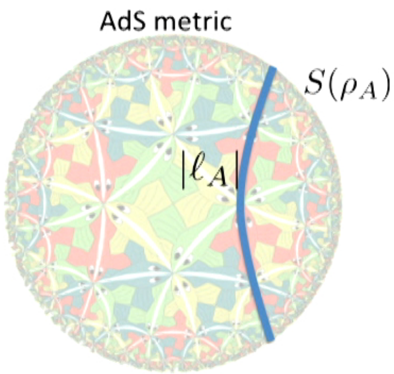


remove short-range entanglement

- AdS/CFT correspondence can be explained by a tensor network ?

(Swingle 09)





Bulk operator vs boundary operator

- [Entanglement wedge reconstruction]

  A bulk operator ϕ can be represented by some integral of local boundary operators supported on A if ϕ is contained inside the entanglement wedge of A.

* HKLL for single intervals * Conjectured for multiple intervals by Aron Wall.

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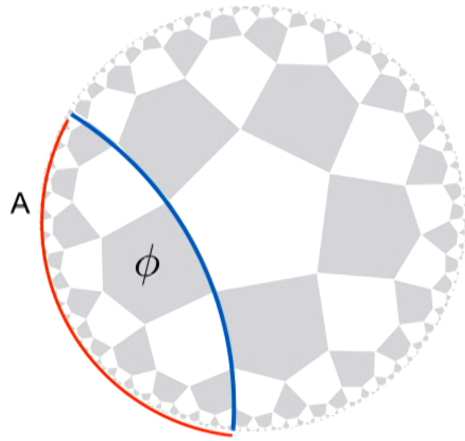

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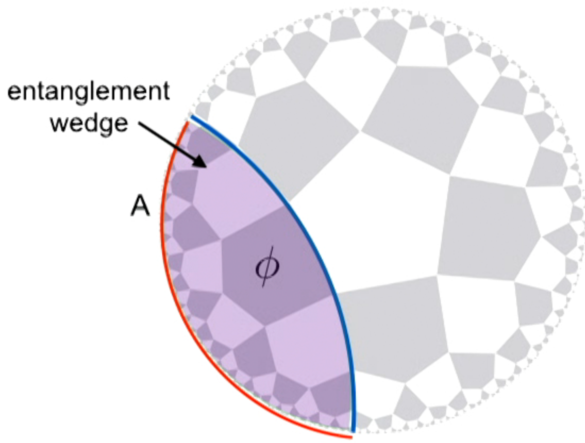

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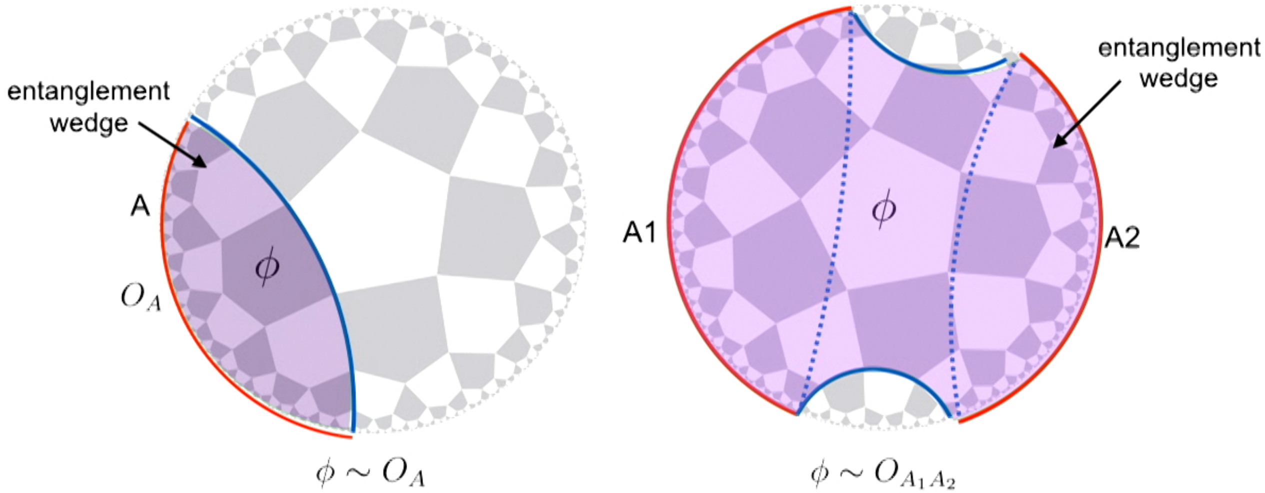

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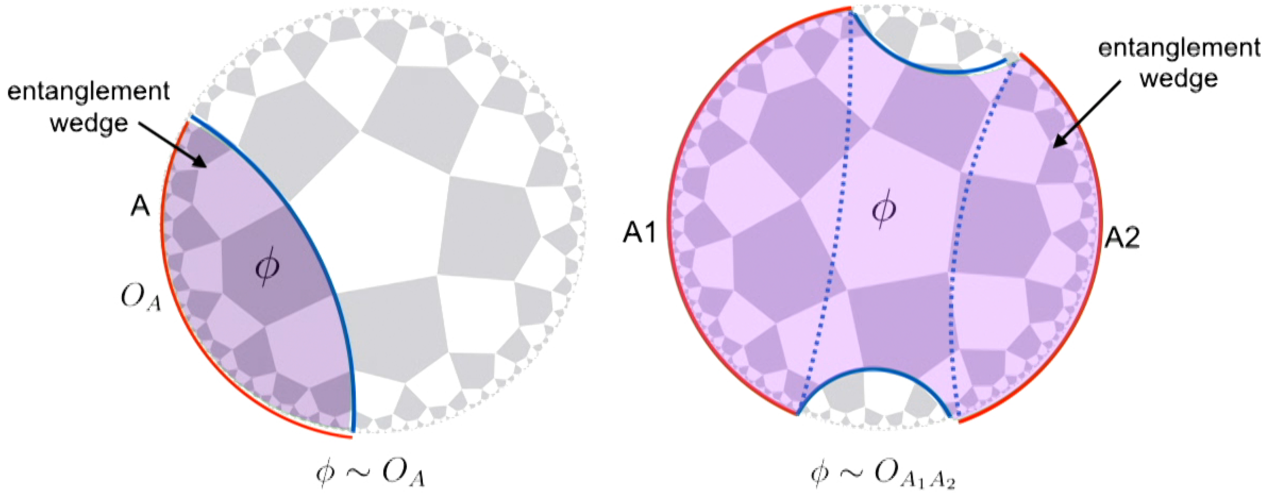

Remarks for experts

- Entanglement wedge may go beyond black hole horizons (i.e. no firewall ?).
- “Proven” by using a “generalized” RT formula (Jefferis et al, Dong et al, Bao et al 2016)
- No explicit recipe is known for more than one intervals (AdS3) (For higher-dimensions, it’s a bit more subtle).

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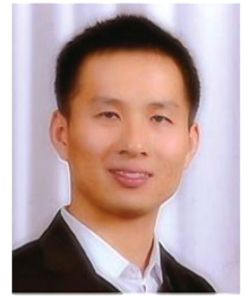
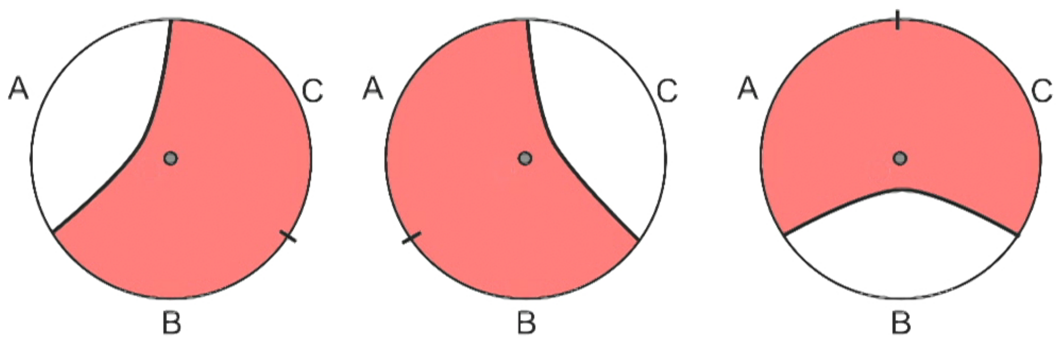
Bulk locality paradox

- The reconstruction leads to a paradox !

[Almheiri-Dong-Harlow 14]

* Uses Schur's lemma, assuming finite-dimensional factorizable Hilbert space

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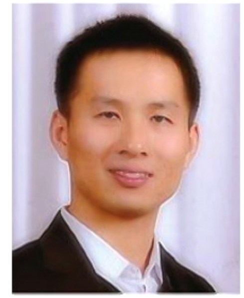
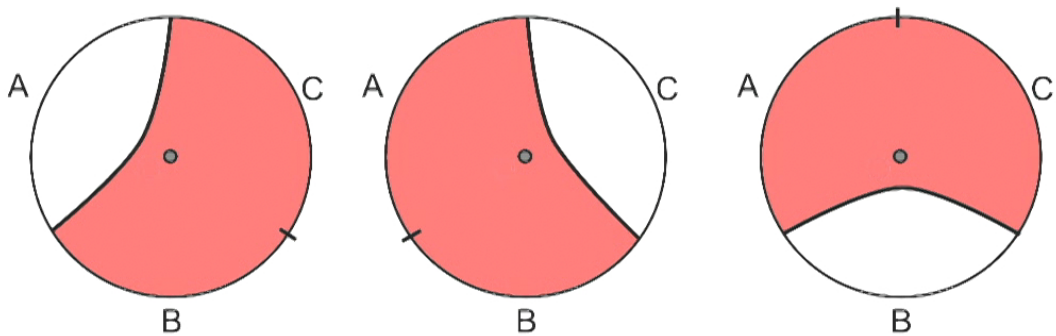


The bulk operator ϕ has non-trivial supports only on AB, BC, CA.

⟶ All the bulk operators must correspond to identity operators on the boundary ?



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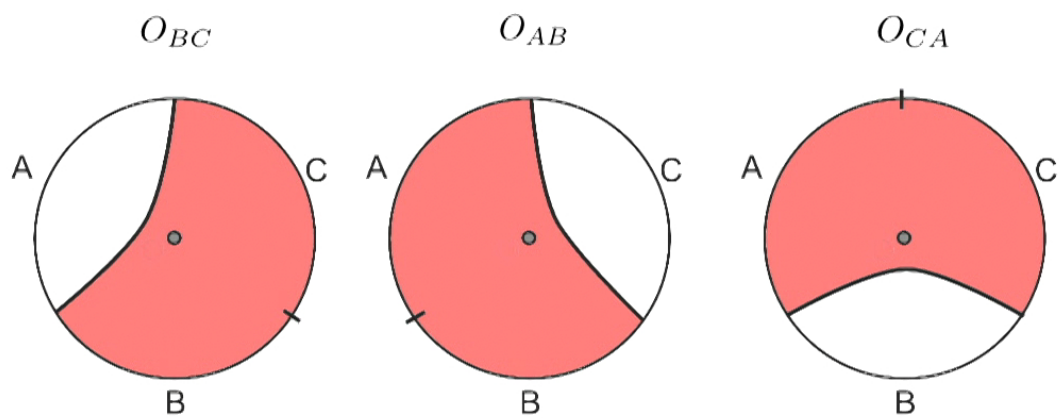
Quantum error-correction in AdS/CFT ?

- The AdS/CFT correspondence can be viewed as a quantum error-correcting code !

These operators may be different, but act in the same manner in a low energy subspace.



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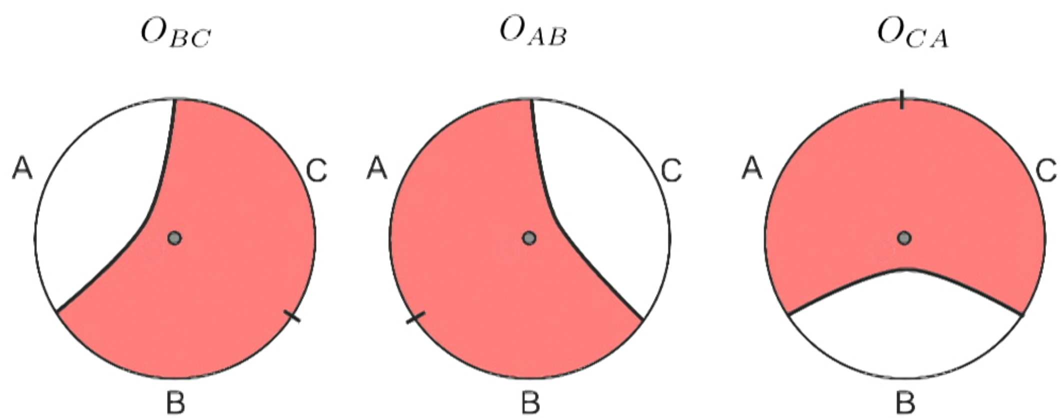
Recall string operators in lattice gauge theory (or Z2 spin liquid).



