Title: Quantum Thermodynamics in the Gaussian Regime

Date: Feb 16, 2017 03:30 PM

URL: http://pirsa.org/17020097

Abstract: <p>The study of thermodynamics in the quantum regime has in recent years experienced somewhat of a renaissance in our community. This excitement is fueled both by the fundamental nature of the subject as well as the potential for heat machines designed with quantum advantages. Here, I will suggest the study of quantum thermodynamics restricted to a Gaussian regime, with two primary goals in mind. The first is to understand just how restrictive Gaussian states and operations are in a thermodynamic context; as such operations are often the most easily implemented in the laboratory it is pertinent to understand what limitations they impose on us. The second is to use the computational simplicity of Gaussian quantum mechanics to examine thermodynamics under less ideal but more realistic assumptions, such as with finite bath sizes and time scales. We will present two works in this seminar, one focusing on each of these motivations.</p>

Quantum Thermodynamics

in the

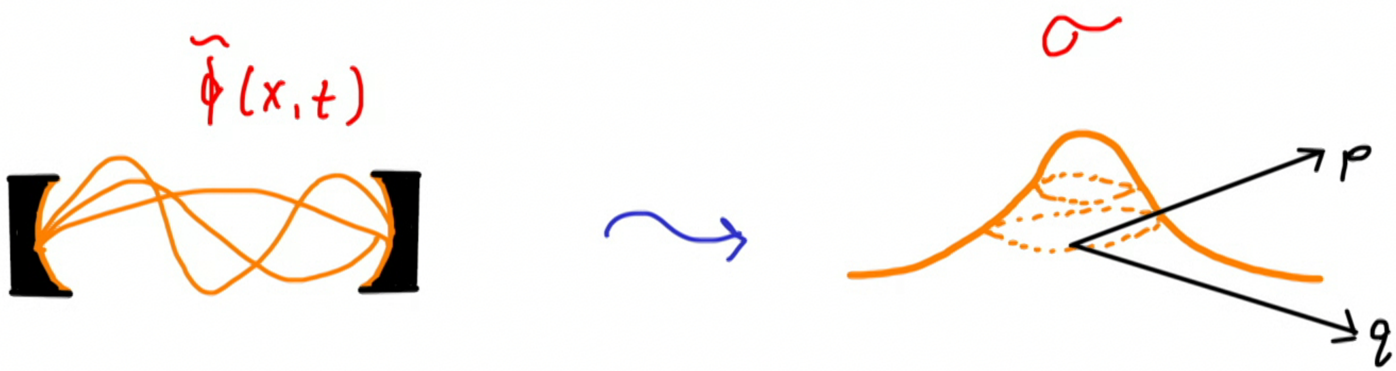
Gaussian Regime

Eric Brown

ICFO

My background:

Using Gaussian QM for field theory.



What does a Gaussian regime give us, thermodynamically?

i.e.

- How thermodynamically restrictive is a Gaussian regime?

- How can Gaussian QM help us analyse Q-thermo?

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Quantum Thermodynamics (in a nutshell)

- Equilibriation.
- Storage and extraction of work.
- Quantum thermal machines.
- Structure of thermal operations.
- Resource theories
- Fluctuation theorems
- Information (e.g. Landauer)

Gaussian Quantum Mechanics (in a nutshell)

→ Essentially GQM is normal QM, here as applied to a collection of continuous variable harmonic modes $(\hat{q}_i, \hat{p}_i)$, where we restrict all relevant Hamiltonians to be quadratic or less, and thermal states thereof.

→ And the resulting open systems formulation.

Gaussian Quantum Mechanics (in a nutshell)

Hilbert space $\longleftrightarrow$ Phase space (dim = 2N)

$\hat{\rho}$ $\longleftrightarrow$ σ (Covariance matrix)

Unitary $\hat{U}$ $\longleftrightarrow$ Symplectic S ($\sigma \to S \sigma S^T$)

Why Gaussian?

This is pretty restrictive, so why use it?

- In optical laboratory setups, linear transformations (Gaussian) tend to be the easiest to implement.

- The math is easy!

  ($\oplus$ rather than $\otimes$)

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Gaussian Passivity

- Work with Nicolai Friis and Marcus Huber.

NJP 18, 2016 (1608.04977)

Passivity?

Defn: A quantum state ρ is <u>passive</u> if there does not exist any unitary that will, on average, lower the energy of the state.

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i.e. If $\exists\ U$ s.t. $\tilde{E} = Tr(HU\rho U^{\dagger}) < Tr(H\rho) = E$,

then ρ is not passive.

Thm: The state ρ is passive iff it is diagonal in the energy basis, $\rho_{pass} = \sum_n p_n |n\rangle\langle n|$, such that $p_n \leq p_m$ for $E_n \geq E_m$.

Pusz and Woronowicz, Commun. Math. Phys. 58, 273 (1978)

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e.g. a thermal state

$$\tau(\beta) = \frac{1}{Z} \exp(-\beta H) = (1 - e^{-\beta\omega}) \sum_n e^{-n\beta\omega} |n\rangle\langle n|$$

Oscillator of freq ω

In fact, thermal states are the unique set of Completely passive states.

But, for example, the product of two thermal states of different temperatures, $\tau(\beta_1) \otimes \tau(\beta_1)$, is <u>not</u> passive.

↳ Good news everyone!

Thm: The state ρ is passive iff it is diagonal in the energy basis, $\rho_{pass} = \sum_n p_n |n\rangle\langle n|$, such that $p_n \leq p_m$ for $E_n \geq E_m$.

But, this theorem for passivity assumes that we may utilize <u>any</u> unitary $U \in \mathcal{L}(\mathcal{H})$.

$\rightsquigarrow$ Not always the easiest in real life!

Thm: The state ρ is passive iff it is diagonal in the energy basis, $\rho_{pass} = \sum_n p_n |n\rangle\langle n|$, such that $p_n \le p_m$ for $E_n \ge E_m$.

$\rightsquigarrow$ How does the theorem change upon restricting the operations we can perform?

Gaussian?

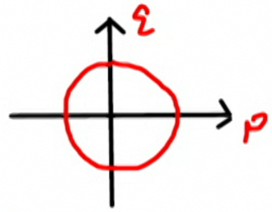
Associated with ρ is its covariance matrix σ:

$$\boxed{\sigma_{ij} = \langle \hat{x}_i \hat{x}_j + \hat{x}_j \hat{x}_i \rangle},$$

where $\hat{\vec{x}}^{T} = (\hat{q}_1, \hat{p}_1, \hat{q}_2, \hat{p}_2, \dots, \hat{p}_n)$.

