Title: PSI 2016/2017 Quantum Field Theory III - Lecture 11

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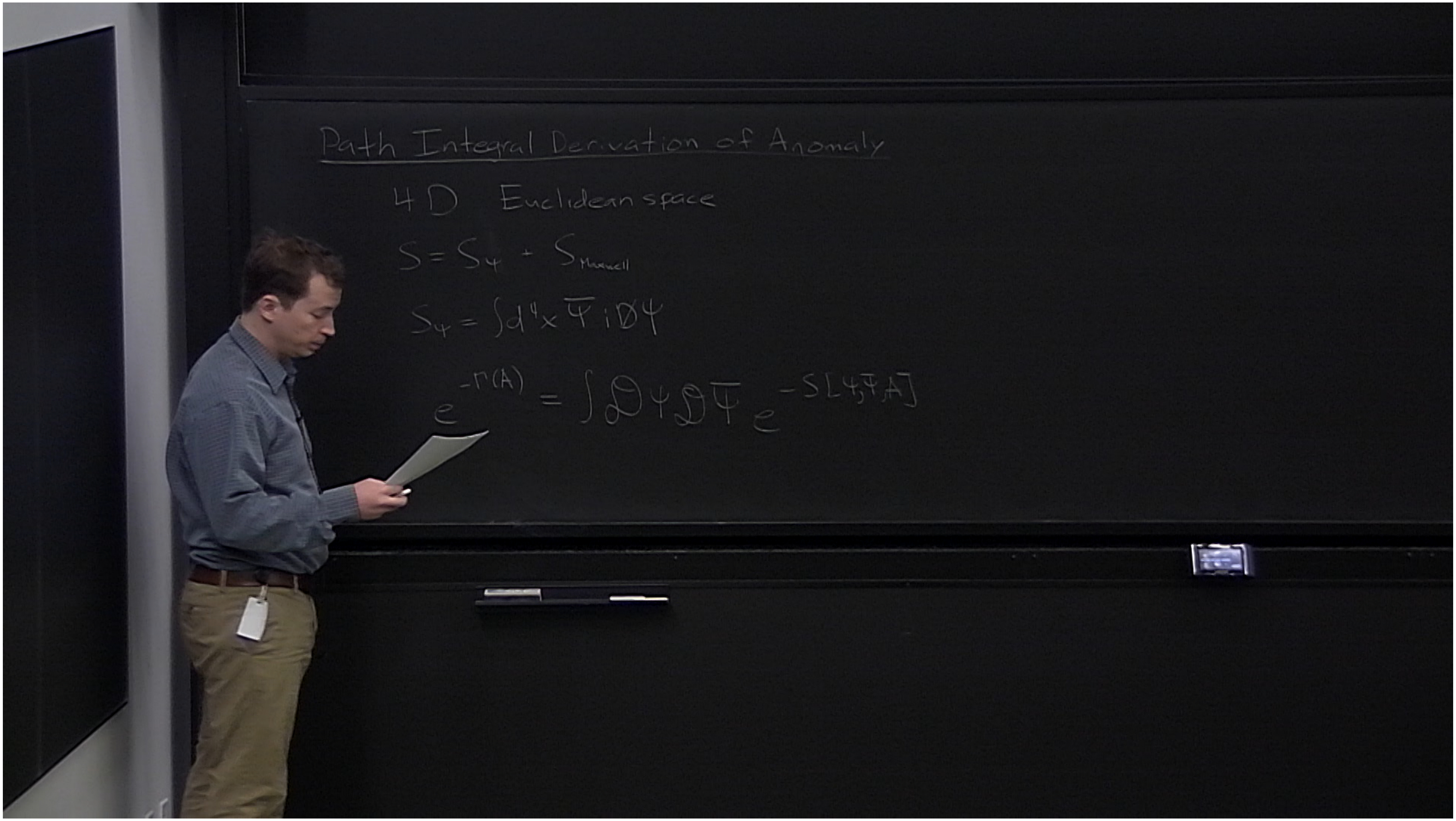
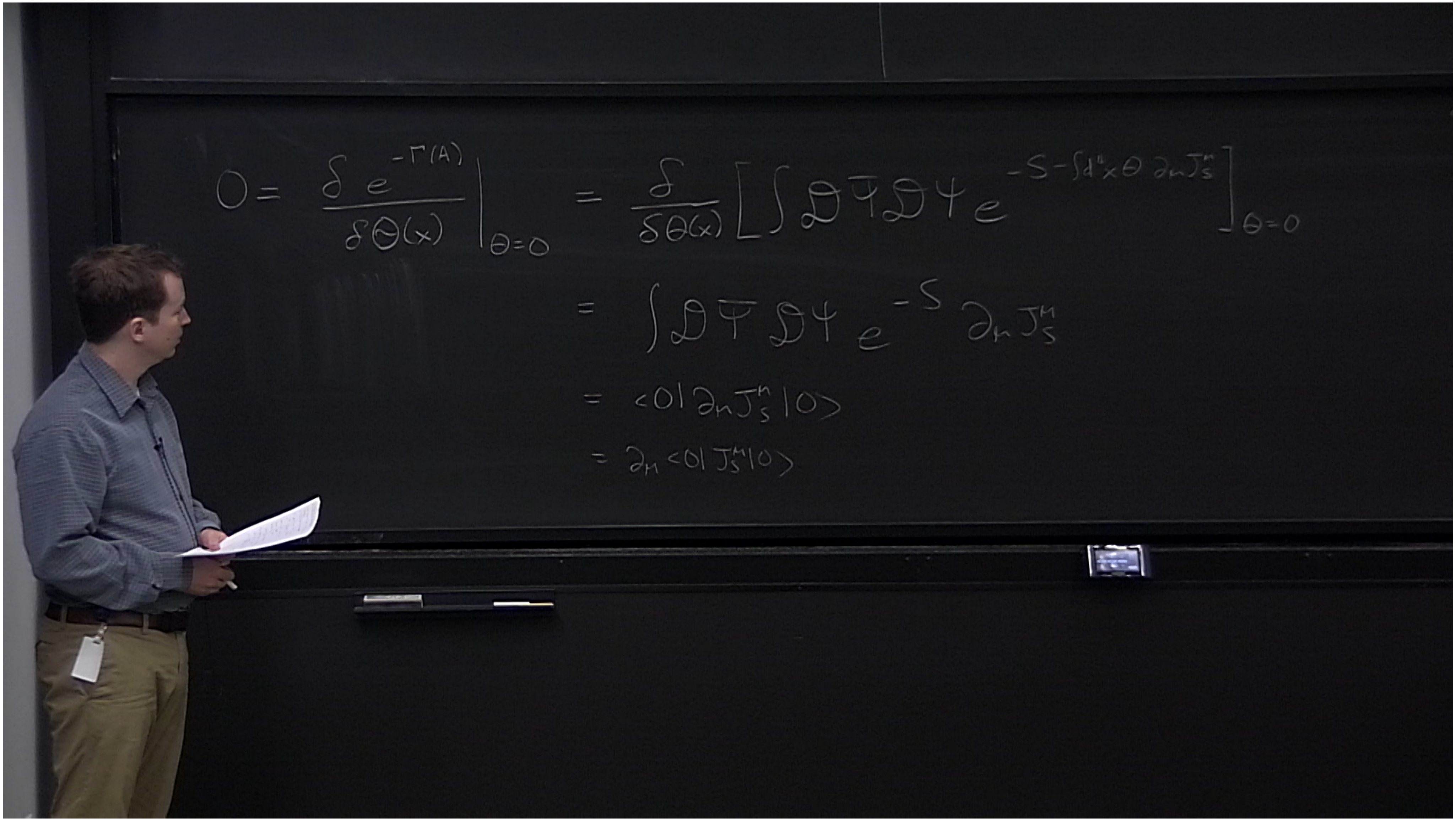
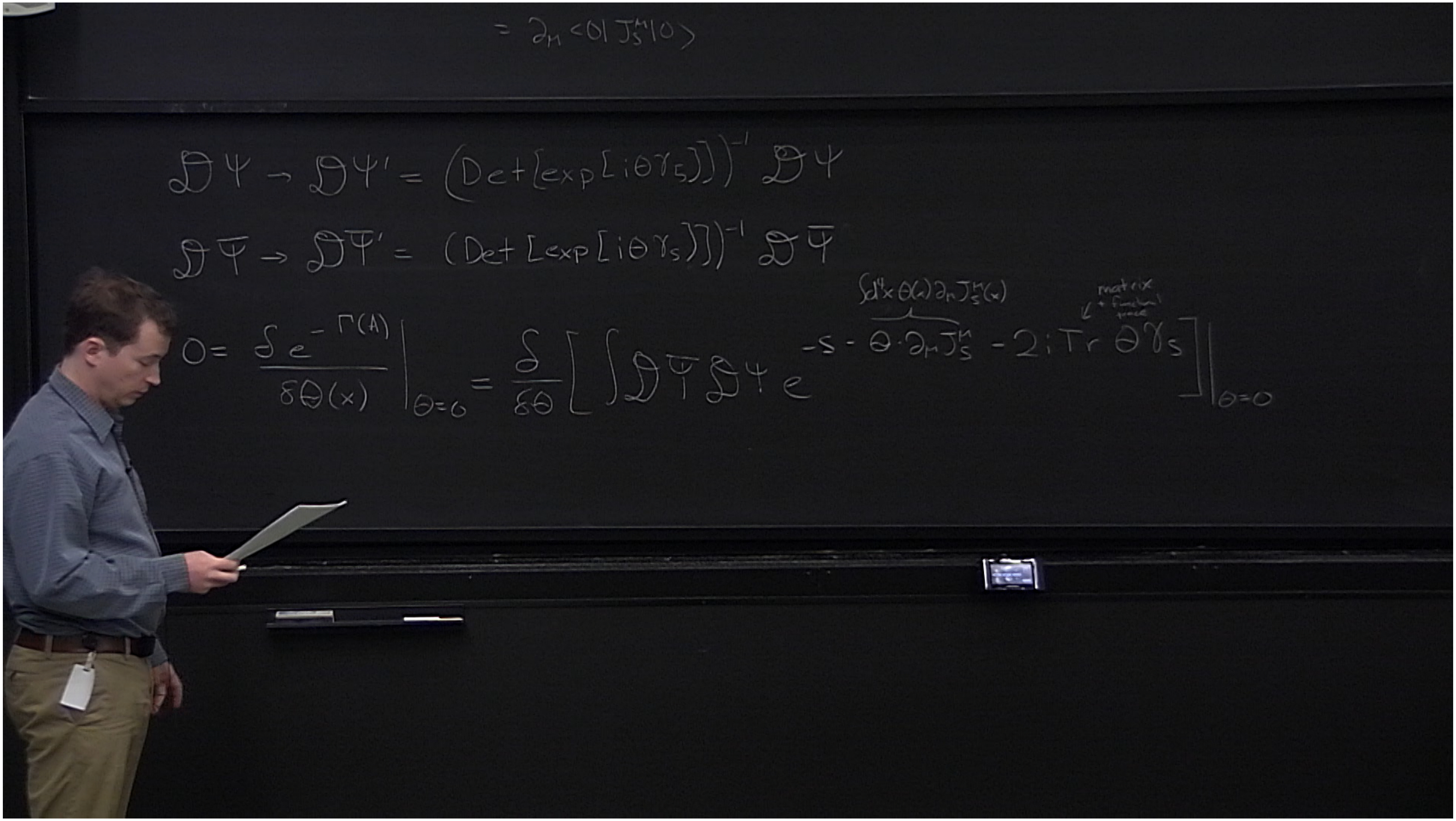
Abstract:


