Title: Superconductivity near a quantum-critical point --- the special role of the first Matsubara frequency

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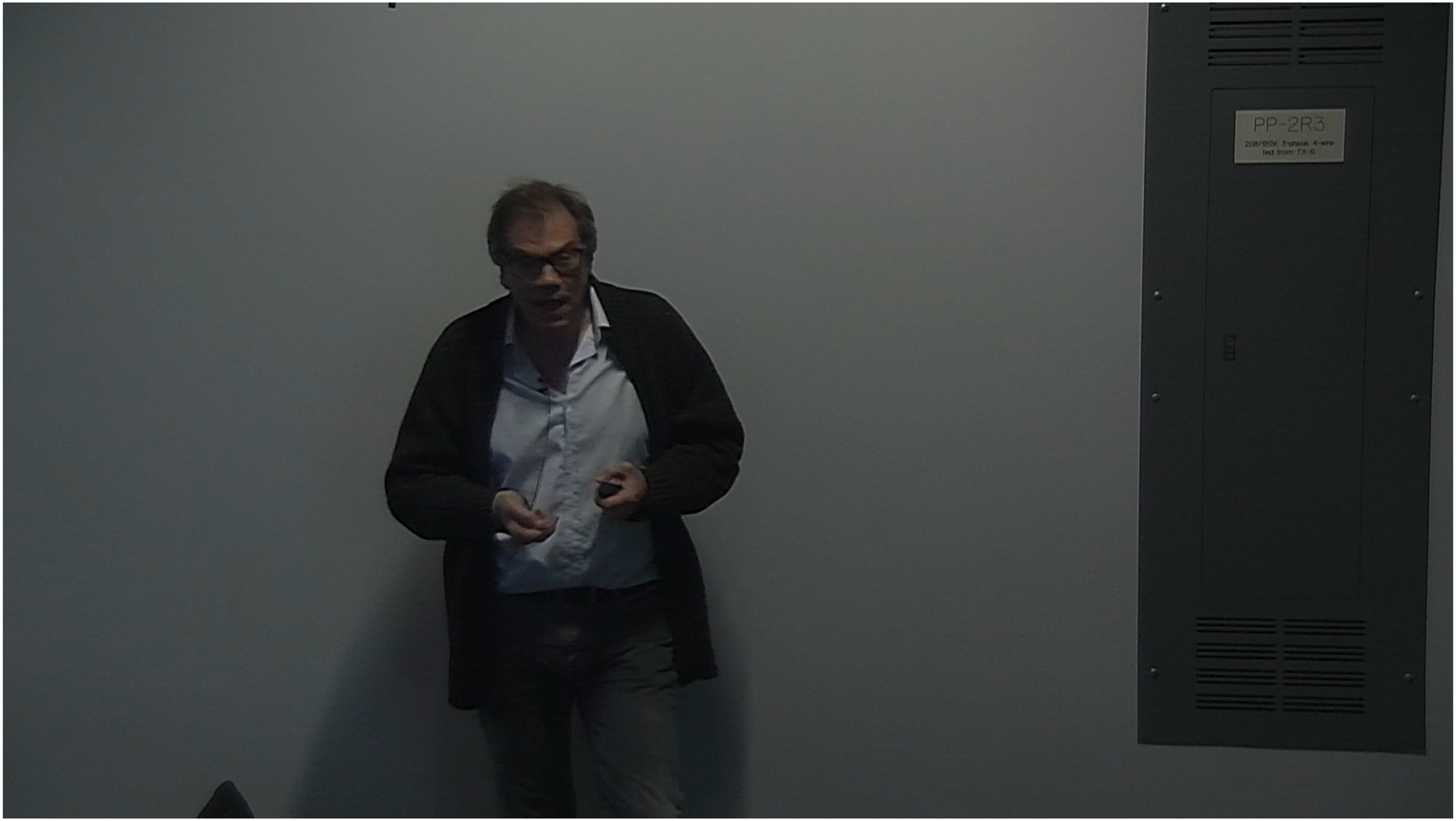
Abstract: <p>Near a quantum-critical point in a metal strong fermion-fermion interaction mediated by a soft collective boson gives rise to incoherent, non-Fermi liquid behavior. It also often gives rise to superconductivity which masks the non-Fermi liquid behavior. We analyze the interplay between the tendency to pairing and fermionic incoherence for a set of quantum-critical models with effective dynamical interaction between low-energy fermions. We argue that superconducting Tc is non-zero even for strong incoherence and/or weak interaction due to the fact that the self-energy from dynamic critical fluctuations vanishes for the two lowest fermionic Matsubara frequencies $\omega_m = \pm \pi T$. We obtain the analytic formula for T_c which reproduces well earlier numerical results, including the ones for the electron-phonon model at vanishing Debye frequency.</p>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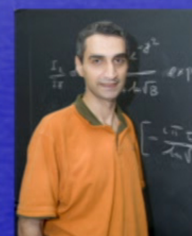
Superconductivity vs non-Fermi liquid at a Quantum Critical Point

Andrey Chubukov

University of Minnesota



Artem Abanov — Texas A&M

Yuxuan Wang — University of Illinois

Emil Yuzbashyan — Rutgers U.

B. Altshuler — Columbia U

NHMFL, Feb. 3, 2017

PRL 117, 157001 (2016)

The talk is about a clean, metallic system of itinerant fermions with short-range interactions

I. Normal state:

Fermi liquid paradigm: interactions change the behavior of fermions in a quantitative, but not qualitative way

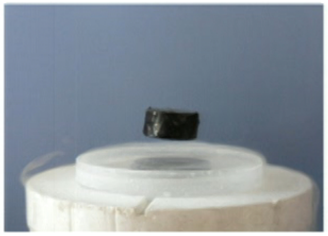
At T=0 fermions remain well defined quasiparticles

Non-Fermi liquid behavior: interactions change the system behavior qualitatively

At T=0, fermions loose coherence even at the lowest frequencies (fermionic Green's function has a branch cut, but no pole)

A clean system of itinerant fermions with a short-range interaction

II. Superconductivity:



If there exists an attractive interaction between fermions (mediated by either phonons or collective modes) the normal state at T=0 is unstable against pairing

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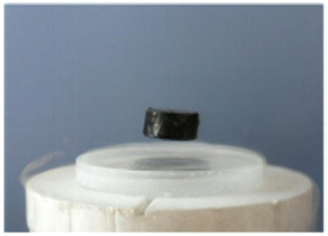

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In a Fermi liquid, superconductivity involves coherent fermions

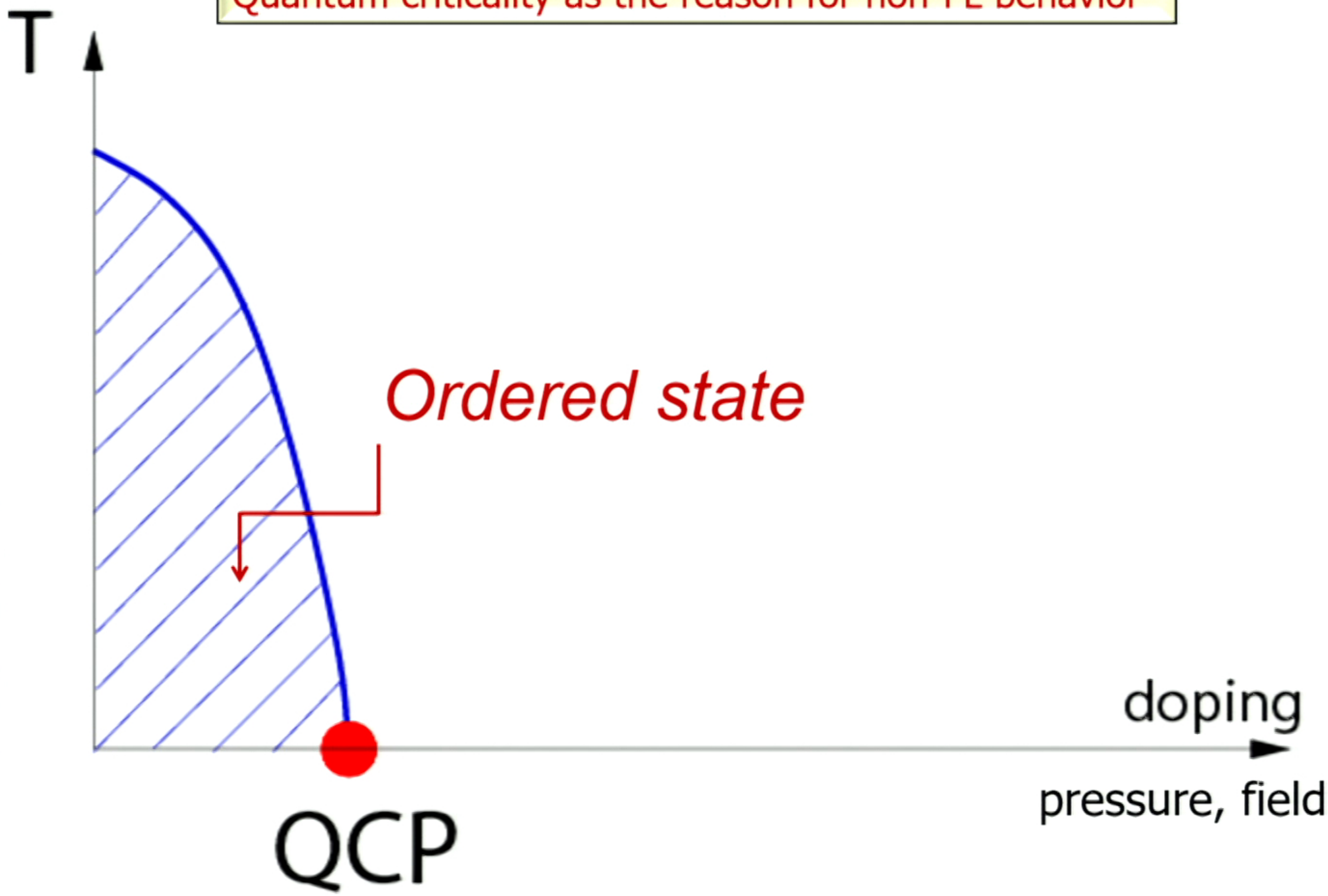
In a non-Fermi liquid, strong fermionic self-energy acts against pairing (it is more difficult to pair incoherent, diffusive fermions, than propagating ones)

This is the talk about the interplay between superconductivity and a non-FL behavior

I will consider the specific reason for non-Fermi liquid behavior – closeness to a metallic quantum-critical point, at which a Fermi liquid develops spin or charge density-wave order