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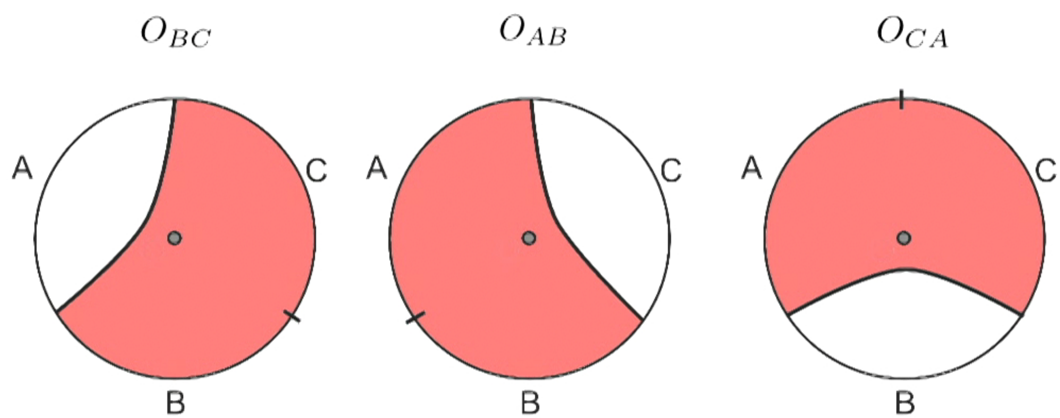
- Quantum secret-sharing code: Alice, Bob and Charlie share a quantum secret.
- Error-correction: Quantum information is protected against erasure of one party.



@ Caltech, 2014 November

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“Construct a toy model !”

A simple toy model

1 bulk qubit

5 boundary qubits

in total, just 6 qubits

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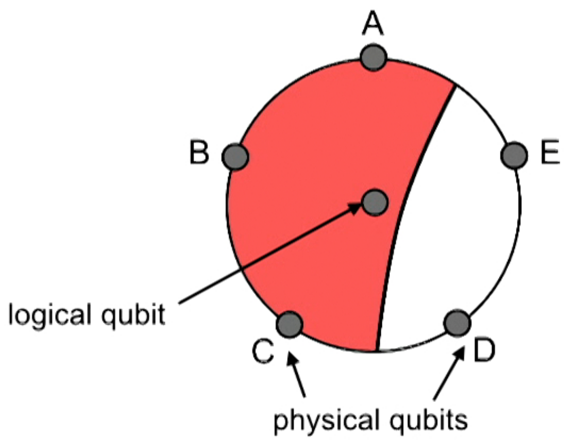


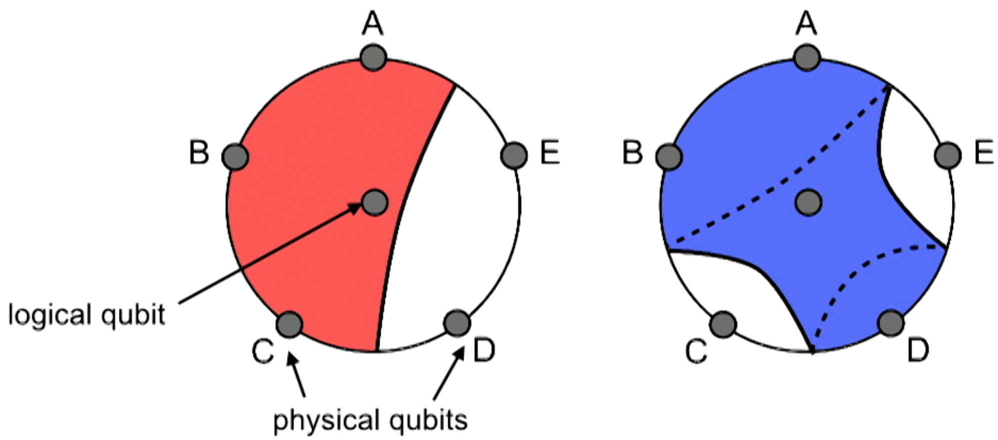
A simple toy model

1 bulk qubit

5 boundary qubits

in total, just 6 qubits

A bulk operator must have corresponding boundary operators on any region with three qubits.



Five-qubit quantum code

- The "simplest" quantum code which is made of qubits.

$$|\xi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow |\xi\rangle = \alpha|c_0\rangle + \beta|c_1\rangle$$

1-qubit input state → 5-qubit output state

[DiVincenzo-Shor, Laflamme-Miquel-Paz-Zurek 1996]

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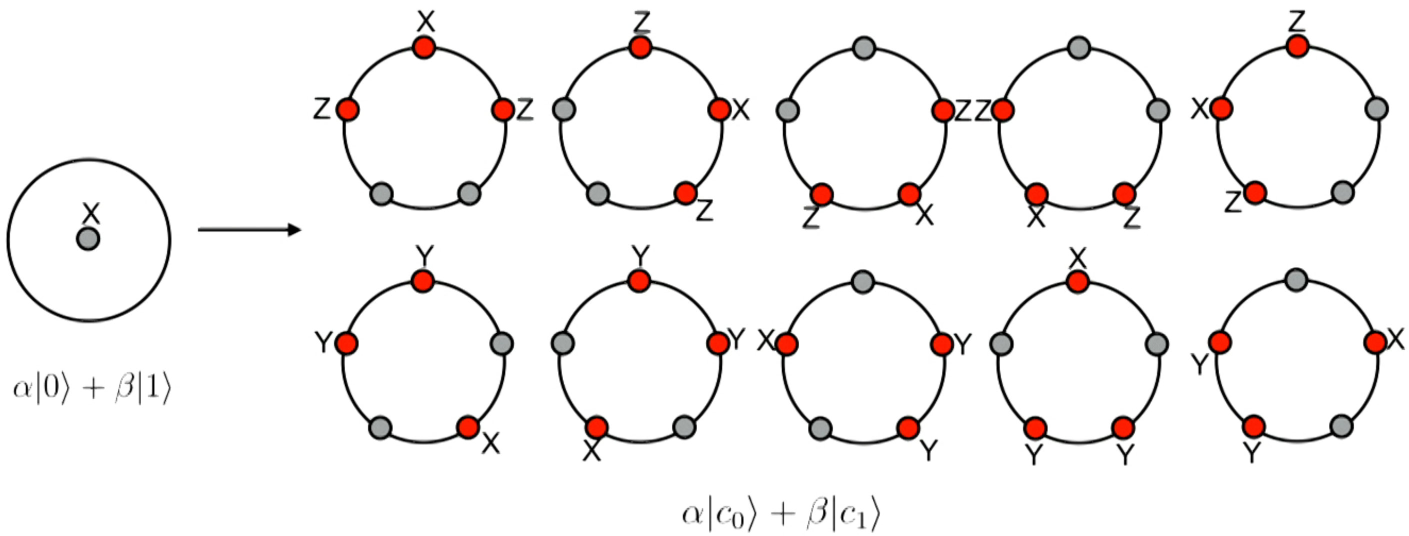
where

$$
\begin{aligned}
|c_0\rangle = {} & |00000\rangle \\
& + |11000\rangle + |01100\rangle + |00110\rangle + |00011\rangle + |10001\rangle \\
& - |10100\rangle - |01010\rangle - |00101\rangle - |10010\rangle - |01001\rangle \\
& - |11110\rangle - |01111\rangle - |10111\rangle - |11011\rangle - |11101\rangle \\
|c_1\rangle = {} & |11111\rangle \\
& + |00111\rangle + |10011\rangle + |11001\rangle + |11100\rangle + |01110\rangle \\
& - |01011\rangle - |10101\rangle - |11010\rangle - |01101\rangle - |10110\rangle \\
& - |00001\rangle - |10000\rangle - |01000\rangle - |00100\rangle - |00010\rangle
\end{aligned}
$$

[DiVincenzo-Shor, Laflamme-Miquel-Paz-Zurek 1996]

Operator correspondence in five-qubit code

- Pauli X operator in the input (bulk) corresponds to following 3-body operators.



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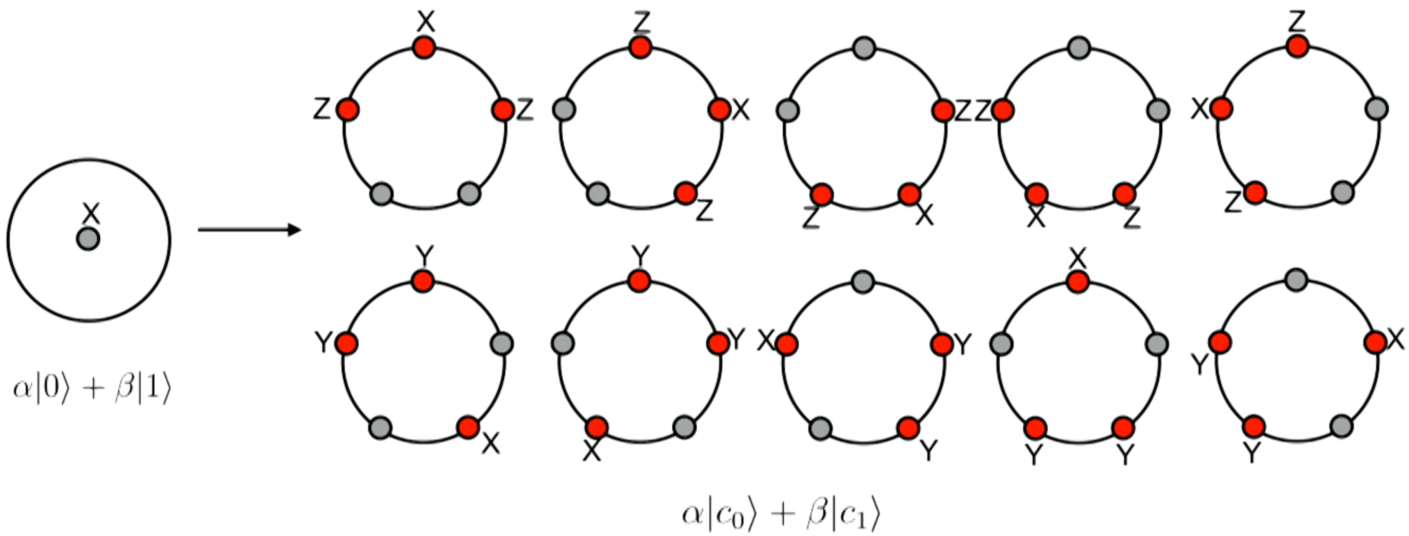

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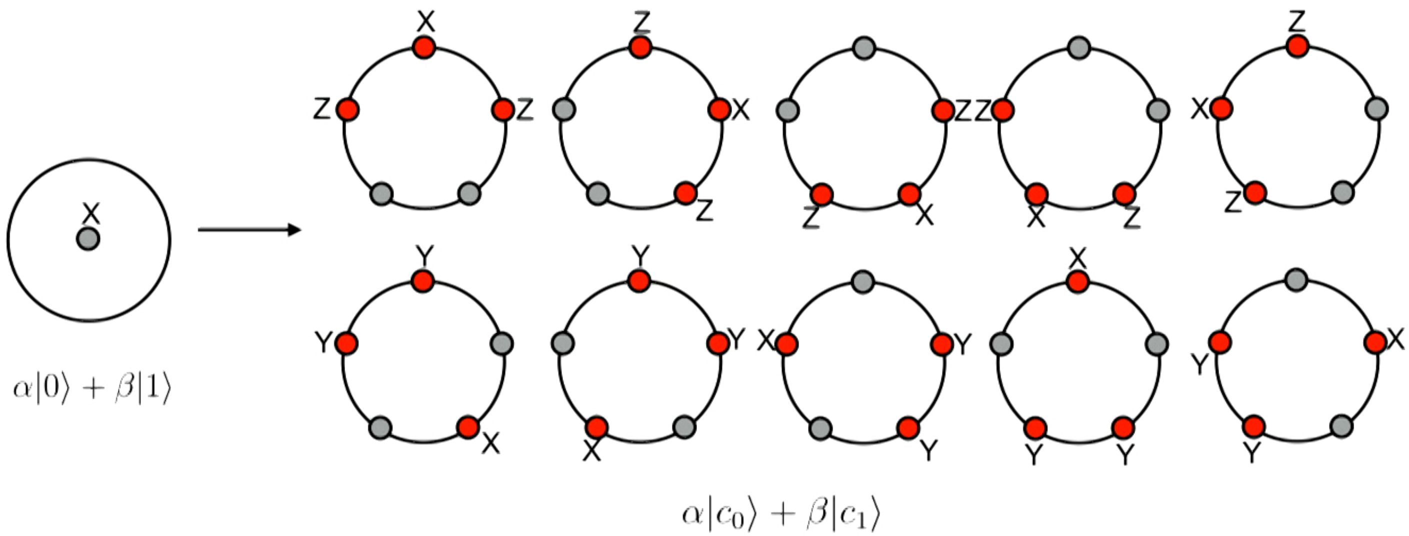