Path Integral Derivation of Anomaly
4D Euclidean space
$S = S_\psi + S_{\text{Maxwell}}$
$S_\psi = \int d^4x \, \bar{\psi} \, i \not{D} \psi$
$e^{-\Gamma(A)} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \, e^{-S[\psi, \bar{\psi}, A]}$



$0 = \frac{\delta e^{-\Gamma(A)}}{\delta\theta(x)}\Big|_{\theta=0} = \frac{\delta}{\delta\theta(x)}\left[\int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-S-\int d^4x\,\theta\,\partial_\mu J^\mu_s}\right]_{\theta=0}$
$= \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-S}\,\partial_\mu J^\mu_s$
$= \langle 0|\partial_\mu J^\mu_s|0\rangle$
$= \partial_\mu \langle 0|J^\mu_s|0\rangle$


$= \partial_\mu \langle 0 | J_5^\mu | 0 \rangle$
$\mathcal{D}\Psi \to \mathcal{D}\Psi' = \left(\mathrm{Det}[\exp[i\theta\gamma_5]]\right)^{-1} \mathcal{D}\Psi$
$\mathcal{D}\bar{\Psi} \to \mathcal{D}\bar{\Psi}' = \left(\mathrm{Det}[\exp[i\theta\gamma_5]]\right)^{-1} \mathcal{D}\bar{\Psi}$
$\int d^4x\, \theta(x)\, \partial_\mu J_5^\mu(x)$
matrix + functional trace
$0 = \left.\frac{\delta e^{-\Gamma(A)}}{\delta\theta(x)}\right|_{\theta=0} = \frac{\delta}{\delta\theta}\left[\int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-S - \theta\cdot\partial_\mu J_5^\mu - 2i\,\mathrm{Tr}\,\theta\gamma_5}\right]_{\theta=0}$

Go to eigenbasis of $\not{D}$

$$\not{D}\phi_n = \lambda_n \phi_n \qquad \lambda_n \in \mathbb{R} \qquad D_\mu = \partial_\mu + A_\mu$$