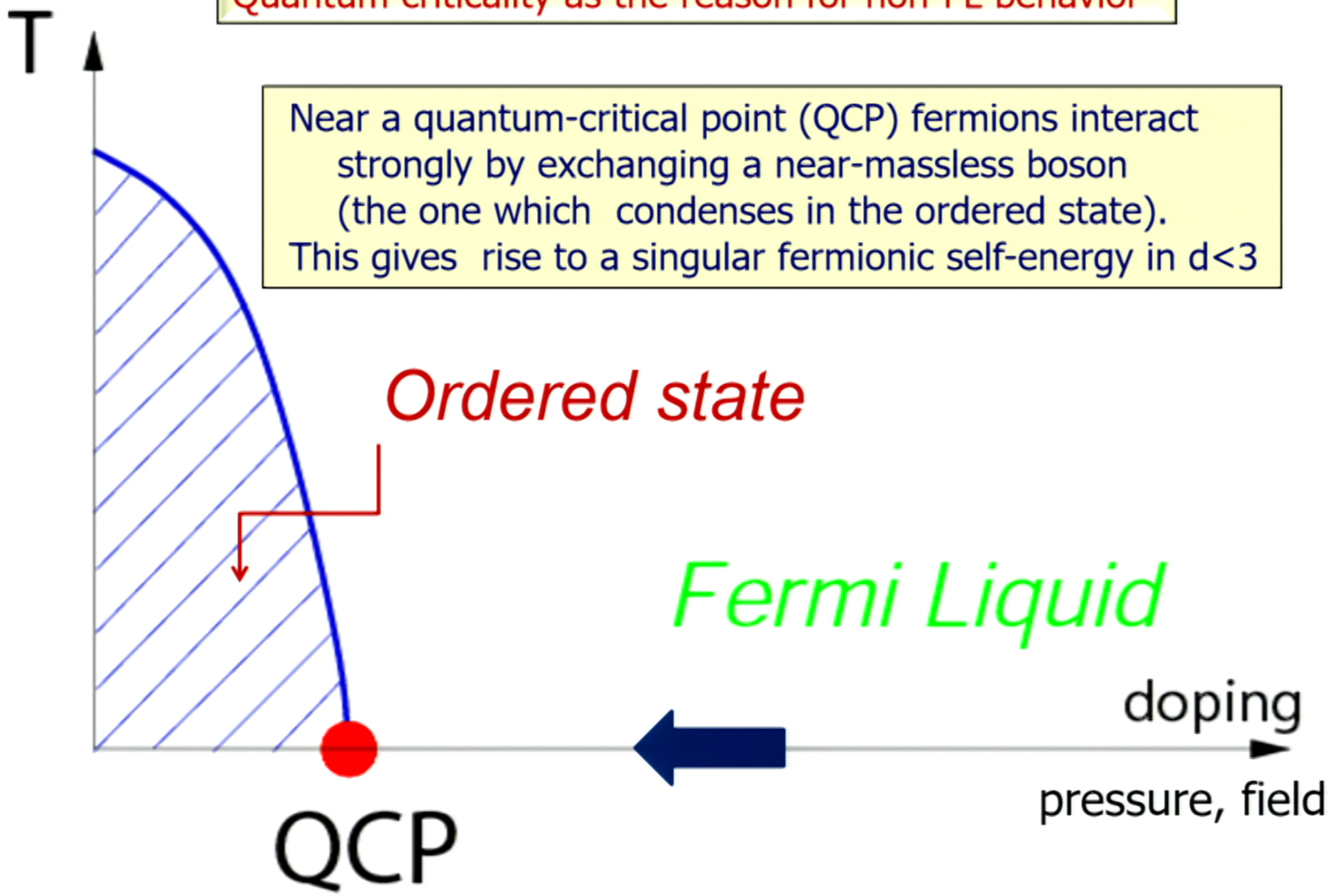
Mott physics is NOT a part of the story

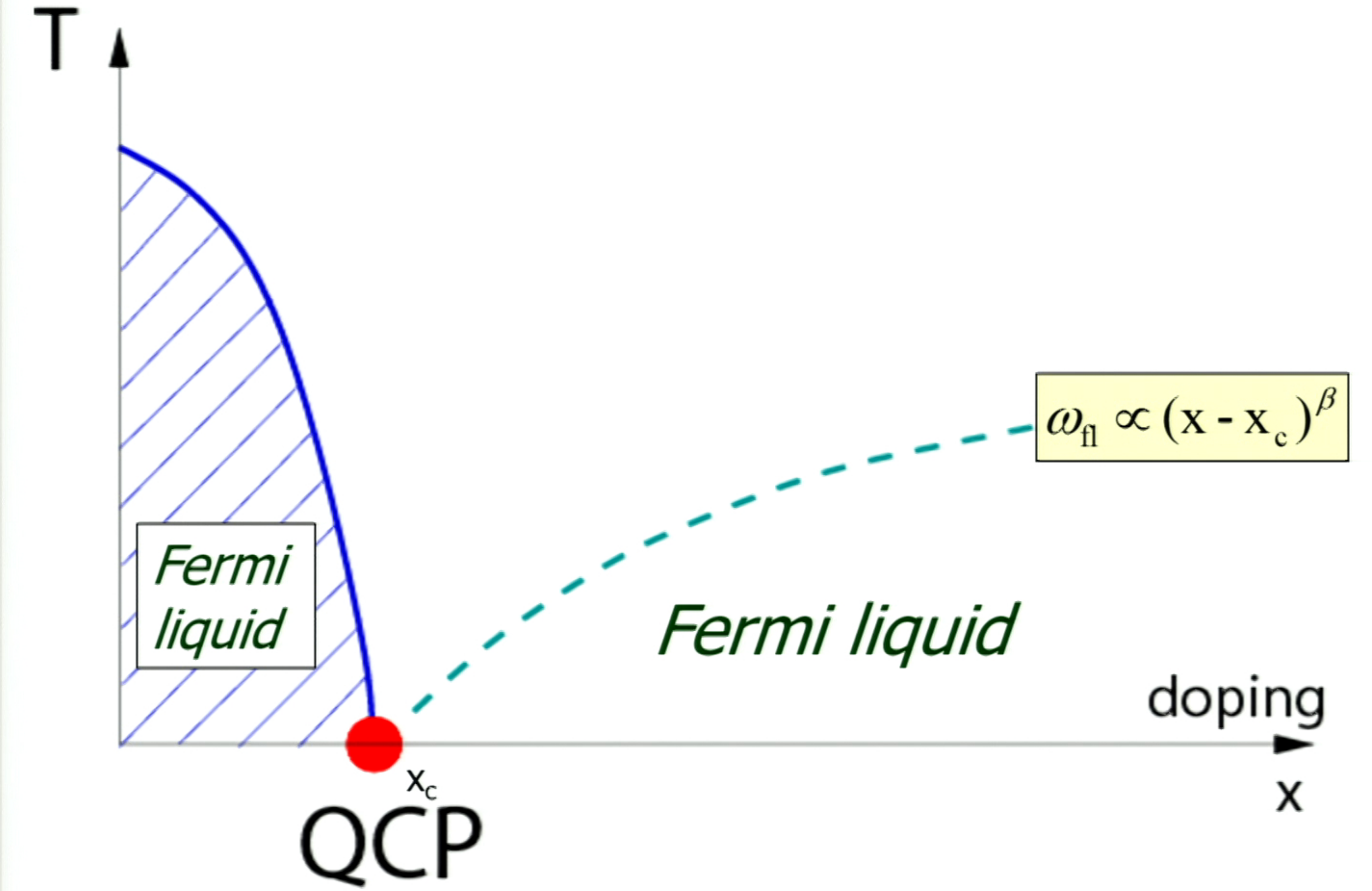
Quantum criticality as the reason for non-FL behavior

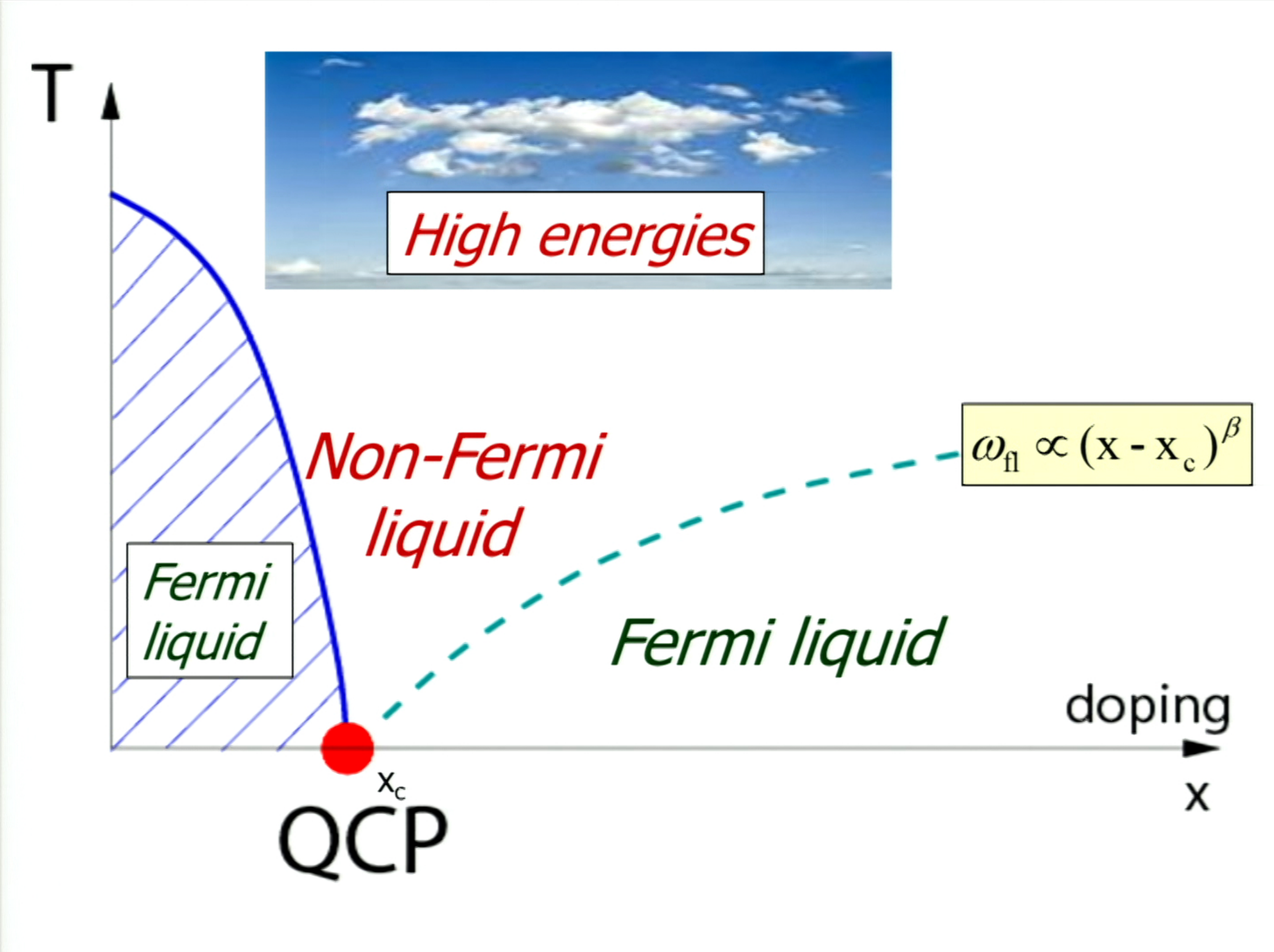


Quantum criticality as the reason for non-FL behavior

Near a quantum-critical point (QCP) fermions interact strongly by exchanging a near-massless boson (the one which condenses in the ordered state).
This gives rise to a singular fermionic self-energy in d<3




T
Fermi
liquid
$\omega_{fl} \propto (x - x_c)^{\beta}$
Fermi liquid
doping
x
x_c
QCP


T
High energies
Non-Fermi liquid
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Peculiarity of a metallic non-FL due to quantum criticality:

the same strong electron-electron interaction that gives rise to non-FL behavior, also gives rise to an attraction in one of the pairing channels

Electron-electron interaction is repulsive, but the system overcomes this by choosing a non-s-wave superconducting channel (d-wave for AFM case, p-wave for FM case, s,p,d… for nematic case…)

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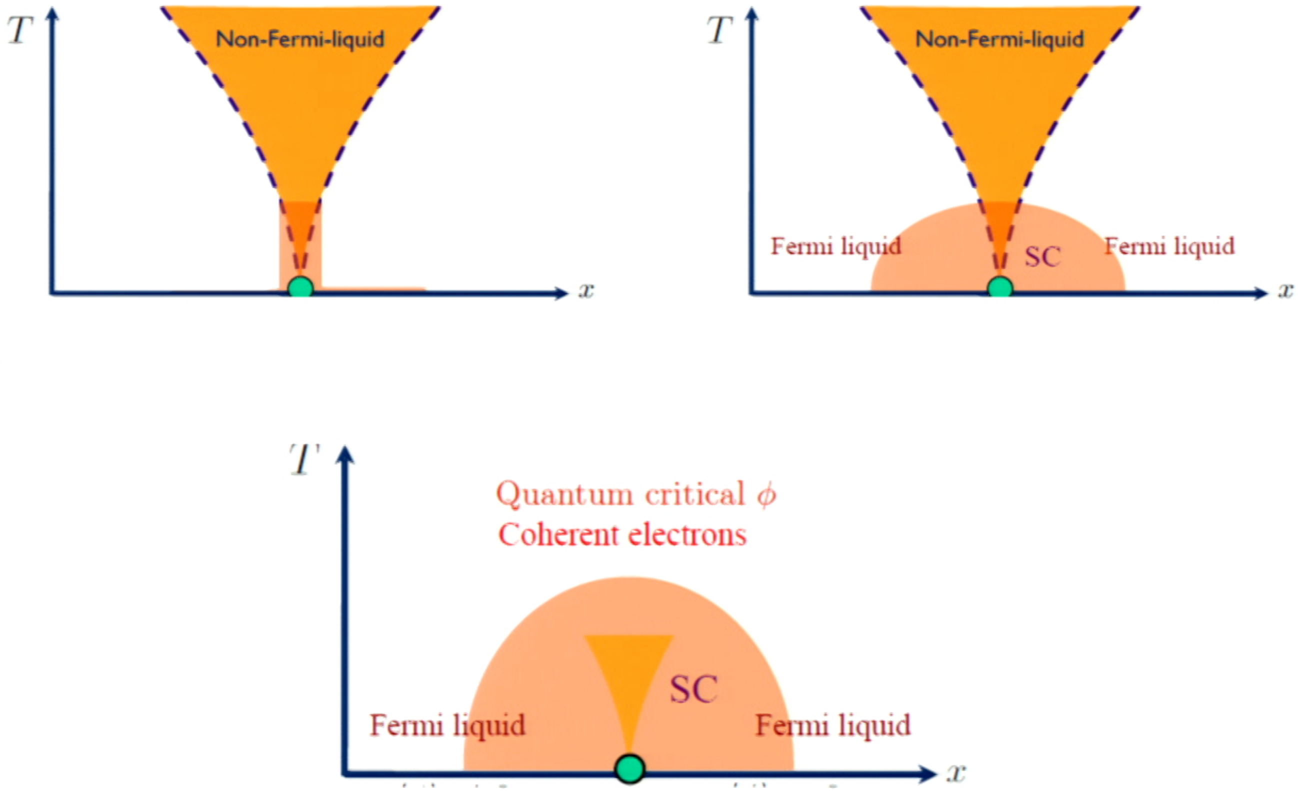
Superconductivity and non-FL come from the same interaction and compete with each other

- Incoherence in a non-FL prevents superconductivity to develop
- Superconductivity gaps low-energy excitations and prevents non-FL behavior to develop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I consider a particular class of itinerant fermionic systems in which soft bosons are slow compared to fermions

- $v_F >> v_s$ for electron-phonon interaction (Eliashberg theory)
- $v_F >> v_{coll}$ for interaction mediated by overdamped collective modes of electrons

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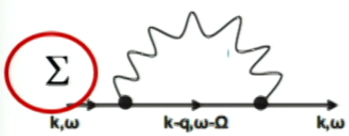
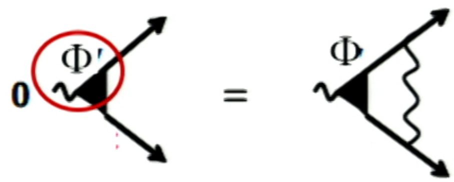
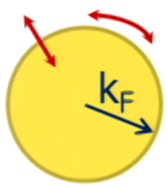




$\Phi(k, \omega) = \Phi(\omega) * f_{\Phi}(\theta_k)$

$\Sigma(k,\omega) = \Sigma(\omega) * f_{\Sigma}(\theta_k)$

θ_k is the angle along the FS



- Neglect vertex corrections (ladder diagrams for $\Phi(k, \Omega)$)
- factorize the momentum integration along and transverse to the Fermi surface between fermionic and bosonic propagators

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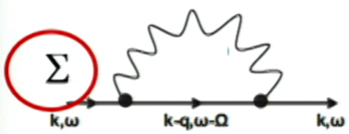

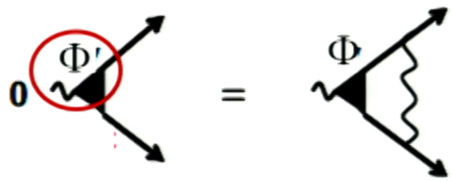

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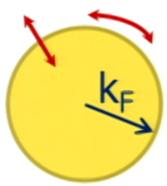

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Not included: new ideas from P. Lunts, A. Schlief, S-S Lee

Bottom line: the problem to find the pairing instability reduces to solving the set of 1D equations for the pairing vertex $\Phi(\Omega)$ and fermionic self-energy $\Sigma(\omega)$

Linearized "gap" equation for spin-singlet SC (small $\Phi(\Omega)$)

Solution sets Tc

$$\Phi(\Omega) = \pi T \sum_{\omega \neq \Omega} \frac{\Phi(\omega)}{|\omega + \Sigma(\omega)|} \chi_L(\omega - \Omega)$$

$$\Sigma(\omega) = \pi T \sum_{\omega' \neq \Omega} \text{sign}(\omega') \chi_L(\omega - \omega')$$

$\chi_L(\Omega)$ is a bosonic propagator integrated along the Fermi surface

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$\chi_L(\Omega)$ is a bosonic propagator integrated along the Fermi surface

$\chi_L(\Omega = 0)$ is a constant in a FL,

but at a QCP, $\chi_L(\Omega) = (g/\Omega)^{\gamma}, \gamma > 0$

The physics of the pairing at a QCP: the competition between two opposite trends

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- Strong attractive pairing interaction (strong in the sense that $\chi_L(\Omega = 0)$ diverges)

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Examples (all in 2D)

1. Near a nematic transition

$$\chi(q,\Omega) = \frac{1}{m^2 + q^2 + \alpha \frac{|\Omega|}{q}}$$

$$\chi_L(\Omega) = \int \chi(q,\Omega)\, dq \propto m^{-1},\ m \neq 0, \qquad \Omega^{-1/3},\ m = 0$$

Bonesteel, McDonald, Nayak; Metlitski, Mross, Sachdev, Senthil; Kivelson, Berg, Lederer, Schattner; Gerlach, Trebst; Metzner...