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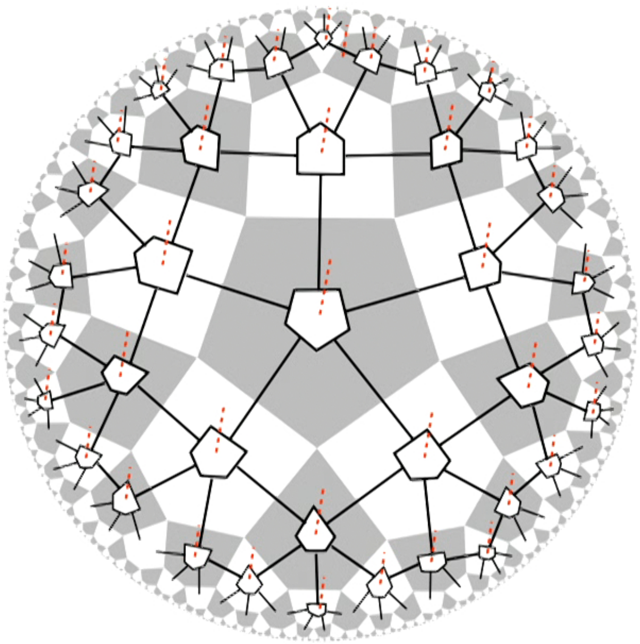
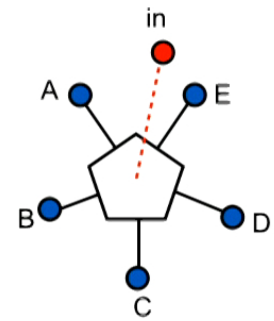
• Error-correction : losing 2 qubits is OK.

A holographic quantum error-correcting code

- A tiling of the five qubit code via tensor network technique





five-qubit code

* Tensor network and AdS/CFT : Vidal07, Swingle12, Qi13, Czech et al15 …

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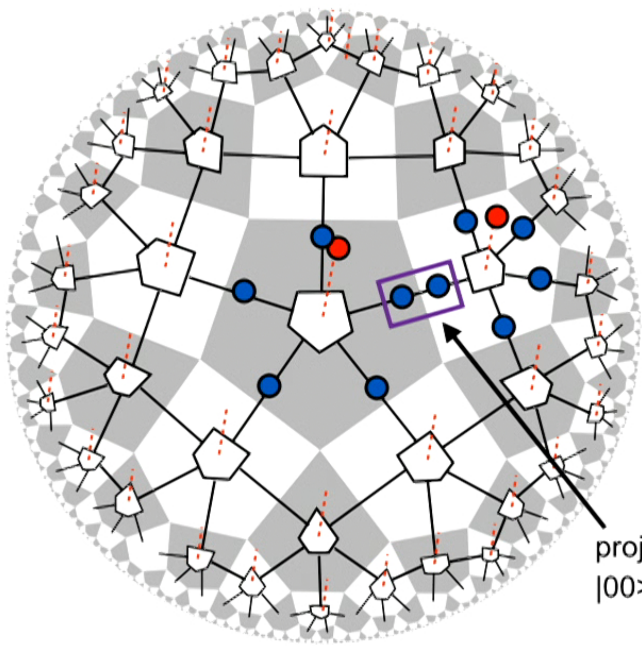

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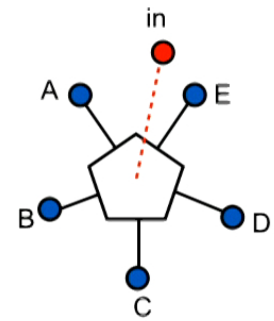

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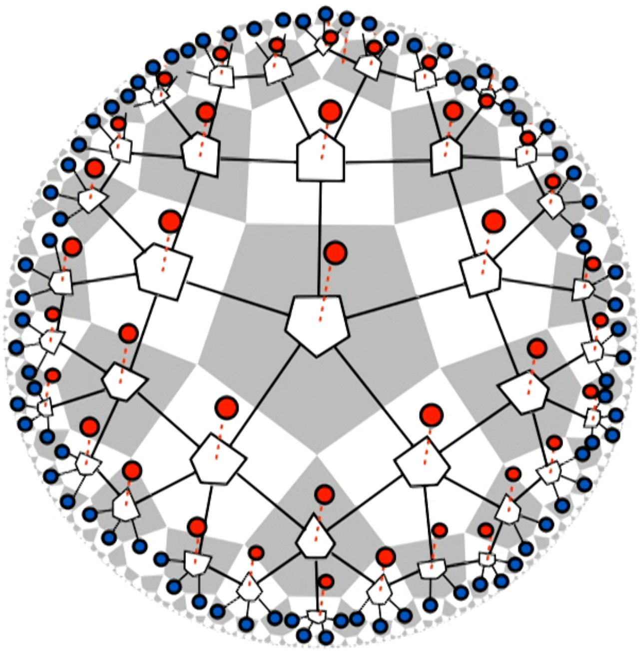

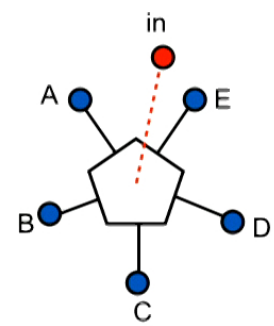

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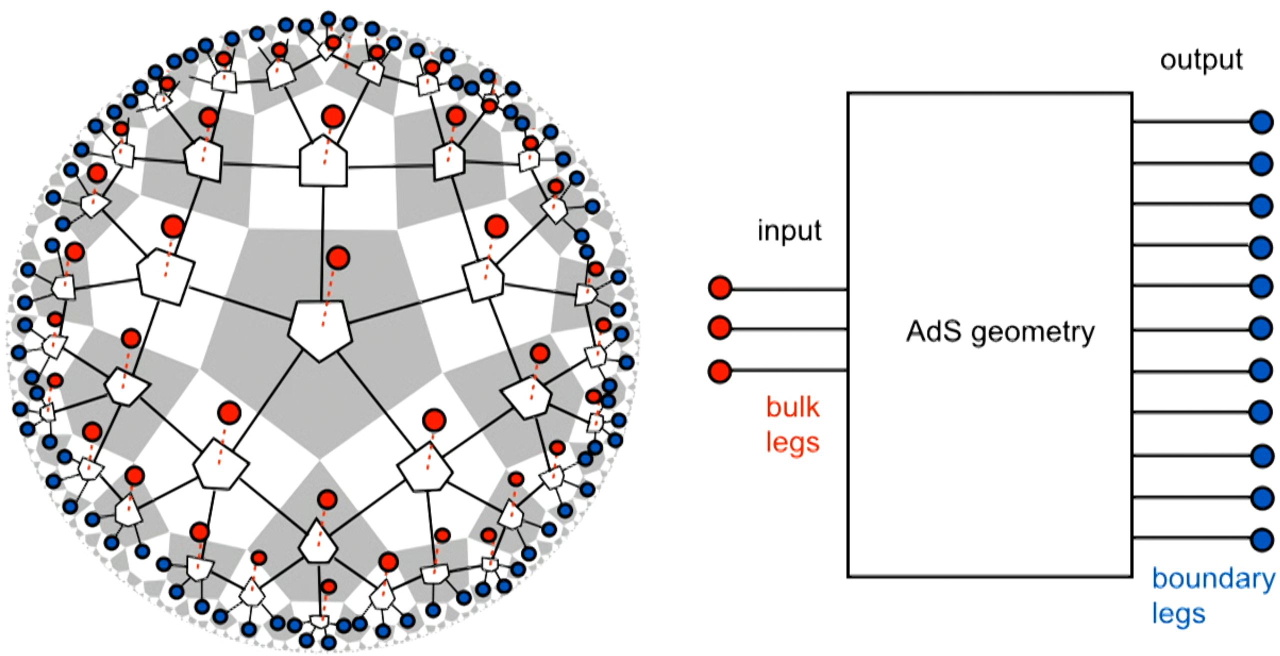
A holographic quantum error-correcting code

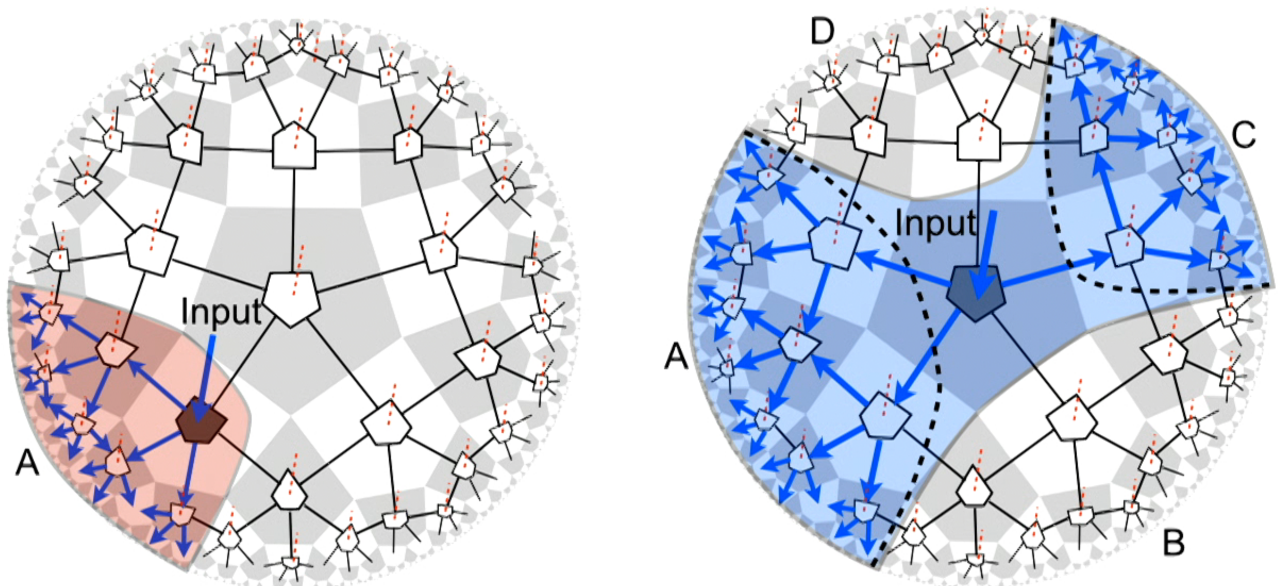
- A tiling of the five qubit code via tensor network technique



* Tensor network and AdS/CFT : Vidal07, Swingle12, Qi13, Czech et al15 …

Entanglement wedge reconstruction

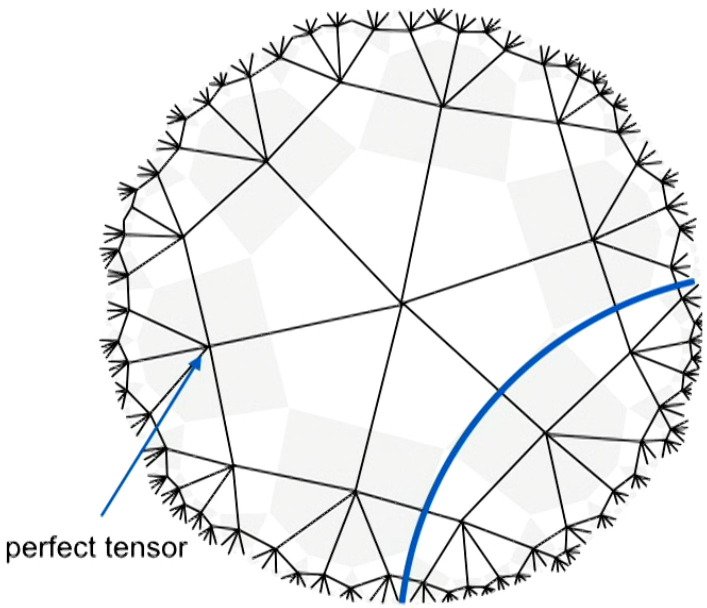
- 1 in & 3 out (operator pushing)