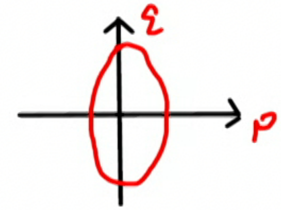
e.g. Single mode

$$\sigma = \begin{bmatrix} \nu & 0 \\ 0 & \nu \end{bmatrix}:$$

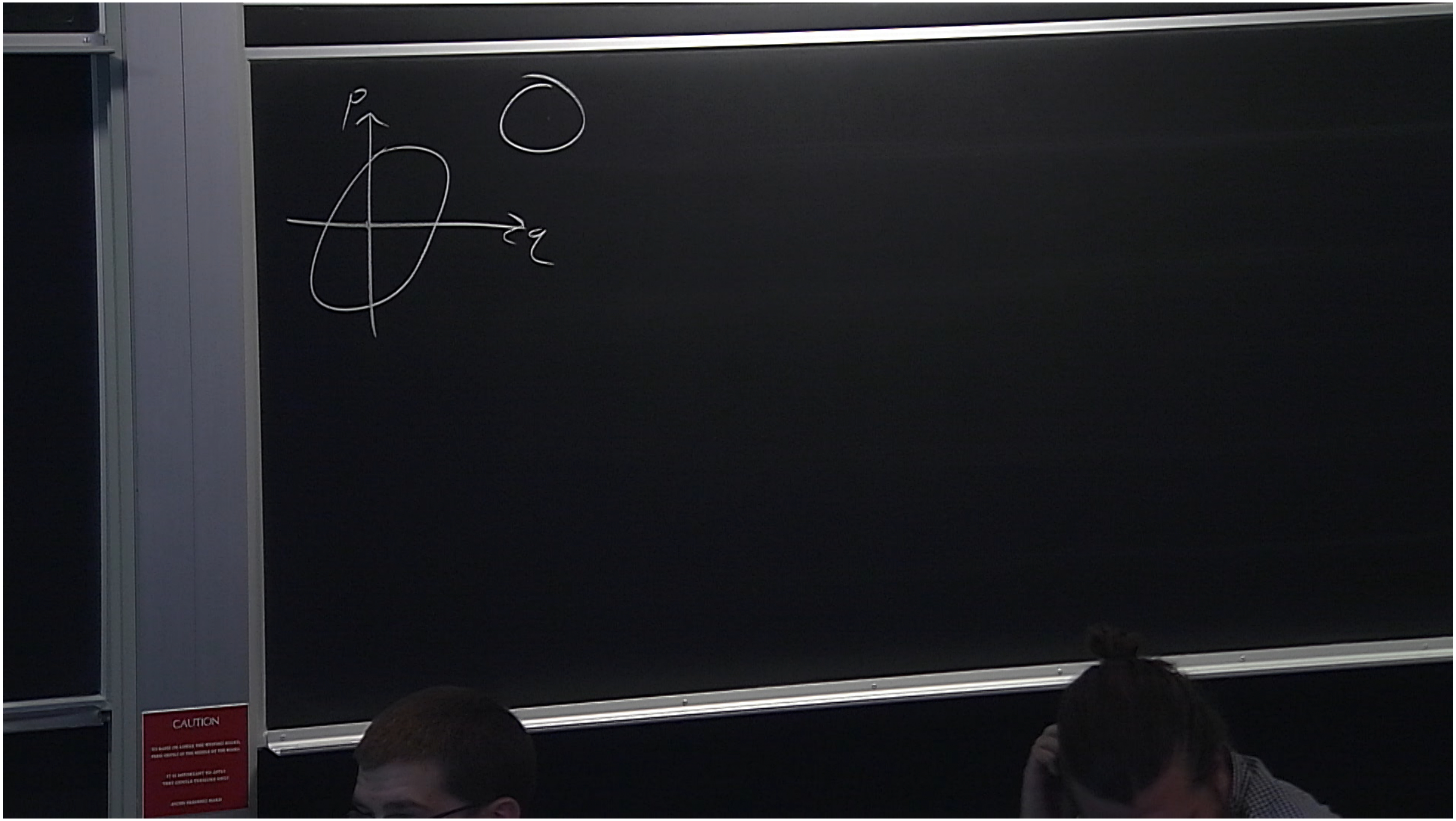


Thermal

$$\sigma = \begin{bmatrix} \nu e^{r} & 0 \\ 0 & \nu e^{-r} \end{bmatrix}:$$



Squeezed thermal


p
q
CAUTION

Associated with ρ is its covariance matrix σ:

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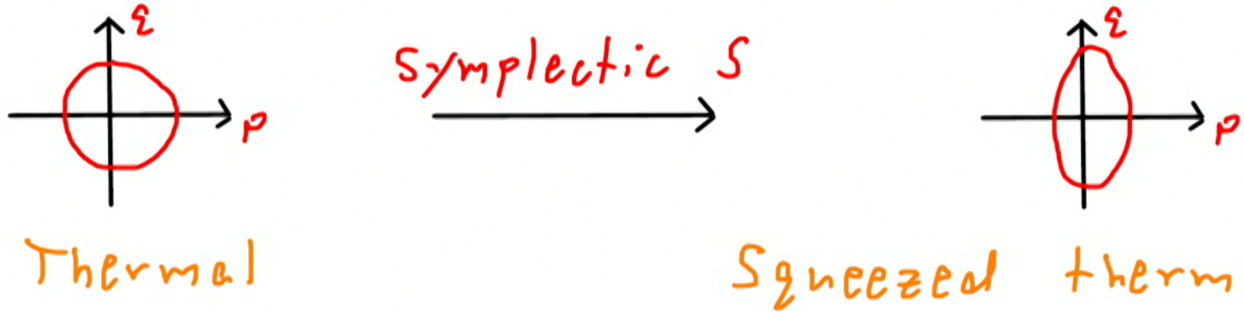
<u>e.g.</u> <u>Single mode</u>



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$$\boxed{\sigma_{ij} = \langle \hat{x}_i \hat{x}_j + \hat{x}_j \hat{x}_i \rangle}.$$

$\rightsquigarrow$ Since H is quadratic, energy depends only on σ and $\langle \vec{x} \rangle$, whether or not ρ is Gaussian.

Let us consider the case where we may only use Gaussian unitaries; from which quantum states of oscillators may we extract energy?

Defn: Assuming a quadratic Hamiltonian H, the state ρ is Gaussian-Passive if it is passive wrt Gaussian unitaries.

Note: ρ need not be Gaussian!

Associated with ρ is its covariance matrix σ:

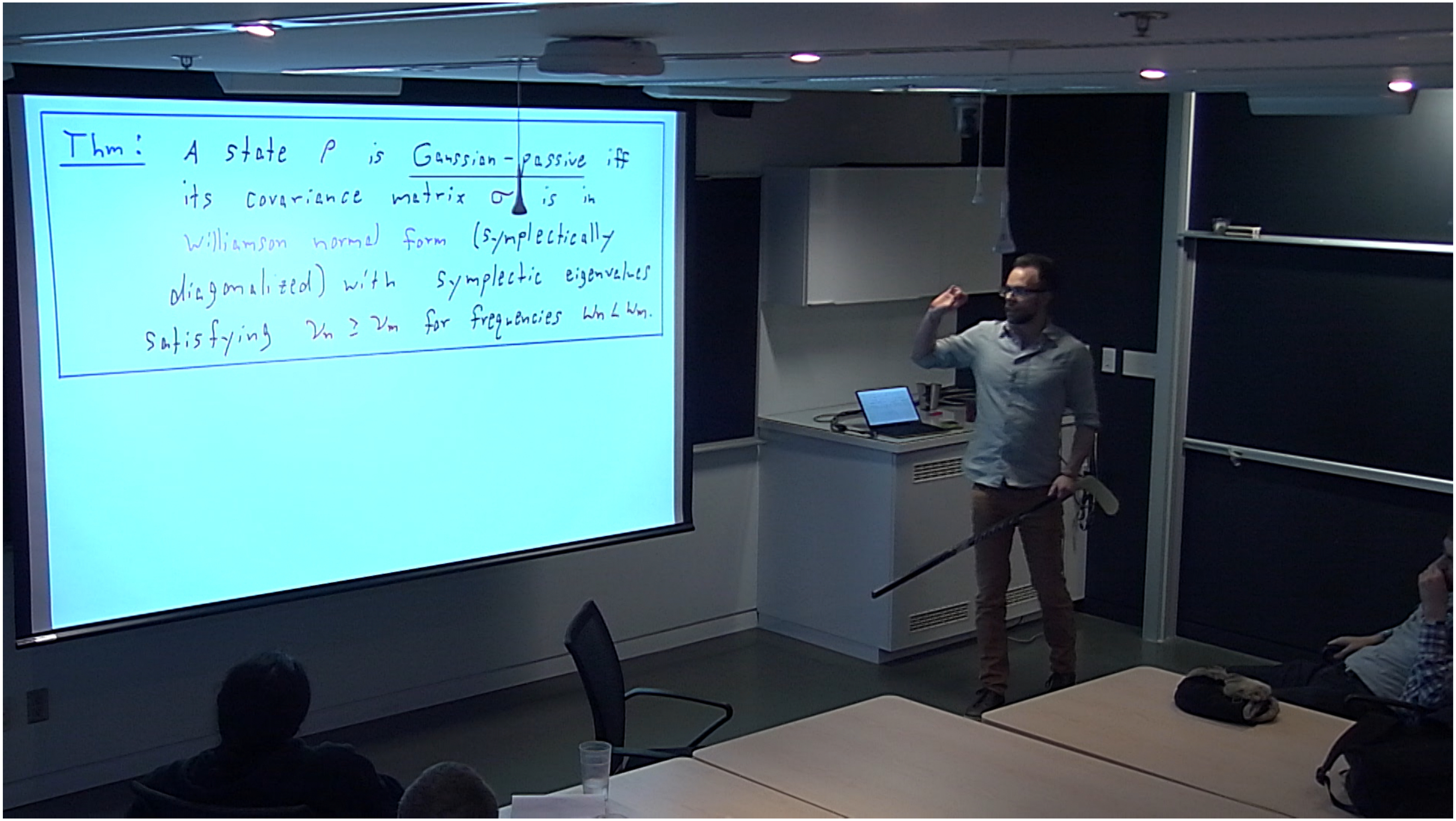
$$\boxed{\sigma_{ij} = \langle \hat{x}_i \hat{x}_j + \hat{x}_j \hat{x}_i \rangle} .$$

→ Since H is quadratic, energy depends only on σ and $\langle \vec{x} \rangle$, whether or not ρ is Gaussian.