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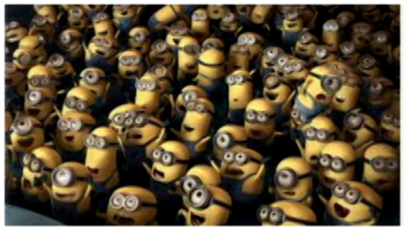
Millis, Sachdev, Varma; Abanov, A.C, Finkelstein, Metlitski & Sachdev; Wang & A.C.....
P. Lunts, A. Schlief, S-S Lee (new ideas about γ at the smallest frequencies)

There is a variety of known QC theories with different γ

$\gamma = 1/2$	2D antiferromagnetic QCP
$\gamma = 1/3$	2D ferromagnetic QCP/interaction with gauge field/nematic
$\gamma = 1/4$	2D $2k_F$ QCP
$\gamma = 2$	QCP of Einstein phonons
$\gamma = 1$	2D QCP of fermions interacting with undamped bosons
$\gamma = +0 \ (\log \omega)$	3D QCP, Color superconductivity
$\gamma = 0(\varepsilon)$	QCP in 3-ε dimension



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The corresponding self-energy at T=0:

$$\Sigma(\omega) = \omega^{1-\gamma}(\omega_0)^\gamma \quad \omega_0 \sim g$$

$$\omega_0 = g\left(\frac{2}{1-\gamma}\right)^{1/\gamma}$$

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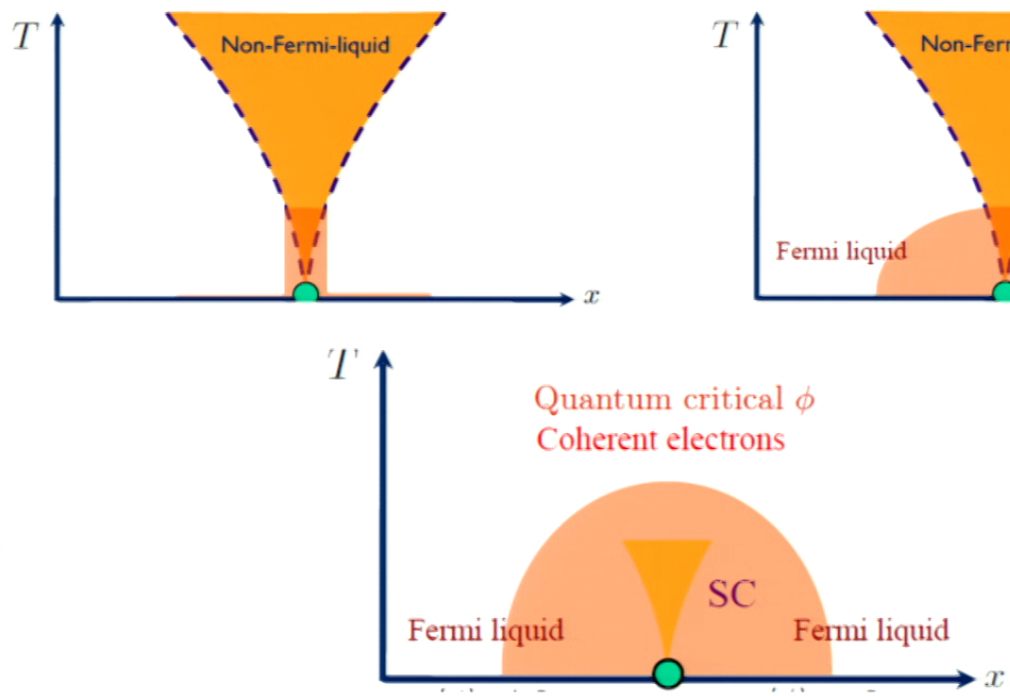
Quantum-critical | Fermi gas ("high energies")

0 | $\Sigma(\omega) > \omega$ | ω_0 | $\Sigma(\omega) < \omega$ | ω

ω_0 -- upper energy edge of non-FL behavior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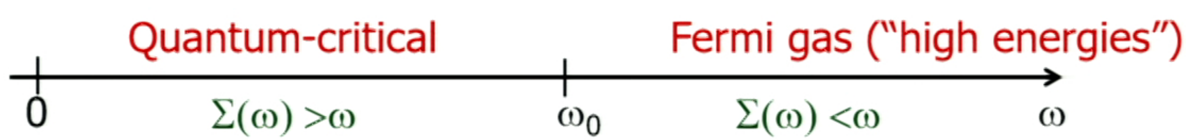
Quantum-critical
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$\Sigma(\omega) > \omega$
ω_0
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Alternatively, we can add bare Φ_0 to the r.h.s of eqn. for $\Phi(\Omega)$ and obtain the pairing susceptibility Φ/Φ_0

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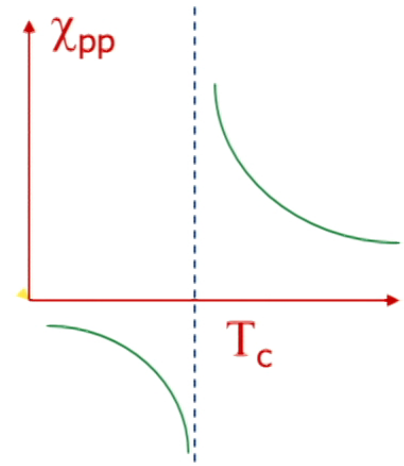

Let's move one step
away from a Fermi liquid:

Consider $\gamma = 0+$, $\chi(\Omega) = g_{eff} \log|\Lambda/\Omega| = g_{eff} L$ (Son's problem)

$g_{eff} << 1$

D. Son, 1999

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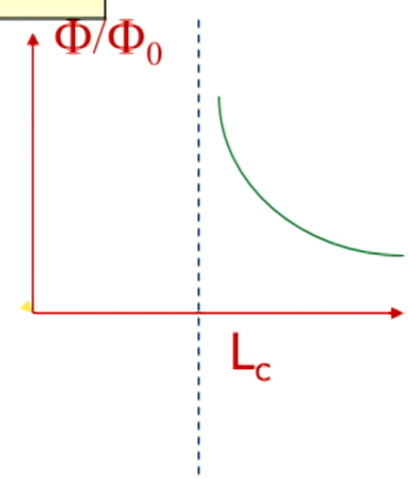

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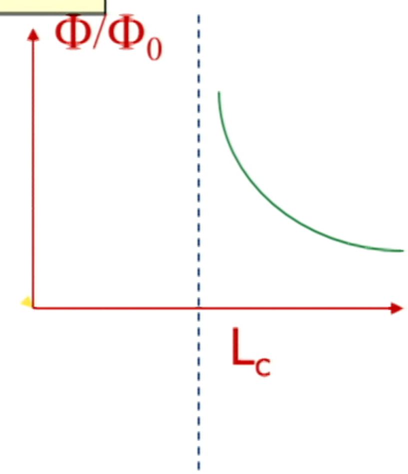

SC vs non-FL in Son's solution:

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fermionic self-energy

Self-energy destroys fermionic coherence when $g_{eff} \log(\Lambda/\omega) > 1$, i.e., when $\omega < \omega_0$

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$$T_c = \Lambda\, e^{-\frac{\pi}{2\sqrt{g_{eff}}}} >> \omega_0$$

SC vs non-FL in Son's solution:

$$\Phi(\Omega) = g_{eff}\, \pi T \sum_{\omega} \frac{\Phi(\omega)\, \log \frac{\Lambda}{|\omega - \Omega|}}{|\omega| (1 + g_{eff} \log \frac{\Lambda}{|\omega|})}$$

fermionic self-energy

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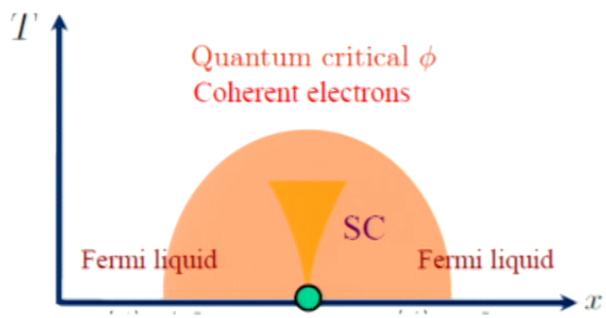
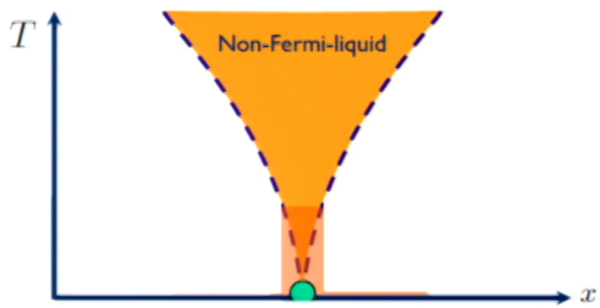
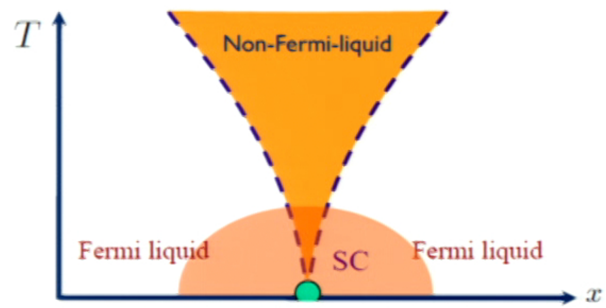


$\chi_L(\Omega) = (g/\Omega)^\gamma$

Next: consider finite γ

Can we get non-FL ahead of superconductivity?





Abanov, A.C. Finkelstein, Yuzbashyan, Altshuler, Wang;
Bonesteel, McDonald, Nayak;
S. Raghu, J. Kaplan, S. Kachru, A. Torroba, A. Fitzpatrick
M. Metlitski, S. Sachdev, J. McGreevy, D. Mross, T. Senthil…

Let's see what happens at a finite γ

$$\Phi(\Omega) = \omega_0 \, \pi \, T \sum_{\omega} \frac{\Phi(\omega)\ (1-\gamma)}{|\omega| + \omega_0^{\gamma} |\omega|^{1-\gamma}} \frac{1}{|\omega - \Omega|^{\gamma}}$$

Now the frequency sum converges on its own, and one can set the cutoff Λ to infinity

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Once Λ is set to infinity, the pairing problem becomes universal in the sense that once we rescale T by ω_0, the only parameter left is γ

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Once Λ is set to infinity, the pairing problem becomes universal in the sense that once we rescale T by ω_0, the only parameter left is γ

Then we expect, in general, $T_c = \omega_0 f(\gamma)$

Let's introduce one more parameter, just to have some freedom

$$\Phi(\Omega) = \pi T \sum_{\omega \neq \Omega} \frac{\Phi(\omega)}{|\omega + \Sigma(\omega)|} \chi_L(\omega - \Omega)$$

$$\Sigma(\omega) = \pi T \sum_{\omega' \neq \Omega} \text{sign}(\omega') \chi_L(\omega - \omega')$$

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Let's artificially reduce the strength of the pairing interaction

Extend the theory to large N (SU(N) global symmetry),
then the coupling in SC channel acquires prefactor 1/N,
but the equation for the self-energy remains intact

A Torroba et al

Consider the pairing problem inside the quantum-critical regime, at $\omega<\omega_0$

$$\chi_L(\Omega) = (g/\Omega)^\gamma$$

$$\Sigma(\omega) = \omega_0^\gamma \, \omega^{1-\gamma} >> \omega$$

$$\omega_0 = g\,\left(\frac{2}{1-\gamma}\right)^{1/\gamma}$$

$$\Phi(\Omega) = \frac{1-\gamma}{2N} \pi T \sum_{\omega}^{\omega_0} \frac{\Phi(\omega)}{|\omega - \Omega|^\gamma |\omega|^{1-\gamma}}$$

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Universality:
Tc = ω_0 * number

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A Torroba et al

Let's check whether non-FL can destroy SC, at least at large N

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Let's see whether we can obtain Tc at large N by summing up the logs

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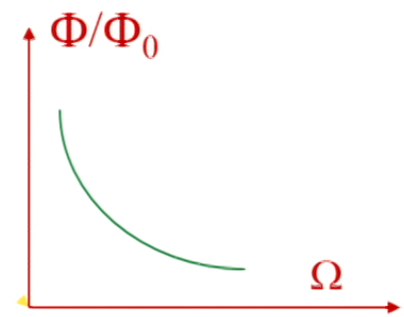

We can actually do better than just sum up the logs:

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exactly the same power that we obtained by summing the logarithms

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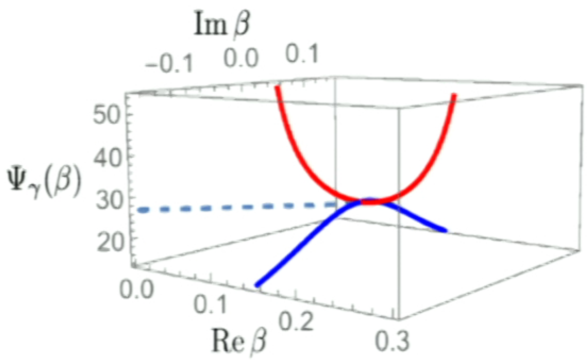

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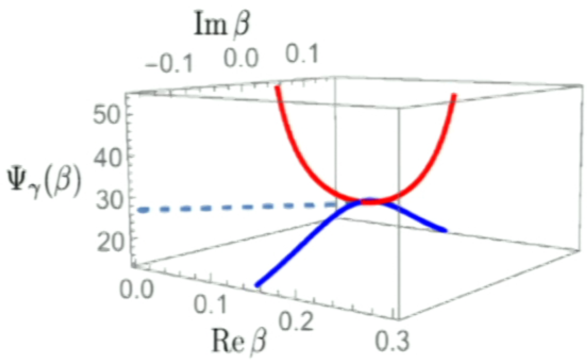

Below some critical N =Ncr, the solution (i.e., β) moves to the complex plane:

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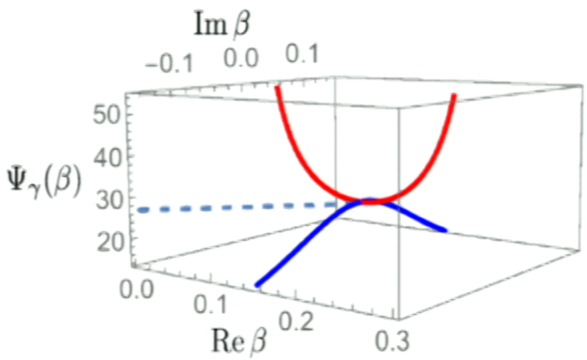