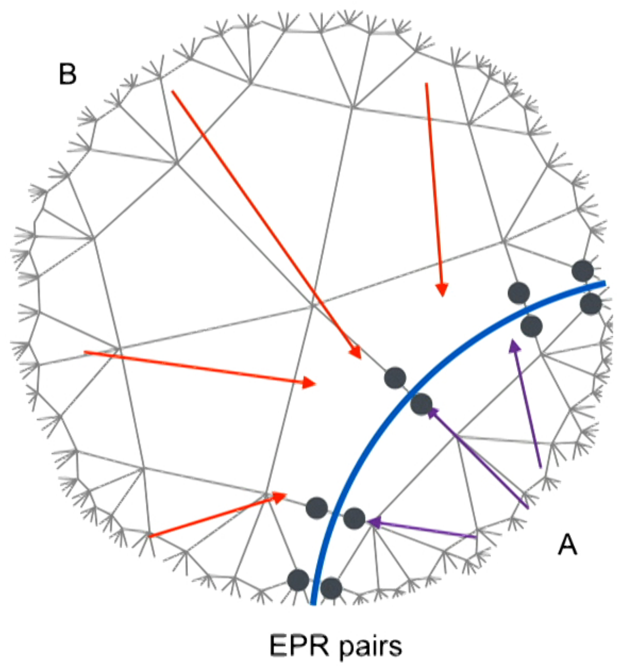


Holographic state

- The Ryu-Takayanagi formula holds (tiling without bulk legs)

Coarse-graining (RG transformation) = Distillation of EPR pairs along the geodesic





Physical interpretation: Matter and Geometry

- Qubits on the bulk represent matter fields, coupled to the geometry (via 5-qubit code tensors).

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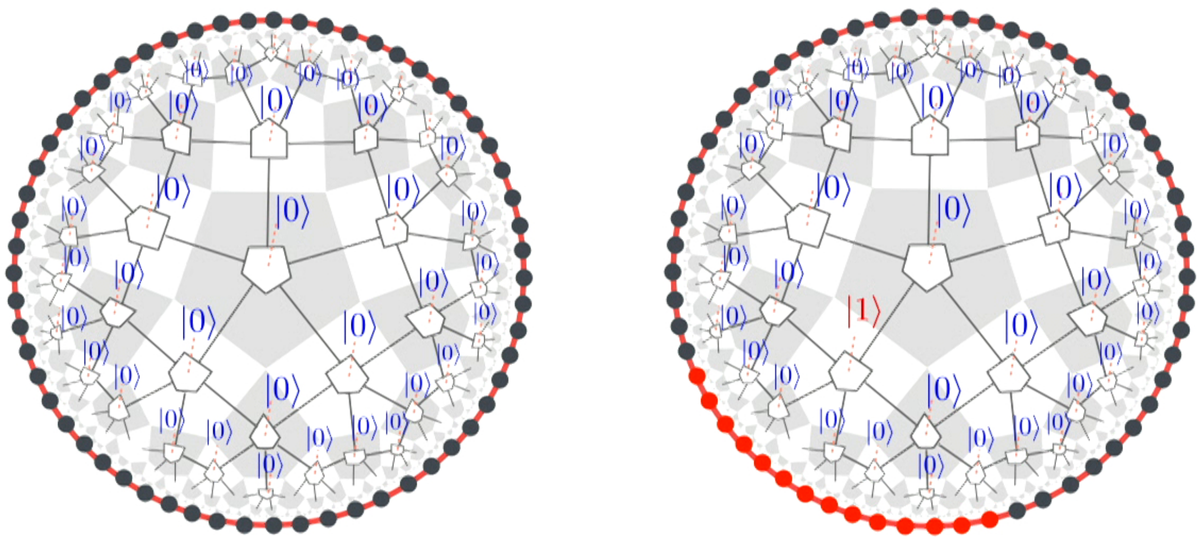
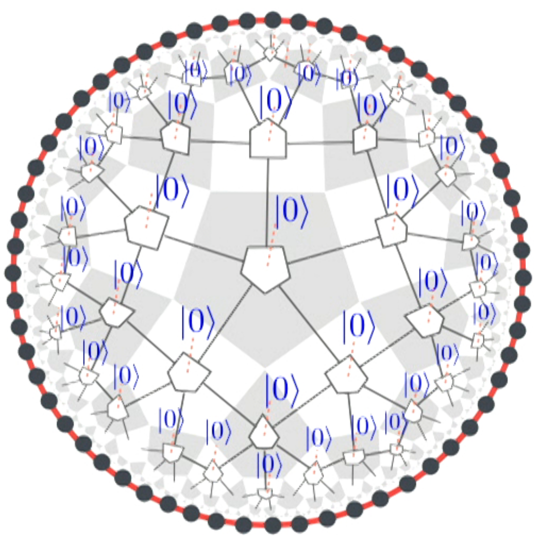


Vacuum Excited state

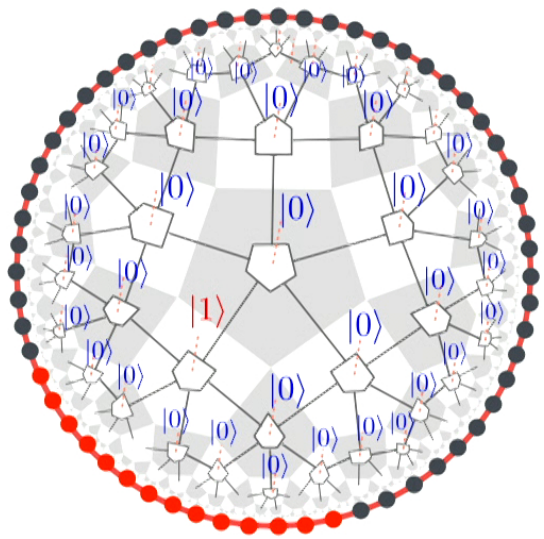
Physical interpretation: Matter and Geometry

- Qubits on the bulk represent matter fields, coupled to the geometry (via 5-qubit code tensors).
- Bulk Hilbert space $\mathcal{H}_{\rm bulk}$ is much smaller than boundary Hilbert space $\mathcal{H}_{\rm bdy}$.

⟶ The model captures "perturbations" around a fixed geometry.



Vacuum



Excited state

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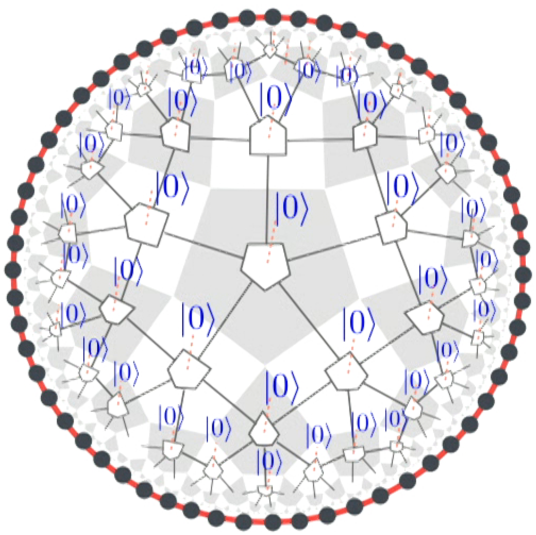
- Going outside the codeword space = changing geometries (eg. micro black holes)



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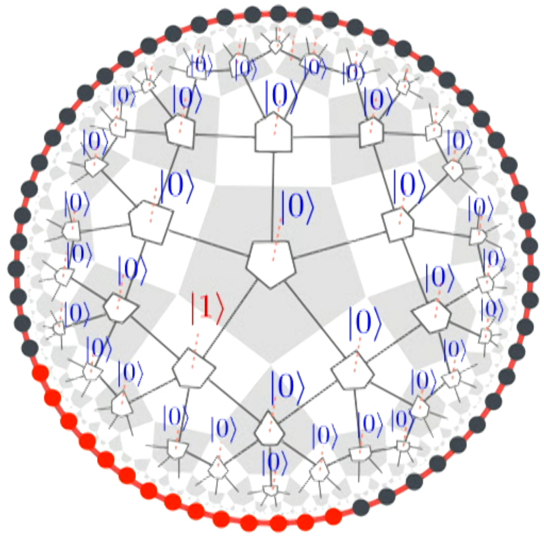
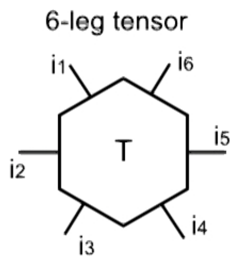


Excited state

What is the key property which makes these possible?

State-Tensor duality [Abstract, but important !]

- A tensor with 6 legs



$$T_{i_1 i_2 i_3 i_4 i_5 i_6} \in \mathbb{C}$$

$$i_k = 0, 1$$

[Choi-Jamilkowski isomorphism]

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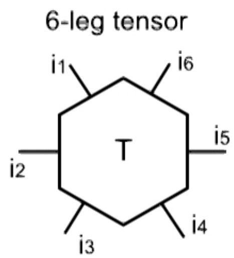

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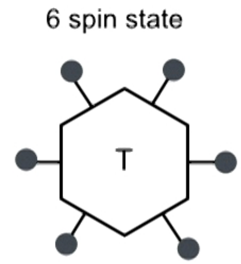

$$|\psi\rangle = \sum_{i_1 i_2 \ldots i_6} T_{i_1 i_2 \ldots i_6} |i_1, i_2, \ldots, i_6\rangle$$

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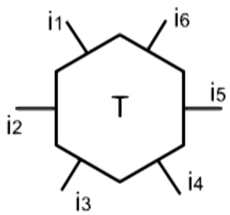

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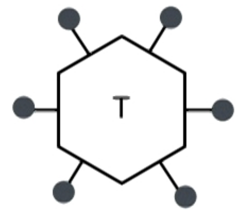
6 spin state



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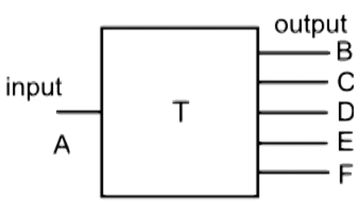
- A quantum code with 1 input & 5 output

5-qubit code



A and B are not entangled.

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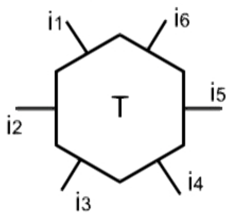

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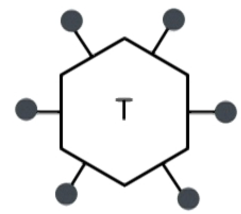

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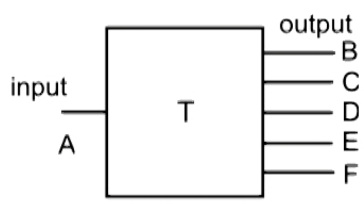
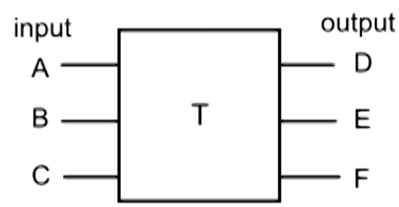


A and B are not entangled.

- A linear operator with 3 input & 3 output

3-qubit quantum operation



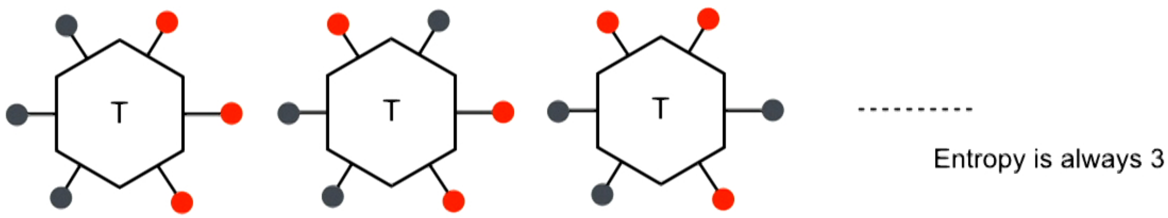
unitary if ABC is maximally entangled with DEF

[Choi-Jamilkowski isomorphism]

5-qubit code is "perfect"

- The 6-qubit state (or 5-qubit code) is "perfect".