→ If $\exists$ symplectic matrix S s.t. $\tilde{E} = E(S\sigma S^T) < E(\sigma) = E$, then ρ is <u>not</u> Gaussian-Passive.

So,

what does a Gaussian-Passive state look like?


Thm: A state ρ is Gaussian-passive iff
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Williamson normal form (symplectically
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satisfying νn ≥ νm for frequencies ωn < ωm.

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WNF: $\forall\ \sigma\ \exists\ S$ s.t.

$$\sigma \rightarrow \hat{\sigma} = S\sigma S^T = \bigoplus_{n=1}^{\sim} \begin{pmatrix} \nu_n & 0 \\ 0 & \nu_n \end{pmatrix}$$

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Product of thermal states

<u>symplectic eigenvalues</u>

- ν_j quantifies mixedness of mode j.
- For thermal state, $\nu_j = 1 + 2\langle n_j \rangle_{th}$.

<u>i.e.</u> ρ is Gaussian-passive iff it is (at level of 2nd moments) a product of thermal states in which $\langle n_j \rangle_{th}$ decreases monotonically with mode frequencies ω_j.

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$$\sigma \rightarrow \hat{\sigma} = S \sigma S^T = \bigoplus_{n=1}^{\sim} \begin{pmatrix} \nu_n & 0 \\ 0 & \nu_n \end{pmatrix}$$

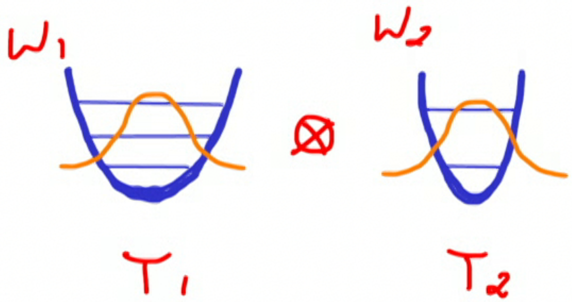
Compare with condition for general passivity:

$$\rho_{pass} = \sum_n p_n |n\rangle\langle n|$$

with $p_n < p_m$ for $E_n > E_m$.

__i.e.__ ρ is Gaussian-passive iff it is (at level of 2nd moments) a product of thermal states in which $\langle n_j \rangle_{th}$ decreases monotonically with mode frequencies ω_j.

__e.g.__ __Two modes__



Unrestricted, we can always extract some work if $T_1 \neq T_2$.

$\rightsquigarrow$ Restricted to Gaussian, we only can if $\boxed{\dfrac{T_1}{T_2} < \dfrac{\omega_1}{\omega_2}}$.

Final note: It can be shown that the energy gap between passivity and Gaussian passivity is maximal.

Details of this, and the proof of main theorem, are in

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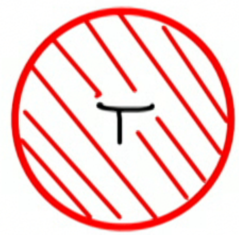
A Quantum Harmonic Heat Engine

Work with Alejandro Pozas and Karen Hovhannisyan.

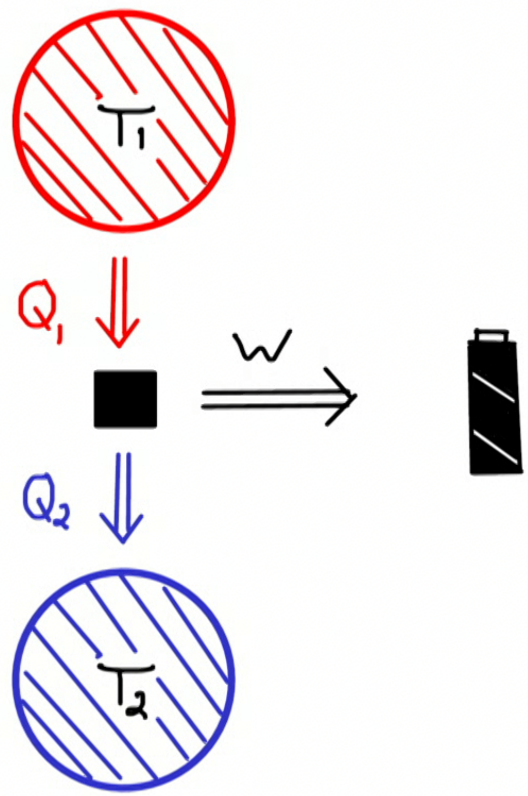
In progress!

You can't get work out of a thermal bath.



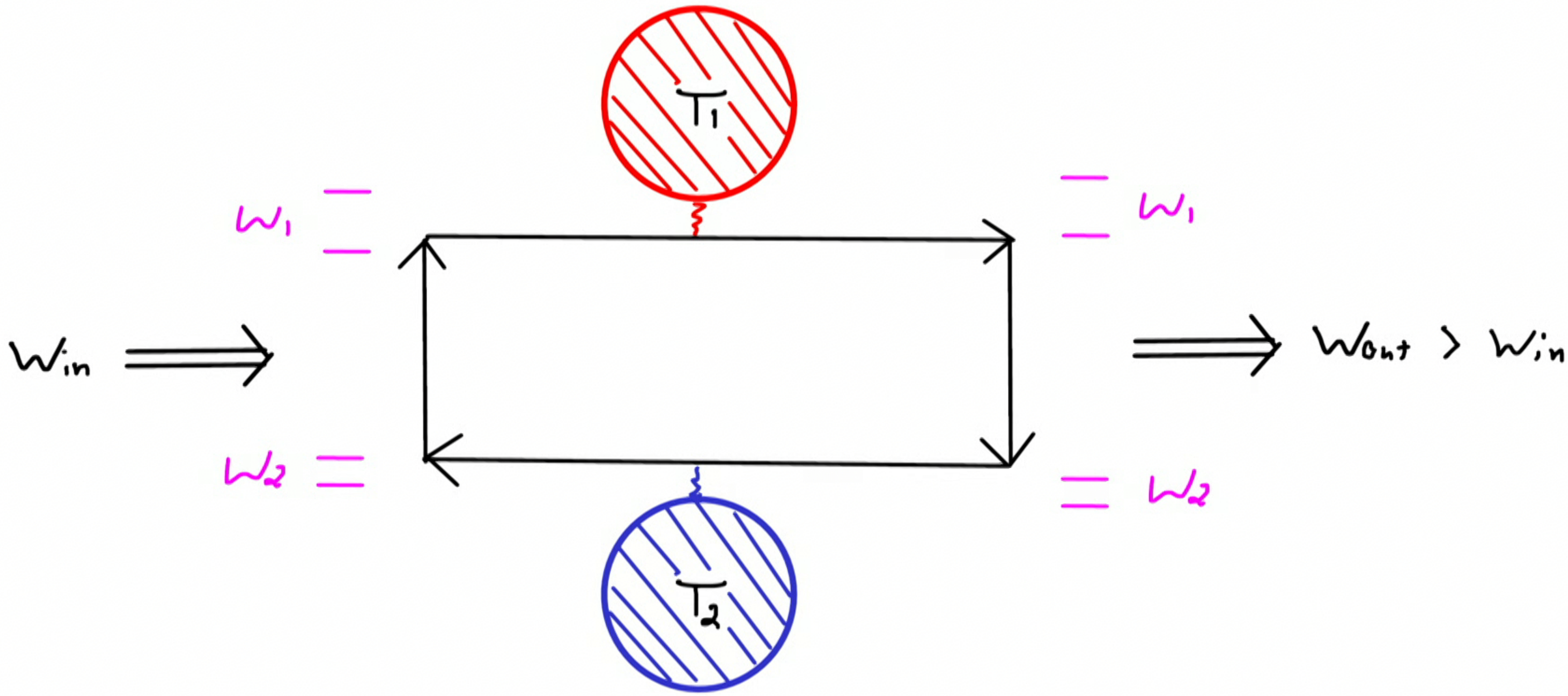
Precious energy

Basic idea behind all heat engines:



The Quantum Otto Cycle

Take a qubit (or oscillator) as our working body.



This works great in idealized scenarios.

i.e. Infinite heat baths, long thermalization times, smooth interaction switching...