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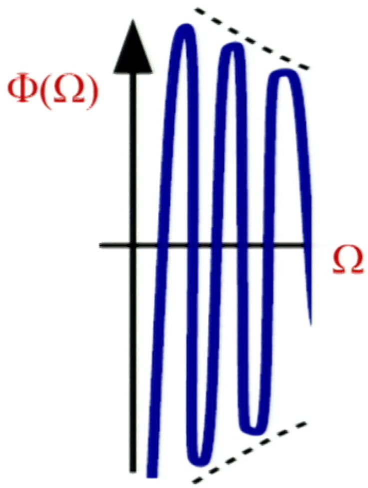

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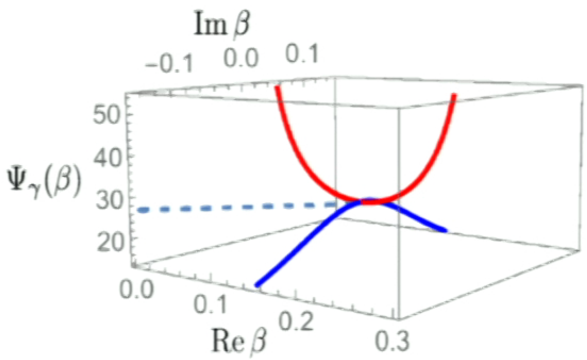

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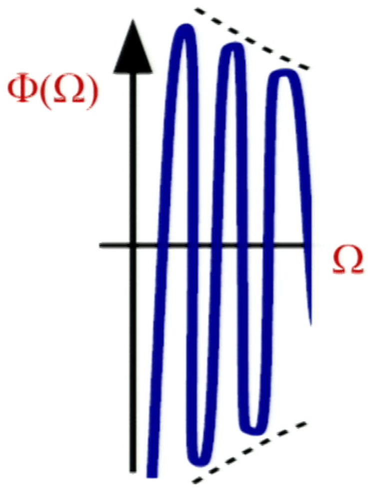
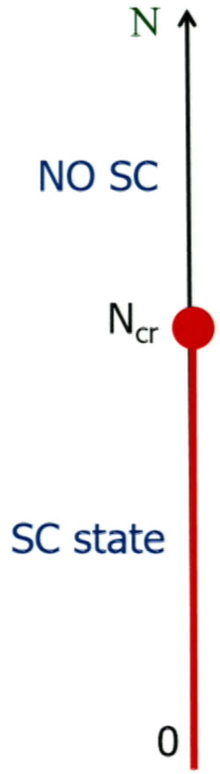
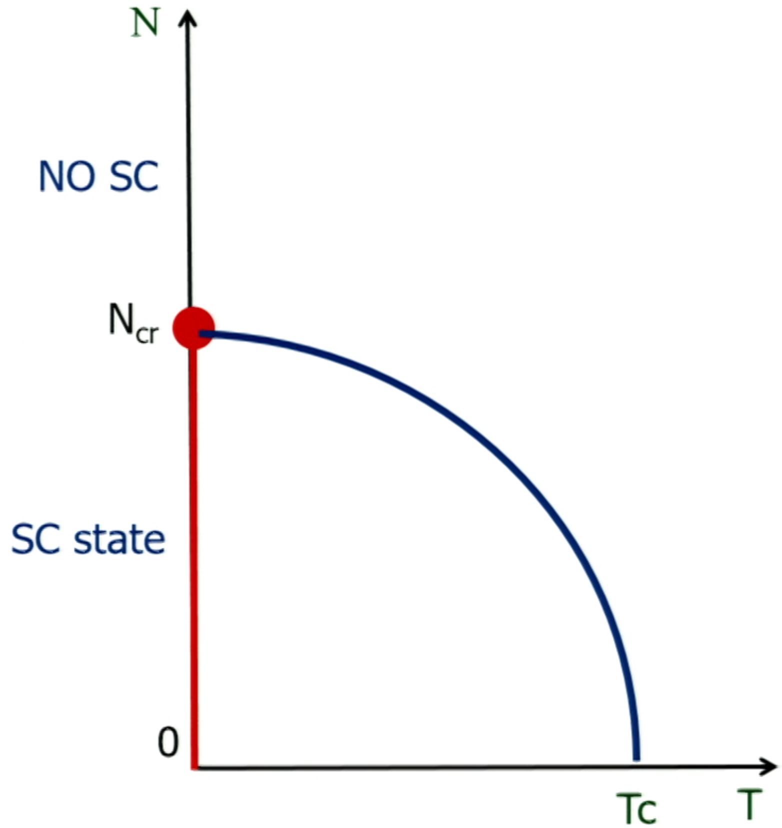
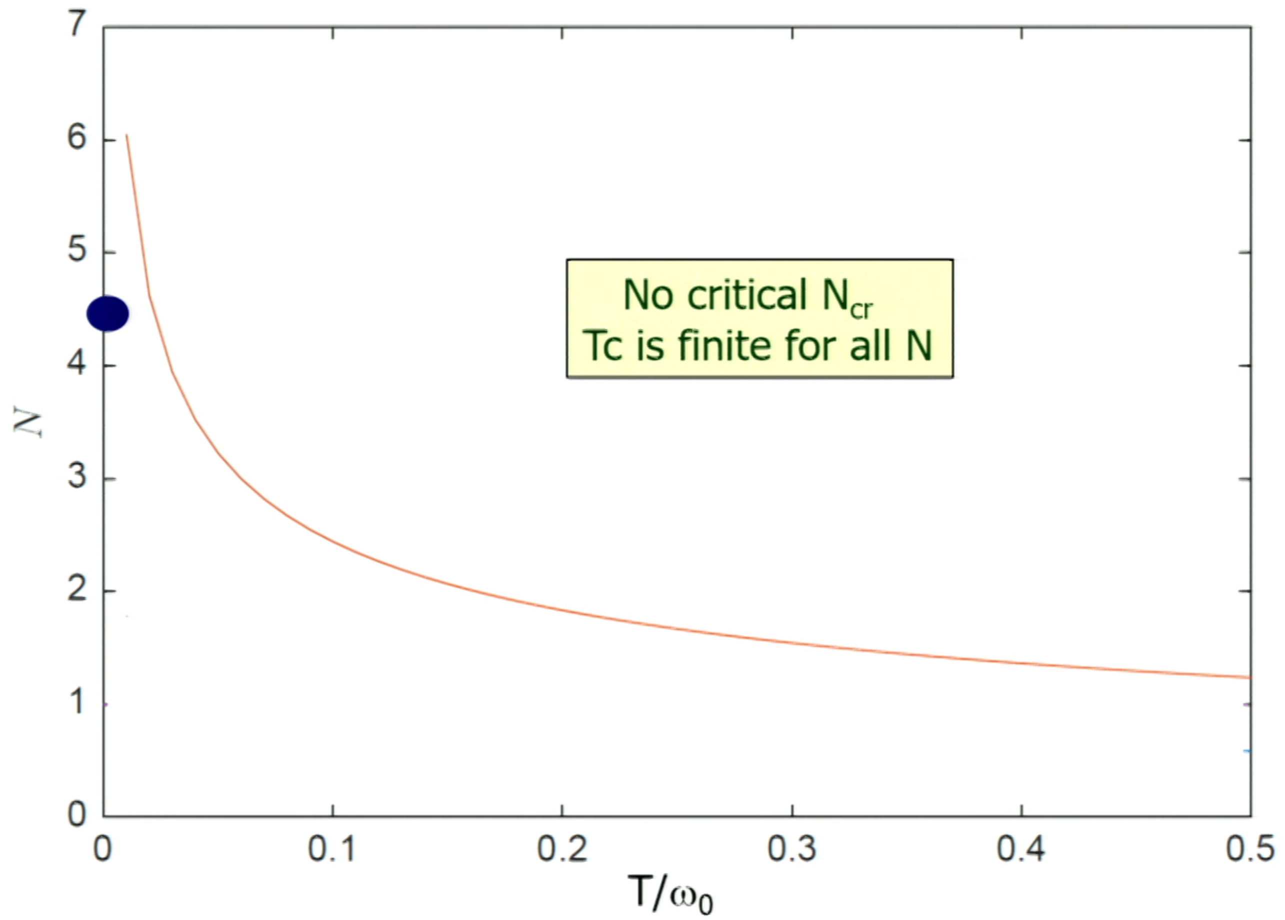


Instability towards SC when N < Ncr
Normal state down to T=0 when N > Ncr

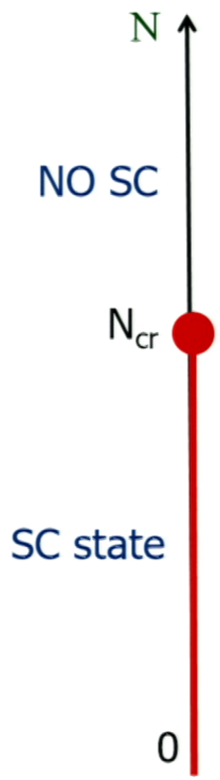

N
NO SC
N_cr
SC state
0


N
NO SC
N_{cr}
SC state
0
Tc
T

Tc for γ=0.5, finite T analysis (summation over Matsubara frequencies)



There is more than that!



T=0, change γ

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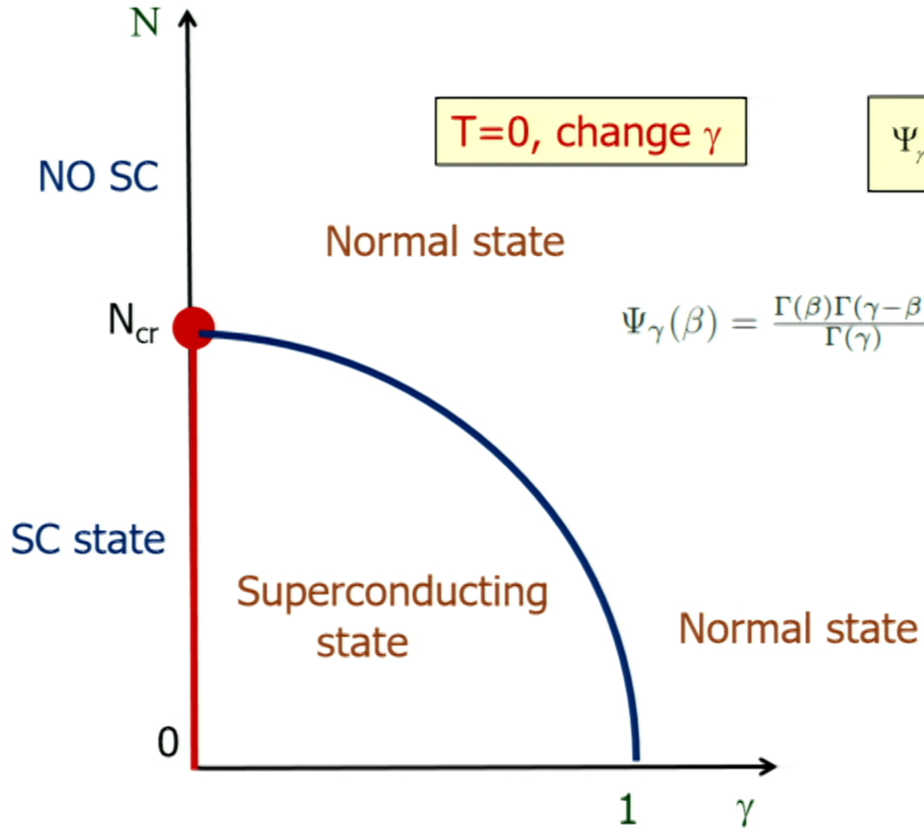
T=0, change γ

$$\Psi_\gamma(\beta) = \frac{2N}{1-\gamma}$$



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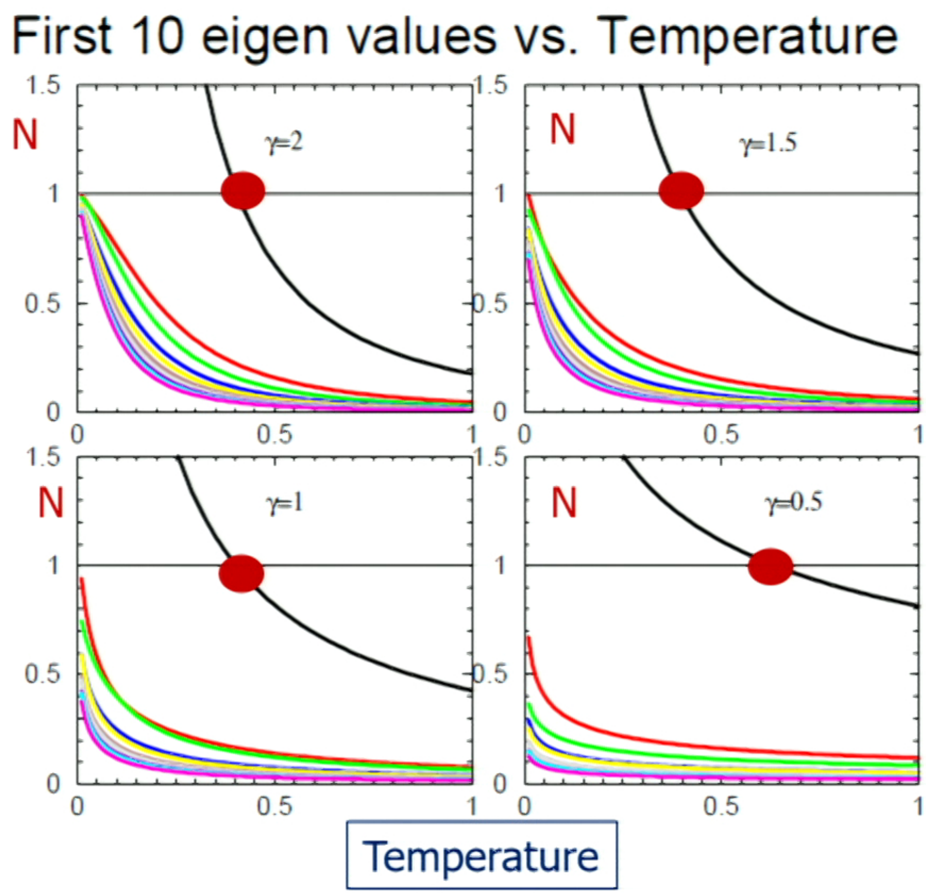
Numerics: the solution of the linearized gap equation at T >0 (!)