$$\gamma^{\mu\dagger} = -\gamma^\mu$$

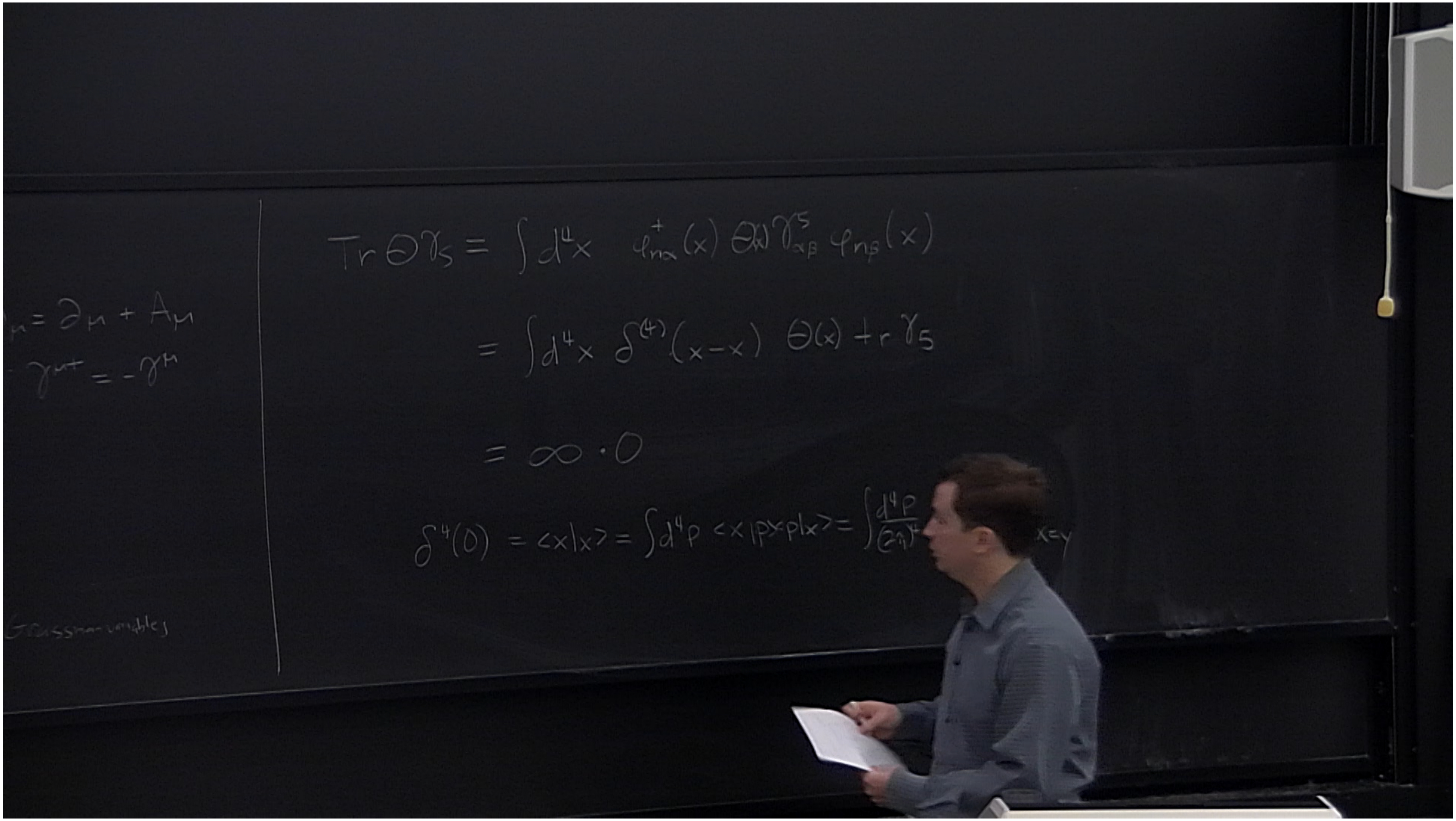
$$\sum_n \varphi_n^\dagger(x)\,\varphi_n(y) = \delta(x-y)\,\underset{\text{spinor}}{\mathbb{1}}$$

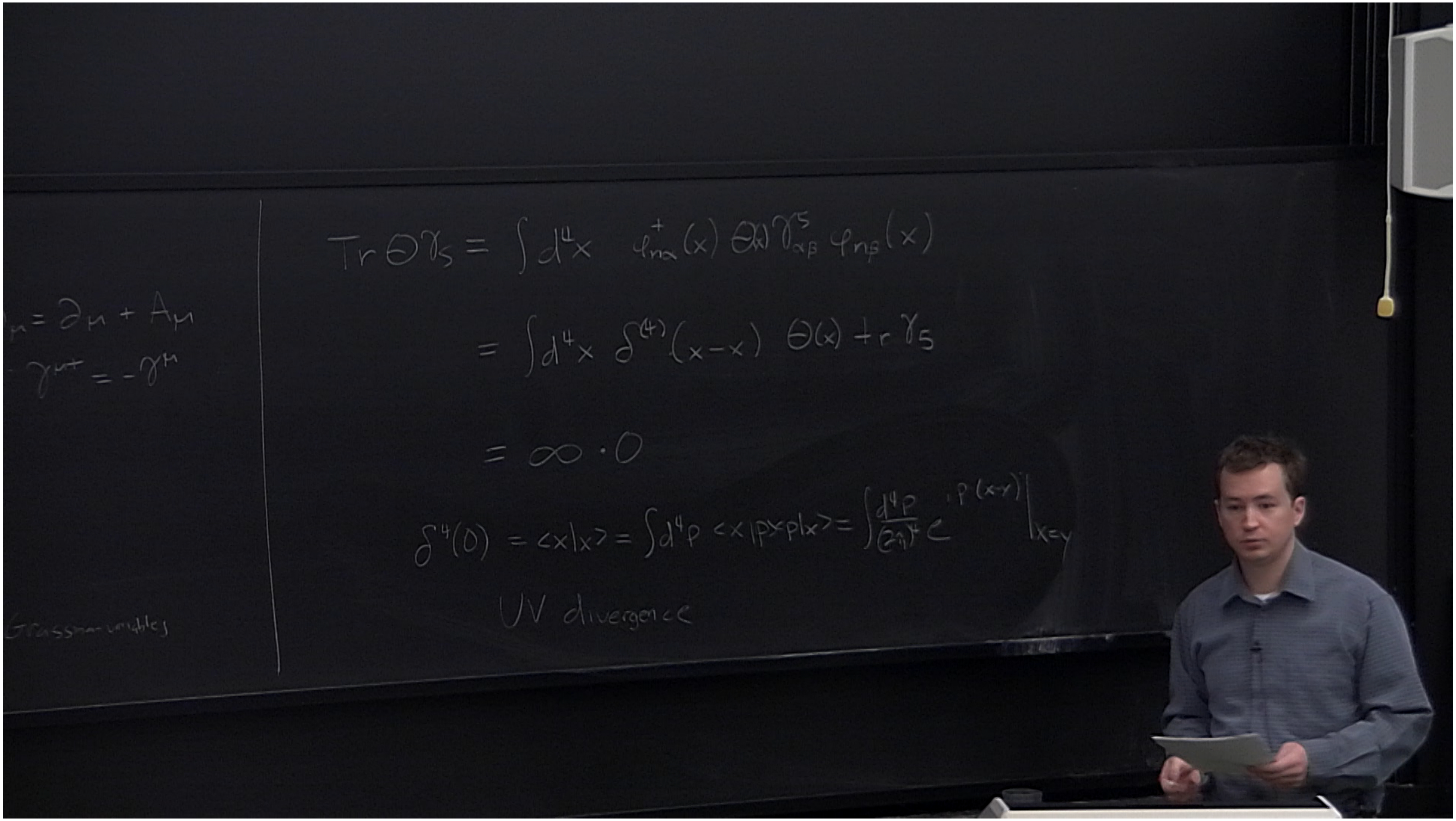
$$\int d^4x\, \varphi_n^\dagger(x)\,\varphi_m(x) = \delta_{nm}$$