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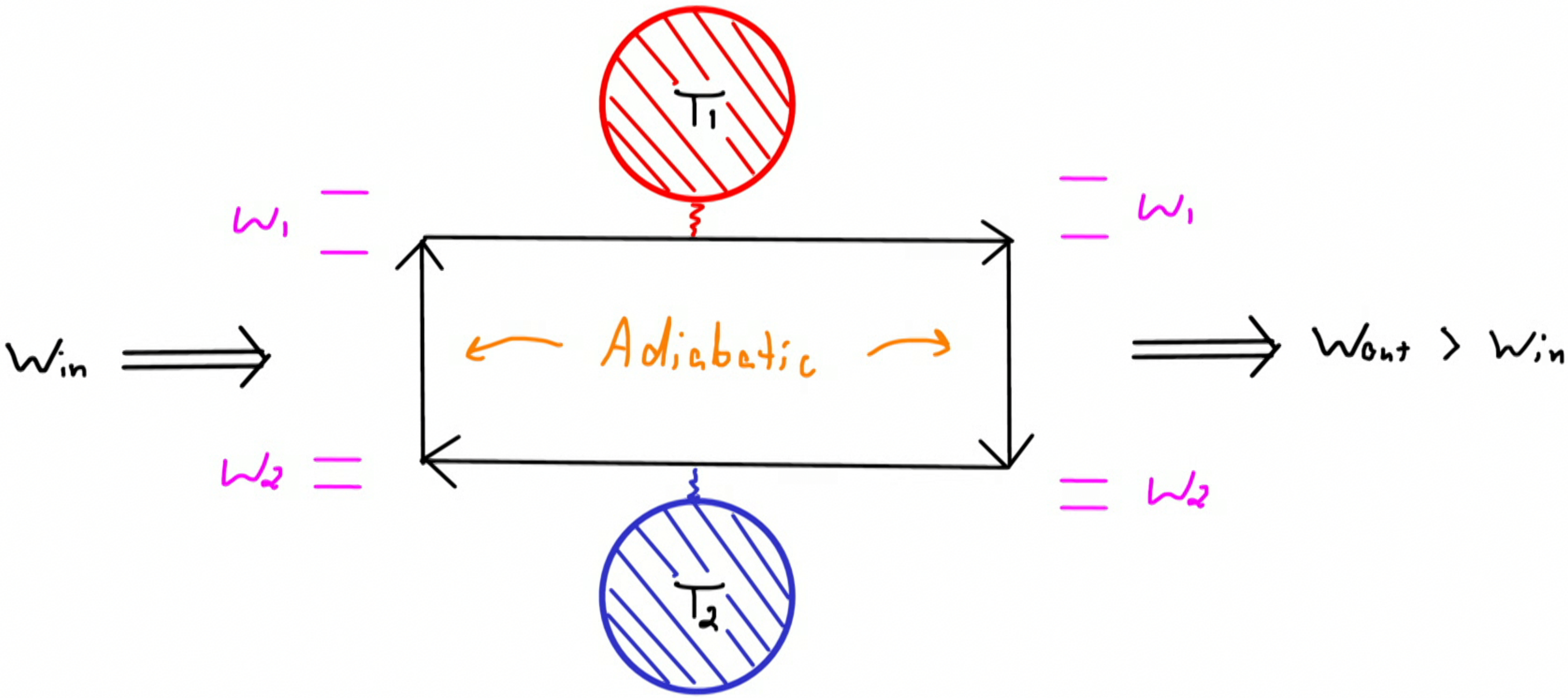

This works great in idealized scenarios.

i.e. Infinite heat baths, long thermalization times, smooth interaction switching...

~> Let's look at less ideal cases, and use Gaussian QM to help solve the dynamics.

~> We take both baths to consist of oscillators, as well as working body.

→ How is the efficiency, lifetime, and total energy output affected by the following?

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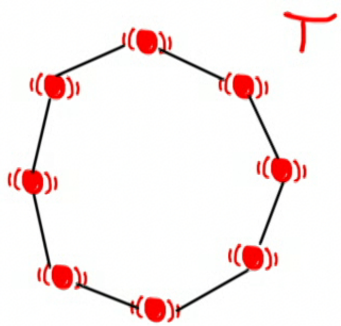
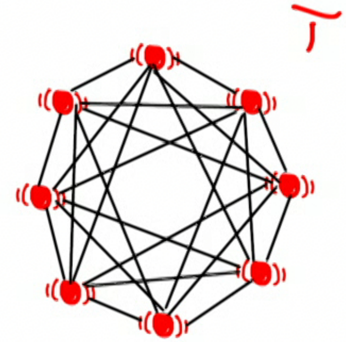
- Size and structure of baths

e.g.



or



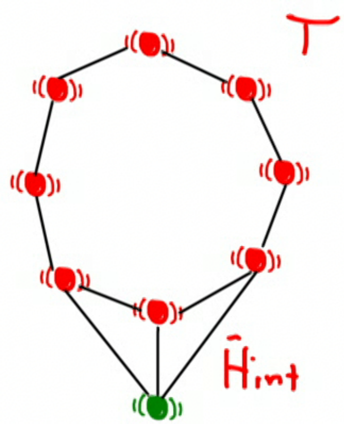
?

N oscillators

→ How is the efficiency, lifetime, and total energy output affected by the following?

- Type, extent, and timescale of interaction

e.g.



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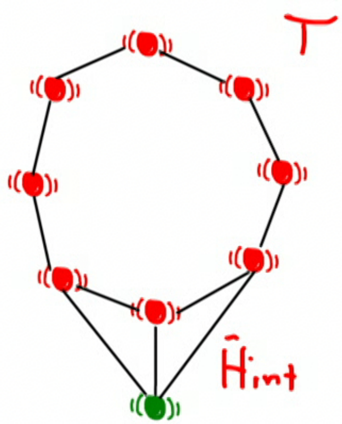
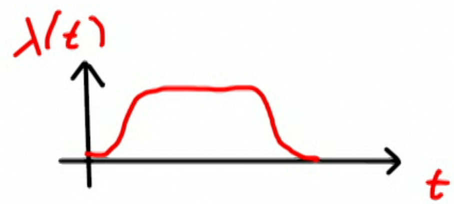
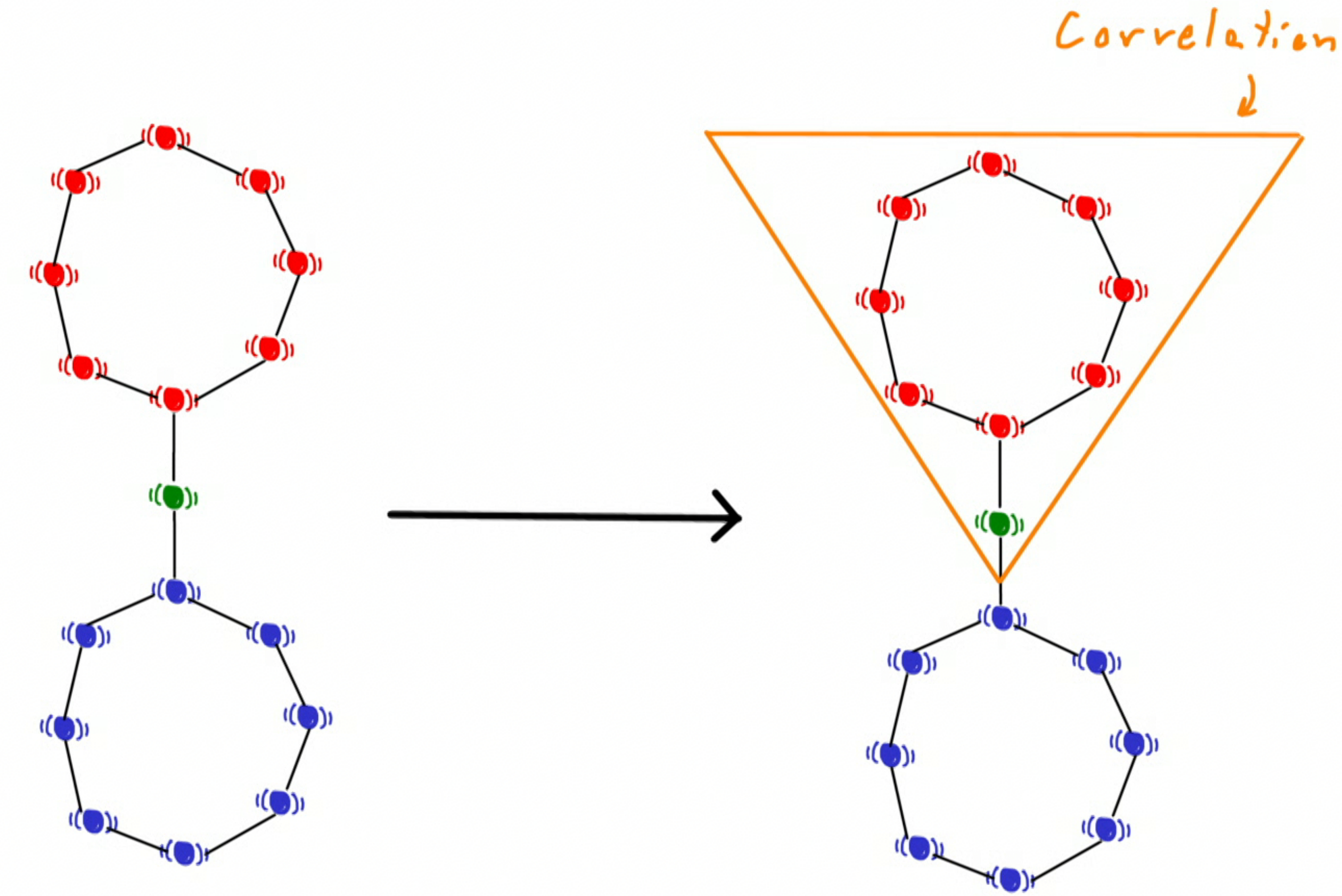