The solution exists for any N and any g

Let's set T to be finite and understand why Tc does not vanish at N >>1

$$\Phi(\Omega_m) = \frac{\pi T}{N} \sum_{\omega \neq \Omega} \frac{\Phi(\omega_m)}{|\omega_m + \Sigma(\omega_m)|} \left(\frac{g}{|\omega_m - \Omega_m|} \right)^{\gamma}$$

$$\Sigma(\omega_m) = \pi T \sum_{\omega' \neq \omega} \text{sign}(\omega_m') \left(\frac{g}{|\omega_m - \omega_m'|} \right)^{\gamma}$$

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By scaling analysis, $\Sigma(\omega_m) \sim (\omega_m)^{1-g}$, same as at T=0

But for two Matsubara frequencies: $\omega_m = \pi$ T and $\omega_m = -\pi$ T, we find $\Sigma(\pi T) = \Sigma(-\pi T) = 0$

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$$\Phi(\pi T) = \Phi(-\pi T) \left(\frac{g}{2\pi T N^{1/\gamma}} \right)^{\gamma} + \frac{a_1}{N} \Phi(3\pi T) + \ldots$$

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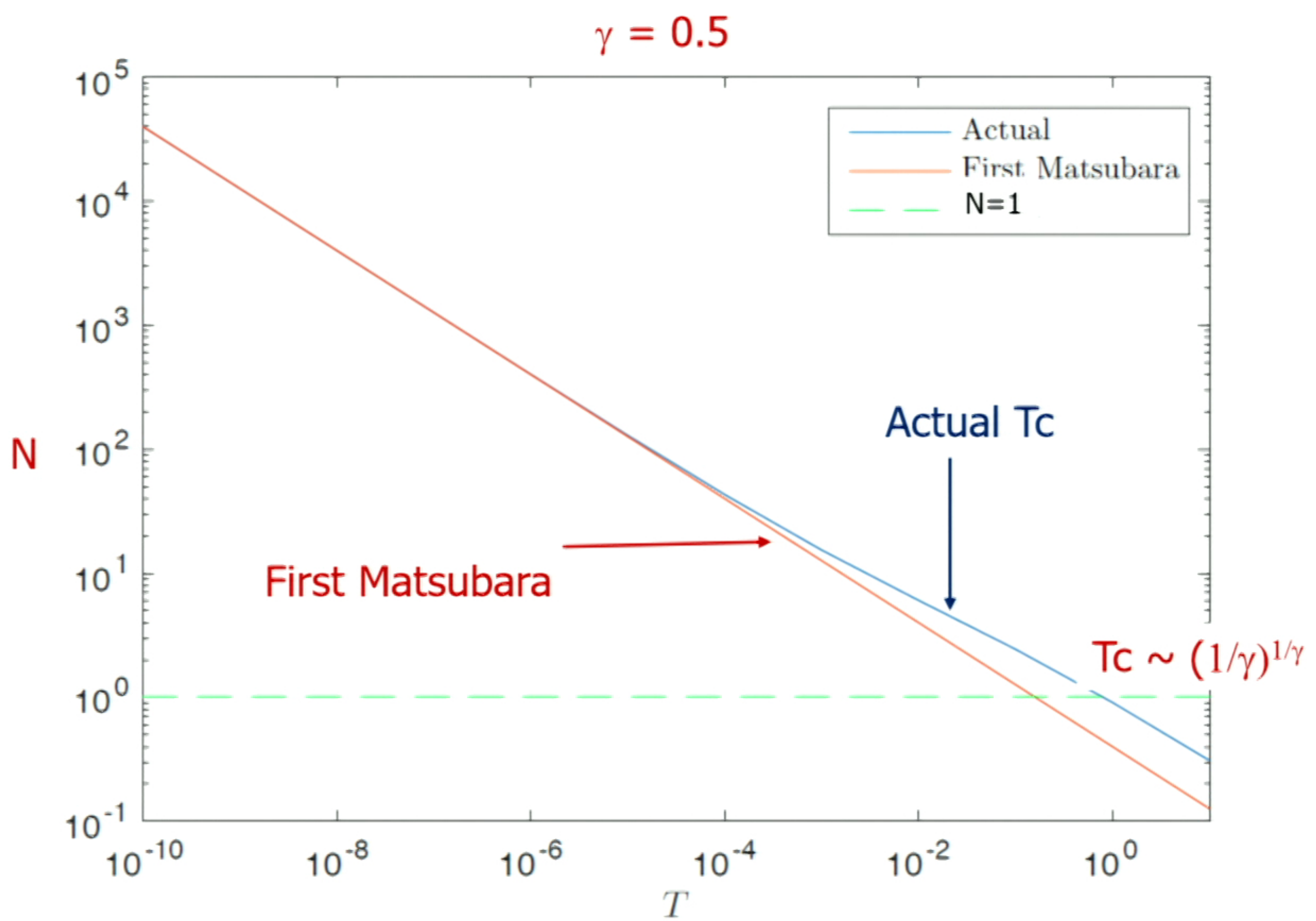
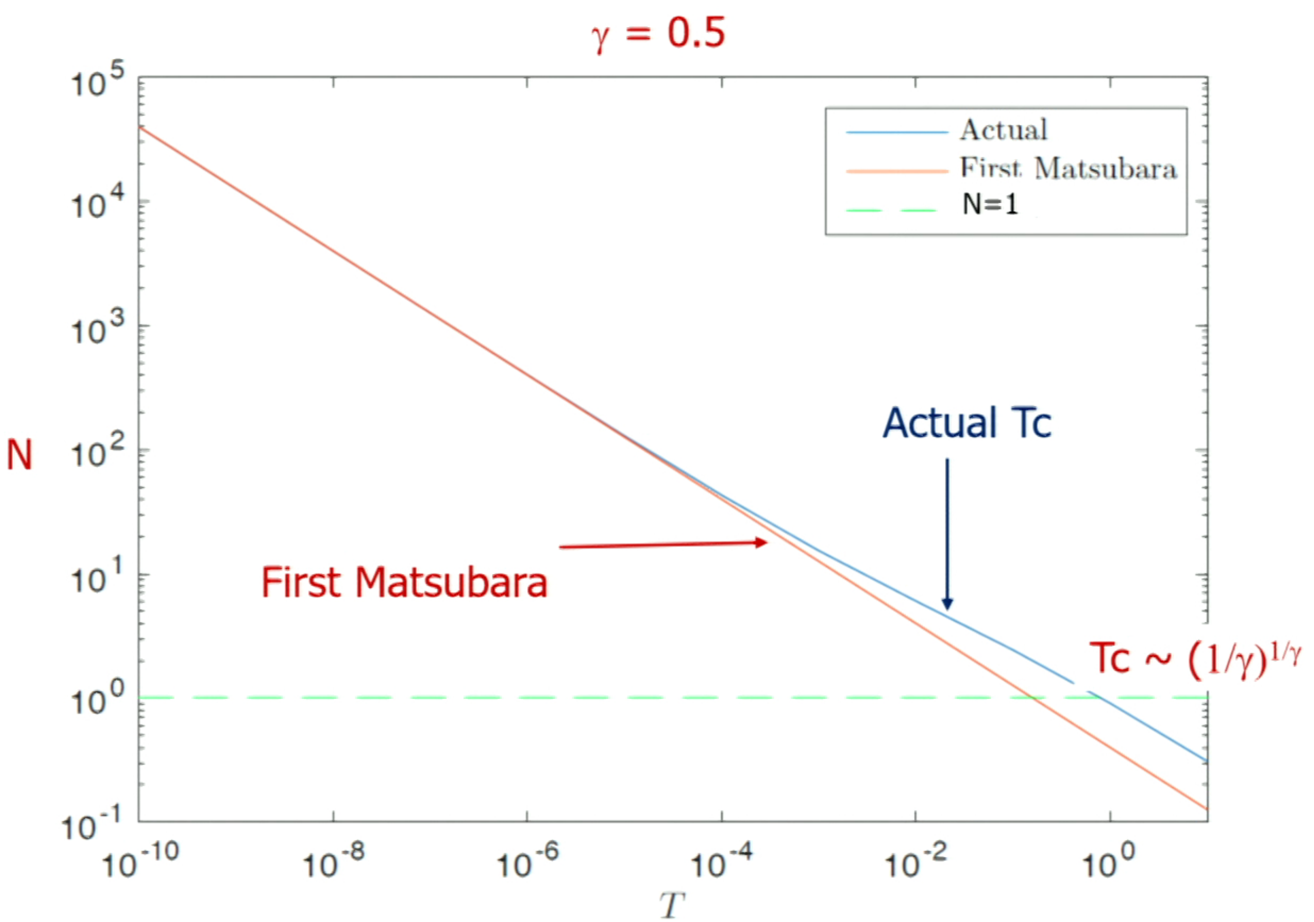

γ = 0.5
Actual
First Matsubara
N=1
Actual Tc
N
First Matsubara
Tc ~ (1/γ)^{1/γ}
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-10
10^-8
10^-6
10^-4
10^-2
10^0
T



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10^4
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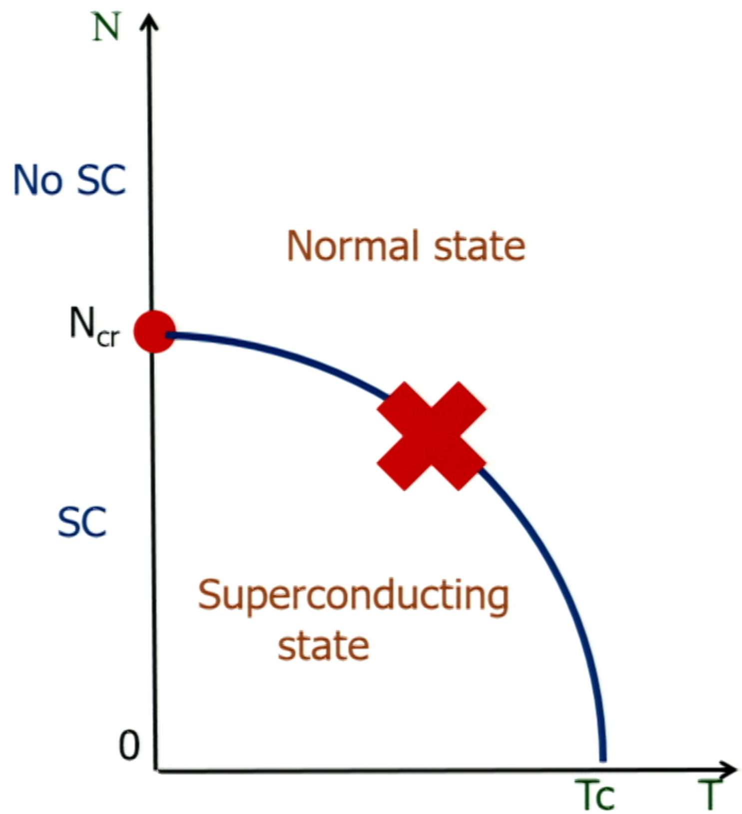
10^1
10^0
10^-1
10^-10
10^-8
10^-6
10^-4
10^-2
10^0
T

At large N, superconductivity comes exclusively from fermions with Matsubara frequencies $\omega = \pi T$ and $\omega = -\pi T$

For these fermions, $\Sigma(\omega) = 0$, while interaction between $\omega = \pi T$ and $\omega = -\pi T$ is still strong!

In other words, superconductivity doesn't care about quantum criticality

And how to relate this to what we had at T=0?



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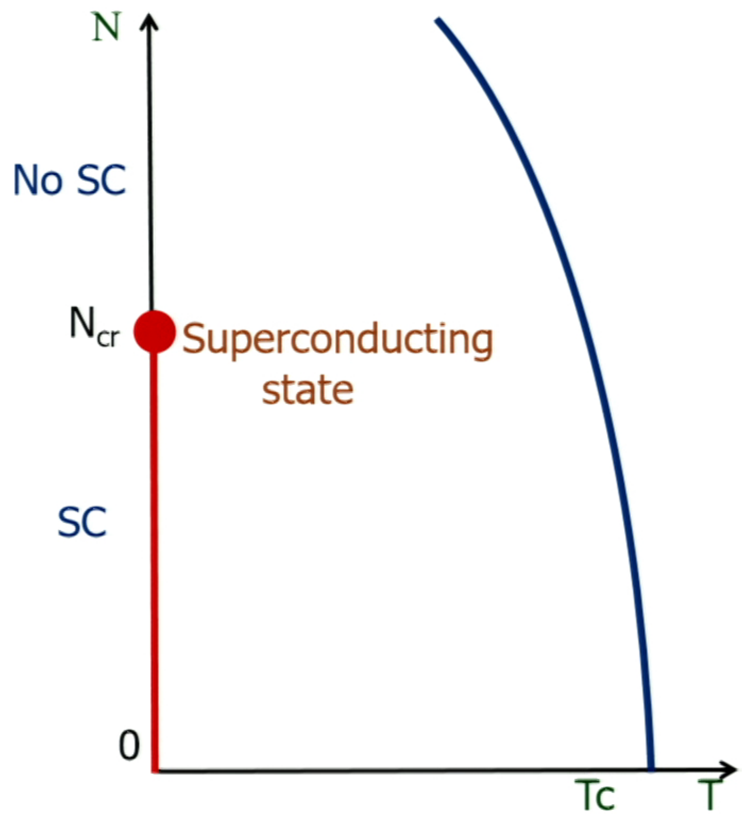