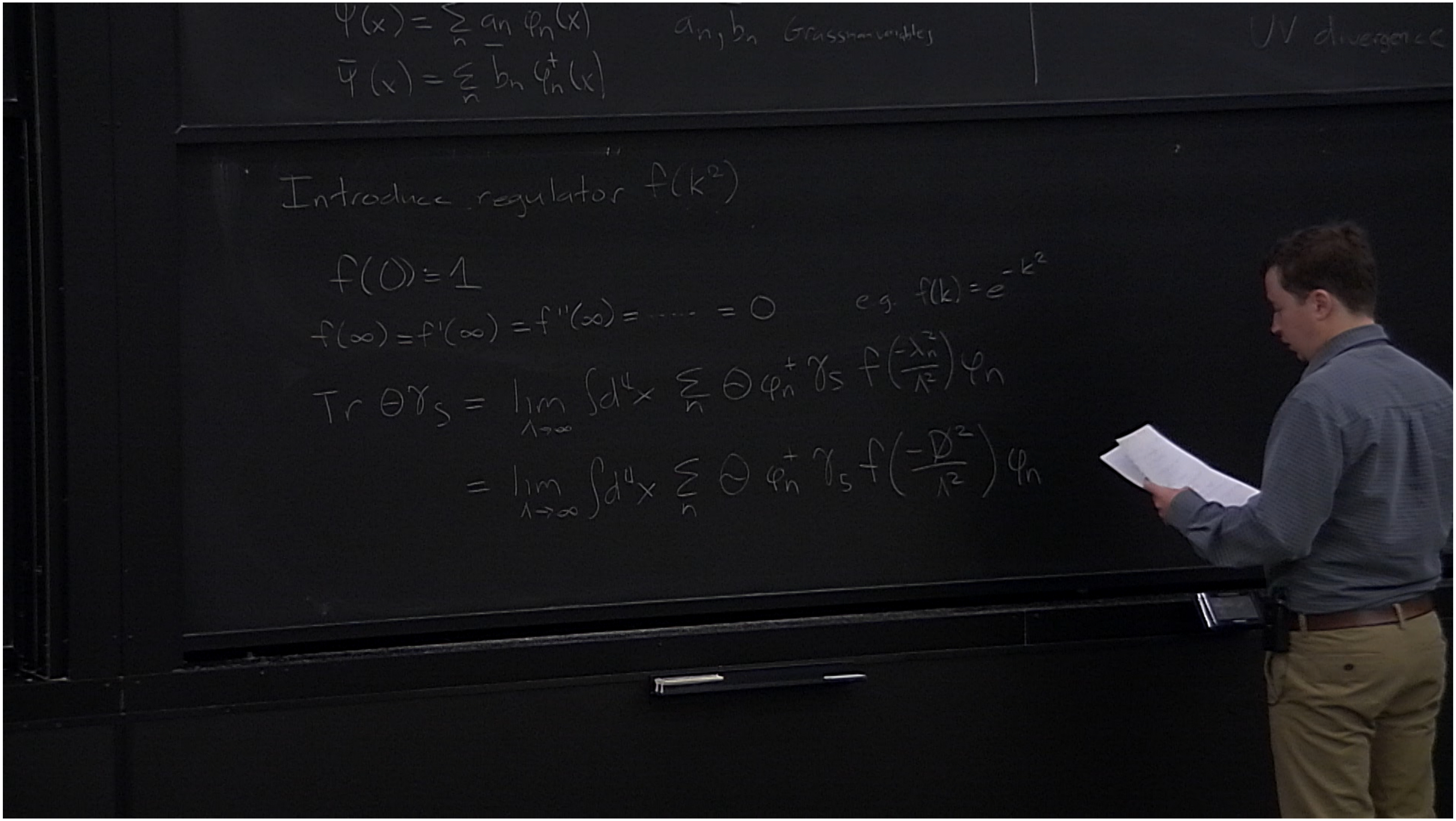
$$\psi(x) = \sum_n a_n \varphi_n(x) \qquad a_n, \bar{b}_n \text{ Grassmann variables}$$

$$\bar{\psi}(x) = \sum_n \bar{b}_n \varphi_n^\dagger(x)$$

$$\mathrm{Tr}\,\Theta\gamma_5 =$$


$\mathrm{Tr}\,\Theta\gamma_5 = \int d^4x \; \varphi^{+}_{n\alpha}(x)\, \Theta(x)\, \gamma^5_{\alpha\beta}\, \varphi_{n\beta}(x)$
$= \int d^4x \; \delta^{(4)}(x-x)\; \Theta(x)\, \mathrm{tr}\,\gamma_5$
$= \infty \cdot 0$
$\delta^4(0) = \langle x|x\rangle = \int d^4p \,\langle x|p\rangle\langle p|x\rangle = \int \frac{d^4p}{(2\pi)^4}$
$x=y$
$D_\mu = \partial_\mu + A_\mu$
$\gamma^{\mu +} = -\gamma^\mu$
Grassmann variables


$D_\mu = \partial_\mu + A_\mu$
$\gamma^{\mu +} = -\gamma^\mu$
Grassman variables
$\mathrm{Tr}\,\Theta\gamma_5 = \int d^4x \; \varphi^+_{n\alpha}(x)\,\Theta(x)\,\gamma^5_{\alpha\beta}\,\varphi_{n\beta}(x)$
$= \int d^4x \; \delta^{(4)}(x-x) \; \Theta(x)\,\mathrm{tr}\,\gamma_5$
$= \infty \cdot 0$
$\delta^4(0) = \langle x|x\rangle = \int d^4p \, \langle x|p\rangle\langle p|x\rangle = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-y)}\Big|_{x=y}$
UV divergence


$\psi(x) = \sum_n a_n \varphi_n(x)$
$\bar{\psi}(x) = \sum_n b_n \varphi_n^+(x)$
a_n, b_n Grassmann variables
UV divergence
Introduce regulator $f(k^2)$
$f(0) = 1$
$f(\infty) = f'(\infty) = f''(\infty) = \dots = 0$
e.g. $f(k) = e^{-k^2}$
$\mathrm{Tr}\, \Theta \gamma_5 = \lim_{\Lambda \to \infty} \int d^4x \sum_n \Theta \varphi_n^+ \gamma_5 f\left(\frac{-\lambda_n^2}{\Lambda^2}\right) \varphi_n$
$= \lim_{\Lambda \to \infty} \int d^4x \sum_n \Theta \varphi_n^+ \gamma_5 f\left(\frac{-\not{D}^2}{\Lambda^2}\right) \varphi_n$

UV divergence

e^{-k^2}

n

φ_n

$$\mathrm{Tr}\,\Theta\gamma_5 = \lim_{\Lambda\to\infty}\int d^4x \int \frac{d^4k}{(2\pi)^4}\int \frac{d^4k'}{(2\pi)^4} \sum_n \Theta\, e^{-ik'\cdot x}\,\tilde{\varphi}_n^{\dagger}(k')$$

$$\cdot\,\gamma_5 f\left(\frac{-\not{D}^2}{\Lambda^2}\right)\left[\tilde{\varphi}_n(k)\,e^{ik\cdot x}\right]$$

$$\not{D}\left[g(x)e^{ikx}\right] = e^{ikx}\left(i\not{k} + \not{D}\right)g(x)$$

$$\mathrm{Tr}\,\Theta\gamma_5 = \lim_{\Lambda\to\infty}\int d^4x \int \frac{d^4k}{(2\pi)^4}\int \frac{d^4k'}{(2\pi)^4} \sum_n \Theta\,\tilde{\varphi}_n^{\dagger}\, e^{ikx}\, e^{-ik'\cdot x}\,\gamma_5 f\left(\frac{-(\not{D}+i\not{k})^2}{\Lambda^2}\right)\varphi_n$$

$$= \lim_{\Lambda\to\infty} \int d^4x \sum_n \Theta\, \varphi_n^{\dagger}\, \gamma_5\, f\!\left(\frac{-\not{D}^2}{\Lambda^2}\right) \varphi_n$$

$$\mathrm{Tr}\,\Theta\gamma_5 = \lim_{\Lambda\to\infty} \int d^4$$

use $\sum_n \varphi_n^{\dagger}(k')\,\tilde{\varphi}_n(k) = (2\pi)^4\,\delta(k-k')\,\mathbb{1}$

$$\mathrm{Tr}\,\Theta\gamma_5 = \lim_{\Lambda\to\infty} \int d^4x \int \frac{d^4k}{(2\pi)^4}\, \mathrm{tr}\,\Theta\gamma_5\, f\!\left(\frac{-(\not{D}+i\not{k})^2}{\Lambda^2}\right)$$

Taylor expand

$$= \lim_{\Lambda\to\infty} \int d^4x \int \frac{d^4k}{(2\pi)^4}\, \mathrm{tr}\,\Theta\gamma_5 \sum_{N=0}^{\infty} \frac{1}{N!} \left(\frac{-\not{D}}{\Lambda}\right)^{2N} f^{(N)}\!\left(\frac{k^2}{\Lambda^2}\right)$$

$\mathrm{tr}\,\gamma_5 = \mathrm{tr}\,\gamma_5 \gamma^\mu \gamma^\nu = 0$ so $N=2$ $\not{D}^4$ is the first term