[Def] The state is maximally entangled in any bipartition.



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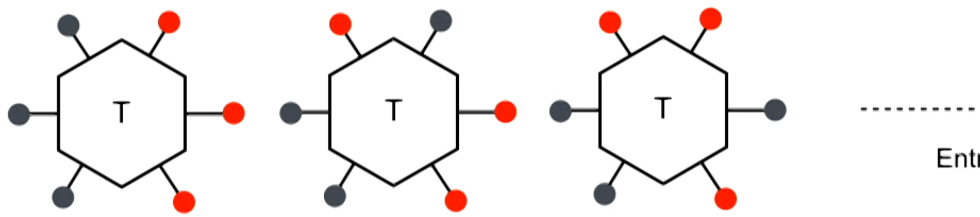
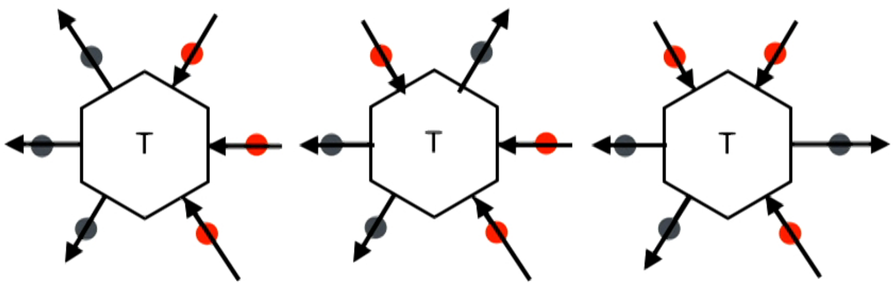


Entropy is always 3

- Any "3-in to 3-out" defines unitary operator.



"Flow" of entanglement

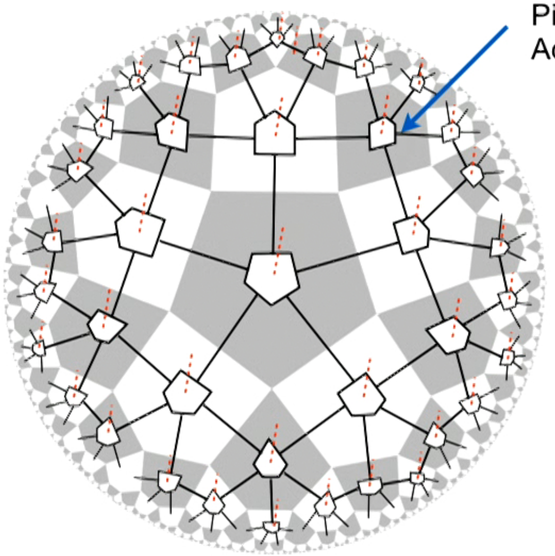
- Perfect tensor networks generate quantum codes with similar AdS/CFT properties.

["Bit threading" Freedman-Headrick]

Space-time from "something random" ?

- Perfect tensors are easy to create !

A random state is nearly maximally entangled along any bipartition (eg: Page's argument, Canonical typicality).



Pick random tensors at each AdS scale patch

- Analytical solution of random tensor network [Hastings15, Hayden et al 16]

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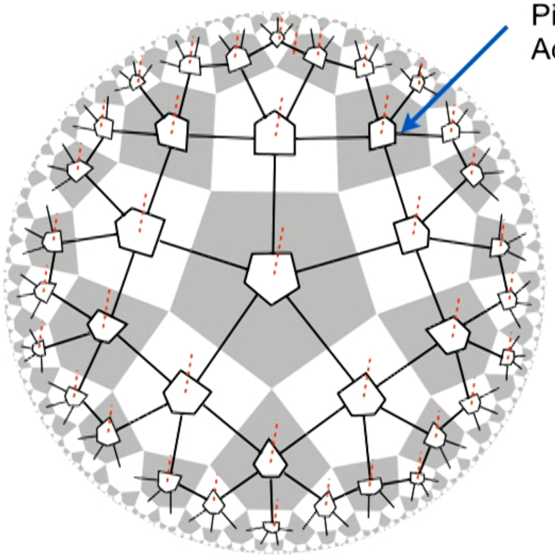

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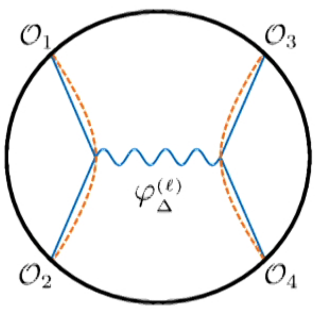
Interesting "1/S" behaviours (S: spin dimensions)



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Aspects of holographic tensor networks

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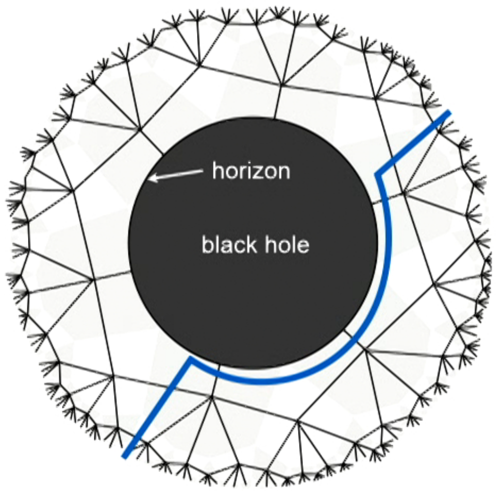
* Toy models of black holes (smoothness of the horizon? interior of black holes?)

One-sided black hole



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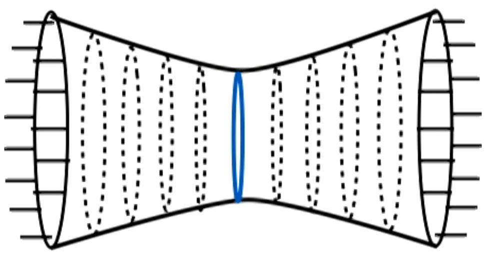

Einstein-Rosen bridge

Aspects of holographic tensor networks

* Toy models of black holes (smoothness of the horizon? interior of black holes?)

* Complexity = Volume = Action? [Susskind and his friends] (number of tensors = complexity?)

* Generalizations: symmetries, “matrix QM”-like network, sub-AdS locality, kinematic space

* Tensor network in de Sitter space ? (Dark matter problem, as mentioned by Eric Verlinde)

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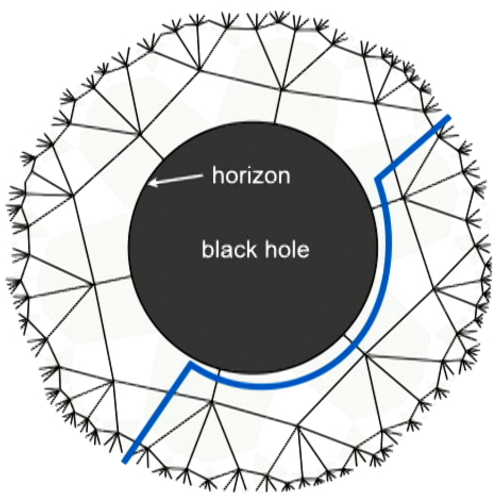

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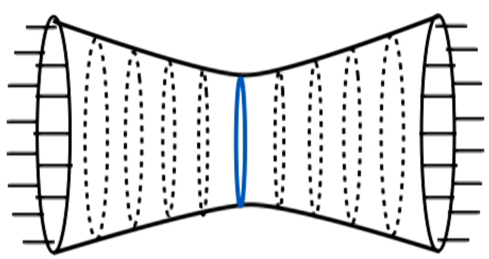

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* Difficulty: No Hamiltonian, No dynamics. Need more inputs to be more realistic…

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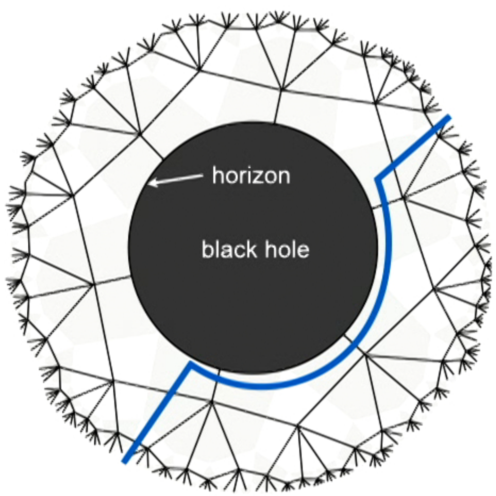

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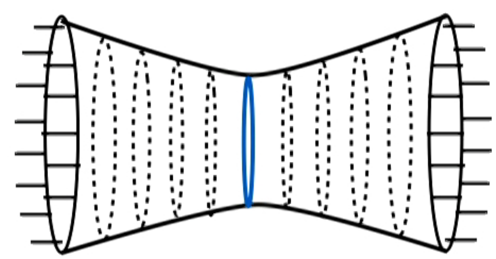


Einstein-Rosen bridge

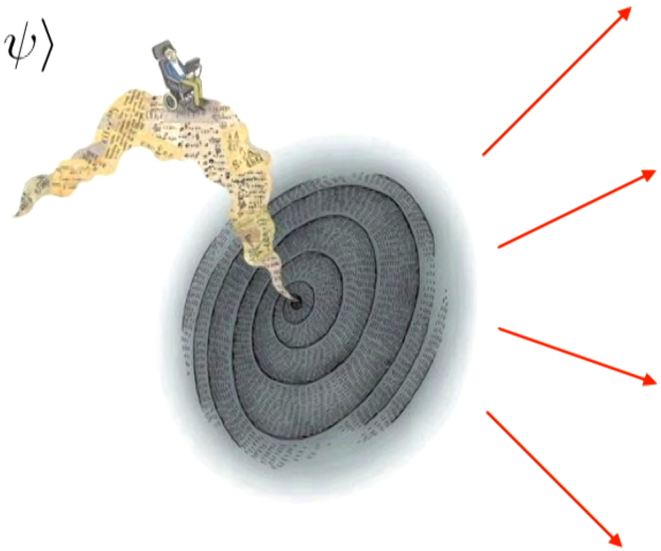
Part 2 :

Black hole is a quantum error-correcting code

Information loss puzzle

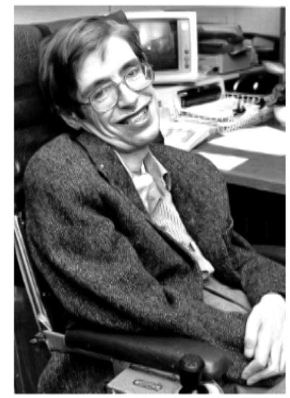
- Is quantum information lost ?

$|\psi\rangle$



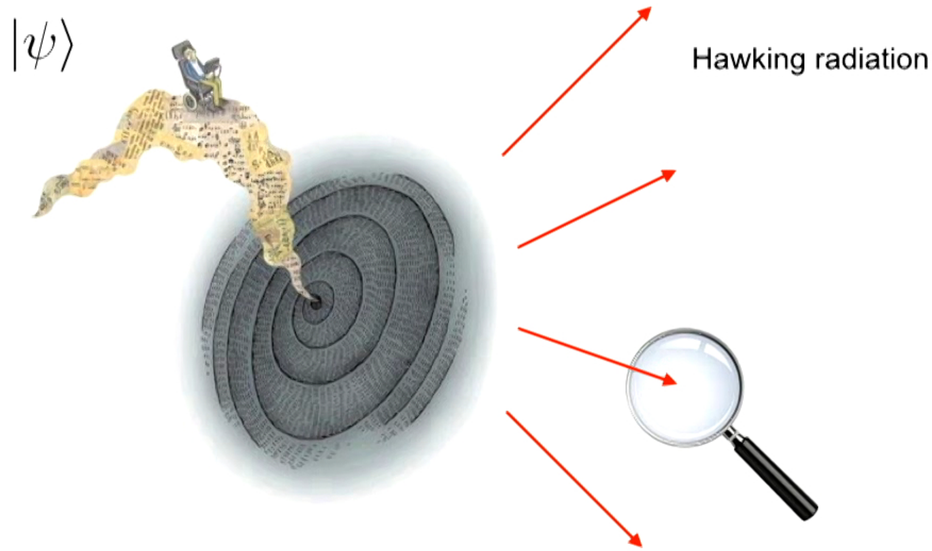
Hawking radiation

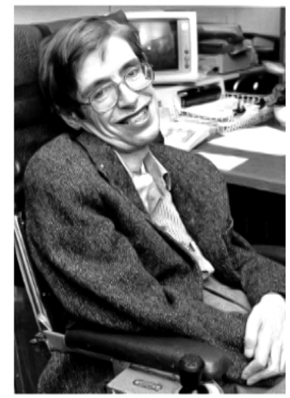
$$“e^{-\beta H}”$$



Information loss puzzle

- Or hidden into some non-local degrees of freedom ? (Scrambling !)