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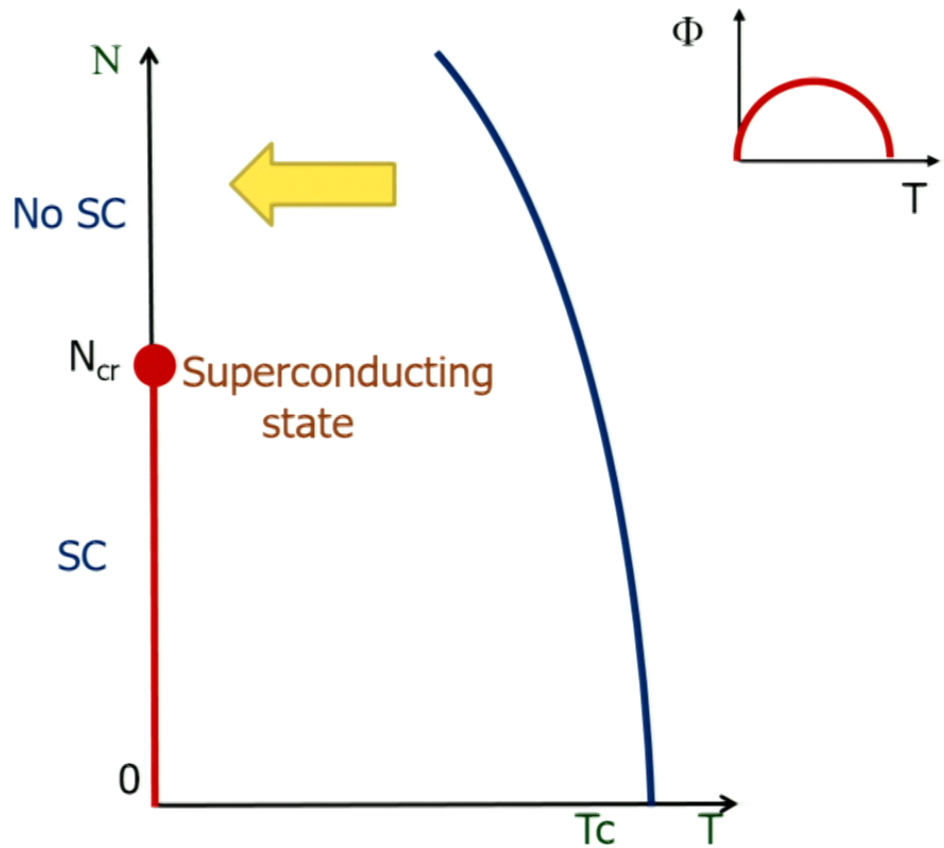
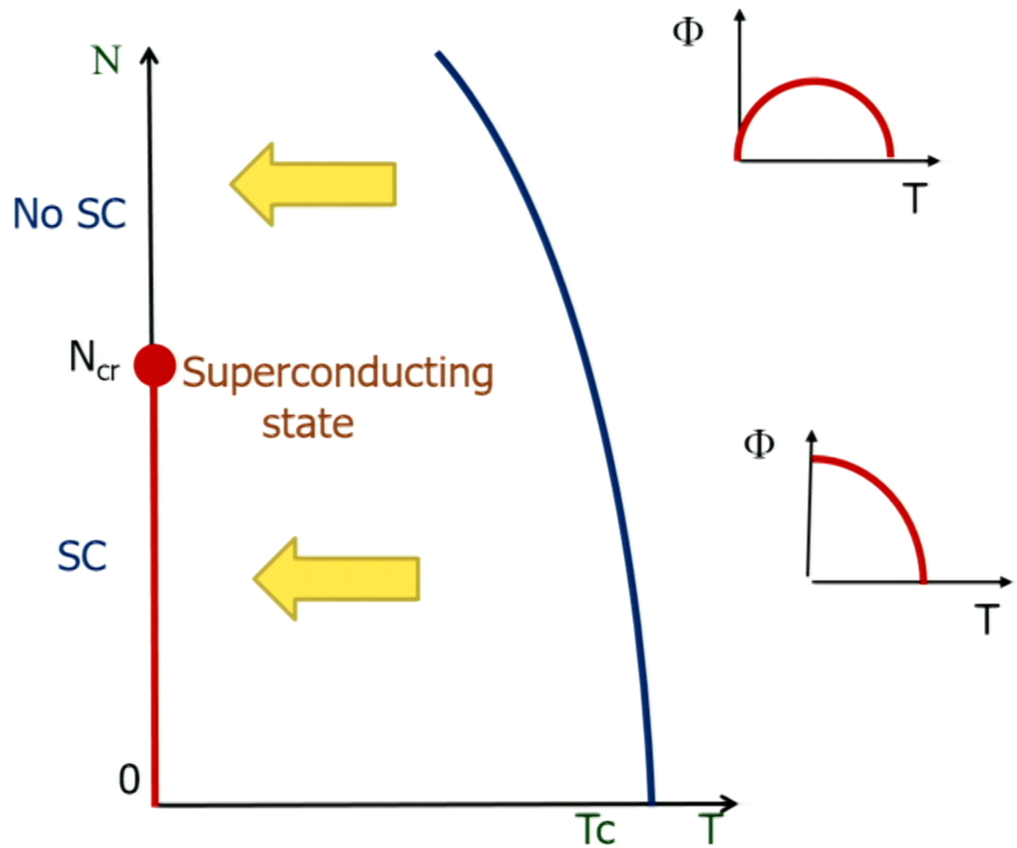
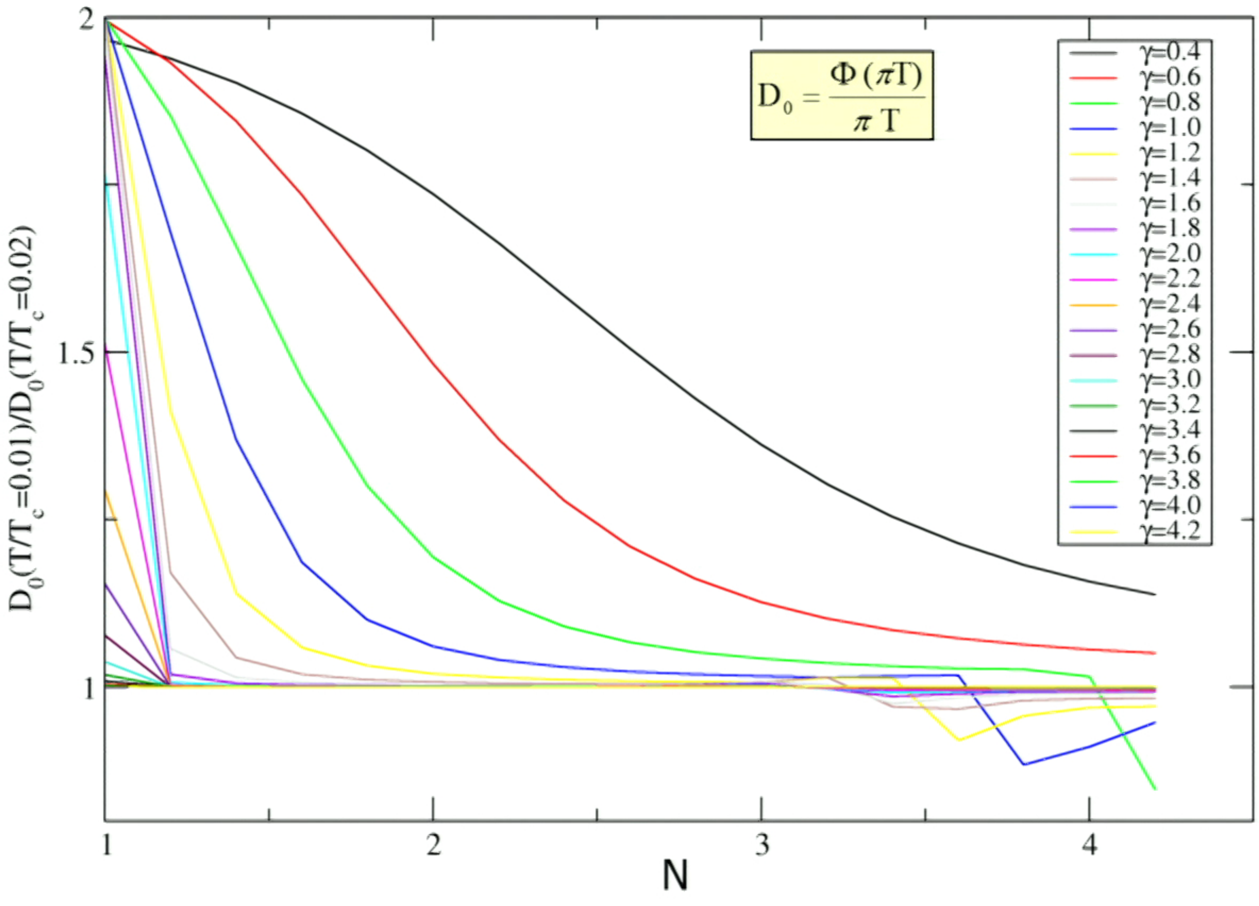
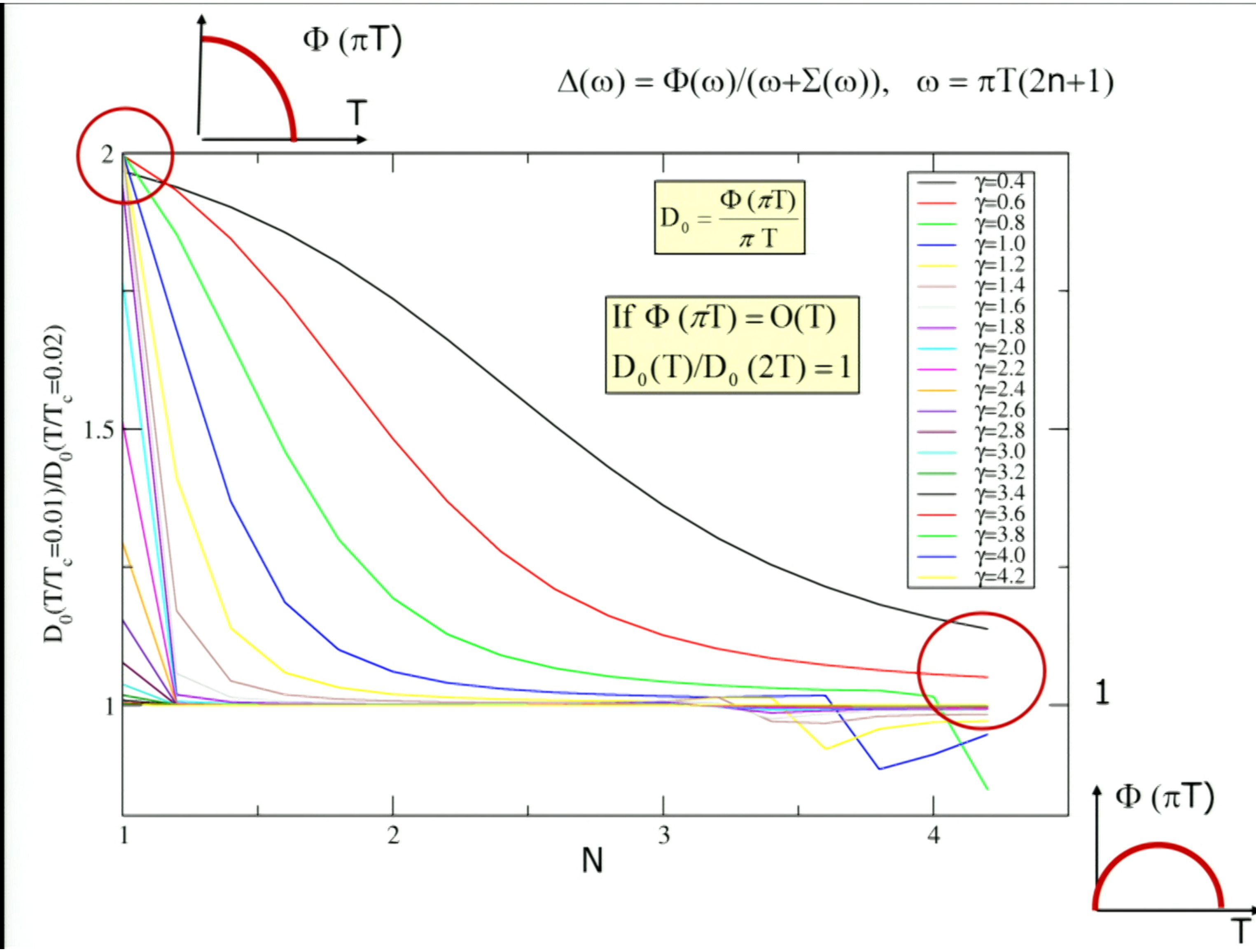


And how to relate this to what we had at T=0?

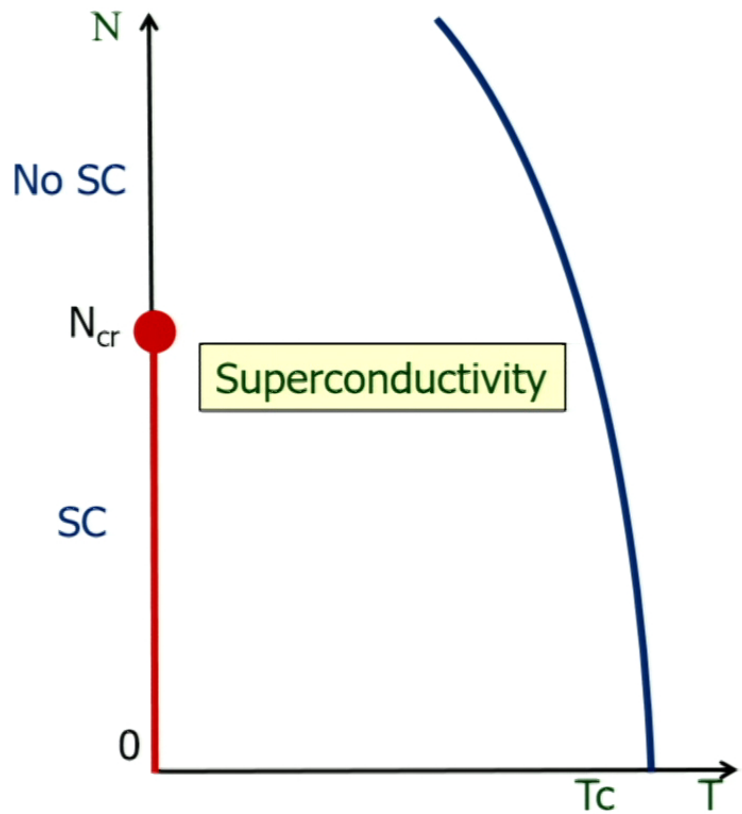


$$\Delta(\omega) = \Phi(\omega)/(\omega+\Sigma(\omega)), \quad \omega = \pi T(2n+1)$$

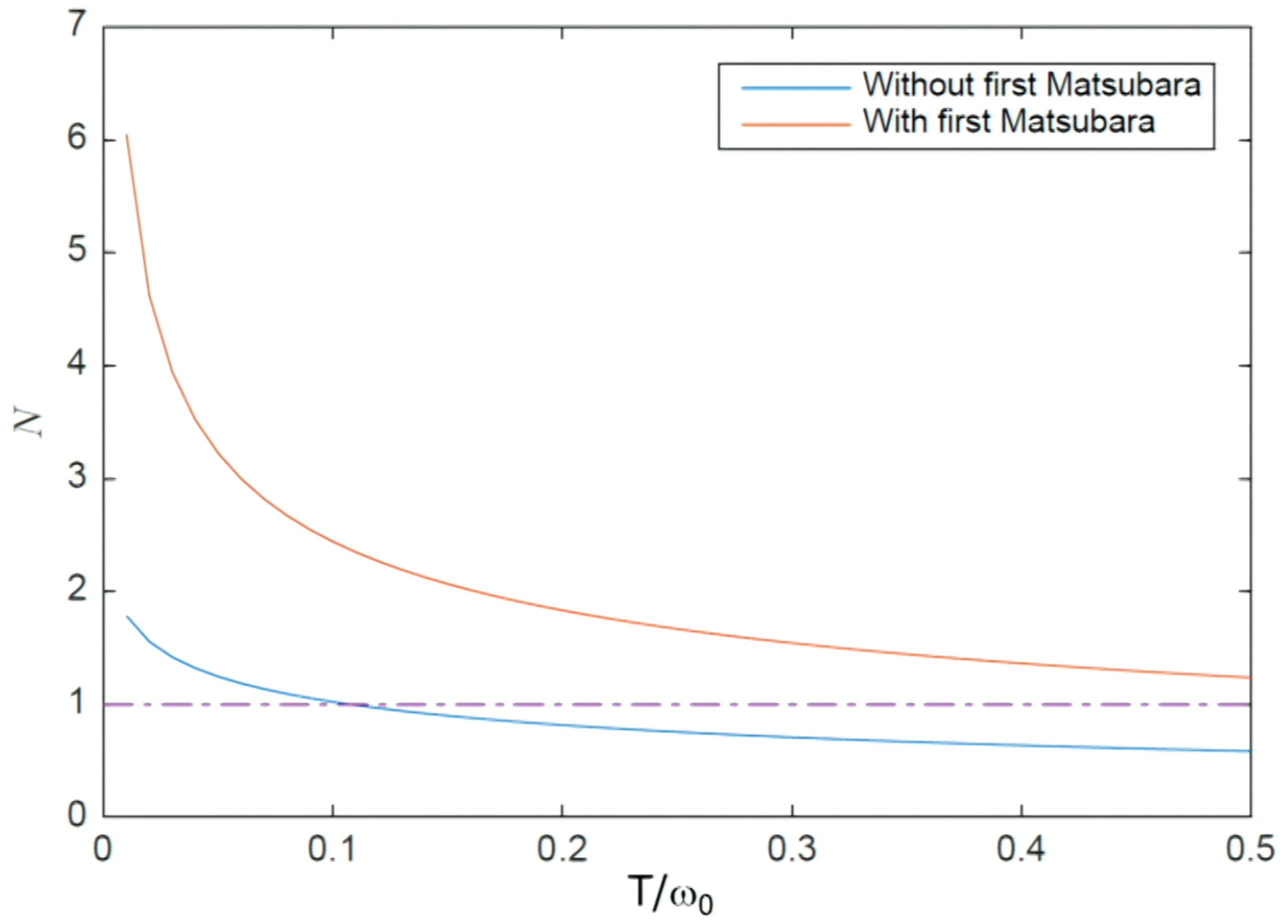



Φ (πT)
T
Δ(ω) = Φ(ω)/(ω+Σ(ω)),   ω = πT(2n+1)
D₀ = Φ(πT) / πT
If Φ (πT) = O(T)
D₀(T)/D₀ (2T) = 1
γ=0.4
γ=0.6
γ=0.8
γ=1.0
γ=1.2
γ=1.4
γ=1.6
γ=1.8
γ=2.0
γ=2.2
γ=2.4
γ=2.6
γ=2.8
γ=3.0
γ=3.2
γ=3.4
γ=3.6
γ=3.8
γ=4.0
γ=4.2
D₀(T/Tc=0.01)/D₀(T/Tc=0.02)
2
1.5
1
1
2
3
4
N
1
Φ (πT)
T