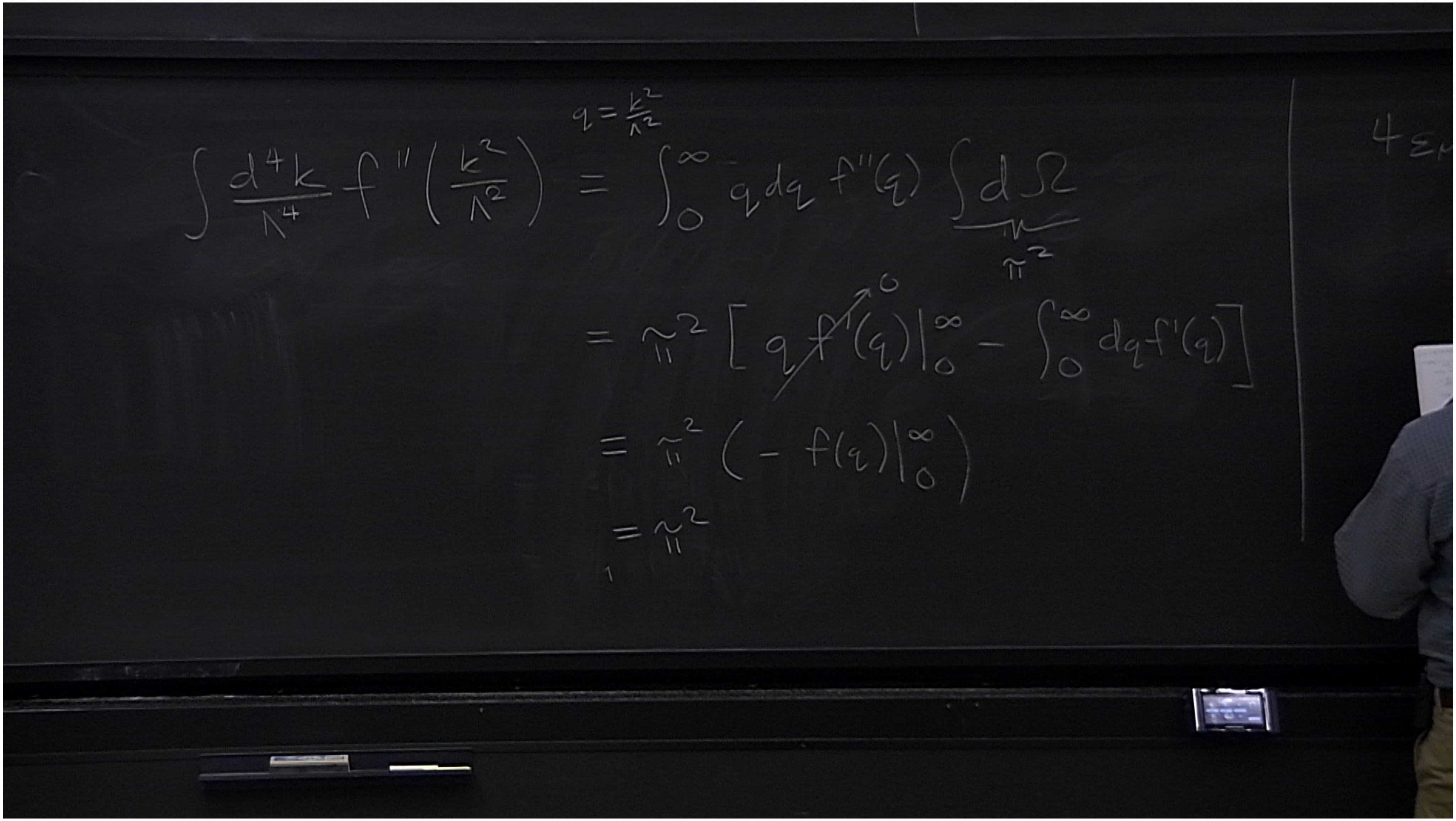
$$\mathrm{Tr}\,\Theta\gamma_5 = \lim_{\Lambda\to\infty} \int d^4x \int \frac{d^4k}{(2\pi)^4}\, \Theta\, \mathrm{tr}\,\gamma_5\, \frac{1}{2} \frac{\not{D}^4}{\Lambda^4} f''\left(\frac{k^2}{\Lambda^2}\right) + \mathcal{O}\left(\frac{1}{\Lambda^6}\right)$$

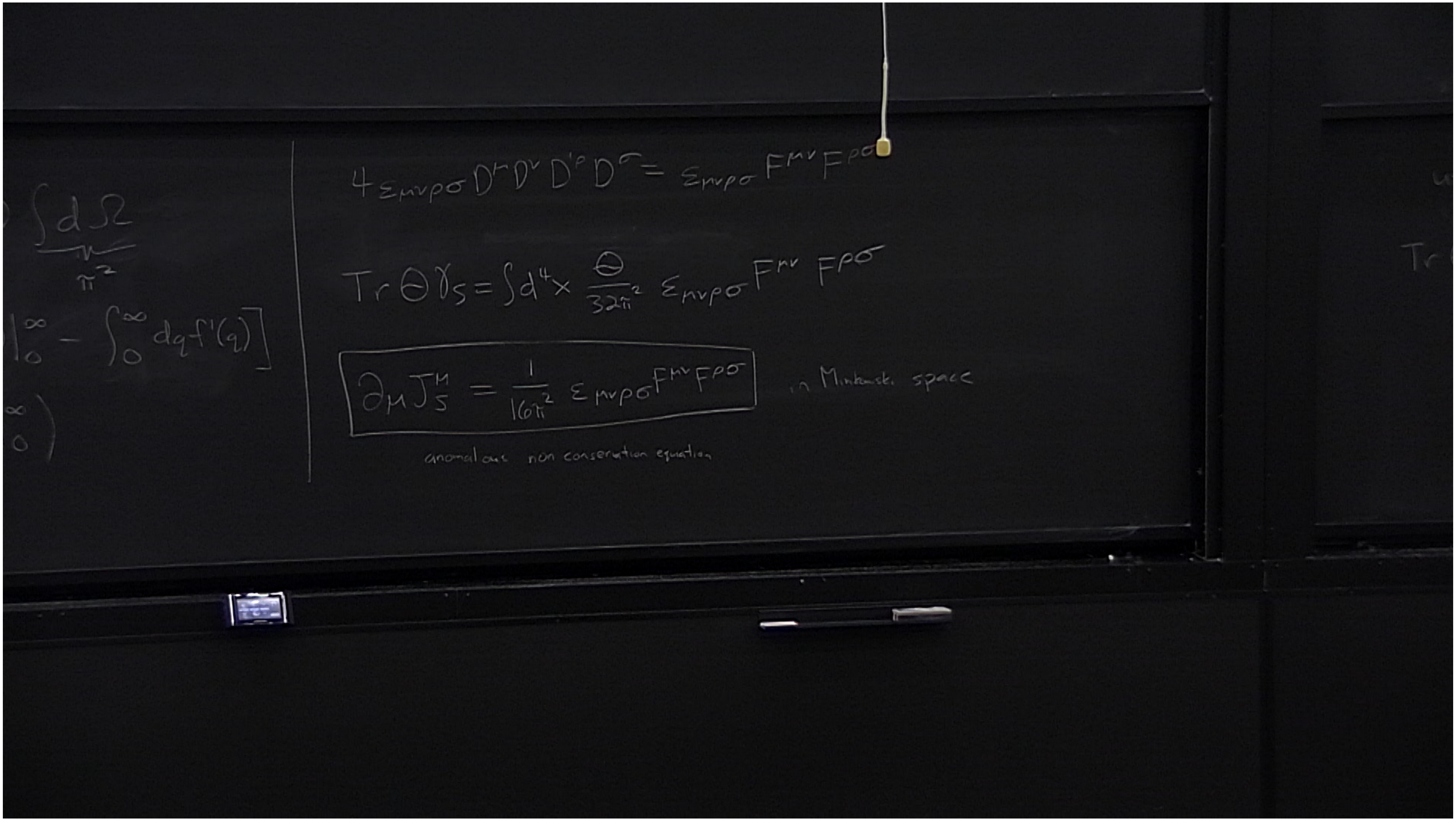
$$\mathrm{tr}\,\gamma_5 \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma = 4\,\varepsilon_{\mu\nu\rho\sigma}$$ no i in Euclidean