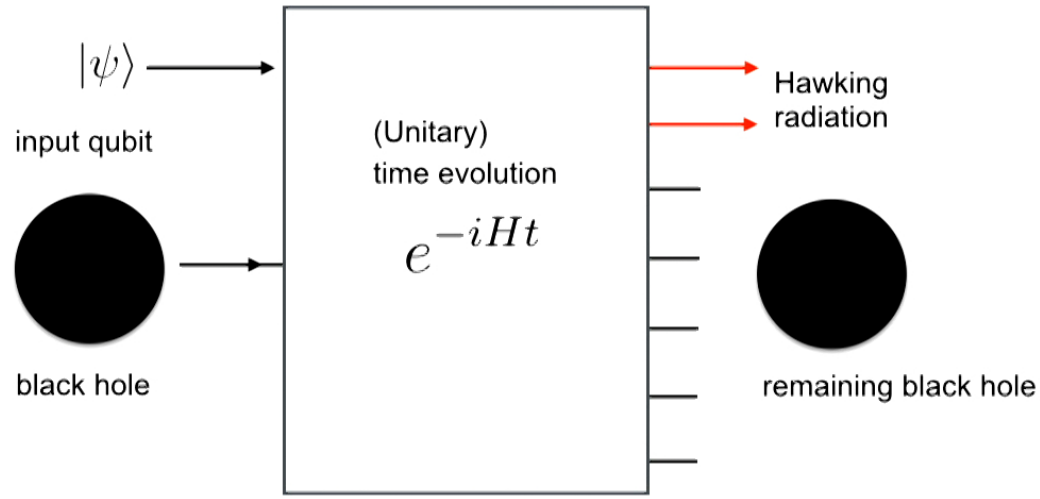




Locally it looks like "$e^{-\beta H}$", but globally it is not .

Black hole is a quantum error-correcting code ?

- Scrambling is very similar to how quantum error-correcting codes work.



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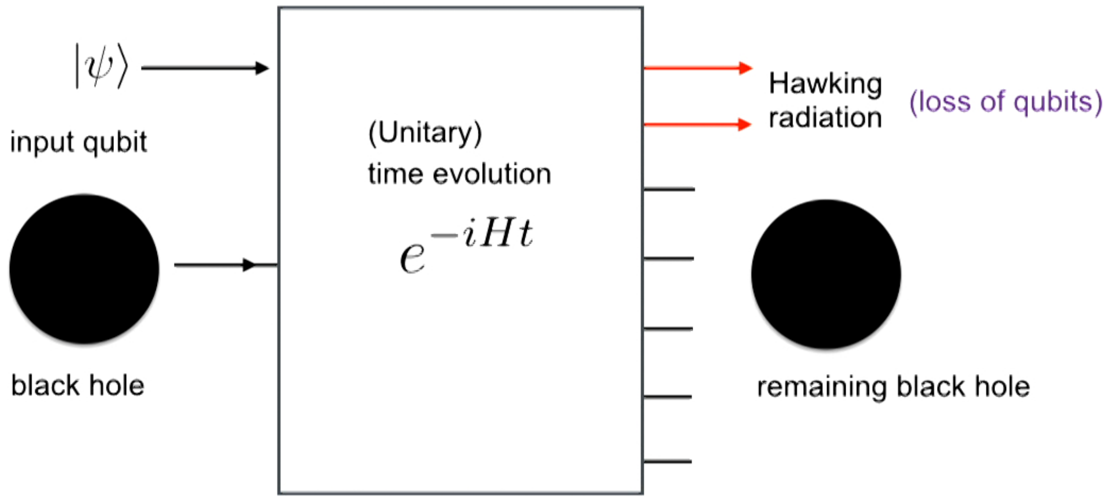
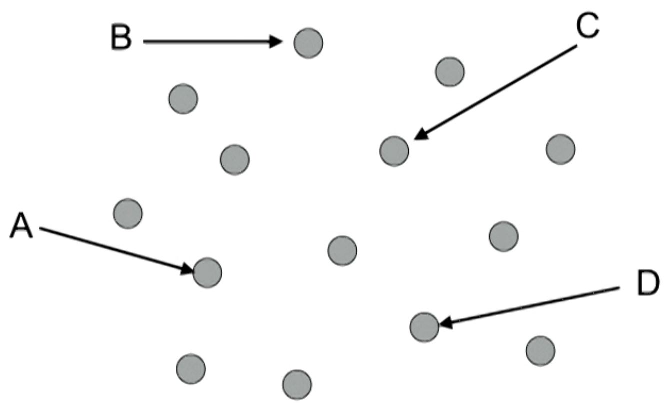


- Input quantum state is protected from loss of qubits (Hawking radiation)

Out-of-time ordered correlation functions

- Black holes have some "hidden" correlations (Kitaev14, Shenker-Stanford13)



- Previously considered by Larkin and Ovchinikov in 1960s

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$$\mathrm{OTOC} = \langle O_A(0) O_B(t) O_C(0) O_D(t) \rangle$$

$$O_B(t) = e^{-iHt} O_B(0) e^{iHt}$$
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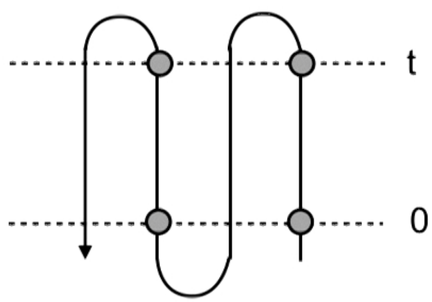

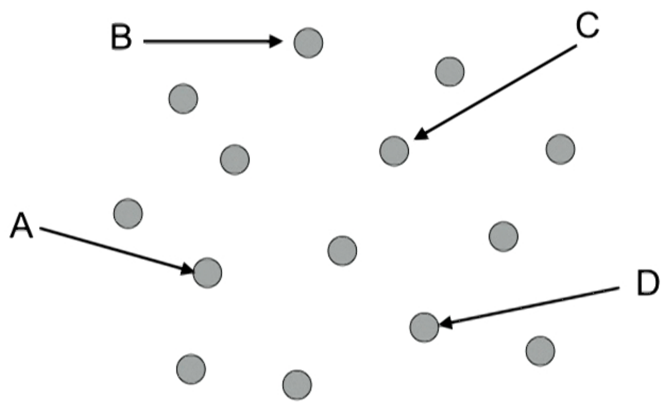

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Out-of-time ordered correlation functions detect scrambling of quantum information.

(joint with Hosur, Qi and Roberts 2015)

State-Tensor duality, again

- Unitary operator acting on n qubits can be viewed as a state on 2n qubits.

[Choi-Jamilkowski 1960s, Hayden-Preskill,Hartman-Maldacena]

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$$U = \sum_{i,j} U_{ij} |i\rangle\langle j|$$

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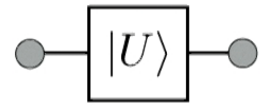

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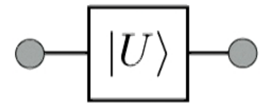
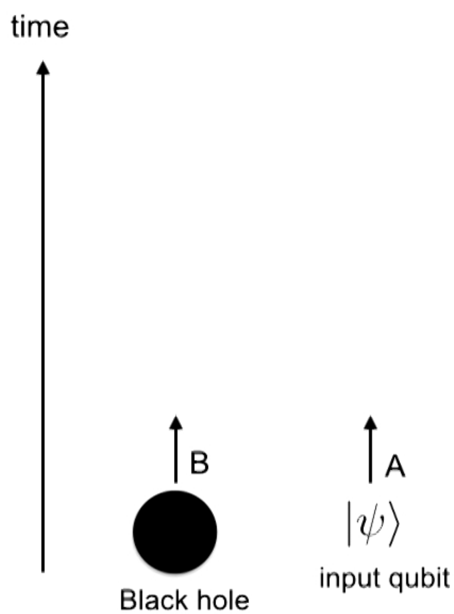


- Viewing the black hole dynamics as a quantum state.



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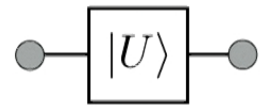

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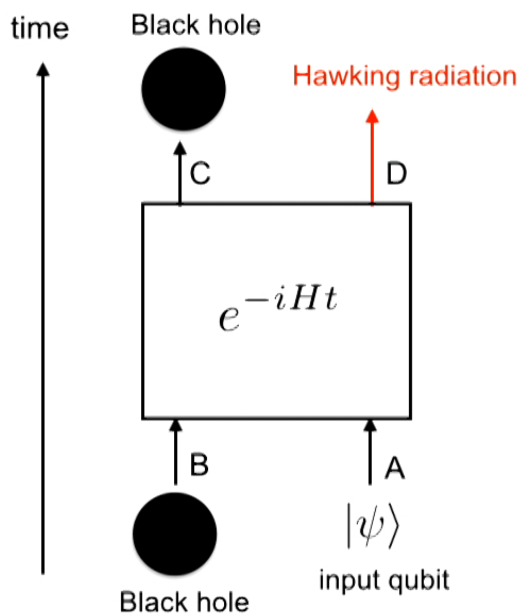
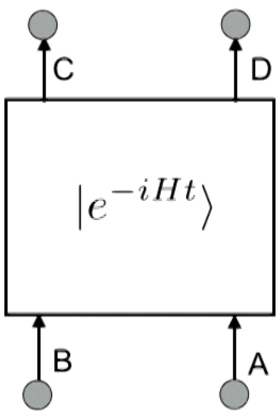


Entanglement between input and output

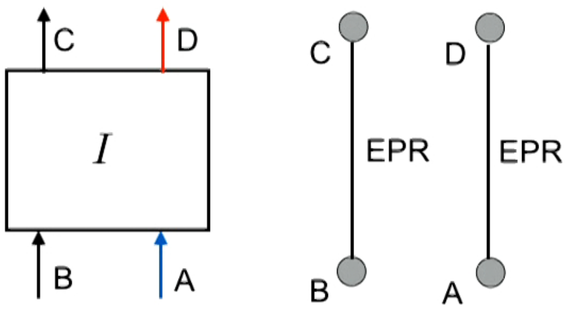
eg. between A and D = radiation and input



[Choi-Jamilkowski 1960s, Hayden-Preskill,Hartman-Maldacena]

Input and output entanglement

- An identity operator



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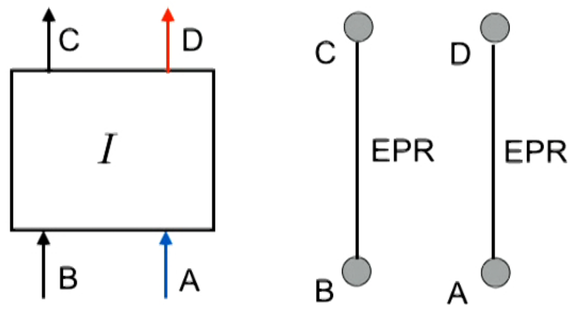

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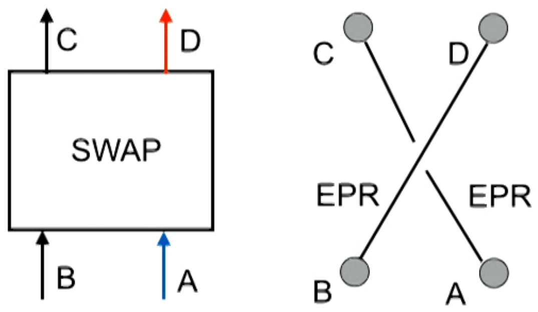

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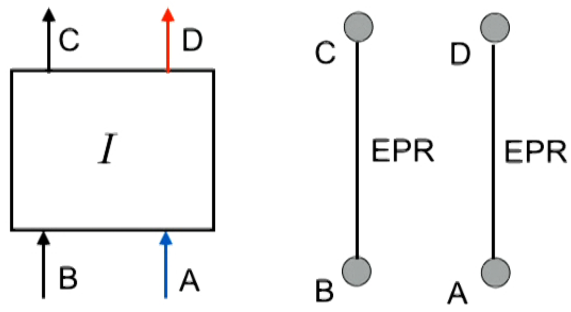