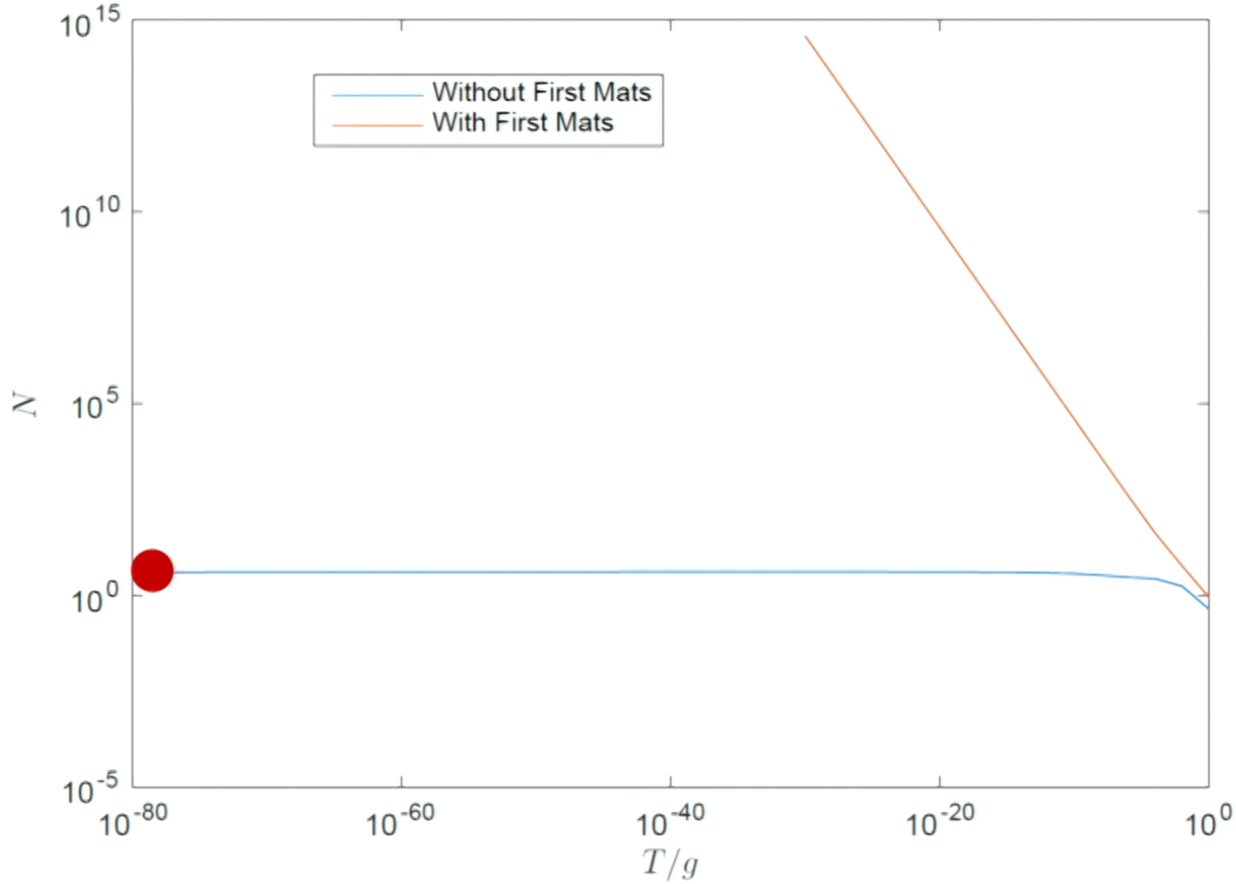
Phase diagram with an isolated critical point.



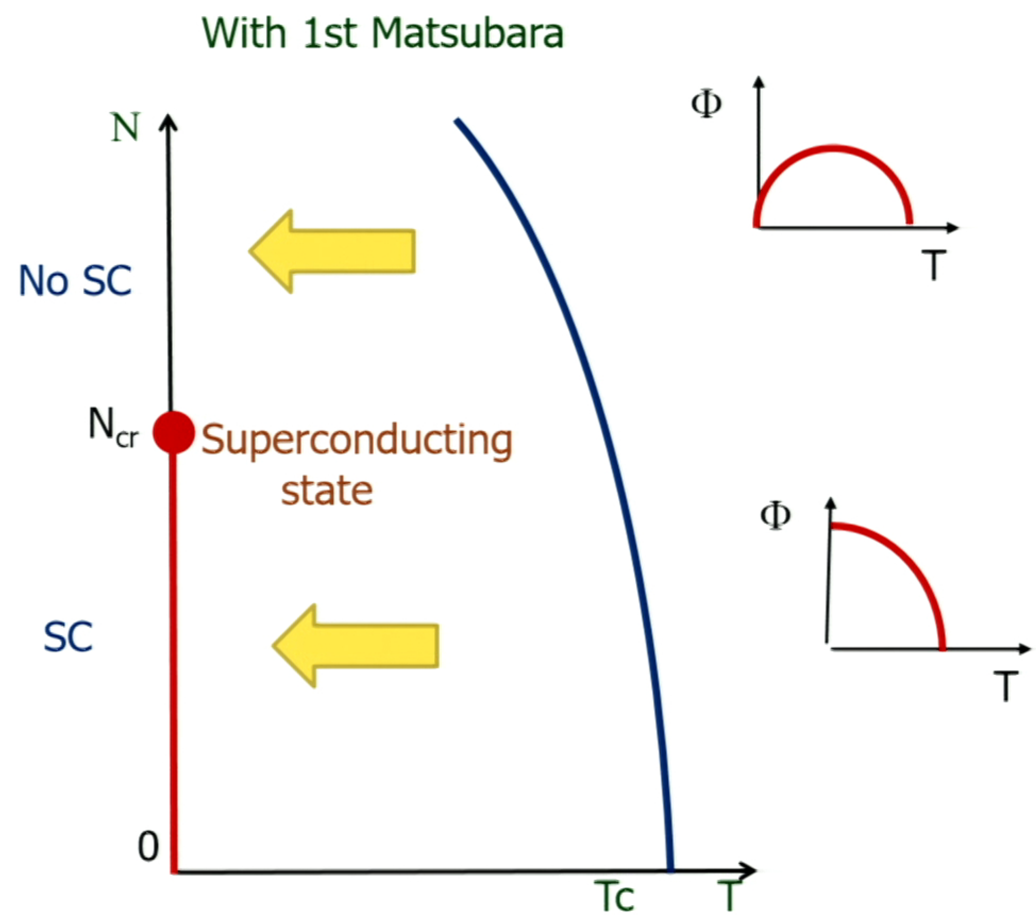
Tc with and without first Matsubara frequency, γ=0.5



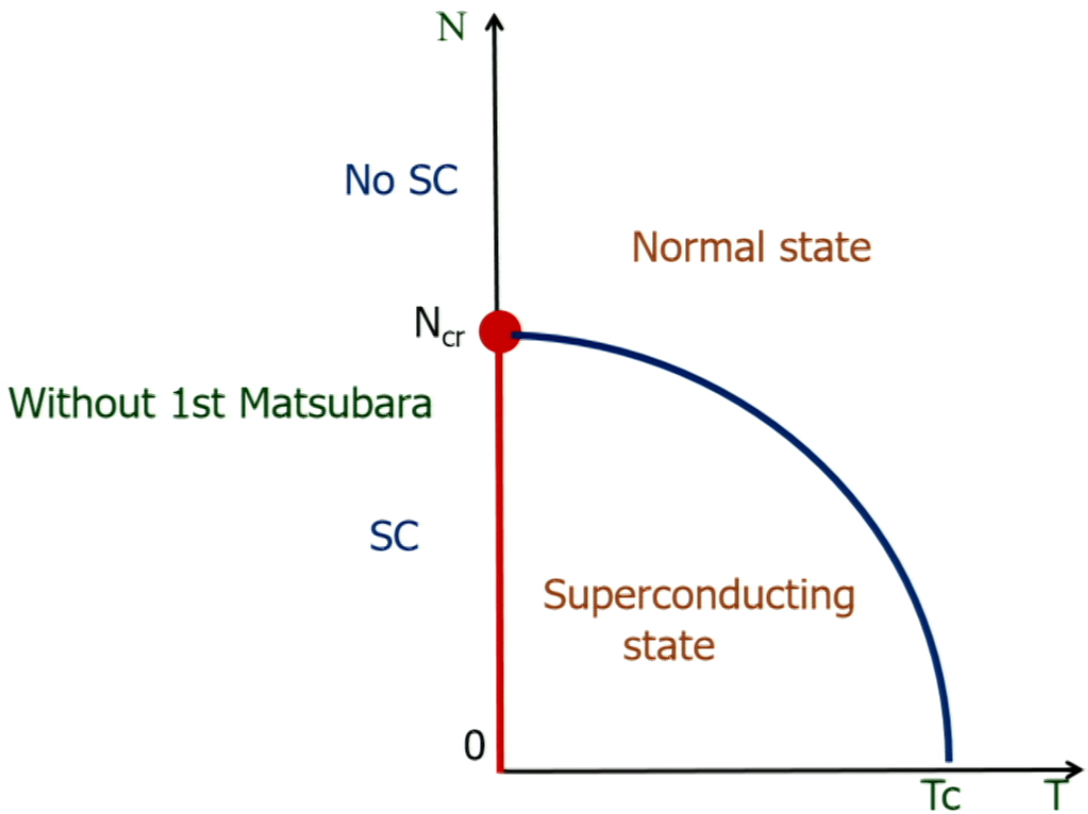
Tc with and without first Matsubara frequency, γ=0.5



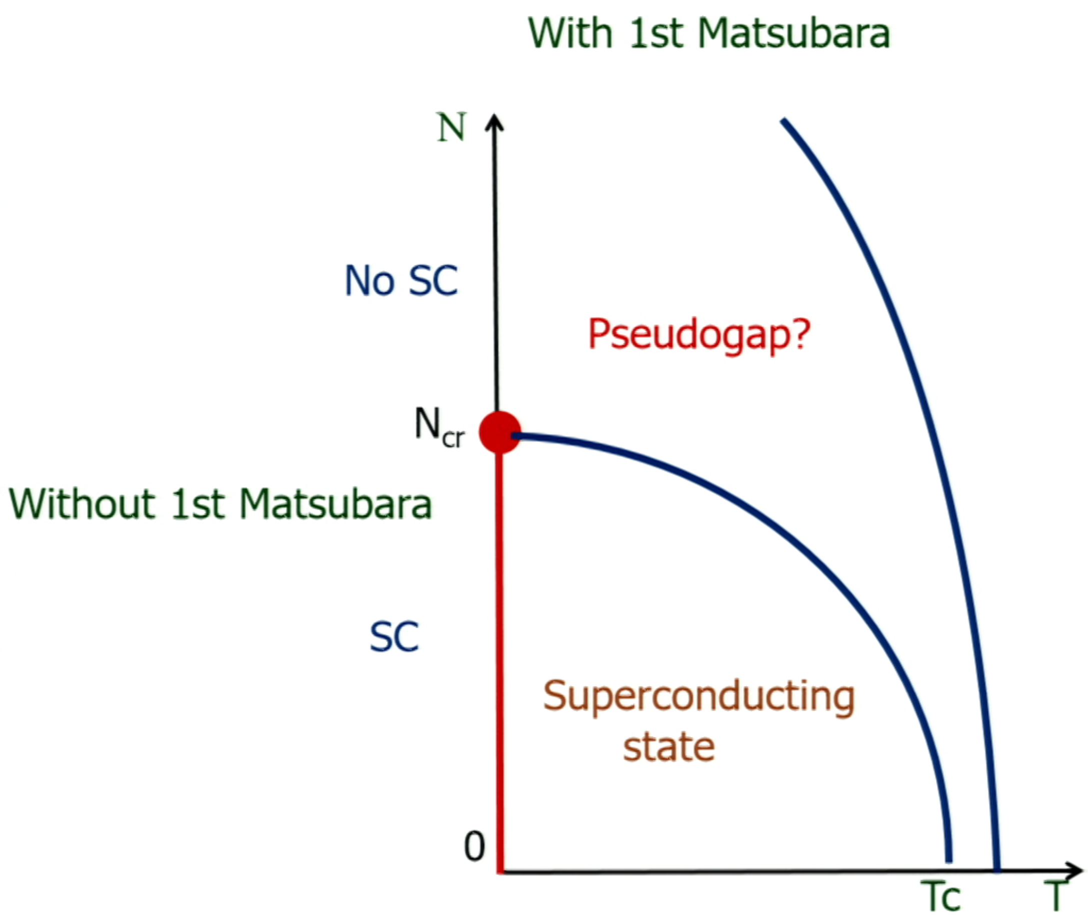
Pairing driven by the first Matsubara frequency, vs the conventional pairing