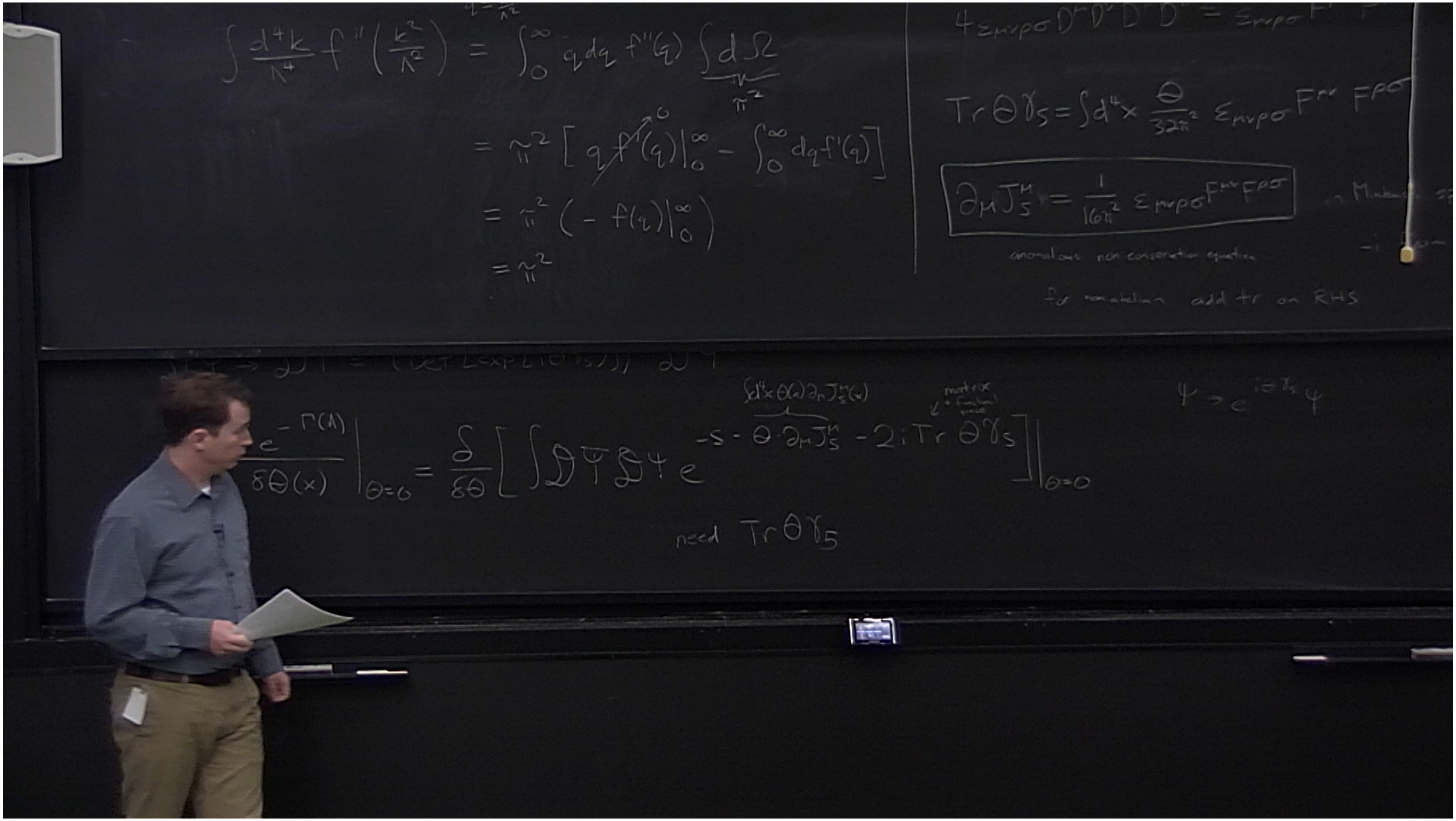
$$= \frac{1}{32\pi^4} \int d^4x\, \Theta \int \frac{d^4k}{\Lambda^4} \left(4\,\varepsilon_{\mu\nu\rho\sigma} D^\mu D^\nu D^\rho D^\sigma\right) f''\left(\frac{k^2}{\Lambda^2}\right)$$

$$0 = \frac{\delta e^{-\Gamma(A)}}{\delta\Theta(x)}\bigg|_{\Theta=0} \neq \frac{\delta}{\delta\Theta(x)}\left[\int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-S-\int d^4x\, \Theta\, \partial_\mu J_5^\mu}\right]_{\Theta=0}$$ measure no

$$= \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\, e^{-S}\, \partial_\mu J_5^\mu$$


$q = \frac{k^2}{\Lambda^2}$
$\int \frac{d^4k}{\Lambda^4} f''\left(\frac{k^2}{\Lambda^2}\right) = \int_0^\infty q\,dq\, f''(q) \underbrace{\int d\Omega}_{\pi^2}$
$= \pi^2 \left[\cancel{q f'(q)\Big|_0^\infty}^{\,0} - \int_0^\infty dq\, f'(q) \right]$
$= \pi^2 \left(- f(q)\Big|_0^\infty \right)$
$= \pi^2$


$4\varepsilon_{\mu\nu\rho\sigma} D^\mu D^\nu D^\rho D^\sigma = \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$
$\mathrm{Tr}\,\Theta\gamma_5 = \int d^4x \frac{\Theta}{32\pi^2} \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$
$\boxed{\partial_\mu J^\mu_5 = \frac{1}{16\pi^2} \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}}$
in Minkowski space
anomalous non conservation equation

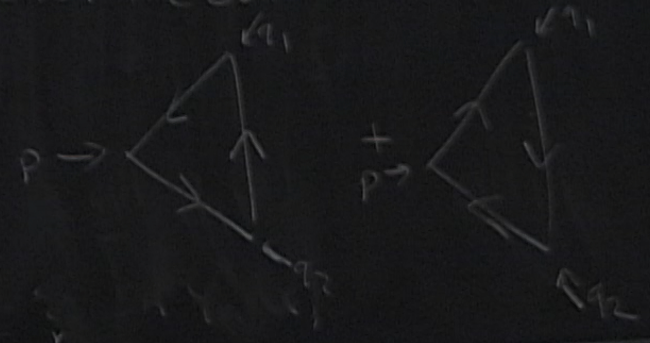

∫ d⁴k/Λ⁴ f''(k²/Λ²) = ∫₀^∞ q dq f''(q) ∫dΩ/π²
= π² [q f'(q)|₀^∞ − ∫₀^∞ dq f'(q)]
= π² (− f(q)|₀^∞)
= π²
Tr Θγ5 = ∫d⁴x Θ/32π² εμνρσ F^μν F^ρσ
∂μ J5^μ = 1/16π² εμνρσ F^μν F^ρσ
anomalous non conservation equation
for non abelian add tr on RHS
ψ → e^(iθγ5) ψ
need Tr Θγ5