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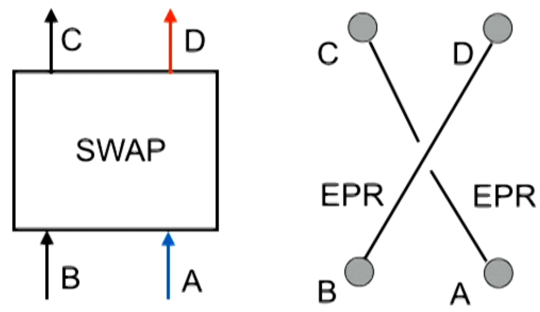
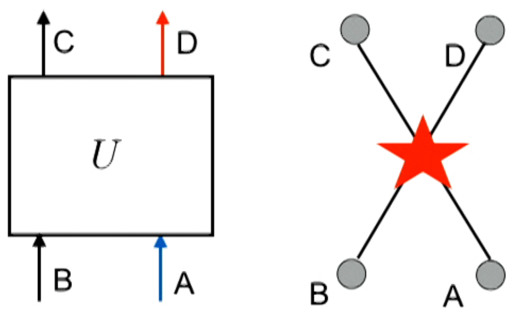


- Scrambling



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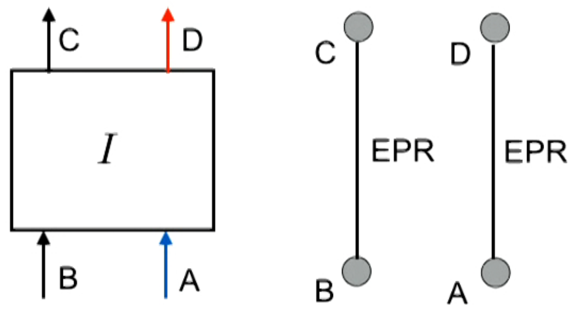
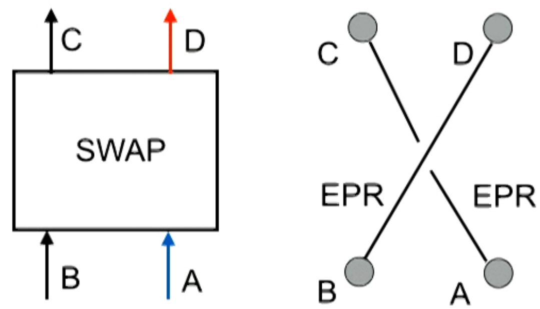
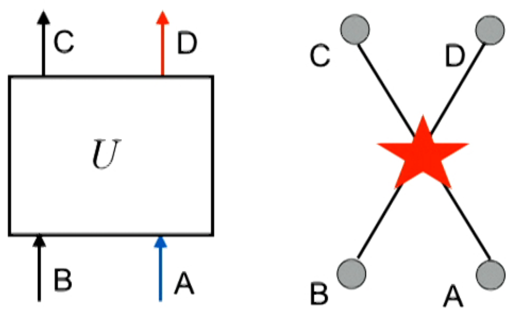


- SWAP operator



- Scrambling



<u>Quantum error-correcting code postulate</u>

No correlation between local inputs and local outputs

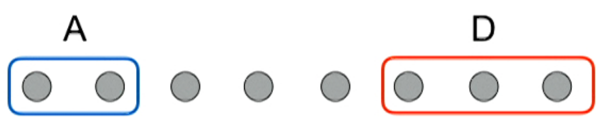
"mutual information"

I(A,D) = 0 I(A,C) = 0

[Formula] Average value of OTO

- Average of OTO over local operators A and D at T=infty [Hosur-Qi-Roberts-BY15]

$$|\langle O_A(0) O_D(t) O_A(0) O_D(t) \rangle|_{\rm ave}$$



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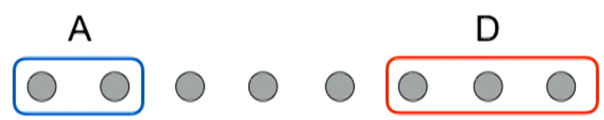
average over O_A, O_D



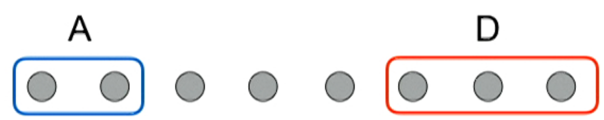
[Formula] Average value of OTO

- Average of OTO over local operators A and D at T=infty [Hosur-Qi-Roberts-BY15]

$$|\langle O_A(0)O_D(t)O_A(0)O_D(t)\rangle|_{\text{ave}} = \frac{1}{4^{a+d}} \sum_{O_A,O_D} \langle O_A(0)O_D(t)O_A(0)O_D(t)\rangle$$

average over O_A, O_D

O_A, O_D: Pauli operators (unitary 1-design)



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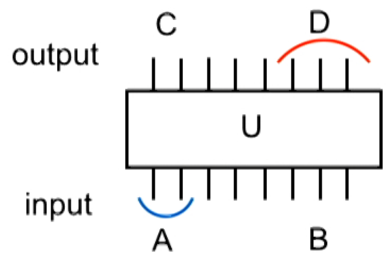

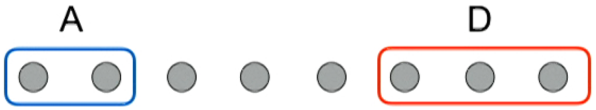

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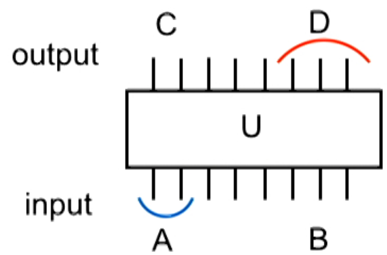
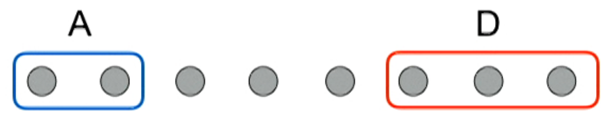
average over OA, OD

Pauli operators (unitary 1-design)

$$= 2^{n-a-d-S^{(2)}_{BD}}$$

Renyi-2 entropy



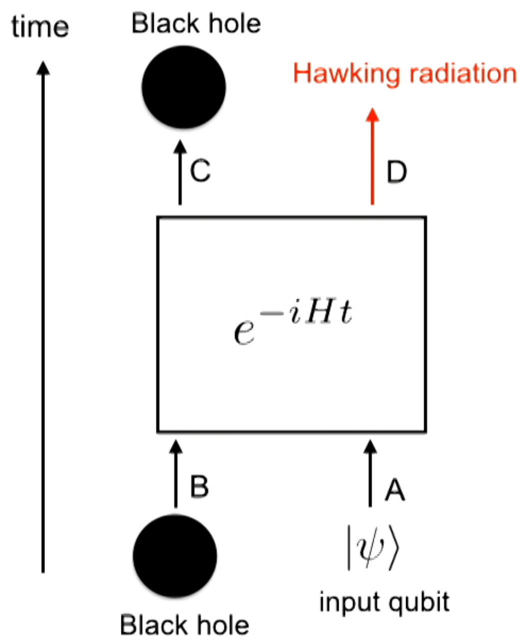


- If $\text{OTO} \simeq 0$ then, $S^{(2)}_{BD}$ is large

  This implies the mutual information $I^{(2)}(B,D) = S^{(2)}_B + S^{(2)}_D - S^{(2)}_{BD}$ is small

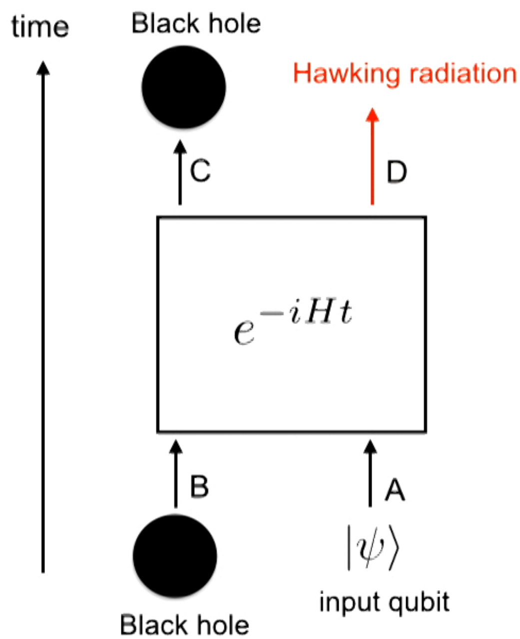
- If $\text{OTO} \simeq 0$ then, $I^{(2)}(A,C) = S^{(2)}_A + S^{(2)}_C - S^{(2)}_{AC}$ is also small.

Quantum error-correction in black holes



[Hosur-Qi-Roberts-BY15, Roberts-BY16, BY in prep]

Quantum error-correction in black holes



- Hawking-Unruh semiclassical calculation

$$\langle O_A(0)O_A(t)\rangle \to 0$$

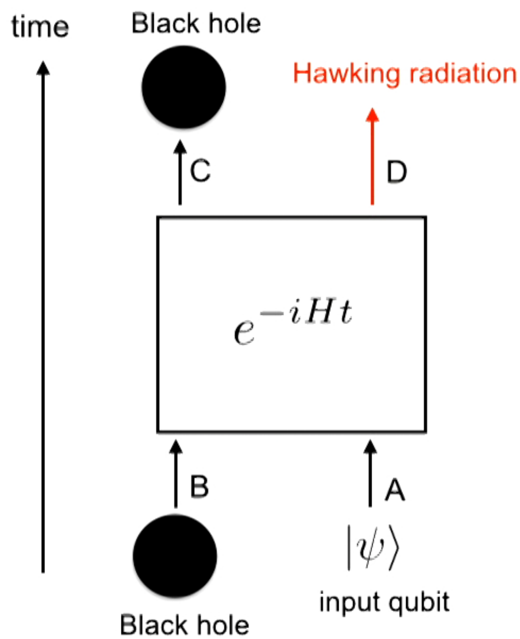
$$\Rightarrow \quad I(A,D) \to 0$$

- Kitaev-Shenker-Stanford shockwave calculation

$$\langle O_A(0)O_D(t)O_A(0)O_D(t)\rangle \to 0$$

[Hosur-Qi-Roberts-BY15, Roberts-BY16, BY in prep]

Quantum error-correction in black holes



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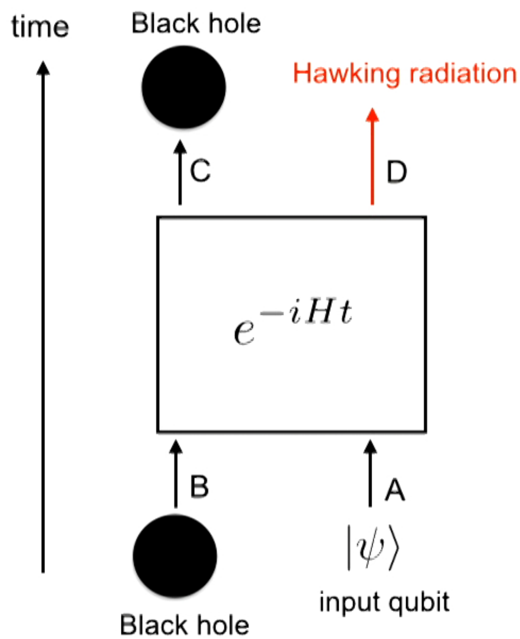
[Hosur-Qi-Roberts-BY15, Roberts-BY16, BY in prep]

Quantum error-correction in black holes



- Hawking-Unruh semiclassical calculation

$$\langle O_A(0) O_A(t) \rangle \to 0$$

$$\Rightarrow \quad I(A, D) \to 0$$

- Kitaev-Shenker-Stanford shockwave calculation

$$\langle O_A(0) O_D(t) O_A(0) O_D(t) \rangle \to 0$$

$$\Rightarrow \quad I(A, C) \to 0$$

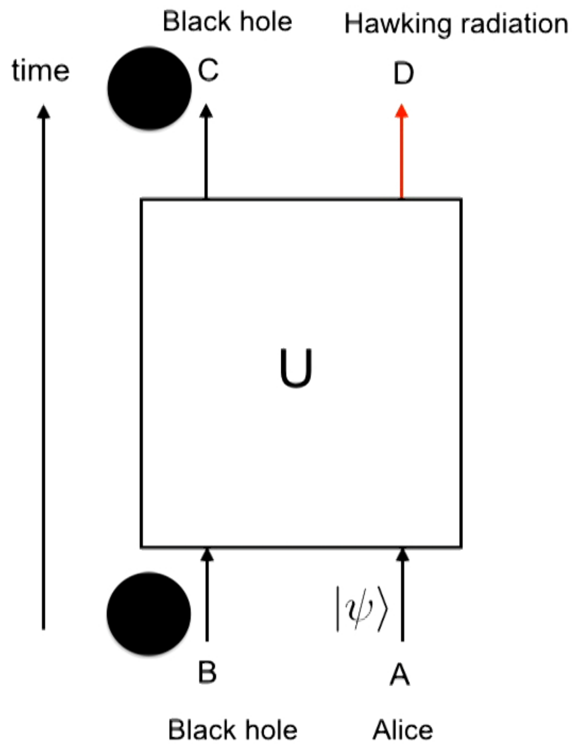
So, a black hole is a quantum error-correcting code.