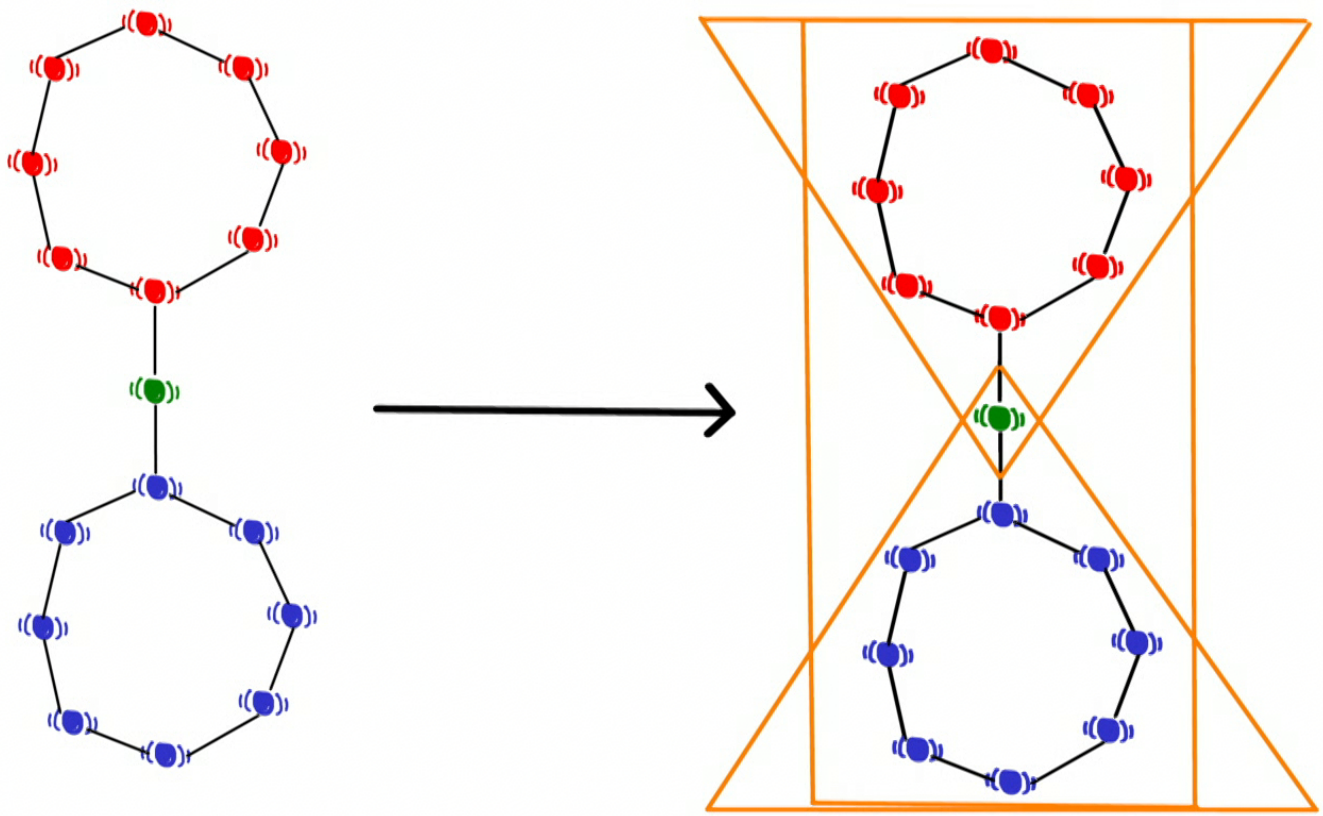
$$\hat{H}_{int} = \lambda(t)\, \hat{q} \otimes \sum_j \hat{Q}_j$$

↖ some subset of bath oscillators.



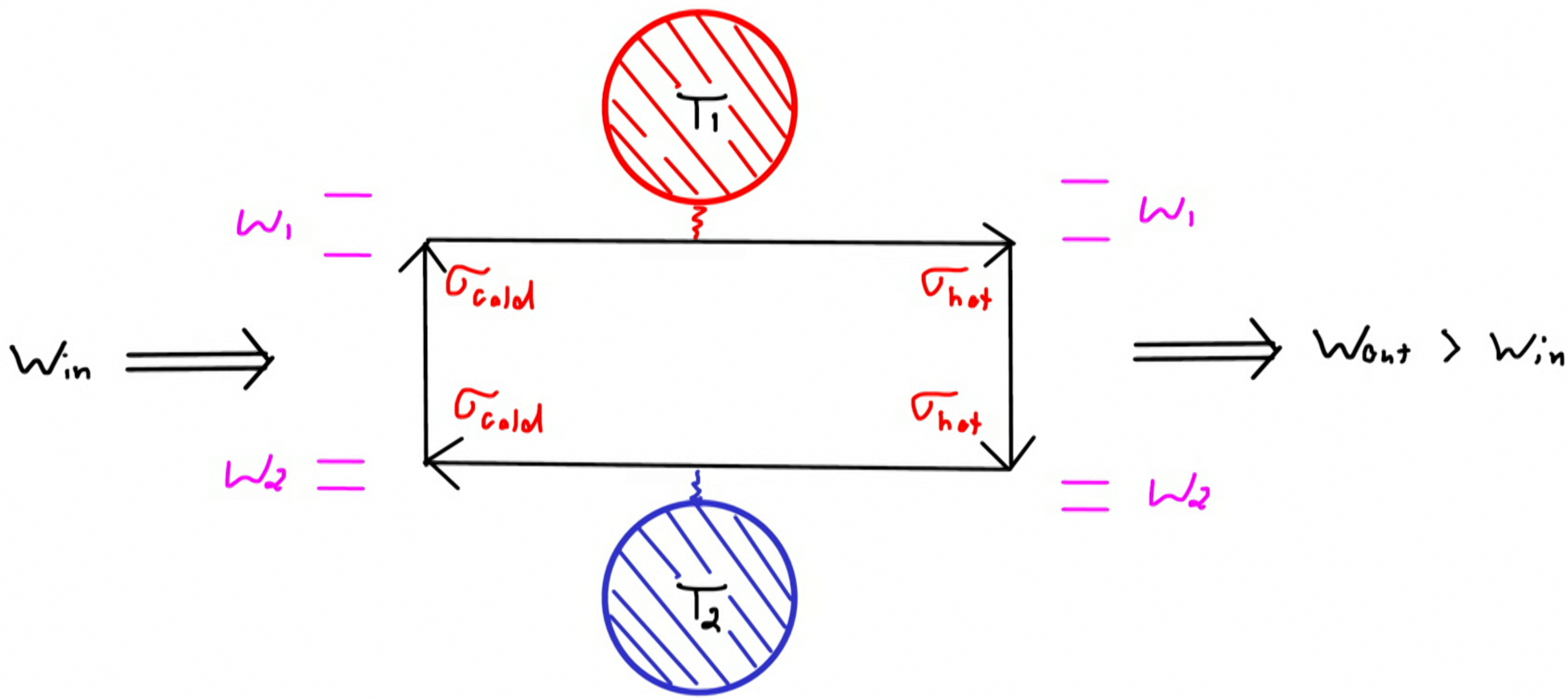
← A single thermalization stroke.


Correlation



Wait for another day I'm afraid

Here we assume adiabatic strokes are ideal.