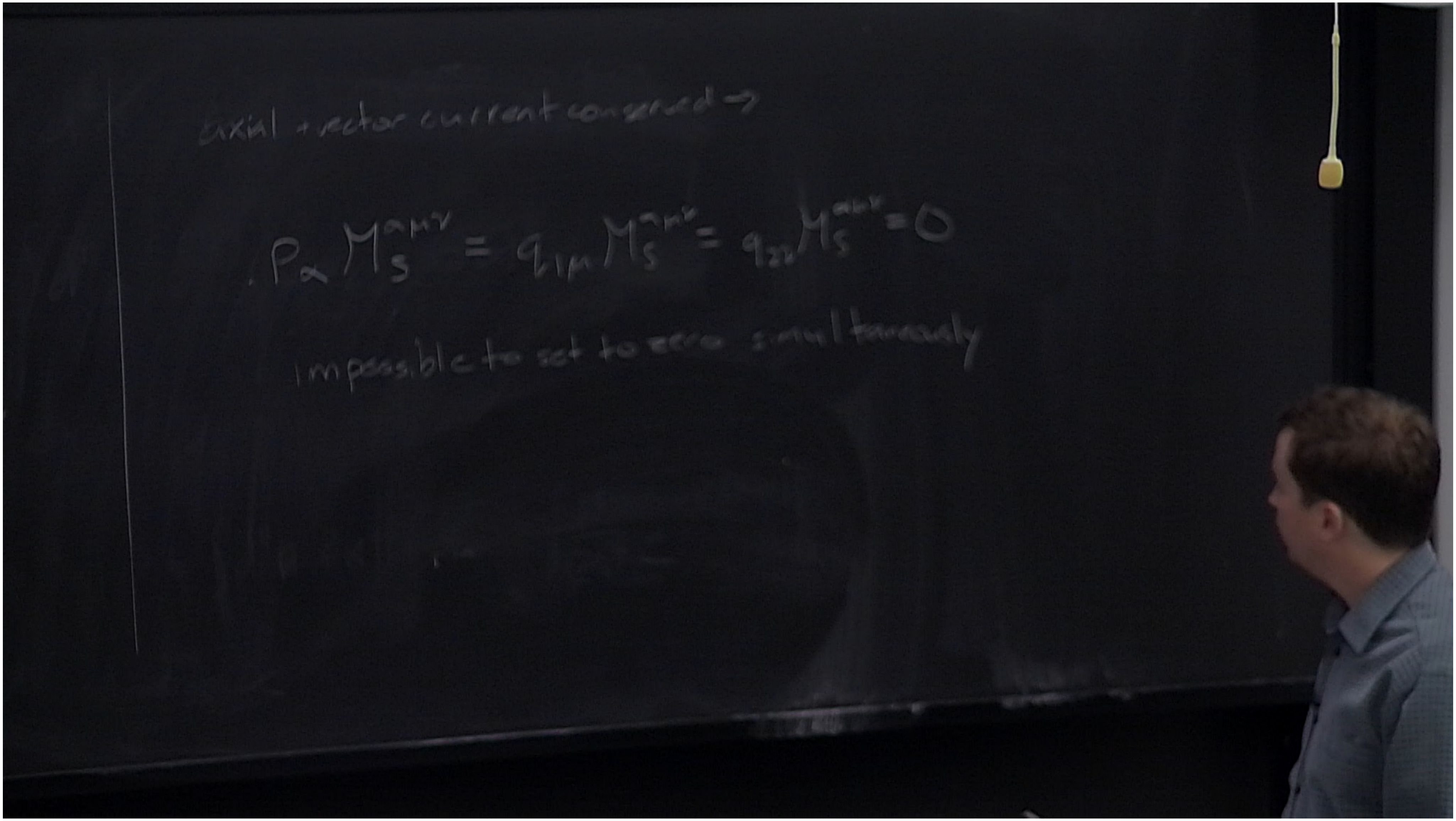
Anomaly from Triangle Diagrams

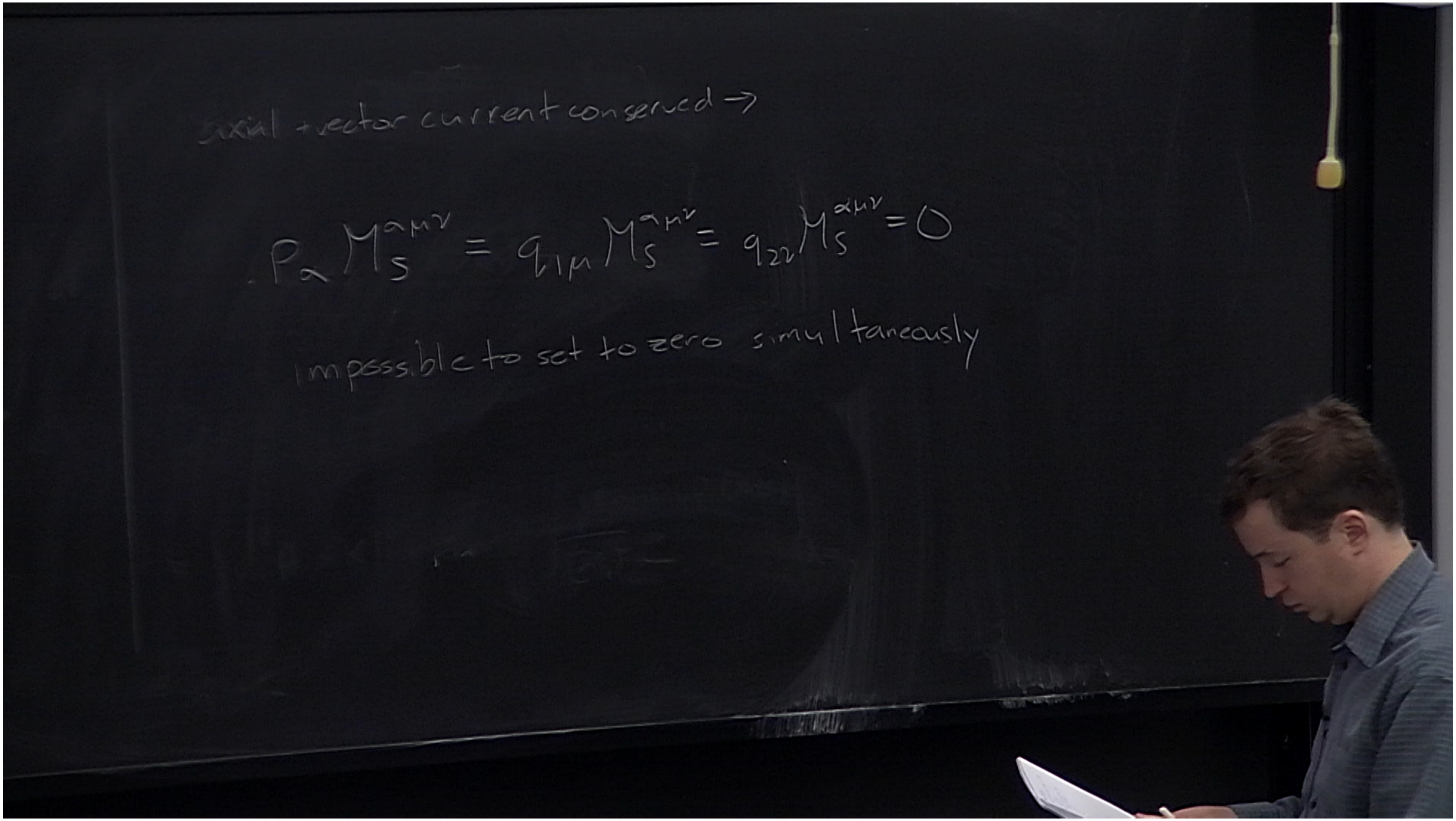
in 4D

$$\langle J_5^{\alpha}(p)\, J^{\mu}(q_1)\, J^{\nu}(q_2) \rangle \equiv i \mathcal{M}_5^{\alpha\mu\nu}$$