[Hosur-Qi-Roberts-BY15, Roberts-BY16, BY in prep]

Any new insight into information loss puzzle ?

Hayden-Preskill thought experiment (2007)

- Can Bob reconstruct Alice's quantum state ?



Hayden-Preskill thought experiment (2007)

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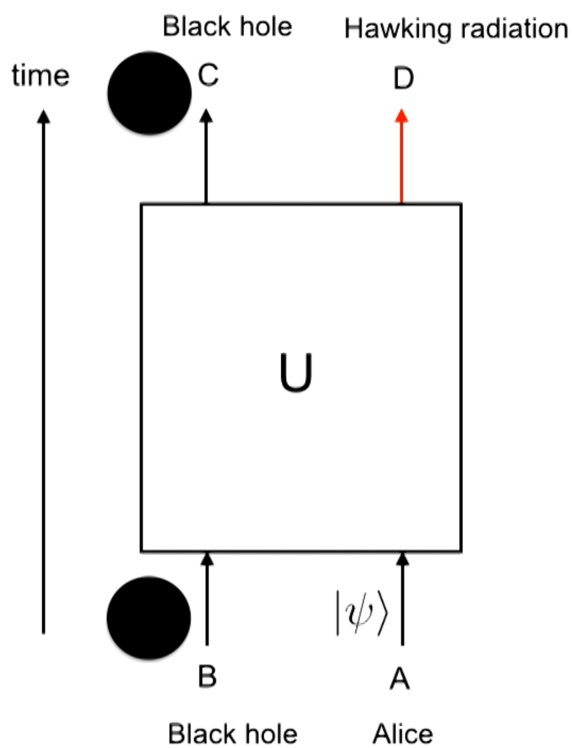


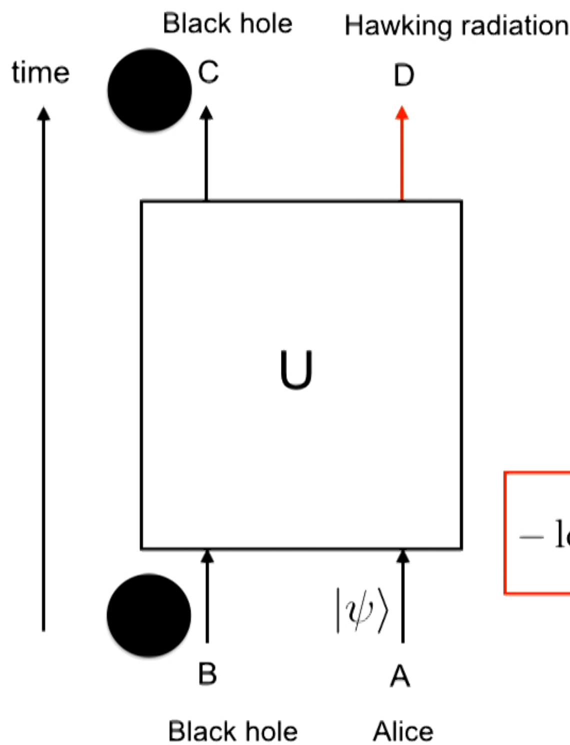
Bob has an access to

B : Black hole initial state

D : Hawking radiation

Hayden-Preskill thought experiment (2007)

- Can Bob reconstruct Alice's quantum state ?



Bob has an access to

B : Black hole initial state

D : Hawking radiation

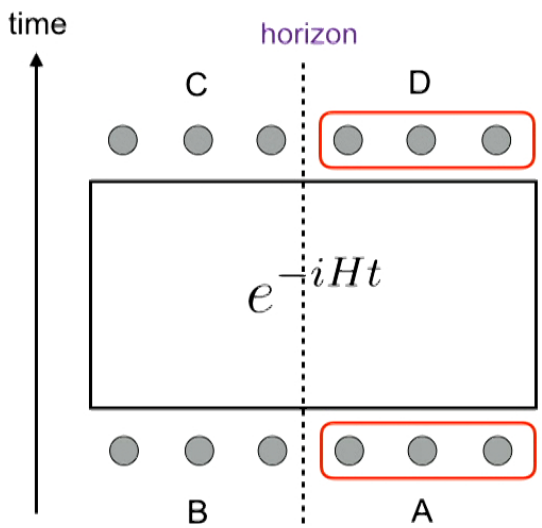
$$-\log_2 |\langle O_A(0) O_D(t) O_A(0) O_D(t) \rangle| = I^{(2)}(A, BD)$$

[Roberts-BY16]

Small OTOC ⟶ Bob's success

Seeing "behind the horizon" with OTOCs ?

- OTOCs provide a way of seeing behind the black hole horizon.



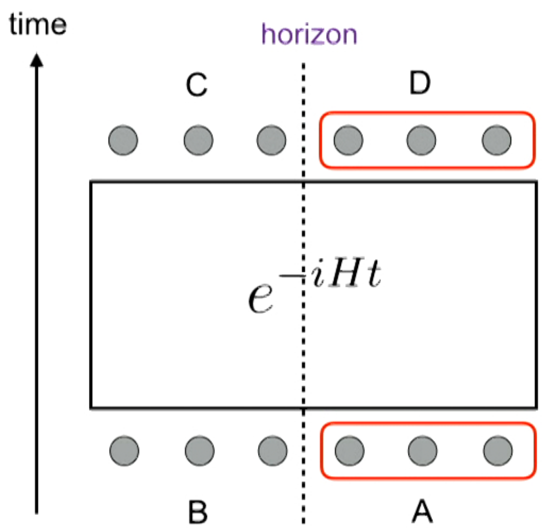
Seeing “behind the horizon” with OTOCs ?

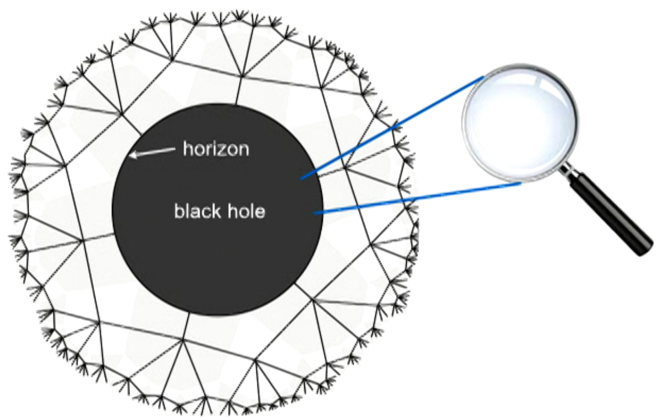
- OTOCs provide a way of <u>seeing</u> behind the black hole horizon.

Assume that we have an access only to A

$$|\langle O_A(0)O_D(t)O_A(0)O_D(t)\rangle|_{\text{ave}} \longrightarrow I(A,C)$$

We can study cross-horizon correlations.





Seeing “behind the horizon” with OTOCs ?

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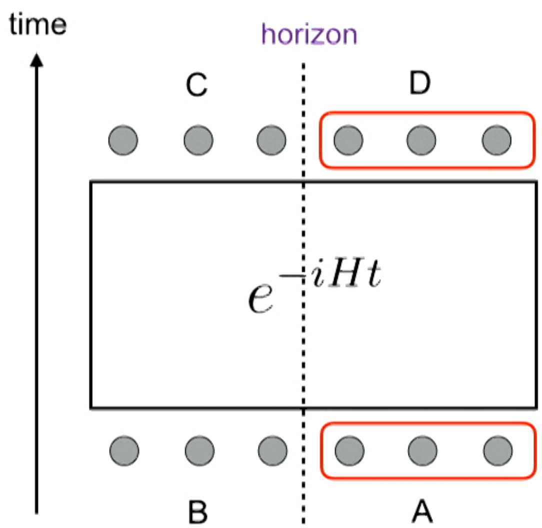
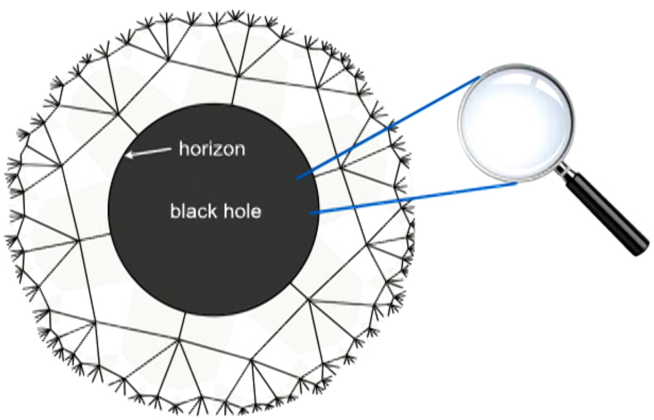
$$|\langle O_A(0) O_D(t) O_A(0) O_D(t)\rangle|_{\text{ave}} \longrightarrow I(A, C)$$

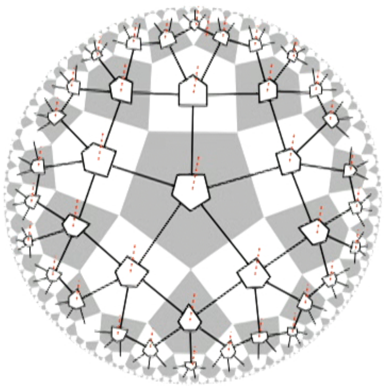
  We can study cross-horizon correlations.

- But, we need a finite-temperature generalization… (ongoing work with Mozgunov and Kitaev)





Part 1: AdS/CFT correspondence is a quantum error-correcting code.



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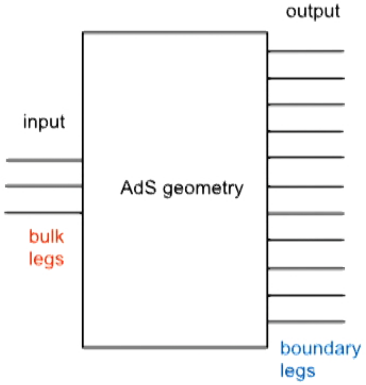
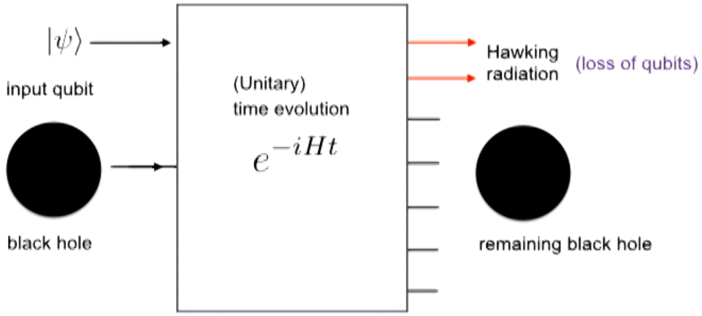
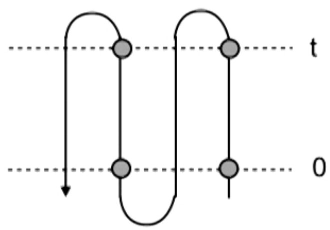


Part 2: Black hole is a quantum error-correcting code.



$\langle O_A(0) O_D(t) O_A(0) O_D(t) \rangle$



21st century quantum information theory

21st century quantum information theory

Entanglement entropy (1932)

von Neumann

21st century quantum information theory

Entanglement entropy (1932)

Factoring algorithm (1994)

von Neumann

Peter Shor