Gaussian Dynamics (in a nutshell)

By using a quadratic $\hat{H}$, we ensure the dynamics are Gaussian.

Let $\hat{H}(t) = \hat{\vec{X}}^{T} F(t)\, \hat{\vec{X}}$, where $\hat{\vec{X}}^{T} = (\hat{q}_1, \hat{p}_1, \hat{q}_2, \hat{p}_2, \dots, \hat{p}_n)$

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$$\hat{\rho}(t) = \hat{U}(t)\,\hat{\rho}_0\,\hat{U}(t)^\dagger \longleftrightarrow \sigma(t) = S(t)\,\sigma_0\,S(t)^T$$

$$i\frac{d\hat{U}(t)}{dt} = \hat{H}(t)\hat{U}(t) \longleftrightarrow \frac{dS(t)}{dt} = \Omega F(t) S(t)$$

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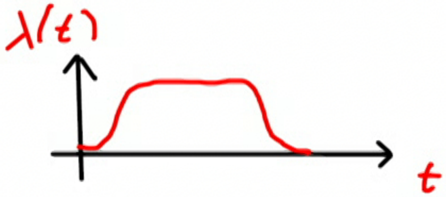
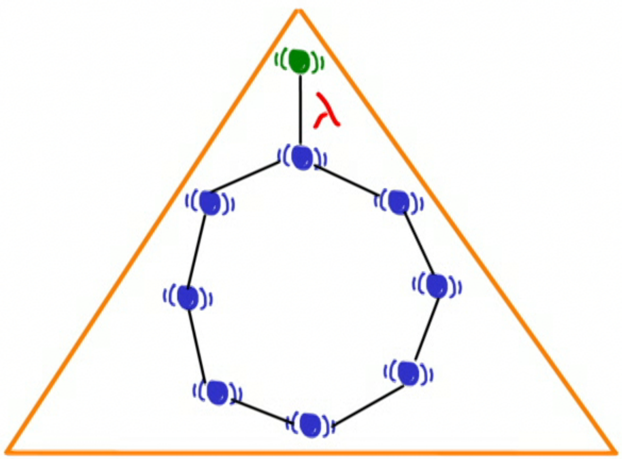
$(N\cdot\infty)\times(N\cdot\infty)$ $\qquad$ $2N\times 2N$

During a thermalization stroke:



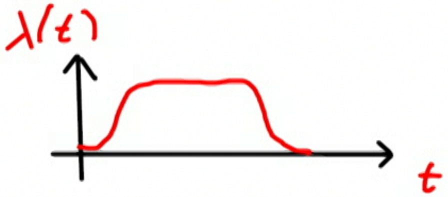
$$\sigma = \begin{bmatrix} \sigma_S & \gamma \\ \gamma^T & \sigma_{cold} \end{bmatrix}$$



$$\longrightarrow \quad S \sigma S^T = \begin{bmatrix} \sigma_S' & \gamma' \\ \gamma'^T & \sigma_{cold}' \end{bmatrix}$$

direct sum structure

During a thermalization stroke:



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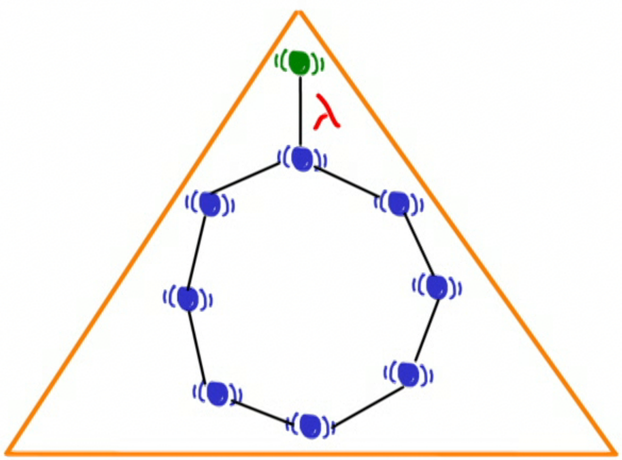

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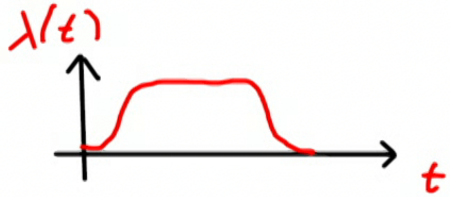

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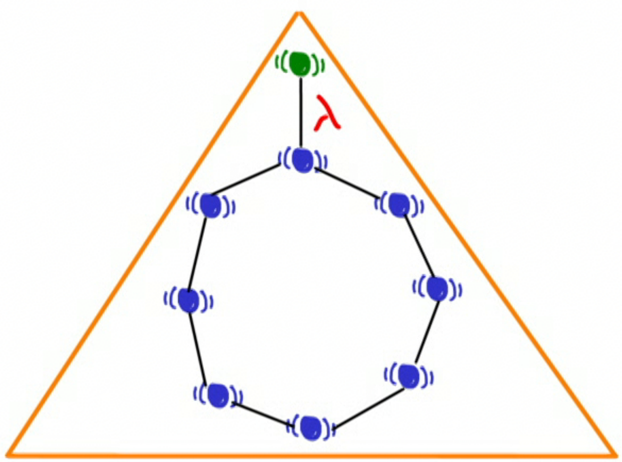
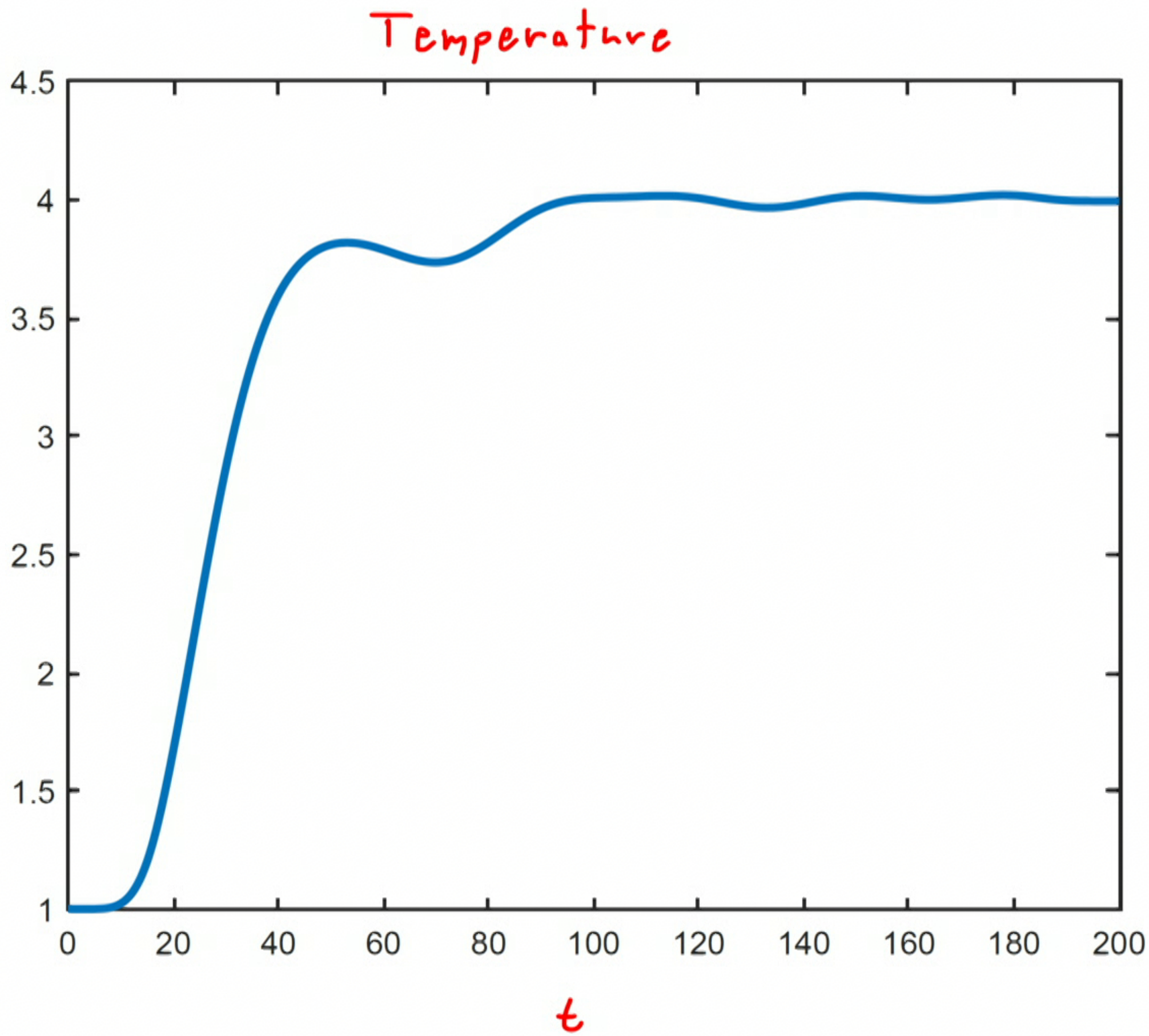