each of the currents involves 2 fermion fields




axial + vector current conserved →
$P_\alpha M_5^{\alpha\mu\nu} = q_{1\mu} M_5^{\alpha\mu\nu} = q_{2\nu} M_5^{\alpha\mu\nu} = 0$
impossible to set to zero simultaneously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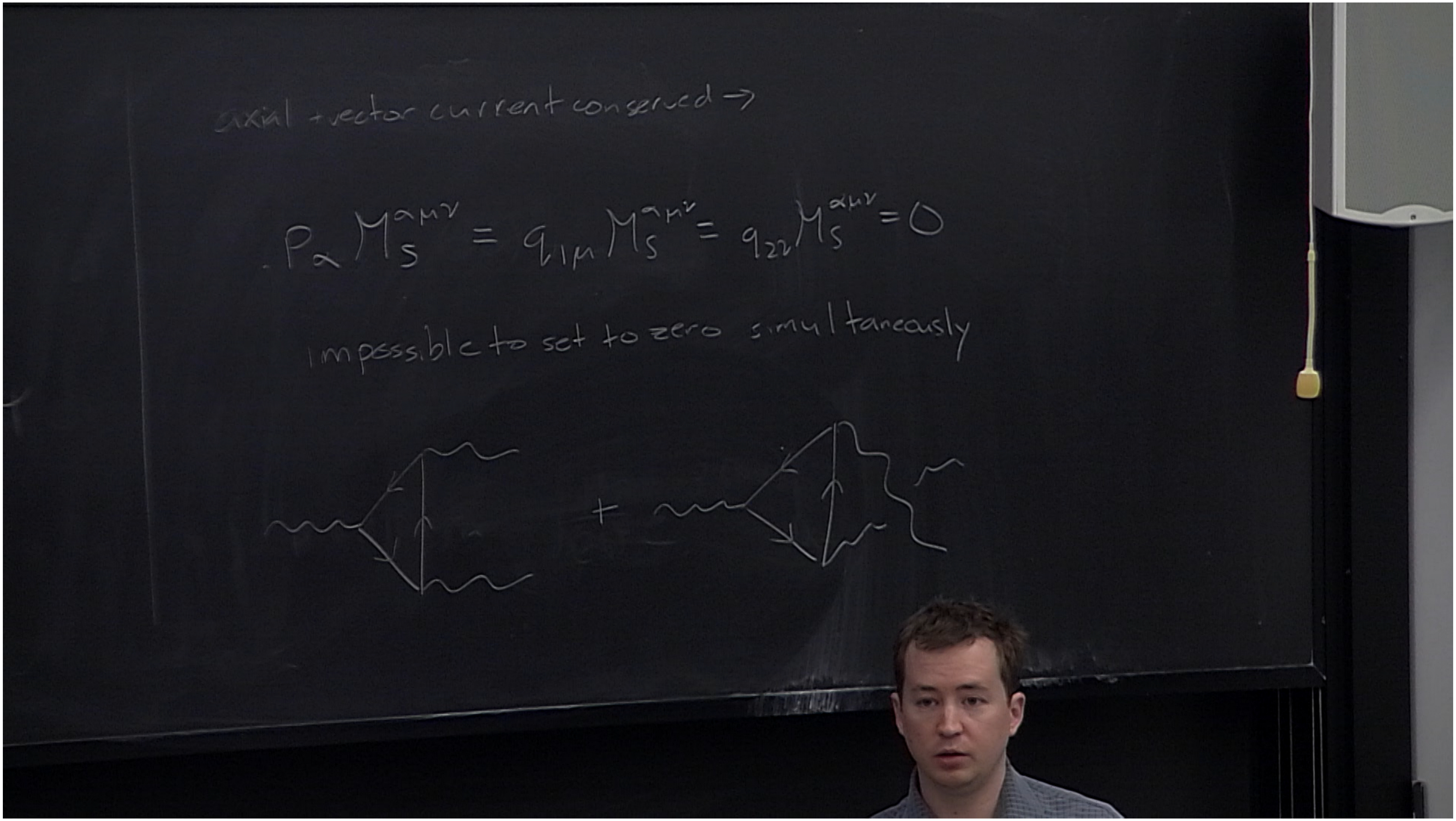
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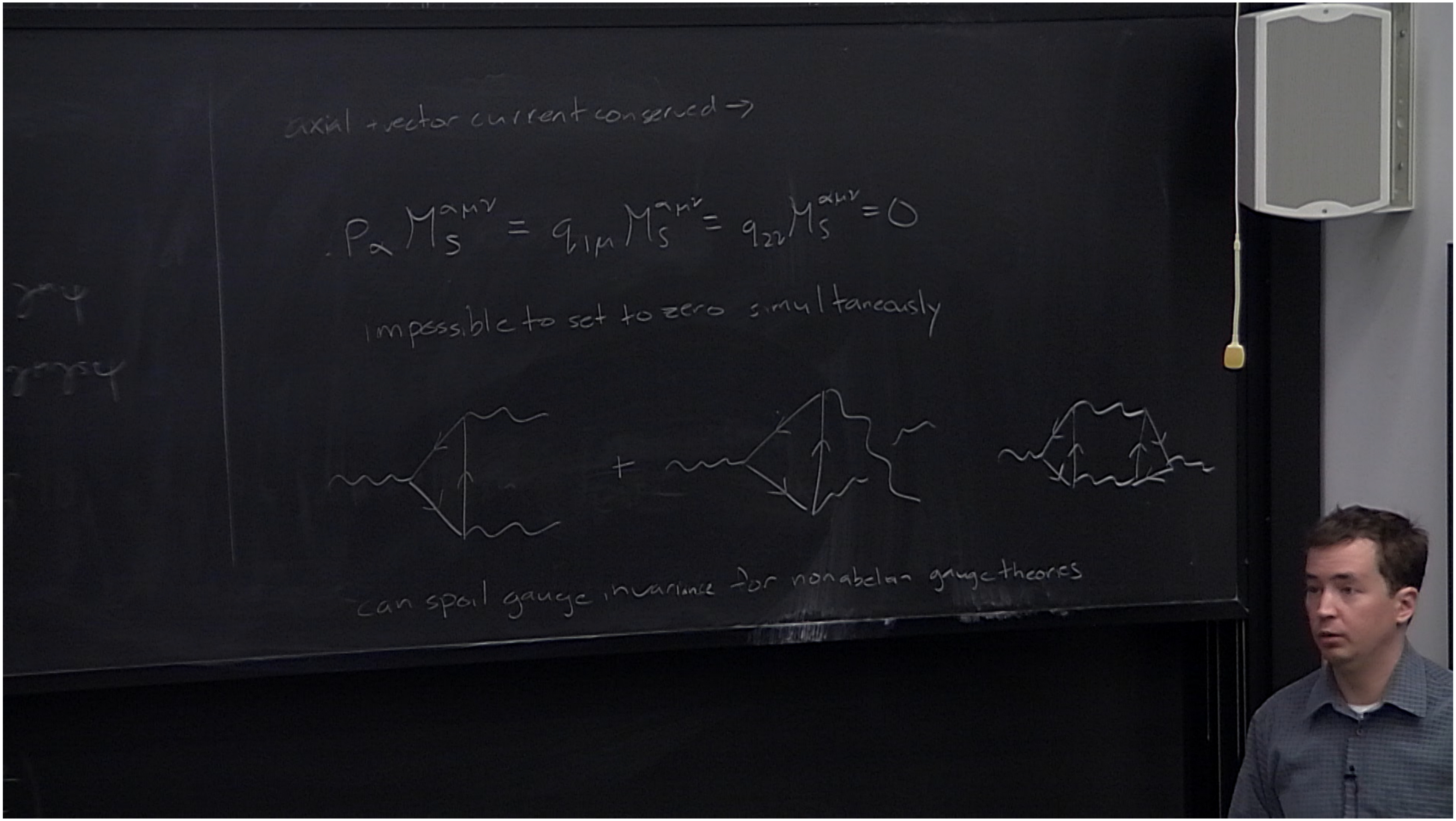
impossible to set to zero simultaneously
+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impsssible to set to zero simultaneously
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can spoil gauge invariance for nonabelian gauge theories