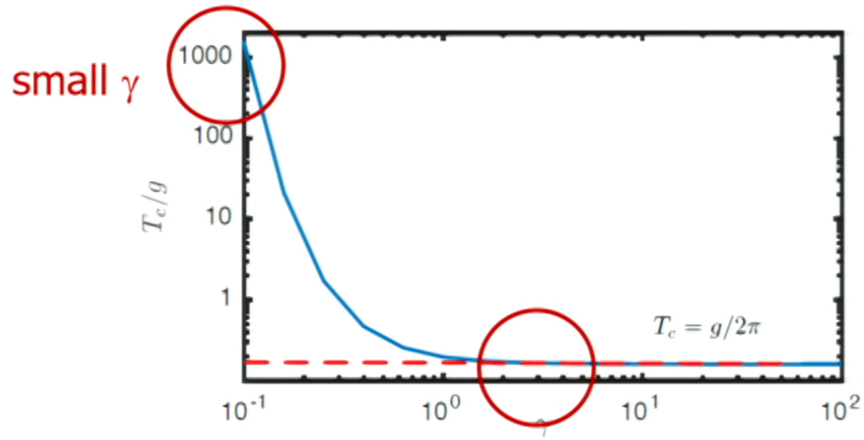
Pairing driven by the first Matsubara frequency, vs the conventional pairing



Pairing driven by the first Matsubara frequency, vs the conventional pairing



The actual case N=1, arbitrary γ



The lower boundary for Tc is $g/(2\pi)$

At large γ, $\chi_L(\omega) = (g/\omega)^\gamma$ drops rapidly, only ω = +- πT matter,

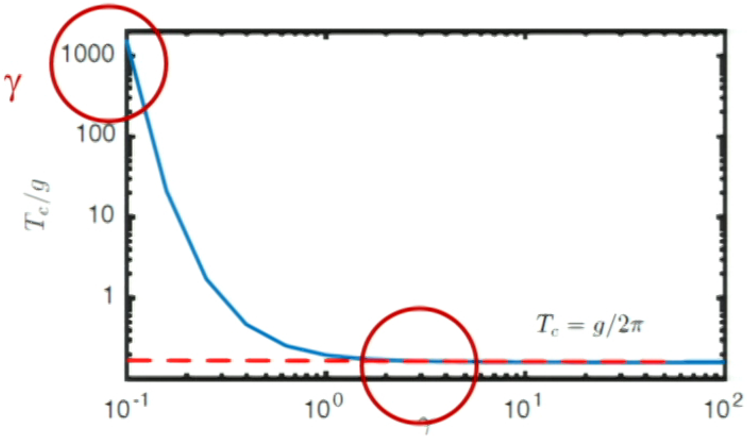
and $T_c \approx \frac{g}{2\pi} = 0.16\,g$

There was a lot of numerics for γ =2 (el-phonon problem): Tc = 0.18 g

R Combescot, …

The actual case N=1, arbitrary γ

small γ



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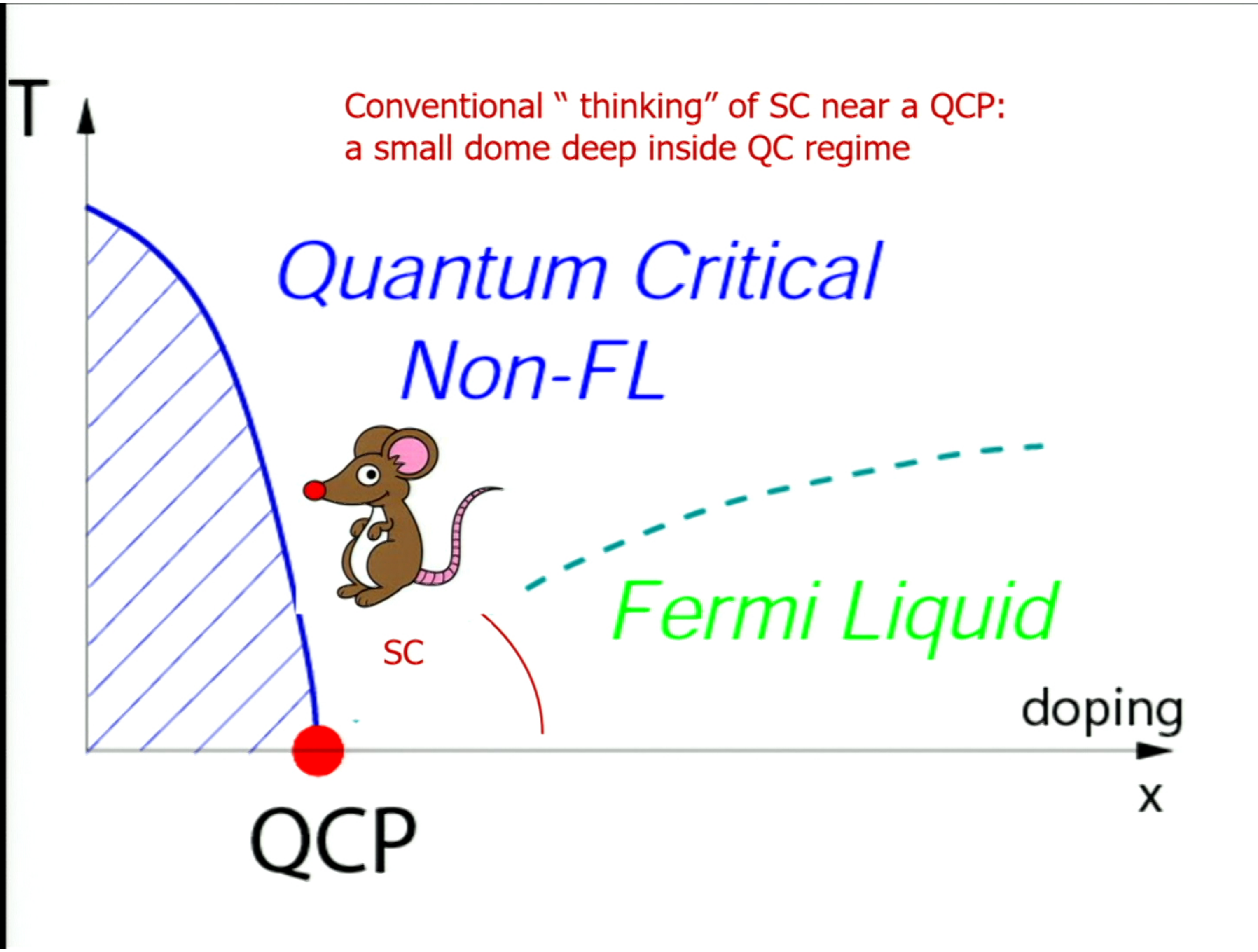
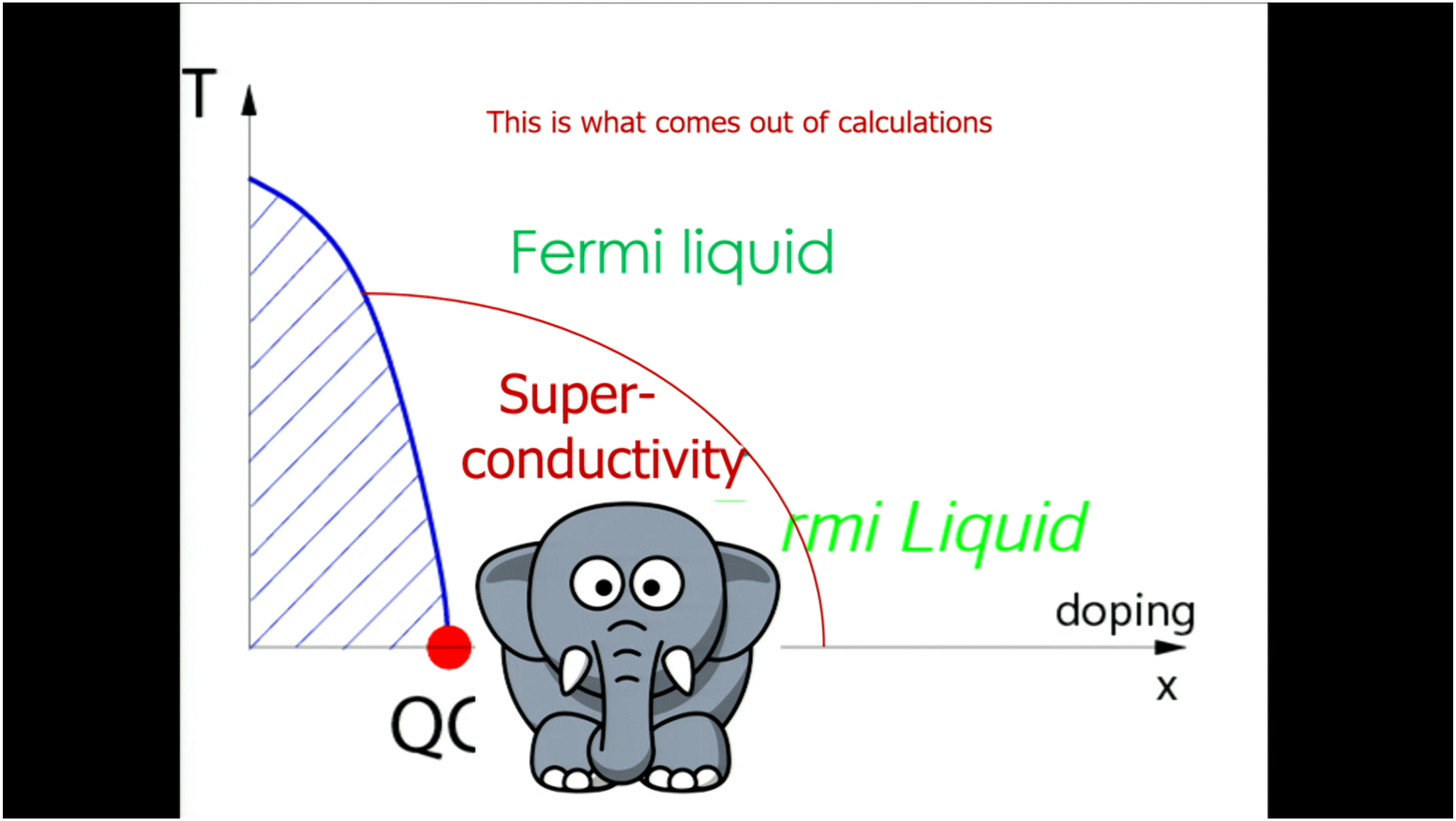
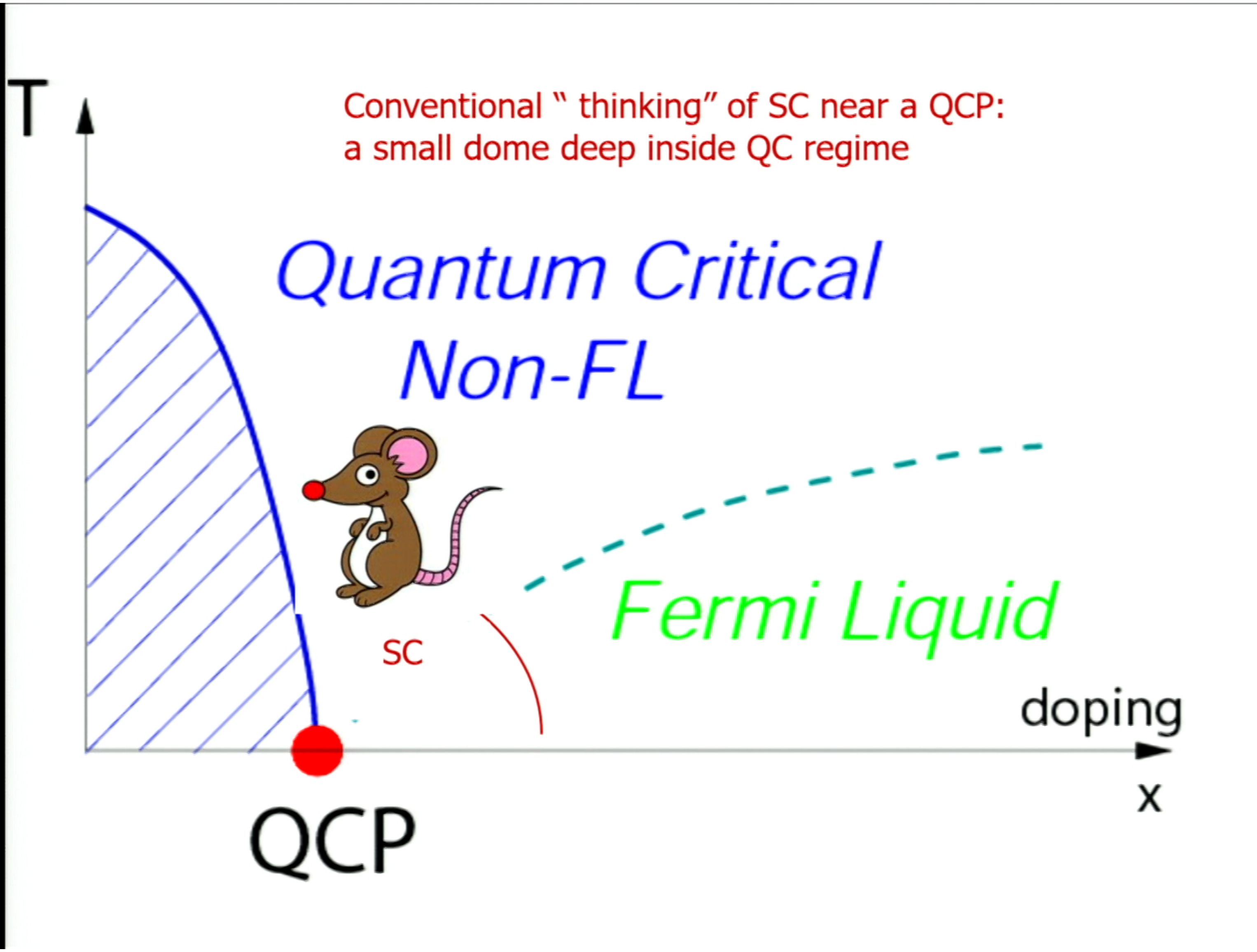
There was a lot of numerics for γ =2 (el-phonon problem): Tc = 0.18 g

$$T_c \approx \frac{g}{2\pi}\left(1+\frac{1}{\gamma}\log(1.18432)\right) = 0.173\,g \text{ for } \gamma = 2$$

R Combescot, …


Conventional " thinking" of SC near a QCP:
a small dome deep inside QC regime
T
Quantum Critical
Non-FL
SC
Fermi Liquid
doping
x
QCP


T
This is what comes out of calculations
Fermi liquid
Super-
conductivity
rmi Liquid
doping
x
QC


T
Conventional " thinking" of SC near a QCP:
a small dome deep inside QC regime
Quantum Critical
Non-FL
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THANK YOU