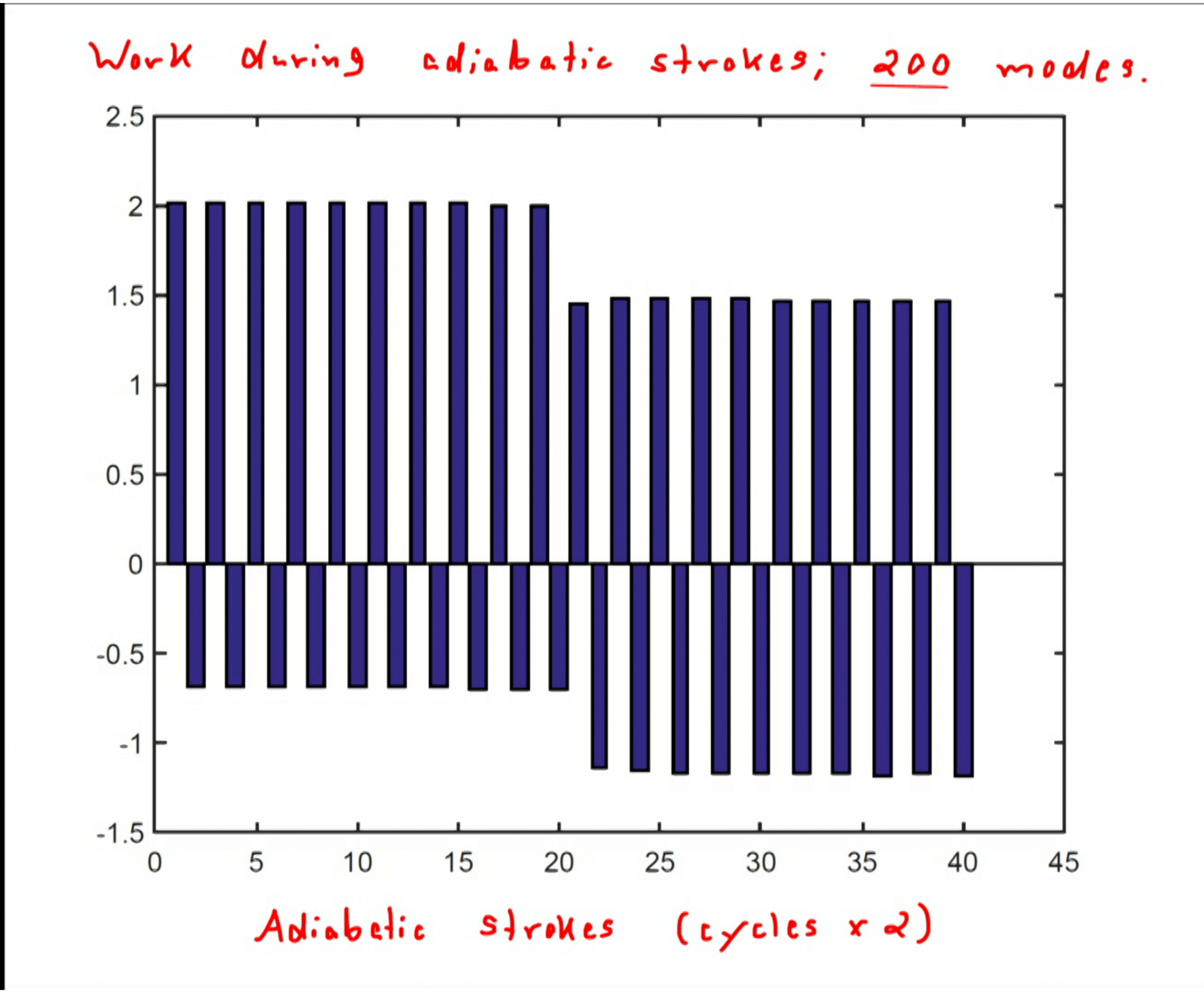
$$\longrightarrow S \sigma S^T = \begin{bmatrix} \sigma_s' & \gamma' \\ \gamma'^T & \sigma_{cold}' \end{bmatrix}$$

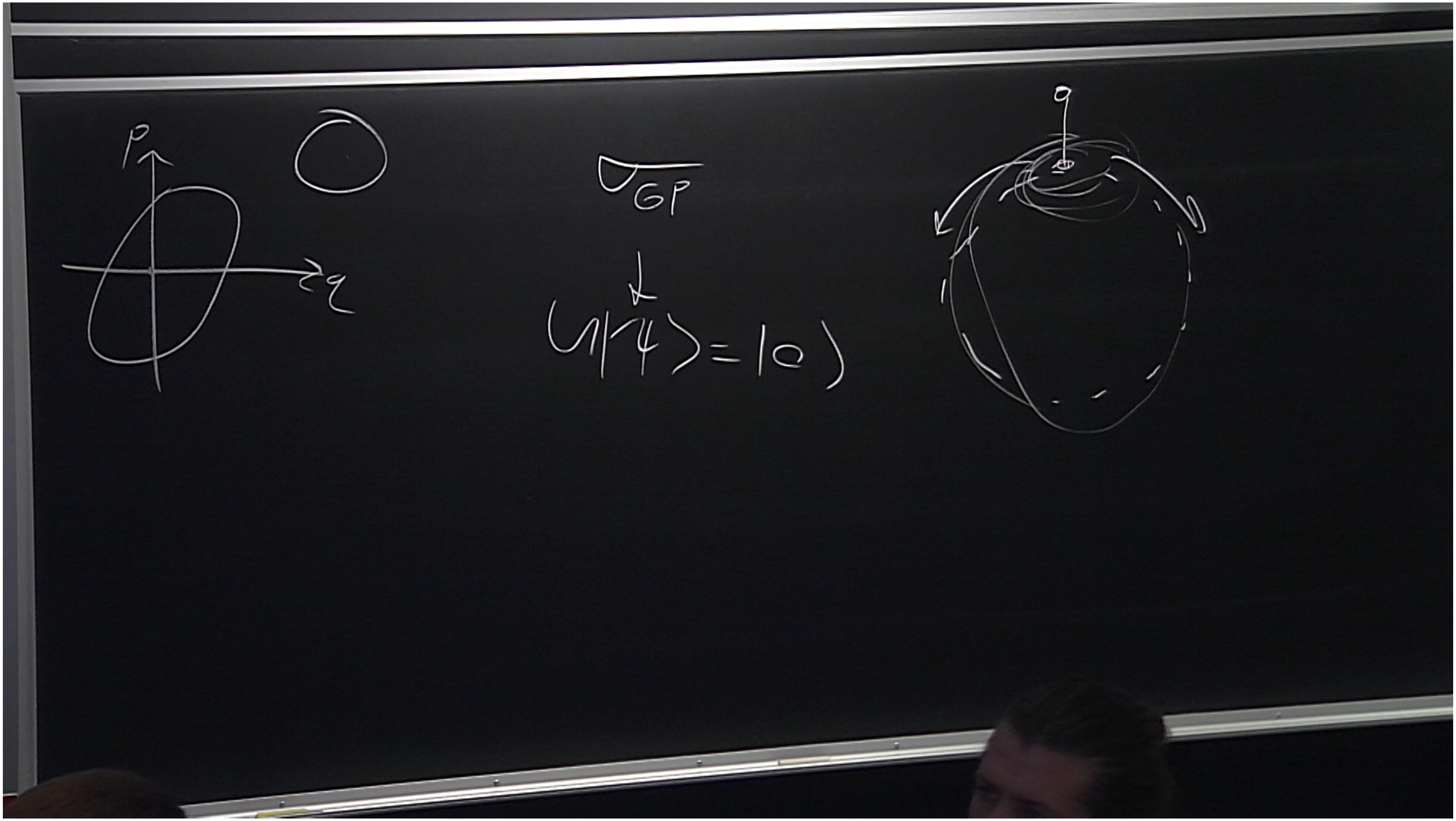
$\downarrow$

direct sum structure

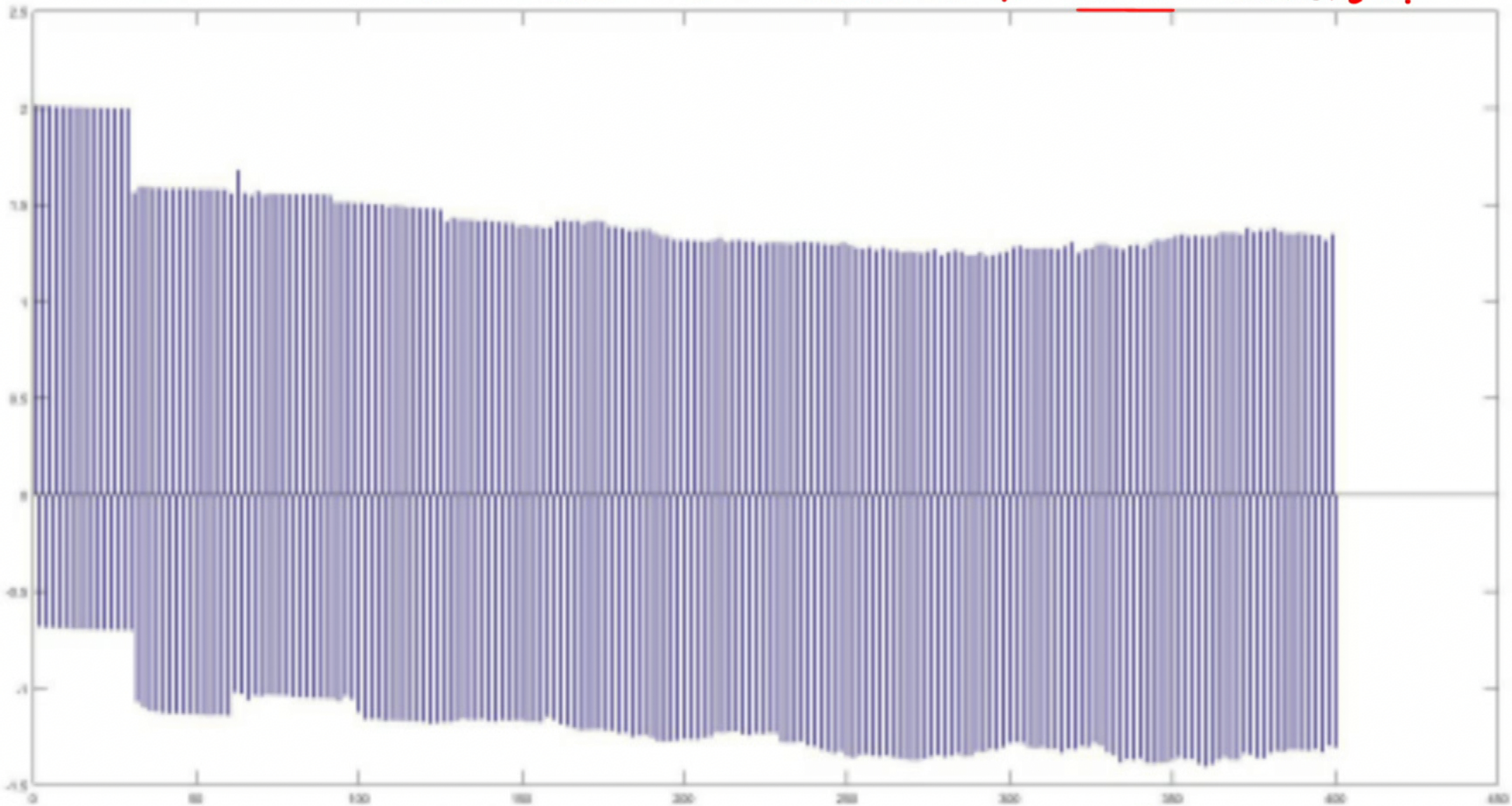
Plots!


Temperature
4.5
4
3.5
3
2.5
2
1.5
1
0
20
40
60
80
100
120
140
160
180
200
t

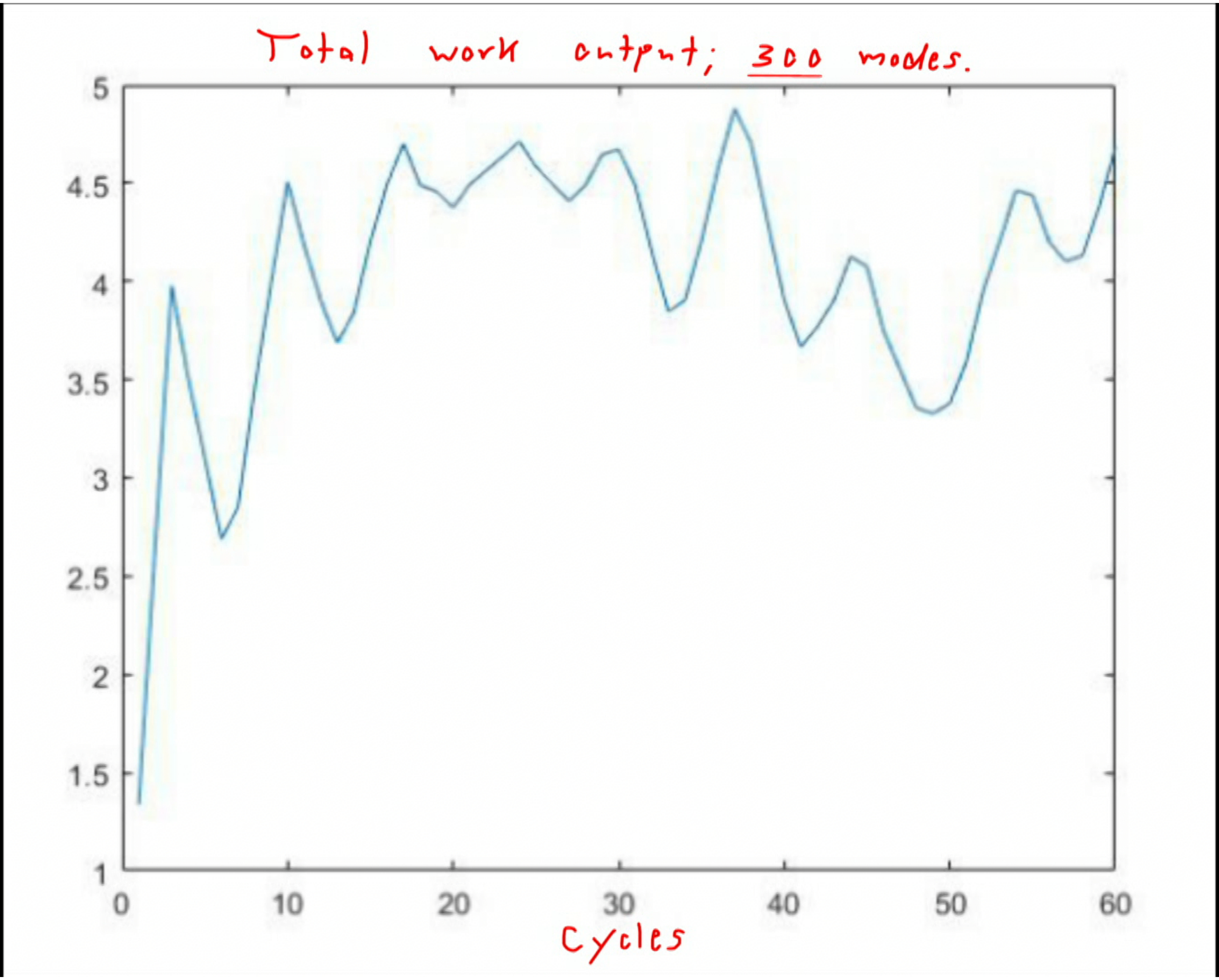

Work during adiabatic strokes; 200 modes.
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25
30
35
40
45
Adiabatic strokes (cycles x 2)


p
q
U|ψ⟩=|0⟩

Work during adiabatic strokes; 300 modes.



Adiabatic strokes (cycles x 2)


Total work output; 300 modes.
5
4.5
4
3.5
3
2.5
2
1.5
1
0
10
20
30
40
50
60
Cycles

What does a Gaussian regime give us, thermodynamically?

i.e.

- How thermodynamically restrictive is a Gaussian regime?

- How can Gaussian QM help us analyse Q-thermo?

I find this stuff cool; I hope I've managed to share that with you.

Please stay tuned for further results on the Otto engine!

Thank you again for this invitation,

and to all of you for listening! :)